A PREDICTIVE TEST OF THE NBD MODEL THAT CONTROLS FOR NON-STATIONARITY

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N° 88 / 36

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Director of Publication :

Charles WYPLOSZ, Associate Dean for Research and Development

Printed at INSEAD, Fontainebleau, France

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June 28, 1988

Draft Copy: Please do not quote.

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The authors wish to thank Donald Morrison, Ambar Rao, and Bob Shoemaker for many helpful comments on earlier versions of this work. Also, thanks are due to Todd Johnson of NPD Research, Inc. for making available the data that were analyzed in this study.

Abstract

In this paper a technique is proposed that largely corrects for the effects of non-stationarity in purchase incidence studies that use the Negative Binomial Distribution model. With non-stationarity controlled in this way, it is possible to assess the validity of the NBD model's remaining assumptions of Poisson purchasing at the individual level and a gamma mixing of Poisson rates across the population. A test of the predictive fit of the NBD model is also derived. Using this predictive test and the proposed technique, the authors show how to diagnose some failures of the NBD assumptions. The effects of the technique on the results of conditional trend analyses are explored. The technique is illustrated using simulations and then applied to consumer panel data for seven product classes.

A Predictive Test of the NBD Model that Controls for Non-stationarity

The Negative Binomial Distribution (NBD) model was introduced to marketing by Ehrenberg (1959) as a way of describing the pattern of consumer purchases. Almost thirty years later, the model continues to be of interest, as is evidenced by its appearance in recent articles by Morrison and Schmittlein (1981,1988), Dunn, Reader, and Wrigley (1983) and Wagner and Taudes (1986). It has also been used as a component in composite models of brand choice and purchase timing, including those of Bass, Jeuland and Wright (1980), Zufryden (1978, 1981) and Schmittlein, Bemmaor, and Morrison (1985). Thus, perhaps because of its relative simplicity and ease of estimation, the NBD enjoys a continued presence in the marketing research literature.

Goodhardt and Ehrenberg (1967) use the NBD to perform "conditional trend analysis." Here buyers are first segmented by their observed number of purchases in a predictor period; their expected purchase frequencies are then predicted for a subsequent criterion period. As discussed by Morrison and Schmittlein (1988), the NBD assumes that purchase frequencies are in part random, so these conditional predictions include a measure of regression to the mean. Conditional trend analysis thus allows a researcher to make baseline predictions for both heavy and light buyers, and these predictions are corrected for an assumed randomness in the purchase process. Deviations from these baseline predictions might then be interpreted as the results of specific marketing efforts. In this way the differential effects of a marketing activity on heavy and light buyers can be examined.

To use the NBD we must make three assumptions. The first is that each individuals purchase in a Poisson fashion with unobservable rate λ. The second is that the dispersion in these individual rates across the population, $f(\lambda)$, is accurately described by the gamma distribution; though fairly flexible, the gamma is constrained to being uni-modal. The last assumption is stationarity, namely, that each individual retains the same purchase rate in the criterion period that he or she held in the predictor period. The first two assumptions have drawn much comment and some criticism, and various gereralizations to them have been proposed. With the exception of a fine review by Morrison and Schmittlein (1988), however, the stationarity assumption has received little attention in the literature.

In this paper, we examine the stationarity assumption and its implications for assessing the performance of the NBD as a predictive model of consumer purchase incidence. There is good reason for doing this. Marketing activities are usually designed to accelerate consumers' purchases, that is, to induce non-stationarities.

These non-stationarities produce deviations between predicted and actual conditional purchase frequencies, causing the NBD predictions to fail. But *any* non-stationarity, not just those induced by a marketing activity, will cause the NBD predictions to fail. And, given the usual level of promotional activity, non-stationarities are probably present in most data sets. Indeed Sabavala (1988) recently noted that "There is *no* stable period!" on which to validate NBD predictions.

How, then, are we to test the NBD so that we can put some faith in its conditional predictions? Rather than search for a stable period, our approach is to control for possible non-stationarities in the data so that we can test the remaining Poisson and gamma assumptions. If the NBD performs well in this context, then we can ascribe its failure in a non-controlled context to non-stationarity. Of course, such non-stationarity may or may not be the result of a particular marketing activity. That judgment we leave to the researcher. Our procedure consists of two parts:

i) We develop the sampling properties of NBD conditional predictions under conditions of stationarity. These allow us to perform a formal statistical test of the NBD predictions, as was suggested by Morrison and Schmittlein (1981).

ii) We devise a data manipulation that largely corrects for the effects of non-stationarity in conditional trend analyses. This allows us to create data sets that appear to have come from stationary markets. With non-stationarity controlled for, we can use the result from part (i) to test the remaining Poisson and gamma assumptions.

But, as we will show, our technique does more than just control for non-stationarities. We can use it to identify *which* of the three NBD assumptions is being violated, if indeed any one of them is. Thus we can use it to make a more informed choice between the simple NBD and more complex models like the Condensed NBD of Chatfield and Goodhardt (1973). In this sense our technique builds on the analytical results presented by Morrison and Schmittlein (1988).

The rest of this paper is organized as follows: First, we discuss more fully the NBD assumptions and their implications. Next, we present our method to control for the effects of non-stationarity. In essence, this method uses the memoryless property of the Poisson purchase assumption to average individual non-stationarities across the sample. We illustrate the method on four simulated data sets, then apply it to consumer panel data from seven product classes. We are able to show which of these data sets do, and don't, meet the NBD assumptions. Discussion and conclusions follow.

2.The NBD: Assumptions and Issues

The requisite assumptions of the NBD model are well known, but a brief review here will enhance the reader's appreciation of the discussion to follow.

Poisson Purchases

For a Poisson individual with unobservable rate λ, the probability of a purchase occuring within any very small interval of time, dt, is simply the product λdt. Further, the behavior of the process on any one such interval of time is independent of that on any other, so what happens in the prior instant does not affect the present. In this sense the process is memoryless, as discussed by Meyer (1970, pp 165-66) and Heyman and Sobel (1982, p 511). We can characterize the observable results of a Poisson purchase process in three ways: (i) by the distribution of purchase frequencies for a unit time, (ii) by the density function of interpurchase times, and (iii) by the distribution of a fixed number of purchases across any given interval. The Poisson process implies certain properties for each.

To begin, the distribution of observed purchase frequencies in any given unit time follows the Poisson distribution

$$P(x; \lambda) = e^{-\lambda} \lambda^{x}/x!, \tag{1}$$

where λ is the rate of purchase and x is the observed frequency. The mean of this distribution equals λ, as does the variance, so purchases are highly irregular. This irregularity is also evident in the density of interpurchase times implied by the Poisson process, which is the exponential

$$f(t; \lambda) = \lambda e^{-\lambda t}, \tag{2}$$

where t is the interpurchase time. This density is memoryless: for a given rate, the timing of future purchases is independent of those in the past. Thus it implies *no purchase regularity*. Also, its mode is at zero, implying that the most likely time for the next purchase, given that one has just occurred, is the following instant.

Finally, because the probability of a Poisson event on any very small interval equals λdt, the distribution of a fixed number of Poisson purchases across an observation period can be viewed as a number of iid draws from a uniform distribution. This property is discussed more fully by Parzen (1962, pp 140-41) and by Schmittlein and Morrison (1985).

The major criticism of the Poisson assumption is that it implies too little regularity in purchase timing (Herniter 1971). If people *consume* goods in a regular fashion, then we would expect them to *purchase* goods in a regular fashion as well.

With this in mind, Chatfield and Goodhardt (1973) proposed the "condensed Poisson model," where the density of individual interpurchase times is Erlang-2, which has a mode greater than zero and implies some purchase regularity. In many situations, the condensed Poisson process would seem to be quite plausible. However, neither Chatfield and Goodhardt (1973) nor Morrison and Schmittlein (1981) found it to yield consistently better fit or predictions than the Poisson model. For more discussion on this point see Morrison and Schmittlein (1988).

Gamma Heterogeneity

The heterogeneity in rates across the population is assumed to be described by the gamma density

$$f(\lambda; r\ \alpha) = (1/\Gamma(r))\ \alpha^r\ \lambda^{r-1}\ e^{-\alpha\lambda} \tag{3}$$

where r and α are known as the shape and scale parameters, respectively. Because the gamma density is uni-modal, there are certain forms of heterogeneity that it will not capture very well. Accordingly, generalizations of this mixing model have been proposed. Morrison (1969) combined a gamma mixing density with a "spike" at zero to describe the presence of hard-core non-buyers in the sample. Robbins (1977) proposed an empirical Bayes estimator that allows for an arbitrary mixing of Poisson processes. A recent analysis of Navy recruiter productivity by Carroll, Lee, and Rao (1986) applies both of these generalizations.

NBD Model

For a fixed time interval, a gamma mixing of individuals with Poisson purchase rates produces an observed aggregate mixture of purchase frequencies that follows the Negative Binomial Distribution

$$P(x; r, \alpha) = (\Gamma(r+x)/\Gamma(r)\ x!)\ (1/(\alpha+1))^x\ (\alpha/(\alpha+1))^r \tag{4}$$

where r and α are the same parameters that define the gamma mixing density. The NBD was first specified as a gamma mixing of Poisson processes by Greenwood and Yule (1920), and its sampling properties have been explored by both Fisher (1941) and Anscombe (1950).

Stationarity

To appreciate the importance of the stationarity assumption, we must first briefly describe the technique of conditional trend analysis as performed with the NBD. The basic result employed by Goodhardt and Ehrenberg (1967) is a conditional expectation. If the aggregate purchase process is NBD with parameters r and α, then an individual who has made x purchases in a predictor period of unit length has an expected rate of purchase given by

$$E\{\lambda \mid r, \alpha, x\} = r/(\alpha+1) + x/(\alpha+1) \quad (5)$$

for a criterion period of equal duration. This conditional expectation is the NBD predictor for the number of purchases the individual will make in the criterion period. It assumes that individuals' rates do not change across the two periods - this is the stationarity assumption. If stationarity is violated, the conditional expectation will be a biased predictor of furture purchase behavior. Thus, as noted by Sabavala (1988) and Montgomery (1988), any predictive test of the NBD requires either an assumption of stationarity or some means to control for its violation.

Table 1 is a conditional trend analysis from Morrison and Schmittlein (1981). Its first column identifies the purchase class, and its second contains the number from the sample who made that many purchases in the predictor period, which in this case is the first 6 months of the observation year. The third column contains the conditional expectations for each purchase class. These are obtained by first estimating the NBD parameters using data from the predictor period, then applying these estimates to the conditional expectation in equation (5). The fourth column contains the observed average purchase frequencies for each purchase class. For example, we see that those making 0 purchases were predicted to make 0.55 purchases each in the criterion period, but in fact they each made 0.89, on average, an under-prediction of about 60%. Is this discrepancy due to a violation of the Poisson, gamma, or stationarity assmption? This is the question our technique seeks, in part, to answer.

INSERT TABLE 1 HERE

Other researchers have tried to control for the effects of non-stationarity. Morrison and Schmittlein (1981) did so by rescaling the expected rate of purchase across all sample members. For example, in one product class (food bags), they found that the average rate of purchase between predictor and criterion periods declined by 32%, so they multiplied each conditional prediction by 68% (pp 1016-17). A similar correction is used by Dalal, Lee, and Sabavala (1984). Such rescaling assumes that all individuals are non-stationary in equal proportion, but this is probably not the case. In fact, according to Goodhardt and Ehrenberg (1967) the primary purpose of conditional trend analysis is to examine the *differential* effects of non-stationarity on heavy and light buyers. And Ehrenberg (1972, p. 12) notes that even product classes that exhibit stability on average are subject to "variable and quite complex patterns of individual purchasing behavior." Thus a technique is needed to control for non-stationarity at the individual level so that we can test the conditional predictions of the NBD model and the gamma-Poisson assumptions that produce them. The technique described in the following section does control non-stationarity in this way.

3. Method

For a Poisson process, an individual's expected number of purchases in a time interval depends only upon the length of that interval, and not on the way in which the interval is selected. Applying this concept to the NBD, we see that if the period of analysis is 12 months, it does not matter which 6 months we call the predictor period and which we call the criterion. The statistical properties of an individual's purchases for any 6 months drawn from the 12 will be identical; that is, the distribution of purchase frequencies around the 6 month mean will be Poisson.

For example, prior predicitve tests of the NBD, such as those of Morrison and Schmittlein (1981), have used the first six months of the year as the predictor period and the second six months as the criterion. But if individuals are Poisson, there is no reason to confine ourselves to this scheme. We could just as well use the last six months as the predictor period and the first six months as the criterion, or take the even numbered months as the predictor period and the odd numbered months as the criterion. In fact, we could use any six months to predict behavior in the remaining six months.

Further, we could randomly select six months *for each individual* and use these months to constitute that individual's predictor period; the remaining six months would then become his or her criterion period. We can perform this random selection independently for each individual in the sample, thereby providing each with idiosyncratic, independently derived predictor and criterion periods.

If individuals purchase in a Poisson manner, and if their behavior is stationary, then our selection technique should have no effect, other than sampling variation, upon the conditional trend analysis. This follows from the memoryless property of the Poisson process. But if individuals are Poisson with rates that vary over time, then our selection technique will tend to average out this non-stationarity across individuals. To see this consider an example from Goodhardt and Ehrenberg (1967, pp158-59) where seasonality has different effects on heavy and light buyers. A conventional first-6-months/second-6-months conditional trend analysis yields predictions that diverge widely from the observed purchase frequemcies. But if we select predictor and criterion periods randomly, and independently, for each individual, then the non-stationarity due to seasonality will tend to be averaged out. That is, for some the predictor period will include some of the heavy months, and for some it will include some light months, but on average, across the sample, the effects of seasonality will be equally partitioned into the preditor and criterion periods. Thus, for purposes of conditional trend analysis, the entire sample would tend to behave as a stationary compound Poisson process.

Also, we must ask: what is the effect of our selection technique on a sample where purchases are stationary but more regular than Poisson? A number of simulation studies reported in Tibrewala (1987) indicate that the random selection of predictor and criterion periods tends to disrupt regular purchase behavior and to make it seem more irregular (i.e., more like Poisson) than it actually is. This lowers the power of any statistical test of a null hypothesis of Poisson behavior. However, for the large sample sizes (on the order of several thousand consumers) that are typical of diary panels, the technique still retains discriminant validity. That is, it will tend to reject a stationary but regular purchase process while it accepts a non-stationary but Poisson process. We will illustrate this in the simulated examples that follow.

This technique does not eliminate entirely the effects of non-stationarity but averages them out across the sample. The degree of attenuation depends upon the number of time units used in the selection. In this study we select randomly 6 of 12 months as the predictor period. We could use more, smaller units (e.g, 26 out of 52 weeks) to better control for non-stationarities, but this would tend to make regular purchasing look more Poisson. In the limit, if we used an extremely small time unit, any purchase process, no matter how regular, would behave as if it were Poisson in a conditional trend analysis (in this case, the unit would behave as the small interval dt). Thus some judgment is required in applying the method. The simulation studies in the following section indicate that selecting 6 months from 12 controls well for non-stationarity and retains adequate discriminatory power.

Based upon this line of reasoning, we construct predictor and criterion periods using the following procedure:

i) For each individual in the sample, pick 6 months at random without replacement and assign the individual's total purchases in these months to the predictor period. Assign the remaining purchases to the criterion period. Select the 6 predictor months independently for each individual.

ii) Aggregate the predictor and criterion purchases across individuals.

iii) Estimate by maximum likelihood the NBD parameters using the predictor period histogram.

iv) Use these estimated parameters and equation (5) to make conditional predictions for the criterion period.

v) Evaluate the quality of the conditional predictions.

This procedure is subject to sampling variation because we use a probability rule to select the predictor and criterion periods for each individual. We can control for this sampling variation by repeating steps (i) through (v) a large number of times, say 100, then looking at the typical performance of the predictions across the 100 runs. For each run a χ^2 goodness-of-fit statistic is calculated; the number of times that this statistic exceeds its critical value under the NBD null hypothesis is taken as a measure of predictive fit.

4. Simulated Examples

Here we present four cases to show how the technique performs under different violations to the NBD assumptions. For each case, we constructed a simulated sample of 3,000 individuals as follows:

NBD: Individuals make purchases in a Poisson fashion and the distribution of rates across the sample is gamma with $r=1$ and $\alpha=1/4$, so the resulting mixture is NBD.

Spike at Zero: We remove 20% of the original sample at random and set their purchase rates to 0, thereby creating a "spike" at zero, as modelled by Morrison (1969). The remaining 80% of the sample purchase in a Poisson fashion with rates distributed gamma $r=1$, $\alpha=1/4$.

Condensed NBD: We retain the same gamma mixing of rates but individuals make purchases in a condensed Poisson fashion, as modelled by Chatfield and Goodhardt (1973), so their interpurchase times are distributed Erlang-2. The resulting mixture of observed purchase frequencies is Condensed NBD.

Non-Stationary NBD: Individuals' rates are distributed gamma with $r=1$ and $\alpha=1/4$, and they purchase in a Poisson fashion, but at the end of the first six months, 20% of the sample draws a new purchase rate at random from the original gamma mixing distribution. Their rates in the second six months are thereby independent of those in the first. Thus 80% of the sample is stationarity and 20% is not.

For each simulated sample, we performed a conventional first-6months/second-6-months conditional trend analysis. Like Morrison and Schmittlein (1981), we adjusted the conditional predictions to control for differences in the average purchase rate, but these adjustments were minor. We had 15 purchase classes ranging from 0 purchases in the predictor period up through 14+ purchases. We aggregated the highest purchase classes, as did Goodhardt and Ehrenberg (1967) and Morrison and Schmittlein (1981), because the number of individuals within each was small.

Next, we performed a conditional trend analysis on these simulated data sets using the random selection of predictor and criterion periods described in the prior section. Every individual's predictor and criterion periods were chosen independently. For each data set, 100 such conditional trend analyses were performed. In this way we could control for the sampling error induced by the probabilistic selection of predictor and criterion periods.

Summary of χ^2 Analyses

Our χ^2 test of predictive fit for NBD conditional trend analyses is derived in the Appendix. This test is based on the fact that the conditional distribution in the criterion period for those who have made x purchases in the predictor period is NBD with updated parameters (r+x, a+1), which allows us to derive the sampling distribution of the conditional mean. Normalizing and squaring predictive errors for each purchase class, and then summing across classes we obtain a χ^2 statistic with k-1 degrees of freedom, where k is the number of purchase classes and one degree of freedom is lost because we adjust the overall mean rates of purchase. If the value exceeds the critical value for k-1 degrees of freedom, it indicates that one or more of the assumptions of the NBD model have been violated.

The χ^2 values for the simulated examples are summarized in Table 2. All four simulations involve 14 degrees of freedom so all share a common critical value (at the 0.05 level) of 23.7. Each row in the table corresponds to a different simulated sample. We consider each in turn:

<u>NBD:</u> The χ^2 value for the first-6-month/second-6-month conditional trend analysis (third column, first row) is 16.7, below the critical value of 23.7, indicating that the data satisfy the assumptions of the NBD model. The fourth column is the median χ^2 value for the 100 conditional trend analyses (we have used the median instead of the mean because it is less likely to be influenced by extreme values). The fifth and sixth columns contain the lowest and highest values. The median value is 16.1, virtually the same as that obtained for first-6/second-6 analysis. Thus it appears that the random selection of predictor periods has little systematic effect on the quality of conditional predictions, if the underlying process is NBD. The range in the values across the 100 runs reflects the sampling variation due to the probabilistic selection of predictor periods.

<u>Spike at Zero:</u> The second row of Table 2 contains similar data for the sample with a 20% spike at zero. The χ^2 value of 66.0 for the first-6-month/second-6-month analysis shows that the model does not fit. This accords with the non-gamma mixing distribution. The median χ^2 value for the randomly selected conditional trend analyses is 63.8, quite close to 66.0, and columns five and six show that 66.0 falls well within the range of χ^2 values obtained across the 100 runs. This suggests that the random selection of predictor periods does not improve predictive accuracy when individuals are Poisson but the gamma mixing assumption is violated.

<u>Condensed NBD:</u> Here the χ^2 value for the first-6/second-6 conditional trend analysis is 93.3. Notice, however, that the median value across the 100 randomly selected conditional trend analyses is 45.6. While this is still above the critical value, it is well below that obtained on the first-6/second-6 analysis. This indicates that the random selection of predictor and criterion periods tends to make condensed Poisson behavior appear more Poisson-like. Studies by Tibrewala (1987) show that if we used a smaller time unit for selection of predictor periods this effect would be more pronounced. Still, for this sample size and population, which are representative of the real data we will analyze in the next section, the technique is clearly able to distinguish between Poisson and condensed Poisson processes. We need only compare the median χ^2 value of the NBD analysis (16.1) to that of the CNBD analysis (45.6) to verify this.

<u>Non-stationarity:</u> For the first-6/second-6 analysis, the χ^2 value is 342.9, while among the 100 conditional trend analyses, the median χ^2 value is 16.6. Thus where a conventional conditional trend analysis would lead us to reject the NBD, our technique shows that the lack of predictive fit is due to non-stationarity in the data. Further, once this non-stationarity has been controlled for, the NBD model provides good predictive fit, indicating that the gamma and Poisson assumptions are met.

INSERT TABLE 2 HERE

Conditional Trend Analyses

The conditional trend analyses for each simulated data set where an NBD assumption is violated are summarized in Tables 3 through 5. In each Table, the first column designates the purchase class, the second is the number of individuals who were in this purchase class for the predictor period of the first-6/second-6 analysis, and the third is the percentage predictive errror for this class. Thus a value of -10 indicates that the conditional trend analysis under-predicted purchases for that class by 10%. Finally, the fourth column contains the median percentage predictive error for that purchase class across the 100 conditional trend analyses.

<u>Spike at Zero:</u> Table 3 shows that when there is a "spike" at zero the first-6/second-6 analysis over-predicts the purchases for the zero class by 43.8%. This happens because 600 of the 972 buyers in the zero class never purchase, but the NBD model assumes they do. The over-prediction of the zero class necessitates a compensating under-prediction of other purchase classes. This occurs in the 1 and 2 classes and in many others. Finally, there is some over-prediction again for the highest two classes. The pattern of errors indicates that, in attempting to fit this non-gamma distribution, the NBD model has produced a linear prediction rule that is "too steep" such that it under-predicts the lower classes (except for the 0 class) and over predicts the higher classes. Such a pattern of errors for the "NBD with a Spike" has been discussed by Morrison and Schmittlein (1988; see Figure 3). The fourth column shows that the purchase class errors obtained in the first-6/second-6 analysis are typical of those obtained when predictor and criterion periods are selected randomly. Thus when individuals are Poisson but the mixing distribution is non-gamma the errors in conditional prediction are not substantially attentuated by the random selection of predictor and criterion periods.

INSERT TABLE 3 HERE

<u>Condensed Poisson:</u> Table 4 shows that when individuals purchase in a condensed Poisson fashion, the NBD will over-predict for purchase classes below the mean (classes 0 up to 4) and under-predict for classes above. This occurs because the NBD assumes that individuals are Poisson, not condensed Poisson, so it allows for too much regression to the mean; see Morrison and Schmittlein (1988; Figure 2). The errors for the randomly selected predictor periods in the fourth column show the same pattern of under- and over-prediction. These errors are somewhat smaller, however, because the random selection disrupts the regularity of the condensed Poisson purchases and makes consumers appear to be more Poisson-like than they are.

INSERT TABLE 4 HERE

<u>Non-stationary NBD:</u> The first-6/second-6 conditional trend analysis in Table 5 shows extreme under-prediction for purchase classes below the mean and over-prediction for classes above the mean. The non-stationarity induces much more regression to the mean than is implied by a stationary Poisson process. Individuals with high λ values for the first 6 months tend to have lower values in the second six months, while those with low λ values tend to have higher values. The purchase class data for the randomly selected predictor periods show that, while the pattern of errors is unchanged, the magnitude of the errors is severely attenuated, so the technique mostly corrects for the predictive errors induced by the non-stationarity.

Also, note that we would expect to see the pattern of errors shown in Table 5 for any case of non-stationarity as long as the mean and variance of the mixing distribution are roughly the same in both the predictor and criterion periods. For all such cases, individuals unobservable rates regress to the mean thereby causing greater regression to the mean in observed purchase frequencies than is implied by a stationary Poisson process; see Morrison and Schmittlein (1988; section 6.1).

INSERT TABLE 5 HERE

Putting together both the χ^2 and conditional trend analyses, we can summarize the way in which the technique will perform for each of four scenarios:

NBD: The χ^2 value for the first-6/second-6 analysis should fall well within the range of values obtained through the random selection of predictor and criterion periods, and on the whole these values should fall below the critical value.

Non-gamma Mixing Distribution: There should be no improvement in predictive fit due to the random selection of predictor and criterion periods; the χ^2 value for the first-6/second-6 analysis should fall within the range obtained across the 100 runs. Most of these χ^2 values should be above the critical value, indicating a failure of assumptions.

Condensed NBD: The χ^2 value for the first-6/second-6 analysis should be above the critical value, and the pattern of errors in the conditional trend analysis should indicate less regression to the mean than is implied by the Poisson assumption. The same pattern of errors should obtain in the analysis based on random selection of predictor periods; however, the level of predictive fit should be somewhat improved.

Non-stationarity NBD: The χ^2 value for the first-6/second-6 analysis should be very far above the critical value, and provided a proper adjustment of the mean purchase rate has been made, the conditional trend analysis should under-predict for purchase classes below the mean and over-predict for those above. The predictive fit should be greatly improved by the random selection of predictor and criterion periods.

A real data set might violate several assumptions at once, which would make the application of these guidelines more difficult. Nevertheless, they do serve as a baseline for evaluating the predictive fit of conditional trend analyses from real data.

5. Empirical Results

The data are from a consumer mail panel operated by NPD Research, Inc. for the year 1975. The sample consists of 3,003 households who returned completed diaries for each week during the year. We applied our technique to seven product classes.

Test of Predictive Fit

The χ^2 analyses for the seven product classes are shown in Table 6. The first six columns of this table are the same as in Table 2. The seventh contains the number of χ^2 values from the 100 runs that fall above the critical value. We can use this number to perform an hypothesis test of the gamma and Poisson assumptions, controlling for non-stationarity.

To see this, note that our 100 χ^2 values may be viewed as a sample drawn from the total population of χ^2 values determined by the set of all possible random selections of predictor and criterion periods. Due to sampling variation, some proportion of this population of χ^2 values will exceed the critical value, even when the NBD assumptions are met. Denote this proportion π. It seems reasonable to say that the NBD fails when π exceeds 0.5; that is, when more than half the population distribution of χ^2 values exceeds the critical value. Thus we can test the predictive fit of the NBD while controlling for non-stationarity by formulating the following hypothesis:

$$H_0\ \pi \leq 0.5,$$

$$H_1\ \pi > 0.5.$$

Rejection of H_0 indicates that more than half of the total population of χ^2 values exceed the critical value, hence a failure of either the gamma or Poisson assumptions. Let p be the proportion of our 100 χ^2 values exceeding the critical value. We can reject the null hypothesis at the 95% confidence level (α =0.05) when $p > p^*$, where

$$p^* = 0.5 + (0.25/100)^{1/2}\, Z_\alpha = 0.5 + (0.25/100)^{1/2}\, 1.65 = 0.5825.$$

Thus if more than 58 of 100 χ^2 values exceed the critical value, we can be 95% sure that there is a failure in either the gamma or Poisson assumption, or both.

INSERT TABLE 6 HERE

Turning now to the data, we see that the first three products in Table 6 - facial tissue, deodorant, and shampoo - seem to conform to the stationary NBD scenario. In all three cases, the χ^2 value obtained from the first-6/second-6 analysis is well within the range of those obtained through random selection of predictor and criterion periods. Thus there does not appear to be any great non-stationarity. Also for all products fewer than 58 χ^2 values fall above the critical value, so we cannot reject the gamma-Poisson assumptions.

For paper towels and cooking oil the χ^2 value for the first-6/second-6 analysis is within the range obtained from the randomly selected predictor and criterion periods, so there is not much evidence of non-stationarity. But for both products roughly 90% the χ^2 values obtained are above their respective critical values, indicating a failure of either the gamma or Poisson assumptions.

Hairspray and cereal seem to display non-stationarity but to obey the gamma-Poisson assumptions. For both products the first-6/second-6 χ^2 values are well above the range of values obtained through the random selection of predictor and criterion periods, and since neither product has more than 58 χ^2 values falling above C*, so we cannot reject the gammà-Poisson assumptions.

Conditional Trend Analyses

Table 7 summarizes the conditional trend analyses for paper towels and cooking oil. The two products display different error patterns that suggest different violations of the gamma-Poisson assumptions. For paper towels, the gamma-Poisson prediction rule seems to under-estimate the lowest and highest purchase classes and to over-estimate the purchase classes near the mean value of 4.66. This indicates that the mixing distribution is non-gamma such that the proper prediction rule should be convex upward, not linear. Cooking oil displays the pattern of prediction errors similar to those associated with condensed Poisson purchase behavior; that is, it tends to over-predict purchase classes below the mean value, which for cooking oil is 3.79, and to under-predict purchase classes above the mean. Individuals thus show less regression to the mean than the Poisson process implies. Such purchasing behavior is not surprising in a regularly consumed staple good like cooking oil. In sum, paper towels appear to violate the gamma assumption while cooking oil violates the Poisson assumption.

INSERT TABLE 7 HERE

Table 8 shows the conditional trend analyses for hairspray and cereal. The first-6/second-6 analysis for hairspray shows a large over-prediction in the 0 class and an under-prediction in the highest purchase class that are largely corrected by the random selection of predictor and criterion periods. But other than these no clear pattern of errors emerge. By contrast, the cereal errors are similar to those for the non-stationarity NBD simulation in Table 5. The first-6/second-6 analysis under-predicts virtually all purchase classes up to the mean of 15.69 and over-predicts all purchase classes above it, and some of the errors are severe. When predictor and criterion periods are selected randomly, these errors are attenuated; thus when we control for non-stationarity, the cereal data appear to meet the gamma-Poisson assumptions.

INSERT TABLE 8 HERE

Of the seven product classes, a first-6/second-6 conditional trend analysis would lead us to reject the NBD model for hairspray, shampoo, cooking oil, and cereal. However, our technique shows that of these four only cooking oil fails the gamma-Poisson assumptions; the other three apparent failures are due to non-stationarity in the data. Also, a conventional first-6/second-6 analysis for facial tissue, deodorant, and paper towels would lead us to believe that the NBD works for all three, but our technique shows that it does not for paper towels. Without the proposed technique, then, we would reach an incorrect assessment in four out of seven product classes.

6. Discussion and Conclusions

We believe this investigation has produced some useful ideas. Most importantly, we have described the role that the often implicit assumption of non-stationarity plays in predictive testing of purchase incidence models. Poor performance in predictive tests, such as that found by Goodhardt and Ehrenberg (1967), Morrison and Schmittlein (1981), and Dalal, Lee, and Sabavala (1984) could be the result of non-stationarity, not failures of the the gamma-Poisson assumptions. As this study shows, without some method of controlling for the effects of non-stationarity it is nearly impossible to say which assumptions of the model are not being met.

In the Appendix, we have presented a formal predictive test of the Negative Binomial Distribution model, such as was suggested by Morrison and Schmittlein (1981). Using this χ^2 test and a conventional predictor/criterion conditional trend analysis, a researcher can perform a joint test of the three NBD assumptions.

We have used the memoryless property of the Poisson process to devise a technique for randomly selecting predictor and criterion periods in conditional trend analysis. The technique averages whatever non-stationarity exists across the entire sample, so when the sample is large, as is typical for diary panels, the effects of non-stationarity are attenuated. With these effects controlled for, we can perform a predicitve test of the remaining gamma-Poisson assumptions. By reasoning and simulation we have shown that this method will have no effect on predictive accuracy as long as the individual level process is Poisson. When individuals behave in a condensed Poisson fashion, the random selection tends to disrupt the regularity that exists; still, the method retains discriminatory power for common sample sizes.

Prior tests of the Poisson purchase assumption, such as variance ratio test devised by Chattfield and Goodhardt (1973) must be performed on individual level data, which causes two problems: (i) each individual produces few data points so the tests have little statistical power; (ii) individual level tests can only be performed for those individuals who have made a certain number of purchases in the observation period, so they exclude much of the sample and are thus subject to selection bias. Our proposed technique is performed simultaneously on all members of the sample so it has greater power while avoiding the selection bias inherent in the variance ratio test.

Though our technique jointly tests both the gamma and the Poisson assumptions, we have shown how it can be used to determine which of these is violated - if indeed only one is being violated. Of course, both assumptions could be violated, in which case the interpretation of results is more difficult, but the technique does provide useful diagnostics when one of the NBD assumptions is not met.

Still, the proposed technique does have limitations. Most importantly, it does not control perfectly for non-stationarity. It is, essentially, a means of averaging out non-stationarity across the sample, which becomes more effective when (i) the sample size increases, and (ii) we divide the total observation period into more time units. So the technique controls *mostly* for non-stationarity. This has implications for the test of the gamma-Poisson assumptions described in section 5; that is, the test assumes that non-stationarity has been perfectly controlled for and so is somewhat conservative.

Also, while we have proposed a technique to control for non-stationarity, we have proposed no model of non-stationarity itself. With such a model, we could generalize the NBD and perhaps make more realistic predictions. For example, the cereal data display much more regression to the mean than is implied by a stationary Poisson process, suggesting that the individuals' unobservable rates, λ, are themselves regressing to the mean over time. Such a scenario could be described by a renewal process where (i) individuals purchase in a Poisson fashion with rate λ; (ii) they draw their λ values from a stationary gamma distribution; (iii) they retain their λ values for a exponentially distributed period of time. Such a gamma-Poisson-exponential model, which is the purchase incidence analog to the beta-binomial-geometric model of Sabavala amd Morrison (1981), might provide a better fit to cases like the cereal data where individual rates appear to regress over time. This model will be developed in a further study.

Finally, the primary purpose of conditional trend analysis is to set purchase class "baselines" for evaluating future purchase incidence behavior. If the gamma-Poisson assumptions are met, these baselines are useful because systematic deviations will be due to non-stationarity, which, presumably, is caused by marketing activity. But without a joint test of the gamma-Poisson assumptions we can not know how useful the NBD baselines are. Our data demonstrate the practical significance of this: a first-6/second-6 conditional trend analysis leads to an incorrect assessment in four of the seven product classes in our study, while our technique separates the effects of non-stationarity from failures in the gamma-Poisson assumptions. Thus we believe the technique can help researchers to use the NBD model in a more informed way.

APPENDIX

It is shown by Goodhardt and Ehrenberg (1967) that when purchases are NBD the conditional distribution of purchase frequencies for a criterion period, given j purchases in the predictor period, is also NBD with updated parameters $(r + j, \alpha + 1)$. This conditional distribution, $P(x_2 \mid r, \alpha, j)$, has variance

$$\mathrm{Var}\{ x_2 \mid r, \alpha, j \} = \mu_{21}((r+j)/(\alpha+1)) + \mu_{21}{}^2((r+j)/(\alpha+1)^2) \qquad (A1)$$

where $\mu_{21} = \mu_2/\mu_1$ is a scaling factor that accounts for the difference in average purchase rates between the two periods.

From this we know that the conditional sample mean in the criterion period, $x_{2|j}$, will have expectation and variance given by

$$E\{ \bar{x}_{2|j} \} = \mu_{21}(r+j)/(\alpha+1) \qquad (A2)$$

$$\mathrm{Var}\{\bar{x}_{2|j} \} = \{ \mu_{21}(r+j)/(\alpha+1) + \mu_{21}{}^2(r+j)/(\alpha+1)^2 \}/n_j \qquad (A3)$$

where n_j is the number of individuals making j purchases in the predictor period. Also, for $n_j > 30$ we know from the central limit theorem that $\bar{x}_{2|j}$ is distributed approximately normal.

The NBD conditional predictions are unbiased, if all assumptions are met. Letting $\lambda_{NBD|j}$ denote the NBD conditional prediction and using the results above we know that

$$(\lambda_{NBD|j} - \bar{x}_{2|j})/ SD\{\bar{x}_{2|j} \}$$

is distributed approximately normal with zero mean and unit variance. The square of this quantity is distributed approximately χ^2 with 1 degree of freedom. Summing across all k purchase classes we have

$$\sum_{\text{all } j} (\lambda_{NBD|j} - \bar{x}_{2|j})^2/ \mathrm{Var}\{\bar{x}_{2|j} \}$$

which is distributed approximately χ^2 with k degrees of freedom. In our application we estimate μ_{21} using the unconditional mean from the criterion period, which uses up one degree of freedom, so we have k-1 degrees of freedom in the test.

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Conditional Trend Analysis Using the NBD Model[1]

Purchase Class j	Number n_j	Prediction $E\{\lambda_{NBD} \mid j\}$	Actual $E\{x_2 \mid j\}$
0	4517	0.55	0.89
1	2019	1.24	1.05
2	1294	1.93	1.69
3	862	2.62	2.31
4	594	3.30	2.95
5	357	3.99	3.71
6	284	4.68	4.48
7+	580	*	6.84

1. Adapted from Morrison and Schmittlein (1981, Table 6, p. 1017); the product class is peanut butter.

* Value not reported by Morrison and Schmittlein.

Table 1

χ^2 Analysis - Simulated Data Sets

Data Set	d.f	C*	$\chi^2_{6/6}$	χ^2_{median}	χ^2_{low}	χ^2_{high}
NBD	14	23.7	16.7	16.1	6.2	32.3
NBD/Spike at 0	14	23.7	66.0	63.8	32.7	120.9
Condensed NBD	14	23.7	93.3	45.6	20.4	79.4
Non-stat. NBD	14	23.7	342.9	16.6	5.4	38.5

Table 2

Conditional Trend Analysis for Simulated Sample with Spike at Zero

Purchase Class	# in Class	% Error$_{6/6}$	%Error$_{median}$
0	972	+43.8[1]	+46.5[2]
1	335	-21.3	-20.8
2	294	-8.3	-8.7
3	245	+1.6	-3.9
4	199	-4.0	-0.7
------	------	------	------[3]
5	150	-2.3	-1.9
6	115	-3.9	-2.9
7	107	-7.7	-1.2
8	96	+10.0	-1.1
9	64	+5.5	-1.3
10	79	-12.6	-0.4
11	58	+11.5	-0.1
12	43	-0.5	+0.3
13	34	+2.5	+1.3
14+	208	+3.0	+3.9

Table 3

1. Should be read: for those individuals making 0 purchases in the predictor period, the NBD over-predicted their purchases in the criterion period by 43.8%; that is 100(predicted - actual)/actual = 43.8.

2. Should be read: across the 100 runs with predictor and criterion periods chosen randomly, the median over-prediction for those making 0 purchases in the predictor period was 46.5%.

3. Dotted line divides purchase classes into those below and those above the mean.

Conditional Trend Analysis for Simulated Sample with Condensed NBD

Purchase Class	# in Class	% $Error_{6/6}$	% $Error_{median}$
0	489	+47.8	+15.7
1	525	+25.8	+19.1
2	399	+15.9	+9.8
3	333	+2.6	+4.9
4	260	+1.8	+3.7
5	211	-5.8	-0.2
6	146	-5.8	-1.4
7	131	-4.3	-2.3
8	107	-2.5	-2.1
9	92	-4.1	-3.2
10	66	-17.6	-7.5
11	45	-13.7	-7.8
12	34	-6.1	-9.3
13	33	-10.1	-6.9
14+	128	-8.4	-7.3

Table 4

Conditional Trend Analysis for Simulated Sample with Non-stationary NBD

Purchase Class	# in Class	% $\text{Error}_{6/6}$	% $\text{Error}_{\text{median}}$
0	567	-48.0	-3.9
1	480	-21.3	-4.9
2	405	-18.4	-2.9
3	324	-0.9	-0.2
4	232	-2.1	+1.2
5	188	+9.8	-0.2
6	153	-2.5	+0.2
7	116	+12.0	-0.1
8	104	+11.6	+1.7
9	81	+8.3	+1.2
10	64	+18.5	+2.9
11	70	+16.5	+1.9
12	52	+20.0	+0.3
13	39	+13.2	+2.1
14+	124	+19.1	+1.8

Table 5

χ^2 Analysis - Empirical Results

Product Class	d.f	C*	$\chi^2_{6/6}$	χ^2_{median}	χ^2_{low}	χ^2_{high}	# > C*
Facial Tissue	11	19.7	13.4	14.0	1.8	33.7	14
Deod rant	7	14.1	8.9	14.1	4.0	38.0	50
Shampoo	6	12.6	14.6	10.6	1.3	43.9	38
Paper Towels	11	19.7	18.6	28.6	11.7	47.9	90
Cooking Oil	10	18.3	28.0	28.8	6.5	55.2	88
Hair Spray	5	11.1	32.7	8.6	2.5	24.1	27
Cereal	20	31.4	247.2	31.9	14.1	63.2	51

Table 6

Conditional Trend Analyses for

Products that Violate the Gamma-Poisson Assumptions

Paper Towels		
Pur. Class	% $\text{Error}_{6/6}$	% Error_{median}
0	+2.5	+2.9
1	-6.2	-7.5
2	+2.9	+0.6
3	+2.3	+4.0
4	+2.5	+5.8
5	+6.6	+6.8
6	+11.8	+6.5
7	+2.8	+5.5
8	+0.9	+2.8
9	-1.2	+1.6
10	+2.2	-0.2
11+	-5.3	-6.1

Cooking Oil		
Pur. Class	% $\text{Error}_{6/6}$	% Error_{median}
0	+15.0	+1.1
1	+3.1	+2.4
2	+1.9	+4.8
3	+9.5	+7.3
4	+3.7	+3.9
5	+1.5	+2.3
6	-2.6	+1.1
7	-5.0	+0.4
8	-1.0	-2.9
9	-1.2	-5.9
10+	-12.4	-9.9

Table 7

Conditional Trend Analyses for Products Exhibiting Non-stationarity

Cereal		
Pur. Class	% $Error_{6/6}$	$\%Error_{median}$
0	-50.1	-3.6
1	-35.0	-15.6
2	-35.2	-10.3
3	-10.8	-3.4
4	-8.8	-3.9
5	-18.0	+0.7
6	-9.6	+0.4
7	-4.2	-0.7
8	-2.1	+2.3
9	-1.9	+2.9
10	-5.8	-2.8
11	-4.3	+0.2
12	-4.6	-2.1
13	-1.2	-1.7
14	-3.5	-1.2
15	+6.1	+0.1
16	+2.4	+1.3
17	+1.5	+2.0
18	+0.4	+1.5
19	+5.2	+2.2
20+	+3.8	+0.1

Hair Spray		
Pur. Class	$\%Error_{6/6}$	$\%Error_{median}$
0	+33.4	+7.7
1	+2.1	-1.6
2	-3.1	+1.3
3	-0.5	+4.0
4	+6.5	+5.8
5+	-13.2	-6.9

Table 8

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