"EQUIVALENCE OF SIMULATIONS: A GRAPH THEORETIC APPROACH"

by

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ABSTRACT

We establish the equivalence of different simulation models from a graph theoretic point of view. The development shifts the emphasis from model behavior to model structure. To this end, the concept of a Simulation Graph Model is introduced; structural equivalence and the associated techniques are defined. It is also shown that structural equivalence, as defined in this paper, is a sufficient but not a necessary condition for behavioral equivalence. Finally, Schruben's rules for event reduction are revised, and stated and proved as theorems.

1. Motivation and Background

The ability to identify "equivalent" simulations is highly desirable from various points of view. For instance, logistical considerations such as ease of implementation on a computer, data requirements, or suitability for the intended application may point out that it is more advantageous to use one model over another once their "equivalence" is established. It might also be the case where two models of the same system are implemented in different simulation programming languages and the user needs to determine whether the implementations can be used interchangeably. Another possible scenario is when a modeler embellishes an existing model and needs a metric to verify that the underlying structure of both models is the same.

To provide a reliable answer to the questions raised above, we need a reasonable definition of "equivalence." Even though the idea is conceptually simple, it is difficult to formalize in an acceptable fashion. For instance, any practically useful definition must be *testable.* That is, it should enable the user to identify "equivalent" simulations possibly implemented in different simulation programming languages without actually running both simulations under the same initial conditions and comparing their output behavior.

Various definitions of "equivalence" have appeared in the literature. Overstreet (1982) offers three such definitions. *Structural Equivalence* is concerned with the objects in a model and their impact on model attributes. For two models to be structurally equivalent, they must have the same objects and the impact of these objects on model attributes should be the same in both models. *External Equivalence* is concerned with the input/output behavior of models. Two models are externally equivalent if they exhibit the same output behavior whenever provided with the same initial conditions. *Derivability* is similar to external equivalence. In this case, the outputs need not be similar. However, the output behavior of one model must be derivable from that of the other model for them to be declared equivalent.

Schruben (1983) offers a definition of equivalence which is based on the behavior of conditioning state variables over simulated time. Sargent's (1988) Type I and Type II equivalences are also based on the behavior of the model variables during execution. We will collectively refer to these definitions as *behavioral equivalence.*

Although behavioral equivalence is intuitively appealing, it is untestable. That is, there exists no general algorithm which would determine whether any two simulation models are behaviorally equivalent. In this paper, we introduce a definition of "equivalence" that exploits structural properties of simulation models. In so doing, we will shift the emphasis from model behavior to model structure. We will also show that structural equivalence (defined shortly and *differently* from Overstreet) is a sufficient but not a necessary condition for behavioral equivalence.

Our development will be based on the concept of a *Simulation Graph Model.* The latter is a mathematical formalization of the Event Graphs introduced by Schruben (1983) as a graphical representation of the event-scheduling approach in discrete event simulation. This concept is introduced and discussed next.

2. Simulation Graph Models

The elements of a discrete event simulation model are *state variables* that describe the entities in a system, *events* that change the values of state variables, and the *relationships* between events. A *Simulation Graph* is a structure of the objects in a discrete event system that facilitates the development of correct simulation models.

Events are represented as vertices on the graph. Each event vertex is associated with a set of changes to state variables.

Relationships between events, on the other hand, are represented as directed edges between pairs of vertices. Each edge depicts under what conditions and after how long of a time delay an event will schedule or cancel another event. More specifically,

$$\boxed{A(\ell)} \xrightarrow{\;t:j \qquad\qquad (i)\;} \boxed{B(k)}$$

indicates that *"t time units after the occurrence of event A, event B will be scheduled to occur, with parameter string k ← j, provided that condition (i) holds at the time event A occurs."* The parameter string carries information pertaining to a particular event instance as well as the execution order priority of that event to break any possible time ties. These strings can be passed in a model through the vertex and edge attributes.

More formally, a Simulation Graph is defined as an ordered quadruple, $\mathcal{G} = (\mathcal{V}(\mathcal{G}), \mathcal{E}_s(\mathcal{G}), \mathcal{E}_c(\mathcal{G}), \Psi_{\mathcal{G}})$, where

$\mathcal{V}(\mathcal{G})$ is the *vertex set* of $\mathcal{G}$,

$\mathcal{E}_s(\mathcal{G})$ is the *set of scheduling edges* of $\mathcal{G}$,

$\mathcal{E}_c(\mathcal{G})$ is the *set of cancelling edges* of $\mathcal{G}$, and

$\Psi_{\mathcal{G}}$ is the *incidence function*.

We then define the objects in a simulation model as the following ordered sets:

$\mathcal{F} = \{f_v \mid v \in \mathcal{V}(\mathcal{G})\}$, set of *state transitions*,

$\mathcal{C} = \{C_e \mid e \in \mathcal{E}_s(\mathcal{G}) \cup \mathcal{E}_c(\mathcal{G})\}$, set of *edge conditions*,

$\mathcal{T} = \{t_e \mid e \in \mathcal{E}_s(\mathcal{G})\}$, set of *edge delay times*,

$\Gamma = \{\gamma_v \mid v \in \mathcal{V}(\mathcal{G})\}$, set of *event execution priorities.*

A Simulation Graph *Model* is then defined as:

$$\mathcal{S} = (\mathcal{F}, \mathcal{C}, \mathcal{T}, \Gamma, \mathcal{G}).$$

The key idea here is that *a Simulation Graph specifies the relationships between the elements of the sets of objects in a simulation model.*

We conclude this section with a simple example to illustrate the concepts discussed above. For a complete treatment of Simulation Graph Models, the reader is referred to [Schruben and Yucesan, 1987].

<u>Example: Single-Server Queueing System</u>

We will develop a Simulation Graph Model of a single-server queueing system. Suppose that customers arrive into the system every t_a units of time, and it takes the server t_s units of time to attend to each customer. The state variables used in this model are:

Q, the number of customers waiting for service, and

S, the status of the server ($0 \equiv$ busy, $1 \equiv$ idle).

The edge conditions in the model are:

(i) (The server is idle) $S = 1$,

(ii) (Customers are waiting to be served) $Q > 0$.

The event descriptions are presented in Table 1; the Simulation Graph is given in Figure 1.

FIGURE 1: SINGLE-SERVER QUEUEING SYSTEM

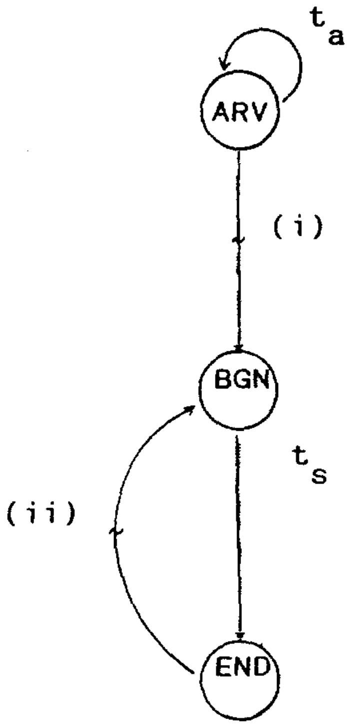


TABLE 1: EVENT DESCRIPTIONS

Event Type	Event Description	State Changes
ARV	Customer Arrival	$Q \leftarrow Q + 1$
BGN	Beginning of Service	$S \leftarrow 0$ $Q \leftarrow Q - 1$
END	End of Service	$S \leftarrow 1$

The concept of structural equivalence is developed next after introducing some preliminary definitions.

3. Equivalence of Simulation Models

3.1 Preliminary Definitions

In a Simulation Graph, an edge condition will be called *simple* if it consists of two arithmetic expressions connected by a relational operator. In other words, a simple edge condition is a relation. The arithmetic expressions may simply be a constant or a variable, whereas the relational operators are "less than," "less than or equal to," "greater than," "greater than or equal to," "equal to" and "not equal to." On the other hand, the edge condition will be called *compound* if it consists of two or more relations joined by Boolean operators AND or OR. For example, [QSIZE>0] is a simple condition, while [(QSIZE=0)AND(S=1)] is a compound edge condition.

Similarly, a vertex will be considered *simple* if there is at most one state variable change associated with it. In other words, vertex E is a simple event vertex if its execution alters the value of at most one state variable. That is, if $X_1,..,X_k,..,X_n$ are the state variables of a model, the execution of event E can be viewed as follows:

$$f_E(x_1,..,x_k,...,x_n) = (x_1,...,x'_k,..,x_n)$$

Otherwise, the vertex will be called a *compound* vertex. A vertex with no state variable changes will be referred to as the *identity* vertex.

A Simulation Graph $\mathcal{G}$ will be called an *Elementary Simulation Graph* and denoted $\mathcal{G}^E$, if it contains only simple event vertices and the edge conditions are all simple. Given a Simulation Graph $\mathcal{G}$, one can always construct an associated Elementary Simulation Graph, $\mathcal{G}^E$, by *expansion*. This is the process of replacing a single vertex with m state variable changes ($m \geq 1$) by m vertices in series, each with a single state variable change. It is also the process of replacing an edge with a compound condition by a series of identity vertices, each connected with simple edges. During this process, the logical structure of the original model is always preserved.

Analogously, $\mathcal{S}^E = (\mathcal{F}', \mathcal{C}', \mathcal{T}', \Gamma', \mathcal{G}^E)$ is an *Elementary Simulation Graph Model*, where the Simulation Graph is expanded and the remaining sets are redefined appropriately.

Recall that we defined a Simulation Graph Model as $\mathcal{S}=(\mathcal{F},\mathcal{C},\mathcal{T},\Gamma,\mathcal{G})$, where $\mathcal{F}$ is the set of state transitions in the model, $\mathcal{C}$ is the set of edge conditions, $\mathcal{T}$ is the set of edge delay times, Γ is the set of event execution priorities, and $\mathcal{G}=(\mathcal{V}(\mathcal{G}),\mathcal{E}_s(\mathcal{G}),\mathcal{E}_c(\mathcal{G}),\Psi_{\mathcal{G}})$ is the Simulation Graph. Two Simulation Graph Models $\mathcal{S}_1 = (\mathcal{F}_1, \mathcal{C}_1, \mathcal{T}_1, \Gamma_1, \mathcal{G}_1)$ and $\mathcal{S}_2 = (\mathcal{F}_2, \mathcal{C}_2, \mathcal{T}_2, \Gamma_2,$

$\mathcal{G}_2$) will be called isomorphic if there exist bijections of the form:

$$\Theta: \mathcal{V}(\mathcal{G}_1) \rightarrow \mathcal{V}(\mathcal{G}_2)$$
$$\Phi_s: \mathcal{E}_s(\mathcal{G}_1) \rightarrow \mathcal{E}_s(\mathcal{G}_2)$$
$$\Phi_c: \mathcal{E}_c(\mathcal{G}_1) \rightarrow \mathcal{E}_c(\mathcal{G}_2)$$
$$\Lambda: \mathcal{F}_1 \rightarrow \mathcal{F}_2$$
$$\Omega: \mathcal{C}_1 \rightarrow \mathcal{C}_2$$
$$\chi: \mathcal{T}_1 \rightarrow \mathcal{T}_2$$
$$\Delta: \Gamma_1 \rightarrow \Gamma_2.$$

The mappings $(\Theta, \Phi_s, \Phi_c, \Lambda, \Omega, \chi, \Delta)$ form an *isomorphism.* Note that the first three mappings basically establish a correspondance between the underlying structure of the two models. Simply stated, these mappings indicate that names given to objects in a Simulation Graph are not important since isomorphic graphs form an equivalence class. Next, we examine the remaining four mappings in more detail.

$\Lambda: \mathcal{F}_1 \rightarrow \mathcal{F}_2$. This mapping establishes a correspondance between the state changes associated with different event vertices on the two Simulation Graphs. For two state transition functions to be declared equivalent, their application to the same state variables must yield identical outcomes. For instance, the state transition functions (Y = 2 * X) and (Y = X + X) are equivalent. It is also required that the state variables have the same range. In case where these functions have any stochastic components (random variables), we require that these variables follow the same probability distribution.

$\Omega: \mathcal{C}_1 \rightarrow \mathcal{C}_2$. This mapping establishes a correspondance between the edge conditions of the two models. We do not require the form of these conditions be the same. Instead, we require that they assume the same Boolean value (0 or 1) when evaluated within a given state of the model. For instance, (QSIZE > 0) and (Q $\geq$ 1) are equivalent edge conditions. If the conditions involve a stochastic component, we simply require that those particular variables follow the same probability distribution.

$\lambda: \mathcal{T}_1 \rightarrow \mathcal{T}_2$. This mapping establishes a correspondance between edge delay times of the two models. If the times are deterministic, they are required to have the same value. If they are stochastic, they are required to follow the same probability distribution.

$\Delta: \Gamma_1 \rightarrow \Gamma_2$. This mapping establishes a correspondance between the event execution order priorities in the two models. Recall that priorities are non-negative real numbers, which are assigned dynamically during the execution of a model [Schruben and Yucesan, 1987]. Hence, we require that the same values be assigned to the respective events at the same instants during execution.

It is also possible to define an isomorphism with respect to a subgraph or a subset of the state variables. This provision allows the modeler to focus only on those components of the model that are relevant in the scope of the simulation study.

3.2 Equivalent Simulation Models

<u>Definition :</u> Simulation Graphs Models $\mathcal{G}_1$ and $\mathcal{G}_2$ are *equivalent* if their Elementary Simulation Graph Models $\mathcal{G}_1^E$ and $\mathcal{G}_2^E$ are isomorphic.

The procedure to determine whether two simulation models are equivalent involves two stages. First, the Elementary Simulation Graph Model associated with each model is constructed. Then, an isomorphism between these Elementary Models is sought. If such an isomorphism is found, then the two simulation models are declared equivalent.

Efficient algorithms exist for establishing graph isomorphism. One such algorithm for planar graphs is presented in [Hopcroft and Tarjan, 1971]. (The surprising fact that Simulation Graphs have planar representations is established in [Yucesan, 1989]. Hence, this definition of equivalence, which is based on model structure, is indeed a testable definition as opposed to definitions based on model behavior. One should note, however, that establishing the mappings Λ, Ω, $\aleph$, and Δ can be a difficult task for large models.

Next, we will show that structural equivalence defined by graph isomorphism, is a sufficient but not a necessary condition for behavioral equivalence. First, let us define the latter term in a more precise manner.

In discrete event simulations, events occur at discrete points in simulated time. Following Sargent's notation (1988), let

$$T_{I\mathcal{M}} = \{ t_1, t_2, \ldots, t_n \}$$

be the ordered set of these points in time for a given execution of model $\mathcal{M}$ with initial conditions I. Analogously, let

$$S_{I\mathcal{M}} = \{ S_1, S_2, \ldots, S_n \}$$

be an ordered set where S_j depicts the state of the system at t'_j, where $t_j < t'_j < t_{j+1}$ for $1 \leq j < n$ and $t'_n > t_n$.

Two Simulation Models X and Y are *behaviorally equivalent*, if, for the same realization of the random processes in the two models and for the same initial conditions I,

$$T_{IX} = T_{IY},$$
$$S_{IX} = S_{IY}.$$

This is similar to Type I equivalence in [Sargent, 1988]. The following proposition states that structural equivalence implies behavioral equivalence.

Theorem: Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two Simulation Graph Models that are structurally equivalent; that is, there exists an isomorphism between the Elementary Simulation Models. Then the two simulation models must be behaviorally equivalent.

Proof: (By contraposition) Suppose the two models $\mathcal{G}_1$ and $\mathcal{G}_2$ are not behaviorally equivalent. That is, either (i) there exists an event time, t_k, which results in $T_{I\mathcal{G}_1} \neq T_{I\mathcal{G}_2}$, or (ii) there exists a state, S_k, which results in $S_{I\mathcal{G}_1} \neq S_{I\mathcal{G}_2}$.

The first case implies that the time delays used in scheduling future events are not the same in the two models. Thus, the mapping $\lambda:\mathcal{T}_1 \rightarrow \mathcal{T}_2$ fails to hold. *Only* the following conditions, *individually or collectively*, may cause the second case:

1. Scheduling or cancelling different events at the same instant in simulated time due to model structure. In this case, any one of the seven mappings that make up the formalism $(\mathcal{R},\Phi_S,\Phi_C,\Lambda,\Omega,\lambda,\Delta)$ may fail to hold.

2. Different state transitions from the execution of the same events. In other words, if E and F are event vertices in $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively, then $f_E \neq f_F$. Hence, the mapping $\Lambda:\mathcal{F}_1 \rightarrow \mathcal{F}_2$ fails to hold.

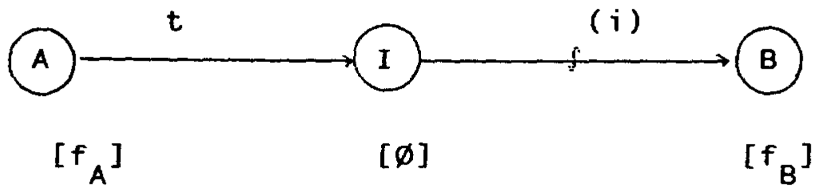
3. Arbitrary execution order of simultaneous events due to different event execution order priorities. In this case, the mapping $\Delta:\Gamma_1 \rightarrow \Gamma_2$ fails to hold.

4. Scheduling or cancelling different events at the same instant in simulated time due to different logical conditions being true at that instant. Then, the mapping $\Omega:\mathcal{C}_1 \rightarrow \mathcal{C}_2$ fails to hold.

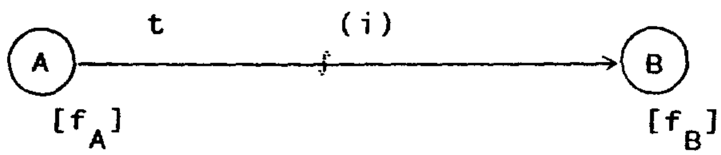
In each of the above cases, it is not possible to establish an isomorphism between different objects in the two Simulation Graph Models. Since the Elementary Simulation Graph Models, Δ^E_1 and Δ^E_2 are not isomorphic, then Δ_1 and Δ_2 are not structurally equivalent.

Structural equivalence is a sufficient but not a necessary condition for behavioral equivalence. A simple example where two models are behaviorally but not structurally equivalent is given by two models having the following subgraphs:

G_1:



G_2:



The terms in square brackets denote the state changes associated with a vertex. Since vertex I in G_1 is an identity vertex, which does not alter the values of any state variable, its inclusion in (or exclusion from) the model does not affect the behavior of the model. Hence, even though the models with and without vertex I are behaviorally equivalent, they are not isomorphic.

Recall that establishing structural equivalence requires the generation of an Elementary Simulation Graph Model. This is achieved through *expansion*, which is the process of replacing compound vertices and/or compound edge conditions by a construction of simple edges and vertices while preserving the logical structure of the original model. For brevity, the expansion rules along with proofs of their validity are not included in this paper. Instead, the reader is referred to [Yucesan, 1989].

An application of model equivalence is illustrated next.

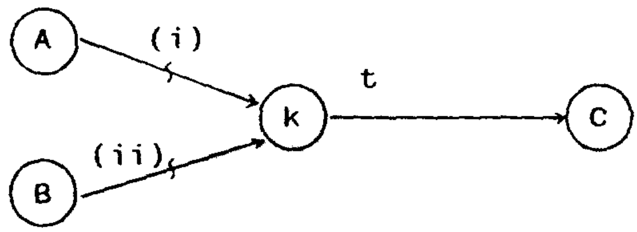
4. Rules for Event Reduction

While illustrating the use of event graphs in model analysis, Schruben (1983) offers several *rules of thumb* to assist the modeler in various modeling tasks. They are called rules of thumb since they are conceptual rules that work when applied "with care." Moreover, since they are not based on a formal framework, their validity cannot be meaningfully tested. In this section, within the context of Simulation Graph Models, we formally state the rules for event reduction as theorems and prove that two of them are indeed correct for *our* definition of equivalence and the third one is not, but is true for Schruben's definition. We will also discuss the impact of event execution priorities and cancelling edges on the application of these rules.

Rule 4(a): Equivalent Simulation Graph Models are possible with and without event vertex k, if vertex k has no conditional exiting edges and all edges entering vertex k have zero delay times.

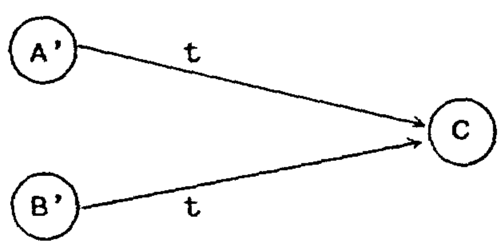
If Rule 4(a) applies, then vertex k may be combined with the originating vertices of entering edges. State variable changes in vertex k are added to changes for these entering event vertices. Delay times on each of the edges exiting vertex k are added to each of the edges entering vertex k.

Proof: Suppose the following is a subgraph in the Simulation Graph Model, $\mathcal{G}$:

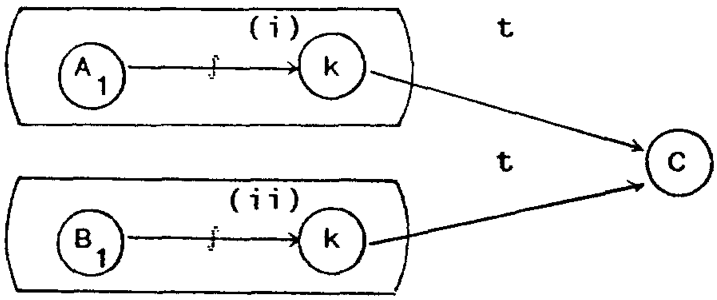


Vertex k satisfies the conditions of Rule 4(a). Define I(k) to be the set of vertices in $V(\mathcal{G})$ with an outgoing edge that terminates at vertex k. In other words, this is the set of predecessors of vertex k. In the above subgraph, I(k) = {A, B}. To avoid the discussion from needlessly getting complex, we will assume that there is exactly one state variable change associated with every event vertex and all edge conditions are simple. That is, $\mathcal{G}$ is assumed to be an Elementary Simulation Graph Model.

Applying Rule 4(a), we merge vertex k with the vertices in I(k), namely vertices A and B. This yields the Simulation Graph Model $\mathcal{G}_1$ with the following subgraph.



where $f_{A'} = f_A \cup f_k$, $f_{B'} = f_B \cup f_k$. Next, we construct the Elementary Simulation Graph Model, $\mathcal{G}_1^E$. We will draw the associated subgraph as follows (the subgraph is drawn in the following manner to clearly demonstrate the implementation of vertex A' as an event routine):



where $f_{A_1} = f_A$ and $f_{B_1} = f_B$. We then seek an isomorphism between $\mathcal{G}$ and $\mathcal{G}_1^E$. We will highlight only those mappings that pertain to the particular subgraph since the rest of the graph is not affected by this reduction.

Θ: $\mathcal{V}(\mathcal{G}) \rightarrow \mathcal{V}(\mathcal{G}_1)$. $\Theta(A) = A_1$, $\Theta(B) = B_1$, $\Theta(k) = k$, and $\Theta(C) = C$.

Φ_s: $\mathcal{E}_s(\mathcal{G}) \rightarrow \mathcal{E}_s(\mathcal{G}_1)$. $\Phi_s((A,k)) = (A_1,k)$, $\Phi_s((B,k)) = (B_1,k)$, and $\Phi_s((k,C)) = (k,C)$.

Φ_c: $\mathcal{E}_c(\mathcal{G}) \rightarrow \mathcal{E}_c(\mathcal{G}_1)$. The mapping is vacuous since there are no cancelling edges in the subgraph.

Λ: $\mathcal{F} \rightarrow \mathcal{F}_1$. $\Lambda(f_A) = f_{A_1}$, $\Lambda(f_B) = f_{B_1}$, $\Lambda(f_k) = f_k$, and $\Lambda(f_C) = f_C$.

Ω:$\mathcal{C} \rightarrow \mathcal{C}_1$. $\Omega(i) = i$, $\Omega(ii) = ii$.

λ: $\mathcal{T} \rightarrow \mathcal{T}_1$. $\lambda(t) = t$.

Δ:$\Gamma \rightarrow \Gamma_1$. Schruben's original rules do not consider event execution priorities. With this added dimension, the merging of vertex k with those vertices in I(k) under Rule 4(a) can be allowed only if vertex k has a *higher* execution priority than all of its predecessors; that is, $\gamma_k < \gamma_j \quad \forall j \in I(k)$. Under this convention, a bijection, Δ, can always be found between Γ and Γ_1.

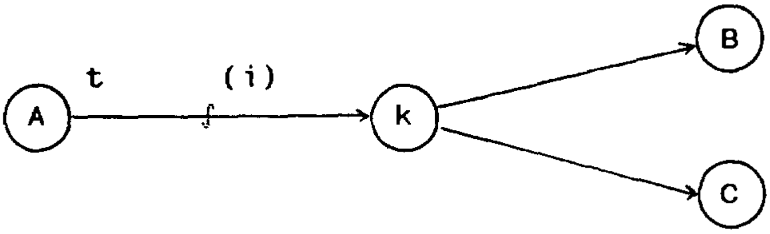
$(\Theta,\Phi_s,\Phi_c,\Lambda,\Omega,\lambda,\Delta)$ is the needed isomorphism and $\mathcal{G} \cong \mathcal{G}_1^E$. Thus, we conclude that the application of Rule 4(a) with the added convention for event execution priorities yields equivalent simulation models.

Next, we consider Rule 4(c), which is "symmetric" to Rule 4(a).

Rule 4(c): Equivalent Simulation Graph Models are possible with and without event vertex k, if vertex k has no conditional exiting edges and all edges exiting vertex k have zero delay times.

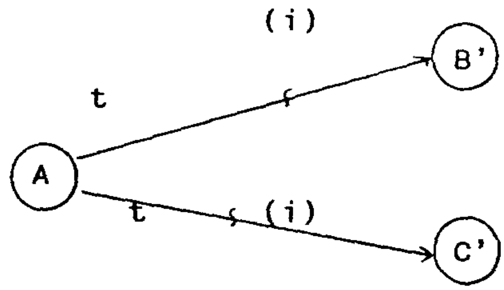
If Rule 4(c) applies, then vertex k may be combined with the termination event vertices of exiting edges. State variable changes in vertex k are added to changes for these succeding event vertices.

Proof: Suppose we have the following subgraph in a Simulation Graph Model, $\mathcal{G}$:

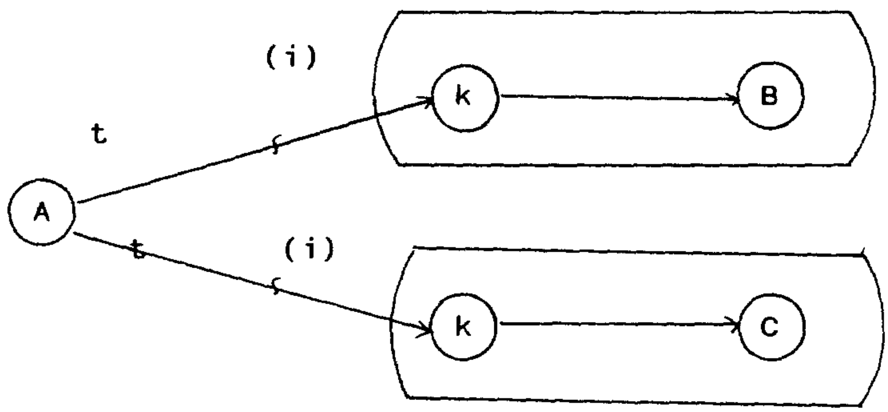


Vertex k satisfies the conditions of Rule 4(c). Also, define O(k) to be the set of vertices in $\mathcal{V}(\mathcal{G})$ that have an incoming edge originating at vertex k. Alternatively, this is the set of successors of vertex k. In the above subgraph, O(k) = {B, C}.

Applying Rule 4(c), we merge vertex k with the verices in O(k), namely vertices B and C. Again, for exposition purposes, we assume that $\mathcal{G}$ is an Elementary Simulation Graph Model. The application of the rule yields the Simulation Graph Model $\mathcal{G}_1$ with the following subgraph:



where $f_{B'} = f_k \cup f_B$ and $f_{C'} = f_k \cup f_C$. The subgraph can be redrawn as:



This also corresponds to $\mathcal{G}_1^E$, the Elementary Simulation Graph Model. From the above drawing, the symmetry with the conditions underlying Rule 4(a) is apparent. Hence, we can find the necessary bijections in the same manner and conclude that $\mathcal{G} \cong \mathcal{G}_1^E$.

We also need to settle the issue of event execution priorities not addressed in Schruben's original work. Under Rule 4(c), vertex k can be allowed to merge with the vertices in O(k) only if it has a *lower* execution priority than all the successor vertices; that is, $\gamma_k > \gamma_j \; \forall j \in O(k)$.

Rule 4(b) states: "Equivalent event graphs are possible with and without event vertex k, if vertex k has no conditional exiting edges and S_k contains no edge conditioning state variables ($E_i \cap$

$S_k = \emptyset, \forall i \in V(\mathcal{G})$)." Recall that Schruben's definition of equivalence is based on the behavior of the edge conditioning state variables (E_i, $\forall i \in V(\mathcal{G})$). More precisely, two event graphs are declared equivalent "if, for a given realization of the random processes in the graph, both graphs have identical sequences of event conditioning state variable changes." It turns out that this is a fairly restrictive definition. For instance, suppose vertex k is associated with a performance monitoring state variable, which is updated periodically during the execution of the simulation model. Then, by definition, $E_i \cap S_k = \emptyset \; \forall i$. However, eliminating vertex k and appending the associated state change (performance update) to either the preceding or succeding vertex will result in the update of the performance monitoring variable at the *wrong* instant in simulated time. This, in turn, may bias the measure of performance. Therefore, Rule 4(b) holds only if there are no state variable changes associated with vertex k ($f_k = \emptyset$); that is, only if vertex k is the identity vertex, which is used solely to make the underlying model structure more transparent. Rule 4(b) is then revised as follows: "Equivalent Simulation Graph Models are possible with or without event vertex k, if vertex k has no conditional exiting edges and there are no state variable changes associated with vertex k."

If Rule 4(b) applies the delay time for each edge exiting vertex k is added to the delay time for each entering edge before vertex k is removed.

Note that, since the application of Rule 4(b) results in the deletion of a vertex and at least one edge, it is not possible to establish the mappings $\Theta : V(\mathcal{G}) \to V(\mathcal{G}')$ and $\Phi_s : \mathcal{E}_s(\mathcal{G}) \to \mathcal{E}_s(\mathcal{G}')$, where $\mathcal{G}$ is the original Simulation Graph and $\mathcal{G}'$ is the reduced graph. Hence, Rule 4(b) cannot be proved using our definition of equivalence.

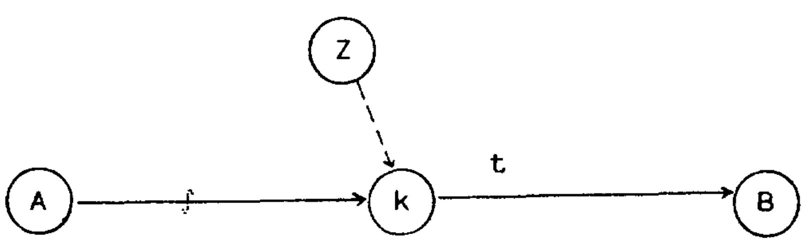
In the light of the above assertion, Rule 5 should also be restated as follows: "If $f_k = \emptyset$ for all interior vertices k of an unconditional event tree, then only the leaf vertices of the tree need be included in a Simulation Graph Model." It is important to recognize that the original Rule 4(b) is not in general a theorem; however, the modification presented here can often be applied with a valid reduction of events [Schruben and Yucesan, 1989].

In the above rules, the discussion of cancelling edges is omitted. In other words, the question "is it possible to remove vertex k if it has an incoming or outgoing cancelling edge?" is not addressed. This is because cancelling edges have been shown to be a modeling convenience rather than a necessary modeling tool (see [Yucesan, 1989]). Nevertheless, if the user chooses to include them in a model, then the following additions should be made to the reduction rules:

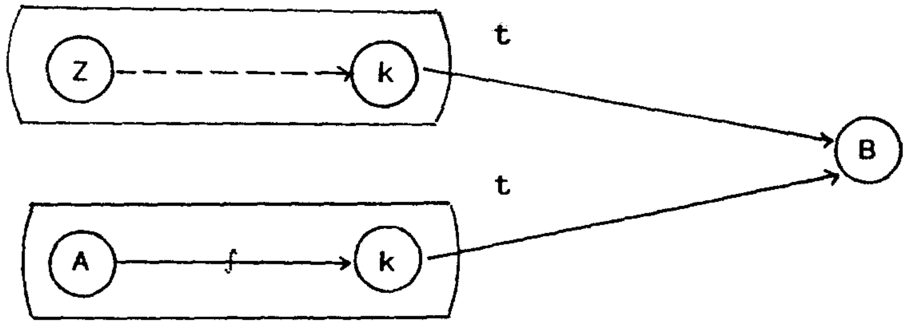
1. Rule 4(a) does not apply if vertex k has an incoming cancelling edge.

2. Rule 4(c) does not apply if vertex k has an outgoing cancelling edge.

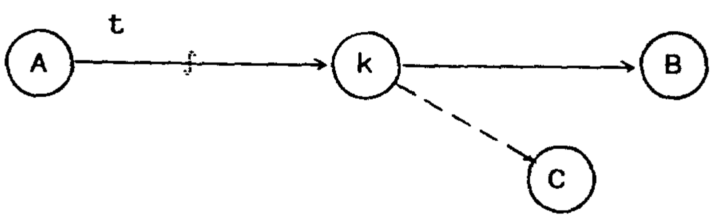
To see why the reduction rules do not directly carry over to cases where cancelling edges are used, consider the following subgraphs. In the first case, vertex k satisfies the conditions of Rule 4(a).



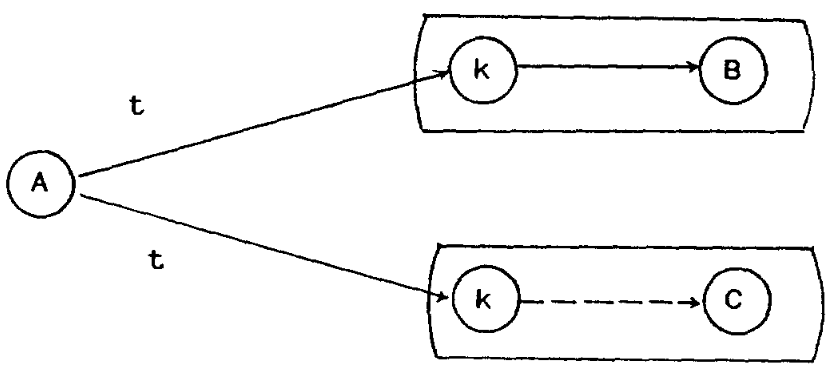
The application of the rule yields:



where the first event vertex (with Z and k connected by a cancelling edge) is meaningless. Now, suppose we have the following subgraph:



Vertex k satisfies the conditions of Rule 4(c). Hence, the application of the rule yields:



where the last vertex (with k and C joined by a cancelling edge) is meaningless. The modified reduction rules are summarized in Table 2. below.

TABLE 2: EVENT REDUCTION RULES

Rule 4(a): Equivalent Simulation Graph Models are possible with and without vertex k if *all* of the following conditions hold:

(i) vertex k has no conditional exiting edges,
(ii) all edges entering vertex k have zero delay times,
(iii) vertex k has no incoming cancelling edges,
(iv) vertex k has a higher execution priority than all of its predecessors.

Rule 4(b): Equivalent Simulation Graph Models are possible with and without vertex k if *all* of the following conditions hold:

(i) vertex k has no conditional exiting edges,
(ii) there are no state variable changes associated with vertex k.

Rule 4(c): Equivalent Simulation Graph Models are possible with and without vertex k if *all* of the following conditions hold:

(i) vertex k has no conditional exiting edges,
(ii) all edges exiting vertex k have zero delay times,
(iii) vertex k has no exiting cancelling edges,
(iv) vertex k has a lower execution priority than all of its successors.

Rule 5: If $f_k = \emptyset$ for all interior vertices k of an unconditional event tree, then only leaf vertices of the tree need be included in a Simulation Graph Model.

5. Concluding Remarks

The concept of model equivalence has also been applied, with limited success, to queueing networks with blocking. This is a set of arbitrarily linked queues in which a server can become blocked when the capacity limitation of the destination queue is reached. In a recent paper, Onvural and Perros (1986) compare commonly used blocking mechanisms in order to obtain equivalencies between different types of blocking under various network configurations. Their results, which are *heavily* dependent on the Markovian nature of the arrival and the service processes, could be confirmed only for the trivial cases. (See [Yucesan, 1989] for a detailed discussion.)

The results of Onvural and Perros were confirmed, however, when the underlying Markov chains were analyzed. To this end, Simulation Graph Models of the associated transition rate diagrams were constructed. Reported equivalencies were then verified by establishing the necessary isomorphism between these models.

This exercise shows that it is a viable alternative to simulate continuous-time Markov chains using their rate matrices - hence, their transition rate diagrams. Note that it is still possible to obtain a *finite* Simulation Graph representation of the transition rate diagram through the use of vertex and edge attributes. This, in turn, allows one to analyze stochastic models through their transition rate diagrams even if the number of states is *infinite*.

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