

"POSITIVE PRICES IN CAPM"

by

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Positive Prices in CAPM

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Abstract

In the mean-variance capital asset pricing model (CAPM), some equilibrium prices may be negative because of non-monotonicity of preferences. This paper identifies several sets of sufficient conditions for prices to be positive. The nature of the central conditions is to impose bounds on the investors' degree of risk aversion, as measured by the marginal rate of substitution between mean and standard deviation of return. These bounds do not need to hold globally but only in a relevant range of portfolios or combinations of mean and standard deviation. The relevant range is specified on the basis of exogenously given parameters and variables, and it is such that it must contain any endogenously determined equilibrium. The bounds on risk aversion ensure that the preferences for assets are sufficiently well-behaved within the relevant range.

1 Introduction

The two-period mean-variance capital asset pricing model (CAPM) has the peculiar property that preferences may not be monotone. The expected return to an investor's portfolio increases as he holds more and more shares of the assets, but so does the variance of return. It may be that at some point, the additional expected return gained from adding more shares to the portfolio is not sufficient to compensate for the increase in variance. If so, then the induced preferences for assets are not monotone. Non-monotonicity of preferences in the mean-variance model is analyzed in Nielsen (1987).

Because portfolio preferences are not necessarily monotone, equilibrium asset prices may be negative or zero. In fact, it is possible that the value of the market portfolio of risky assets is negative. This is demonstrated in Nielsen (1985, 1990a) and in two examples in the present paper.

When the CAPM is used as a model of the prices of stocks, which in reality have limited liability, it is disturbing that it may predict negative prices. This paper analyzes various conditions on the exogenous parameters which ensure positive prices. The exogenous parameters in question are the expectations, variances and covariances of total returns, the initial allocations of the assets (endowments), and the investors' utility functions for mean and variance of return. The analysis applies to the classical version of the CAPM with a riskless asset, as developed by Sharpe (1964), Lintner (1965) and Mossin (1966), as well as the CAPM with risky assets only, which was introduced by Black (1972) and has been much used in empirical work under the name the "zero-beta" or "two-factor" CAPM.

To identify conditions which ensure positivity of prices, we compute the gradient of the derived utility function for portfolios and exploit the fact that at an equilibrium, all investors' gradients point in the direction of the price vector. This leads to a number of sufficient conditions in the form of inequalities which involve expected returns, an asset's covariance with the return either on the market portfolio or on a particular investor's portfolio, and a function of the risk aversion and risk (standard deviation) taken by either all investors or one particular investor. The nature of the inequalities is to impose a bound on risk aversion, the bound being related to expected returns, covariances, etc.

A first approach would be to impose these restrictions globally, that is,

to require that they hold at every portfolio. For example, if it is assumed that some investor's risk aversion, as measured by the slope of his indifference curve for combinations of standard deviation and mean, is everywhere less than the ratio of mean to standard deviation available from each of the assets, then his preferences for assets are strictly monotone (cf. Proposition 6). In that case, all equilibrium prices must be positive. However, this condition is not satisfied by for example utility functions which are linear in mean and variance. These utility functions correspond to the popular case of negative exponential von Neumann-Morgenstern utility functions and normal returns distributions.

Even though utility functions such as these lead to non-monotonicity, they may still be useful if the model is treated as an approximation. The approximation will be reasonable—in the sense that predicted asset prices will be positive—for some values of the exogenous parameters but not for others. Thus, we search for conditions that are weaker than the requirement of global strict monotonicity.

It may be that the unreasonable properties of preferences occur only at levels of mean and standard deviation so large as to be irrelevant for the process of establishing an equilibrium. There is a “relevant range” of portfolios and a corresponding range of combinations of mean and standard deviation of return, such that the shape of preferences outside that range does not matter. Specifically the relevant range of allocations is the “individually rational” allocations, which are allocations that are consistent with the total supply of assets and are preferred to the initial allocation by all investors. For each investor, the relevant range of portfolios is those portfolios that form part of individually rational allocations. This idea has been used in Nielsen (1989, 1990b) to study the issue of existence of equilibrium.

Exploiting this idea, we observe that it is enough that the inequality conditions hold in the relevant range of portfolios, specifically at all individually rational allocations. That is significantly weaker than the global requirement, because the set of individually rational allocations is compact.

The set of individually rational allocations is difficult to identify in practice because it is defined by joint reference to the preference maps of all the investors. For this reason, it is desirable to find some conditions for positive prices that can be imposed in a manner which is independent across

investors.

One such condition for the equilibrium value of an asset to be positive is that some investor's risk aversion, measured as the marginal rate of substitution between mean and standard deviation, is less, over a range specified with reference only to that investor, than the ratio of the mean to standard deviation available from the asset (Proposition 7). This condition means that the investor would want to acquire some of the asset if he initially had a portfolio which was uncorrelated with it and was in the specified range.

The various conditions can be simplified if the investors have decreasing risk aversion, which means that the higher the mean return is, the less additional mean return is required to compensate for a given increase in the standard deviation of return.

When risk aversion is decreasing, a particularly appealing version of the condition for the equilibrium value of an asset to be positive is that all investors have a risk aversion at their initial portfolio holdings which is less than the ratio of the mean to standard deviation available from the asset (Theorem 1). This means that the investor would want to acquire some of the asset if his endowment was uncorrelated with it.

If there is a riskless asset, one can exploit the "separation theorem" to derive variations of the sufficient conditions for positive prices. These variations are specific to the case of a riskless asset.

The mean-variance CAPM may be questioned on several grounds: It is a non-trivial exercise to construct a reasonably powerful empirical test that does not reject the model, cf. Gibbons (1982), Huang and Litzenberger (1988, Chapter 10). The assumption of unrestricted short sales may be unrealistic. Mean-variance behavior is consistent with expected utility only in special cases, cf. Chamberlain (1983) and Owen and Rabinovitch (1983).

Despite the objections, the CAPM continues to be an important part of the standard equipment of financial economists because of its simplicity and its striking implications. It provides a useful first approach to problems of risk and return, which is why it dominates the teaching of finance at the introductory levels. It is often used in empirical studies, and it continues to be an object of theoretical research. The usefulness and importance of the model makes it important to understand exactly the nature of its explanation of asset prices.

The possible non-monotonicity of preferences in CAPM may lead not only to positive equilibrium prices but also to non-existence of equilibrium. The unboundedness of choice sets resulting from short-selling also may lead to existence problems. Allingham (1988) and Nielsen (1985, 1990a, 1990b) study conditions which ensure the existence of equilibrium in CAPM.

The plan of the paper is this: Section 2 sets up the model. Section 3 discusses preference restrictions which are “joint” in the sense that they involve all investors’ preference maps jointly. Section 4 discusses preference restrictions which are independent across investors. Section 5 studies the case where there is a riskless asset.

2 Prerequisites

There are n assets. A *portfolio* is represented by a n -vector x , where the j 'th entry indicates the number of shares of the j 'th asset included in the portfolio. Short-selling is allowed, so that the number x_j of shares of asset j held in a portfolio may be negative.

There are m *investors* $i = 1, \dots, m$. All investors have *choice set* \Re^n , which means that there are no short sales constraints. They all have the same beliefs about the total (gross) returns per share of the assets, which they summarize in a mean vector \bar{R} and a covariance matrix Ω . The mean return to a portfolio x is $x' \bar{R}$, and the standard deviation is $\sigma(x) = (x' \Omega x)^{1/2}$.

A portfolio x is *riskless* if $\sigma(x) = 0$. It will be assumed that either there is no riskless asset or else the first asset is riskless while the remaining assets are risky. More specifically, in the first case (no riskless asset), it is assumed that the full covariance matrix Ω is positive definite. In the second case (where the first asset is riskless), the total return to the first (riskless) asset is assumed to be positive, and the covariance matrix of returns to the remaining assets is assumed to be positive definite. These assumptions imply that there are no *redundant assets or portfolios*: all portfolios $e \neq 0$ have $(\sigma(e), e' \bar{R}) \neq (0, 0)$.

Investor i has a utility function $W_i(v, \mu)$ which is a function of the variance and mean of total portfolio return. It is defined for $v \geq 0$ and for all values of μ . The corresponding utility function for standard deviation and mean is $U_i(\sigma, \mu) = W_i(\sigma^2, \mu)$.

Assumption 1 W_i is continuously differentiable (also at $v = 0$) with $W'_{iv} < 0$ and $W'_{i\mu} > 0$, and U_i is quasi-concave.

Continuous differentiability of W_i is not strictly necessary, but it is very convenient, and little interesting generality is lost by maintaining Assumption 1. In fact, the main reason for introducing the function W_i instead of just working with U_i is the convenience of the differentiability assumption on W_i . It implies differentiability of U_i , but differentiability of U_i does not imply differentiability of W_i when $v = 0$.

The assumption implies that U_i is *strongly quasi-concave*, which means that if (σ, μ) and (σ', μ') are points with $U_i(\sigma', \mu') > U_i(\sigma, \mu)$, then

$$U_i(t(\sigma, \mu) + (1 - t)(\sigma', \mu')) > U_i(\sigma, \mu)$$

for all t with $0 < t < 1$.

The investor's utility function for portfolios is

$$V_i(x) = W_i(x' \Omega_i x, x' \bar{R}^i) = U_i(\sigma_i(x), x' \bar{R}^i).$$

It is quasi-concave (in fact, strongly quasi-concave) and continuously differentiable. If U_i is concave, then so is V_i .

Mean-variance behavior is consistent with expected utility maximization with general utility functions if the total returns follow the distributions described by Chamberlain (1983) and Owen and Rabinovitch (1983), which include the normal distributions. In the present paper, mean-variance behavior is treated not as a special case of expected utility maximization but as an alternative, normal distributions being a special case of both.

Concavity of U_i is a stronger assumption than convexity of the induced preferences in (σ, μ) -space. For example, concavity of U_i implies that all the indifference curves in (σ, μ) -space have the same asymptotic slope at large values of σ . Kannai (1977) discusses the assumptions involved in representing a preference relation by a concave utility function. If U_i results from expected utility based on normal distributions and a risk averse von Neumann–Morgenstern utility function, then U_i is concave.

As discussed in Nielsen (1987), V_i may not be monotone, and it may exhibit satiation if there is no riskless asset. *Satiation* refers to the situation where V_i has an unconstrained global maximum at some portfolio x . Such a maximum is called a *satiation portfolio* for i . The possibility of satiation and the unboundedness of the investors' choice sets may lead to non-existence of a general equilibrium, cf. Nielsen (1990a). Nielsen (1989, 1990b) has provided several sets of sufficient conditions for existence of equilibrium in CAPM.

An *allocation* is an m -tuple $(x^i) = (x^1, \dots, x^m)$ consisting of a portfolio x^i for each i . The investors are endowed with an *initial portfolio allocation* (ω^i) . The *market portfolio* $\omega = \sum_i \omega^i$ indicates the total number of shares available of each asset. An *attainable allocation* is an allocation (x^i) such that $\sum_i x^i = \omega$.

An allocation is *individually rational* if it is attainable and Pareto dominates the initial allocation. Let A denote the set of individually rational allocations, i.e.,

$$A = \{(x^i) : \sum_i x^i = \sum_i \omega^i, x^i \in X^i, V_i(x^i) \geq V_i(\omega^i) \text{ for all } i\}.$$

Say a portfolio x is *Pareto attainable* for investor i if it is part of some allocation in A , i.e., it belongs to the set

$$A^i = \{x : x = x^i \text{ for some } (x^1, \dots, x^m) \in A\}.$$

Proposition 1 *The set A of individually rational allocations and the sets A^i of Pareto attainable portfolios are compact.*

PROOF: It is enough to show that A is compact. Since A is closed and convex, it suffices to show that it has no non-zero directions of recession, cf. Rockafellar (1970, Theorem 8.4). This follows from Proposition 1 of Nielsen (1989). \square

The conditions to be derived ensure positive prices not only at general equilibria but, more generally, at price equilibria or at individually rational price equilibria.

A *price equilibrium* is a pair $(p, (x^i))$ of a price system $p \neq 0$ and an attainable allocation (x^i) such that for each i , if y^i is a portfolio with $p'y^i \leq p'x^i$, then $V_i(y^i) \leq V_i(x^i)$. The price equilibrium is *individually rational* if the allocation (x^i) is individually rational.

A *general equilibrium* is a pair $(p, (x^i))$, where $p \neq 0$ is a price system (a k -vector) and (x^i) is an attainable allocation, such that for each i , $p'x^i \leq p'\omega^i$, and if y^i is a portfolio with $p'y^i \leq p'\omega^i$, then $V_i(y^i) \leq V_i(x^i)$. The initial allocation (ω^i) is exogenously given, while the asset price vector p and the equilibrium portfolio allocation (x^i) are endogenous.

Every general equilibrium is an individually rational price equilibrium. A price equilibrium $(p, (x^i))$ is a general equilibrium if and only if $p'x^i = p'\omega^i$ for all i .

3 Joint Restrictions

In the mean-variance model, some of the prices associated with an equilibrium may be negative. The reason for this is that preferences for portfolios may not be monotone (which is also why satiation may occur). In this and the following sections, we shall identify some restrictions on the exogenous parameters of the model which ensure that all equilibrium prices are positive. The restrictions derived in the present section are “joint restrictions” in the sense that they involve all investors’ preference maps jointly. Specifically, they are restrictions of the shape of the preference maps on the set A of individually rational allocations or the sets A^i of Pareto attainable allocations. Recall that the definition of the set A , and hence also the definition of the sets A^i , involves all the investors’ utility functions. The restrictions derived in the following section are “independent restrictions” which can be checked for each investor individually without reference to the preference maps of the others.

The idea will be to use the gradients of the utility functions to derive an expression for the equilibrium prices. In order to compute and exploit an expression for the gradient of V_i , it is useful to introduce some notation. Set

$$\gamma_i(\sigma, \mu) = -2W'_{iv}(\sigma^2, \mu)/W_{i\mu}(\sigma^2, \mu).$$

Then $\gamma_i(\sigma, \mu) > 0$. If x is a portfolio, let

$$\begin{aligned}\tilde{\gamma}_i(x) &= \gamma_i(\sigma(x), x' \bar{R}) \\ &= -2W'_{iv}(x' \Omega x, x' \bar{R})/W_{i\mu}(x' \Omega x, x' \bar{R}).\end{aligned}$$

The gradient of V_i is

$$\begin{aligned}V'_i(x) &= 2W'_{iv} \Omega x + W'_{i\mu} \bar{R} \\ &= W'_{i\mu} [\bar{R} - \tilde{\gamma}_i(x) \Omega x].\end{aligned}$$

The expression for the gradient has the following implication. The induced utility function V_i is *locally strictly monotone* at x in the direction of asset j (formally, $V'_{ij}(x) > 0$) if and only if

$$\bar{R}_j > \tilde{\gamma}_i(x) \text{cov}(R_j, x' R).$$

Suppose $(p, (x^i))$ is a price equilibrium. It follows from the first-order condition for utility maximization and the expression above for the gradient that for each i , there exists $\lambda_i \geq 0$ such that

$$\lambda_i p = \bar{R} - \tilde{\gamma}_i(x^i) \Omega x^i, \quad (1)$$

where $V'_i(x^i) \neq 0$ and $\lambda_i > 0$ if and only if investor i is not satiated at x^i .

For each asset j and each investor i , the function $\tilde{\gamma}_i(x^i) \text{cov}(R_j, x^i R)$ is continuous. Since the set A^i of Pareto attainable portfolios is compact, the function has a well defined maximum on A^i .

Proposition 2 *If*

$$\bar{R}_j > \min_i \max\{\tilde{\gamma}_i(x^i) \text{cov}(R_j, x^i R) : x^i \in A^i\},$$

then $p_j > 0$ in every individually rational price equilibrium.

PROOF: Let $(p, (x^i))$ be an individually rational price equilibrium. By assumption, there is an investor i such that $\bar{R}_j > \tilde{\gamma}_i(x^i) \text{cov}(R_j, x^i R)$. It follows from Equation 1 that $p_j > 0$. \square

The assumption in Proposition 2 says that at every individually rational allocation, at least one investor's induced utility function is locally strictly monotone in the direction of asset j .

If $x = (x^i)$ is an allocation, set

$$\gamma(x) = \left[\sum_i (\tilde{\gamma}_i(x^i))^{-1} \right]^{-1}.$$

Then $\gamma(x) > 0$. Divide Equation 1 by $\tilde{\gamma}_i(x^i)$, sum over i , and multiply by $\gamma(x)$ to get

$$\lambda p = \bar{R} - \gamma(x) \Omega \omega; \quad (2)$$

where

$$\lambda = \gamma(x) \sum_i \frac{\lambda_i}{\tilde{\gamma}_i(x^i)} \geq 0.$$

Equation 2 holds at all price equilibria, and λ is positive if and only if at least one investor i is not satiated at x^i .

Since $\gamma(x) \text{cov}(R_j, \omega' R)$ is a continuous function of the allocation x , it has a well defined maximum value on the set A of individually rational allocations.

Proposition 3 If

$$\bar{R}_j > \max\{\gamma(x)\text{cov}(R_j, \omega'R) : x \in A\}, \quad (3)$$

then $p_j > 0$ at every individually rational price equilibrium.

PROOF: Let $(p, (x^i))$ be an individually rational price equilibrium. By assumption, $\bar{R}_j > \gamma(x)\text{cov}(R_j, \omega'R)$. It follows from Equation 2 that $p_j > 0$. \square

If $\text{cov}(R_j, \omega'R) = 0$, then Inequality 3 holds. If $\text{cov}(R_j, \omega'R) > 0$, then Inequality 3 is equivalent to

$$\bar{R}_j/\text{cov}(R_j, \omega'R) > \max\{\gamma(x) : x \in A\}.$$

If $\text{cov}(R_j, \omega'R) < 0$, then Inequality 3 is equivalent to

$$\bar{R}_j/\text{cov}(R_j, \omega'R) < \min\{\gamma(x) : x \in A\}.$$

Example 1 Utility linear in mean and variance, no riskless asset. Suppose $W_i(v, \mu) = \mu - a_i v/2$. In this case, $\tilde{\gamma}_i(x^i) = a_i$ independently of the portfolio x^i , and

$$\gamma(x) = \gamma = [\sum_i a_i^{-1}]^{-1}$$

independently of the allocation (x^i) , so that the prices associated with a price equilibrium must, apart from a positive multiplicative constant, be given by the price system

$$q = \bar{R} - (\sum_i a_i^{-1})^{-1} \Omega \omega.$$

If Ω is regular (no riskless asset), then each investor has the unique satiation portfolio $s^i = a_i^{-1} \Omega^{-1} \bar{R}$. It is shown in Nielsen (1989) that a general equilibrium exists if and only if $q' \omega^i \leq q' s^i$ for all i . The associated equilibrium price system q will be positive if and only if $\bar{R} \gg \gamma \Omega \omega$. Note that this inequality is expressed entirely in terms of exogenous variables and parameters. If it does not hold, then some of the equilibrium prices are zero or negative. \square

For each asset j , the continuous function $\frac{1}{m}\tilde{\gamma}_i(x^i)\text{cov}(R_j, \omega'R)$ takes a maximum value on the set of Pareto attainable portfolios, which is compact.

Proposition 4 *If*

$$\bar{R}_j > \frac{1}{m} \max_i \max\{\tilde{\gamma}_i(x^i)\text{cov}(R_j, \omega'R) : x^i \in A^i\} \quad (4)$$

then $p_j > 0$ in all individually rational price equilibria.

PROOF: This follows from Proposition 3 since for each allocation $x = (x^i)$,

$$\frac{1}{m} \max_i \tilde{\gamma}_i(x^i)\text{cov}(R_j, \omega'R) \geq \gamma(x)\text{cov}(R_j, \omega'R).$$

□

If $\text{cov}(R_j, \omega'R) = 0$, then Inequality 4 holds. If $\text{cov}(R_j, \omega'R) > 0$, then Inequality 4 is equivalent to

$$\bar{R}_j/\text{cov}(R_j, \omega'R) > \frac{1}{m} \max_i \max\{\tilde{\gamma}(x^i) : x^i \in A^i\}.$$

If $\text{cov}(R_j, \omega'R) < 0$, then Inequality 4 is equivalent to

$$\bar{R}_j/\text{cov}(R_j, \omega'R) < \frac{1}{m} \min_i \min\{\tilde{\gamma}(x^i) : x^i \in A^i\}.$$

4 Independent Restrictions

The sufficient conditions for positive prices developed in the previous section rely on maximizing various functions on the set of Pareto attainable portfolios or the set of individually rational allocations. Whether an allocation is individually rational depends on all investors' indifference maps, and whether a portfolio is Pareto attainable for particular investor depends not only on his own indifference map but also on those of the other investors. The present section exhibits some sufficient conditions that are independent across investors.

Proposition 5 *Consider a particular asset j . Make the following assumptions:*

1. $U_i(\sigma(\omega^i), \omega^i \bar{R}) \geq U_i(0, 0)$ for all i .
2. For all i , $\bar{R}_j > \frac{1}{m} \gamma_i(\sigma, \mu) \text{cov}(R_j, \omega' R)$ for all (σ, μ) with $\mu \leq E_M$ and $U_i(\sigma, \mu) \geq U_i(\sigma(\omega^i), \omega^i \bar{R})$.

Then $p_j > 0$ at all individually rational price equilibria (in particular, at all general equilibria).

PROOF: Suppose (x^i) is an individually rational allocation. Then

$$U_i(\sigma(x^i), x^i \bar{R}) \geq U_i(\sigma(\omega^i), \omega^i \bar{R}) \geq U_i(0, 0),$$

and hence $0 \leq x^i \bar{R}$ for all i . Since $\sum_i x^i = \omega$, it follows that $0 \leq x^i \bar{R} \leq E_M$ for all i . So,

$$\bar{R}_j > \frac{1}{m} \tilde{\gamma}_i(x^i) \text{cov}(R_j, \omega' R).$$

The result follows from Proposition 4. \square

The second assumption of Proposition 5 restricts the value of $\gamma_i(\sigma, \mu)$ for values of (σ, μ) in a certain bounded range. That range is illustrated in Figure 1.

For each asset j , let σ_j denote the standard deviation of total return per share to asset j (so that σ_j^2 is the j 'th diagonal entry in the covariance matrix Ω). Let α_j denote the ratio of mean to standard deviation of total return per share of asset j . Formally, $\alpha_j = \bar{R}_j / \sigma_j$ (if $\sigma_j = 0$ then $|\alpha_j| = \infty$).

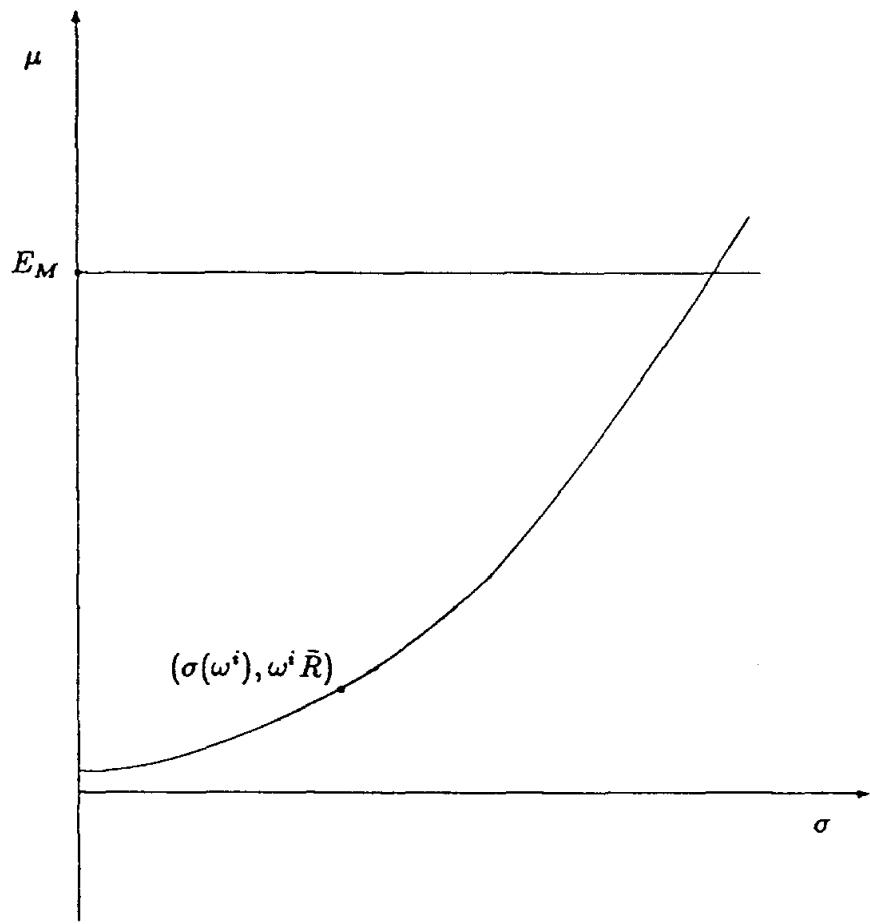


Figure 1: The range of (σ, μ) in the second assumption of Proposition 5

Let

$$S_i(\sigma, \mu) = -U'_{i\sigma}(\sigma, \mu)/U'_{i\mu}(\sigma, \mu)$$

denote the slope of investor i 's indifference curve in (σ, μ) -space at (σ, μ) . Then $S_i(0, \mu) = 0$ since $U'_{i\sigma} = 2\sigma W'_{i\sigma}$. If x is a portfolio, let

$$\tilde{S}_i(x) = S_i(\sigma_i(x), x' \bar{R}^i)$$

be the slope of the indifference curve at the standard deviation and mean of return to x .

Since $U'_{i\sigma} = 2\sigma W'_{iv}$, and $U'_{i\mu} = W'_{iv}$, it follows that

$$\sigma(x)\tilde{\gamma}_i(x) = \tilde{S}_i(x).$$

The joint restrictions in Proposition 2 imposed local strict monotonicity of the induced utility function at individually rational allocations. One way to make this restriction independent across investors would be to impose strict monotonicity everywhere. This would be undesirably restrictive, because it is not satisfied by utility functions that are linear in mean and variance. We can do better, but it is useful first to relate local and global strict monotonicity.

Proposition 6 Consider a particular asset j and an investor i .

1. If $\tilde{S}_i(x) < \alpha_j$ at some particular portfolio x , then V_i is locally strictly monotone at x in the direction of asset j .
2. If V_i is globally strictly monotone in the direction of asset j , then $\tilde{S}_i(x) < \alpha_j$ at all portfolios x .

PROOF: The first statement follows from the following computation:

$$\begin{aligned} \tilde{\gamma}_i(x)\text{cov}(R_j, x'R) &\leq |\tilde{\gamma}_i(x)|\text{cov}(R_j, x'R)| \\ &\leq |\tilde{\gamma}_i(x)\sigma_j\sigma(x)| \\ &= |\tilde{S}_i(x)\sigma_j| \\ &< |\alpha_j\sigma_j| \\ &= |\bar{R}_j|. \end{aligned}$$

To prove the second statement, let e^j be a portfolio consisting of one share of asset j . For $t > 0$ increasing, the curve $(\sigma(x + te^j), (x + te^j)' \bar{R})$ in (σ, μ) -space cuts across ever higher indifference curves. The slope of the curve decreases strictly toward the limit α_j . It stays above the indifference curve through $(\sigma(x), x' \bar{R})$, whose slope increases strictly as σ and μ increase. Hence, $\tilde{S}_i(x) < \alpha_j$. \square

Proposition 7 Consider a particular asset j . Make the following assumptions:

1. $U_i(\sigma(\omega^i), \omega^i \bar{R}) \geq U_i(0, 0)$ for all i .
2. There is some investor k for whom $S_k(\sigma, E_M) < \alpha_j$ for all σ with $U_k(\sigma, E_M) \geq U_k(\sigma(\omega^k), \omega^k \bar{R})$.

Then $p_j > 0$ at all individually rational price equilibria (in particular, at all general equilibria).

PROOF: Choose k as in (2). Let (x^i) be an individually rational allocation. As in the proof of Proposition 5, $x^i \bar{R} \leq E_M$ for all i . Choose σ with $U_k(\sigma, E_M) = V_k(x^k)$. Then $U_k(\sigma, E_M) \geq V_k(\omega^k)$, so $S_k(\sigma, E_M) < \alpha_j$. Since $(\sigma(x^k), x^k \bar{R})$ and (σ, E_M) lie on the same indifference curve but $x^k \bar{R} \leq E_M$, it follows that $\tilde{S}_k(x^k) \leq S_k(\sigma, E_M) < \alpha_j$. By Proposition 6,

$$\tilde{\gamma}_k(x^k) \text{cov}(R_j, x^k R) < \bar{R}_j.$$

The result now follows from Proposition 2. \square

The second assumption of Proposition 7 restricts the investor's risk aversion $S_k(\sigma, \mu)$ on the top line segment of the region where $\gamma_k(\sigma, \mu)$ is restricted in Proposition 5. The argument is illustrated in Figure 2. The restriction can be interpreted like this: If the investor had a portfolio with standard deviation and mean (σ, E_M) , then he would like to add a small uncorrelated portfolio with the same standard deviation and mean as asset j . Note that the assumption is considerably weaker than requiring that at least one investor's induced utility function is globally strictly monotone.

The slope S_i of the investor's indifference curve in (σ, μ) -space is a measure of his risk aversion. It indicates how much additional mean return

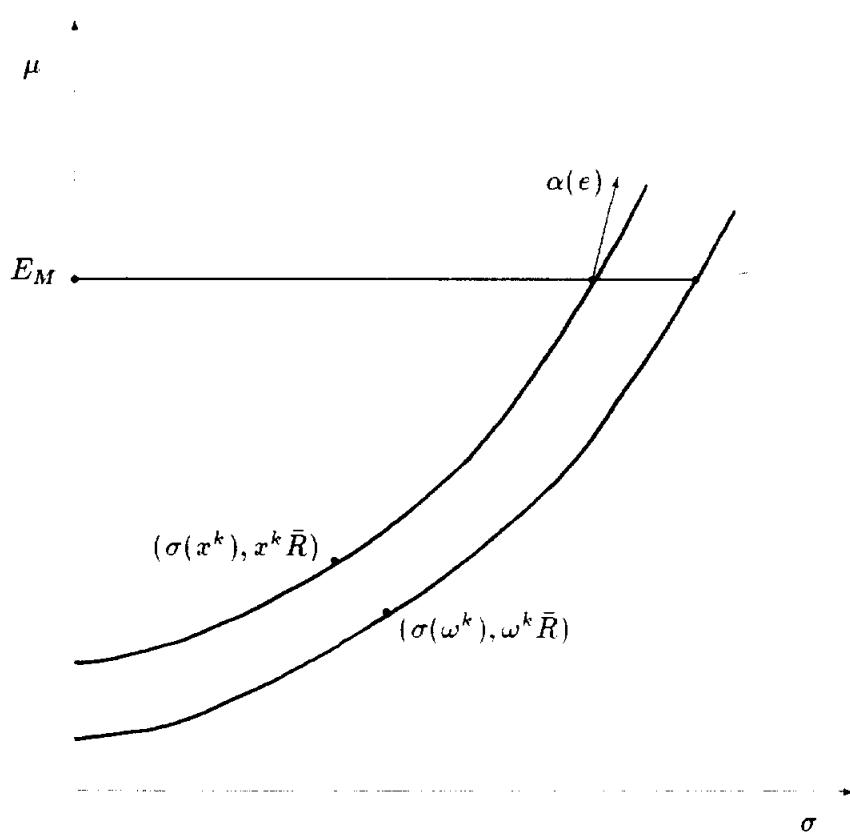


Figure 2: The situation in Proposition 7

he requires as compensation for accepting an infinitesimal increase in standard deviation of return. A high value of S_i implies high risk aversion. Say that U_i exhibits *decreasing risk aversion* if S_i is a non-increasing function of μ . This appealing property means that the higher the investors' mean return is, the less additional mean return does he need in order to accept a given increase in standard deviation of return. If U_i exhibits decreasing risk aversion, then for every $\sigma \geq 0$, $\sigma' \geq 0$, there exists μ' such that $U_i(\sigma, \mu) = U_i(\sigma', \mu')$; and S_i is a non-decreasing function of σ . If U_i is derived from expected utility based on normal distributions and a von Neumann–Morgenstern utility function u_i , then U_i exhibits decreasing risk aversion (in the sense defined here) if u_i exhibits decreasing absolute risk aversion (in the ordinary sense). For an example of an application of the concept of decreasing risk aversion in the mean-variance model, see Nielsen (1988).

Exploiting the concept of decreasing risk aversion in the present context, the second assumption in Proposition 7 can be replaced as follows:

Proposition 8 Consider a particular asset j . Make the following assumptions:

1. $U_i(\sigma(\omega^i), \omega^i \bar{R}) \geq U_i(0, 0)$ for all i .
2. There is some investor k with decreasing risk aversion, for whom $S_k(\sigma, E_M) < \alpha_j$ at the unique σ where $U_k(\sigma, E_M) = U_k(\sigma(\omega^k), \omega^k \bar{R})$.

Then $p_j > 0$ at all individually rational price equilibria (in particular, at all general equilibria).

The second assumption of Proposition 8 restricts the investor's risk aversion only at the right end-point of the line segment shown in Figure 2.

The following theorem is possibly the most satisfactory of our results about positive portfolio values. It shows that when the investors have decreasing risk aversion, the only additional restriction on their risk aversion that is necessary in order to ensure a positive price of asset j is that $\tilde{S}_i(\omega^i) < \alpha_j$ for all investors i . This says that all investors would like to add a small amount of j to their initial portfolio if j were uncorrelated with it.

Theorem 1 Consider a particular asset j . Make the following assumptions:

1. $U_i(\sigma(\omega^i), \omega^i \bar{R}) \geq U_i(0, 0)$ for all i .
2. For all i , $\tilde{S}_i(\omega^i) < \alpha_j$.
3. All investors have decreasing risk aversion.

Then $p_j > 0$ at all individually rational price equilibria.

PROOF: Let (x^i) be an individually rational allocation. As in the proof of Proposition 5, $x^i \bar{R} \leq E_M$ for all i . Since $\sum_i x^i \bar{R} = \sum_i \omega^i \bar{R}$, there is at least one investor i such that $x^i \bar{R} \leq \omega^i \bar{R}$. For that investor,

$$U_i(\sigma(\omega^i), \omega^i \bar{R}) \leq V_i(x^i) \leq U_i(0, \omega^i \bar{R}).$$

It follows that there exists $\sigma \leq \sigma(\omega^i)$ such that $U_i(\sigma, \omega^i \bar{R}) = V_i(x^i)$. Then

$$\tilde{S}_i(x^i) \leq S_i(\sigma, \omega^i \bar{R}) \leq S_i(\sigma(\omega^i), \omega^i \bar{R}) < \alpha_j.$$

As in the proof of Proposition 7, it follows that $p_j > 0$. \square

The proof of Theorem 1 is illustrated in Figure 3.

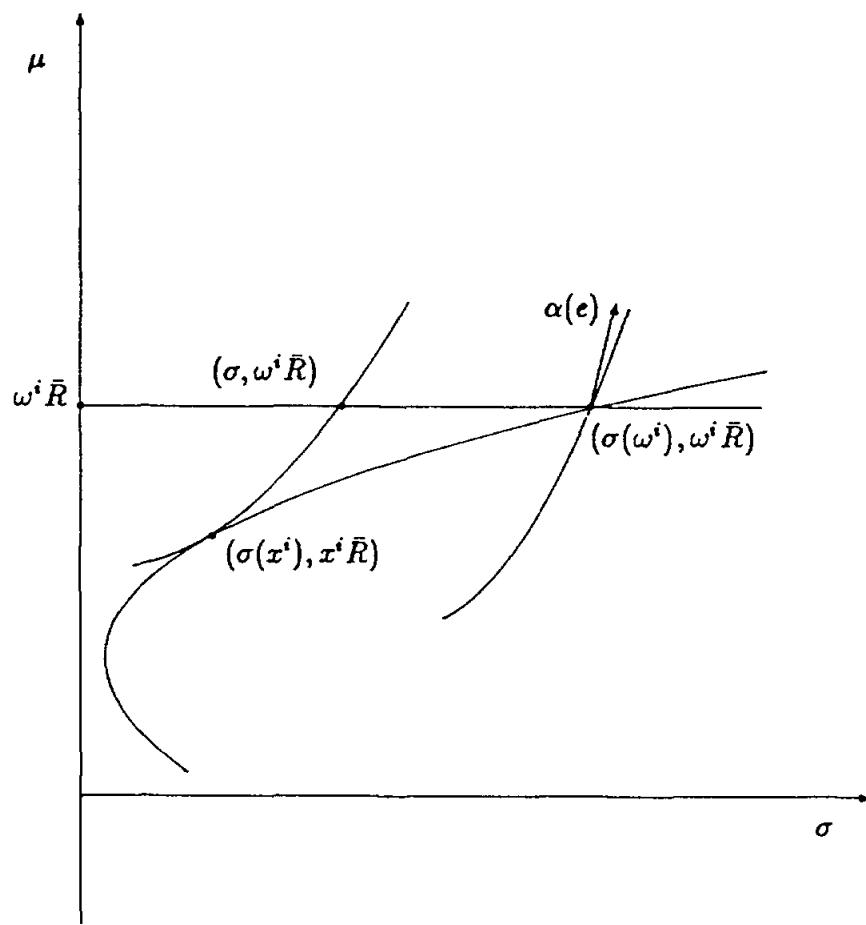


Figure 3: The proof of Theorem 1

5 Riskless Asset

From now on, it will be assumed that the first asset is riskless. Correspondingly, we make a few changes in the notation. Now, Ω will denote the covariance matrix of returns per share only to the risky assets, and R and \bar{R} will denote the vector of total returns per share and the vector of expected total returns per share to those assets. The total return per share of the riskless asset, which is assumed to be positive, will be denoted R_f (subscript “*f*” for *risk-free*). A portfolio will be written in the form (x, y) , where x is the number of shares of the riskless asset in the portfolio, and where the entries in the vector y are the numbers of shares of the risky assets included in the portfolio. The total return to a portfolio (x, y) is $xR_f + y'R$, and the expected total return is $xR_f + y'\bar{R}$. The prices of the assets will be normalized in such a manner that the price per share of the riskless asset is one; and p will denote the vector of prices per share only of the risky assets. The value (cost) of a portfolio (x, y) is $x + p'y$. The investor's initial portfolio is written (ϕ^i, ω^i) , and the total supply of risky asset shares is $\omega = \sum_i \omega^i$. Let $E_M = \omega'\bar{R}$ be the expected return to the market portfolio of risky assets only, and let $\sigma_M = \sqrt{\omega'\Omega\omega}$ be the standard deviation of return to the market portfolio of risky assets.

When the first asset is riskless, if $(p, (x^i, y^i))$ is a price equilibrium, then Equation 2 takes the following form for the system p of prices of the risky assets:

$$p = \frac{1}{R_f} [\bar{R} - \gamma \Omega \omega].$$

When the investors have homogeneous beliefs and the first asset is riskless (with positive expected return), this equation holds at all price equilibria.

Example 2 Utility linear in mean and variance, first asset riskless. Suppose $W_i(v, \mu) = \mu - a_i v / 2$, as in Example 1, but suppose the first asset is riskless. Then $\tilde{\gamma}_i(x^i, y^i) = a_i$ independently of the portfolio (x^i, y^i) , and

$$\gamma(x, y) = \gamma = [\sum_i a_i^{-1}]^{-1}$$

independently of the allocation (x, y) . The prices of risky assets associated

with a price equilibrium must be

$$p = \frac{1}{R_f} (\bar{R} - (\sum_i a_i^{-1})^{-1} \Omega \omega).$$

In fact, there is always a unique general equilibrium, and the associated prices are as above. The price system p will be positive if and only if $\bar{R} \gg \gamma \Omega \omega$. If this inequality does not hold, then some of the equilibrium prices are zero or negative. Note that the inequality is expressed entirely in terms of exogenous variables and parameters. It is possible that some prices are negative; in fact, it is possible that the value of the market portfolio ω of risky assets is negative. Since $p' \omega = (E_M - \gamma \sigma_M^2) / R_f$, $p' \omega$ is negative if and only if $E_M / \sigma_M^2 < (\sum_i a_i^{-1})^{-1}$. This is more likely to be the case the lower is the expected return to the market, the higher is the variance of return to the market, and the more risk averse are the investors. \square

The results of the previous sections can easily be translated to the present situation where there is a riskless asset. The following proposition is specific to the case with a riskless asset, because it relies on the fact that equilibrium portfolios are combinations of the riskless asset and the market portfolio of risky assets.

Proposition 9 *CAPM with a riskless asset. Consider a particular risky asset j . Assume that for some investor k , $S_k(\sigma_M, \mu) < \alpha_j$ for all μ with $U_k(\sigma_M, \mu) \geq U_k(\sigma(y^k), \phi^k R_f + \omega^k \bar{R})$ (and there exist such μ). Then $p_j > 0$ at all price equilibria.*

PROOF: If $(p, ((x^i, y^i)))$ is a price equilibrium, then each y^i has the form $y^i = \lambda_i \omega$, $\lambda_i \geq 0$, by the “separation theorem.” Here, $\lambda_i \leq 1$ for all i because $\sum_i \lambda_i = 1$, and so $\sigma(y^i) = \lambda_i \sigma_M \leq \sigma_M$ and $U_i(\sigma_M, x^i R_f + y^i \bar{R}) \leq V_i(x^i, y^i)$. Consider a particular investor k satisfying the assumption. Choose μ such that $V_k(x^k, y^k) = U_k(\sigma_M, \mu)$. In order to show that this is possible, first choose $\hat{\mu}$ such that $U_k(\sigma_M, \hat{\mu}) \geq U_k(\sigma(y^k), \phi^k R_f + \omega^k \bar{R})$. If $\hat{\mu}$ is large enough, then the line through $(\sigma_M, \hat{\mu})$ with slope α_j passes above $(\sigma(y^k), \phi^k R_f + \omega^k \bar{R})$. Since $S_k(\sigma_M, \hat{\mu}) < \alpha_j$, the indifference curve through $(\sigma_M, \hat{\mu})$ also passes above $(\sigma(y^k), \phi^k R_f + \omega^k \bar{R})$, and so $V_k(x^k, y^k) \leq U_k(\sigma_M, \hat{\mu})$. Since

$$U_k(\sigma_M, x^k R_f + y^k \bar{R}) \leq V_k(x^k, y^k) \leq U_k(\sigma_M, \hat{\mu}),$$

it is possible to choose μ such that $V_k(x^k, y^k) = U_k(\sigma_M, \mu)$. Now, $U_k(\sigma_M, \mu) \geq U_k(\sigma(\omega^k), \phi^k R_f + \omega^k \bar{R})$ and $\tilde{S}_k(x^k, y^k) \leq S_k(\sigma_M, \mu) < \alpha_j$. As in the proof of Proposition 7, it follows that $p_j > 0$. \square

The assumption in Proposition 9 restricts the investor's risk aversion on a half-line in (σ, μ) -space. The half-line is illustrated in Figure 4.

Exploiting the concept of decreasing risk aversion, the assumption in Proposition 9 can be replaced as follows:

Proposition 10 *CAPM with a riskless asset. Consider a particular risky asset j . Assume that for some investor k with decreasing risk aversion, $S_k(\sigma_M, \mu) < \alpha_j$ at the unique μ where $U_k(\sigma_M, \mu) = U_k(\sigma(\omega^k), \phi^k R_f + \omega^k \bar{R})$. Then $p_j > 0$ at all price equilibria.*

The assumption in Proposition 10 restricts the investor's risk aversion only at the end-point of the half-line in Figure 4.

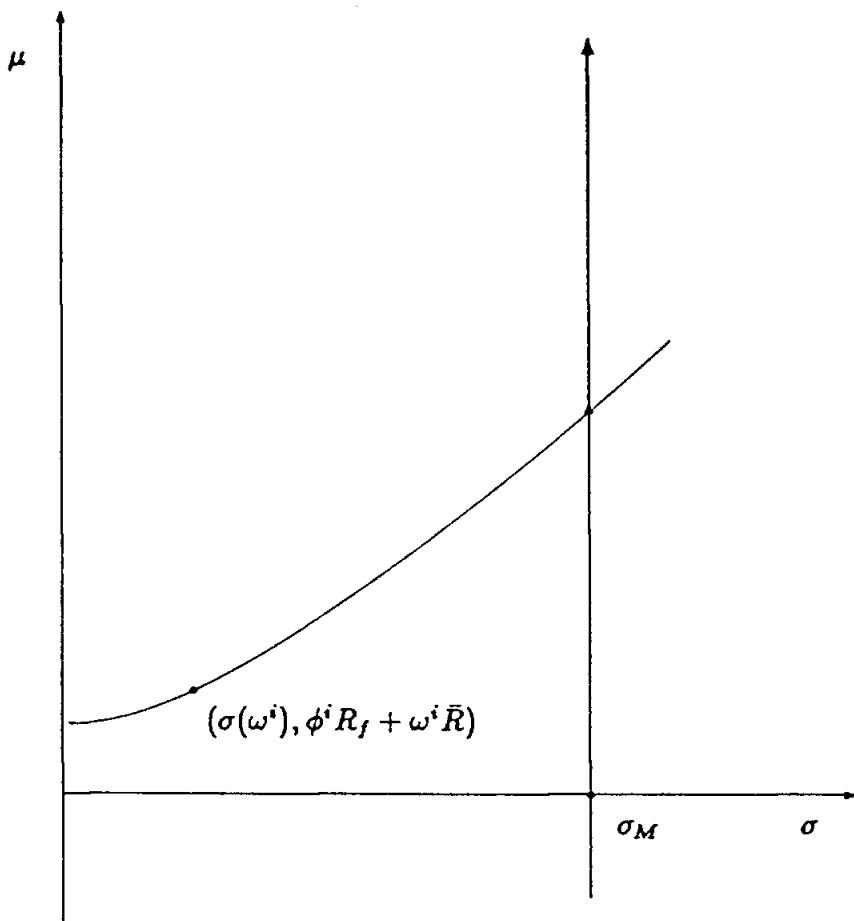


Figure 4: The half-line in the assumption of Proposition 9

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