

**"THE DEMAND FOR RETAIL PRODUCTS AND  
THE HOUSEHOLD PRODUCTION MODEL:  
NEW VIEWS ON COMPLEMENTARITY AND  
SUBSTITUTABILITY**

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**N° 87 / 23**

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Director of Publication :

Charles WYPLOSZ, Associate Dean  
for Research and Development

Printed by INSEAD,  
Fontainebleau, France

The Demand for Retail Products and the Household Production Model:  
New Views on Complementarity and Substitutability\*

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\*The ideas in this paper at various stages of elaboration were presented in seminars at Georgetown University and the University of Maryland. The paper was also presented at the INSEAD Telecommunications Conference. We thank the participants in these seminars, the conference discussant, M. Perry, and Ira Horowitz for their comments without incriminating them for the results. Both authors also gratefully acknowledge INSEAD's financial support for this research project.

## ABSTRACT

This paper integrates the economic literature on the household production model with the marketing literature on retailing to derive the demand for retail products in a rigorous framework. Retail firms are viewed as providing customers with a variety of distribution services, where higher levels of these services entail higher costs to the firms that provide them and lower costs for the patrons of these firms. We develop the household production model in terms of a two-stage formulation in which the distribution services provided by retailers are treated as fixed inputs into the household's production activities. The two-stage formulation permits a useful decomposition of a price change that is reminiscent of the Slutsky decomposition of standard consumer theory. Adopting the decomposition for cross item effects and for item-service effects allows us to draw implications from our model for leader pricing, retail composition, and agglomeration of disparate retail activities.

In this paper we bring together the economic literature on the household production model and the marketing literature on retailing to derive the demand for retail products in a rigorous framework. As with any cross-fertilization effort, mutual benefits arise. Hence, some characteristics of the household production model, either unknown or ignored in the prior literature, are brought to the fore; and a critical feature stressed in the retailing literature in an informal manner is formally incorporated into the analysis of demand.

The household production model was introduced in the 1960's by Muth (1966), Becker (1965) and Lancaster (1966). Becker's emphasis on the role of time in consumption activities spurred a large number of applications, especially to nonstandard topics such as the economics of the family (Willis, 1987). Muth and Lancaster were concerned with applications to standard economic topics, for example the use of specific market goods or retail items to produce commodities such as food or nutrition, but the use of restrictive assumptions on the specifications of the household's technology has limited their applicability. The analysis in this paper follows the specifications of the model in the Becker tradition but applies this specification to the analysis of topics in the Muth and Lancaster tradition. Some of the technical properties of the Becker formulation of the household production model were clarified in a contribution by Pollak and Wachter (1975) and in a subsequent exchange (1977) with Barnett (1977). More recently, Deaton and Muellbauer (1980) set out the model in terms of a two-stage procedure which takes advantage of developments in duality theory. This formulation provides the point of departure for our own analysis.

Contributions to the marketing literature have often made the point that the role of distribution is more than just providing market goods. For

instance, Bucklin (1966) views the 'product' of distribution "as a mix of market goods in conjunction with an array of services..." More specifically, in the context of retailing, Ingene (1984) notes the possibility of a shifting of distribution costs between consumers and retailers. Nevertheless, until very recently, these ideas were put forth in an informal manner which limited their applicability and further development. Several recent papers have moved the topic closer to formal analysis. For instance, Bliss (1985) postulates a demand side for a retail market in which an indirect utility function depends on the (same) wholesale prices plus a mark-up, which varies according to the store's location, and an income net of transport cost. On the other hand, Lal and Matutes (1986) postulate a demand side for a retail market in which there is a uniform distribution of consumers with respect to various combinations of distribution services offered by stores. Since both papers focused on explaining particular features of a retail market, neither one develops the demand side of the analysis in any detail.

Our emphasis in this paper stems from earlier work, Betancourt and Gautschi (1987), on the nature of retail markets. Retail firms are viewed as providing consumers with a variety of distribution services, as in Lal and Matutes, which were identified in our work, for example, as accessibility of location, product assortment, ambiance, assurance of product delivery at the desired time and in the desired form, and information. Higher levels of these services entail higher costs to the firms that provide them whereas, at the same time, they entail lower distribution costs for the consumers that patronize these firms, as in Bliss. In order to formalize the role of these distribution costs on the demand side, we postulated a household production model in which the distribution services provided by retailers act as fixed inputs into the household production activities. Since our earlier work also

focused on explaining particular features of a retail market, it limited the analysis to the demand for a single market good and it did not explore the formal implications of this formulation for demand analysis in the general case. This last issue is the focus of the present paper.

In the next section of the paper (I), we present the household production model in terms of the two-stage formulation mentioned above and we include the distribution services of retailers as fixed inputs into the household's production activities. In Section II, we analyze the own (uncompensated) price elasticity of demand for retail products. The two-stage formulation of the model leads to a natural decomposition of a price change into a production effect and a consumption effect, which can be analyzed separately and are reminiscent of the Slutsky decomposition of standard consumer theory. Not surprisingly, the production effect of a change in the price of a retail item is always negative. On the other hand, there are now two sets of sufficient conditions that ensure a negative consumption effect. In the standard analysis of demand, the assumption of no inferiority ensures a negative income effect of a price change. Similarly, the assumption of no inferiority of commodities assures a negative consumption effect of a price change for retail items used exclusively in the production of a single commodity. In contrast to the standard analysis, however, a condition parallel to the Cournot aggregation condition also ensures a negative consumption effect for retail items used in the production of every commodity. Moreover, such retail items are also shown to have elastic demands.

Section III contains an analysis of the (uncompensated) cross-price elasticity of demand for retail items. The thrust of this section is to explain why the majority of items in the assortment of a retailer will be gross complements from the point of view of the household production model.

Once again, the two-stage formulation is extremely useful in presenting this result. First, the conventional arguments about particular items being substitutes or complements, for example coffee and sugar, are shown to be well defined in terms of the production effect and usually for items that are used in the production of the same commodity. Hence, these arguments can merely establish net substitutability or net complementarity. Gross substitutability or complementarity must also take account of the consumption effect and the same two forces driving this effect to be negative in the analysis of the own price elasticity of demand will be at work in this case. For instance, if two items are used by the household in the production of the same commodity no inferiority of the demand for commodities ensures a negative consumption effect. Hence, if the items are (net) independents or complements in production they will be gross complements. Even if the items are (net) substitutes in production they will be gross complements if the consumption effect outweighs the production effect. Similarly, if two items are used in the production of two different commodities they will in general be net independents. Hence, they will be gross complements if the consumption effect is negative. If the two commodities are net independent or complements in consumption the items will be gross complements. Even if the two commodities are net substitutes in consumption, the retail items will be gross complements if the substitution effect is outweighed by the income effect.

One reason for emphasizing this tendency toward gross complementarity among the items purchased by the household from a retailer is the importance of its economic implication. Namely, this tendency is one of the principal factors on the demand side generating the economic incentives for retail firms to offer many different items for sale. Moreover, the provision of a variety of items for sale is an intrinsic characteristic of most modern retail institutions.

Another implication of the tendency toward gross complementarity is market basket pricing. The possibility of pricing items in an assortment in such a way as to generate store traffic is very useful in practical settings, provided one can identify the characteristics of items that are likely to have high (absolute) values of the uncompensated cross-price elasticity of demand. Section IV contains an analysis of the characteristics of items with this property in three different cases, according to the usage of the item in household production. Despite the level of generality of our argument, we are able to identify one of the main characteristics of items that will be useful as traffic builders: namely, items whose costs are a large share of the consumer's budget will be good traffic builders. An illustration of these ideas in terms of an example shows that our procedure for identifying what the retailing literature calls "one-way cross-price elasticities of demand" (Albion, 1983) is more precise and useful than appeals to 'product salience.'

In Section V we analyze the distribution services elasticity of demand. This concept represents a unique feature of our model. By including the distribution services of a retailer as fixed inputs into the household's production activities, we formalize the informal notions available in the literature in a manner that generates results perfectly comparable to those obtained with respect to price changes. Our analysis generalizes to any other determinant of consumption behavior that can be viewed as a fixed input into the household's production activities. Incidentally, Barten (1977, pp. 36-37) notes a related feature in the standard approach to the analysis of consumer behavior and Dreze and Hagen (1978, Appendix) provide a similar formalization with respect to the input-output coefficient in Lancaster's linear technology model.

Just as in previous cases, the two-stage formulation of the model is useful in separating the effect of a change in a distribution service into a production effect and a consumption effect. This segmentation allows us to define distribution services as net substitutes, independent or complements with items in an assortment according to the sign of the production effect. While in principle it is possible for distribution services to be net substitutes with items in the assortment employed by the household, in practice such cases will be indeed rare. It would mean, for example, that greater accessibility of location by a retailer would lead the household to demand less of an item in the assortment of this retailer given that the prices of all items sold by this and other retailers, as well as the household's level of production of commodities and wage rate, remain the same. Hence, the widely prevalent situation is one where distribution services are net independent or complements with items in an assortment. Similarly, we can define distribution services as gross substitutes, independents or complements according to the sign of the services elasticity of demand (negative, zero or positive, respectively). Since the same forces driving the consumption effect of a price change to be negative, namely commodities having a positive income effect and Cournot aggregation, operate to drive the consumption effect of a change in the quantity of a distribution service to be positive, the distribution services of a retailer will be gross complements with almost every item in this retailer's assortment employed by the household.

As a mechanism for keeping the notation simple, the interpretation of the results in the sections previously discussed is developed in terms of a single retailer. While the formal results describing the effect of a change in price or distribution service of one retailer on the quantity demanded of an item in another retailer's assortment are the same as in the case of a single

retailer, the interpretation is somewhat different. Hence, the concluding section (VI) brings out these implications of our approach. On the substantive side, the main results are that gross substitutability, when it exists, is likely to take place between items in the assortments of different retailers or, to a lesser extent, through the relation between items in one retailer's assortment and another retailer's distribution services. Nevertheless, even in the context of different retailers gross complementarity remains a widespread phenomenon; therefore, it provides a powerful incentive on the demand side for the creation of retail agglomerations including, for example, shopping centers and shopping malls.

To conclude, the approach presented here should serve as a useful formal characterization of the demand side of retail markets in future theoretical and empirical work. Furthermore, if one wishes to engage in policy evaluations, the positive analysis of the household production model in goods space provided here can be integrated with the welfare analysis of the model in goods space provided in Bockstael and McConnell (1983, Section III).

## I. The Model

For our purposes it is useful to develop the household production model in terms of the two-stage procedure mentioned in the introduction (Deaton and Muellbauer (1980, pp. 245-54)). The first stage can be described as follows:

$$\text{Min } pQ \quad \text{s.t.} \quad h(Q, D, Z) = 0 \quad (1)$$

$Q$

where  $Q$  is a vector of all the goods and services employed by the household in production, including the goods and services purchased from different retailers as well as the time employed by the household in production activities.  $p$  is a corresponding vector of prices, including the opportunity cost of the household's time.  $D$  is a vector of distribution services provided

by the retailers which the household patronizes in its purchase activities.  $Z$  is the vector of commodities produced by the household, which are the ones that yield satisfaction or utility directly.  $h$  is a transformation function with the usual properties.

The result of this optimization procedure is the cost function below

$$C = C(p, D, Z) \quad (2)$$

This function has the following properties: nondecreasing concave and linear homogeneous in prices, increasing in outputs (the elements of  $Z$ ) and nonincreasing in distribution services (the elements of  $D$ ). The last property follows from assuming that the distribution services provided by a retailer act as fixed inputs into the household production activities. It is in this manner that the shifting of distribution costs between consumers and retailers can be captured formally in the model. It follows from Shephard's Lemma that the conditional or Hicksian demand function for a good purchased from a particular retailer will be given by

$$Q_k = C_k = \partial C / \partial p_k = g_k(p, D, Z) \quad k = 1, \dots, K \quad (3)$$

In the second stage the household maximizes utility, by choosing the optimal levels of the commodities that yield satisfaction, subject to the constraint that the household's full income ( $W$ ) be sufficient to cover the costs of producing these levels of the commodities. This second stage can be described as follows:

$$\text{Max } U(Z) \quad \text{s.t.} \quad W \geq C(p, D, Z) ,$$

where  $U(Z)$  is a quasi-concave utility function. The first-order conditions for an interior solution are given by

$$U_i(Z) = \lambda C_i(p, D, Z) \quad i = 1, \dots, I \quad (4)$$

$$W = C(p, D, Z) , \quad (5)$$

where  $U_i = \partial U / \partial Z_i$ ,  $C_i = \partial C / \partial Z_i$  and  $\lambda$  is the usual Lagrange multiplier. The solution of (4) and (5) yields the demand functions for the commodities, i.e.,

$$Z_i = f_i(p, D, W) \quad i = 1, \dots, I \quad (6)$$

Finally, substitution of (6) into (3) yields the Marshallian or uncompensated demand functions for any item purchased from a retailer. To wit,

$$Q_k = g_k[p, D, f(p, D, W)] \quad k = 1, \dots, K \quad (7)$$

## II. The Own-Price Elasticity of Demand for Retail Items

One advantage of the two-stage formulation of the model is that it leads to a natural decomposition of the effects of a price change which is analogous to the Slutsky decomposition of standard consumer theory. That is, the own price elasticity of demand for a retail item can be obtained from (7) and written as

$$\epsilon_{kk} = \epsilon_{kk}^* + \sum_i \omega_{ki} \eta_{ik} , \quad (8)$$

where  $\epsilon_{kk} = (\partial Q_k / \partial p_k)(p_k / Q_k)$ ,  $\epsilon_{kk}^* = (\partial Q_k / \partial p_k | Z)(p_k / Q_k)$ ,  $\omega_{ki} = (\partial Q_k / \partial Z_i)(Z_i / Q_k)$  and  $\eta_{ik} = (\partial Z_i / \partial p_k)(p_k / Z_i)$ .

The first term on the RHS of (8) is the production effect of a price change. Namely, it represents the percentage change in the quantity demanded of the  $k^{\text{th}}$  item or input given the levels of production of the commodities that yield satisfaction and, from (3), it must always be nonpositive by the concavity of the cost function. The second term in (8) is the consumption

effect of a price change and it captures the percentage change in the demand for the  $k^{\text{th}}$  item as a result of the changes in the demand for commodities induced by the change in the costs of producing these commodities.  $\omega_{ki}$  represents the percentage change in the usage of input  $Q_k$  as a result of a percentage change in the output of  $Z_i$ . Throughout we will assume that there are no regressive inputs in production (Hicks, 1946); consequently,  $\omega_{ki} \geq 0$ .  $\eta_{ik}$  is the elasticity of demand of the  $i^{\text{th}}$  commodity with respect to a change in the price of the  $k^{\text{th}}$  item or input.

Just as in the analysis of the income effect of standard consumer theory, it is useful to derive sufficient conditions for the consumption effect to be negative. In contrast to that case, however, there are two sets of sufficient conditions for the consumption effect to be negative which are of interest. These conditions will be stated in the form of two theorems.

Theorem 1: When an item in a retailer's assortment is used by the household in the production of every commodity, the consumption effect of a change in the price of that item will always be negative.

Proof: Differentiation of the budget constraint in (5) with respect to the price of the  $k^{\text{th}}$  product and manipulation of the results leads to the following condition.

$$\sum_i \theta_i \eta_{ik} = -S_k, \tag{9}$$

where  $\theta_i = C_i Z_i / C$ ,  $S_k = p_k Q_k / C$ <sup>1</sup> and  $\eta_{ik}$  has already been defined. In terms of interpretation  $\theta_i$  is the marginal budget share of the  $i^{\text{th}}$  commodity and  $S_k$  is the budget share of the  $k^{\text{th}}$  input or item in an assortment.

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<sup>1</sup>Incidentally, equation (9) is simply the Cournot aggregation or adding up condition in the household production model.

If the weights in the consumption effect,  $\omega_{ki}$ , are proportional to the marginal budget shares of the  $i^{\text{th}}$  commodity, the consumption effect will be negative by (9). This is indeed the case. These weights will either be zero, if the  $k^{\text{th}}$  product is not used in the production of the  $i^{\text{th}}$  commodity, or proportional to the marginal budget shares. For

$$\begin{aligned}\omega_{ki} &= (\partial Q_k / \partial Z_i) Z_i / Q_k - (\partial Q_k / \partial Z_i) (p_k / C_i) C_i Z_i / p_k Q_k \\ &= [(\partial Q_k / \partial Z_i) (p_k / C_i)] \theta_i / S_k\end{aligned}\quad (10)$$

The term in square brackets will either be zero or unity, since  $(\partial Q_k / \partial Z_i) p_k$  is the marginal cost of input  $k$  in the production of the  $i^{\text{th}}$  commodity and it must, therefore, equal  $C_i$ , or the marginal cost of producing the  $i^{\text{th}}$  commodity, if it is used in the production of the  $i^{\text{th}}$  commodity or zero if it is not used.

Q.E.D.

Corollary: If the conditions of Theorem 1 are met, the demand for the  $k^{\text{th}}$  item in the assortment must be elastic.

Proof: Substitution of (10) in (8) and use of (9) yields

$$\epsilon_{kk} = \epsilon_{kk}^* - 1 \quad (11)$$

Q.E.D.

A second set of sufficient conditions for the consumption effect to be negative provides a result analogous to the standard case. Namely,

Theorem 2: When an item in a retailer's assortment is used exclusively in the production of a single commodity, the consumption effect will be negative if the commodity is a normal commodity in the sense of having a positive income effect.

Proof: If the  $k^{\text{th}}$  product in the assortment is used exclusively in the production of a single commodity (n), the consumption effect can be written as

$$\sum_i \omega_{ki} \eta_{ik} = \omega_{kn} \eta_{nk} , \quad (12)$$

since  $\omega_{ki} = 0$  for  $i \neq n$ . In the Appendix we show that

$$\eta_{nk} = \eta_{nk}^* - S_k \eta_i < 0 , \quad (13)$$

where  $\eta_{nk}^* = (\partial Z_n / \partial p_k | U)(p_k / Z_n)$  and  $\eta_n = (\partial Z_n / \partial W)W / Z_n$ ,  $\eta_{nk}^*$  is the compensated price elasticity of demand for commodity n as a result of a change in the price of input k and it will always be negative for the same reason as in the standard case. Therefore when commodity n has a positive income effect ( $\eta_n > 0$ ),  $\eta_{nk}$  must be negative.

Q.E.D.

These two theorems identify two basic forces leading to a negative consumption effect. First, each product price elasticity of demand for a commodity ( $\eta_{ik}$ ) will tend to be negative for the same reason as in the case of standard consumer theory. Secondly, even if some of them are not, a weighted average of these elasticities must lead to a negative number by Cournot aggregation (equation 9). Given that the production effect is always negative, one comes to the unsurprising conclusion that the own uncompensated price elasticity of demand for retail items ( $\epsilon_{kk}$ ) is expected to be negative. Less transparently, the Corollary to Theorem 1 suggests that retail items which are used by the household in the production of many different commodities will have more elastic demands than those which are used by the household in the production of a single commodity, other things equal. An important input that also falls into this category is the household's time.

### III. The Cross-Price Elasticity of Demand for Retail Items

From the uncompensated demand function in (7), we can also obtain the cross-price elasticity of demand for any items in a retail assortment, i.e.,

$$\epsilon_{kl} = \epsilon_{kl}^* + \sum_i \omega_{ki} \eta_{il}, \quad k = 1, \dots, K; \quad \ell = 1, \dots, K \quad (14)$$

where  $\epsilon_{kl} = (\partial Q_k / \partial p_\ell)(p_\ell / Q_k)$  and  $\epsilon_{kl}^* = (\partial Q_k / \partial p_\ell | Z)(p_\ell / Q_k)$ .

Equation (14) suggests the following definitions.

Definition 1: Two items in an assortment are net substitutes, independent or complements as the production effect ( $\epsilon_{kl}^*$ ) is positive, zero, or negative, respectively.

Definition 2: Two items in an assortment are gross substitutes, independent or complements as the sum ( $\epsilon_{kl}$ ) of the production effect and the consumption effect is positive, zero, or negative, respectively.

When two items are used in the production of the same commodity, the full range of possibilities encompassed by Definition 1 is available and only a discussion of specific circumstances allows the classification of items into one category or another. Indeed, we would contend that most discussions in the literature about specific items, for example coffee and tea or coffee and sugar, being substitutes or complements are well defined only when these items are viewed as inputs into the production of the same commodity, and the relevant definition being used is Definition 1. In order to strengthen the case for this contention, we establish the following theorem.

Theorem 3: If there is no joint production of commodities, any two items used exclusively in the production of different commodities will be net independents.

Proof: In the absence of joint production, the total costs of producing two commodities, say,  $Z_1, Z_2$ , can be specified as

$$C(p_k, p_l, \bar{p}, Z_1, Z_2, D) = C^1(p_k, \bar{p}, Z_1, D) + C^2(p_l, \bar{p}, Z_2, D) ,$$

where  $\bar{p}$  is a vector of prices other than  $p_k$  and  $p_l$ . The conditional demand for item  $k$  will be given by  $Q_k = \partial C^1 / \partial p_k$ , since  $\partial C^2 / \partial p_k = 0$  because there is no joint production.<sup>2</sup> Hence,

$$(\partial Q_k / \partial p_l | Z) = 0 = (\partial Q_l / \partial p_k | Z) , \text{ and}$$

$$\epsilon_{kl}^* = 0 = \epsilon_{lk}^* (S_l / S_k) .$$

Q.E.D.

Turning to the consumption effect, i.e., the second term on the RHS of (14), a number of possibilities must be considered. It is convenient to start by considering two possibilities that correspond to the two theorems established in the previous section.

Theorem 1': If two items in the retail assortment are used by the household in the production of every commodity, the consumption effect will be negative.

Proof: In this situation  $\omega_{ki} = \theta_i / S_k$  for all  $i$ ; hence, the second term in (14) becomes

$$\sum_i \theta_i \eta_{il} / S_k = -S_l / S_k < 0 , \tag{15'}$$

where the equality follows from Cournot aggregation, (9).

Q.E.D.

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<sup>2</sup>Actually, all we need for the result in the theorem is that both inputs  $k$  and  $l$  be net independents with the inputs that are the source of jointness in production.

Theorem 2': If two items in the retail assortment are used exclusively in the production of the same commodity, (n), the consumption effect will be negative if the commodity has a positive income effect.

Proof: Since  $\omega_{ki} = 0$  for  $i \neq n$ , the second term in (14) becomes

$$\sum_i \omega_{ki} \eta_{il} = \omega_{kn} \eta_{nl} \quad (16)$$

But, as we show in the Appendix (equation A8),  $\eta_{nl}$  must be negative if the income effect ( $\eta_n$ ) is positive.

Q.E.D.

An implication of these two results is that the consumption effect, by being negative, is a force driving all items in the assortment toward gross complementarity. Indeed, a similar force exists in the standard analysis of the consumer under the assumption that goods are normal. This tendency toward gross complementarity, which has not been stressed in the literature, is further illustrated by considering the following situation in our model. Namely, the nature of the consumption effect when two items in the assortment are used exclusively in the production of different commodities. Our point will be made in terms of the following remark.

Remark 1: If two items in a retail assortment are used exclusively by the household in the production of different commodities (n, m), in the absence of joint production these items will be gross complements even if the commodities are (net) substitutes in consumption, provided that the commodities are normal goods and that the income effect is "sufficiently strong."

Proof: By Theorem 3, the two items are net independents. Hence, by Definition 2, gross complementarity will be determined by the consumption effect which is given by

$$\sum_i \omega_{ki} \eta_{il} = \omega_{kn} \eta_{nl} . \quad (17)$$

In the Appendix (equation A11) we show that

$$\omega_{kn} \eta_{nl} = \omega_{kn} (p_{nm}^* \pi_{m\ell} - S_\ell \eta_n) < 0 , \quad (18)$$

where  $p_{nm}^*$  is the compensated cross price elasticity of demand of commodity  $n$  with respect to a change in the shadow price of commodity  $m$  and  $\pi_{m\ell}$  is the percentage change in the shadow price of commodity  $m$  as a result of a percentage change in the price of item  $\ell$  in the assortment. If the two commodities are net or Hicksian independent or complements in consumption ( $p_{nm}^* < 0$ ), the sign of (18) follows from the commodities being normal goods. If the two commodities are net substitutes in consumption, however, it is also required that the income effect ( $S_\ell \eta_n$ ) be "sufficiently strong" to dominate the substitution effect ( $p_{nm}^* \pi_{m\ell}$ ).

Q.E.D.

While the analysis of the previous three situations does not exhaust all the possibilities, as we shall see in the next section, it spans the range of these possibilities. Hence, it allows us to draw the main economic implication of the analysis in this section for the demand for retail products. This implication is summarized in terms of the following proposition.

Proposition 1: Most items purchased by the household from a retailer will be gross complements.

Since this proposition must be, per force, somewhat imprecise, we merely provide an argument for its validity. Consider first the situations underlying Theorems 1' and 2'. That is, items that are used by the household

either in the production of every commodity or exclusively in the production of the same commodity. If these items are net independent or complements in production, they will also be gross complements. Moreover, even if these items are substitutes in production they will still be gross complements if the consumption effect dominates the production effect. Next consider the situation underlying Remark 1. That is, items used by the household exclusively in the production of different commodities. If these items are used in the production of commodities that are net independent or complements in consumption, the items will be gross complements. Even if these items are used in the production of commodities that are net substitutes in consumption, the items will still be gross complements if the income effect dominates the substitution effect. While situations in which two items purchased from a retailer are gross substitutes can in principle be constructed, the analysis in this section suggests that such situations are far less common than situations where gross complementarity prevails.

#### IV. Characteristic of Traffic Building Items in an Assortment.

One issue that is of particular interest from the point of view of retail institutions is the identification of the characteristics of items that are gross complements with many products and that have high (absolute) values of the cross-price elasticity of demand. This issue underlies discussions of loss leader behavior and market basket pricing, for example Preston (1962), or traffic building products, for example Albion (1983). In this section, we identify some of these characteristics.

In order to proceed effectively, it is convenient to consider various cases.

Case i. Two items (k, l) used exclusively in the production of the same commodity (n). The cross-price elasticities can be written as

$$\epsilon_{kl} = \epsilon_{kl}^* + (\theta_n/S_k)(p_{nn}^* \pi_{nl} - S_l \eta_n) \quad (19a)$$

$$\epsilon_{lk} = \epsilon_{lk}^* (S_k/S_l) + (\theta_n/S_l)(p_{nn}^* \pi_{nk} - S_k \eta_n) \quad (19b)$$

Clearly, if an item represents the largest share in the household's budget and leads to the largest increases in the marginal costs of producing a commodity, it is the one most likely to be gross complements with all other items used in the production of that commodity, i.e., it will have the largest absolute value of the consumption effect (the second term in (19a) and (19b)).

Moreover, if this item is used in the production of a commodity which has the highest income elasticity, represents the largest marginal share of the budget and has the highest compensated own price elasticity of demand, it will have the highest possible (absolute) value of the cross-price elasticity of demand out of all the items that are gross complements and are used exclusively in the production of a single commodity.

Case ii. Two items (k, l) used exclusively in the production of different commodities, respectively (n, m). The cross-price elasticities can be written as

$$\epsilon_{kl} = \epsilon_{kl}^* + (\theta_n/S_k)(p_{nm}^* \pi_{ml} - S_l \eta_n) \quad (20a)$$

$$\epsilon_{lk} = \epsilon_{lk}^* + (\theta_m/S_l)(p_{mn}^* \pi_{nl} - S_k \eta_m) \quad (20b)$$

If we assume that neither item (k, l) is the source of any jointness in production,  $\epsilon_{kl}^* = 0 = \epsilon_{lk}^*$  by Theorem 3, and we use the symmetry of the compensated cross-price effects in consumption,  $p_{mn}^* = p_{nm}^*(\theta_n/\theta_m)$ , we have

$$\epsilon_{kl} = (\theta_n p_{nm}^* \pi_{ml} - S_l \theta_n \eta_n)/S_k \quad (21a)$$

$$\epsilon_{lk} = (\theta_n p_{nm}^* \pi_{nl} - S_k \theta_m \eta_m) / S_l \quad (21b)$$

Given that any two items are used in the production of commodities that are net complements in consumption ( $p_{nm}^* < 0$ ), the highest value of the cross-price elasticity of demand will be generated by that item which constitutes the largest percentage of the budget and makes the largest contribution to the marginal costs of the commodity in which it is used. This cross-price elasticity will be largest with respect to those other items that are used in the production of different commodities that have large marginal shares in the budget and a high income elasticity of demand.

Case iii. One item ( $l$ ) used in the production of every commodity and another item ( $k$ ) used exclusively in the production of a single commodity ( $n$ ). The cross-price elasticities can be written as

$$\epsilon_{kl} = \epsilon_{kl}^* + (\theta_n / S_k) \eta_{nl} = \epsilon_{kl}^* + (\theta_n / S_k) (\sum_s p_{ns}^* \pi_{sl} - S_l \eta_n) \quad (22a)$$

$$\epsilon_{lk} = \epsilon_{lk}^* + \sum_i (\theta_i / S_l) \eta_{ik} = \epsilon_{kl}^* (S_k / S_l) - (S_k / S_l) \quad (22b)$$

If the change in the price of the  $l^{\text{th}}$  item increases the marginal costs of producing all commodities in the same proportion, ( $\pi_{sl} = \pi$  for all  $s$ ), then  $\sum_s p_{ns}^* \pi_{sl} = \pi \sum_s p_{ns}^* = 0$  in (22a), because of the homogeneity of degree zero in prices of the compensated demand functions for commodities. With this assumption, the item used in the production of every commodity will have the largest (in absolute value) cross-price elasticity when its share of total costs is largest, assuming that the production effect is dominated by the consumption effect, and with respect to items used exclusively in the production of commodities with high marginal budget shares and income elasticities of demand. Similarly, the item used exclusively in the production of a single commodity will have large absolute values of its cross-

price elasticity, the larger its share in total costs. Interestingly, this will be the case even if the two items are substitutes in production, as long as the compensated cross-price elasticity ( $\epsilon_{kl}^*$ ) is less than unity.

Recently, the literature on retailing has stressed what it calls "one way cross-price elasticity of demand," for example Albion (1983, p. 9). This notion in terms of our model is that the uncompensated cross-price elasticity of demand for an item is not symmetric ( $\epsilon_{kl} \neq \epsilon_{lk}$ ) and that some items or products enjoy high values of these elasticities. Albion argues that products with these high elasticities are important because they act as traffic builders and that these high values for the elasticities are due to "product salience: the degree to which consumers notice and care about the terms of sale of a product" (p. 11). Subsequently, the example of meat and meat sauce, where meat has product salience, is mentioned (p. 103) without elaboration.

Our analysis provides various avenues in which "one-way cross-price elasticities" arise as well as a more precise way of identifying items or products that have this characteristic than an appeal to 'product salience.' To illustrate, consider the example of meat and meat sauce provided by Albion. Both of these items can be viewed as inputs used exclusively in the production of a commodity, a tasty (red) meal. Since meat constitutes a much larger share of the budget than meat sauce and a change in its price increases the marginal costs of producing a tasty (red) meal substantially more than a change in the price of meat sauce, the analysis in case i explains why meat has a much higher "one-way cross-price elasticity of demand" than meat sauce. Moreover, many of the other inputs used in the production of a tasty (red) meal, for example peppers, onions, salt, etc., can be viewed as inputs used in the production of every commodity or activity associated with the broad category nutrition-food; therefore, the analysis in case iii) is also

applicable and once again meat will have a high one-way cross-price elasticity because it constitutes a large share of the budget for this broad category whereas any of the other items will have a small share of the budget for this category. Finally, the analysis in case ii) suggests that meat will also have a high "one-way cross-price elasticity of demand" with items used exclusively in the production of other commodities that have high marginal shares of the food budget or high income elasticities of demand, for example fresh shrimp or lobster in the production of tasty (white) meals, due to its being a large share of the food budget, i.e.,  $S_j$  or  $S_k$  in equations (21a) or (21b). It is this latter characteristic which contributes the most to meat being an excellent traffic builder or to put it another way: an item's share in the consumer's budget is one of the main determinants of "product salience."

#### V. The Services Elasticity of Demand for Retail Items

In this section we turn to consider the most novel and perhaps important feature of the model for the analysis of retail markets. Hence, a few explanatory remarks will be useful. Our claim, Betancourt and Gautschi (1987), is that an intrinsic feature of a retail market is that retail firms offer consumers a set of explicit products or services for purchase together with various levels of distribution services such as accessibility of location, information, etc. This idea is formalized in the model presented here by arguing that the levels of these distribution services offered by any retailer that the household patronizes appear as fixed inputs in the household's production activities, i.e., as elements of the D vector. Hence, if a retailer increases the levels of these services the costs to the household of producing a given level of commodities must not increase and will usually decrease. These distribution services will be classified as common or

specific in the subsequent discussion. A common distribution service is one that is available to all the explicit products offered by a retailer, for example accessibility of location. If two stores instead of a single store are located in a given market area, the increased accessibility of location is provided for all items that the household may purchase from each store. By contrast a specific distribution service is one that is available to one particular item that the household may purchase, for example information on the price of an item.

From the uncompensated demand function in (7), we can also obtain the service elasticity of demand for any item in a particular retailer's assortment with respect to the  $j^{\text{th}}$  distribution service. Namely

$$\epsilon_{kj} = \epsilon_{kj}^* + \sum_i \omega_{ki} \eta_{ij} \quad , \quad k = 1, \dots, K^* \quad (23)$$

$$j = 1, \dots, J$$

where  $\epsilon_{kj} = (\partial Q_k / \partial D_j) (D_j / Q_k)$  ,  $\epsilon_{kj}^* = (\partial Q_k / \partial D_j | Z) D_j / Q_k$  ,  $\omega_{ki} = (\partial Q_k / \partial Z_i) (Z_i / Q_k)$  and  $\eta_{ij} = (\partial Z_i / \partial D_j) D_j / Z_i$  .

Equation (23) suggests the following definition.

Definition 3: A distribution service of a retailer and an item in this retailer's assortment that may be purchased by the household, i.e.,  $k = 1, \dots, K^*$  , are net complements, independent or substitutes as the production effect ( $\epsilon_{kj}^*$ ) is positive, zero or negative.

Incidentally, this definition is perfectly consistent with Definition 1 of Section III. The reason for the difference in sign is that the elasticity is being defined with respect to a change in a quantity rather than a price. For instance, in a conceptual experiment where the distribution service were offered by a retailer at an explicit price ( $p_j$ ), we would have: an increase

in  $p_j$  decreasing  $D_j$  (by the concavity of the cost function), which in turn would imply a decrease in  $Q_k$  if  $Q_k$  and  $D_j$  are complements. Since the behavior of the restricted cost function must be consistent with the behavior of the unrestricted one, Definition 3 is consistent with this conceptual experiment. While one would expect the distribution services of a retailer to be net independent or complements with the items in the retailer's assortment that the household may purchase, it is difficult to rule out in general, except by assumption, special sets of circumstances in which a relation of net substitutability may arise.

One important relation of net substitutability arises with respect to the input of the household's time in the production of commodities. Several distribution services, by their very nature, are net substitutes in production with the household's own time in purchase activities. We are referring in particular to accessibility of location, extent of product assortment, degree of assurance of product delivery and level of information services. Higher levels of these distribution services diminish the need for the household to employ its own time in travel and search activities at any given level of  $Z_i$ 's.

Equation (23) also suggests the following definition.

Definition 4: A distribution service of a retailer and any item in this retailer's assortment that may be purchased by the household are gross complements, independent or substitutes as the sum ( $\epsilon_{kj}$ ) of the production effect and the consumption effect is positive, zero or negative.

Given this definition, it is of interest to explore the circumstances generating positive consumption effects. To summarize these circumstances, we establish the following theorems:

Theorem 4: If an item in an assortment is used in the production of every commodity by the household, the consumption effect of a change in distribution services will always be positive.

Proof: Under the hypothesis of the theorem, the second term in (23) becomes

$$\sum_i (\theta_i/S_k) \eta_{ij} - (1/S_k) \sum_i \theta_i \eta_{ij} - S_j/S_k > 0 , \quad (24)$$

where  $S_j = (\partial C/\partial D_j) D_j/C$ .  $S_j$  represents the percentage reduction in costs as a result of percentage increase in the levels of the  $j^{\text{th}}$  distribution service, hence it will be negative; indeed,  $(\partial C/\partial D_j)$  may be viewed as the negative of the shadow price of the  $j^{\text{th}}$  distribution service. The second equality in (24) follows from differentiation of the budget constraint in (5) with respect to the  $j^{\text{th}}$  distribution service and some manipulation. It is exactly analogous to the Cournot aggregation condition that arises in the household production model as a result of a price change. Since  $S_j$  is negative, the consumption effect will be positive.

Q.E.D.

Theorem 5: If an item is used exclusively in the production of a single commodity ( $n$ ), the consumption effect of a change in a specific distribution service will always be positive if the commodity has a positive income effect.

Proof: Under the hypothesis of the theorem, the second term in (23) becomes

$$(\theta_n/S_k) \eta_{ij} - (\theta_n/S_k) (p_{nn}^* \pi_{nj} - S_j \eta_n) > 0 , \quad (25)$$

where  $\pi_{nj}$  is the proportionate change in the marginal costs of the  $n^{\text{th}}$  commodity as a result of a proportionate change in the level of the  $j^{\text{th}}$  distribution service. The equality in (25) follows from the comparative statics analysis performed in the Appendix, equation A14. Note that  $p_{nn}^* < 0$

,  $S_j < 0$ , and  $\pi_{nj} < 0$ ; hence, if the  $n^{\text{th}}$  commodity has a positive income effect, the consumption effect will be positive.

Q.E.D.

Theorem 5': If an item is used exclusively in the production of a single commodity ( $n$ ), the consumption effect of a change in a common distribution service, which affects all marginal costs in the same proportion, will always be positive if the commodity has a positive income effect.

Proof: Under the hypothesis of the theorem, the second term in (23) becomes

$$(\theta_n/S_k)\eta_{ij} = (\theta_n/S_k)(\sum_s p_{ns}^* \pi_{sj} - S_j \eta_n), \quad (26)$$

where  $\pi_{sj}$  is the proportionate decrease in the marginal costs of commodity  $s$  as a result of a proportionate increase in the level of common distribution service. The equality follows from the comparative statics analysis in the Appendix, equation A16. If for simplicity, we assume that the common distribution service affects the marginal costs of producing all commodities and in the same proportion we have  $\sum_s p_{ns}^* \pi_{sj} = \pi \sum_s p_{ns}^* = \pi \cdot 0 = 0$ .<sup>3</sup> Hence (26) becomes

$$-\theta_n S_j \eta_n/S_k > 0, \quad (27)$$

where the inequality follows from assuming a positive income effect for the commodity.

Q.E.D.

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<sup>3</sup>Even if all marginal costs do not change in the same proportion, there will be a tendency for this term to be close to zero since if the marginal costs were to change in the same proportion the condition would have to be satisfied. Therefore, this term is likely to be outweighed by the income effect in almost every instance.

The most direct economic implication of these results is best summarized in terms of the following proposition.

Proposition 2: The distribution services provided by a retailer will tend to be gross complements with every item in this retailer's assortment that may be purchased by the household.

While this proposition requires no proof, it is useful to examine some of the conditions under which the result will be strongest, in the sense of leading to the largest (positive) values of the distribution services elasticity of demand. First, the percentage reduction in the costs to the household of an increase in a distribution service ( $S_j$ ) by the retailer is likely to be much higher for distribution services that are common inputs and affect many of the household activities than for those that are specific inputs and only affect the costs of purchasing a particular item. Second, the same common input, for example accessibility of location, may be a greater source of gross complementarity in an area where the opportunity cost of time is high than in an area where it is low, because an increase in this service generates a larger reduction in costs for a household with a high opportunity cost of time than for one with a low valuation of time. Finally, specialty retailers who provide items that are a small share of the consumer budget may be able to increase demand for these items considerably by providing a distribution service that is a net complement with these items and that contributes to a substantial reduction in the household costs.

## VI. Implications

In the previous sections, we have analyzed the effects of changes in a particular retailer's prices or distribution services on the demand for the items in that retailer's assortment. Such an analysis suggests that the

consumption effect is a powerful force driving the results toward gross complementarity. With respect to price changes, gross substitutability is indeed possible, but it requires the net substitution effect in production to be stronger than the consumption effect for items used in the production of the same commodity or the net consumption effect to dominate the income effect for items used in the production of different commodities that are net substitutes in consumption. In the case of distribution services, since the net production effect is almost always likely to be positive, gross substitutability is far less likely. These results raise the question of how competition takes place in the context of our model.

From the results developed in the previous sections, one obtains the analytical framework for answering this question, although some care must be exercised in the interpretation. For instance, a change in the price of an item in the assortment of retailer B will have an effect on the demand for an item in the assortment of retailer A that will also be described by equation (14) in Section III. The difference in interpretation lies in the fact that the magnitudes are likely to be different. Thus, a rise in B's price of brand X of coffee is likely to generate a larger positive net production effect on brand X of coffee in A's assortment than on brand Y of coffee in B's assortment. Therefore, it is more likely to dominate the consumption effect and thus lead to gross substitutability. Indeed, it is entirely possible for retailer B to drive an item out of the bundle that the household purchases from A's assortment through price decreases, since the size of the consumption effect is directly related to the value of the usage term for A,  $\omega_{ki}$ , which will be driven to zero as a result of a series of price decreases when there is gross substitutability between the items in the two assortments.

With respect to changes in distribution services, one can similarly analyze the effect of a change in a distribution service by retailer B on the demand for an item in the assortment of retailer A through the use of equation (23). In this case, however, the need to interpret the results carefully is more pronounced than in the previous one. For instance, a change in a distribution service in the assortment of retailer B will usually have two types of net production effects on the items in B's assortment,  $\epsilon_{ki}^*(B,B) \geq 0$ , as the items are net complements or independents with the  $j^{\text{th}}$  distribution service, respectively. This means that in the case of net independence, there will be no net production effect on the items in A's assortment, i.e.  $\epsilon_{kj}^*(A,B) = 0$ . Since the consumption effect will be positive, however, the demand for items in A's assortment will increase as a result of the increase in B's  $j^{\text{th}}$  distribution service. Those items in B's assortment that are net complements with the  $j^{\text{th}}$  distribution service will generate a more complex pattern of changes as a result of an increase in B's  $j^{\text{th}}$  distribution services. First, the net production effect on the items in A's assortment,  $\epsilon_{kj}^*(A,B)$ , will be positive, zero or negative as these items are net production substitutes, independent or complements with the items in B's assortment,  $\epsilon_{kl}^*(A,B) \geq 0$ , that are net complements with the  $j^{\text{th}}$  distribution service,  $\epsilon_{kj}^*(B,B) > 0$ . Secondly, the demand for those items in A's assortment will tend to increase, since the consumption effect is positive, unless the net production effect for those items that are substitutes,  $\epsilon_{kj}^*(A,B) < 0$ , is sufficiently strong to dominate the consumption effect. Hence, allowing for the role of different retailers generates opportunities for a distribution service of one retailer to exhibit gross substitutability with items in another retailer's assortment, if these items are net substitutes in production with items in the assortment of the retailer that alters the level of the distribution service.

Our characterization of the demand for retail products suggests three main general insights into the interactions between retailers in a market. First, distribution services provide retailers with tools for nonprice competition. Nevertheless, these tools have limits on their usefulness: they must work through the same channels as price competition, in the sense that net substitutability in production between items in different assortments is a necessary condition for the nonprice competition to take place; with respect to common distribution services, these tools are not precise instruments because they can affect the demand for many different items in the different assortments and frequently in the wrong direction, i.e., if gross complementarity prevails. Second, the effects of competition between retailers, both price and nonprice, are asymmetric, especially when the assortment and the mix of distribution services offered by any two retailers are very different in the initial situation. Last but not least, the tendency toward gross complementarity between retailers through the provision of distribution services is one of the most important forces leading to the creation of retail agglomerations. In addition, the model also helps to explain why two department stores with very broad but similar assortments might agree to locate in the same mall but two specialty stores with very narrow and similar assortments might be reluctant to do so. Namely, net substitution in production between all the items in the narrow assortment of the specialty stores is far more prevalent than between all the items in the broad assortments of the department stores. Therefore, gross substitutability and intense price and nonprice competition is more likely to arise for the specialty stores than for the department stores.

By the way of a conclusion we note that, since most distribution services are net substitutes in production with the household's use of its own time in

consumption activities, a significant part of the 'competitive effect' of increases in most distribution services by any one retailer is likely to be borne by the household through its time allocation. Since there is a given level of prices in this experiment, such changes always enhance welfare, because the household can maintain the same pattern of consumption as before the increase in distribution services. Alternatively, increases in the opportunity cost of time, through rising wage rates, provide an incentive for retail institutions to offer higher levels of distribution services that economize on this household resource directly or indirectly, i.e., by allowing a relaxation on the constraints that may limit the timing of its employment.

## Appendix. Comparative Statics Analysis

For simplicity of exposition, let us consider the case in which there are two commodities ( $Z_1, Z_2$ ), but we allow  $s = 1, \dots, K$  items used by the household and  $j = 1, \dots, J$  distribution services provided by a particular retailer.

Total differentiation of the first-order conditions, (4) and (5) in the text, yields after rearrangement

$$(U_{11} - \lambda C_{11})dZ_1 + (U_{12} - \lambda C_{12})dZ_2 - C_1 d\lambda = \lambda \sum_s C_{1s} dp_s + \lambda \sum_j C_{1j} dD_j \quad (A1)$$

$$(U_{21} - \lambda C_{21})dZ_1 + (U_{22} - \lambda C_{22})dZ_2 - C_2 d\lambda = \lambda \sum_s C_{2s} dp_s + \lambda \sum_j C_{2j} dD_j \quad (A2)$$

$$- C_1 dZ_1 - C_2 dZ_2 + 0 \cdot d\lambda = \sum_s Q_s dp_s + \sum_j C_j dD_j - dW \quad (A3)$$

This system can be written more compactly as

$$HdZ = dx, \quad (A4)$$

and its solution will be given by

$$dZ = H^{-1}dx. \quad (A5)$$

If only the price of the  $k^{\text{th}}$  product is allowed to change, (A5) yields

$$\partial Z_1 / \partial p_k = h^{11} \lambda C_{1k} + h^{12} \lambda C_{2k} + h^{13} Q_k \quad (A6)$$

Since by the assumptions of Theorem 2, the  $k^{\text{th}}$  product is used exclusively in the production of the first commodity, we have, given the continuity of the cost function,  $C_{2k} = C_{k2} = \partial Q_k / \partial Z_2 = 0$ . Thus, (A6) reduces to an expression analogous to the income and substitution effects of standard consumer theory, i.e.,

$$\partial Z_1 / \partial p_k = (\partial Z_1 / \partial p_k | U) - Q_k (\partial Z_1 / \partial W) , \quad (A7)$$

which expressed in elasticity terms yields

$$\eta_{1k} = \eta_{1k}^* - S_k \eta_1 , \quad (A8)$$

where  $\eta_1$  is the income elasticity of demand of commodity 1. The substitution effect,  $\eta_{1k}^*$ , can also be expressed as

$$\eta_{1k}^* = p_{11}^* \pi_{1k} , \quad (A9)$$

where  $p_{11}^* = (\partial Z_1 / \partial C_1 | U) C_1 / Z_1$  and  $\pi_{1k} = C_{1k} P_k / C_1$ . In terms of interpretation  $p_{11}^*$  is simply the compensated own (shadow) price elasticity of demand of commodity 1 and  $\pi_{1k}$  is the percentage change in the shadow price of commodity 1 ( $C_1$ ) as a result of a percentage change in the price of product k. Under the assumption that commodity 1 is a normal good ( $\eta_1 > 0$ ), (A8) must be negative.

If only the price of the  $l^{\text{th}}$  product, which is used exclusively in the production of let us say the second commodity ( $Z_2$ ), changes we have from (A6)

$$\partial Z_1 / \partial p_l = h^{12} \lambda C_{2l} + h^{13} Q_l , \text{ or}$$

$$\partial Z_1 / \partial p_l = \partial Z_1 / \partial p_l | U - Q_l \partial Z_1 / \partial W . \quad (A10)$$

This expression can be rewritten in elasticity terms as

$$\eta_{1l} = \eta_{1l}^* - S_l \eta_1 = p_{12}^* \pi_{2l} - S_l \eta_1 , \quad (A11)$$

where  $p_{12}^*$  is the compensated cross price elasticity of demand of commodity 1 with respect to a change in the shadow price of commodity 2. Since we only have two commodities, they must be net substitutes in consumption ( $p_{12}^* > 0$ ) although in the general case they can also be net independent or complements.

In any event, even though the commodities are net substitutes in consumption, the consumption effect can still be negative if the commodities are normal goods ( $\eta_1 > 0$ ) and the income effect is "sufficiently strong," i.e., the magnitude of the second term in (A11) exceeds in absolute value the magnitude of the first term.

If there is only a change in a distribution service, the effect on the first commodity, let us say, is given from (A5) as

$$\partial Z_1 / \partial D_j = h^{11} \lambda C_{1j} + h^{12} \lambda C_{2j} + h^{13} C_j . \quad (A12)$$

If the distribution service is a specific one in that it affects only the  $k^{\text{th}}$  item used exclusively in the production of the first commodity, we then have  $C_{2j} = 0$  and (A12) becomes

$$\partial Z_1 / \partial D_j = \partial Z_1 / \partial D_j | U - (\partial Z_1 / \partial W) C_j . \quad (A13)$$

This result can be expressed in elasticity terms as

$$\eta_{1j} = \eta_{1j}^* - S_j \eta_1 - p_{11}^* \pi_{1j} - S_j \eta_1 . \quad (A14)$$

Since  $S_j < 0$ ,  $\pi_{1j} < 0$  and  $p_{11}^* < 0$ , (A14) must be positive if  $\eta_n > 0$ .

If the distribution service is a common one and it affects the marginal costs of producing the second commodity, then (A12) implies

$$\partial Z_1 / \partial D_j = (\partial Z_1 / \partial C_1 | U) C_{1j} + (\partial Z_1 / \partial C_2 | U) C_{2j} - (\partial Z_1 / \partial W) C_j \quad (A15)$$

or in elasticity terms

$$\eta_{1j} = p_{11}^* \pi_{1j} + p_{12}^* \pi_{2j} - S_j \eta_1 . \quad (A16)$$

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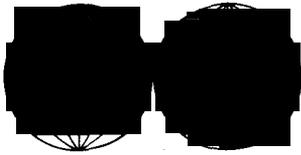
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