"METHOD OF MOMENTS TESTS OF CONTINGENT CLAIMS ASSET PRICING MODELS"

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Comments Welcome

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Abstract

This paper develops and applies formal statistical tests of contingent claims asset pricing models. The Generalized Method of Moments (GMM) estimator and the Method of Simulated Moments (MSM) estimator are used to test models that have known and unknown analytical solutions, respectively. The GMM and MSM estimators have the same asymptotic properties of consistency and asymptotic normality. Both estimators are used to test option pricing models on a panel data set of gold futures options. The traditional "exercise price" and "time–to–maturity" biases are shown to be statistically significant. A third bias, an "interest-rate" bias, is also uncovered. Violations of the assumption of a constant interest rate is a possible explanation for the rejection of the different models tested.

1 Introduction

The purpose of this paper is to develop and apply formal statistical tests of contingent claims asset pricing models. The tests are based on the Generalized Method of Moments (GMM) and Method of Simulated Moments (MSM) estimators. The former estimator is well known in the financial economics literature.[1] The latter has been applied in certain areas of economics, but its usefulness for testing asset pricing models has not been appreciated yet.[2,3]

Empirical tests of contingent claims asset pricing models involve the calculation of deviations of observed contingent claims prices from theoretical prices, referred to, henceforth, as the pricing errors. The problem is to assess their statistical significance. The errors have several potential sources. One is nonsynchroneity, i.e., the contingent claim price and the underlying security price are not observed simultaneously. Data inaccuracies and market imperfections, such as transaction costs, bid–ask spreads, generate a similar error. A second source of error emerges when the parameters of the contingent claims pricing model are estimated from historical data. This procedure introduces sampling error. A third source arises when solutions to contingent claims asset pricing models cannot be calculated analytically, and are obtained by numerical approximation. Numerical approximation procedures introduce an additional source of error. Even though these sources of errors are well acknowledged, the statistical significance of the deviations of observed from theoretical prices is difficult to assess because of the inherent nonlinearity of contingent claims asset pricing models.

The GMM and MSM estimators can be used to assess the statistical significance of deviations of observed from theoretical prices. The hypothesis that the errors originate from the first source, *i.e.*, nonsynchroneity, data inaccuracies, market imperfections, or bid–ask spreads, can be tested with

[1] See for example the papers of Hansen and Singleton (1982), Dunn and Singleton (1986), and Gibbons and Ramaswamy (1986).

[2] See for example the papers of Bong–Soo Lee (1986), McFadden (1987), and Pakes and Pollard (1986).

[3] A notable exception is Duffie and Singleton (1988) who develop a simulation estimator of the parameters of diffusion models. The differences between the present paper and theirs is discussed below.

the GMM estimator, provided the contingent claims asset pricing model has an analytical solution. The GMM tests do not require estimating the parameters from historical data. Therefore, the second source of error, *i.e.*, the sampling error of the parameter estimates, is avoided. The theoretical prices of contingent claims asset pricing models, that cannot be solved analytically, can be approximated by simulation with the following procedure. Prices of the underlying security at the maturity date of the contingent claim are drawn from their conditional distribution. Alternatively, if the conditional distribution has no known analytical expression, the underlying security prices are simulated by discrete–time approximation of the continuous–time process. For each outcome, the payoffs on the contingent claim are discounted, and then averaged. This mean value is a simulation estimate of the theoretical price of the contingent claim. MSM tests of contingent claims asset pricing models are obtained by substituting the simulation estimate for the analytically uncomputable theoretical price of the contingent claim in the GMM moment conditions.[4] The MSM estimator allows for the third type of errors, *i.e.*, those arising from the approximation of solutions to contingent claims prices.[5]

The GMM and MSM estimators are based on the mimimization of the distance of a vector of "orthogonality conditions," also known as "moment conditions," from zero. The sum of the deviations of observed contingent claims prices from theoretical prices is one such orthogonality condition. Other moment conditions can be obtained by multiplying the deviations by the value of instrumental variables. Asymptotically, the distance of the vector of orthogonality conditions from zero follows a chi–square distribution which can be used to evaluate the goodness–of–fit of the model. Hence, the chi–square test determines the statistical significance of the average deviation from model prices, and of the relationship between the deviations and the instruments. Therefore, the GMM and MSM estimators can be used to test the statistical significance of the biases previously uncovered in the contingent claims pricing literature, like for example the "exercise

[4]The simulated variable is the terminal price of the underlying security and not the price of the contingent claim. This explains why "simulation estimate" rather than "simulated estimate" is used thoughout the paper.

[5]The approximation error is a simulation and not a numerical approximation error in the traditional sense.

price" and the "time–to–maturity" biases, using the exercise price and the time to maturity as instrumental variables. The statistical significance of other potential biases, like an "interest rate" bias, can also be investigated.

In GMM and MSM estimation, the distance of the vector of orthogonality conditions is mimimized with respect to the parameters of the model. The model yields estimates of the parameters and their standard errors. The parameter estimates are implied from the model instead of being estimated from historical data, typically from a time–series of changes in the price of the underlying asset, as in Duffie and Singleton (1988). Hansen (1982) shows that the GMM parameter estimates are consistent and asymptotically normally distributed. The MSM estimator shares the consistency and asymptotic normality properties of the GMM estimator. However, the power of MSM tests to reject the null hypothesis is lower than the power of GMM tests. The variance of the simulation error explains the difference. Techniques to reduce the variance of the simulation error are introduced to minimize the number of simulations and simultaneously maximize the power of MSM tests.

Several contingent claims asset pricing models are tested with the GMM and MSM estimators on a panel data set of gold futures options. Two different versions of the Black (1976) pricing model are tested with the GMM estimator. Most contingent claims pricing models are derived under the assumption of a constant continuous–time interest rate. The matched maturity interest rate, *i.e.*, the yield on a T–bill whose maturity coincides with the maturity of the contingent claim, is generally used as a proxy for the unobservable constant interest rate. The Black model is first tested with the matched maturity interest rate. The original version of the Black model, which assumes a constant interest rate, is also tested. There, the interest rate is treated as an unknown parameter and is implied from the model. The MSM estimator is then used to test two stochastic variance contingent claims asset pricing models. The first one is the Constant Elasticity of Variance (CEV) model of Cox (1975). The second model is *ad hoc*, and constraints the conditional standard deviation of futures price changes to be proportional to the level of the futures price, unlike the CEV model which imposes *inverse* proportionality.

The paper is organized as follows. Section 2 introduces the GMM estimator, discusses its merits, and contrasts it with other estimators used

in the contingent claims pricing literature. As an illustration, the GMM estimator is used to test the Black model. Section 3 presents the MSM estimator, discusses its asymptotic properties, examines its advantages and disadvantages relative to the GMM estimator, and finally compares it to the estimator used by Duffie and Singleton (1988). Section 4 concludes the paper.

2 Tests based on the Generalized Method of Moments estimator

2.1 The null hypothesis

Various problems are associated with testing the validity of contingent claims pricing models. Tests of contingent claims pricing models are known to be tests of joint hypotheses about 1) model validity, 2) its correct parameter estimation, 3) market efficiency, 4) market synchronization, and 5) data accuracy.[6] A clear formulation of the hypotheses being tested is essential to draw reliable conclusions about the validity of the contingent claims pricing model being investigated.

In this paper, the joint null hypothesis is that 1) the contingent claims pricing model is correct, *i.e.*, the particular contingent claims pricing model being tested is the model the market uses to determine contingent claim prices, 2) markets are non–synchronous, and 3) data is accurate. The set of unknown parameters that investors employ to determine the contingent claims prices are implied from the model. The use of "implied" instead of "historical" parameter estimates solves the potential problems of incorrect parameter estimation. Also, no assumptions are made about the validity of the model given the true stochastic process followed by the price of the underlying security or any other relevant variables. In particular, the contingent claims pricing model need not be consistent with the process followed by the price of the underlying security. Consequently, no assumption

[6] See Galai (1983) and Rubinstein (1985).

is made about the efficiency of the option market.[7][8]

The joint hypothesis of model validity and data nonsynchroneity replaces the joint hypothesis of model validity and market efficiency traditionally tested in the asset pricing literature. Under the null hypothesis, the deviations of market prices from theoretical prices, *i.e.*, the pricing errors, are assumed to be be generated by nonsynchroneity in the observation of the contingent claims prices and the underlying security prices, and/or other security prices like the risk–free rate. More specifically, the pricing errors are assumed to originate from the use of closing prices in the computation of the the theoretical prices.[9] The major issue is how to empirically test the null hypothesis. The null hypothesis is shown to be expressable in terms of moment conditions imposed on the pricing error. Two sets of pricing errors are estimated and the moment conditions associated with each set are derived below.

2.1.1 Two alternative formulations of the null hypothesis

Let m be the number of contingent claims written on the same underlying security. The parameter m need not be equal to one. The parameter m is larger than one whenever contingent claims written on the same instrument differ with respect to characteristics like the time to maturity, the exercise price, or the type of contingent claim, *i.e.*, calls versus puts, for example. Contingent claims written on the same underlying security are referred to, henceforth, as a panel data set of contingent claims. This data set can be

[7] An example will clarify this point. Suppose the market assumes that the price of the underlying security follows a Geometric Brownian motion with a constant variance, *i.e.*, the market uses the Black–Scholes (BS) (1973) model to compute contingent claims prices. Suppose, however, that the true process followed by the price of the underlying security is a Geometric Brownian motion with a stochastic variance. Then, the null hypothesis that the BS model is true will not be rejected by the data. Yet, the option market is inefficient since arbitrage opportunities exist given the knowledge about the stochastic variance.

[8] The efficiency of the option market is not tested in the paper. Tests based on trading strategies are better suited for that purpose. The tests performed in this section correspond to what Geske and Trautman (1985) define as "tests of unbiasedness", that is tests which focus on identifying systematic behavior in the prediction error of the valuation models.

[9] Deviations may also stem from data inaccuracies or from market imperfections, such as transaction costs or bid–ask spreads.

pictured as a $(T \times m)$ matrix where T is the number of contingent claims prices and m is the number of contingent claims.

Let Ω be a set of characteristics that differentiates the contingent claims written on the same underlying instrument, and $\underline{\omega}_{it}$ be the vector of characteristics associated with contingent claim i at time t. The row vector $\underline{\omega}_{it}$ includes the time to maturity, and the exercise price of contingent claim i at time t, denoted by τ_{it} and x_{it}, respectively. The row vector $\underline{\omega}_{it}$, may also contain additional characteristics. Most contingent claim asset pricing models are derived under the assumption of a constant continuous–time interest rate R. One property of R is to be unobservable. The matched maturity interest rate, namely the yield at time t on a T–bill whose maturity coincides with the maturity of contingent claim i, denoted by r_{it}, can be used as a proxy for the unobservable constant continuous–time interest rate R. The contingent claims asset pricing model model can be tested under the assumption that investors use the matched–maturity interest rate r_{it} instead of the unobservable R. In that case, $\underline{\omega}_{it}$ includes r_{it}.[10] The row vector $\underline{\omega}_{it}$ is, therefore, defined as,

$$\underline{\omega}_{it} = (\tau_{it}, x_{it}, r_{it}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T]. \tag{1}$$

Unlike the first two characteristics, the risk–free rate, r_{it}, is generally observed with error.[11] The observable risk–free rate is denoted by r^o_{it}, and the row vector $\underline{\omega}^o_{it}$ is defined as,

$$\underline{\omega}^o_{it} = (\tau_{it}, x_{it}, r^o_{it}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T]. \tag{2}$$

Alternatively, the constant continuous–time interest rate, R, can be treated as an unknown parameter and be inferred from the model. In that case, the row vector $\underline{\omega}'_{it}$ is defined as,

$$\underline{\omega}'_{it} = (\tau_{it}, x_{it}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T]. \tag{3}$$

Let s_t and s^o_t denote the theoretical price and the observable price of the underlying security at time t, respectively. Let g be a functional relationship between the contingent claim price, the price of the underlying security

[10] The use of r_{it} instead of R is correct only if the term structure is flat.

[11] This error emerges because it is not possible to find a bond that matures the same day and at the same time as contingent claim i. The difference between r_{it} and r^o_{it} represents a fraction of the nonsynchroneity error.

and the vector of characteristics. The functional relationship g corresponds to the contingent claims pricing model that investors use to determine contingent claims prices. Finally, let c_{it} and c_{it}^o denote the theoretical price and the market price of contingent claim i at time t.

The theoretical price of contingent claim i at time t, which is a function of the observed variables s_t^o and r_{it}^o at time t, is equal to,

$$c_{it} = g(s_t^o, \underline{\omega}_{it}^o; \underline{\theta}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T], \tag{4}$$

where $\underline{\theta}$ is a vector of parameters that remain constant across contingent claims and over time.[12] Similarly, the market price of contingent claim i at time t, which is a function of the unobservable variables s_t and r_{it} at time t, is equal to

$$c_{it}^o = g(s_t, \underline{\omega}_{it}; \underline{\theta}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T]. \tag{5}$$

The pricing error associated with contingent claim i at time t can be estimated as,

$$\begin{aligned} \epsilon_{it} &= c_{it}^o - c_{it}, \\ &= c_{it}^o - g(s_t^o, \underline{\omega}_{it}^o; \underline{\theta}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T]. \end{aligned} \tag{6}$$

The pricing error can also be expressed as a function of the price of the underlying security. By defining the implied stock price, s_{it}^{imp}, which unlike s_{it} is indexed by i, as,

$$s_{it}^{\text{imp}} = g^{-1}(c_{it}^o, \underline{\omega}_{it}^o; \underline{\theta}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T], \tag{7}$$

where g^{-1} is the inverse function of g if g is invertible, the percentage deviation associated with contingent claim i at time t can be estimated as,

$$\begin{aligned} \epsilon_{it}' &= \log s_t^o - \log s_{it}^{\text{imp}}, \\ &= \log s_t^o - g^{-1}(c_{it}^o, \underline{\omega}_{it}^o; \underline{\theta}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T]. \end{aligned} \tag{8}$$

As shown below, the formulation of the moment conditions in terms of the first difference of the ϵ_{it}''s justifies the *log* transformation. Equations (6) and

[12]For example, when the Black–Scholes model is tested, $\underline{\theta}$ includes the variance of the underlying security. Further, if the constant continuous interest–rate of the Black–Scholes model is treated as an unknown parameter, $\underline{\theta}$ also includes R.

(8) suggest two possible approaches to test the null hypothesis. The first approach is based on a pricing error estimated as the difference between the observed and the theoretical contingents claim prices. The second approach is based on a pricing error estimated as the difference between the the observed and the implied theoretical underlying security prices. The benefits and limitations of these two approaches and their complementarity are discussed next.

2.1.2 Tests of the null hypothesis based on contingent claims prices

The pricing error is estimated from equation (6) as the difference between the observed and the theoretical contingent claim prices. The functional form that investors use to determine contingent claim prices is assumed to be known and the following subsequent assumptions are made.

- Assumption 1: The error is assumed to follow a stationary stochastic process. Stationarity is achieved when 1) the unconditional mean is equal to a constant, 2) the unconditional second moment, *i.e.*, the unconditional variance, is also equal to a constant, and 3) the autocovariances are equal to constants independent of time.

- Assumption 2: The first unconditional moment is assumed to be equal to zero,
$$E[\epsilon_{it}] = 0, \qquad \forall i \in [1, m], \tag{9}$$
that is the average nonsynchroneity is assumed to be equal to zero, where $E[\cdot]$ denotes the expectation operator.

- Assumption 3: Let Z be a set of L instrumental variables, where z_{it}^j denotes the value of instrumental variable j associated with contingent claim i at time t. The unconditional second cross-moments,
$$E[\epsilon_{it}\, z_{it}^j] = 0, \qquad \forall i \in [1, m], \quad \forall j \in [1, L], \tag{10}$$
are assumed to be equal to zero.[13] The variables in the set Ω can be employed as instrumental variables, though other variables can also

[13]Note that the instruments are not demeaned. The tests can also be performed using demeaned instruments, however.

be used.

The implications of the above assumptions need to be carefully examined. Assumptions 2 and 3 state that the pricing errors have mean zero and cross–moments zero. Suppose the null hypothesis is rejected. Assuming away data inaccuracies, the model can be rejected because investors use a different function g to price contingent claims than the one being investigated and/or because nonsynchroneity between s_t^o and s_t and/or r_{it}^o and r_{it} is not correctly specified. From assumptions 2 and 3, nonsynchroneity is hypothesized to be non–systematic on average and to be non–systematically related to the selected instrumental variables. The null hypothesis might be rejected because nonsynchroneity is systematic instead of non–systematic.

There are quite a few reasons to suspect nonsynchroneity to be systematic. Nonsynchroneity is likely to be systematic if contingent claim prices are systematically observed with a time lag with respect to the underlying security prices. For example a non–zero mean price change of the underlying security is likely to induce a non–zero mean nonsynchroneity error.[14] Similarly, nonsynchroneity may be systematically related to some instruments. For example, the time lag between the contingent claims prices and the underlying security prices may be related to the exercise price. The assumption that nonsynchroneity is non–systematic is one weakness of this approach. This assumption can, however, be relaxed by estimating the errors as the difference between the observed and the implied theoretical prices of the underlying security.

2.1.3 Tests of the null hypothesis based on the underlying security prices

The pricing error is estimated from equation (8) as the difference between the observed and the implied theoretical underlying security prices. Two

[14]Suppose that instead of observing the true price of the underlying security at say 5 o'clock, the econometrician only observes the closing price at 2 o'clock, a three–hour difference. If the mean return on the security is positive, the underlying security price should drift upward slightly during that time period. If all the parameters of the contingent claim remain constant over this three–hour period, the estimate of the contingent claim price at 5 o'clock will be below the investors' estimate on average. Hence, the estimate is biased.

of the previous assumptions are maintained. Namely, it is assumed that 1) the functional form that investors use to determine contingent claims prices is known, and 2) the error, ϵ'_{it}, follows a stationary stochastic process. The assumptions concerning the first moment and the second cross-moments are modified to allow for the presence of systematic nonsynchroneity. They are reformulated as follows:

- Assumption 2': The first unconditional moment is equal to a constant denoted by γ_i,

$$E[\epsilon'_{it}] = \gamma_i, \qquad \forall i \in [1, m]. \tag{11}$$

- Assumption 3': The second unconditional cross-moments are equal to the constants ς_i^j,

$$E[\epsilon'_{it}\, z_{it}^j] = \varsigma_i^j, \qquad \forall i \in [1, m], \quad \forall j \in [1, L]. \tag{12}$$

The constant γ_i can be easily eliminated from equation (11). By taking the first difference, the first moment condition can be rewritten as,

$$E[\epsilon'_{it} - \epsilon'_{it-1}] = 0, \qquad \forall i \in [1, m]. \tag{13}$$

Provided the unconditional mean percentage change in the price of the underlying security is equal to the constant μ,

$$E[\log s_t^o - \log s_{t-1}^o - \mu] = 0, \tag{14}$$

the substitution of equations (8) and (14) in (13) yields the following moment condition,[15]

$$E[\log s_{it}^{\text{imp}} - \log s_{it-1}^{\text{imp}} - \mu] = 0, \qquad \forall i \in [1, m]. \tag{15}$$

However, it is not true that the second cross-moment conditions are equal to,

$$E[(\epsilon'_{it} - \epsilon'_{it-1}) z_{it-1}^j] = 0, \qquad \forall i \in [1, m], \quad \forall j \in [1, L], \tag{16}$$

[15]This explains why in equation (8), the pricing error is estimated as the difference between the *log* of the observed price and the log of the implied price of the underlying security. Without the log transformation, $\log s_{it}^{\text{imp}} - \log s_{it-1}^{\text{imp}}$ would not be equal to μ.

after taking the first difference. Hence, the second cross-moment conditions *cannot* be rewritten as,

$$E[(\log s_t^o - \log s_{t-1}^o - \mu) z_{it-1}^j] = 0, \quad (17)$$

$$E[(\log s_{it}^{\text{imp}} - log s_{it-1}^{\text{imp}} - \mu) z_{it-1}^j] = 0, \quad \forall i \in [1, m], \quad \forall j \in [1, L]. \quad (18)$$

The estimation of the null hypothesis based on the underlying security prices instead of the contingent claims prices offers two major benefits. First, nonsynchroneity is not constrained to be non-systematic. Second, an estimate of the expected return on the underlying security, μ is obtained. In past empirical tests of contingent claims pricing models, one parameter of the underlying security distribution *at most*, in general the variance, is implied from the contingent claims pricing model. Here, an additional parameter, *i.e.*, the mean change of the price of the underlying security, is implied from the pricing model. If the benefits of testing contingent claims asset pricing models with the underlying security instead of the contingent claims are obvious, a certain number of problems have first to be solved. These problems pertain to 1) the conditions under which the cross-moment conditions can be tested, and 2) the power of the tests to reject the null hypothesis. An additional moment condition should be added to increase the presumed low power of the tests. However, this moment condition must have very specific properties. These issues are left for future research.

Regardless of how pricing errors are estimated, the null hypothesis is specified in terms of moment conditions. The null hypothesis can, therefore, be conveniently tested with the Generalized Method of Moments estimator of Hansen (1982) which is introduced below.

2.2 Testing contingent claim asset pricing models with the Generalized Method of Moments estimator

2.2.1 The Generalized Method of Moments estimator

The essence of the GMM estimator is succintly presented here. Let y_t, z_{1t}, and z_{2t} be three time-series, with $t = 1, \ldots, T$, where y_t, z_{1t}, and z_{2t} represent a dependent variable, and two instrumental variables, respectively. Let β be an unknown parameter and $g(z_{1t}; \beta)$ be a known function not

necessarily linear in z_{1t} and β. Let e_t be an unobservable stochastic process defined as,

$$e_t = y_t - g(z_{1t};\beta), \qquad \forall t \in [1,T], \tag{19}$$

and suppose that the following two moment conditions,

$$E(z_{1t}\, e_t) = 0, \tag{20}$$

$$E(z_{2t}\, e_t) = 0, \tag{21}$$

are imposed on the model.[16]

Least squares can be used to estimate β. By minimizing,

$$\sum_{t=1}^{t=T} e_t^2, \tag{22}$$

with respect to the parameter β, one obtains,

$$\sum_{t=1}^{t=T} g'(z_{1t};\beta)\, e_t = 0, \tag{23}$$

where $g'(z_{1t};\beta)$ denotes the first derivative of $g(\cdot)$ with respect to β. Equation (23) can be viewed as a sample moment condition. However, unless $g(z_{1t};\beta)$ is linear in z_{1t} and β, as with the linear least squares estimator, this sample moment condition, does not correspond to (20). Moreover, regardless of the linearity or non–linearity of $g(\cdot)$, the least squares estimator fails to impose the second moment condition.

The problem is to determine the parameter β by imposing simultaneously the two moment conditions (20) and (21). This problem does not always have a solution. In finite samples, no beta will ever solve simultaneously both moment conditions, since the number of moment conditions exceeds the number of parameters. The model is said to have one overidentifying restriction. In finite samples, the problem can be solved by forming a vector composed of the two sample moment conditions,

$$h = \begin{bmatrix} \sum z_{1t} e_t \\ \sum z_{2t} e_t \end{bmatrix}, \tag{24}$$

[16] In principle, GMM works directly with the errors, the e_t's. Everything else are instrumental variables including y_t.

and by minimizing the length of this vector with respect to the parameter β, *i.e.*, by minimizing the distance of this vector from zero. This distance is defined by the quadratic norm,

$$\|h\| = h' A h, \tag{25}$$

where A is a weighting matrix, possibly depending on sample size. The optimal weighting matrix, that is the matrix that minimizes the standard errors of the estimated parameters, is S^{-1}, the inverse of the variance covariance matrix of the sample moments, mean–adjusted or not, adjusted for possible autocorrelation and heteroskedasticity. Intuitively, this matrix weights each moment condition by its precision.

Hansen (1982) demonstrates that the estimated β converges almost surely to the true β, even when the identity matrix is used as the weighting matrix. Moreover, the estimated β obeys a central limit theorem, *i.e.*, is asymptotically normally distributed. The variance covariance matrix of the errors in beta is equal to $(d_o' S^{-1} d_o)$, where d_o is the expectation of the matrix of partial derivatives of the moment conditions with respect to the parameters, evaluated at the true β. Intuitively, d_o measures the sensitivity of the moment conditions to small changes in the parameters.[17] Hansen (1982) also provides a "goodness–of–fit" statistic which can be perceived as a test of the overidentifying restriction, or alternatively, as a test of the statistical significance of the distance h from zero. He shows that $(T h' S^{-1} h)$ converges weakly to a chi–squared variable with number of degrees of freedom equal to the number of overidentifying restriction.[18]

GMM can be viewed as a generalized non–linear instrumental variables procedure. The GMM estimator is particularly suited to test contingent claims asset pricing models, since these models are all non–linear functions of the parameters in the pricing formulas. Further, as the previous section indicates, tests of contingent claims pricing models can be easily formulated in terms of moment conditions. Therefore, GMM can be used in a

[17]Entries of d_o close to zero mean that changing the parameters will not change the quadratic form $(h' S^{-1} h)$ very much. This situation is similar to a flat likelihood function in maximum likelihood analysis.

[18]When the number of parameters is equal to the number of orthogonality restrictions, then the number of degrees of freedom and the χ^2 statistic are equal to zero, since there exists a parameter value for β that satisfies simultaneously all the orthogonality conditions.

straightforward way to test the two null hypotheses defined in the previous section by minimizing with respect to the unknown parameters the distance from zero of the vector formed of *all*, or *any subset*, of the sample moment conditions. The parameters and the instrumental variables remain to be determined. This issue is examined in a subsequent section since the relevant parameters and the optimal instrumental variables are specific to the model being tested.

2.2.2 Benefits of using the GMM estimator

The benefits of using the GMM estimator to test contingent claims asset pricing models are numerous. The comparison of the GMM procedure with the two recent statistical methodologies of Rubinstein (1985) and Lo (1986) will illustrate this point. These papers have in common to suggest different ways of testing the statistical significance of deviations of observed contingent claims prices from theoretical prices.

The GMM approach can be viewed as a generalization in several respects of the binomial tests suggested by Rubinstein (1985). His null hypothesis is that the Black–Scholes model yields unbiased contingent claims prices. To test the null hypothesis, Rubinstein selects two "instrumental" variables to construct pairs of contingent claims. These two variables are the time–to–maturity and the ratio of the underlying security price to the exercise price of the contingent claims.[19] The contingent claims in each pair satisfy the following requirement. They are written on the same underlying security but differ with respect to one of the two instruments, *i.e.*, the time–to–maturity or the exercise price. Using the Black–Scholes model, Rubinstein then computes two estimates of the implied volatility, one from each of the two contingent claims and repeats the procedure for each pair. The probability that the contingent claim with say the shorter maturity has a higher implied volatility than the contingent claim with the longer maturity should be equal to 1/2 under the null hypothesis, provided the mean and the median pricing errors are equal. More generally, the null hypothesis that the market uses the BS model to price contingent claims

[19] He tests the statistical significance of two biases known in the option literature as the "time–to–maturity" bias and the "exercise price" bias. See the summary papers of Galai (1983) and Geske and Trautman (1985).

should be rejected if a systematic relationship is found between the implied volatility estimates associated with each pair and any of the two instrumental variables. The null hypothesis is tested with non-parametric tests which make no assumptions about the data generating process apart from the symmetry of the underlying distribution.[20]

Rubinstein's approach can be extended and substantially improved in three directions with a slightly more stringent parametric assumption. The parametric assumption concerns the pricing errors, namely the ϵ_{it}'s of equations (6) or (8), which are hypothesized to be stationary. The assumption is satisfied when 1) the unconditional mean of the pricing errors is equal to a constant, 2) the pricing errors are unconditionally homoskedastic, and 3) the autocovariances are equal to constants independent of time. No additional assumptions are required. The GMM procedure yields autocorrelation-adjusted weighting matrices and standard errors when the pricing errors are correlated over time. Further, the pricing errors can be heteroskedastic, conditional on the information set at time $t-1$, and they can be non-normal. Finally, unlike the requirement of Rubinstein's sign test, the distribution of the pricing error need not be symmetric.

The first improvement is related to the way and the number of biases that are being tested. Rubinstein tests separately the statistical significance of the time-to-maturity and exercise price biases. His methodology fails to take into account the possible dependence between the biases. This can be a problem as the evidence in Geske and Roll (1984) indicates.[21] Second, his methodology cannot be easily adapted to test the existence of other potential biases, like for example an "interest rate" bias. The GMM approach is more general. The most important statistic associated with the GMM estimator is the χ^2 statistic testing the significance of the overidentifying restrictions. This statistic is essential to assess the significance of biases. It is valid for a whole set of potential biases *individually* or *simultaneously*, and can explicitly take into account the sample dependency between the

[20]The validity of the critical assumption, that the mean is equal to the median, was not investigated.

[21]Geske and Roll (1984) find that when the time-to-maturity, the exercise price, and the volatility biases are tested simultaneously instead of independently that the sign of the time bias is reversed from what has been previously observed.

different biases. Additional biases can be easily explored by employing new instrumental variables. As initially suggested by Rubinstein, macroeconomic variables could be such instruments.

The second improvement concerns the sample size. The non–parametric tests of Rubinstein are based on pairs of contingent claims. The GMM tests of biases with respect to instrumental variables are not constrained to a sample of pairs. The GMM tests are performed on a panel data set of contingent claims, namely on all the contingent claims written on the same underlying instrument regardless of their nature, *i.e.*, calls or puts, and of their characteristics like the time–to–maturity or exercise price.

The third improvement deals with the number of parameters and the information about the parameters. Rubinstein's procedure yields one parameter, namely the volatility of the underlying security. The volatility is estimated to achieve the best fit between the contingent claims model and the data. No estimate of the precision of this implied volatility parameter estimate can be computed. The GMM procedure yields the estimates of the parameters of the assumed model used by investors to price contingent claims and also their corresponding heteroskedasticity–adjusted standard errors. As shown in the previous section, the parameter estimates and their standard errors are derived by minimizing equation (24) with respect to the parameters. Further, when the pricing errors are computed from the underlying security prices, as in equation (8), the unconditional mean change in the price of the underlying security, μ, can also be estimated. Like the implied volatility, the implied expected return estimate is likely to be more precise than any historical estimate computed from a time series of price changes of the underlying asset. Finally, the continuous time constant interest rate, R, can also be estimated with the GMM procedure.

The GMM tests can be viewed as a pseudo–parametric way to extend the binomial tests proposed by Rubinstein. They significantly differ from the tests of contingent claims pricing models developed by Lo (1986). Lo suggests a test of contingent claims asset pricing model by appealing to asymptotic statistical theory. The parameter estimates are obtained independently of the contingent claims model and contingent claims data. His only assumption involves the stochastic specification of the underlying asset price, namely a general diffusion process. The test of his null hypothesis is a joint test of the contingent claims asset pricing model and of the associated

price dynamics. His null hypothesis is that the pricing error is generated by sampling error in the estimate of the historical volatility.

Lo's procedure can be used to test contingent claims asset pricing models only under certain conditions. Lo assumes that the pricing error, calculated as the difference between the observed and theoretical contingent claims prices, is *solely* generated by the sampling error in the estimate of historical volatility. Given that 1) the contingent claims are written on the same underlying security, and 2) one single historical volatility estimate is computed, the pricing errors at time $t, t+1, t+2, \cdots, t+\tau$, with $T = t+\tau$, will all be a function of the sampling error in the historical volatility estimate. Given a sample of T pricing errors, the knowledge of any pricing error, say the first, can be used to predict the remaining $(T-1)$ pricing errors. The effective sample size is not equal to T but to 1. Inferences about the validity of contingent claim pricing models drawn from tests performed on one observation are questionable. There are several ways to circumvent the degeneracy property of the pricing errors. One way is to assume that the pricing errors have other sources than the sampling error in the estimate of historical volatility. Nonsynchroneity is one such source. A second way is to perform cross-sectional tests that involve different underlying securities. A third way is to calculate at each date t a different estimate of the historical volatility of the underlying asset using a "rolling" procedure, and obtain a time-series of T different volatility estimates. The issue of the degeneracy of the pricing errors needs to be solved before Lo's methodology can be used to test contingent claims asset pricing models.

These problems are avoided with the GMM procedure. The pricing error is not assumed to be generated by the sampling error in the estimate of historical volatility but from nonsynchroneity. Though arbitrary to a certain extent, this assumption about the origins of the pricing error is not unreasonable. Under this assumption, the asymptotic properties of the parameter estimates and a goodness-of-fit statistic can be easily derived.[22]

[22]Kalaba and *al.* suggest another procedure to test contingent claims asset pricing models. Their estimation procedure is based on a linearisation of the highly non-linear contingent claims pricing models. The linearisation introduces an approximation error which obscures inferences about the validity of the contingent claims pricing model being tested.

2.3 An empirical application

2.3.1 Description of the model

The GMM procedure can be used to test any contingent claims asset pricing model. As an illustration, GMM is applied to test a European option pricing model on a futures contract.[23] The null hypothesis is that investors employ the Black (1976) model to price European call options on futures contracts and markets are nonsynchronous.[24]

Since the underlying instrument is a futures instead of a spot contract, the notation of section 2.1 needs to be slightly modified. Let F be the set of all the futures contracts written on the same spot contract possibly differing with respect to the time to maturity and/or the futures price. The sample is constructed by randomly drawing m call options written on the set of futures contracts F at every instant of time t. Any element in this sample is indexed by i and t with $i = 1, \ldots, m$, and $t = 1, \ldots, T$. The parameter m has to be smaller than or equal to the minimum number of call options written on the set of futures contracts F traded at any point in time t.[25] Let f_{it} and f_{it}^{o} denote the unobservable and the observable price of futures contract i at time t. Unlike s_t of section 2.1, the underlying instrument, is indexed by i since more than one futures contracts are generally written on any spot contract.

The functional form $g(\cdot)$ used by the market to price European call options on futures contracts is assumed to be the Black model, that is,

$$\begin{aligned} c_{it}^{o} &= g(f_{it}, \underline{\omega}_{it}; \underline{\theta}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T], \\ &= [f_{it}\mathcal{N}(d_1) - x_{it}\mathcal{N}(d_2)] \exp^{-r_{it}\tau_{it}}, \end{aligned} \tag{26}$$

where c_{it}^{o} is the observed price of the call option with exercise price x_{it}, and time–to–maturity τ_{it}, assuming that 1) the unobservable price of the underlying futures contract at time t is equal to f_{it} and 2) the continuous–time

[23]Options on futures are examined in the papers of Brenner and *al.* (1985), Ramaswamy and Sundaresan (1985), and Ball and Torous (1986). Gold futures options are investigated by Bailey (1987).

[24]No assumption is made about the stochastic process driving futures prices.

[25]Otherwise, missing observations in one of the m series of call prices will be obtained at some dates t.

constant interest rate is proxied by r_{it}, the yield on a Treasury–Bill whose maturity matches the maturity of the option. The Normal distribution function is denoted by $\mathcal{N}(\cdot)$, and d_1 is equal to,

$$d_1 = \frac{\log(f_{it}/x_{it}) + \sigma^2 \tau_{it}/2}{\sigma\sqrt{\tau_{it}}},$$

and d_2 is equal to $d_1 - \sigma\sqrt{\tau_{it}}$.

The vector of parameters $\underline{\theta}$ is composed of σ, the volatility of the underlying futures contract. Also, when the pricing errors are computed from the underlying futures prices instead of the futures options prices, $\underline{\theta}$ includes μ, the unconditional mean percentage change in the futures price. The parameter μ is assumed to be constant across the set of futures contracts F. The assumption that prices of futures contracts written on the same spot contract exhibits the same drift is consistent with the assumption underlying the Black model that interest rates are nonstochastic.

In the above formulation of the Black model, the constant continuous interest rate R is proxied by the matched maturity interest rate r_{it}, and is treated as a known variable. An alternative way to test the Black model is to treat the constant continuous interest rate R as an unknown parameter and to imply it from the model,

$$\begin{aligned} c^o_{it} &= g(f_{it}, \underline{\omega}'_{it}; \underline{\theta}'), \qquad \forall i \in [1,m], \quad \forall t \in [1,T], \\ &= [f_{it}\mathcal{N}(d_1) - x_{it}\mathcal{N}(d_2)]\exp^{-R\tau_{it}}, \end{aligned} \tag{27}$$

where, $\underline{\omega}'_{it}$ contains the two known variables x_{it} and τ_{it}, and $\underline{\theta}'$ contains the two unknown parameters σ and R.

Tests of the Black model where the constant continuous interest rate R is implied from the model may be considered to be more valuable than those using r_{it} as a proxy for R. This judgement may be incorrect, however. It is true that the use of r_{it} istead of R in the computation of the theoretical option prices is inconsistent with the Black model when the term structure of interest rates is not flat. However, this model may be consistent with contingent claims pricing models derived under the assumption of a *stochastic* interest rate. Under the five following assumptions,

- Assumption 1: The continuous–time interest rate follows the mean–reversion model of Cox, Ingersoll, and Ross (1985).

- Assumption 2: The default–free bonds are priced according to the local expectations hypothesis, *i.e.*, their instantaneous expected returns are equal and equal to the continuous–time interest rate.

- Assumption 3: Futures prices follow Geometric Brownian motions, as in the Black model.

- Assumption 4: Instantaneous changes in the continuous–time interest rate are uncorrelated with instantaneous changes in the futures prices.

- Assumption 5: The futures contracts do not have any costs–of–carry.

a stochastic interest rate version of the Black model is obtained by substituting r_{it} for R.[26] Stated differently, equation (26) correctly prices a gold futures call option under the hypothesis of stochastic interest rates.

The null hypotheses presented in section 2 are specified in terms of moment conditions. The pricing errors are hypothesized to have mean zero. Also, the pricing errors, are hypothesized to be uncorrelated with instrumental variables. The most efficient choice of instrumental variables is a difficult issue. From a GMM perspective, the optimal instruments, namely those maximizing efficiency, can be found by taking the first derivatives of the pricing formula $g(\cdot)$ with respect to the parameters in the vector $\underline{\theta}$.[27] These derivatives are known to be a function of the exercise price, the time–to–maturity, and the interest rate, *i.e.*, all the known parameters in the set Ω. Substituting these variables for the derivatives roughly approximates the optimal instruments.[28] Therefore, obvious choices for the instruments are the time–to–maturity, the exercise price, and the interest rate. To satisfy the GMM requirement of stationarity, the ratio of the exercise price to

[26]The proof is available from the authors upon request.

[27]For a certain instrument matrix, there is a close relationship between GMM estimation and maximum likelihood estimation. Chamberlain (1983) demonstrates that in an *i.i.d.* setting, GMM estimation using this instrument matrix is equivalent to non–parametric maximum–likelihood estimation, where the probability distribution of the data is estimated non–parametrically with the parameters of interest. However, these instruments cannot be used in practise because the matrix of instruments depends on the underlying parameters in the vector $\underline{\theta}$ which are unknown.

[28]The first derivative of g with respect to the parameters is a function of the instruments. The linear approximation of this function for values of the instruments equal to zero yields the moment conditions.

the price of the underlying futures, *i.e.*, $\log(f_{it}^0/x_{it})$, is used as instrumental variable instead of the exercise price. The first two instruments have been extensively studied in the option literature and have been shown to be highly correlated with the pricing errors. These correlations are known as the "time–to–maturity" bias and the "exercise price" bias.[29] The third instrument, namely the interest rate, has not been investigated. Yet, it should be used as an additional instrument since the first derivative of g depends on it.[30]

2.3.2 The data

The GMM estimator is used to test the Black (1976) futures options pricing model on a panel data set of Comex daily gold futures options prices.[31] The daily closing prices of options on gold futures contracts and the corresponding gold futures contracts are collected from the WSJ for the 210 trading days in the time period extending from July 1st 1983 to April 29 1984. The mid–afternoon T–bill rates with a closing matching maturity are also collected from the WSJ. Three series of options are formed by drawing every trading day the first three quoted gold futures options

[29]It must be added, though, that these tests are to a large extent inconclusive, since these biases are tested using ordinary t–statistics which fail to take into account obvious properties of the pricing errors such as errors–in–variables and heteroskedasticity. Rubinstein is a major exception, since his tests are non–parametric. More importantly, the statistical significance of biases, like the "exercise price" bias and the "time–to–maturity" bias, is traditionally assessed by regressing with *linear* procedures, such as linear ordinary least squares, the "implied standard deviation" on instruments like the exercise price and the time–to–maturity. However, the implied standard deviation is *non–linearly* related to the instruments. The use of linear regression techniques is inaccurate. Though acknowledged in the literature, see Geske and Roll (1984), footnote 3 page 448, the effects of this inaccuracy on the tests of contingent claims asset pricing models do not seem to have been investigated.

[30]Another famous bias in the contingent claims pricing literature is the so–called "variance" bias, namely the existence of a systematic relationship between the pricing errors and the variance of the underlying securities. See the survey papers of Galai (1983) and Geske and Trautman (1985). The existence of this bias is not tested here since all the options used in this analysis are written on the same underlying security. Nonetheless, the variance bias can be tested with the GMM estimator by devising cross–sectional instead of time–series tests. See Bossaerts and Hillion (1988)

[31]We would like to thank W. Bailey for kindly providing us with the data.

differing with respect to the time–to–maturity or the exercise price. The number of series, *i.e.*, m=3, is a compromise between computational ease and having enough power to reject the null hypothesis. Given the way the dataset is constructed, the sample exhibits substantial variation with respect to the time–to–maturity but little with respect to the exercise price.[32] A (210×3) matrix of prices is, therefore, obtained. The means, variances, and first–order serial correlation of call prices, call price changes and of the three time–series corresponding to the instrumental variables, namely, r_{it}, $\log(f_{it}^{o}/x_{it})$, and r_{it}, with $t = 1, \ldots, 210$, appear in table 1.[33]

2.3.3 Results of the tests

The results of two sets of tests, corresponding to the cases where the interest rate is treated as a known or unknown parameter are presented below.

<u>Case 1</u>: The matched–maturity interest rate is used as a proxy for the constant interest rate

The interest rate is first treated as a known parameter, *i.e.*, is included in the set Ω and excluded from $\underline{\theta}$. The volatility of the gold futures contract is the only unknown parameter in $\underline{\theta}$. For each of the three series of contingent claims, the following four orthogonality conditions are tested,

$$E(\epsilon_{it}) = 0, \quad \forall i \in [1,3], \tag{28}$$

$$E[\epsilon_{it}(r_{it} - E(r_{it}))] = 0, \quad \forall i \in [1,3], \tag{29}$$

$$E[\epsilon_{it}(\log(f_{it}^{o}/x_{it}) - E(\log(f_{it}^{o}/x_{it})))] = 0, \quad \forall i \in [1,3], \tag{30}$$

$$E[\epsilon_{it}(r_{it} - E(r_{it}))] = 0, \quad \forall i \in [1,3], \tag{31}$$

[32]The small variation with respect to the exercise price is a potential problem to test the existence of the so–called "exercise price" bias. Yet, as the subsequent sections indicate, the model is rejected when the exercise price is taken as instrumental variable. In a future version of the paper, the sample will be extended to include five or six daily observations randomly drawn from the set of all put and call options written on the same underlying security differing with respect to the time–to–maturity and/or the exercise price.

[33]GMM requires the instruments to be stationary. This property is checked by estimating the first serial correlation coefficients of the three instrumental variables.

where the pricing error ϵ_{it} is estimated as,

$$\begin{aligned}\epsilon_{it} &= c^o_{it} - c_{it}, \\ &= c^o_{it} - g(f^o_{it}, \omega^o_{it}; \sigma), \qquad \forall i \in [1,3], \quad \forall t \in [1,210],\end{aligned}$$

with $g(\cdot)$ given by the Black model. The orthogonality conditions are first tested one by one, then two by two, three by three and finally jointly. The results appear in table 2.

The results obtained for the four orthogonality conditions tested separately are first examined. The number of degrees of freedom is equal to 2, since the GMM tests are performed on three series of options and only one parameter is estimated. The probability value associated with the χ^2 test indicates whether the model is rejected or not. Table 2 reveals that the Black model is rejected at p–levels exceeding .999 regardless of the moment condition. The empirical evidence in table 2 confims the presence of the "time–to–maturity" and "exercice price" biases uncovered in past empirical tests of contingent claims asset pricing models.[34] The results obtained for the last cross–moment condition, where r_{it} is used as instrumental variable, also support the existence of an "interest rate" bias, apparently left unnoticed in the literature. The volatility parameter estimate of the underlying futures contract, σ, varies between .167 and .170 across moment conditions. The estimates are extremely tight.[35]

The finding that the instrumental variables are correlated with the pricing error is consistent with the hypothesis that the pricing error is generated by nonsynchroneity. This conjecture is correct only if nonsynchroneity has

[34]These results are consistent with the findings of Bailey (1987) for gold futures options. He reports that the model overprices in–the–money calls by \$3.59 on average and out–of–the money calls by \$1.97. Also, the model appears to generate larger errors for longer maturity options and is less accurate for at–the–money options than for other options. Also, he finds that the model's errors are strongly correlated with the time to expiration of calls and the degree to which calls are in– or out–of the money.

[35]The tightness of the volatility parameter estimate follows from the 3 to 1 ratio of the number of moment conditions to the number of parameter to be estimated and the high p-level. Since the null hypothesis is rejected, each of the three moment conditions tries to pull the parameter estimate in its direction. This cannot be done without violating even more other moment conditions. Hence, marginal changes in the parameter estimate lead to a large jump in the GMM criterion function. This explains the extremely small standard errors.

a different property than the one assumed in the null hypothesis, namely is systematic rather than nonsystematic. The systematic property of nonsynchroneity is not surprising. Out–of–the money options are less frequently traded than in–the–money options. Therefore, one would expect, nonsynchroneity to be related to the extent the option is in– or out–of–the money, *i.e.*, to the exercise price. Similarly, close–to–maturity options are more frequently traded than far–from–maturity options. One would also expect nonsynchroneity to be related to the maturity of the options. The empirical evidence in table 3 supports these hypotheses. However, no inference about the origins of the pricing error can be drawn from the result that the average pricing error differs from zero. A way to infer whether the pricing error in the first moment condition is related to nonsychroneity is to test the first moment and the cross–moment conditions *jointly*. The comparison of the χ^2 statistics obtained by testing the first moment and any cross–moment condition *simultaneously* with the sum of the χ^2 statistics obtained by testing the same two moment conditions *individually* is useful for that purpose. The pricing error is likely to be generated by nonsynchroneity if the former χ^2 statistic is significantly lower than the latter.

The moment conditions are, therefore, tested two by two. This procedure yields 6 pairs of moment conditions. The number of degrees of freedom is equal to 5 since 6 moment conditions are tested and 1 parameter is estimated for each pair. Table 2 indicates that the Black model is rejected regardless of the pair of moment conditions tested. This result is not surprising since the moment conditions are all rejected when tested separately. The χ^2 statistics obtained for the joint tests of the first moment condition with any of the cross–moment conditions are not significantly higher than any of the χ^2 statistics obtained for the individual moment conditions. This finding gives more support to the hypothesis that the pricing error is generated by systematic nonsynchroneity. The ability of the GMM procedure to take into account the possible dependency between the different biases may also be appreciated. The χ^2 statistics obtained for pairs of moment conditions relative to those obtained for individual moment conditions suggest that the "time to maturity" bias is not highly correlated either with the "exercise price" bias or the "interest rate" bias. However, the "exercise price" bias seems to be more correlated with the "interest rate" bias. The volatility parameter estimates vary between .167 and .175 across moment

conditions. The estimates are even tighter than those obtained by testing the moment conditions separately.[36]

The moment conditions are then tested three by three. Four triplets of moment conditions are formed. The number of degrees of freedom is equal to 8. Consistent with the previous findings, each triplet of moment conditions is rejected at a high rejection level. The lowest χ^2 statistic is obtained for the triplet formed of the first moment condition and the two cross-moment conditions where the exercise price and the interest rate are used as instrumental variables. The χ^2 statistics obtained for certain triplets are sometimes lower than those obtained for pairs of moment conditions. This result can be explained by the correlation between the pricing error and the instrumental variables, likely to be generated by systematic nonsynchroneity. The sharpest rejection is obtained when the three cross-moments are tested jointly. The volatility parameter estimate varies between .169 and .176 across moment conditions and is even tighter than the previous estimates. The whole set of orthogonality conditions is finally tested. The length of the vector formed with the 12 sample moment conditions is minimized with respect to the parameter σ and. The number of degrees of freedom is equal to 11. As table 2 indicates, the Black model is rejected at a high significance level.

Case 2: The constant interest rate is implied from the model

Other arguments than systematic nonsynchroneity can be advanced to explain the rejection of the Black model. The use of the proxy r_{it} for the unobservable constant continuous interest rate R is one possibility. To determine whether the use of this proxy has any effect on the rejection of the model, the constant continuous interest rate R is treated as an unknown parameter and is estimated from the model. The variable r_{it} is dropped from $\underline{\omega}_{it}$, and R is consequently added to $\underline{\theta}$. The four orthogonality conditions specified in equations (28) through (31) remain unchanged. The moment conditions are tested individually, two by two, three by three, and then jointly. The number of degrees of freedom is equal to 1, 4, 7, and 10, respectively. There is one degree of freedom less than in the previous tests

[36]The 6 to 1 ratio of the number of moment conditions to the number of parameter estimates is twice as large as in the previous case.

since one additional parameter is estimated. The results are presented in table 3.

Table 3 indicates that the use of a proxy for the constant continuous interest rate has non-trivial implications for tests of contingent claims pricing models. When tested individually, none of the four moment conditions is rejected. The p-levels vary between a low of .32 and a high of .52 across moment conditions. The pricing error is on average zero and the "time-to-maturity" bias, the "exercise price" bias and the "interest rate" bias disappear. The volatility parameter estimates tend to be lower, between .10 and .14, and are less precise than those obtained in table 2.[37] The most interesting result concerns the estimate of the constant continuous interest rate which consistently comes up with a negative sign in the four moment conditions. This last finding may suggest that the constant interest Black model is inadequate.

The results concerning the rejection and non rejection of the individual first moment conditions in tables 3 and 4 are intriguing. They can be interpreted as follows. The results in table 4 can be attributed to a lack of power of the tests to reject the null hypothesis. Alternatively, they can be ascribed to the systematic nonsynchroneity error between r_{it} and r^o_{it}. It was suggested earlier that r_{it}, the proxy for the continuous-time interest rate R, was observed with error, and that the difference between r_{it} and r^o_{it} could represent a fraction of the total nonsynchroneity error. The nonsynchroneity error between r_{it} and r^o_{it} emerges since it is generally impossible to find a bond that matures exactly the same day and at the same time as contingent claim i. The nonsynchroneity error between r_{it} and r^o_{it} is implicitely eliminated when R, the constant continuous-time interest rate, is directly inferred from the model, as in the tests reported in table 4. The effects of nonsynchroneity between the contingent claim price and the underlying security price on the tests of contingent claims asset pricing models have been carefully investigated and monitored.[38] However, the effects of the interest rate induced nonsynchroneity error do not seem to have ever been examined. The empirical evidence in tables 3 and 4 suggests

[37]The drop in the ratio of the number of moment conditions to the number of estimates, 3/2 against 3/1 previously, and the low p-levels explain the decrease in the tightness of the volatility parameter estimates.

[38]See Rubinstein (1985).

that the fraction of the interest rate induced nonsynchroneity error relative to the total nonsynchroneity error is not necessarily negligible.[39]

These results change significantly when the moment conditions are tested two by two, three by three, and simultaneously. An increase in the power of the tests when more than one moment condition is tested might explain this finding. The moment conditions, except two, are now systematically rejected at p–levels exceeding .999. One of these exceptions concerns the two cross–moment conditions with respect to the time–to–maturity and the exercise price. These two moment conditions are not rejected when tested jointly. Also, the χ^2 statistics are systematically lower than those presented in table 2.[40] This suggests that, though indequate since rejected, the constant continuous interest rate Black model is better than the version which uses r_{it} as a proxy for R. These tests, unlike the previous ones, are not affected by the interest rate induced nonsynchroneity error. This might explain the difference between the two sets of χ^2 statistics. The volatility parameter estimates and their precision are close to those found in table 2. The range of σ varies between .14 and .17 and is equal to .16. More interestingly, the estimates of the constant interest rate remain negative. However, they are substantially lower and less statistically significant than those obtained with the tests of the individual orthogonality conditions. The range obtained for R varies between a low of -.09 and a high of -.02. and is equal to -.04 when the moment conditions are tested jointly.

Valuable inferences can be drawn from the empirical tests of the Black model. The Black model, based on the assumption of a constant continuous–time interest rate, is rejected by the data. Negative estimates of R are systematically obtained when the constant interest rate is inferred from the model. This result suggests that the constant interest rate model is inadequate. A model that relaxes the assumption of a constant interest rate might be more appropriate.[41] However, under a stochastic interest

[39]Further, the interest rate induced nonsynchroneity is likely to be sytematic rather than non–systematic. Bailey (1987) reports, in footnote 2 page 1188, that T–bills mature on Thursdays, while the options mature on Fridays.

[40]This comparison is not exactly correct since it does not take into account the one degree of freedom difference between the two tests.

[41]For gold futures options, Bailey (1987) finds that a stochastic interest rate version of the Black model produces errors with absolute values substantially less than for the model which assumes constant interest rates.

rate model, the early exercise feature of the option may become highly valuable as the papers of Brenner and *al.* (1985), and Ramaswamy and Sundaresan (1985) indicate. Early exercise may explain some of the biases in table 3. This explanation is not entirely satisfactory at first, since the early exercise premium is likely to be a decreasing function of the time left to maturity. The pricing error should, therefore, be systematically related to the time–to–maturity. However, the empirical evidence in table 3 shows that the hypothesis that the pricing error is uncorrelated with the time–to–maturity is not rejected by the data. The lack of a systematic relationship between the pricing error and the time–to–maturity *is consistent* with the early exercise argument only if the early exercise premium is too small to be detectable. This is what the empirical evidence of Brenner and *al.* (1985), and Ramaswamy and Sundaresan (1985) seems to indicate. The exercise premium is small and, therefore, early exercise might explain why the Black model is rejected.

The use by investors of a stochastic rather than a constant interest rate model was suggested as a potential reason for the rejection of the Black model. Yet, the Black model tested with r_{it} as a proxy for R is rejected at higher significance levels than the constant interest rate Black model. This rejection might at first look inconsistent with the claim of section 2.3 that the Black model tested with a proxy for the interest rate is similar to a stochastic interest rate version of the Black model. However, the equivalence between the two models was established under five assumptions. A violation of any of these assumptions can potentially explain the inconsistency. For example, the assumption that gold futures prices and bond prices are uncorrelated may be incorrect. A further investigation of stochastic interest rate models is left for future research.

The rejection of the null hypothesis was attributed earlier to systematic nonsynchroneity. This conjecture can be explicitly tested with equation (8), where the pricing error is estimated as a function of the underlying security price instead of the contingent claim price. Substituting f^o_{it} for s^o and f^{imp}_{it} for s^{imp}_{it}, in equations (14) and (15), respectively, yields,

$$E[\log f^o_{it} - \log f^o_{it-1} - \mu] = 0, \qquad \text{with,} \quad i = 1, \tag{32}$$

$$E[\log f^{\text{imp}}_{it} - \log f^{\text{imp}}_{it-1} - \mu] = 0, \qquad \forall i \in [1,3], \tag{33}$$

where f^o_{it} and f^{imp}_{it} are the observed futures price, and the implied futures

price of contract i at time t, respectively.[42] The two unknown parameters to be estimated by GMM are μ and σ, the mean percentage change in the price of the gold futures contract, and the volatility of the gold futures contract, respectively.

Though not reported here, the results of the tests of the four moment conditions can be briefly summarized as follows.[43] While the null hypothesis is not rejected, the parameter estimates are highly inaccurate, suggesting that the power of the tests is low. The low power displayed by the tests of the moment conditions specified in equations (32) and (33) stems from the high correlation between the *sample* moment versions of these moments.[44] Tighter estimates can be obtained by adding a moment condition different from those specified in equations (32) and (33). This moment condition is arbitrarily chosen to be,

$$E[(\log f_{1t}^{\text{imp}} - \log f_{1t-1}^{\text{imp}} - \mu) - \sigma_{\text{his}}^2] = 0, \tag{34}$$

where σ_{his} is the historical volatility of the gold futures contract. The sample moment version of this moment condition is unlikely to be highly correlated with the sample moment versions of (32) and (33). Adding this moment condition considerably improves the tests.[45] An increase in the accuracy of the parameter estimates is observed. The estimates of σ and μ are equal to .143 and -.33 with t–statistics equal to 26.41 and -1.88, respectively. The model is now marginally rejected since the p–value associated with the χ^2 test is equal to .93. The rejection of the above moment conditions suggests that systematic nonsynchroneity does

[42]Only the first series, *i.e.*, $i = 1$, is used to test the moment condition specified in equation (32), because the two other moment conditions obtained by successively substituting $i = 2$ for $i = 1$ and $i = 3$ for $i = 1$ in (32) are perfectly correlated with the moment condition obtained by setting i equal to 1. Adding these two moment conditions would decrease the power of the tests.

[43]The moment condition specified in equation (32), and the three moment conditions specified in equation (33) are tested. The number of degrees of freedom is equal to 2 since two parameters, namely σ and μ, are estimated.

[44]The sample moment conditions are highly correlated because the variance of the change in the futures price overwhelms the variance of the mispricing error. However, the change in the futures price is the same in the four moment conditions whereas the nonsynchroneity error is not.

[45]Five moment conditions are tested and two parameters are estimated. The number of degrees of freedom is now equal to three.

not entirely explain the rejection of the Black model. However, only the first moment condition is tested. Reliable inferences about the effects of systematic nonsynchroneity on the Black model can be drawn only after specifying and testing the cross–moment conditions, namely if systematic nonsynchroneity relates to the instrumental variables.

Stochastic volatility models may also explain why the Black model is rejected. The Black model assumes that the volatility of the underlying security is constant. Instead, the market may use a model based on stochastic volatility. However, stochastic volatility models cannot be tested with the GMM estimator since they do not have analytical solutions. Tests of contingent claims asset pricing models that do not have closed formed solutions are examined next.

3 Tests based on the Method of Simulated Moments estimator

3.1 The Method of Simulated Moments estimator

3.1.1 Principles

Moment conditions, as formulated in equations (9) and (10), or equation (14), can be tested only if the functional form $g(\cdot)$, representing the contingent claims pricing model being tested, has an analytical solution. This is not always the case. Many contingent claims pricing models do not have a closed form solution.

The Method of Simulated Moments (MSM) can be viewed as an extension of the GMM estimator when the expression, *i.e.*, $g(\cdot)$, entering the moment conditions cannot be calculated analytically. The idea is to replace $g(\cdot)$ by a *simulated* $g(\cdot)$. This substitution is possible because asset prices in general, and contingent claims prices in particular, can always be expressed as expectations, that is $g(\cdot) = E(\cdot)$, where E denotes the expectation operator.

The idea behind the MSM is the following. Let c_{it} be the price of contingent claim i at time t with exercise price x_{it} and maturity $t + \tau_{it}$, written on

the underlying security denoted by s. The contingent claim is assumed to have no early exercise feature.[46] Let $c_{it+\tau}$ and $s_{t+\tau_{it}}$ be the price of contingent claim i and the underlying security at time $t+\tau_{it}$, respectively. Then, $c_{it+\tau}$ is equal to,

$$c_{it+\tau} = \mathrm{MAX}\,(0, s_{t+\tau_{it}} - x_{it}), \qquad \forall i \in [1,m]. \tag{35}$$

Then, since asset prices are expectations, the price of contingent claim i at time t, c_{it}, is equal to,

$$c_{it} = E_t[PV(c_{it+\tau})], \qquad \forall i \in [1,m], \quad \forall t \in [1,T], \tag{36}$$

where E_t and PV denote the expectation at time t and the present value operators, respectively. A difficulty arises from the presence of the $PV(\cdot)$ operator in equation (35). This problem can be easily solved, however. Harrison and Kreps (1979) show that prices are independent of preferences when assets are priced by arbitrage. This result applies to contingent claims, and consequently investors can be assumed to be risk–neutral. Under risk–neutrality, risky payoffs can be discounted at the risk–free rate. Also, the mean return of the underlying security is equal to the risk–free rate for securities that require an initial investment like stocks, or zero for securities that do not require an initial investment like futures contracts. Consequently,

$$c_{it} = E_t[\exp^{-r_{it}\tau_{it}}\, c_{it+\tau}], \qquad \forall i \in [1,m], \quad \forall t \in [1,T]. \tag{37}$$

The contingent claim price c_{it} can be estimated by randomly drawing K values of $c_{it+\tau}$, denoted by $c^j_{it+\tau}$ with $j = 1,\ldots,K$. Once sampled, the mean present value, denoted by $\hat{c}_{it}$,

$$\hat{c}_{it} = \frac{1}{K}\sum_{j=1}^{j=K} PV(c^j_{it+\tau}), \qquad \forall i \in [1,m], \quad \forall t \in [1,T], \tag{38}$$

is calculated as an estimate of the population mean c_{it}. Using the definition of the present value operator, equation (38) can be rewritten as,

$$\hat{c}_{it} = \frac{1}{K}\sum_{j=1}^{j=K} \exp^{-r_{it}\tau_{it}}\, c^j_{it+\tau}, \qquad \forall i \in [1,m], \quad \forall t \in [1,T]. \tag{39}$$

[46]Early exercise considerably complicates the simulation procedure. See Bossaerts (1988).

The MSM is then defined by substituting $\hat{c}_{it}$ for c_{it} in the GMM moment conditions. The idea of using a simulation procedure to obtain estimates of moments is not new. Monte Carlo analysis are based on that principle. However, the idea of using simulations *jointly* with the GMM estimator, as implemented in the papers of Lee and Ingram (1988), McFadden (1987), and Pakes and Pollard (1986)), is more recent.

The MSM estimator is presented in detail below. Two cases are successively examined. The distribution of the terminal value of the underlying asset, $s_{t+\tau_{it}}$, conditional on the value s_t at time t is first assumed to be known. This assumption is then relaxed. The knowledge of the conditional distribution is replaced by a stochastic differential equation for the value of the underlying asset. Techniques to reduce the variance of the simulation error are finally discussed.

3.1.2 Case 1: The conditional distribution is known

Let s_t^o and $s_{t+\tau_{it}}^j$ with $j = 1, \ldots, K$, denote the *initial* value, and K possible *terminal* values of the underlying security at time t and $t + \tau$, respectively. Let's assume that the $s_{t+\tau_{it}}^j$'s given s_t^o can be written as a function of the random variable v. Let $q(\cdot)$ be that function. Then, the terminal values of the underlying security can be expressed as,

$$s_{t+\tau_{it}}^j = q(v^j; \underline{\theta}, s_t^o), \qquad \forall j \in [1, K]. \tag{40}$$

Further, let $h(v)$ be the known distribution function of the random variable v, assumed to be independent of the vector of parameters $\underline{\theta}$ and the initial value s_t^o.[47]

[47] Suppose that the price of the underlying asset, s, follows a geometric Brownian motion,

$$\frac{ds}{s} = \mu\, dt + \sigma\, dz,$$

where dz is a Wiener process. Under this assumption, v is a standard normal variable, and $h(\cdot)$ is the standard normal distribution. Then,

$$s_{t+\tau_{it}} = s_t^o \exp\left\{\sigma\sqrt{\tau_{it}}\, v + \mu - \frac{1}{2}\sigma^2 \tau_{it}\right\},$$

where the superscript j is dropped for sake of simplicity.

The simulation estimator of c_{it}, denoted $\hat{c}_{it}$, can be obtained by following a four-step procedure.

- Step 1: Draw K numbers v^j's, with $j = 1, \ldots, K$ from the distribution $h(\cdot)$.
- Step 2: Compute $s^j_{t+\tau_{it}}$, with $j = 1, \ldots, K$, using equation (40).
- Step 3: Compute $c^j_{it+\tau}$, with $j = 1, \ldots, K$, using equation (35).
- Step 4: Compute the mean present value $\hat{c}_{it}$ using equation (39).

The MSM estimator follows from the substitution of $\hat{c}_{it}$ for c_{it} in the GMM moment conditions. For each iteration in the minimization of the criterion function, *i.e.*, for each new value of $\underline{\theta}$, the steps 2 through 4 are repeated since the transformation of v^j to $s^j_{t+\tau_{it}}$ depends on $\underline{\theta}$, as equation (40) indicates.

As shown below, the substitution of $\hat{c}_{it}$ for c_{it} does not affect the validity of the moment conditions. Let η_{it} be the simulation error,

$$\eta_{it} = \hat{c}_{it} - c_{it}. \qquad \forall i \in [1, m], \quad \forall t \in [1, T], \tag{41}$$

Substituting $\hat{c}_{it}$ for c_{it} in the GMM moment condition $E[c^o_{it} - c_{it}] = 0$ yields,

$$E[(c^o_{it} - \hat{c}_{it})] = E[(c^o_{it} - c_{it}) - (\hat{c}_{it} - c_{it})] = E[\epsilon_{it} - \eta_{it}]. \tag{42}$$

However, by construction $\hat{c}_{it}$ is an unbiased estimate of c_{it},

$$E(\eta_{it}) = 0, \qquad \forall i \in [1, m]. \tag{43}$$

Further, from the null hypothesis,

$$E(\epsilon_{it}) = 0, \qquad \forall i \in [1, m],$$

therefore,

$$E[c^o_{it} - \hat{c}_{it}] = 0, \qquad \forall i \in [1, m]. \tag{44}$$

Equation (44) shows that the first unconditional moment of the difference between observed and *simulated* prices is equal to zero, like in equation (9), where ϵ_{it} is estimated as the difference between observed and

theorical prices. The results concerning the first moment condition can be extented to the other moment conditions. The cross–moment conditions are also unaffected by the replacement of the c_{it}'s by the $\hat{c}_{it}$'s since the simulation error is independent on the value of the instrumental variables z_{it}^j, with $j = 1, \ldots, L$.

The MSM and GMM estimators are virtually identical. Instead of being applied to a single time–series of random variables, *i.e.*, the ϵ_{it}'s, the MSM estimator is applied to a time–series of *pairs* of random variables, *i.e.*, the $(\epsilon_{it}, \eta_{it})$'s. Since the MSM estimator involves one additional random variable relative to the GMM estimator, namely the η_{it}'s, the next issue is to determine if the MSM estimator shares the same asymptotic properties as the GMM estimator. The MSM and GMM estimators have similar asymptotic properties because the η_{it}'s obey a law of large number *uniformly* in the vector of parameters $\underline{\theta}$, that is,[48]

$$\frac{1}{T}\sum_{t=1}^{t=T} \eta_{it} \xrightarrow{a.s.} 0. \tag{45}$$

Therefore, Hansen's (1982) arguments still apply to the MSM estimates. The estimates of the parameter vector θ converge in probability, and obey a central limit theorem. Further, the expression for the MSM variance-covariance matrix of the parameter estimates is the same as the one obtained for the GMM variance–covariance matrix. A major attractiveness of the MSM estimator is the independence of the asymptotic properties on the number of simulations K per observation. Since the simulation error, η_{it} is averaged out across observations by the law of large numbers, the asymptotic properties hold regardless of the number of simulations per observation.[49] However, this result does not imply that the number of simulations, K, is irrelevant. Unlike the asymptotic properties, the efficiency

[48]Uniformity means that for any given number, there exists a sample sise T^*, such that $(\frac{1}{T}\sum_t \eta_{it})$ is below that number for $T \geq T^*$ regardless of $\underline{\theta}$. Uniformity follows because of the smoothness of the transformation $q(v; \theta, s_i^o)$. See Andrews (1987) for a generic uniform law of large numbers.

[49]Unfortunately, the this property does not hold when considering the moment condition formulated in equation (14). Under this formulation of the null hypothesis, the pricing error is estimated as the difference between the *log* of the observed price and the *log* of the implied price of the underlying security. The log transformaton makes the simulation error enter non–linearly in the moment conditions. Hence, a law of large numbers cannot

of the parameter estimates and the power of the tests to reject the null hypothesis depend on the number of simulations per observation.

The variance–covariance matrix of the sample moment conditions is affected by the substitution of the c_{it}'s by the $\hat{c}_{it}$'s.[50] It can be shown that for a finite K, the MSM variance–covariance matrix is strictly larger than its GMM counterpart, meaning that the difference between the two variance–covariance matrices is equal to a positive definite matrix.[51] This result has two important implications. First, the MSM standard errors of the parameter estimates will always be larger than their GMM counterparts. Second, the power of MSM tests is likely to be lower than the power of GMM tests. The adverse effects of the substitution of the c_{it}'s by the $\hat{c}_{it}$'s can be explained as follows. The moment conditions under GMM and MSM involve the deviation of the observed from the theoretical price, *i.e.*, the pricing error ϵ_{it}. However, η_{it}, the deviation of the simulated price from the theoretical but analytically uncomputable price also appears in the moment conditions under MSM. The mean simulation error is equal to zero by construction. Therefore, statistical tests of whether the deviations of observed from simulated prices have mean zero are possible despite the presence of simulation error. However, if the variance of η_{it}, and, hence, of $(\frac{1}{T}\sum_t \eta_{it})$, is large, the simulation error η_{it} may overwhelm the pricing error, ϵ_{it}. Under those conditions, no reliable inferences about the null hypothesis that the average pricing error is zero can be drawn, and tight parameter estimates cannot easily be obtained. The problem can be alleviated, to a certain extent, by increasing K. This will decrease the variance of η_{it}, and, consequently, of $(\frac{1}{T}\sum_t \eta_{it})$. The increase of K will lead to an improvement in

be appealed to. The mean simulation error across observations will generally be nonzero, regardless of the number of observations.

[50]The inverse variance–covariance matrix of the sample moment conditions is used as weighting matrix in the minimisation of the quadratic form and in the calculation of the standard errors of the estimated parameters.

[51]For the reader who is familiar with the results in Lee and Ingram (1988), McFadden (1987), and Pakes and Pollard (1986), it must be pointed out that the MSM weighting matrix in the present context *cannot* be simplified to $(1+1/K)$ times the GMM weighting matrix. In the aforementioned papers, two *similar* errors enter the moment conditions. The first one is the error from nature's simulation and the second one is the error from the econometrician's simulation. Here, two *different* types of errors enter the moment conditions, namely ϵ_{it} and η_{it}, the pricing and simulation error, respectively.

the power of MSM, and, simultaneously, a decrease in the standard error of the parameters.

The determination of K is a critical issue. A possible way to find the apropriate K is to test, both with the GMM and MSM estimators, a contingent claims pricing model that *can* be solved analytically, to set K equal to a small value, and to increase the value of K until the inferences obtained with the MSM estimator match those obtained with the GMM estimator. The first moment conditions,

$$\begin{aligned} E[c_{it}^{o} - c_{it}] &= 0, \qquad \forall i \in [1, m], \\ E[c_{it}^{o} - \hat{c}_{it}] &= 0, \qquad \forall i \in [1, m], \end{aligned}$$

are tested for that purpose, with the GMM and MSM estimators, respectively. The contingent claims asset pricing model used by investors is assumed to be the Black futures option pricing model. The parameter K, the number of simulations per observation, is successively set equal to 10, 20, 40, 60, 80, and 100. The parameter K is determined when the level of rejection of the model and the tightness of the parameter estimate match under GMM and MSM . Table 4 presents the results of the simulation. Table 4 indicates that the MSM and GMM inferences match for K equal to 100. This number is fairly large because the gold futures options are tightly priced.[52] The deviations of observed from theoretical prices are so small that the simulation error per observation needs to be even smaller to obtain enough power to reject the null hypothesis. With 100 simulations per observation, the average simulation error is small enough to reject the hypothesis that the average error is equal to zero. Accordingly, the number of simulations per observation is set equal to 100 in the subsequent MSM tests of contingent claims pricing models.

3.1.3 Case 2: The conditional distribution is unknown

The assumption that the conditional distribution of the terminal value of the underlying asset is known is now relaxed.

[52]Fortunately, the number of necessary iterations for the MSM estimator to converge decreases as K increases.

The price of the underlying asset is assumed to follow a diffusion process. Therefore, its terminal price can be estimated by discrete–time approximations. The process,

$$ds = \mu(s, \underline{\theta})\, dt + \sigma(s, \underline{\theta})\, dz, \tag{46}$$

where dz denotes a Wiener process, is approximated by the difference equation,

$$\Delta s = \mu(s, \underline{\theta})\, \Delta t + \sigma(s, \underline{\theta})\, \varsigma, \tag{47}$$

where ς is a standard normal variable.

The simulation estimate of the contingent claim prices are computed as follows. The time interval $[t, t+\tau_{it}]$ is divided into n equally spaced intervals of length Δt, and $n\,K$ random variates, denoted by $\varsigma^{l,j}$, with $l = 1, \ldots, n$, and $j = 1, \ldots, K$, are drawn from the standard normal distribution. Then K terminal values of the underlying security, denoted by $s^{n,j}_{t+\tau_{it}}$, with $j = 1, \ldots, K$, are calculated *after iterating K times* a n–step procedure, where in,

- Step 1: $s^{1,1}_{t+1}$ is obtained from equation (47) using $\varsigma^{1,1}$ and an initial value for the underlying security,
- Step 2: $s^{2,1}_{t+2}$ is obtained from equation (47) using $\varsigma^{2,1}$ and $s^{1,1}_{t+1}$ determined in the previous step,
- $\vdots$
- Step l: $s^{l,1}_{t+l}$ is obtained from equation (47) computed using $\varsigma^{l,1}$ and $s^{l-1,1}_{t+l-1}$ determined in the previous step,
- $\vdots$
- Step n: $s_{t+\tau_{it}}$ is obtained from equation (47) using $\varsigma^{n,1}$ and $s^{n-1,1}_{t-1+\tau_{it}}$ determined in the previous step.

Let $c^{n,j}_{it+\tau}$, with $j = 1, \ldots, K$, denote the payoff of contingent claim i at time $t + \tau$ calculated from equation (35) with the $s^{n,j}_{t+\tau_{it}}$'s replacing $s_{t+\tau_{it}}$. The simulation estimate of the price of contingent claim i at time t is equal to,

$$\hat{c}^{n}_{it} = \frac{1}{K} \sum_{j=1}^{j=K} \exp^{-r_{it}\tau_{it}}\, c^{n,j}_{it+\tau}, \qquad \forall i \in [1, m], \quad \forall t \in [1, T], \tag{48}$$

where the superscript n appears to distinguish this simulation estimate from the one computed in the previous section where the conditional distribution is assumed to be known. The simulation estimate $\hat{c}_{it}^{n}$ is then substituted for c_{it} in the GMM moment conditions. The parameter estimates of the model and their standard errors are obtained by minimizing a quadratic form of the sample moments with respect to the vector of parameters $\underline{\theta}$.

Unfortunately, $\hat{c}_{it}^{n}$ is not an unbiased estimate of c_{it}. The simulation error has two components. The first one is a sampling error, associated with the estimation of the contingent claim price at time t, originating from the approximation of the population mean c_{it} by the sample mean $\hat{c}_{it}^{n}$. The second one is a sampling error, associated with the estimation of the terminal price of the underlying security at time $t + \tau_{it}$, arising from the approximation of the true terminal value $s_{t+\tau_{it}}^{j}$ by an estimated $s_{t+\tau_{it}}^{n,j}$. Unlike the case which assumes that the conditional distribution of the terminal value of the underlying security is known, the asymptotic properties of the MSM estimator break down because of the second source of error. A law of large numbers cannot be appealed to, even though the simulation errors are independent over time. For finite values of K, the simulation error, *i.e.*, $(\hat{c}_{it}^{n} - c_{it})$, does not have a mean equal to zero. This unfortunate property destroys a very attractive feature of the MSM estimator.

However, Pardoux and Talay (1985) demonstrate that $\hat{c}_{it}^{n}$ converges to c_{it} when the parameters K and n go to to infinity. Therefore, the simulation error disappears asymptotically. Further, the asymptotic properties of the GMM estimator still hold if convergence is *uniform* in the parameter vector $\underline{\theta}$.[53] When K, n, and T, increase at appropriate rates, the parameter estimates converge and obey a central limit theorem with the traditional asymptotic variance–covariance matrix.

3.1.4 Variance reduction techniques

Variance reduction techniques can be employed to lower the variance of the simulation error.[54] They can be applied only if the pricing model being

[53]The rate of convergence of $\hat{c}_{it}^{n}$ to c_{it} needs to be at least $\sqrt{n}$. This issue will be investigated in a future version of the paper, using the results from Pardoux and Talay (1985).

[54]See Boyle (1977), or Hull and White (1987).

tested is close to a model that has an analytical solution.

Let's consider, for example, the Constant Elasticity of Variance (CEV) model of Cox (1975). Though a solution to the CEV model can be approximated with Fourier expansions, a closed-form solution does not exist. Let $c^o_{it}(\text{cev})$ denote the observed price of call i at time t, with $i = 1, \ldots, m$, and $t = 1, \ldots, T$, assumed to be generated by the CEV model. Let $c_{it}(\text{cev})$ denote the theoretical price of call i at time t, given the observed price of the underlying asset at time t, denoted by s^o_t. Let's assume that the Black model is an appropriate closely related pricing model. Let $c^o_{it}(\text{bl})$ and $c_{it}(\text{bl})$ denote the observed and the theoretical call prices under the Black model, respectively, and let τ_{it} be the number of days until maturity.

Two sample paths of the price of the underlying asset are constructed after randomly drawing τ_{it} standard normal variates. One sample path is generated assuming the constant variance Black model. The other one is generated under the assumption that the variance changes every day depending on the price of the underlying asset. The same random draws are used to generate the sample paths. The procedure is repeated K times for each sample path. The daily intervals used to compute price changes are assumed to be small enough to minimize the error introduced by the approximation of a continuous-time process by discrete-time calculations. This approximation error is assumed to be equal to zero. The payoffs on each option is calculated from equation (35) using the respective K sample paths. The payoffs are discounted and sample averages are computed using equation (38). This procedure generates the simulation estimates of $c_{it}(\text{bl})$ and $c_{it}(\text{cev})$, denoted by $\hat{c}_{it}(\text{bl})$ and $\hat{c}_{it}(\text{cev})$, respectively. Let $\eta_{it}(\text{bl})$ and $\eta_{it}(\text{cev})$ be defined as,

$$\eta_{it}(\text{bl}) = \hat{c}_{it}(\text{bl}) - c_{it}(\text{bl}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T], \tag{49}$$

$$\eta_{it}(\text{cev}) = \hat{c}_{it}(\text{cev}) - c_{it}(\text{cev}), \qquad \forall i \in [1, m], \quad \forall t \in [1, T]. \tag{50}$$

The simulation estimates $\hat{c}_{it}(\text{bl})$ and $\hat{c}_{it}(\text{cev})$ are both generated with the *same* random draws. Further, $c_{it}(\text{bl})$ and $c_{it}(\text{cev})$ are assumed to be generated by closely related contingent claims asset pricing models. Therefore, $\eta_{it}(\text{bl})$ and $\eta_{it}(\text{cev})$ should be highly correlated.

Let us consider the following moment conditions,

$$E[c^o_{it} - \hat{c}_{it}(\text{cev}) + \hat{c}_{it}(\text{bl}) - c_{it}(\text{bl})] = 0, \qquad \forall i \in [1, m], \tag{51}$$

$$E[(c_{it}^{o} - \hat{c}_{it}(\text{cev}) + \hat{c}_{it}(\text{bl}) - c_{it}(\text{bl}))z_{it}^{j}] \;=\; 0, \forall i \in [1,m], \forall j \in [1,L], \quad (52)$$

where z_{it}^{j}, with $j = 1, \ldots, L$, are appropriate instrumental variables. These moment conditions hold since,

$$\begin{aligned}[c_{it}^{o} - \hat{c}_{it}(\text{cev}) + \hat{c}_{it}(\text{bl}) - c_{it}(\text{bl})] &= [c_{it}^{o} - c_{it}(\text{cev}) - (\hat{c}_{it}(\text{cev}) - c_{it}(\text{cev})) \\ &\quad + \hat{c}_{it}(\text{bl}) - c_{it}(\text{bl})] \\ &= \epsilon_{it} - \eta_{it}(\text{cev}) + \eta_{it}(\text{bl}),\end{aligned}$$

where ϵ_{it} is the nonsynchroneity error. Noting that,

$$E[\epsilon_{it}] = E[\eta_{it}(\text{cev})] = E[\eta_{it}(\text{bl})] = 0, \qquad \forall i \in [1,m], \qquad (53)$$

and,

$$E[\epsilon_{it}\, z_{it}^{j}] = E[\eta_{it}(\text{cev})\, z_{it}^{j}] = E[\eta_{it}(\text{bl})\, z_{it}^{j}] = 0, \forall i \in [1,m], \forall j \in [1,L], \quad (54)$$

establish the proof.

The error $\eta_{it}(\text{bl})$ is introduced in the moment conditions to reduce the variance error of the *sum* of the simulation errors. Since,

$$\text{var}\,[\eta_{it}(\text{bl}) - \eta_{it}(\text{cev})] = \text{var}\,[\eta_{it}(\text{bl})] + \text{var}\,[\eta_{it}(\text{cev})] - 2\text{cov}\,[\eta_{it}(\text{bl}), \eta_{it}(\text{cev})]. \quad (55)$$

the last term on the right hand side is large and positive. This implies that,

$$\text{var}\,[\eta_{it}(\text{bl}) - \eta_{it}(\text{cev})] < \text{var}\,[\eta_{it}(\text{cev})]. \qquad (56)$$

The simulation error is reduced. The use of variance reduction techniques should, therefore, substantially increase the power of MSM tests and the precision of the parameter estimates.

3.2 Advantages of the MSM estimator

The MSM estimator can be used to estimate and test contingent claims asset pricing models that have unknown analytical solutions. Further, the conditional distribution of the terminal value of the underlying security

does not have to be known.[55] The use of simulation procedures to calculate contingent claims prices is not new.[56] These procedures, however, are applied on *individual* contingent claims, and the parameters of the models are generally estimated from historical data.[57] The MSM estimator can be used to *imply* the parameters of contingent claims asset pricing models from *panel data sets of contingent claims.* The information embodied in more than one time series of contingent claims prices is used to estimate the parameters.

However, the MSM estimator requires the knowledge of the state variables that drive the price of the underlying security. The value of the underlying security and the state variables at time t should be observable to simulate different terminal values $s^j_{t+\tau_{it}}$, with $j = 1, ..., K$. Therefore, models, like for example the stochastic variance option pricing model of Hull and White (1987), Scott (1987), and Wiggins (1987), cannot be tested with the MSM estimator. These papers consider pricing an option on a non-dividend-paying security. The price of the underlying asset is assumed to follow the process,

$$\frac{ds}{s} = \mu\, dt + \sigma\, dz, \tag{57}$$

$$\frac{d\sigma^2}{\sigma^2} = \alpha\, dt + \delta\, dw. \tag{58}$$

The resulting option pricing model could be estimated with the MSM estimator, by simulating the above process with discrete approximations, if the instantaneous variance at time t, σ^2_t were observable. However, σ^2_t, is not observable.

An approach that could potentially solve the problem of unobservable state variables is presently being investigated by Duffie and Singleton (1988).

[55]The number of simulations per observation can be limited when the conditional distribution is known but must grow as the number of observations increases, when unknow. The determination of the optimal values of the parameters K, and n, the number of simulations per observation, and the number of intervals over which the continuous-time process is discretely approximated, respectively, is, therefore, an important issue in the latter case.

[56]See Boyle (1977), or Hull and White (1987).

[57]This is not an easy task when the unconditional distribution of prices is analytically unknown, and discrete approximations of the continuous-time process are used instead. See Duffie and Singleton (1988).

They use *unconditional* moments of prices, and simulate stochastic processes by discrete approximation.[58] Their estimation strategy is based on simulating time–series of prices of observable and unobservable state variables. The simulation results are insensitive to arbitrarily chosen initial values of the unobserved state variables by imposing certain restrictions, like geometric ergodicity. The parameter estimates converge and obey a central limit theorem under that assumption.

When the state variables are observable, conditional distributions and their corresponding moments can be employed. This is the idea of Gibbons and Jacklin (1988). They estimate the parameters of the CEV model from stock price data using a maximum likelihood procedure. This approach is possible, since the distribution of the future stock price, conditional on today's price, has a known, yet complicated, analytical expression. However, no such analytical expressions exist for truncated moments such as those involved with contingent claims asset pricing models. This justifies the use of simulation estimators. The CEV model is tested in next section using contingent claims prices instead of the underlying security prices as in Gibbons and Jacklin. However, rather than using the conditional distribution of the terminal value of the underlying asset derived in Gibbons and Jacklin, the terminal price is estimated with the discrete–time procedure of section 3.1.3.

The MSM estimator can be used to estimate and test contingent claims asset pricing models that do not have known analytical solutions, regardless of whether the conditional distribution of the terminal value of the underlying security is known or unknown. The models are testable only when the state variables driving the value of the underlying security are observable. Further research along the lines of Duffie and Singleton (1988) is necessary to extend the present methodology to the case of unobservable variables.

[58]This contrasts with the procedure used in this paper, where *conditional* moments, namely the expected discounted payoffs on contingent claim at maturity *i.e.*, $t + \tau_{it}$, are calculated given a value of the underlying asset at time t.

3.3 Empirical tests using the MSM estimator

The MSM estimator is used to test contingent claims asset pricing models that relax the assumption of constant volatility over the life of the contingent claim. Two models are examined and tested on gold futures option contracts. These models assume that the variance at time t is a function of the underlying futures price at time t. As stressed in the previous section, the knowledge of the state variables is required. The assumption that the variance is a known function of the futures price satisfies this requirement.

The first model is a discrete time version of the Constant Elasticity of Variance (CEV) model of Cox (1975). The CEV model assumes that the standard deviation is inversely proportional to the futures price. However, the CEV model is not entirely appropriate for futures contracts, since it implies a positive probability of default. Default occurs with certainty if the futures price drifts downwards. Further, the variance process associated with the CEV model exhibits very little variability over time for gold futures options. The second model tested with the MSM estimator is designed to increase the variability of the variance process. The model is *ad hoc* and constrains the conditional standard deviation of futures price changes to be proportional to the level of the futures price, unlike the CEV model which imposes *inverse* proportionality. Though arbitrary, this model is selected to overcome the problem of the lack of variability displayed by the variance process of gold futures contracts.[59]

The constant elasticity of variance model posits that the conditional standard deviation of price changes of the underlying security at time t, here a gold futures option contract, is a function of the price of the underlying security at time t,

$$\sigma_{it} = \sigma f_{it}^{\rho}, \qquad \forall i \in [1, m], \quad \forall t \in [1, T], \tag{59}$$

where f_{it} is the price of futures contract i at time t, and σ, and ρ are two unknown parameters to be estimated. Two well-known special cases

[59]Ball and Torous (1986) test, with futures option contracts, Samuelson's (1965,1976) hypothesis that the variance per unit of time of futures prices increases monotonically as the time to maturity of the underlying futures contract decreases. For gold futures, they find that seven of the contracts are consistent with implied average futures price variance with decreasing futures contract maturity, while two of the contracts are consistent with implied average futures price variance increasing with decreasing futures contract maturity.

are the Black model and the so-called "square root" model. When ρ is equal to zero, the CEV model reduces to the Black model which assumes constant volatility over the life of the option. When ρ is equal $-\frac{1}{2}$, the CEV model reduces to the square root model which assumes that the variance's elasticity is equal to -1.[60]

A solution to the continuous-time call options pricing model under CEV processes are available only in terms of a Fourier Series expansion. The infinite sums can be approximated by finite sums. Though computionally feasible, this procedure introduces an approximation error. This error obscures the interpretation of the statistical tests of significance of deviations of observed from theoretical contingent claims prices. In contrast, the effects of the approximation, or simulation, error on the overall goodness-of-fit, *i.e.*, the χ^2 test, can be explicitely controlled for with the MSM estimator. Gibbons and Jacklin (1988) show that the conditional distribution of the futures price at maturity is a complicated expression. Therefore, rather than drawing from this distribution, the continuous-time process of the futures price is approximated by a finite-difference stochastic equation, as suggested in section 3.1.3.

The properties of consistency and normality hold when the parameters n, K, and T have large values. The number of intervals over which the futures price is approximated, n, is set equal to the number of days until the option matures. Stated differently, a discrete interval of one day is assumed to be a fairly accurate approximation to continuous-time processes. The number of sample paths simulated per observation, K, is set equal to 100. The number of observations, T, remains equal to 210. To see whether stochastic variance models lead to an improvement over the Black constant variance model, namely, one set of moment conditions, namely,

$$E[c_{it}^{o} - \hat{c}_{it}^{n}] = 0, \qquad \forall i \in [1,3],$$

is tested with the MSM estimator. In the above moment conditions, minimized with respect to the parameters σ and ρ, the pricing error is defined as the difference between the observed price and the simulated price. The results are presented in table 5. The CEV model is rejected at a p-level

[60]Hence a 10% decrease in the futures price is accompanied by a 10% increase in the variance of the futures price changes.

equal to .975. As found in past studies, ρ is negative, σ positive, and both have a reasonable magnitude.[61] The parameter estimates are equal to -.121 and .352, respectively. Though jointly significant, neither ρ nor σ is estimated with precision. The t–statistics are equal to -.019 and .026, respectively. Certain characteristics of the sample explain these results. As table 1 indicates, the range of f_{it} varies between \$367.7 and \$447.9. Therefore, σ_{it} exhibits very little variation over time for any reasonable ρ. The conditional standard deviation of the underlying security σ_{it} is virtually constant, meaning that the Black model is actually being retested.

The variance reduction technique described in section 3.1.4 are employed to increase the precision of the estimates. The number of simulations per observation is set equal to 10, *i.e.*, is reduced by a factor of 10 relative to the previous case. The results are similar to those obtained without the use of variance reduction techniques. The CEV model is rejected at a higher significance level. The p–value is higher than .999. The estimate of σ is high being equal to 1.726. The value obtained for the parameter θ is equal to -.388, which suggests that the CEV model is close to being a square root process. The t–statistics are larger than in the previous case but the parameter estimates remain insignificant. The limited variability displayed by the conditional standard deviation is a possible explanation.

The CEV model, which imposes an inverse relationship between the futures price and its conditional standard deviation, is not necessarily appropriate to price gold futures options. First, as suggested earlier, the CEV model implies a positive probability of bankruptcy. This implication is clearly unrealistic for gold futures options. Second, the limited range of the gold futures prices prohibits the conditional standard deviation to display much variability over time. The CEV model is, therefore, very close to a model which assumes constant variance. More variability can be obtained with a model that imposes a positive relationship between the futures price and its conditional standard deviation. The second model, tested with the MSM estimator, is based on the assumption that the conditional standard deviation is related to the price of the underlying futures according to the

[61]See Christie (1982) and Gibbons and Jacklin (1988).

functional form,

$$\sigma_{it} = \sigma \log\left(\frac{f_{it}}{100}\right), \qquad \forall i \in [1,m], \quad \forall t \in [1,T]. \tag{60}$$

The model is successively estimated without and with the variance reduction technique of section 3.1.4, with K, the number of simulations per observation, equal to 100 and 10, respectively. In both cases, n, the number of intervals over which the futures price is approximated, is set equal to, τ_{it}, the number of days until the option matures. The results appear in table 5. Unlike the tests of the CEV model, the estimate of the volatility parameter σ, is highly significant, regardless of the value of K. The point estimates are equal to .125, and .126 with t–statistics equal to 32.36 and 39.93, with K equal to 100 and 10, respectively. The estimates of σ are extremely tight. The model is rejected at the 10% level with K set equal to 100. Similarly to the result obtained with the CEV model, the model is rejected at higher significance level when the variance technique is used. This finding provides additional evidence on the usefulness of variance reduction techniques. Table 5 also indicates that the CEV and the *ad hoc* models are rejected at very similar significance levels with K equal to 10. Finally, when compared to the GMM tests of the first moment condition, the *ad hoc* model is rejected at a marginally lower significance level than the Black model which assumes constant variance.

Based on the empirical evidence in table 5, models that relax the assumption of constant variance do not seem to fit gold futures options any better than the Black constant variance model. These models are all rejected by the data. However, it is too early to draw definite conclusions about the validity of stochastic variance contingent claims asset pricing models. Two *specific* stochastic variance processes were examined above. They do not necessarily correspond to the true but unobservable variance process followed by gold futures. The model tested by Ball and Torous (1986), based on the assumption that the volatility of futures prices is related to the time–to–maturity of the futures contract, is maybe more realistic than the two models examined here. This model can also be easily tested with the MSM estimator.

4 Conclusion

This paper shows how the GMM and MSM estimators can be used to estimate and test contingent claims asset pricing models. The GMM estimator is used when the model has a known analytical solution. Simulation is used to estimate the price of contingent claims when the model cannot be solved analytically. The simulation estimates are then substituted for the theoretical price in the GMM criterion function. The use of simulation jointly with the GMM estimator is known as the MSM estimator. When the conditional distribution of the terminal value of the underlying security is known, the MSM estimator shares the same asymptotic properties as the GMM estimator. Except under certain conditions, these asymptotic properties break down when the terminal price of the underlying security is estimated with discrete–time approximation procedures. The power of MSM tests to reject the null hypothesis is generally lower than the power of GMM tests unless the number of simulations is large. Fortunately, the use of variance reduction techniques considerably decreases the number of simulations necessary to match the power of MSM tests with that of GMM tests.

The GMM and MSM are simple, yet powerful, estimators to test contingent claims asset pricing models that have, or do not have, analytical solutions. The use of these two procedures offers substantial benefits. First, the statistical significance of the pricing error can be assessed, and valuable information on the possible causes of rejection, such as systematic versus non–systematic nonsynchroneity, can be obtained. Second, the presence of biases and their possible dependency can be explicitely tested. Biases, other than the "time–to–maturity" or the "exercise price" bias, can be easily explored by employing new instruments, like the interest rate or macroeconomic variables. Third, more than one parameter of the contingent claims asset pricing model can be estimated. Examples are the volatility of the underlying security and the constant continuous–time interest rate. Under certain conditions, the expected return on the underlying security can also be estimated. These parameters are efficiently estimated since the GMM and MSM tests are performed on panel data sets of contingent claims. Fourth, the GMM and MSM estimators yield the standard error of the

parameter estimates.

The GMM and MSM estimators are applied to estimate and test different contingent claims asset pricing models on a panel data set of gold futures options. The GMM tests reject the Black model which assumes constant interest rate and volatility. The traditional "time–to–maturity" and "exercise price" biases are significant. An "interest rate" bias, previously left unnoticed in the contingent claims literature, is also uncovered. Further, the GMM tests reject the Black model under stochastic interest rates and the local expectation hypothesis of the term structure. Though formal empirical tests are necessary, systematic nonsynchroneity does not seem to explain the rejection of the models. Two stochastic variance models are tested with the MSM estimator. Both models are rejected at high significance levels. The parameter estimates of the CEV model are found to be highly inaccurate with and without a variance reduction technique. The low variability displayed by the price of gold futures contracts over time may explain this last result. Additional tests are necessary to conclude that stochastic volatility contingent claims asset pricing models do not perform better than constant variance models. It is clear from the evidence in tables 2 through 5 that the GMM and MSM estimators produce powerful tests of contingent claims asset pricing models. Even though options are tightly priced, the models tested in the paper are all rejected by the data. This contrasts with results obtained from trading–based tests such as those reported in Galai (1983).

Certain arguments can be advanced to explain these rejections. The GMM tests of the Black model yield a negative estimate of the constant continuous–time interest rate. Therefore, a violation of the assumption of a constant interest rate may explain why the different models are rejected. The validity of these conjectures can be assessed with tests of stochastic interest rate models under a different assumption than the the simple local expectation of the term structure tested in the paper. Tests of stochastic interest rate models are difficult, since premature exercise may become highly valuable. The GMM and MSM estimators should, therefore, be modified to account for the probability of early exercise.[62] The GMM and MSM estimators have considerable applications for estimating and testing

[62]This problem is being examined by one of the authors. See Bossaerts (1988).

contingent claims asset pricing models. One application, currently investigated by the authors, is the estimation of the continuous–time interest rate at a given moment in time from a cross–section of contingent claims written on *different* securities.[63]

[63]See Bossaerts and Hillion (1988).

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Table 1

Descriptive statistics

Series	Variable	Mean	Median	Std err	Min	Max	Autocorrelation ρ_1	ρ_3	ρ_3	ρ_6	ρ_9	ρ_{12}
1	f^o_{1t}	402.04	398.60	19.59	372.00	447.90	.966	.942	.917	.852	.780	.722
1	$\log(f^o_{1t}/f^o_{1t-1})$	-.0009	-.0011	.0107	-.0369	.0469	-.039	-.041	-.010	-.011	-.103	.046
1	$\log(f^o_{1t}/z_{1t})$	.1164	.1201	.0332	.0331	.1806	.770	.644	.646	.538	.455	.454
1	z_{1t}	357.90	360.00	18.13	340.00	400.00	.936	.901	.895	.849	.802	.779
1	T_{1t}	46.62	42.50	30.78	.00	120.00	.823	.719	.636	.419	.376	.398
1	c^o_{1t}	45.49	46.00	12.00	16.50	70.30	.761	.639	.639	.535	.464	.461
1	r^o_{1t}	.095	.095	.004	.086	.104	.942	.905	.859	.737	.686	.626
2	f^o_{2t}	402.99	401.55	18.12	367.70	443.30	.948	.910	.872	.759	.676	.616
2	$\log(f^o_{2t}/f^o_{2t-1})$	-.0008	-.0009	.0109	-.0386	.0474	-.043	-.033	-.016	-.018	-.089	.063
2	$\log(f^o_{2t}/z_{2t})$	.0872	.0766	.0442	.0060	.1974	.832	.727	.652	.417	.325	.298
2	z_{2t}	369.52	360.00	20.63	340.00	400.00	.943	.912	.891	.833	.798	.781
2	T_{2t}	56.35	51.00	37.79	.00	177.00	.685	.561	.460	.307	.114	.006
2	c^o_{2t}	35.24	32.75	15.96	5.00	76.00	.819	.720	.649	.413	.299	.272
2	r^o_{2t}	.096	.096	.004	.087	.104	.926	.893	.839	.727	.617	.521
3	f^o_{3t}	405.52	402.10	17.12	372.10	447.90	.950	.915	.880	.768	.661	.584
3	$\log(f^o_{3t}/f^o_{3t-1})$	-.0010	-.0008	.0105	-.0295	.0475	-.027	-.029	-.018	-.078	-.133	[illegible]
3	$\log(f^o_{3t}/z_{3t})$	.0736	.0719	.0408	-.0210	.2084	.812	.708	.661	.511	.347	[illegible]
3	z_{3t}	376.76	360.00	19.17	340.00	420.00	.944	.911	.900	.879	.845	.817
3	T_{3t}	78.84	77.50	18.59	.00	232.00	.818	.694	.597	.383	.160	-.049
3	c^o_{3t}	32.30	32.00	54.71	4.00	78.00	.803	.696	.652	.488	.313	.164
3	r^o_{3t}	.097	.096	.005	.086	.106	.921	.869	.820	.730	.655	.529

Table 2

GMM tests of the Black model

$$c_{it}^{o} = [f_{it}\mathcal{N}(d_1) - x_{it}\mathcal{N}(d_2)]\exp^{-r_{it}\tau_{it}}$$

Orthogonality conditions	Parameter estimates		Goodness-of-fit		
	$\hat{\sigma}$	t-stat.	χ^2	d.o.f	p-val.
$E[\epsilon_{it}] = 0$	.170	56.21	26.29	2	> .999
$E[\epsilon_{it}\, T_{it}] = 0$	.167	48.71	29.27	2	> .999
$E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = 0$	.169	45.07	28.94	2	> .999
$E[\epsilon_{it}\, r_{it}] = 0$	.170	57.09	26.99	2	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, T_{it}] = 0$	.172	72.51	34.81	5	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = 0$	.172	64.36	28.94	5	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, r_{it}] = 0$	.175	71.09	34.84	5	> .999
$E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = 0$	.167	49.67	45.89	5	> .999
$E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, r_{it}] = 0$	.169	68.63	45.64	5	> .999
$E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = E[\epsilon_{it}\, r_{it}] = 0$	.171	67.45	29.93	5	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = 0$	.173	74.36	43.06	8	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, r_{it}] = 0$	.175	80.64	44.71	8	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = E[\epsilon_{it}\, r_{it}] = 0$	.176	80.40	37.04	8	> .999
$E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = E[\epsilon_{it}\, r_{it}] = 0$	.169	71.45	56.97	8	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, r_{it}] =$ $E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = 0$	.174	82.29	47.03	11	> .999

Table 3

GMM tests of the Black model
Continuous-time interest R estimated from the data

$$c_{it}^{o} = [f_{it}\mathcal{N}(d_1) - x_{it}\mathcal{N}(d_2)]\exp^{-R\tau_{it}}$$

Orthogonality conditions	Parameter estimates				Goodness-of-fit		
	$\hat{\sigma}$	t-stat.	$\hat{R}$	t-stat.	χ^2	d.o.f	p-val.
$E[\epsilon_{it}] = 0$	.135	14.18	-.176	-5.15	.31	1	.425
$E[\epsilon_{it}\, T_{it}] = 0$	.114	6.58	-.150	-5.32	.50	1	.522
$E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = 0$	.100	4.29	-.150	-6.51	.42	1	.485
$E[\epsilon_{it}\, r_{it}] = 0$	.135	14.03	-.174	-5.25	.17	1	.319
$E[\epsilon_{it}] = E[\epsilon_{it}\, T_{it}] = 0$	.166	38.38	-.017	-.837	29.91	4	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = 0$	.168	39.65	-.028	-1.66	18.53	4	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, r_{it}] = 0$	.157	25.05	-.077	-3.72	15.26	4	.996
$E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = 0$	.140	16.98	-.091	-5.79	6.65	4	.845
$E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, r_{it}] = 0$	.160	27.02	-.028	-1.25	36.15	4	> .999
$E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = E[\epsilon_{it}\, r_{it}] = 0$	.164	26.08	-.033	-1.94	18.86	4	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = 0$	.162	45.99	-.034	-2.22	33.04	7	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, r_{it}] = 0$	.163	42.63	-.042	-2.60	38.03	7	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = E[\epsilon_{it}\, r_{it}] = 0$	.170	41.84	-.027	-1.68	27.39	7	> .999
$E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = E[\epsilon_{it}\, r_{it}] = 0$	.153	31.32	-.041	-2.86	47.38	7	> .999
$E[\epsilon_{it}] = E[\epsilon_{it}\, r_{it}] =$ $E[\epsilon_{it}\, T_{it}] = E[\epsilon_{it}\, \log(f_{it}^{o}/x_{it})] = 0$	.164	50.54	-.037	-2.62	39.06	10	> .999

Table 4

MSM tests of the Black model
Tests of the first moment condition $E[c^o_{it} - \hat{c}_{it}] = 0$,
with $c^o_{it} = [f_{it}\mathcal{N}(d_1) - x_{it}\mathcal{N}(d_2)]\exp^{-r_{it}\tau_{it}}$

Number of simulations	Parameter estimates		Goodness-of-fit		
K	$\hat{\sigma}$	t-stat.	χ^2	d.o.f.	p-val.
20	.198	24.194	1.805	2	.594
40	.177	30.920	1.597	2	.550
60	.186	32.826	9.859	2	.993
80	.183	34.475	10.612	2	.995
100	.183	33.204	18.287	2	> .999
GMM results	.170	56.214	26.286	2	> .999

Table 5

MSM tests of two call pricing models with stochastic variance
Tests of the first moment condition $E[c^o_{it} - \hat{c}^m_{it}] = 0$

Model	Variance reduced	K	Parameter Estimates				Goodness-of-fit		
			$\hat{\sigma}$	t-stat.	$\hat{\beta}$	t-stat.	χ^2	d.o.f.	p-val.
$\sigma_{it} = \sigma f^{\beta}_{it}$	no	100	.352	.026	-.121	-.019	5.023	1	.975
$\sigma_{it} = \sigma f^{\beta}_{it}$	yes	10	1.726	.078	-.388	-.181	22.642	1	> .999
$\sigma_{it} = \sigma \log\left(\frac{f_{it}}{100}\right)$	no	100	.125	32.365	–	–	4.766	2	.908
$\sigma_{it} = \sigma \log\left(\frac{f_{it}}{100}\right)$	yes	10	.126	39.93	–	–	20.424	2	> .999

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