"SIMULATION GRAPHS FOR DESIGN AND ANALYSIS OF DISCRETE EVENT SIMULATION MODELS"

by

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ABSTRACT

This paper contains a rigorous definition of discrete event dynamical systems using the mathematics of graph theory. The generality of these graphs is established. This result has implications for the design of discrete event simulation modeling languages as well as for the analysis of discrete event dynamical models.

1. INTRODUCTION

Many intuitively appealing ways of representing the relationships between events in a discrete **event dynamical system** (DEDS) exist. Evans et al. (1967) use schematic chains of event occurrences to depict the general logic flow in a discrete event simulation model implementation. Gordon (1969) uses flow charts to model the behavior of a DEDS. Suri (1984) uses a tree representation to describe the logic for a specific simulation run.

A DEDS network model, that gives quantitative attributes to the elements of a relationship graph, is presented in Schruben (1983). This graph structure, called an *event graph*, has logical and temporal expressions attributed to the edges, while state transition expressions are attributed to the vertices. Several variations of this event graph have appeared in the literature [Pegden, 1987; Sargent, 1988; Law and Kelton, 1989].

In this paper, we present a rigorous mathematical definition of DEDS using graphs. It is shown that these graphs can represent any discrete event simulation. Indeed, they can represent any computer program whatsoever. This result permits us to define a *Simulation Graph* that can be used in the design of discrete event simulation languages as well as the analysis of discrete event simulation models.

The goal of research on the mathematical foundations of discrete event simulation modeling is the development of a general and practically useful mathematical base for this important class of models. The lack of such a base accounts in part for the apparent *ad-hoc* and complex reputation of these models. In a recent report of the *Panel on Future Directions in Control Theory* (1988), Wendel H. Fleming, the panel's chairman, writes: "discrete event dynamical systems exist in many technological applications, but there are no models of discrete event systems that are as concise or computationally as feasible as are differential equations for continuous variable dynamical systems. (...) A complete theory of discrete event dynamical systems may require fundamentally new approaches, involving techniques from algebra, graph theory, discrete mathematics, computational mathematics, and combinatorics. More specifically, the following modeling questions should be answered: how does one formally specify and reason about discrete event dynamical systems?" Here, we propose graph theory as an effective base for discrete event systems.

There have been recent advances in obtaining a general understanding discrete event models. See, for instance, Nance (1981), Overstreet (1982), Radiya and Sargent (1987) and Zeigler (1984). While theoretical structures for discrete event simulations and practical model building environments have been developed [eg., TESS (Pritsker and Standridge, 1987) and CINEMA (Systems Modeling Corp., 1985)], an explicit general mathematical formulation of discrete event simulations with practical utility has not yet emerged. *This paper formalizes the concept of using graph structures to represent DEDS models.*

Modeling tools can be evaluated by two characteristics: their generality and their utility. Generality refers to the ability to construct a *valid* representation of a wide class of systems. Utility is the ability to analyze specific instances of a modeled system and make valid inferences. In the parlance of theoretical Computer Science, generality of a modeling tool is more precisely defined as its *modeling power* whereas its utility is known as its *decision power* [Peterson, 1977]. These characteristics usually represent a trade-off. As we increase the generality of our modeling tools, they tend to become more abstract and conceptual. This, in turn, makes it more difficult to determine various properties of specific systems, reducing the tool's utility.

This trade-off is more explicitly depicted in Figure 1. The Turing Machine [Hopcroft and Ullman, 1979] is general; it is accepted as being able to model any modern computational procedure ("Church's thesis"). However, it has limited utility, since even the simplest system can be difficult to represent as a Turing Machine and most specific questions are undecidable. The highest levels of Zeigler's abstract conceptual DEVS hierarchy represents an extreme example of this trade-off in favor of generality [Zeigler, 1984]. Examples of modeling tools with limited modeling power but excellent decision power include most of the analytical modeling tools of Operations Research (OR) and Computer Science (for example, game theory, queueing theory, linear programming, finite automata). Typically, OR methodologies have high decision power but are limited in generality. Much of OR research is directed at increasing the applicability of these tools. Most analytical techniques can be observed to have evolved from being very specific to being more general (eg., M/M/1 queues to GI/GI/s systems). However, as these techniques have become more general, they have become more complex often impairing their decision power. The development of simulation languages, on the other hand, has been typically driven by the needs of

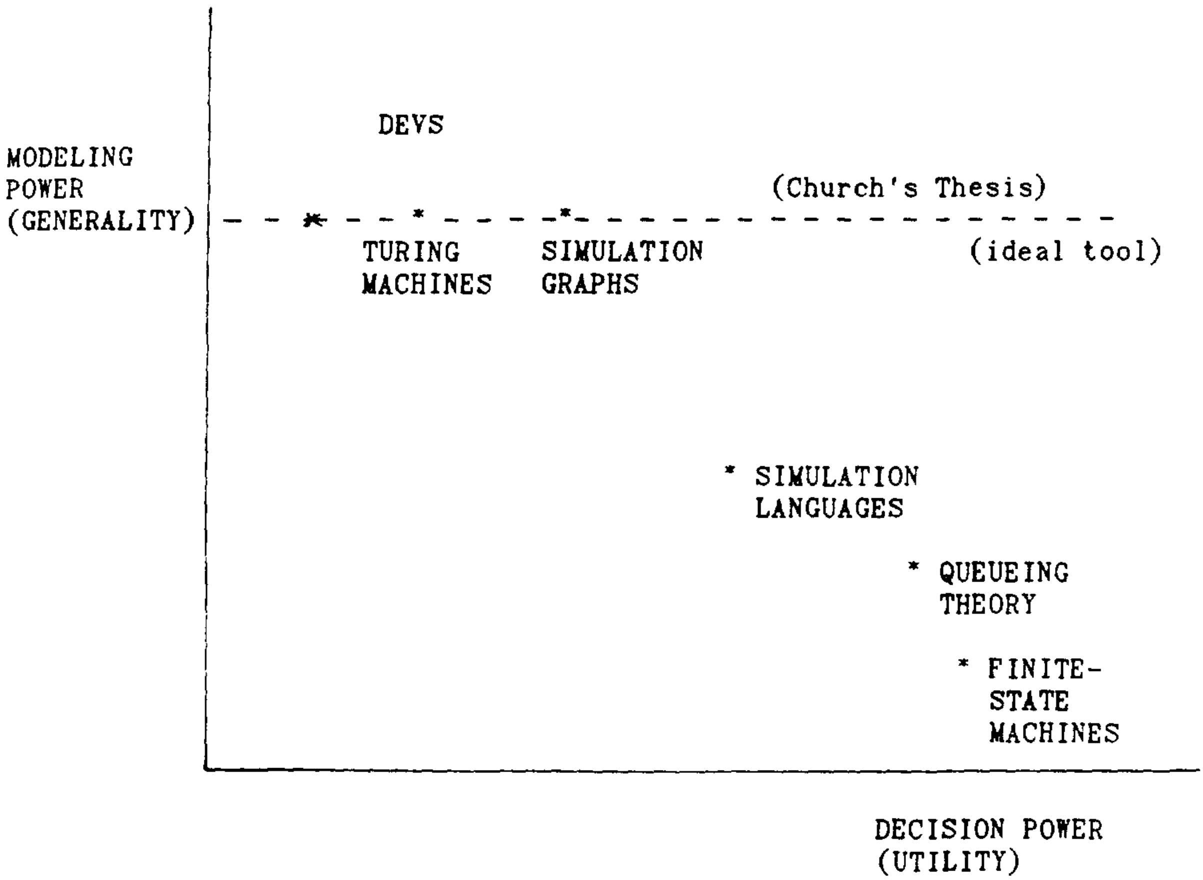


FIGURE 1: MODELING POWER vs DECISION POWER

the users (see, for instance, Gordon (1979)). As more and more features were built into these languages, both their modeling and decision power increased.

We will show that the Simulation Graphs presented here retain the modeling power of Turing Machines while achieving considerably better decision power for a special class of models: discrete event dynamical simulations.

This paper is organized as follows: Section 2 defines Simulation Graphs and exhibits its decision power. An example is also included. The modeling power of these graphs is discussed in Section 3. Section 4 Section 4 discusses some implications. Conclusions are presented in Section 5.

2. THE DECISION POWER OF THE SIMULATION GRAPHS

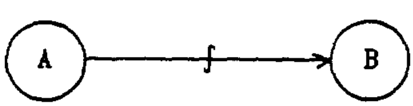
2.1. Simulation Graphs:

The elements of a DEDS model are state variables that describe the system, events that change the values of state variables, and the relationships between events. A Simulation Graph is a structure of the objects in a discrete event system that facilitates the development of a correct simulation model. Hence, the emphasis is directly on system events; system entities are represented implicitly.

Events are represented on the graph as vertices. Here, each vertex is associated with a set of changes to state variables.

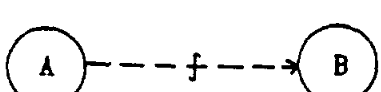
Relationships between events are represented in a Simulation Graph as directed edges between pairs of vertices. Each edge is associated with sets of logical and temporal expressions. Basically, the edges define under what conditions and after how much of a time delay one event will schedule another event to occur.

Two classes of edges are distinguished: a (solid) *scheduling* edge and a (dashed) *cancelling* edge. Specifically,



indicates that *event A may schedule event B with some time delay*.

Similarly,



indicates that *event A may cancel any currently scheduled occurrence of event B*.

In defining a Simulation Graph ***Model*** in Section 2.4, we will associate with each edge a set of conditions that must be true in order for the edge origination vertex to schedule or cancel the edge termination vertex. Also associated with each edge will be an expression or function that will give a delay time telling how long until the scheduled event will occur. There can be multiple edges between any pair of vertices; the edges can point in either direction or may simply point from a vertex to itself. To simplify the notation, if $t \equiv 0$ (no time delay) or if $i \equiv (1=1)$ (condition always true), then these parameters are omitted from the graph. Similarly, if $k \equiv 1$ (same event instances), then j is also omitted from the graph.

In a Simulation Graph Model, one vertex may also pass a string of parameters to another vertex. This parameter string carries information pertaining to a particular event instance as well as the execution order priority of that event.

Note that the customary notion of an *event procedure* in a discrete event simulation program may involve a subgraph, typically a set of vertices connected by edges with no time delays.

Before we introduce a mathematical definition of Simulation Graphs, we present an example.

2.2. An Example: Oil Tanker Problem:

A sample study problem from the texts by Schriber (1977), and Law and Kelton (1989) is used to illustrate the construction of a Simulation Graph. This system is represented as a process block diagram in Schriber, as a process network in Pritsker (1984) and as a modified Petri net graph in Torn (1981). The system description is adapted from Law and Kelton (Problem 2.23):

> A port with three docks is used to load oil. Three different types of tanker ships use the port. The distribution for the random time between arrivals of a type j tanker, $t_a(j)$, is known. The random time required to load a type j tanker, $t_l(j)$, also has a known distribution. There is a single tugboat in the harbor that is required for all tanker berthing or deberthing. It takes one hour for a tanker to be berthed or deberthed. The area experiences storms of random frequency and random duration. The time between storm occurrences is t_s and the duration of a storm is t_d. No berthing or deberthing operation can be initiated while the storm is in progress and any berthing operation in progress when the storm hits is aborted. However, the storm does not interrupt any deberthing in progress. A fleet of another tanker type wishes to begin using the port for shipment of oil on a specific route. The loading times, $t_l(4)$, and (random) round-trip time, $t_a(4)$, for this fourth type of tanker have known distributions. The travel time for the tug when not towing a tanker is insignificant. The tug will attend to tankers requesting berthing first.

The model is taken from Schruben (1982). The state variables we selected for this model are:

1. B represents the number of tankers waiting for berthing; the range of B is the set of non-negative integers.

2. D represents the number of tankers waiting for deberthing; the range of D is also the set of non-negative integers.

3. P represents the number of empty docks at the port; the range of P is the set $\{0,1,2,3\}$.

4. T denotes the status of the tug; it has a range of $\{-2,-1,0,1\}$ with $-2 \equiv$ berthing aborted by storm, $-1 \equiv$ deberthing a tanker, $0 \equiv$ berthing a tanker, $1 \equiv$ available.

5. S denotes the status of the storm; it has a range of $\{0,1\}$ with $0 \equiv$ storm in progress and $1 \equiv$ no storm.

6. QB and QD are lists of tankers requesting berthing and deberthing, respectively. Both variables are single dimensional arrays denoting the type of the tanker. More precisely, QB[i] = j (QD[i] = j) denotes that the i^{th} tanker requesting berthing (deberthing) is of type j. Here, $i \in \{1,2,3,...\}$ and $j \in \{1,2,3,4\}$.

The edge conditions for the model are:

(i) (No storm in progress, a berth is available, the tug is available) S = 1, P > 0, T = 1, i.e., (S*P*T>0) (* denotes multiplication).

(ii) (No storm in progress, a berth is available, berthing queue is non-empty) S = 1, P > 0, B > 0, i.e., (S*P*B>0).

(iii) (No storm in progress, deberthing queue is non-empty, berthing queue is empty) S = 1, D > 0, B = 0, i.e., (S*D>0)&(B=0).

(iv) (No storm in progress, the tug is available, berthing queue is empty) S = 1, T = 1, B = 0, i.e., (S*T=1)&(B=0).

(v) (No storm in progress, berthing queue is empty, deberthing queue is non-empty) S = 1, B = 0, D > 0, i.e., (S*D>0)&(B=0).

(vi) (No storm in progress, berthing queue is non-empty) S = 1, B > 0, i.e., (S*B>0).

(vii) (The tug is berthing a tanker) (T=0).

(viii) (Berthing is aborted by a storm) (T=-2).

(ix) (The tug is available, berthing queue is non-empty, a berth is available) T = 1, B > 0, P > 0, i.e., (T*B*P>0).

(x) (The tug is available, berthing queue is empty, deberthing queue is non-empty) T = 1, B = 0, D > 0, i.e., (T*D>0)&(B=0).

The event descriptions are presented in Table 1.

TABLE 1

Event Type	Event Description	Parameters	State Changes
AR(j)	Type j tanker arrival	j = k	B = B + 1 QB[tail] = j
BB(j)	Begin berthing tanker of type j		T = 0 B = B - 1 k = QB[head]
EB(j)	End berthing tanker of type j	j = k	P = P - 1 T = 1
BL(j)	Begin loading tanker of type j	j = k	
EL(j)	End loading tanker of type j	j = k	D = D + 1 QD[tail] = j
BD(j)	Begin deberthing tanker of type j		T = -1 D = D - 1 P = P + 1 k = QD[head]
ED(j)	End deberthing tanker	j = k	T = 1
BS	Begin storming		S = 0
ES	End storming		S = 1
BS2	Abort berthing tanker		T = -2

The associated Simulation Graph is presented in Figure 2. Note that there may be several possible representations of this model as a Simulation Graph. For example, by applying the "rules of thumb" in Schruben (1983), this model can be represented as in Figure 7 in Schruben (1982).

2.3. Definition of Simulation Graph Models:

A directed graph is characterized as an ordered triple $(V(G),E(G),\Psi_G)$ consisting of a nonempty set $V(G)$ of vertices, a set $E(G)$, disjoint from $V(G)$, of edges, and an incidence function Ψ_G that associates with each edge of G an ordered pair of (not necessarily distinct) vertices of G. If e is a directed edge joining vertices u and v, then $\Psi_G(e) = uv$ [Bondy and Murty, 1976].

We define a *Simulation Graph* as an ordered quadruple $\mathcal{G} = (\mathcal{V}(\mathcal{G}),\mathcal{E}_s(\mathcal{G}),\mathcal{E}_c(\mathcal{G}),\Psi_\mathcal{G})$, where $\mathcal{V}(\mathcal{G})$ is the *vertex set* of $\mathcal{G}$, not necessarily finite, $\mathcal{E}_s(\mathcal{G})$ is the *set of scheduling edges* of $\mathcal{G}$, $\mathcal{E}_c(\mathcal{G})$ is the *set of cancelling edges* of $\mathcal{G}$, and $\Psi_\mathcal{G}$ is the *incidence function*.

We then define the objects in a simulation model as the following ordered sets:

$\mathcal{F} = \{ f_v : v \in \mathcal{V}(\mathcal{G}) \}$, which is the set of *state transition functions* associated with (event) vertex v;

$\mathcal{C} = \{ C_e : e \in \mathcal{E}_s(\mathcal{G}) \cup \mathcal{E}_c(\mathcal{G}) \}$, which is the set of *edge conditions*.

$\mathcal{T} = \{ t_e : e \in \mathcal{E}_s(\mathcal{G}) \}$, which is the set of *edge delay times*; note that $t_e \in R^+_{0,\infty}$ for all edges.

$\Gamma = \{ \gamma_e : e \in \mathcal{E}_s(\mathcal{G}) \}$, which is the set of *event vertex execution*

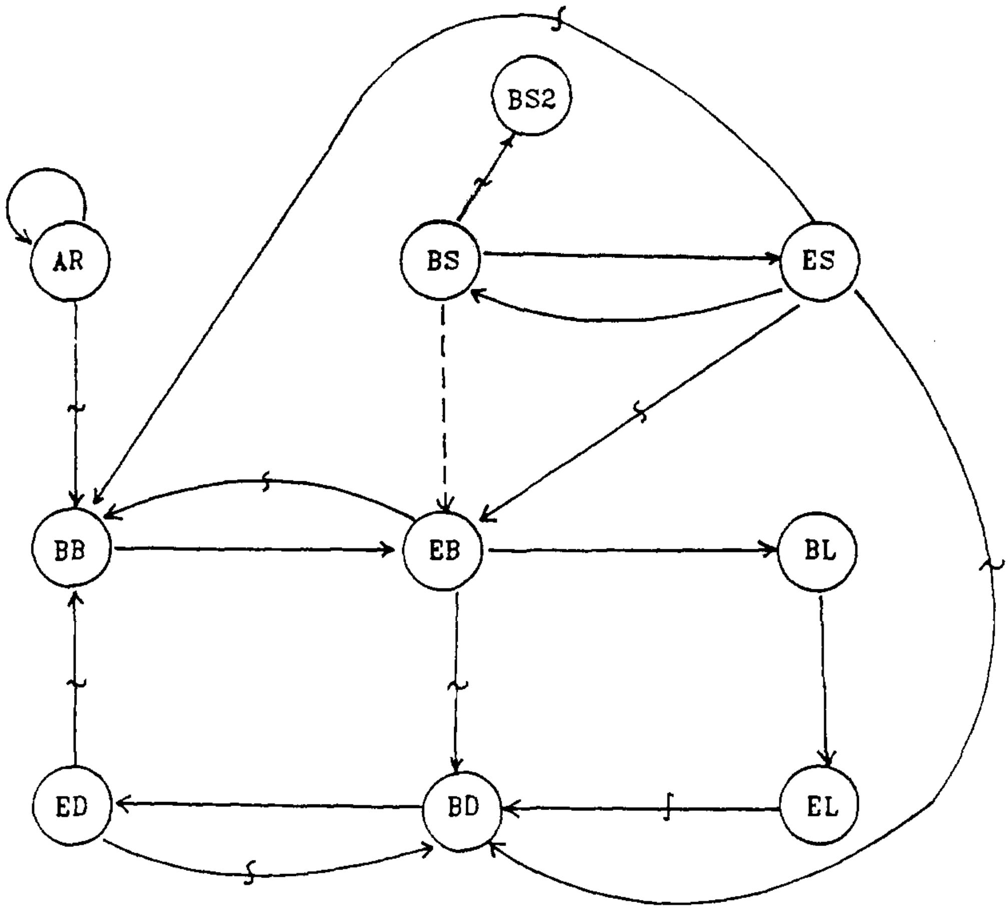


FIGURE 2: SIMULATION GRAPH FOR THE OIL TANKER PROBLEM

priorities. γ_e is a scalar assuming values from the set of non-negative real numbers extented to include positive infinity. Notice that the event execution priorities are associated with edges, not vertices. Therefore, the priority of a vertex can change dynamically depending on when and how it is scheduled. This assignment yields a *complete* ordering of execution priorities. Schemes of static event execution priorities, attaching static priorities to event vertices (Som and Sargent, 1988), are ambiguous in that they provide only a partial ordering of event execution. Each vertex has an execution priority with smaller values corresponding to higher priorities, and with 0 representing the highest possible event execution priority.

The key idea here is that a Simulation Graph specifies the relationships between the elements of the sets of objects in a simulation model. Then, we can formally define a Simulation Graph *Model* as:

$$\mathcal{S} = (\mathcal{F}, \mathcal{C}, \mathcal{T}, \Gamma, \mathcal{G})$$

The first four sets in the above five-tuple define the objects in a simulation. The role played by the Simulation Graph, $\mathcal{G}$, in the definition of a Simulation Graph Model, $\mathcal{S}$, is analogous to the role of the incidence function in the definition of a directed graph. The incidence function organizes the individual objects (vertices and edges) into a meaningful graph. The Simulation Graph, G (a graph of *indices*), organizes the objects ($\mathcal{F}$, $\mathcal{C}$, $\mathcal{T}$, Γ) into a meaningful simulation model.

In a Simulation Graph Model, parameter strings can be passed from one vertex to another through vertex and edge attributes. A *vertex attribute list* is a string of state variables associated with a particular vertex. An *edge attribute list*, on the other hand, is a string of expressions associated with a particular edge. When the origination vertex of an edge is executed, the expressions in the edge attribute list are evaluated. When the destination vertex is subsequently executed, the state variables in its attribute list take on the values that had been computed for the expressions in the scheduling edge's attribute list. The vertex and edge attribute lists are useful in scheduling or cancelling specific instances of system events.

Example: The Oil Tanker Problem (Continued):

We will use our example for a concrete illustration of the sets in a simulation model, $\mathcal{S}$. The following objects make up the model (for exposition purposes, the vertex indices are not included):

$\mathcal{V}(\mathcal{G}) = \{v_1, v_2, \ldots, v_{10}\} = \{AR, BB, EB, BL, EL, BD, ED, BS, ES, BS2\}$,

$\mathcal{E}_s(\mathcal{G}) = \{e_1, e_2, \ldots, e_{17}\} = \{(AR,AR), (AR,BB), (BB,EB), (EB,BB), (EB,BL), (EB,BD), (BL,EL), (EL,BD), (BD,ED), (ED,BD), (ED,BB), (BS,ES), (ES,BS), (BS,BS2), (ES,BB), (ES,EB), (ES,BD)\}$,

$\mathcal{E}_c(\mathcal{G}) = \{e_{14}\} = \{(BS,EB)\}$,

$\Psi_{\mathcal{G}}$: see figure 2,

$\mathcal{F}$: State Transition Functions

$f_{AR} = \{ j = k, B = B + 1, QB[tail] = j \}$,

$f_{BB} = \{ B = B - 1, T = 0, k = QB[head] \}$,

$f_{EB} = \{ j = k, P = P - 1, T = 1 \}$,

$f_{BL} = \{ j = k \}$,

$f_{EL} = \{ j = k, D = D + 1, QD[tail] = j \}$,

$f_{BD} = \{ D = D - 1, T = -1, P = P + 1, k = QD[head] \}$,

$f_{ED} = \{ j = k, T = 1 \}$,

$f_{BS} = \{ S = 0 \}$,

$f_{ES} = \{ S = 1 \}$,

$f_{BS2} = \{ T = -2 \}$.

$\mathcal{C}$: Edge Conditions

$C_{AR,BB} = \{ S * P * T > 0 \}$,

$C_{EB,BB} = \{ S * P * B > 0 \}$,

$C_{EB,BD} = \{ S * D > 0, B = 0 \}$,

$C_{EL,BD} = \{ S * T = 1, B = 0 \}$,

$C_{ED,BD} = \{ S * D > 0, B = 0 \}$,

$C_{ED,BB} = \{ S * B > 0 \}$,

$C_{BS,BS2} = \{ T = 0 \}$,

$C_{ES,EB} = \{ T = -2 \}$,

$C_{ES,BB} = \{ T*B*P > 0 \}$,

$C_{ES,BD} = \{ T*D > 0, B = 0 \}$.

$\mathcal{T}$: Edge Delay Times

$t_{AR,AR} = t_a$,

$t_{BB,EB} = 1$,

$t_{BL,EL} = t_1$,

$t_{BD,ED} = 1$,

$t_{BS,ES} = t_d$,

$t_{ES,BS} = t_s$,

$t_{ES,EB} = 1.$

$\Gamma = \{\gamma_{AR}, \gamma_{BB}, \gamma_{EB}, \gamma_{BL}, \gamma_{EL}, \gamma_{BD}, \gamma_{ED}, \gamma_{BS}, \gamma_{ES}, \gamma_{BS2}\} = \{2, 1, 2, 2, 2, 2, 2, 1, 2, 1\}$.

This model can be executed using the algorithm presented in the next section.

2.4. Execution of a Discrete-Event Simulation

The simulation (execution) of a Simulation Graph model requires the services of a support routine, which will be referred to as the *Event Scheduling Function* (ESF). Basically, ESF maintains two crucial variables: τ, the global simulation clock, and $\mathcal{L}$, the list of scheduled events (the events list). In addition, it also supports a scalar stochastic process of independent uniform random variables, U, on [0,1]. Appropriate transformations of U provide any random process needed in the simulation. Notice that the *non-model-specific objects* such as the events list and the global simulation clock are *not* included in the definition of a Simulation Graph Model, $\mathcal{S}$.

The events list is an ordered set of triples. That is,

$$\mathcal{L} = \{ (t_1,\gamma_1,v_1(k)),\ (t_2,\gamma_2,v_2(k)), \ldots.\ \}.$$

where t_i represents the *event execution time*, γ_i represents the *event execution priority* and v_i represents the associated event vertex. The index k denotes specific instances of these events. The triples, which will be referred to as *event records*, are ordered by the event execution time. Those events that are scheduled to occur at the same simulated time are further ranked by their event execution priorities. Recall that event execution priorities can be assigned dynamically using the event and edge attribute lists.

For convenience, we also define the following sets:

S_v is the set of state variables possibly altered at vertex v ($v \in \mathcal{V}(G)$);

E_v is the set of state variables involved in the conditions on the edges emanating from vertex v ($v \in \mathcal{V}(G)$);

A_v, *the vertex attribute list*, is a set of state variables associated with vertex v ($v \in \mathcal{V}(G)$). Similarly, A_e, the *edge attribute list*, is simply a string of expressions associated with edge e ($e \in \mathcal{E}_s(G) \cup \mathcal{E}_c(G)$).

Based on the above framework, the execution of a Simulation Graph model is carried out in the following manner:

<u>INITIALIZE (Run Initialization)</u>

<u>Step 1.</u> Initialize global simulation clock:

$$\tau := 0$$

<u>Step 2.</u> Insert the first event record into the events list: this is usually the event where the initial condition of the model is specified by assigning the initial values to the state variables:

$$\mathcal{L} := \mathcal{L} \cup \{ (0, \Upsilon_0, v_0) \}$$

<u>EXECUTE (Execution of the Model)</u>

<u>Step 1.</u> Remove the first event record from $\mathcal{L}$; this is the event with the smallest event execution time (ties are resolved based on the

event execution priorities; remaining ties are broken arbitrarily):

$$\mathcal{L} := \mathcal{L} \setminus \{(t_i, \gamma_i, v_i(k))\}$$

Step 2. Update the simulation clock:

$$\tau := t_i$$

Step 3.1. Assign the values of the state variables in the vertex attribute list, $A_{v_i(k)}$, if the list is not empty.

Step 3.2. Evaluate the state variables in $S_{v_i(k)}$:

$$S_{v_i(k)} := f_{v_i(k)}(S_{v_i(k)})$$

Step 4. Schedule and/or cancel further events:

for all edges emanating from vertex v_i:

if $C_{v_i v_j}(E_{v_i}) = 1$, then

compute values of expressions in the

corresponding edge attribute list

(including execution priorities),

generate the inter-event time, t_j and

$$\mathcal{L} := \mathcal{L} \cup \{(\tau + t_j, \gamma_j, v_j(k))\}$$

(Note that for cancellations, we have

$$\mathcal{L} := \mathcal{L} \setminus \{(\tau, \gamma_j, v_j(k))\} \text{ .)}$$

For completeness, in Step 4, any event scheduling is done before any event cancellation.

Step 5. Terminate the execution of the simulation if any of the following situation is reached:

(i) $\tau \geq T_{STOP}$,

(ii) $(t_{end}, \gamma_{end}, v_{end})$ has just been executed,

Otherwise, go to Step 1 of EXECUTE.

Here, T_{STOP} represents a pre-determined stopping time for the simulation and v_{end} represents an end-of-simulation event.

2.5 Computer Implementation of Simulation Graphs

Simulation Graph models can be implemented using a high level programming language or a general purpose simulation modeling language by possibly coding subgraphs into separate procedures [Som and Sargent, 1988]. Alternatively, they can be directly implemented using Σ [Schruben and Briskman, 1988]. Σ is an interactive graphics program specifically designed to build, test, and experiment with discrete event dynamical systems on personal computers using Simulation Graphs. The development of Σ has been different from other discrete event simulation languages. For the latter, the addition of built-in functions has increased both their modeling and decision power, moving in the northeastern direction in Figure 1. Σ, however, starts with a basic structure having complete modeling power; any addition simply increases its decision power, moving in the eastern direction along the horizontal dotted line in Figure 1. Building a model, implementing the model, and experimenting with its implmentation are some of the steps in a simulation study. Σ is intended to facilitate model implementation by allowing the user to construct executable models by drawing their graphs with a mouse.

3. THE MODELING POWER OF SIMULATION GRAPHS

In Figure 1 of Section 1, it is claimed that Simulation Graph Models have the same modeling power as Turing Machines. In this section, we will formalize this assertion. This has implications both on the design of discrete event simulation languages and the analysis of simulation models as we will illustrate.

3.1 Turing Machine Representation:

We first define the basic Turing Machine. For other variants, the reader is referred to Hopcroft and Ullman (1979). The Turing machine is a simple mathematical model of a computer. Despite its simple structure, it captures the essential features of real computing machines, and is accepted (by Church's Thesis) as the formalization of any algorithm or computational procedure.

Following Hopcroft and Ullman (1979), we define a basic Turing Machine as consisting of a finite control, an input tape that is divided into cells and a tape head that scans one cell of the tape at a time. The tape has a leftmost cell, but is infinite to the right. Each cell of the tape may hold exactly one of a finite number of tape symbols. Formally, the Turing Machine is defined as:

$$\mathcal{M} = (Q, \Sigma, T, \delta, q_0, B, F)$$

where

Q is the *finite set of states*,

T is the *finite set of allowable tape symbols*,

B is the symbol for *blank*, $(B \in T)$,

Σ is the *set of input symbols*, $(\Sigma \subset T \setminus \{B\})$,

δ is the *next-move function*, $(\delta : Q \times T \to Q \times T \times \{L,R\})$,

q_0 is the *start state*, $(q_0 \in Q)$,

F is the *set of final states*, $(F \subset Q)$.

To demonstrate that Simulation Graphs and Turing Machines have the same modeling power, one needs to show that every system specified in one formalism can be simulated by a system specified in the other one.

The fact that a Turing Machine, $\mathcal{M}$, can simulate a Simulation Graph Model, $\mathcal{S}$, is established through Church's Thesis. A constructive proof is also presented in Schruben and Yucesan (1987). We next establish the fact that a Simulation Graph Model, $\mathcal{S}$, can simulate a Turing Machine, $\mathcal{M}$.

This is achieved through the construction of a a Simulation Graph Model of a basic Turing Machine. The *tape symbols* of $\mathcal{M}$ will be represented by *state variables* in $\mathcal{S}$. Hence, the *input symbols* for $\mathcal{M}$ will simply consist of a subset of the state variables of $\mathcal{S}$. Another state variable will be used to denote the *state of the finite control* of the Turing Machine. Hence, STATES of $\mathcal{S}$ will completely describe the set of states of $\mathcal{M}$, Q. Table 3 summarizes the correspondance between the elements of $\mathcal{M}$ and $\mathcal{S}$. Here, β_i denotes the i^{th} state variable of the model, whereas RANGE[β_i] depicts the set of possible values that may be assumed by that state variable. Figure 3 depicts the Simulation Graph representation of a Turing Machine. Here, vertex INT is the *initialization* vertex, where the initial state of the finite control, FC, is established by assigning the initial values to appropriate state variables. An input string of length L is also read in. Notice that the

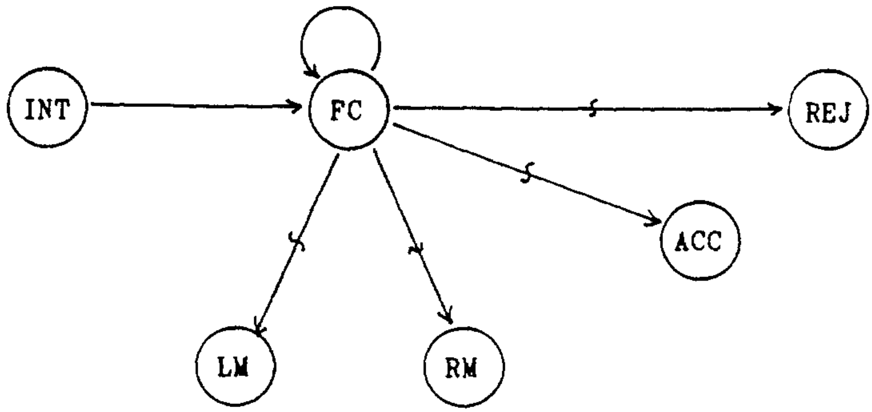


FIGURE 3: SIMULATION GRAPH REPRESENTATION OF TURING MACHINES

tape of $\mathcal{M}$ is represented in $\mathcal{S}$ by a one-dimensional array (a state variable) of infinite length. Each element of the array will correspond to a cell of the tape. The input string will be stored in this state variable, ARRY(.). Another state variable, say HEAD, will also keep track of the current position of the tape head. Finally, the first element of the array, ARRY(1), is read and HEAD is assigned the value of 1 corresponding to the placement of the tape head on the first cell of the input tape.

The vertex FC represents the *finite control*. Note that the *next move function* of $\mathcal{M}$

$$\delta : Q \times T \rightarrow Q \times T \times \{L,R\}$$

is represented *implicitly* in $\mathcal{M}$. That is, depending on the current state of the finite control, which is depicted by the current values of the state variables, and the current input symbol, given by the array element just read, the edge conditions, $\mathcal{C}$, determine the next vertex to be scheduled. This vertex specifies the move of the tape head as well as the new state of the finite control.

Vertices LM and RM represent *the left move (L)* and *the right move (R)*, respectively. At these vertices, the values of the state variables, hence, the state of the finite control, are updated. The tape head is also moved one cell to the right or to the left by pointing at the proper array element. For instance, a right move will be represented by HEAD = ARRY(CURRENT + 1), where CURRENT denotes the array element that was read last.

Vertices ACC and REJ represent the set of *final states*. More specifically, vertex ACC represents the final state where $\mathcal{M}$ stops with a "YES" answer, accepting the input string. Vertex REJ, on the other hand, represents a final state where the input string is rejected.

The execution of the above described Simulation Graph model, $\mathcal{S}$, of a Turing Machine, $\mathcal{M}$, is carried out until the first execution of either vertex ACC or vertex REJ.

Thus, with the above construction, we have shown the simulation of a Turing Machine by a Simulation Graph model.

TABLE 3

$\mathcal{M}$	$\mathcal{S}$
Q	STATES $\subset$ RANGE[$\mathcal{E}_1$]X..XRANGE[$\mathcal{E}_m$]
T	$\cup_i$RANGE[$\mathcal{E}_i$] ($1 \leq i \leq m$), where $\mathcal{E}_i$ is a state variable
Σ	Subset of $\cup_i$RANGE[$\mathcal{E}_i$] ($1 \leq i \leq m$)
δ	$\mathcal{C}$, set of edge conditions
q_0	Set of initial values for the state variables
F	Set of final values for the state variables

This result shows that any simulation, indeed any computer program as we know it today, can be modeled using Simulation Graphs.

An example of a Simulation Graph Model of a Turing Machine used for bit parity checking executed using Σ is presented in Schruben and Yucesan (1987).

3.2 DEVS Representation

As Zeigler (1984) points out, "the concept of *simulation* employed here is a rather weak one, and states little more than that a [Turing Machine] can be used as a computer to generate the behavior" of a Simulation Graph Model, $\mathcal{S}$. "It leaves the burden of writing and interpreting this simulation to the user."

A perspective on this result can be obtained by considering the hierarchical system specification scheme that Zeigler introduces (1984). At the "lower" levels of this hierarchy, the system is considered as a black box. Hence, only the system's input/output (I/O) behavior is apparent to the observer/user. As we move "up" the hierarchy, this I/O behavior is more precisely defined, leading us to the "top" level of the hierarchy where the internal structure of the system is completely apparent. This internal structure, in turn, fully accounts for the observed I/O behavior of the system. For a detailed discussion of the hierarchical system specification, the reader is referred to Zeigler (1984). Zeigler also discusses the DEVS (Discrete Event System Specification) formalism in this hierarchical approach. DEVS is the following structure:

$$\langle \text{INPUTS, STATES, OUTPUTS}, \delta, \lambda, ta \rangle.$$

From our previous discussion, it is clear that Simulation Graph Models comfortably fit into the DEVS framework. These models are autonomous. Then, with slight modifications for this case, the definition of STATES, OUTPUTS, δ, and λ directly carry over from DEVS to $\mathcal{S}$. In the context of Simulation Graph Models, INPUTS = $\emptyset$. The time advance mechanism needed to execute (simulate) the model is provided by the support routine, the

Event Scheduling Function (ESF). ESF also maintains the global simulation clock, τ, as well as the events list, $\mathcal{L}$ (see Schruben and Yucesan (1987) for details).

Since we have just discussed how Simulation Graphs fit into the DEVS framework, we can conclude that they are not only useful for depicting the I/O behavior of any system (Turing machine representation), but also sufficiently powerful to describe the internal structure of any autonomous discrete event system (DEVS representation).

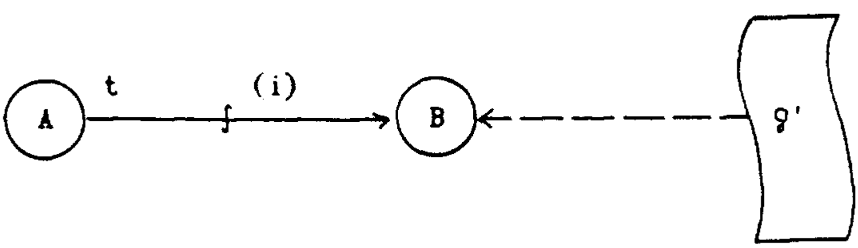
4. THE DESIGN OF SIMULATION LANGUAGES

4.1. Cancelling Edges

The correspondance between Simulation Graphs and Turing Machines permits the classification of the various features of a simulation language (or environment) as being *"necessary"* or *"a modeling convenience."* We consider here three examples: event cancelling, event attribute passing, and event execution priorities.

For instance, in establishing the equivalence between $\mathcal{S}$ and $\mathcal{M}$, it is observed that cancelling edges in Simulation Graphs are not necessary. Cancelling edges are typically used to model *disruptions* in a process. A machine breakdown that shuts down a production line or a storm that closes an airport are examples. When such a disruption occurs, it may be that certain events that were scheduled will be cancelled. Event cancellation is a feature of most modern simulation languages. It is implemented using a "CANCEL" statement in Simscript II.5 [CACI, 1976], an "RMOVE" subroutine or a "PREEMPT" node in SLAM II [Pritsker, 1984], a "PREEMPT" or "ABORT" node in Insight [Roberts, 1983] and an "REMOVE" block in SIMAN [Pegden, 1985].

A Simulation Graph Model of such a system takes the following form:



Here $\mathcal{G}'$ denotes the subgraph which eventually causes the cancellation of event B.

Instead of using cancelling edges as shown above, one can use a "look-ahead" feature along with a secondary list of *pending events*, where decisions are based on what is going to happen in the future. This is due to the unique advantage of computer simulation where the user has complete control over the events throughout simulated time. Therefore, in the generic situation depicted above, the use of cancelling edges can be eliminated as follows: let T denote the current value of the simulation clock (τ = T). Before scheduling event B to occur at T+t, one simulates the subgraph, $\mathcal{G}$, up to that point in simulated time. Within this *test window*, [T, T+t], event B is kept in this secondary list of pending events, say $\mathcal{L}_p$. If the cancellation of event B would indeed occur within this test window, then one avoids the scheduling of event B altogether. If event B is never cancelled within this period, than its event record is transferred from the secondary list, $\mathcal{L}_p$, to the future events list, $\mathcal{L}$.

As this "look-ahead" feature requires the execution of a subgraph within a test window and the maintenance of a secondary list structure, it tends to be far less appealing than the use of cancelling edges. Thus, cancelling edges are indeed a valuable modeling convenience. This feature of various simulation languages, while affording a great modeling convenience, is technically not necessary.

4.2. Vertex and Edge Attribute List

As discussed earlier, vertex and edge attribute lists are simple but powerful modeling tools. Recall that an edge attribute list is a string of expressions. The vertex attribute list, on the other hand, is a string of state variables. The main value of the attribute lists is in defining the particular system entities to which an event pertains. For example, suppose that there are two identical machines in a simulated factory. The same "start processing" event vertex may be used for both machines if the event has an attribute telling which machine is to start. In fact, it is possible to simulate a factory with n (n > 1) machines with only three event vertices provided that we define vertex and attribute lists in the model. Another common use for attribute lists is in initializing a simulation program. In Σ, for instance, the use of an edge attribute list for the "initialization vertex" creates an interactive environment where the user can specify the starting conditions for the model. For example, the user can specify the number of servers and their service rates while initializing a multi-server queueing simulation.

A Simulation Graph model with vertex and edge attributes really represents an *array of graphs*. For example the Simulation Graph model of the Oil Tanker Problem actually embodies four copies of the same graph, one for each type of tanker. During the execution of the model, for instance, vertex AR may execute the arrival event of a tanker of type 1, while vertex BB executes the berthing of another tanker of type 3. Hence, the use of vertex and edge attributes eliminates unnecessary duplication of similar event vertices; and in so doing, may prevent a

Simulation Graph from becoming infinite (consider a Simulation Graph of an $M/M/\infty$ queue without attributes where the identity of individual servers must be preserved).

It is possible to construct simulation models without using vertex and edge attributes. In this case:

1. It would be necessary to duplicate the vertices (together with their edges) to accomodate all of the different classes of entities that were previously distinguished by a vertex and/or edge attribute. However, this may result in a very large (possibly infinite) Simulation Graph.

2. It would be necessary to code the initial conditions of the model within the body of the simulation program. This would further necessitate the alteration (and subsequent recompilation) of the simulation program every time a different set of initial conditions need to be incorporated into the model (see the discussion of the "experimental frames" in Zeigler).

We, therefore, observe that vertex and edge attributes are indeed valuable modeling conveniences. Moreover, most of the modern simulation languages mentioned above provide facilities to index event routines as well as other entities in simulation models. For example, the expression "WITH" in Simscript, "entity attributes" in GPSS, event and queue subscripts in SLAM, and edge and vertex attributes in Σ are such modeling conveniences.

4.3 Event Execution Priorities

In establishing the equivalence between $\mathcal{S}$ and $\mathcal{M}$, it is also observed that event execution priorities in Simulation Graph Models are not necessary. In fact, many simulation programming languages do not offer such a facility. One exception is SIMSCRIPT [CACI, 1976] which has the "PRIORITY ORDER" statement for breaking ties among simultaneously scheduled events. This is a static priority scheme that must be established *apriori* in the so-called "PREAMBLE" whereas Simulation Graph models allow dynamic priority assignments which can be altered during the execution of a model implementation. As we will see in the next section, even though it is not technically necessary, an event execution priority scheme is useful in the construction of an unambiguous model.

4.4 Further Implications

The fact that every Turing Machine can be represented as a Simulation Graph Model further implies that the *undecidability results* pertaining to Turing Machines also apply to Simulation Graph Models. These results, which have important implications for model analysis, were first reported by Overstreet (1982). Here, we will review some of the issues in the context of Simulation Graph Models.

First, we recast some preliminary definitions in the context of Simulation Graphs. A Simulation Graph Model, $\mathcal{S}$, is said to be *finite*, if, in any implementation of it, when provided with valid initial conditions, only a finite number of events are executed before the termination condition is satisfied. $\mathcal{S}$ is said to be *complete*, if, at each instant during the execution of any implementation of it, either the termination condition is satisfied or the events list contains at least one more event record. An event vertex is *accessible* if it can be executed during the execution of some implementation of the Simulation Graph Model. Two events are *order independent* if their execution in any particular sequence yields identical results. If two implementations of a Simulation Graph Model, $\mathcal{S}$, are possible in which their output behavior would differ, then $\mathcal{S}$ is said to be *ambiguous*.

Overstreet has shown that *"one cannot construct (by Church's thesis) a computer program that can determine,"* for every Simulation Graph Model, *"whether it is complete, finite, accessible or ambiguous."* He has also shown that no general algorithm exists to determine whether two events are order independent. The importance of these results is that no algorithmic solution exists to determine some of the most vital properties of simulation models. Another implication is that the *"rules of thumb"* offered by Schruben (1983) for "event initialization" and "simultaneous event precedence" as well as procedures constructed by Sargent (1988) are the best heuristic tools we can hope for. In other words, most of the rules given for model analysis contain necessary, but not sufficient, conditions for their application.

5. CONCLUDING REMARKS

As Nance (1981) points out, "... simulation still carries the label of an expensive and uncertain problem solving technique that represents 'the court of last resort.' (...) The concern with simulation extends beyond those who use the technique or those who underwrite the project costs. Others, admittedly a much smaller group, are concerned with the lack of recognition of a fundamental structure - a theory - after so many years of practice."

This paper represents a discrete event simulation modeling structure. The graphs described here offer a general and practical means of mathematically defining a discrete event dynamical system and its simulation model as a language free structure. Furthermore, Simulation Graphs are designed to offer the decision power of high level simulation languages as well as the modeling (expressive) power of the abstract formalisms.

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