"APPROXIMATE REASONING BY ANALOGY TO ANSWER NULL QUERIES"

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Approximate Reasoning By Analogy To Answer Null Queries

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Abstract

Analogy refers to recognizing certain similarities between a source situation/object and a target situation/object and deriving some properties of the target based on the observed similarities with the source. Analogy is an important inference method in human cognition and is a powerful computational tool for general inference. Null queries are queries which elicit a null answer from the database system, often due to the presence of incomplete information in the database, e.g., the absence of certain attribute values for some objects. Analogy is useful for obtaining an approximate answer to a null query. In this paper, we develop the theoretical basis for the application of analogical reasoning to obtain approximate answers for null queries in the context of a fuzzy relational data model. Incorporating analogical reasoning in data models enhances their user-friendliness. Our proposed model of analogy incorporates fuzzy logic and is a natural generalization of models of analogy researched in the domain of artificial intelligence.

1 Introduction

Null queries are a common source of frustration to users while handling data-bases. Informally, we refer to a query as a null query if it elicits a null answer from the database. Null answers are produced when no data items satisfy the conditions expressed in the query. Many null queries are caused by undetected errors in the queries submitted by the users [41, 44]. Such errors could either be simple syntactic mistakes (e.g., misspelling a name) or be a result of misconceptions of the user about the data stored in the database (e.g., asking for tuples from a relation R, where the relation R does not exist in the database). The other source of null queries is incomplete information in the database [41, 42], usually caused by missing values of attributes for some entities.

In this paper we shall only consider null queries caused by incomplete information in the database. Most conventional databases simply indicate that the user query has produced a null answer. Following the principle that *something is better than nothing*, it would be advantageous to have the database produce an approximate answer to the query rather than output a null answer. This shall improve the co-operativeness of the the database and enhance man-machine interaction.

Analogy consists of recognizing certain similarities between a source situation/object and a target situation/object and deriving certain properties of the target on the basis of its observed similarities with the source. Analogy is an important inference method in human cognition and is a powerful computational tool for general inference in artificial intelligence research. In this paper we shall develop a theoretical model for the application of analogy as an effective reasoning tool for producing approximate answers to null queries. We shall consider a fuzzy relational datamodel because it is more powerful than crisp data models in dealing with the inherent imprecision in real world data. Our results also apply to conventional relational data models as the fuzzy relational model is a generalization of the classical relational model.

This paper contains five other sections. Section 2 formalizes the kind of null queries we shall be considering in this paper and describes the relevant prior research in database theory for answering null queries. Section 3 introduces analogical reasoning and describes past work in analogy from the domain of artificial intelligence. Section 4 provides a brief introduction to fuzzy set theory and fuzzy relational data models. Section 5 describes our model for the application of analogical reasoning for answering null queries in the context of a fuzzy relational database. Section 6 concludes the paper.

2 Null Queries

This section describes the kind of null queries considered in this paper and describes some of the relevant research in producing approximate answers to null queries.

In this paper we shall only consider the case of incomplete information in the database caused by missing data. This is a common source of null queries caused by incomplete databases. Strictly speaking, our model can be applied to any sort of incomplete database as we can always treat the available value of an attribute (which may be obsolete or erroneous) as missing and go ahead and reason approximately by analogy. Though it may be beneficial to also utilize the obsolete or erroneous values of the attribute in arriving at an answer, we shall not deal in this paper with the issue of suitably combining the results of analogical reasoning with other available erroneous values to arrive at a more comprehensive answer.

Let D be a database, Q be a query posed to the database and Q(D) be the answer generated by D in response to Q. For a null query Q, Q(D) is the empty set. Our aim in this paper is to develop a theoretical model for obtaining by analogical reasoning, Q'(D), where Q'(D) is an approximation to Q(D), Q being a null query. The system attempts to determine Q'(D) only as a recovery measure after the original user query has produced a null answer and the system has verified that some missing data is the cause of the null answer. An example of a null query caused by missing data is shown in example 1.

Table 1 about here

<u>Example 1</u> : Consider the relation shown in table 1 giving the nationalities, native languages and eye-colors of certain persons. Note that the nationality and eye-color of Mike are known, but his native language is unknown. Now when the query

language(Mike) ?

is presented to the system, a null answer is produced. Here though the value of the attribute language of Mike is missing, the co-operative nature of the data-base would be improved if the data-base could produce an approximate answer to the query by considering the other facts in the database. A reasonable answer to the above query would be

language (Mike) = English

as the nationality of Mike is American and from the knowledge present in the other tuples of the relation, we see that the native language of both the other Americans (John and Barry) is English. Of course this answer may be incorrect as the native language of Mike could in reality be Spanish or something else, but producing an approximate answer is perhaps better than producing no answer at all. What we have done just now is reasoned by analogy to arrive at the conclusion that the native language of Mike may be English. We have some faith in our conclusion due to our knowledge of the fact that usually persons of the same nationalities have the same mother tongue. We shall formalize the idea of analogical reasoning in section 3.

2.1 Prior Research in Answering Null Queries

The issue of producing approximate answers to null queries has not been studied by many researchers in data-base theory. Traditionally, several extensions of the classical relational model have been proposed [45, 46, 47, 48], in some of which a variety of "null values" have been introduced to model unknown or not applicable data values. Various generalized operators were introduced to handle the extended data models and Reiter [47] suggested the usage of first order predicate calculus where Skolem functions are used to represent "null values". However, these models canot be used for reasoning about approximate answers to null queries.

Wong [42] considered queries for which no exact answers could be provided due to incomplete information in the database. He developed a statistical model to produce an approximate answer to the query by exploiting prior information about the imprecision. His model deals with a large spectrum of possible sources of imprecision in the database. He deals with the problem of missing data by projecting the missing value from another tuple. This has a flaw in that the model does not specify which tuple to use as the base from which to project the missing value. A random choice may lead to a poor answer. Selecting the tuple with the maximum number of common values shall also often not lead to the right answer . **In example** 1 above, at most one attribute value of the tuple (Mike, USA, ?, Black) matches with any of the other tuples. If we choose to project the value of the attribute language from the tuple (Ashok, India, Bengali, Black), we shall obtain the (most probably) wrong answer :

language(Mike) = Bengali

What we need to know before making such a projection is that certain attributes are more relevant than others for determining the value of a particular attribute, e.g., the attribute nationality is more relevant for determining the value of the attribute native-language than the attribute eye-color. Such a notion of relevance is an important part of analogical reasoning and is discussed in sections 3 and 5. Another limitation of the model developed by Wong [42] is that the relation between the real and idealized worlds is either in the form of a distortion function or in the form of a conditional distribution and both of these measures are difficult to obtain accurately. The model of analogical reasoning developed in this paper allows us to develop a simple and viable procedure to determine an approximate answer to a null query.

Motro [41] developed query generalization as a method for interpreting null answers. Motro defined generalization as a partial order among the different queries : the answer of any less general query is always a subset of a more general query. When a query results in a null answer, the system tries to produce a suitable interpretation which classifies the query failure as either a user error or as a genuine null. In the former case it attempts to point out the error; in the latter case, it attempts to provide partial answers (of the more general queries which succeeded) and to delimit the scope of the failure (given by the maximally general query which failed). This scheme is useful for providing an interpretation of the misconceptions of the user about the data in the database but is not very useful for cases where some data is missing in the database, e.g., in example 1, a generalization of the query would have resulted in the system printing out the values of language for all other tuples (persons) in the relation; for a large relation, such an answer would have made little sense to the user. It is certainly preferable to have the system figure out the answer to the best of its ability and output only its best (or best few) possible choice(s). Also, generating all the different (possibly large number of) generalizations of a query is a non-trivial task. The works of Motro [41] and Corella [43] share many basic principles, but the latter is more general.

3 Analogical Reasoning

In this section, we provide an introduction to the problem of analogical reasoning and review some prior research in analogy.

3.1 Analogy

Analogy is an important inference method in human cognition and is a powerful computational tool for general inference in current machine learning research [4-6,8-22]. Learning by analogy refers to the process of inferring that a property Q is true of a particular situation or object T (the target) from the fact that T shares a property (or a set of properties) P with another situation or object S(the source) which has the property Q . P is the similarity between S and T and Q is projected from S onto T . Schematically, it can be represented as

$$P(S) \ \& \ Q(S) \ \& \ P(T) \longrightarrow Q(T)$$

An example would be (see example 1)

$$\{Nat(John, USA) \ \& \ Lang(John, Eng) \ \& \ Nat(Mike, USA) \rightarrow Lang(Mike, Eng)\}$$

where *Nat* is an abbreviation for the attribute *Nationality* and *Lang* is an abbreviation for the attribute *Native-language.* Note that P may be shared fully or partially by S and T. In general the match on P between S and T is partial and hence we need some techniques to quantify this partial match. In addition, most real world knowledge is imprecise, both due to the complexity (imprecision) of data itself and due to our limited information. Traditional boolean logic is incapable of handling such imprecision effectively and thus we need to use some form of multi-valued logic. In this paper, we shall use the fuzzy relational model (see section 4) for handling both imprecision in the data and partial matches between the source and the target.

Various researchers [2, 19, 13], have addressed the issue of trying to find the criterion which, if satisfied by any particular analogical inference, establishes the truth of that inference. Others [4, 5, 8-10, 12, 14, 20-22] have tried to measure the plausibility of the conclusion Q in terms of the similarity between the source and the target. However such an approach is not fully correct, e.g., from the similarity that both John and Barry are humans, we can (most probably) conclude that both have two legs each, but if John has blue eyes, we cannot validly infer that Barry also has blue eyes. Thus researchers [6, 11, 15-17, 18] recognized that it is important that P and Q be relevant to each other for the conclusion Q to be valid. For example, the attribute nationality is relevant for drawing a conclusion about the value of the attribute native-language, while the attribute eye-color is not (see example 1).

Analogical inference is an important technique for approximate reasoning. The conclusions of analogical reasoning are approximate due to the following reasons :

[1] The associated theoretical models do not depend on classical deductive logic. The conclusion Q does not follow deductively from the premises.

[2] The universe from which analogical inference is made (the source S, the target T and the properties P and Q) is generally neither exhaustive nor universal.

3.2 Review of Research in Analogical Reasoning

Analogical reasoning (AR) is currently an area of active research in the domain of artificial intelligence. Most of the early work in AR adopted a similarity-counting approach which consisted of the following two steps:

i. Find the source with the highest number of matching facts with the target. A matching fact usually referred to a property or relation with an identical or 'similar' value.

ii. Project the values of the desired attribute from the source (selected in step (i) above) onto the target.

Such an approach seems to be the natural way to go about doing AR, but an important deficiency of this approach is that all matching facts are given equal value. It is obviously not desired to give equal weight to both trivial and important matching facts. Later research tried to rectify this problem by using a more refined calculation of similarity.

Kling [12] used the following ordering in measuring similarity: predicate matches > function symbol matches > constant-symbol matches. Gentner [8-10] proposed the degree of relationality (relation between facts are more important than relations between objects) and the degree of systematicity (similarities bounded by higher order relations are preferred). These measures were primarily descriptive, rather than normative, backed up by psychological evidence. Winston [20-22], proposed salience theories of analogy in which matches between important and unusual features are given more importance. Such adhoc strategies worked reasonably in certain cases, but simple similarity counting methods often do not lead to the right conclusion in AR and no amount of parameter adjustment or changes in the ordering of important matches can fully rectify this problem. Also none of these models captured the notion of relevance, i.e., certain properties are more important than others for specific conclusions in AR.

This brings us back to the problem of relvance discussed earlier. Chouraqui [49] developed the ARCHES system in which he proposed the existence of a Dependency relation for AR. Such dependencies were well defined statements like

climate depends uniquely on location & latitude

and the source and the target had to match exactly on the values of location and latitude to enable the use of the dependency in AR. ARCHES had no algorithm for automatically learning these dependencies and it also could not also handle imprecision in data.

Davies & Russell [6, 15-17] introduced rules of determinations as a measure of relevance. A determination rule(DR) can be represented as :

nationality determines language

or in short as nationality $\rightarrow$ language. Thus this DR states that for inferring the language of a person, we need to find another person with a matching nationality. Each determination has a numerical measure called the partial determination factor (pdf). The pdf can be thought of as a measure of the accuracy or relevancy of the determination. Dutta [40] implemented algorithms for learning these rules of determinations from a data-base. The pdf values of determinations are obtained by a statistical analysis of the data-base. Determinations specify which attributes are relevant for inferring the value of a certain attribute. This is important when an object may be described by several attributes and we need to find the most relevant attribute for inferring the value of another attribute. However, determinations also have limitations. They do not specify which source to use for AR and do not take other similarities between the source and the target (besides that given by the matching LHS of the determination being used) into consideration. They can only be used for crisp data and cannot accommodate partial matches between the source and target.

Determinations as defined by Davies & Russell [6, 15-17] are a weaker form of functional dependencies as studied extensively in data-base literature [38, 39]. A classical functional dependency (fd) can be represented by a statement

$$X \rightarrow Y$$

where X and Y are sets of attributes. A relation r satisfies this fd if all tuples in r having the same value of X also have the same value of Y. If $X \rightarrow Y$ then $X \rightsquigarrow Y$, but the converse is not necessarily true. Raju & Majumdar [29] have extended the classical fd to form a fuzzy functional dependency. We shall describe these concepts in detail in section 5 and use fuzzy fd's to formalize the notion of relevance which is important for successful analogical reasoning.

There are other research efforts in analogical reasoning and some of them are described in [50]. The description of prior research in this section has been restricted to those most relevant to the contents of this paper.

4 Fuzzy Sets And The Fuzzy Relational Data Model

In this section we provide a brief introduction to fuzzy set theory and the fuzzy relational data model considered in this paper. The description of fuzzy set theory in this section is concise and the reader is encouraged to refer to [23-28] for further details.

4.1 Fuzzy Set Theory : A Brief Overview

Let U be a classical set of objects called the universe of discourse and u represent a generic element of U.

<u>Definition 4.1</u>: A fuzzy set F in a universe of discourse U is characterized by a membership function

$$\mu_F : U \rightarrow [0, 1]$$

where $\mu_F(u)$ denotes the membership of u in the fuzzy set F. F can be written concisely as

$$F = \{ \mu(u_1)/u_1, \mu(u_2)/u_2, \dots , \mu(u_n) \}$$

where $u_i \in U$, $1 \leq i \leq n$. A fuzzy set is a generalization of a classical subset as a classical subset A of U can be written as a fuzzy subset with a membership function μ_A taking binary values, i.e.,

$$\begin{aligned} \mu_A(u) &= 1 \text{ if } u \in A \\ &= 0 \text{ else} \end{aligned}$$

The cardinality of a fuzzy set A is denoted by card(A) and is defined as:

$$card(A) = \Sigma\ \mu_A(u) \quad \text{for } u \in U$$

An alternative interpretation of $\mu_F(u)$ is to treat it as a measure of the possibility that a variable X has a value u, where X takes values in U. Zadeh [25] has suggested that a fuzzy proposition X is F, where F is a fuzzy subset of U and X is a variable which takes values from U, induces a possibility distribution π_X which is equal to F, i.e.,π_X = F, which is to be interpreted as

$$\text{Possibility}(X = u) = \mu_F(u) \text{ for all } u \in U$$

For two fuzzy subsets A and B of U, with membership functions μ_A and μ_B respectively, the membership functions for the usual set theoretic operations of union, intersection and complementation are

$$\mu_{A \cup B}(u) = \max(\mu_A(u), \mu_B(u))$$
$$\mu_{A \cap B}(u) = \min(\mu_A(u), \mu_B(u))$$
$$\mu_{\neg A}(u) = 1 - \mu_A(u)$$

Based on these definitions, most of the properties of classical set operations, e.g., DeMorgan's Laws, can be shown to hold for fuzzy sets. The only exception is that the law of the excluded middle of ordinary set theory is no longer true for fuzzy sets,i.e.,

$$A \cap \neg A \neg= 0 \text{ and}$$
$$A \cup \neg A \neg= U.$$

Let $U = U_1 \times U_2 \times \ldots U_n$, be the Cartesian product of n-universes and $A_1, A_2, \ldots, A_n$ be fuzzy sets in $U_1, U_2, \ldots U_n$. Then the Cartesian product $A_1 \times A_2 \times \ldots \times A_n$ is defined as the fuzzy subset of $U_1 \times U_2 \times \ldots U_n$ where

$$\mu_{A1 \times A2 \times \ldots \times An}(u_1, u_2, \ldots, u_n) = \min(\mu_{A1}(u_1), \ldots, \mu_{An}(u_n))$$

For two universes of discourse, U and V, a fuzzy relation R on U X V is defined by

$$R = \{ ((x, y), \mu_R(x, y)) | (x, y) \subseteq X \times Y \}$$

Consider an atomic fuzzy propositions, X is F, and its associated possibility distributions π_X, where F is a fuzzy subset of the universes U, and X is a variable taking values in U. Then the possibility distribution π_X^* of the modified proposition X is σF is given by

$$\pi_X^* = F^*$$

where F^* is a fuzzy subset of U with membership function

$$\mu_{F*} = f_\sigma(\mu_F(u)) \text{ for } u \in U$$

Some commonly used modifiers and their corresponding modification functions are

$$\sigma = \text{not}\ ,\ f_\sigma(x) = 1 - x$$
$$\sigma = \text{very}\ ,\ f_\sigma(x) = x^2$$
$$\sigma = \text{more or less},\ f_\sigma(x) = \sqrt{x}$$

4.2 The Fuzzy Relational Data Model

Fuzzy data models have been considered by many researchers in database theory and numerous variants have been proposed. Buckles and Petry [31] proposed a heterogeneous data model in which domain values were allowed to be fuzzy sets. Raju & Majumdar [29] proposed a fuzzy relational model in which domain values were allowed to be fuzzy sets and a membership value was attached to each tuple in a relation (the membership value showing the membership of the tuple in that relation). The fuzzy relational models suggested in [32-34] primarily use a possibilistic interpretation. Other researchers [30, 35, 36, 37] have studied fuzzy relational databases and proposed query languages to support fuzzy constructs. Discussion of the relative merits of these and other models is beyond the scope of this paper. Our main aim is to have a general data model which can adequately capture the imprecision in the data stored in the database.

We shall consider a heterogeneous relational data model in which data values can range from crisp values to a set of fuzzy sets. For ease of conceptual understanding, we can classify our fuzzy relational data model into the following types, in increasing order of generality .

- Type 0: Only crisp data values are allowed. This corresponds to the conventional relational data model.

- Type 1: The domains of attributes can be fuzzy sets. The value of a particular attribute for a given tuple is defined by the degree of membership of that tuple in the fuzzy set corresponding to the domain of the attribute under consideration.

- Type 2: The domains of attributes can be sets of fuzzy sets. Each tuple can have a fuzzy set as the value of a particular attribute.

- Type 3: The domains of attributes can be power sets of a set of fuzzy sets. Here multiple fuzzy sets can be assigned as the value of an attribute for a particular tuple.

The type 0 data model is the least general while the type 3 data model is the most general. It should be noted that data models of each type encompass all the "lower" (numbered) type data models. These various types of data models shall become clear in example 2.

Table 2 about here

<u>Example 2</u>: Consider a simple Employee relation with four fields (Name, Age, Salary, Experience) as shown in table 2. From table 2, we observe that field #1 (Name) has crisp data (type 0); field #2 (Age) has type 1 data which is the degree of membership in the fuzzy set *youth*; field #4 has type 2 data as each tuple value is described by a fuzzy set (high or *little* or *moderate*) and finally field #3 has type 3 data as each tuple value is described by a set of fuzzy sets (e.g., John's salary is both *high* and *volatile*, where high and volatile are fuzzy sets). The membership functions of fuzzy sets are domain dependent and we need to define the various fuzzy sets used in the above relation for the data to be meaningful. For illustration, we define the fuzzy sets for the domain of the field *Experience*. Note that

dom(experience) = {little, moderate, high}

Define the universe of discourse, U_{Exp} for the above fuzzy sets to be the set of positive integers (representing number of years of work experience) in the range of 0-25. Then possible membership functions for the fuzzy sets, *little*, *moderate* and *high* are as shown in figure 1. Formulations for the other fuzzy sets can be given in similar manner.

Figure 1 about here

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5 A Model of Analogical Reasoning For Answering Null Queries

In this section, we develop the theoretical foundations of our proposed model of fuzzy analogical reasoning to obtain approximate answers for null queries. We chose to develop our model of analogical reasoning in the context of a fuzzy relational data model because of the inherent capability of fuzzy set theory to naturally represent and effectively manipulate the imprecision in real world data. Analogical reasoning is approximate and frequently the match between the source and the target is partial. Classical relational data models are based on binary boolean logic and are unable to handle this imprecision in data efficiently. As fuzzy set theory and fuzzy relational data models are generalizations of their classical counterparts, our results also hold for conventional crisp data models . Our proposed model is also novel because in contrast to other conventional models of analogy studied in artificial intelligence, it uses fuzzy logic to deal with imprecise data. It is a natural, but significant generalization of the prior research in analogy (see section 3.2).

5.1 Basic Terminology

Attributes are symbols taken from a finite set U = { A_1, A_2, ..., A_n }. The domain associated with each attribute A_i is dom(A_i). Elements of dom(A) and dom(B) are represented by a and b respectively with possible suffixes. A, B, C, .. are used for representing attributes while X, Y, Z, .. are used for representing sets of attributes. For a set of attributes X, an X value is an assignment of values to the attributes {A_1, A_2, ..., A_n} in X from dom(A_1) X dom(A_2)X .. X dom(A_n). With each attribute A_i we associate a set U_i which is the universe of discourse for the domain values of A_i. No distinction is made between a single attribute A and the single element set {A}.

A relation scheme on $\{A_1, A_2, \ldots, A_n\}$ is denoted as R_n or simply as R, when there is no confusion. A relational database r is an instance of a relation scheme R and is a subset of the Cartesian product of $dom(A_1) \times dom(A_2) \times \ldots \times dom(A_n)$. Tuples are the individual rows of a relation. "t" represents an individual tuple and T represents a set of tuples. A tuple is defined by the values of the different attributes in R, e.g., ab is a tuple of a relation r on R(AB). A relation r containing m tuples can be represented as $r(t_1, t_2, \ldots, t_m)$ or simply as r_m. t[X] denotes the restriction of t to X, e.g., if t = abc, then t[AB] = ab. Similarly for $T=\{t_1, t_2, \ldots, t_m\}$, $\{t_1, t_2, \ldots, t_m\}[X]$, or in short T[X], denotes the restriction of the set of tuples T to X. The projection of a relation r of R(XYZ) over the set of attributes X is the restriction of the tuples of r over the attributes in X and is denoted by

$$\pi_X(r) = \{ t[X] \mid t \in r \}$$

For analogical reasoning, we shall often need to project the values of a set of attributes from one tuple t_1 onto another tuple t_2. This operation is defined by the assignment

$$t_2[\pi_X(r)] = t_1[\pi_X(r)]$$

and is represented in short as

$$t_1[X] \rightarrow t_2[X]$$

A functional dependency(fd) is an implication statement, $X \rightarrow Y$ where X and Y are sets of attributes. A relation r satisfies this fd if for all t_1 and t_2 in r, $t_1[X] = t_2[X]$ implies that $t_1[Y] = t_2[Y]$.

5.2 The Problem of Analogical Inference

We present below a formal definition of analogical inference in the context of determining an approximate answer for null queries.

Given :

$r(t_1, t_2, .., t_m)$ such that
$(t_1, t_2, ..., t_j)[A_1, A_2, ..., A_n]$ and $(t_{j+1}, ..., t_m)[A_1, ..., A_k]$
$(k < n, j < m)$
are defined (i.e., data values exist) and
$(t_{j+1}, ..., t_m)[A_{k+1}, ..., A_n]$
are not defined (i.e., data values are missing)

To find by analogy approximate values for :

$(t_{j+1}, ..., t_m)[A_{k+1}, ..., A_n]$

In the above definition we are implicitly assuming that the null query is due to the user asking for the values of the missing data values, $(t_{j+1}, ..., t_m)[A_{k+1}, ..., A_n]$. For $k=n-1$ and $j=m-1$, the problem reduces to :

Given :

$r(t_1, t_2, .., t_m)$ such that
$(t_1, t_2, ..., t_{m-1})[A_1, A_2, ..., A_n]$ and $(t_m)[A_1, ..., A_{n-1}]$
are defined (i.e., data values exist) and
$(t_m)[A_n]$
is not defined (i.e., the data value are missing)

To find by analogy approximate values for :

$(t_m)[A_n]$

Two types of inference strategies are possible:

5.2.1 Shallow Inference

Here we adopt the previously mentioned simple similarity-counting (see section 3) approach. For k=n-1 and j=m-1 :

[1] We first find t_i, $i \in (1, m-1)$ such that t_i is most similar to t_m.

[2] Next we project the value of the attribute A_n from t_i onto t_m, i.e., $t_i[A_n] \rightarrow t_m[A_n]$.

Such a shallow inference strategy suffers from the problems mentioned earlier in section 3. We need to incorporate the notion of *relevance* for arriving at a valid conclusion through analogical reasoning.

5.2.2 Deep Inference

Here we augment the process of evaluating similarity between the source and the target with the notion of relevance. For k=n-1 and j=m-1 :

[1] We first find t_i, $i \in (1, m-1)$, such that t_i is most similar to t_m with highest relevancy to A_n.

[2] Next we project the value of the attribute A_n from t_i onto t_m, i.e., $t_i[A_n] \rightarrow t_m[A_n]$.

The difference between these two inference strategies is the introduction of the notion of relevance in step [i] above. This is necessary because not all tuples are equally relevant for analogically inferring the value of $t_m[A_n]$. This can be also seen in example 1 where t_4 and each of the other tuples match in exactly one field, but t_1 and t_3 are more relevant (as compared to t_2) for determining the value of the attribute language for t_4. We explain below methods for measuring similarity and relevance.

5.3 Measuring similarity

For being able to successfully perform AR, we need to be able to measure the similarity between the values of different attributes for different tuples. As stated earlier, the match between the source and the target is usually partial and we need to define a fuzzy resemblance relation [23, 24, 28] EQUAL (EQ) over U_i, which can be used as a fuzzy measure to compare elements of domain dom(A_i).

5.3.1 A Fuzzy Relation for Determining Equality

We use the definition given by Raju & Majumdar [29]:

Definition :A fuzzy relation EQUAL(EQ) over an universe of discourse U is a fuzzy subset of U X U, where μ_{EQ} satisfies the following conditions. For all a, b $\in$ U,

$$\mu_{EQ}(a, a) = 1 \qquad \text{(reflexivity)}$$
$$\mu_{EQ}(a, b) = \mu_{EQ}(b, a) \qquad \text{(symmetry)}$$

Alternatively, $\mu_{EQ}(a, b)$ can be interpreted as the possibility of a being equal to b. The above definition does not require EQUAL to be a similarity relation (as it is not required to satisfy the transitivity property). This helps in capturing the notion of approximate equality of domain values. In fact, transitivity does not hold for most distance/proximity measures used for comparing domain values [28].

Different definitions of the fuzzy equality relation EQUAL are possible. As mentioned in section 4, the domains of the various attributes of a relation r in our fuzzy relational data model can range from crisp data (Type 0) to a power set of a set of fuzzy sets (Type 3). We present below in Example 3 one possible set of definitions for the various different combinations of attribute domains. It can be easily verified that the following membership functions for the fuzzy equality EQUAL are reflexive and symmetric.

<u>Example 3:</u> We have several different cases to consider depending upon the kind of data values (type 0/1/2/3). For $a, b \in U$:

i. If a and b are both crisp, then

$$\mu_{EQ}(a, b) = 1/\{1 + \beta|a - b|\}$$

where β is a domain dependent constant

ii. If a is crisp and b is the degree of membership in a fuzzy subset S then

$$\mu_{EQ}(a, b) = 1/\{1 + |\mu_S(a) - b|\}$$

iii. If a is crisp and b is a fuzzy subset B then

$$\mu_{EQ}(a, b) = \mu_B(a)$$

iv. If a is crisp and b is a set $S=\{s_1, s_2, \ldots, s_1\}$ of fuzzy sets s_i, then

$$\mu_{EQ}(a, b) = \max_{si \in S}(\mu_{si}(a))$$

v. If a is the degree of membership in a fuzzy subset A (with membership function μ_A) and b is a fuzzy subset B then

$$\mu_{EQ}(a, b) = \mu_B(\mu_A^{-1}(a))$$

where μ_A^{-1} is the inverse of μ_A.

vi. If a is the degree of membership in a fuzzy subset A (with membership function μ_A) and b is a set $S=\{s_1, s_2, \ldots, s_l\}$ of fuzzy sets s_i, then

$$\mu_{EQ}(a, b) = \max_{si \in S}(\mu_{si}(\mu_A^{-1}(a)))$$

vii. If a is a fuzzy set A and and b is a set $S=\{s_1, s_2, \ldots, s_l\}$ of fuzzy sets s_i, then

$$\mu_{EQ}(a, b) = \max_{si \in S} \{ \max [(c_i/A), (c_i/s_i)]\}$$

where $c_i = \text{card}(A \cap s_i)$.

viii. If a is a set $S=\{s_1, s_2, \ldots, s_l\}$ of fuzzy sets s_i and b is another set $Q=\{q_1, q_2, \ldots, q_k\}$ of fuzzy sets q_i, then

$$\mu_{EQ}(a, b) = \max_{si \in S,\ qj \in Q} \{\max[(c_{ij}/s_i), (c_{ij}/q_j)]\}$$

where $c_{ij} = \text{card}(s_i \cap q_j)$.

5.3.2 Similarity in Shallow Inference

Assume that we have a fuzzy resemblance relation EQUAL(EQ) (something like that given in example 3) with membership function μ_{EQ}^i in each domain $\text{dom}(A_i)$ to compare elements of that domain (A_i). We can use this definition of EQ to compute the similarity between tuples. We can define the similarity between tuples t_i and t_j as

$$\mu_{EQ}(t_i,t_j)=\max(\mu_{EQ}^1(t_i[A_1],t_j[A_1]),\mu_{EQ}^2(t_i[A_2], t_j[A_2]),\ldots, \mu_{EQ}^n(t_i[A_n], t_j[A_n]))$$

The above definition shall give a high value for $\mu_{EQ}(t_i, t_j)$

even if equality in one attribute domain is high and the rest are low. An analogous problem exists if we use min instead of max. An alternative definition that gives equal weight to the similarities between different attribute domains can be an arithmetic average as shown below:

$$\mu_{EQ}(t_i,t_j) = \frac{\{ \sum_{p=1}^{p=n} \mu_{EQ}^{p}(t_i[A_p],\ t_j[A_p]) \}}{n}$$

Different measures can be similarly defined to better suit particular situations.

5.3.3 Similarity in Deep Inference

Now we have to introduce the idea of relevance into our calculation of the similarity between objects. As explained earlier, certain attributes are more relevant than others for analogically inferring the value of a particular attribute . We shall show in the next sub-section, how we can measure this relevancy between attributes for analogical inference. Assume now that we have a measure

$$R_{(A1 \rightarrow An)},\ 0 < R_{(A1 \rightarrow An)} < R_{max}$$

that quantifies the relevancy of attribute A_1 for inferring the value of the attribute A_n. A high value of $R_{(A1 \rightarrow An)}$ shows that attribute A_1 is very relevant (or important) for determining the value of attribute A_n and vice versa, e.g., $R_{(nat \rightarrow lang)}$ shall be high, while $R_{(eyecolor \rightarrow lang)}$ shall be low (see example 1). While R_{max} may be arbitrarily large, in practice, it is usually more meanigful when less than a small integer around 10. The resons for this become clear in section 5.4.1. The degree of similarity between tuples t_i and t_n for inferring the value of attribute $t_n(A_n)$ is given by:

$$\mu_{EQ}(t_i, t_n, A_n) = \max \{ f(R_{(A1 \rightarrow An)}, \mu_{EQ}^{1}(t_i[A_1], t_n[A_1])),\ f(R_{(A2 \rightarrow An)}, \mu_{EQ}^{2}(t_i[A_2], t_n[A_2])), \ldots,\ f(R_{(An-1 \rightarrow An)}, \mu_{EQ}^{n-1}(t_i[A_{n-1}], t_n[A_{n-1}])) \}$$

The above function f can be defined in many different ways. One can identify some desirable features for f(a, b):

[1] For constant b, f(a, b) should be monotonically increasing in a, i.e., for the same degree of equality, f(a, b) should be greater for more relevant attributes.
[2] For low values of a, f(a, b) should yield low values, i.e., for less relevant attributes equality should be not emphasized.
[3] For moderate to high values of a, f(a, b) should be monotonically increasing in b, i.e., for relevant attributes, a higher degree of equality should be emphasized.

Given the above requirements, one possible definition of f is: $f(a, b) = b*(a/R_{max})$. Other definitions can be obtained to better suit certain domain specific conditions.

An alternative definition for the similarity which gives equal weight to the similarities between the various domains is

$$\mu_{EQ}(t_i, t_n, A_n) = \frac{\{\sum_{p=1}^{p=n-1} f(R_{(Ap \rightarrow An)}, \mu_{EQ}^{p}(t_i[A_p], t_n[A_p]))\}}{n-1}$$

5.4 Measuring Relevancy between attributes

The importance of measuring relevancy for performing sound analogical inferences has been elucidated earlier. Here we consider methods for obtaining these measures. Obviously the simplest technique is to ask the user/expert to input these measures of relevancy, e.g., we may ask the expert to input which attributes (e.g., experience) determine the salary of a person. Such a method may be desired and easy in some cases, but our aim is to develop algorithms and methods by which a database can learn these relevancies automatically from the data contained in it and perform analogical reasoning independently.

5.4.1 Relevance in the Fuzzy Relational Data Model

Functional dependencies of classical data-base research are similar to the determinations and dependencies used by AI researchers for analogical reasoning (see section 3.2). Thus it is natural that to perform analogical inference in the general context of a fuzzy relational model, we look at fuzzy functional dependencies to define fuzzy determinations.

Analogous to a fuzzy functional dependency[29], a fuzzy determination can be considered as a particularization of a fuzzy relation on R due to the fuzzy conditional proposition **If X is equal then Y is equal** where $X = \{A_{i1}, A_{i2}, \ldots, A_{ik}\}$ and $Y = \{A_{j1}, A_{j2}, \ldots, A_{jk}\}$ are subsets of a relation scheme $R(A_1, A_2, \ldots, A_n)$. A fuzzy proposition **X is equal** is a fuzzy subset of $dom(A_{i1})$ X $dom(A_{i2})$ X...X $dom(A_{ik})$ with a membership function as defined in section 5.3.1 for the equality of tuples. For the special case of k=p=1, we have the one attribute fuzzy determination

$$A_i \rightsquigarrow > A_j$$

which can be considered as a particularization of R by the fuzzy conditional proposition : **If A_i is equal then A_j is equal**. The fuzzy proposition **A_i is equal** is a fuzzy subset of $dom(A_i)$ with a membership function defined by the fuzzy equality function EQUAL for testing equality among domain values.

For the fuzzy conditional proposition **IF X is equal then Y is equal**, Zadeh [25, 28] has proposed the compositional rule of inference, based on Lukasiewicz's multi-valued logic [52], for fuzzy conditional inference. Mizumoto and Zimmerman [51] compared different translation rules for fuzzy conditional inference and pointed out that the compositional rule of inference often does not lead to intuitively correct conclusions. Thus they proposed several other new translation rules. We select the R_S translation rule [51] that is based on implication in standard sequence logic. We can define R_S as shown below:

Definition : The possibility distribution $\pi(X \rightarrow Y)$ associated with the conditional fuzzy proposition If X is F then Y is G is given by

$$\pi(X \rightarrow Y) = R_S$$

where F and G are fuzzy subsets of U and V respectively, and R_S is a fuzzy subset of U X V with membership function

$$\begin{aligned} \mu_{RS}(u, v) &= 1 \text{ if } \mu_F(u) \le \mu_G(v) \\ &= 0 \text{ else} \end{aligned}$$

Consequently, we have the following definition of a fuzzy determination (similar to that for fuzzy functional dependencies as given in [29]):

Definition : A fuzzy determination $X \twoheadrightarrow\!\!> Y$ holds for a relation r with a degree of relevance σ if for all tuples t_1, t_2 of r for which X and Y are defined (i.e., data values are present), the following relation holds

$$\mu_{EQ}(t_1[X],\ t_2[X]) \le (\mu_{EQ}(t_1[Y],\ t_2[Y]))^{\sigma}$$

Then

$$R_{(X \rightarrow Y)} = \sigma$$

If $\sigma = 1$, then X determines Y and the following relation holds:

$$\mu_{EQ}(t_1[X],\ t_2[X]) \le \mu_{EQ}(t_1[Y],\ t_2[Y])$$

Note that in the conventional relational model, the above case ($\sigma = 1$) corresponds to a functional dependency (μ_{EQ} in both sides of the above equation is 1). Thus functional dependencies can be integrated seamlessly into the framework of analogical reasoning being presented.

If $\sigma = 0.5$, then X *more or less* determines Y, and the following relation holds :

$$\mu_{EQ}(t_1[X],\ t_2[X]) \le (\mu_{EQ}(t_1[Y],\ t_2[Y]))^{\frac{1}{2}}$$

If $\sigma = 2$, then X *very much* determines Y, and the following relation holds :

$$\mu_{EQ}(t_1[X],\ t_2[X]) \le (\mu_{EQ}(t_1[Y],\ t_2[Y]))^{2}$$

Given the usual range of modifier functions (see section 4.1) discussed in the literature, the value of σ shall usually be at most a small integer. This was the basis for the suggestion (in section 5.3.3) for choosing R_{max} to be a small integer. The variation of the value of σ with a change in the strength of the fuzzy determination is as would be expected intuitively. It is important to note that a simple algorithm is all that is necessary for the database to learn these degree of relevancies autonomously.

5.5 Algorithms for Analogical Reasoning

We consider the problem model (see section 5.2) for the case $j = m-1$:

Given :

$r(t_1, t_2, .., t_m)$ such that
$(t_1, t_2, ..., t_j)[A_1, A_2, ..., A_n]$ and $(t_m)[A_1, ..., A_k]$
$(k < n)$
are defined (i.e., data values exist) and
$(t_m)[A_{k+1}, ..., A_n]$
are not defined (i.e., data values are missing)

To find by analogy approximate values for :

$(t_m)[A_{k+1}, ..., A_n]$

5.5.1 Shallow Inference

Assume that we have a fuzzy relation EQUAL for determining equality of attribute values in a domain (see section 5.3.1). The equality of tuples is given by the equations defined in section 5.3.2.

The solution algorithm is :
[1] Find t_i ($i \neg= m$) such that

$$\mu_{EQ}(t_i, t_m) = \max_{j=1,m-1}(\mu_{EQ}(t_j, t_m))$$

[2] Project the values of the missing attributes :

$$t_i[A_{k+1}, ..., A_n] \rightarrow t_m[A_{k+1}, ..., A_n]$$

Note that in this solution algorithm, we are simply finding the tuple t_i that is **most similar** to t_m and projecting all the missing values from t_i onto t_m. This shall not lead to valid conclusions in general due to the importance of considering *relevance* while analogically inferring conclusions. The next sub-section presents the solution algorithm for deep inference and this is the preferred mode of solution as it accounts for the varying degrees of relevancies of the different attributes for analogically inferring the value of the missing attribute .

5.5.2 Deep Inference

Again assume that we have a fuzzy relation EQUAL (see section 5.3.1) for comparing domain values and that we can measure the similarity of tuples for inferring the value of a particular attribute by the definitions in section 5.3.3. The solution algorithm is given below. Note that in contrast to the solution algorithm for shallow inference, we now infer the value of each missing attribute separately from the tuple which is most similar to t_m (the tuple with missing attribute values) for inferring the value of that particular attribute.

The solution algorithm is :

For $A_j \rightarrow A_{k+1}$ to A_n do

find t_i, $i \neg= m$, such that

$$\mu_{EQ}(t_i, t_m, A_j) = \max_{p=1,m-1}(\mu_{EQ}(t_p, t_m, A_j))$$

Project the value of the missing attribute from the most *relevant* tuple:

$$t_i[A_j] \rightarrow t_m[A_j]$$

loop

This algorithm presents us with a viable computational procedure to determine approximate values for missing data values. A disadvantage of the above algorithm is that for a large relation with many tuples, looking through all the tuples to find the best possible source can be expensive. A minor modification to the above algorithm can reduce this problem significantly. We can set a threshold value of the desired similarity necessary for making analogical inference and stop the search for a suitable source as soon as we reach a tuple with a similarity (with the target tuple, i.e., t_m) exceeding the threshold.

5.5.3 Truth of Analogically Inferred Conclusion

Analogical reasoning is a form of approximate reasoning and it is difficult to come up with a precise measure of the truth of the conclusion reached via analogy (see section 3.1). The problem is compounded by the fact that analogical reasoning is non-deductive and the premises are neither exhaustive nor universal.

However, we can attempt to come up with a conservative measure of the truth of the conclusion reached via analogy. Consider the solution algorithm given in section 5.5.2 and consider one iteration of the loop where we are trying to find the value of $t_m[A_n]$. Assume t_i' is the tuple such that

$$\mu_{EQ}(t_i', t_m, A_n) = \max_{p=1,m-1}(\mu_{EQ}(t_p, t_m, A_n))$$

and that similarity between tuples is given by:

$$\begin{aligned}\mu_{EQ}(t_i', t_m, A_n) = \max \{ & f(R_{(A1\rightarrow An)}, \mu_{EQ}^1(t_i'[A_1], t_m[A_1])), \\ & f(R_{(A2\rightarrow An)}, \mu_{EQ}^2(t_i'[A_2], t_m[A_2])), \ldots, \\ & f(R_{(An-1\rightarrow An)}, \mu_{EQ}^{n-1}(t_i'[A_{n-1}], t_m[A_{n-1}])) \}\end{aligned}$$

Let A_i, $i \neg= n$, $i < n$, be the attribute such that

$$\mu_{EQ}(t_i', t_m, A_n) = f(R_{(Ai \rightarrow An)}, \mu_{EQ}^i(t_i'[A_i], t_m[A_i]))$$

Then a conservative measure of the truth of the conclusion reached while projecting $t_i'[A_n] \rightarrow t_m[A_n]$ is given by :

$$\min(\mu_{EQ}^i(t_i'[A_i], t_m[A_i]), f(R_{(Ai \rightarrow An)}, \mu_{EQ}^i(t_i'[A_i], t_m[A_i])))$$

It is clear that the above is a very conservative measure of the truth of the conclusion. Better measures can be arrived at with a greater study of (and careful formulation of) the definitions of similarity and relevancy used for analogical reasoning.

5.5.4 An Example

In this section, we present a simple example to illustrate some of the ideas expressed in the last few sections. Imagine that a tailor keeps a database of all his customers in which he notes down all relevant data about them. Let us extract a small portion of his database, containing only the following four attributes : *name of customer, height* (in feet), *weight* (in lbs) and the *amount of cloth required for stiching a suit* (in metres). We have reproduced the data for two customers, John and Bob in table 3. Table 3 also contains some partially specified data for customer Gary and we would like to know how much cloth is required for stiching a suit for Gary. Assume that for computing equality, we use defintion (i) of example 3 in section 5.3.1. Assume that the values for β for the different attributes are:

height: $\beta = 1$
weight: $\beta = 10$
cloth: $\beta = 1$

Table 3 about here

For shallow inference, we can compute the similarity between John and Gary as (using the definition given in section 5.3.2) :

$$\mu_{EQ}(\text{John, Gary}) = \max (.77, .33) = 0.77$$

and between Bob and Gary as:

$$\mu_{EQ}(\text{Bob, Gary}) = \max (0.67, .84) = 0.84$$

Thus we shall project the value of cloth from Bob and infer that Gary shall also require 5.5 metres of cloth for getting his suit stiched.

From the data specified for customers John and Bob, we can compute the relevance of attributes height and weight for determining the attribute cloth using the technique specified in section 5.5.1.

$$R_{height \rightarrow cloth} = 2.29$$
$$R_{weight \rightarrow cloth} = 0.38$$

Using the definitions for equality in deep inference given in section 5.3.3, we can perform deep inference according to the algorithm specified in section 5.5.2 (assuming R_{max} = 2.29):

$$\mu_{EQ}(\text{John, Gary, Cloth}) = \max(0.77*1,\ 0.38*0.166) = 0.77$$

$$\mu_{EQ}(\text{Bob, Gary, Cloth}) = \max(0.67*1,\ 0.84*0.166) = 0.67$$

Thus now we shall project the value for cloth from John and infer that Gary shall require **6 meters** of cloth. This answer is more intuitively correct as height is more relevant for determining the cloth required for stiching a suit (generally). A conservative measure of the truth of the analogically inferred conclusion is (according to the definition of section 5.5.3):

$$\text{truth} = \min(0.77,\ 0.77*1) = 0.77$$

As a final note, this example may seem contrived so as to give different results for the two different inference strategies, but we hope that the essential point of the importance of relevance (i.e., deep inference) while performing such approximate inference (via analogy) is driven home.

6 Conclusion

In this paper we have presented a model of analogical reasoning in the context of a fuzzy relational data model. The model of analogy developed in this paper is a generalization of models of analogy developed in the domain of artificial intelligence. It can be successfully used to obtain approximate values to missing data values in an incomplete database. Such a facility shall increase the co-operative nature of databases and enhance man-machine interaction.

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TABLE 1

Name	Nationality	Language	Eye-color
John	USA	English	Blue
Ashok	India	Bengali	Black
Barry	USA	English	Brown
Mike	USA	?	Black

TABLE 2 : Relation Employee

Name	Age	Salary	Experience
John	0.2/youth	high, volatile	high
Barry	0.8/youth	low, steady	little
Mike	0.6/youth	moderate, steady	moderate

TABLE 3

NAME	HEIGHT	WEIGHT	CLOTH
John	6	160	6
Bob	5.8	175	5.5
Gary	6.3	180	?

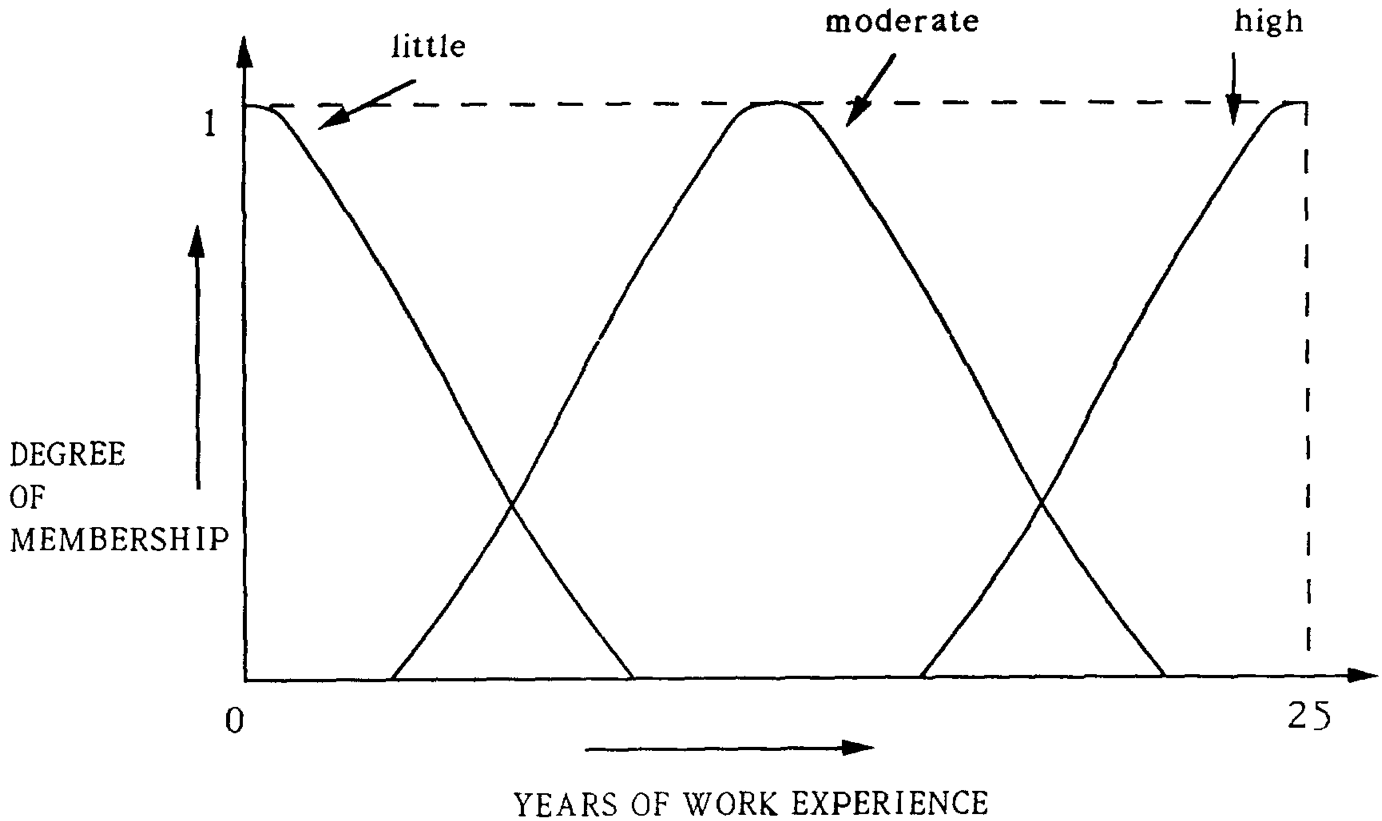

Figure 1: Membership functions of fuzzy sets

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