"INFERENCES WITH AN UNKNOWN NOISE LEVEL IN A BERNOULLI PROCESS"

by

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ABSTRACT

Inferences about a proportion p are often based on data generated from dichotomous processes, which are generally modeled as processes that are Bernoulli in p. In reality, the assumption that a data generating process is Bernoulli in p, the proportion of interest, is often violated due to the presence of various sources of noise. The level of the noise is usually unknown and, furthermore, dependent on the unknown proportion in which one is interested. Two specific models which take into account the existence of noise in a Bernoulli process are developed. In a likelihood analysis, an identification problem arises. The incorporation of prior information via a Bayesian analysis avoids this identification problem, and helps us to formalize a priori dependence between the proportion and the noise level. Empirical data is used to illustrate the models and provide some flavor of the implications of our uncertainty about the noise for inferences about a proportion.

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1. INTRODUCTION

Data are often generated from a dichotomous process to obtain inferences about a proportion. The data generating process is then almost always modeled as a process that is Bernoulli in p, the proportion of interest. However, the assumption that the data generating process is Bernoulli in p is often violated due to the presence of various sources of noise. The level of noise in the process is usually unknown and may also be dependent on the unknown proportion in which one is interested.

An illustrative example involves incorrect responses to dichotomous survey questions regarding sensitive issues such as drug usage, criminal activities, or any other socially undesirable or personal characteristic (see, for example, Sudman and Bradburn, 1974). Also, the level of response bias usually varies with different issues and populations (Sudman and Bradburn, 1974). Attempts to suppress the response bias in surveys by using techniques such as a randomized response procedure, in which noise is intentionally introduced and is therefore known and carefully controlled, are not always successful (see, for example, Locander et al., 1976). Such response biases in a survey are not limited to threatening questions. An empirical study by Wind and Lerner (1979) provides examples where purchase behaviour as measured from the survey method can be highly inaccurate. Another setting in which the sample information is often contaminated is the marketing research where advertising effects are measured based on recognition tests, in which there is a strong tendency on part of the subjects to "recognize ads" irrespective of prior exposure to them (see, for example, Singh and Churchill, 1986). Still more examples of imperfect sample information may include observations of good and defective items in a quality control setting with an imperfect inspection device, or positive and negative results in a medical test with the possibility of false results. Noise may also be present due to errors in recording, coding, and mishandling the data.

In this paper, two specific models are developed that account not only for an unknown level of noise in a Bernoulli process but also for a priori dependence between the level of noise and the underlying proportion of interest. The noise parameter is treated as a probability of misclassifying an observation generated from a simple Bernoulli process, and this unknown misclassification probability is considered to be a priori dependent on the proportion of interest.

Dependence between two or more proportions often arises in many different settings. Thus this research has a wider applicability. In principle, the approach, which is used in both the models that are developed in this paper, is also valid for modeling different forms of dependence that may exist between two or more proportions in a setting other than the one in context of a noisy Bernoulli process.

Inferences from a noisy Bernoulli process under the assumption that the noise parameters are known are studied in Winkler and Franklin (1979) and in Winkler (1985). The results of a likelihood and Bayesian analyses indicate that the noise can have considerable impact on inferences made about the Bernoulli parameter. In a Bayesian analysis, the reduction in effective sample size leads to more weight being given to the prior distribution. In Winkler (1985), loss of information is studied also in context of a normal model with dependent observations. It is shown that dependence can have considerable impact on effective information. A Bernoulli process with an unknown noise level is discussed in Winkler and Gaba (1990). In a likelihood analysis, an identification problem is encountered. The same identification problem is avoided under a Bayesian analysis with a joint prior distribution for the proportion and the noise parameter. The noise parameter is considered a priori independent of the proportion. It is shown that the posterior density for p can be substantially different from that obtained under a noise-free approach.

Dependence among proportions has been discussed in several studies based on dirichlet prior distributions (see, for example, Novick and Grizzle, 1965). Good (1967) proposed a generalized version of a dirichlet distribution, but for the specific purpose of developing a Bayesian significance test for a multinomial distribution. Another form of generalized dirichlet density has been considered, for example, in Lochner (1975), based on an intuitive derivation of the prior density in life-testing situations.

In Section 2, a noisy Bernoulli process is considered with a bivariate beta density (which is the bivariate form of a dirichlet density) for the proportion of interest and a noise parameter. In Section 3, Model 1 from Section 2 is generalized by developing a richer class of prior distributions for the proportion and the noise parameter. In Section 4, Models 1 and 2 are illustrated in the context of actual data from a NBER survey. A brief summary and discussion follows in Section 5.

2. MODEL 1

Consider a large population in which each member belongs to either Group A or Group B, but not to both. Let p denote the proportion of the population in Group A. A random sample of size n is drawn from the population, and each member of the sample is classified as being in Group A or in Group B. Let r be the number of members in the sample who are classified as being in Group A.

If the classification is perfect, the process observed is Bernoulli in p. The maximum likelihood estimator for p is then the sample proportion, and a conjugate Bayesian analysis involves beta prior and posterior distributions.

Now suppose the classification is imperfect. Let λ_a be the probability that a member of the sample who actually belongs to Group A is misclassified as being in Group B. The other misclassification probability is denoted by λ_b, which is the probability that a member of the sample who actually belongs to Group B is incorrectly classified as being in Group A. The noise parameters here are λ_a and λ_b.

Let $x_i = 1$ if the ith member of the sample is classified as being in Group A and $x_i = 0$ if classified as being in Group B. Then the process we actually observe is not Bernoulli in p, but is Bernoulli in q, where

$$q = P(x_i=1 \mid p,\lambda_a,\lambda_b) = p(1-\lambda_a) + (1-p)\lambda_b \tag{1}$$

and

$$1-q = P(x_i=0 \mid p,\lambda_a,\lambda_b) = p\lambda_a + (1-p)(1-\lambda_b). \tag{2}$$

The likelihood of the sample is thus of the form

$$k(r \mid n,p,\lambda_a,\lambda_b) = q^r(1-q)^{n-r}$$

$$= \left[p(1-\lambda_a) + (1-p)\lambda_b\right]^r \left[p\lambda_a + (1-p)(1-\lambda_b)\right]^{n-r}. \qquad (3)$$

This approach is similar to the one used in Winkler and Gaba (1990), where p and the noise parameters are considered a priori independent. The purpose here is to include some form of dependence between p and the misclassification rates.

In the analysis here, it is assumed that $\lambda_b = 0$. Such representation might be reasonable for some applications. Consider a survey in which each respondent answers "Yes" or "No" to a question, "Do you belong to Group A?," where being in Group A is a sensitive issue or is socially undesirable. For example, Group A might consist of women who have had abortions, individuals who have ever used some drug, or people who do not use a prestigious brand. Typically, in such cases, one would expect λ_b (the probability that an individual who is actually in Group B would say that he/she belongs to Group A) to be negligible.

The likelihood of the sample, with $\lambda_b = 0$, reduces to

$$k(r \mid n,p,\lambda_a) = \left[p(1-\lambda_a)\right]^r \left[p\lambda_a + (1-p)\right]^{n-r}. \qquad (4)$$

By expanding the terms in (4),

$$k(r \mid n,p,\lambda_a) = \sum_{t=0}^{n-r} \binom{n-r}{t} p^{n-t} (1-p)^t \lambda_a^{n-r-t} (1-\lambda_a)^r. \qquad (5)$$

In the likelihood function, represented in (5), t is interpreted as the number of correct Group B classifications in the sample (or n-r-t as the number of Group A members in the sample who are incorrectly classified as being in Group B). Since we do not know t the likelihood is expressed as a mixture of the n-r+1 likelihoods that could arise with each possible number of misclassifications in the sample.

The likelihood function for n=10 and r=1 is shown in Figure 1. The shape of the likelihood function illustrates an identification problem that arises in this noisy process. It can be seen that the maximum likelihood is not unique, but is a contour of points (p,λ_a) such that

$[p(1-\lambda_a)] = r/n$. In general, the likelihood function is partitioned into equivalence classes. In a likelihood analysis, an identification problem arises because each equivalence class contains more than one (p,λ_a) pair. These (p,λ_a) pairs are observationally equivalent structures, the respective likelihoods of which are identical; only a locally (over an equivalence class) non-uniform prior distribution for p and λ_a in a Bayesian analysis, is able to discriminate between the (p,λ_a) pairs that have identical likelihoods.

Consider, again, the example in which λ_a is defined as the probability that a respondent who is actually in Group A would "lie" in a survey when asked the question, "Do you belong to Group A?," and p represents the proportion of the population in Group A. Usually, in such cases, one would expect to have some relevant prior information regarding p and λ_a. For instance, $(p,\lambda_a) = (0.1,0.2)$ might be considered much more reasonable than $(p,\lambda_a) = (0.8,0.9)$, although both would have identical likelihoods for any given sample. Besides a priori beliefs about the realistic levels of p and the "lying" rate, the expectation of the "lying" rate conditional on p might be higher for a lower value of p, and the uncertainty surrounding the level of "lying" might also be greater for lower values of p; it seems reasonable, for instance, that individuals feel less threatened or less socially outcast when they are part of a larger group. The appropriate approach, then, is to model such dependence in the prior distribution for p and λ_a.

A beta distribution can usually provide a reasonable approximation for a wide variety of prior information regarding a parameter which has a value between zero and one. Also, in the case of noise-free sampling, the family of beta distributions is conjugate with respect to a Bernoulli process. In Model 1, the a priori beliefs are modeled in the form of a bivariate beta distribution for p and λ_a. The joint density for p and λ_a is then given by

$$f(p,\lambda_a) = f_\beta(p,\lambda_a \mid \alpha_1,\alpha_2,\alpha_3)$$

$$= \frac{\Gamma(\alpha_1+\alpha_2+\alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)} p^{\alpha_1-1} \lambda_a^{\alpha_2-1} (1-p-\lambda_a)^{\alpha_3-1}, \qquad (6)$$

where $p \geq 0$, $\lambda_a \geq 0$, $p+\lambda_a \leq 1$ and $\alpha_2,\alpha_1,\alpha_3 > 0$. From (6), it follows (see, for example, Mardia, 1970) that the marginal distribution for p is beta with parameters α_1 and $\alpha_2+\alpha_3$, and the marginal distribution for λ_a is beta with parameters α_2 and $\alpha_1+\alpha_3$. The conditional distribution of $\lambda_a/(1-p)$, given p, is beta with parameters α_2 and α_3. Hence the conditional distribution of λ_a, given p, is

$$f(\lambda_a \mid p) = \frac{\Gamma(\alpha_2+\alpha_3)}{\Gamma(\alpha_2)\Gamma(\alpha_3)} \left[\frac{\lambda_a}{1-p}\right]^{\alpha_2-1} \left[1-\frac{\lambda_a}{1-p}\right]^{\alpha_3-1} \frac{1}{1-p}, \tag{7}$$

where $0 \leq \lambda_a \leq 1-p$. The conditional expectation of λ_a, given p, is

$$E(\lambda_a \mid p) = \frac{\alpha_2(1-p)}{\alpha_2+\alpha_3} \tag{8}$$

and the conditional variance of λ_a, given p, is

$$\sigma^2(\lambda_a \mid p) = \frac{\alpha_2\alpha_3(1-p)^2}{(\alpha_2+\alpha_3)^2(\alpha_2+\alpha_3+1)}. \tag{9}$$

Note that the prior expectation and standard deviation of λ_a, given p, are linear nonincreasing functions of p.

A bivariate beta density for p and λ_a with $\alpha_1=3$, $\alpha_2=2$ and $\alpha_3=5$ is shown in Figure 2. It can be seen that the prior density is unimodal and correlates lower values of p with higher values of λ_a. The conditional expectation and standard deviation of λ_a, given p, as functions of p, and the conditional density for λ_a, given p, for p=0.1, p=0.3 and p=0.5, are shown in Figure 3. The conditional expectation and standard deviation of λ_a, given p, are linear decreasing functions of p. For a higher given value of p, the conditional distribution for λ_a is tighter and concentrated over lower values of λ_a.

From Bayes' theorem, the prior distribution in (6) and the likelihood in (5) yield the following posterior distribution:

$$f(p,\lambda_a \mid r,n) = \sum_{t=0}^{n-r} \sum_{i=0}^{t} \sum_{j=0}^{r} \omega_{tij}\, f_\beta(p,\lambda_a \mid \alpha_1+n-t+i, \alpha_2+n-r-t+j, \alpha_3), \tag{10}$$

where

$$\omega_{tij} = \frac{w_{tij}}{\phi}, \tag{11}$$

$$w_{tij} = \binom{n-r}{t}\binom{t}{i}\binom{r}{j}(-1)^{i+j}\,\frac{\Gamma(\alpha_1+n-t+i)\ \Gamma(\alpha_2+n-r-t+j)\ \Gamma(\alpha_3)}{\Gamma(\alpha_1+\alpha_2+\alpha_3+2n-2t-r+i+j)}, \tag{12}$$

and

$$\phi = \sum_{t=0}^{n-r}\sum_{i=0}^{t}\sum_{j=0}^{r} w_{tij}. \tag{13}$$

The joint posterior distribution for p and λ_a is a mixture of bivariate beta distributions of the same form as the prior bivariate beta distribution. Here, the parameters j and i are artifical, and the interpretation of t is the same as before, that is, t is the number of correct classifications in Group B. The posterior probability that there were n-r-t misclassifications in the sample (or t out of n-r correct Group B classifications) is given by

$$\omega_t = \sum_{i=0}^{t}\sum_{j=0}^{r} \omega_{tij}. \tag{14}$$

The joint posterior density is, then, expressed as a mixture of n-r+1 possible posterior densities that could arise under the perfect knowledge of the exact number of misclassifications, and each such misclassification density is itself expressed as a weighted mixture of (t+1)(r+1) densities.

The marginal posterior density for p, obtained by taking (10) and integrating out λ_a, is a mixture of beta distributions:

$$f(p \mid r, n) = \sum_{t=0}^{n-r}\sum_{i=0}^{t}\sum_{j=0}^{r} \omega_{tij}\, f_\beta(p \mid \alpha_1+n-t+i,\ \alpha_2+\alpha_3+n-r-t+j). \tag{15}$$

The posterior expectation of p is

$$E(p \mid r, n) = \sum_{t=0}^{n-r} \sum_{i=0}^{t} \sum_{j=0}^{r} \omega_{tij} \frac{(\alpha_1+n-t+i)}{(\alpha_1+\alpha_2+\alpha_3+2n-2t-r+i+j)}, \tag{16}$$

and the posterior variance of p can be obtained from $E(p \mid r, n)$ and $E(p^2 \mid r, n)$where

$$E(p^2 \mid r, n) = \sum_{t=0}^{n-r} \sum_{i=0}^{t} \sum_{j=0}^{r} \omega_{tij} \frac{(\alpha_1+n-t+i)(\alpha_1+n-t+i+1)}{(\alpha_1+\alpha_2+\alpha_3+2n-2t-r+i+j)(\alpha_1+\alpha_2+\alpha_3+2n-2t-r+i+j+1)}. \tag{17}$$

For λ_a, the marginal posterior density is given by

$$f(\lambda_a \mid r, n) = \sum_{t=0}^{n-r} \sum_{i=0}^{t} \sum_{j=0}^{r} \omega_{tij}\, f_\beta(\lambda_a \mid \alpha_2+n-r-t+j,\ \alpha_1+\alpha_3+n-t+i). \tag{18}$$

The first two posterior moments for λ_a are thus given by

$$E(\lambda_a \mid r, n) = \sum_{t=0}^{n-r} \sum_{i=0}^{t} \sum_{j=0}^{r} \omega_{tij} \frac{(\alpha_2+n-r-t+j)}{(\alpha_1+\alpha_2+\alpha_3+2n-2t-r+i+j)}, \tag{19}$$

and

$$E(\lambda_a^2 \mid r, n) = \sum_{t=0}^{n-r} \sum_{i=0}^{t} \sum_{j=0}^{r} \omega_{tij} \frac{(\alpha_2+n-r-t+j)(\alpha_2+n-r-t+j+1)}{(\alpha_1+\alpha_2+\alpha_3+2n-2t-r+i+j)(\alpha_1+\alpha_2+\alpha_3+2n-2t-r+i+j+1)}.$$

(20)

The posterior density for p and λ_a with $(\alpha_1,\alpha_2,\alpha_3) = (3,2,5)$, r=1 and n=10, is shown in Figure 4. Note that in this posterior distribution, one half of the likelihood function (see Figure 1) has had no influence because the entire mass of the prior distribution (see Figure 2) is concentrated in the region where $p+\lambda_a \leq 1$. Also, the prior distribution is non-uniform over the equivalence classes of the likelihood function (see the contour plots in Figures 1 and 2) and, thus, the posterior distribution is unimodal despite the multimodality of the likelihood function.

To apply Model 1 in practice, one must assess values for α_1, α_2 and α_3. The values for α_1 and $\alpha_2+\alpha_3$ can be selected by assessing a beta prior distribution for p, which is identical in principle to the assessment of a beta distribution in a noise-free situation. For example, various fractiles of the prior distribution can be assessed and then, using tables, a beta distribution can be fitted to these fractiles (see, for example, Winkler 1967). However, in the noisy situation, the assessor must remain cognizant of the possibility that any past data pertaining to p may also be contaminated with noise. The value for α_2 and α_3 can be chosen, for example, by assessing a conditional mean for λ_a, given p, for some assumed value of p and then using the expression given in (8) to calculate values for α_2 and α_3. This process could be followed for several values of p to check the consistency of the selected values for α_2 and α_3.

3. MODEL 2

With the choice of appropriate parameters, a bivariate beta distribution may well provide a reasonable representation of our a priori beliefs about p and λ_a in some cases, while such a prior distribution may not be adequate in other cases. To further generalize the approach in Model 1, a richer class of prior distributions for p and λ_a is developed in this Section.

In Model 2, the noise parameter is modeled as follows:

$$\lambda_a = c\, g(p), \tag{21}$$

where $0 \le \lambda_a \le 1$; g(p) is a nonincreasing (linear or nonlinear) function of p such that $0 \le g(p) \le 1$, with $0 \le p \le 1$ and $g(0) = 1$; c is a random variable and can be considered as an unknown intercept term such that $0 \le c \le 1$. Note that g is not a probability density function.

From (21), the conditional expectation of λ_a, given p, can be expressed as

$$E(\lambda_a \mid p) = E(c)\, g(p). \tag{22}$$

Since g(p) is a nonincreasing function of p, the conditional expectation of λ_a, given p, is also a nonincreasing function of p. The values for both p and c are between 0 and 1. I will assume that p and c are a priori independent and that the prior uncertainty regarding each is represented by a beta distribution:

$$f(p) = f_\beta(p \mid \alpha_1, \beta_1) = \frac{\Gamma(\alpha_1 + \beta_1)}{\Gamma(\alpha_1)\,\Gamma(\beta_1)} p^{\alpha_1 - 1} \ (1 - p)^{\beta_1 - 1}, \tag{23}$$

and

$$f(c) = f_\beta(c \mid \alpha_2, \beta_2) = \frac{\Gamma(\alpha_2 + \beta_2)}{\Gamma(\alpha_2)\,\Gamma(\beta_2)} c^{\alpha_2 - 1} \ (1 - c)^{\beta_2 - 1}, \tag{24}$$

where $0 \le p \le 1$ and $0 \le c \le 1$, with α_1, β_1, α_2, and $\beta_2 > 0$.

From (21) and (24), the conditional distribution for $\lambda_a/g(p)$, given p and $0 < g(p) \le 1$, is beta with parameters α_2 and β_2. Hence the conditional distribution of λ_a, given p, is

$$f(\lambda_a \mid p) = \frac{\Gamma(\alpha_2 + \beta_2)}{\Gamma(\alpha_2)\,\Gamma(\beta_2)} \left[\frac{\lambda_a}{g(p)}\right]^{\alpha_2 - 1} \left[1 - \frac{\lambda_a}{g(p)}\right]^{\beta_2 - 1} \frac{1}{g(p)}, \tag{25}$$

where $0 \le \lambda_a \le g(p)$ and $0 < g(p) \le 1$. If $g(p) = 0$, then λ_a, given p, has a degenerate dirac distribution at zero, that is $P(\lambda_a = 0 \mid p) = 1$. The conditional expectation of λ_a, given p, is

$$E(\lambda_a \mid p) = \frac{\alpha_2}{\alpha_2 + \beta_2} g(p), \tag{26}$$

and the conditional variance of λ_a, given p, is

$$\sigma^2(\lambda_a \mid p) = \frac{\alpha_2\,\beta_2}{(\alpha_2 + \beta_2)^2\,(\alpha_2 + \beta_2 + 1)} [g(p)]^2 . \tag{27}$$

With g(p) as a nonincreasing function of p, the prior expectation and variance of λ_a, given p, are both nonincreasing functions of p.

The joint density for p and λ_a is then given by

$$f(p,\lambda_a) = f(\lambda_a \mid p)f(p)$$

$$= \frac{\Gamma(\alpha_2+\beta_2)\,\Gamma(\alpha_1+\beta_1)}{\Gamma(\alpha_2)\,\Gamma(\beta_2)\,\Gamma(\alpha_1)\,\Gamma(\beta_1)} \left[\frac{\lambda_a}{g(p)}\right]^{\alpha_2 - 1} \left[1 - \frac{\lambda_a}{g(p)}\right]^{\beta_2 - 1} \frac{1}{g(p)}\, p^{\alpha_1 - 1}(1-p)^{\beta_1 - 1}\,, \tag{28}$$

where $0 \le p \le 1$, $0 \le \lambda_a \le g(p)$, and $0 < g(p) \le 1$. If g(p) = 0, then the joint distribution of p and λ_a is the product of the marginal distribution of p and the dirac distribution of λ_a, given p, at $\lambda_a = 0$. Note that g(p) defines the upper limit of λ_a for a given value of p.

With g(p) = 0, Model 2 is equivalent to the noise-free approach which assumes that λ_a = 0. When g(p) = 1, the joint density for p and λ_a in (28) reduces to a product of two beta densities, and this model becomes equivalent to the approach that assumes a priori independence between p and λ_a, with $f(p,\lambda_a) = f_\beta(\lambda_a \mid \alpha_2,\beta_2)f_\beta(p \mid \alpha_1,\beta_1)$, where $0 \le \lambda_a \le 1$ and $0 \le p \le 1$. Furthermore, if g(p) = 1-p and $\beta_1 = \alpha_2 + \beta_2$, the same joint density in (28) reduces to a bivariate beta density for p and λ_a with parameters α_1, α_2, and β_2. In this case Model 2 is equivalent to Model 1 from Section 3.

To assess the g(p) function in practice, it is useful to parameterize it. The functional form used here for g(p) is

$$g(p) = \begin{cases} \dfrac{(k-p)^m}{k^m} & \text{, for } 0 \le p < k\,, \\ 0 & \text{, for } k \le p \le 1\,, \end{cases} \tag{29}$$

where $0 \le p \le 1$, $k > 0$, and m = 0, 1, 2 .... The values for m are restricted to integer values in order to analytically derive the posterior distributions of interest; there is no other compelling reason to restrict the values for m to integer values. With this particular form, the function g(p) can take on a wide variety of shapes with respect to p for different values of k and m. Figure 5 shows some of these shapes. The maximum value of g(p) is 1 at $p = 0$, and the minimum is 0 at $p \geq k$ for $0 < k \leq 1$, or $(k-p)^m/k^m$ at $p = 1$ for $k > 1$.

For example, if we define λ_a as the probability that a respondent who is actually in Group A would "lie" in a survey when asked the question, "Do you belong to Group A?," then g(p) can be interpreted as the maximum rate of "lying" that might occur for that given value of p. The parameters that define g(p), besides p, are k and m. In somewhat loose terms, k (when $k \leq 1$) may be interpreted as the proportion such that, when p approaches k, the members of Group A will no longer "lie," and m may be interpreted as a sensitivity factor that contributes to the rate at which the maximum possible rate of "lying" decreases as the size of Group A increases. In practice, of course, one might be uncertain about one or more of the parameters of the g(p) function.

In this model, it is assumed that k and m are known (or that satisfactory point estimates of k and m can be obtained from the assessed conditional distributions of λ_a, given p, for different values of p). Substituting for g(p) in (28), the joint prior density for p and λ_a can now be expressed by

$$f(p, \lambda_a) = \begin{cases} \dfrac{\Gamma(\alpha_2+\beta_2)\,\Gamma(\alpha_1+\beta_1)}{\Gamma(\alpha_2)\,\Gamma(\beta_2)\,\Gamma(\alpha_1)\,\Gamma(\beta_1)} \dfrac{k^m}{(k-p)^m} \left[\dfrac{\lambda_a k^m}{(k-p)^m}\right]^{\alpha_2-1} \left[1-\dfrac{\lambda_a k^m}{(k-p)^m}\right]^{\beta_2-1} p^{\alpha_1-1}(1-p)^{\beta_1-1}, \\ \qquad\qquad \text{for } 0 \leq p < k,\ 0 \leq \lambda_a \leq (k-p)^m/k^m, \\ \dfrac{\Gamma(\alpha_1+\beta_1)}{\Gamma(\alpha_1)\,\Gamma(\beta_1)} p^{\alpha_1-1}(1-p)^{\beta_1-1}, \\ \qquad\qquad \text{for } k \leq p \leq 1,\ \lambda_a = 0. \end{cases} \tag{30}$$

When $m = 0$ and $k \geq 1$ (that is, $g(p) = 1$), the prior density in (30) reduces to a product of beta densities for p and λ_a. If $m = k = 1$ (that is, $g(p) = 1-p$) and $\beta_1 = \alpha_2+\beta_2$, the same density in (30) reduces to a bivariate beta density for p and λ_a with parameters α_1, α_2, and β_2.

With the likelihood in (5) and the prior density in (30), using Bayes theorem, we get the following posterior distribution:

$$f(p,\lambda_a|r,n)=\begin{cases}\displaystyle\sum_{t=0}^{n-r}\sum_{j=0}^{r}\sum_{t=0}^{m(n-r-t+j)} w_{tji}\, f(\lambda_a,p\,|\,r,n,t,j,i)\,, & \text{for } 0\le p<k,\ 0\le\lambda_a\le(k-p)^m/k^m\,,\\[2ex] w_0\ f_\beta(p\,|\,\alpha_1+r,\ \beta_1+n-r)\,, & \text{for } k\le p\le 1,\ \lambda_a=0,\end{cases} \tag{31}$$

where

$$f(\lambda_a,p\,|\,r,n,t,j,i)=f(\lambda_a\,|\,p,r,n,t,j)\ f(p\,|\,r,n,t,j,i)\,, \tag{32}$$

$$f(\lambda_a\,|\,p,r,n,t,j)=\frac{\Gamma(\alpha_2+\beta_2+n-r-t+j)}{\Gamma(\alpha_2+n-r-t+j)\,\Gamma(\beta_2)}\,\frac{k^m}{(k-p)^m}\left[\frac{\lambda_a k^m}{(k-p)^m}\right]^{\alpha_2-1+n-r-t+j}\left[1-\frac{\lambda_a k^m}{(k-p)^m}\right]^{\beta_2-1}, \tag{33}$$

$$f(p\,|\,r,n,t,j,i)=f_\beta(p\,|\,\alpha_1+n-t+i,\ \beta_1+t)=\frac{\Gamma(\alpha_1+\beta_1+n+i)}{\Gamma(\alpha_1+n-t+i)\,\Gamma(\beta_1+t)}\ p^{\alpha_1-1+n-t+i}\,(1-p)^{\beta_1-1+t}, \tag{34}$$

$$w_{tji}=\frac{b_{tji}}{\phi}\,, \tag{35}$$

$$w_0=\frac{1}{\phi}\,, \tag{36}$$

$$b_{tji}=\frac{\Gamma(\alpha_2+n-r-t+j)\ \Gamma(\beta_2)\,\Gamma(\alpha_1+n-t+i)\,\Gamma(\beta_1+t)}{\Gamma(\alpha_2+\beta_2+n-r-t+j)\,\Gamma(\alpha_1+\beta_1+n+i)}\binom{n-r}{t}\binom{r}{j}\binom{m(n-r-t+j)}{i}(-1)^{i+j}k^{-i}, \tag{37}$$

and ϕ is the normalizing constant given by

$$\phi=w_0\delta_0+\sum_{t=0}^{n-r}\sum_{j=0}^{r}\sum_{i=0}^{m(n-r-t+j)} w_{tji}\delta_{tji}\,, \tag{38}$$

with

$$\delta_0 = \int_k^1 f_\beta\left(p \,\middle|\, \alpha_1+r,\ \beta_1+n-r\right)dp, \tag{39}$$

and

$$\delta_{tji} = \int_0^k f_\beta\left(p \,\middle|\, \alpha_1+n-t+i,\ \beta_1+t\right)dp. \tag{40}$$

The parameters j and i are artificial. The interpretation of the parameter t is the same as in Model 1, that is, t is the correct number of classifications in Group B. In other words, there are n-r-t misclassifications in the sample. The value of t, as in Model 1, is unknown to us. Given $0 \le p < k$, the posterior probability that there were n-r-t misclassifications (or t members of the sample were correctly classified in Group B) is given by w_t, where

$$w_t = \sum_{j=0}^{r} \sum_{i=0}^{m(n-r-t+j)} w_{tji}\ . \tag{41}$$

For $0 \le p < k$, the joint posterior density for p and λ_a is then expressed as a mixture of n-r+1 possible posterior densities that could arise under perfect knowledge of the exact number of misclassifications, and each such misclassification density is itself expressed as a weighted mixture of (r+1)[m(n-r-t+j)+1] densities (this second mixture is necessary for obtaining an analytical solution for the posterior density, and has no real interpretation). Given $k \le p \le 1$, $\lambda_a = 0$ and the joint posterior density for p and λ_a is a weighted posterior density for p, a density that would arise for p under the noise-free approach. If we further define

$$\omega_t = \sum_{j=0}^{r} \sum_{i=0}^{m(n-r-t+j)} w_{tji}\, \delta_{tji} \tag{42}$$

and

$$\omega_0 = w_0\,\delta_0, \tag{43}$$

with δ_{tji} and δ_o as defined in (39) and (40) respectively, then ω_t can be interpreted as the unconditional posterior probability that there were n-r-t (t varies from 0 to n-r) misclassifications in the sample and ω_o as the unconditional posterior probability that $\lambda_a = 0$, where $\omega_t + \omega_o = 1$.

For reasons of space, the expressions for the posterior marginal densities and the posterior moments for p and λ_a are not presented here. However, the posterior distribution for p is a mixture of beta distributions, and the posterior conditional distribution for λ_a, given p, is a mixture of conditional distributions of the same form as the prior conditional distribution for λ_a, given p; the posterior marginal distribution for λ_a has to be evaluated numerically.

To apply Model 2 in practice, the user must select the parameters of the prior distribution for p (α_1 and β_1) and of the prior conditional distribution for λ_a, given p (α_2, β_2, k, and m). The assessment of α_1 and β_1 would be the same as suggested for the parameters of the prior distribution for p in Model 1. The values for α_2 and β_2, k, and m can be selected through the assessment of the means and fractiles of the distributions of λ_a, given p, for three or four assumed values of p.

An example is given in the next section to further illustrate Models 1 and 2.

4. EXAMPLES

Self-reported data is a dominant source for obtaining measures of juvenile delinquency in sociological studies. I use such data as an example for illustrating Models 1 and 2. Hindelang, Hirschi, and Weis (1981) conducted an extensive study during 1978-1979 in the city of Seattle using interviews and questionnaires in which subjects were asked if they had committed certain delinquent acts. Also, for the subjects who were officially listed in the police and court records as having committed certain offences, official records were compared with the responses to parallel items in the self-report setting. The data show that the subjects as a whole in a self-report setting are, in general, likely to underreport an offence that falls in the same category as their official offence. The nonreporting rate was higher for serious offences than for nonserious offences. For example, 67% of the black males who were listed in the official records for having committed offences in the categories of robbery and burglary did not report the same offences in their self-report settings. For nonserious offences as a whole (shoplifting, petty larcenies, incorrigibility, and some other miscellaneous offences) the nonreporting rate was 18% among black males.

The NBER survey of Inner City Black Youth (see Freeman and Holzer, 1986) is another study that provides self-reported data on criminal activities. This survey was conducted in 1979-80, with a sample population consisting of inner-city black males (ages 16 to 24) from Boston, Chicago and Philadelphia. In the studies based on the same survey, possibilities of severe underreporting have been recognized, but no serious attempt to assess the overall criminal involvement has been made (see Viscusi 1986).

I use the NBER data for subjects between 16 and 18 years of age as my sample data (with n=1156). The characteristics of this sample then closely parallel those of the sample population in the Hindelang et al. study. To illustrate Models 1 and 2, I chose two questions from the NBER survey: "Have you done any muggings or purse snatchings (without a gun) over the past 12 months?," and "Have you played numbers or other illegal gambling over the past 12 months?." In these examples, p is then the proportion who actually indulged in the illegal act and λ_a is the probability that an individual having done so would not report it in a survey.

Considering information given in the Hindelang et al. study on nonreporting rates and the proportions who responded "Yes" to the questions of the type "Have you ever...(committed a certain offence)?," and in consultation with Professor Kenneth C. Land from the department of Sociology at Duke University, I assessed prior distributions for p and prior conditional distributions for λ_a, given p, for the two examples. In each of the examples, the assessment of a conditional distribution for λ_a, given p, involved, as an intermediate step, assessment of means and fractiles of the distributions for λ_a, given p, with three different values of p.

GAMBLING

For gambling, I use a bivariate beta prior distribution for p and λ_a, with $(\alpha_1,\alpha_2,\alpha_3) = (4, 2.09, 4.91)$. The marginal prior for p is shown in Figure 6; it is a beta distribution with parameters 4 and 7. Thus, a priori, the expected proportion who have gambled over the past 12 months is 0.36. The corresponding standard deviation is 0.14. The 0.05 and 0.95 fractiles of the distribution are 0.15 and 0.61, respectively.

The shape of the conditional expectation for λ_a, given p, with respect to p is also shown in Figure 6. In this case, the marginal prior for λ_a is a beta distribution with parameters 2.09 and 8.91 (i.e, with mean 0.19 and standard deviation 0.113). Thus, a priori, the expected probability that a respondent who has gambled would respond "No" to the survey question about gambling is 0.19.

Using Model 1, with r=106 and n=1156 (r/n=0.092) from the NBER survey, the expected proportion who gambled (the posterior mean for p) is 0.146 and the expected misclassification probability (the posterior mean for λ_a) is 0.33; the corresponding standard deviations are 0.034 and 0.137, respectively.

Of particular interest is the comparison of these inferences from Model 1 to those obtained from a model based on the assumption of perfect classification (i.e., assuming a noise-free process). Assuming $\lambda_a = 0$ and with the same prior distribution for p as in Model 1, the posterior distribution for p is a beta distribution with parameters 110 and 1167. The posterior mean and standard deviation for p in Model 1 are then 1.5 times and 4 times larger, respectively, than those in the noise-free model. The impact of the noise is to shift r/n toward 0, away from the "true" p. For a given r/n, then, the model with noise (Model 1) accounts for this by placing more weight on higher values of p, thereby shifting the posterior mean for p away from 0. Also, the a posteriori uncertainty about p in Model 1 is very much greater than that indicated by the noise-free model; in Model 1, consideration of all the possible misclassifications in the sample that could possibly occur causes much greater uncertainty about p.

The larger posterior standard deviation for p in Model 1 implies that the noise causes a reduction in the information content of the sample regarding p. Another way to investigate this is to determine an "effective sample size" for the noisy model. Such a value is generated by fitting beta distributions to the marginal posterior distributions for p and λ_a. The betas are fitted by equating their means and variances to those of the marginal distributions. The parameters of the beta fits are given as α_i^* and β_i^*, for $i = p$ or λ_a, and the effective sample size for p and λ_a is then defined by

$$n^*(i) = (\alpha_i^* + \beta_i^*) - (\alpha_1 + \alpha_2 + \alpha_3).$$

Under the noise-free approach, n* for p is simply n and n* for λ_a is meaningless since λ_a is non-existent. The marginal posterior distributions may not always be closely approximated by beta distributions, but n* still seems useful as a rough measure of effective sample size. In the gambling example, the effective sample sizes are 98.53 for p and -0.16 for λ_a. When compared with n=1156, the effective sample size for p indicates a substantial loss of information due to the noise. The negative effective sample size for λ_a actually suggests loss of information, though very little, relative to the prior distribution of λ_a.

Another interesting question regarding the inferences from Model 1 is how these compare with inferences from a model based on the assumption of a priori independence between p and λ_a. In the equivalent independent noise approach, the prior distribution for p and λ_a is treated as the product of the marginal prior distributions for p and λ_a in Model 1; the resulting posterior distribution for p and λ_a (and each of the marginal posterior distribution) is a mixture of n-r+1 beta distributions. The inferences from Model 1 and from the equivalent independent noise model are not very different, which could be accounted for by the fact that the a priori dependence between p and λ_a is not very "strong" in the ranges of values for p and λ_a considered in this example. In Figure 7, the marginal posterior distribution for p in Model 1, along with the prior distribution for p and the posterior distributions for p in the corresponding noise-free and the independent noise models, is shown. The posterior distribution for p under the noise-free approach is extremely tight and concentrated near p=r/n=0.092. In comparison to this, the posterior distributions for p in the noisy models (Model 1 and the independent noise model) reflect a rightward shift and a much greater dispersion of the probable values of p. It is clearly evident that the prior distribution plays a much larger role in the models with noise than in the noise-free model. The a priori dependence between p and λ_a does not play much of a role in this example, as seen from the almost overlapping posterior marginal densities for p from Model 1 and from the independent noise model.

Since the prior distribution plays a crucial role in Model 1, the robustness of inferences to variations in the prior distribution is of interest. I vary the prior mean for λ_a from 0.19 to 0.1 and 0.3, the prior mean for p from 0.363 to 0.273 and 0.455, and $\alpha_1+\alpha_2+\alpha_3$ from 11 to 5.5, 22, and 33. The results from these variations are summarized in Table 1.

As expected, greater weight on higher values of λ_a or p in the prior distribution causes an increase in the posterior mean for p, which varies from 0.146 in the base case to between 0.12 and 0.18 with the changes in the prior means of p and λ_a; the posterior mean for p is insensitive to the changes in the tightness of the prior distribution and remains almost unchanged. A higher prior mean for λ_a or p also leads to a greater posterior standard deviation for p (which varies between 0.026 and 0.04, from 0.034 in the base case). For example, with higher values of λ_a, greater number of possibilities of misclassifications have to be considered and, hence, the uncertainty about p increases. Also, a tighter prior distribution leads to a tighter posterior distribution for p since a tighter prior distribution reflects more information. The posterior mean and standard deviation for λ_a are very sensitive to the prior distribution. Not only a higher prior mean for λ_a, but also a higher prior mean for p tends to increase the posterior mean and standard deviation for λ_a; for a given r/n, more weight on higher values of p in the prior distribution is reconciled by putting greater weight on higher values of λ_a.

The posterior mean for p is consistently greater than that in the equivalent noise-free model, and the posterior standard deviation for p is consistently very much greater (between 3 and 5 times greater) than the corresponding standard deviation in the noise-free model. A tighter prior distribution for p and λ_a, for example, leads to somewhat lesser difference between inferences from Model 1 and from the noise-free model. This happens because greater weight is placed on the prior information relative to the sample information, thus dampening the effect of noise in the sample information on the posterior inferences. On the other hand, a higher prior mean for λ_a (that is, more weight, a priori on higher levels of noise), for example, accentuates the difference between inferences from Model 1 and the noise-free model. The loss of information due to the noise is further indicated by the effective sample sizes for p in Model 1 which are very much less than the sample size of 1156. The magnitudes of the effective sample sizes for p are consistent with the ratios of the posterior standard deviations for p in Model 1 and the noise-free model. For example, with a higher prior mean for λ_a, the effective sample size for p is smaller; more weight in the prior distribution on higher levels of noise causes a greater reduction in the information content of the sample. The effective sample sizes for λ_a in Model 1 indicate that the data contain very little information regarding λ_a. The negative effective sample sizes actually suggest loss of information relative to the prior distribution for λ_a.

In this example, the differences between the posterior means (and standard deviations) for p in Model 1 and in the equivalent independent noise model are negligible. The posterior mean for λ_a is generally higher in Model 1 than in the independent noise model. This is because, for a given prior distribution for p and λ_a, higher values of λ_a get more weight for lower values of p in Model 1; the prior distributions for p that have been considered give more weight to lower values of p. The posterior standard deviations for λ_a are in general smaller in Model 1 than in the independent noise model. This is because, in Model 1, information about p in the prior distribution also provides some indirect information about λ_a through the a priori dependence between p and λ_a, thus adding to the direct information that is available about λ_a.

A similar analysis is presented for the mugging example, but using Model 2 instead of Model 1.

MUGGING

The prior density for p in the mugging example is shown in Figure 8; it is a beta distribution with $(\alpha_1, \beta_1) = (2, 14)$. This implies that, a priori, the expected proportion who have done muggings in the past 12 months is 0.125, with the corresponding standard deviation of 0.08. The 0.05 and 0.95 fractiles of the distribution are 0.02 and 0.28, respectively.

In Figure 8, the shape of the prior conditional expectation for λ_a, given p, with respect to p, is also shown for the mugging example. This shape corresponds to the assessed conditional distribution of λ_a, given p, with $(\alpha_2, \beta_2) = (18, 2)$, $k = 1.3$, and $m = 3$. The conditional expectation of λ_a, given p, is, for example, 0.67 with $p = 0.125$, 0.34 with $p = 0.36$, and 0.21 with $p = 0.5$; the corresponding conditional standard deviations are 0.048, 0.025, and 0.015, respectively.

With these prior distributions for p and for λ_a, given p, the a priori expected rate of false "No" responses is 0.67 with the corresponding standard deviation of 0.14.

Using Model 2, with $r = 29$ and $n = 1156$ ($r/n=0.025$), the posterior mean for p is 0.092, with the corresponding standard deviation of 0.014; the posterior mean for λ_a is 0.723, with the corresponding standard deviation of 0.04. Thus, I estimate that 9.2% of the population, almost four times larger than the 2.5% implied by the survey data, have done muggings over the past 12 months, and that 72.3% of the people who have done muggings would lie when asked the mugging question in a survey.

In the equivalent noise-free model, assuming $\lambda_a=0$ and with the same prior distribution for p as in Model 2, the posterior distribution for p is a beta distribution with parameters 31 and 1141. The posterior mean for p is almost 3.5 times larger and the posterior standard deviation for p is 3 times larger than the corresponding posterior quantities in the noise-free model. Ignoring the possibility of false responses to the question leads to severe underestimation of the rate of muggings. Also, the posterior uncertainty regarding the rate of muggings in the noise-free model is much less than what it should be given the presence of noise. The effective sample sizes for p and λ_a are 393.14 and 115.43, respectively. Compared to the sample size of 1156, these effective sample sizes indicate, as in the gambling example, that there is a substantial loss of information regarding p and that not much information is obtained from the data regarding λ_a.

These inferences from Model 2 are also compared to inferences from a model in which p and λ_a are considered to be a priori independent. In the equivalent independent noise model, the joint density for p and λ_a is expressed as a product of two beta densities. The beta distribution for p is the same as the prior marginal distribution for p in Model 2 (which is a beta distribution with parameters α_1 and β_1). The beta distribution used for λ_a is a beta that is fitted to the prior marginal distribution for λ_a in Model 2 (the exact form of which has to be evaluated numerically). The beta fit for λ_a is obtained by equating its first two moments to those of the exact marginal prior distribution for λ_a in Model 2. The joint posterior distribution for p and λ_a (and the resulting marginal distributions) is then a mixture of n-r+1 beta distributions .

The posterior mean for p and λ_a in Model 2 do not differ much from those in the independent noise model. However, the posterior standard deviations for p and λ_a in Model 2 are about 1/3 of those in the independent noise model. This happens because ignoring the dependence between p and λ_a in the prior leads to loss of information regarding both p and λ_a. In Model 2, a priori, any information about λ_a also provides, through the dependence structure between p and λ_a, indirect information about p which is in addition to the direct information available about p and vice versa. This is not the case in the independent noise model.

The analysis under Model 2 for the mugging example is summarized in Table 3. In Figure 9, the marginal posterior distribution for p, along with the prior distribution for p and the posterior distributions for p under the corresponding noise-free and the independent noise models, is shown. The posterior distribution for p in Model 2 is very much different from the posterior distributions for p in the noise-free model and in the independent noise model. The posterior distribution under the noise-free model, as in the gambling example, is extremely tight and concentrated near $p = r/n = 0.025$. As expected, the posterior distributions with noise (from the independent noise model and from Model 2) reflect a substantial rightward shift and much greater dispersion relative to the noise-free distribution. Furthermore, the posterior distribution is much tighter under Model 2 than under the independent noise model. The marginal prior distribution for p has the most influence on the posterior distribution for p under the independent noise model, and almost no influence on the same distribution under the noise-free model.

As with Model 1, the prior distribution plays a crucial role in Model 2. The robustness of posterior inferences to variations in the prior distribution is thus of interest. The prior parameters in the base case are $(\alpha_1, \beta_1) = (2, 14)$, $(\alpha_2, \beta_2) = (18, 2)$, $k = 1.3$, and $m = 3$. For the prior distribution for λ_a, given p, I varied $\alpha_2+\beta_2$ to 10 and 40, $\alpha_2/(\alpha_2+\beta_2)$ to 0.7 and

0.95, k to 1.1 and 1.5, and m to 2 and 4. For the prior distribution for p, I varied $\alpha_1+\beta_1$ to 48, and $\alpha_1/(\alpha_1+\beta_1)$ to 0.063 and 0.188. Also, I obtained results for a diffuse prior on p, that is for $(\alpha_1, \beta_1) = (1, 1)$; a diffuse prior for λ_a, given p, seems highly unrealistic. For these variations in the prior parameters, the prior means and standard deviations for p and λ_a, along with some of the resulting posterior inferences about p and λ_a, are given in Tables 4a and 4b.

The posterior mean and standard deviation for p are quite robust to the variations in the prior parameters mentioned above. The greatest change in the posterior mean occurs when (α_2, β_2) is changed from (18, 2) in the base case to (14, 6); the posterior mean changes from 0.092 in the base case to 0.067. Otherwise, the posterior mean for p remains between 0.083 and 0.1. The posterior standard deviation for p varies between 0.012 (when $m = 4$) and 0.018 (when $m = 2$), from 0.014 (when $m = 3$) in the base case.

Similarly, the posterior mean and standard deviation for λ_a are also quite robust. For all but one of the cases, the posterior mean for λ_a varies between 0.69 and 0.75 (from 0.72 in the base case), and the posterior standard deviation varies between 0.03 and 0.05 (from 0.04 in the base case); when $(\alpha_2, \beta_2) = (14, 6)$, the posterior mean is 0.61 and the posterior standard deviation is 0.07.

With the same changes in the prior parameters, the posterior means and standard deviations for p under Model 2 are consistently very much greater than the corresponding posterior means and standard deviations under the noise-free model. This is not surprising since, with the lower effective sample sizes, greater weight is placed on the prior distribution and less on the sample information. The effective sample sizes for p are consistently much smaller, and the same for λ_a are consistently very much smaller, than the sample size of 1156.

Except in the case with a diffuse prior on p, the posterior means for p and λ_a remain almost identical to those in the independent noise model and the posterior standard deviations for p and λ_a are 2 to 4 times greater in the independent noise model than in Model 2. With a diffuse prior on p, the posterior standard deviation for p under the independent noise model is 12 times greater than the corresponding posterior standard deviation under Model 2; for λ_a, it is about 8 times greater. This is further reflected by the extremely different effective sample sizes, for p and λ_a, in the independent noise model and in Model 2. The effective sample sizes for p and λ_a are 0.66 and -0.70 in the independent noise model, compared to 400 and 126, respectively, in Model 2.

The analysis in this section, although limited, provides some flavour of the implications from Models 1 and 2. In general, for the examples presented in this section, ignoring the existence of lying leads to underestimation, and sometimes severe underestimation, of the rates of criminal behaviour. In the survey, 2.5% of the sample respondents admitted to having done muggings over the past 12 months and 9.2% said that they have gambled over the past 12 months. Using Models 1 and 2, I estimate that 9.2% of the population have done muggings and 14.6% of the population have gambled over the past 12 months. These estimates are about 3.5 times and 1.5 times larger, respectively, than those implied by the noise-free approach. Given the sample, Models 1 and 2 also modify the prior information regarding the lying rates. I estimate that 72.3% of the people who have done muggings and 33% of the people who have gambled, would lie when asked about it in a survey.

Also, I estimate that the uncertainty associated with the percent who have done mugging (as measured by the respective posterior standard deviation) is about 3 times higher than that implied with the noise-free model; in the case of gambling, it is about 4 times higher. The posterior uncertainty regarding the crime rates in the noise-free model is much less than what it should be given the presence of lying. The reduction in the information content of the sample due to the presence of lying is also reflected by the very low effective sample sizes (in comparison to the actual sample size) in Models 1 and 2.

Furthermore, I estimate that by ignoring the a priori dependence between the crime rates and the lying rates (between p and λ_a) we increase our uncertainty (as measured by the posterior standard deviations) about these rates three fold in the case of mugging; though, this effect is almost negligible in the gambling case. In general, ignoring the a priori dependence between p and λ_a leads to an unnecessary loss of information regarding both p and λ_a; this loss of information is greater when the dependence between p and λ_a is "stronger." For mugging and gambling, very little information about the lying rates is obtained from the data in the independent noise model as indicated by the very low effective sample sizes for λ_a; in comparison, much more information is obtained from the sample data in Models 1 and 2.

Overall, the results are quite robust to minor variations in the prior parameters. However, inferences about λ_a in Model 1 (for the gambling example) are sensitive to any variations in the prior distribution.

5. SUMMARY AND DISCUSSION

Data generated from a Bernoulli process may often be contaminated with incorrect observations due to the presence of various sources of noise. The level of noise in the process is usually unknown and, in an a priori sense, may be correlated to the proportion of interest. Attempts to suppress the noise by augmenting the design of the data generating processes are not always successful in reducing the level of the noise. In this paper, two specific models have been developed which take into account the existence of noise in a Bernoulli process. These models also help us to formalize prior information regarding the proportion and the noise level, including a priori information about the dependence between the two.

Actual data is used in Section 4 to illustrate the two models. It is seen that ignoring the noise altogether can cause very misleading inferences about the proportion of interest. The presence of noise leads to loss of information in the sample about the proportion. For example, this is reflected by a reduction in the effective sample size. In general, very little information about the noise parameter is obtained from the data.

Furthermore, ignoring any a priori dependence between the proportion of interest and the noise level can cause unnecessary loss of information about the proportion and about the level of noise. This loss of information can be substantial at times. The greater the dependence between the proportion and the noise level, the more crucial it becomes to appropriately take that dependence into account. A more important issue is, however, one of avoiding a misrepresentation of prior information. If the proportion and the noise level are dependent, assuming that they are independent may simply lead to erroneous inferences regardless of issues involving the amount of information.

The specific models developed in this paper are applicable to much of the research that is based on surveys with dichotomous questions (for example, recognition tests in advertising research, surveys on consumer behavior, and surveys on sensitive issues in sociological research). Other applications may include, for example, quality control situations with imperfect inspection devices and blood testing situations with false positives and false negatives. Though, in some of these cases with a mechanical or a technical classification device, the device may be so well calibrated that there may be little uncertainty regarding the rate of false results. In such situations, using a model that assumes the noise parameters to be known may be more appropriate.

The models presented in this paper can easily be extended to include more than one noise parameter. The underlying approach remains identical in principle. For further research in this direction, it may be of interest to introduce economic considerations in the analysis presented in this paper. The existence of noise in general leads to a reduction in the information content of the sample which, in turn, causes a reduction in the expected value of sample information. This could have implications when an investigator is deciding on the size of a sample from a noisy Bernoulli process.

Other situations where models akin to the models in this paper can be developed are prediction of purchase or voting behavior based on stated intentions, or accounting for non response bias in surveys. Moreover, to the extent that situations with two or more proportions that may be dependent are common, I feel that the methodology presented in this paper can be used, in principle, for a wide variety of problems other than those involving a noisy Bernoulli process.

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TABLE 1

"GAMBLING" EXAMPLE

MODEL 1

p = proportion who have "played street numbers or other illegal gambling over the past 12 months." λ_a = probability that an individual who has actually "played street numbers or other illegal gambling..." will answer "No" to a question, "Have you played street numbers or other illegal gambling...?."
PRIOR: $(\alpha_1, \alpha_2, \alpha_3) = (4, 2.09, 4.91)$, $E(p) = 0.363$, $\sigma(p) = 0.14$, $E(\lambda_a) = 0.19$, $\sigma(\lambda_a) = 0.113$.
SAMPLE: $n = 1156$, $r = 106$, $r/n = 0.092$.
POSTERIOR (with subscript i=1 for Model 1): $E_1(p \mid r,n) = 0.146$, $\sigma_1(p \mid r,n) = 0.034$, $E_1(\lambda_a \mid r,n) = 0.330$, $\sigma_1(\lambda_a \mid r,n) = 0.137$.
With i=1 for Model 1, and i=NF for the Noise-Free Model: $\frac{E_1(p \mid r,n)}{E_{NF}(p \mid r,n)} = 1.550$, $\frac{\sigma_1(p \mid r,n)}{\sigma_{NF}(p \mid r,n)} = 3.930$.
With i=1 for Model 1, and i=IN for the Independent Noise Model: $\frac{E_1(p \mid r,n)}{E_{IN}(p \mid r,n)} = 1.101$, $\frac{\sigma_1(p \mid r,n)}{\sigma_{IN}(p \mid r,n)} = 0.979$, $\frac{E_1(\lambda_a \mid r,n)}{E_{IN}(\lambda_a \mid r,n)} = 1.276$, $\frac{E_1(\lambda_a \mid r,n)}{\sigma_{IN}(\lambda_a \mid r,n)} = 0.971$, $n_1^*(p) = 98.53$, $n_1^*(\lambda_a) = -0.16$, $n_{IN}^*(p) = 85.67$, $n_{IN}^*(\lambda_a) = -2.35$.

TABLE 2 :SENSITIVITY ANALYSIS FOR THE "GAMBLING" EXAMPLE

		PRIOR MEANS AND STANDARD DEVIATIONS (MODEL 1)				POSTERIOR MEANS AND STANDARD DEVIATIONS (MODEL 1)			
	$(\alpha_1, \alpha_2, \alpha_3)$	$E_1(p)$	$\sigma_1(p)$	$E_1(\lambda_a)$	$\sigma_1(\lambda_a)$	$E_1(p \mid r,n)$	$\sigma_1(p \mid r,n)$	$E_1(\lambda_a \mid r,n)$	$\sigma_1(\lambda_a \mid r,n)$
1.	(4, 2.09, 4.91)	0.363	0.139	0.190	0.113	0.146	0.034	0.330	0.137
2.	(4, 1.1, 5,9)	0.363	0.139	0.100	0.087	0.121	0.026	0.197	0.133
3.	(4, 3.3, 3.7)	0.363	0.139	0.300	0.132	0.179	0.039	0.455	0.118
4.	(3, 2.09, 5.91)	0.273	0.129	0.190	0.113	0.130	0.027	0.264	0.126
5.	(5, 2.09, 3.91)	0.455	0.144	0.190	0.113	0.170	0.040	0.415	0.136
6.	(2, 1.045, 2.455)	0.363	0.189	0.190	0.154	0.144	0.043	0.309	0.177
7.	(8, 4.18, 9.82)	0.363	0.100	0.190	0.082	0.149	0.026	0.342	0.101
8.	(12, 6.27, 14.73)	0.363	0.082	0.190	0.067	0.151	0.022	0.344	0.084

COMPARISON OF MODEL 1 TO THE NOISE-FREE MODEL

$E_{NF}(p \mid r,n) \equiv$ posterior mean for p in the noise-free model

$\sigma_{NF}(p \mid r,n) \equiv$ posterior standard variation for p in the noise-free model

$n_1{*}(i) \equiv$ effective sample size for i, where i = p or λ_a, in Model 1

COMPARISON OF MODEL 1 TO THE INDEPENDENT NOISE MODEL

$E_{IN}(. \mid r,n) \equiv$ posterior mean in the independent noise model

$\sigma_{IN}(. \mid r,n) \equiv$ posterior standard deviation in the independent noise model

$n_{IN}{*}(i) \equiv$ effective sample size for i, where i=p or λ_a in the independent noise model

	$\frac{E_1(p \mid r,n)}{E_{NF}(p \mid r,n)}$	$\frac{\sigma_1(p \mid r,n)}{\sigma_{NF}(p \mid r,n)}$	$n_1^*(p)$	$n_1^*(\lambda_a)$	$\frac{E_1(p \mid r,n)}{E_{IN}(p \mid r,n)}$	$\frac{\sigma_1(p \mid r,n)}{\sigma_{IN}(p \mid r,n)}$	$\frac{E_1(\lambda_a \mid r,n)}{E_{IN}(\lambda_a \mid r,n)}$	$\frac{\sigma_1(\lambda_a \mid r,n)}{\sigma_{IN}(\lambda_a \mid r,n)}$	$\frac{n_1^*(p)}{n_{IN}^*(p)}$	$\frac{n_1^*(\lambda_a)}{n_{IN}^*(\lambda_a)}$
1.	1.550	3.930	98.525	-0.159	1.101	0.979	1.276	0.971	1.150	0.068
2.	1.282	3.007	148.752	-3.097	1.080	1.180	1.410	1.165	0.754	1.099
3.	1.898	4.586	83.539	5.734	1.078	0.752	1.158	0.791	2.145	-4.322
4.	1.396	3.175	143.158	0.216	1.044	0.957	1.153	0.982	1.145	-0.165
5.	1.784	4.701	74.523	1.036	1.181	0.926	1.393	0.880	1.410	-0.314
6.	1.552	5.095	59.066	0.350	1.050	0.774	1.200	0.943	1.891	-0.335
7.	1.538	2.998	167.033	-1.136	1.121	1.089	1.320	0.987	0.918	0.235
8.	1.524	2.569	224.887	-2.072	1.123	1.108	1.333	0.993	0.883	0.285

TABLE 3

"MUGGING" EXAMPLE

MODEL 2

p = proportion who have done "muggings or purse snatchings (without a gun) over the past 12 months."

λ_a = probability that an individual who has actually "done muggings or purse snatchings ..." will answer "No" to a question, "Have you done any muggings or purse snatchings?."

PRIOR:

$(\alpha_1, \beta_1) = (2, 14)$, $(\alpha_2, \beta_2) = (18, 2)$, $k = 1.3$, $m = 3$,

$E(p) = 0.125$, $\sigma(p) = 0.08$,

$E(\lambda_a) = 0.67$, $\sigma(\lambda_a) = 0.14$, $E(\lambda_a | p{=}0.125) = 0.67$, $\sigma(\lambda | p{=}0.125) = 0.048$.

SAMPLE:

$n = 1156$, $r = 29$, $r/n = 0.025$.

POSTERIOR (with subscript i=2 for Model 2):

$E_2(p \mid r, n) = 0.092$, $\sigma_2(p \mid r, n) = 0.014$,

$E_2(\lambda_a \mid , n) = 0.723$, $\sigma_2(\lambda_a \mid r, n) = 0.040$.

With i=2 for Model 2, and i=NF for the Noise-Free Model:

$$\frac{E_2(p \mid r, n)}{E_{NF}(p \mid r, n)} = 3.487, \quad \frac{\sigma_2(p \mid r, n)}{\sigma_{NF}(p \mid r, n)} = 3.049 .$$

With i=2 for Model 2, and i=IN for the Independent Noise Model:

$$\frac{E_2(p \mid r, n)}{E_{IN}(p|r,n)} = 0.923, \quad \frac{\sigma_2(p \mid r, n)}{\sigma_{IN}(p|r,n)} = 0.308, \quad \frac{E_2(\lambda_a \mid r, n)}{E_{IN}(\lambda_a|r,n)} = 1.032, \quad \frac{\sigma_2(\lambda_a \mid r, n)}{\sigma_{IN}(\lambda_a|r,n)} = 0.334,$$

$n_2^*(p) = 393.14$, $n_2^*(\lambda_a) = 115.43$, $n_{IN}^*(p) = 24.92$, $n_{IN}^*(\lambda_a) = 3.21$.

TABLE 4A

SENSITIVITY ANALYSIS FOR THE "MUGGING" EXAMPLE (MODEL 2)

					PRIOR MEANS AND STANDARD DEVIATIONS (MODEL 2)				POSTERIOR MEANS AND STANDARD DEVIATIONS (MODEL 2)			
	(α_1, β_1)	(α_2, β_2)	k	m	$E_2(p)$	$\sigma_2(p)$	$E_2(\lambda_a)$	$\sigma_2(\lambda_a)$	$E_2(p \mid r,n)$	$\sigma_2(p \mid r,n)$	$E_2(\lambda_a \mid r,n)$	$\sigma_2(\lambda_a \mid r,n)$
1.	(2, 14)	(18, 2)	1.3	3	0.125	0.008	0.674	0.137	0.092	0.014	0.723	0.040
2.	(2, 14)	(14, 6)	1.3	3	0.125	0.008	0.524	0.125	0.067	0.014	0.607	0.069
3.	(2, 14)	(19, 1)	1.3	3	0.125	0.008	0.711	0.140	0.100	0.013	0.747	0.031
4.	(2, 14)	(9, 1)	1.3	3	0.125	0.008	0.674	0.145	0.092	0.016	0.721	0.053
5.	(2, 14)	(36, 4)	1.3	3	0.125	0.008	0.674	0.133	0.092	0.013	0.724	0.031
6.	(2, 14)	(18, 2)	1.1	3	0.125	0.008	0.639	0.152	0.087	0.013	0.705	0.039
7.	(2, 14)	(18, 2)	1.5	3	0.125	0.008	0.700	0.126	0.097	0.016	0.737	0.040
8.	(2, 14)	(18, 2)	1.3	2	0.125	0.008	0.739	0.111	0.106	0.018	0.759	0.041
9.	(2, 14)	(18, 2)	1.3	4	0.125	0.008	0.617	0.158	0.083	0.012	0.693	0.039
10.	(1, 15)	(18, 2)	1.3	3	0.063	0.059	0.781	0.118	0.090	0.015	0.720	0.042
11.	(3, 13)	(18, 2)	1.3	3	0.188	0.095	0.576	0.144	0.095	0.014	0.725	0.037
12.	(6, 42)	(18, 2)	1.3	3	0.125	0.047	0.668	0.092	0.095	0.013	0.726	0.037
13.	(1, 1)	(18, 2)	1.3	3	0.5	0.289	0.292	0.257	0.093	0.014	0.723	0.039

TABLE 4B

SENSITIVITY ANALYSIS FOR THE "MUGGING" EXAMPLE (MODEL 2)

	COMPARISON OF MODEL 2 TO THE NOISE-FREE MODEL $E_{NF}(p \mid r,n) \equiv$ posterior mean for p in the noise-free model $\sigma_{NF}(p \mid r,n) \equiv$ posterior standard variation for p in the noise-free model $n_2{}^*(I) \equiv$ effective sample size for i, where i = p or λ_a, in Model 2				COMPARISON OF MODEL 2 TO THE INDEPENDENT NOISE MODEL $E_{IN}(. \mid r,n) \equiv$ posterior mean in the independent noise model $\sigma_{IN}(. \mid r,n) \equiv$ posterior standard deviation in the independent noise model $n_{IN}{}^*(i) \equiv$ effective sample size for i, where i = p or λ_a, in the independent noise model					
	$\frac{E_2(p \mid r,n)}{E_{NF}(p \mid r,n)}$	$\frac{\sigma_2(p \mid r,n)}{\sigma_{NF}(p \mid r,n)}$	$n_2^*(p)$	$n_2^*(\lambda_a)$	$\frac{E_2(p \mid r,n)}{E_{IN}(p \mid r,n)}$	$\frac{\sigma_2(p \mid r,n)}{\sigma_{IN}(p \mid r,n)}$	$\frac{E_2(\lambda_a \mid r,n)}{E_{IN}(\lambda_a \mid r,n)}$	$\frac{\sigma_2(\lambda_a \mid r,n)}{\sigma_{IN}(\lambda_a \mid r,n)}$	$\frac{n_2^*(p)}{n_{IN}^*(p)}$	$\frac{n_2^*(\lambda_a)}{n_{IN}^*(\lambda_a)}$
1.	3.487	3.049	393.139	115.434	0.923	0.308	1.032	0.334	15.775	35.948
2.	2.530	3.003	298.451	34.531	1.032	0.581	1.079	0.559	3.458	94.637
3.	3.772	2.746	525.614	187.766	0.901	0.244	1.027	0.266	28.379	44.552
4.	3.495	3.475	299.441	61.353	0.912	0.335	1.028	0.428	13.840	18.807
5.	3.477	2.709	501.358	197.653	0.929	0.282	1.034	0.266	18.514	62.264
6.	3.276	2.739	463.568	124.370	0.923	0.282	1.043	0.298	19.179	48.952
7.	3.676	3.352	338.883	107.475	0.923	0.333	1.024	0.369	13.329	27.793
8.	4.016	3.930	262.970	93.419	0.923	0.374	1.015	0.430	10.486	18.176
9.	3.143	2.564	511.338	129.153	0.924	0.271	1.050	0.282	20.700	59.632
10.	3.497	3.190	359.757	99.389	0.849	0.314	0.986	0.404	13.825	18.190
11.	3.469	2.922	426.502	131.156	0.987	0.304	1.075	0.289	17.395	97.76
12.	3.269	2.759	433.260	121.943	0.985	0.473	1.023	0.476	7.157	14.063
13.	3.582	3.100	399.355	125.859	0.743	0.084	1.466	0.123	605.045	-180.81

FIGURE 1

THE LIKELIHOOD FUNCTION [n=10 and r=1]

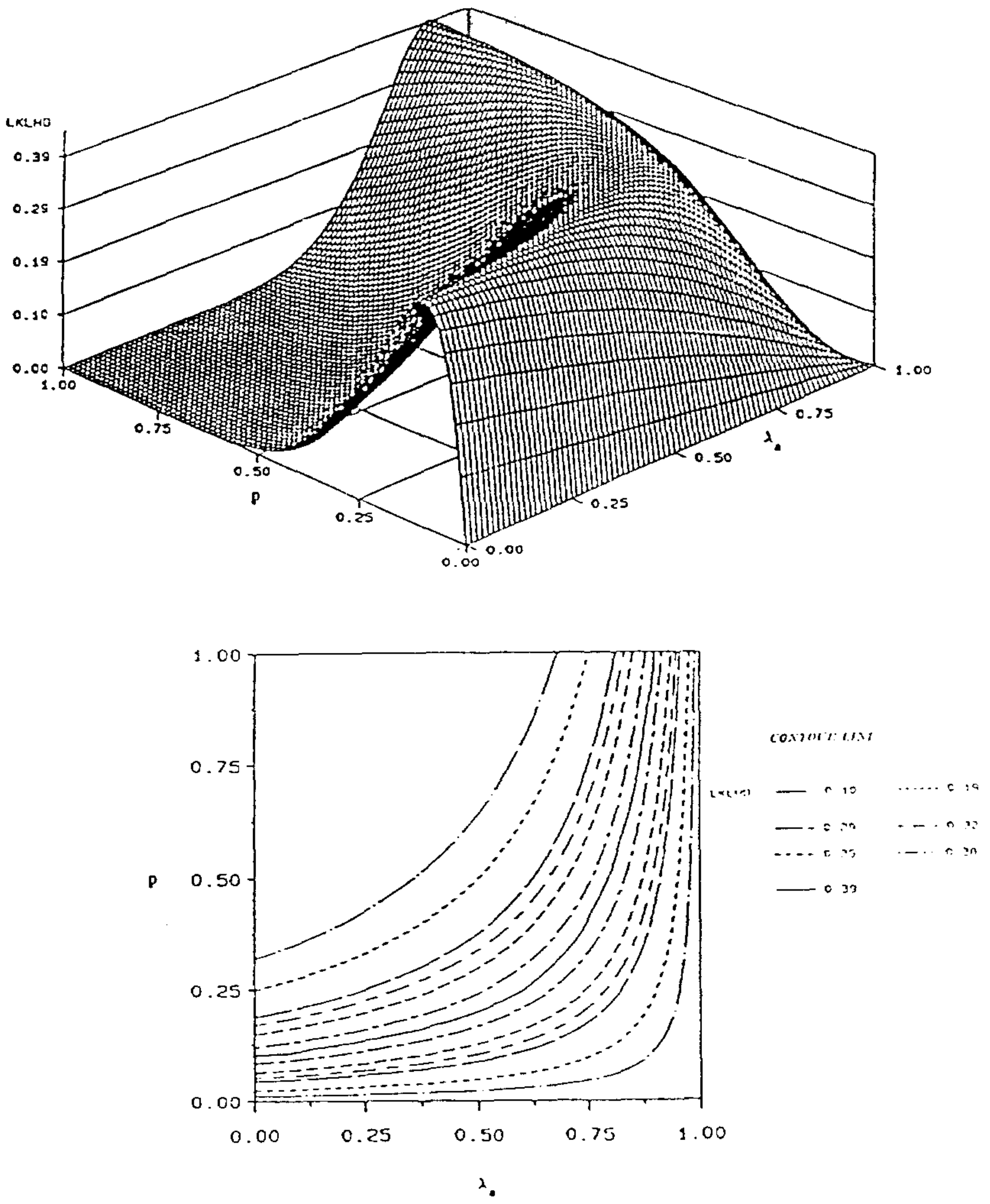


FIGURE 2

PRIOR DENSITY FOR p AND λ_a $[(\alpha_1,\alpha_2,\alpha_3)=(3,2,5)]$

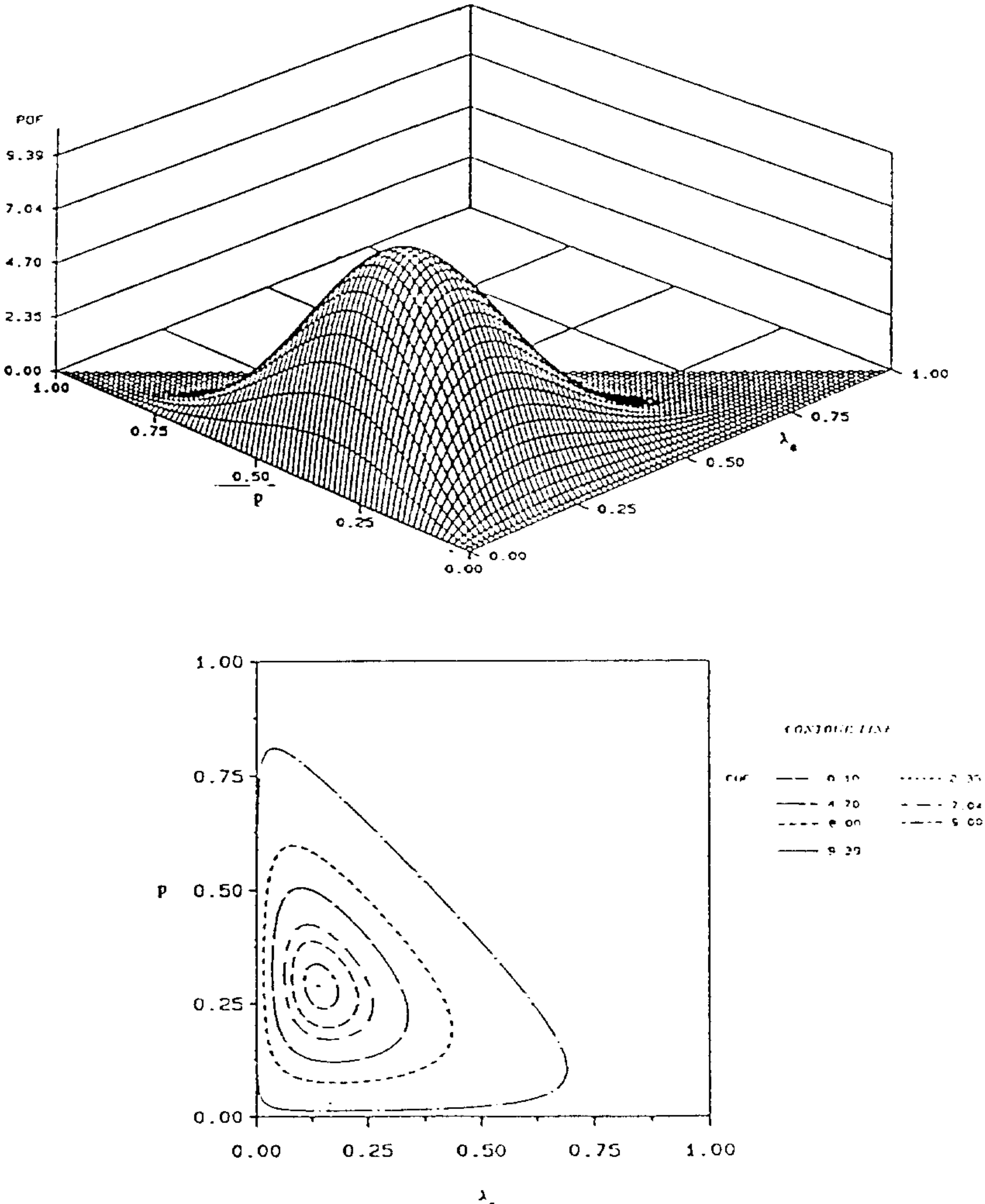
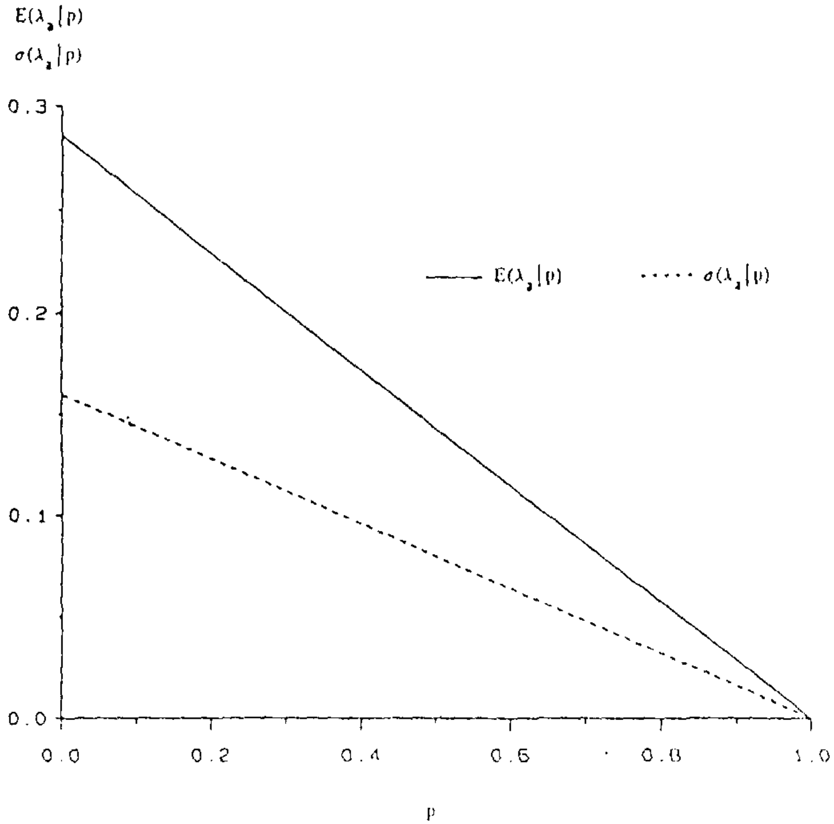


FIGURE 3

PRIOR CONDITIONAL EXPECTATION AND STANDARD DEVIATION FOR λ_a, GIVEN p

MODEL I

$[(\alpha_1,\alpha_2,\alpha_3)=(3,2,5)]$



PRIOR CONDITIONAL DENSITY FOR λ_a, GIVEN p

MODEL I

$[(\alpha_1,\alpha_2,\alpha_3)=(3,2,5)]$

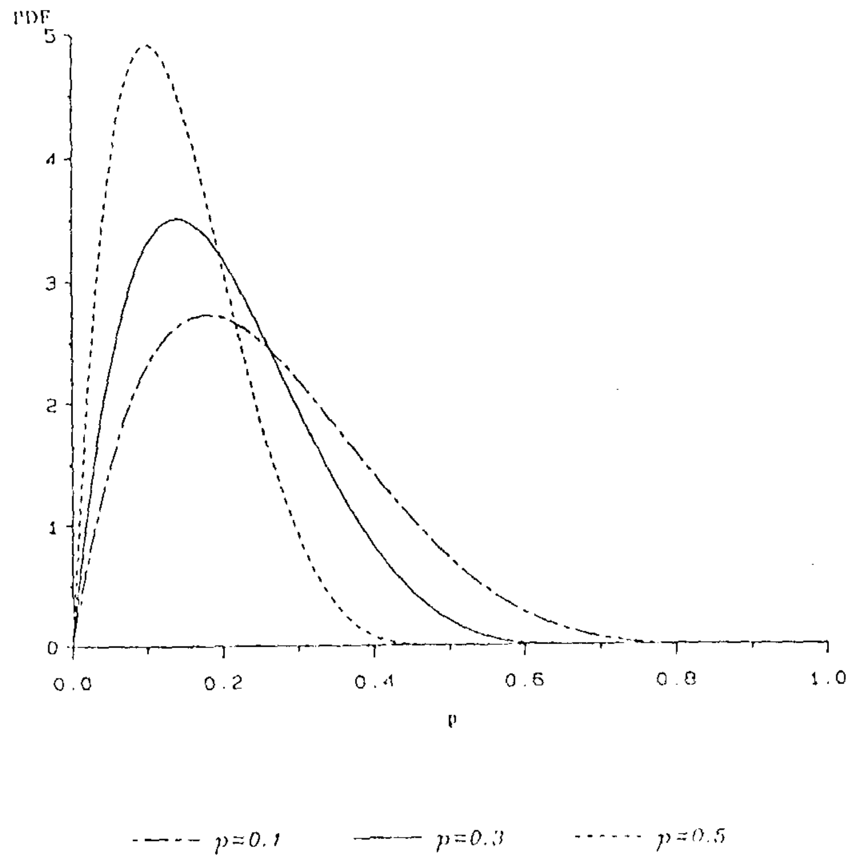
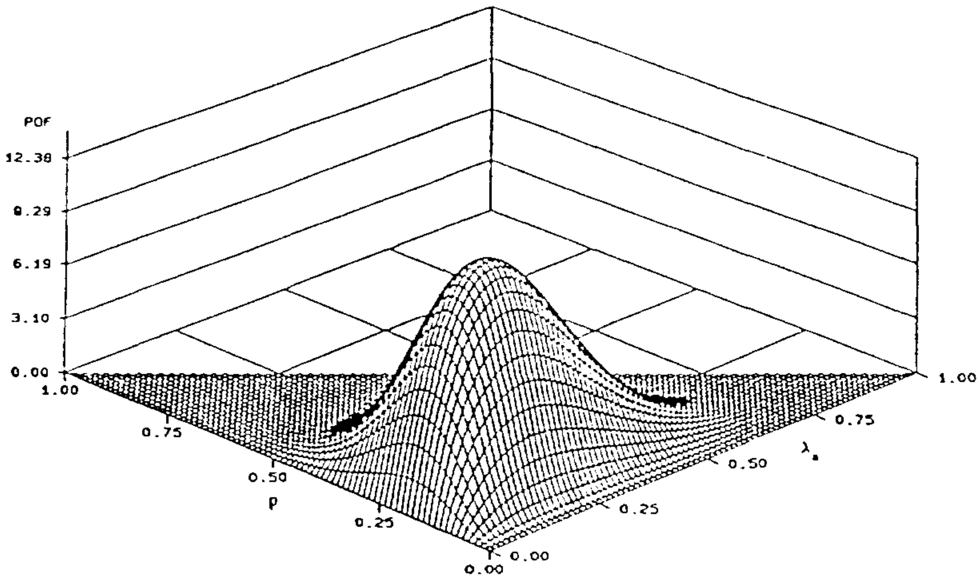


FIGURE 4

POSTERIOR DENSITY FOR p AND λ_2
$[(\alpha_1,\alpha_2,\alpha_3)=(3,2,5)$, n =10 and r=1]



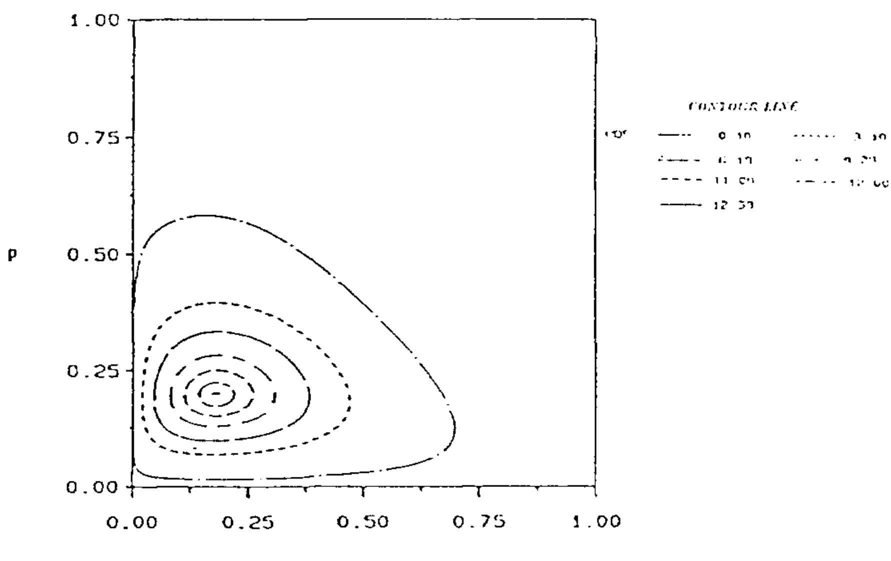


FIGURE 5

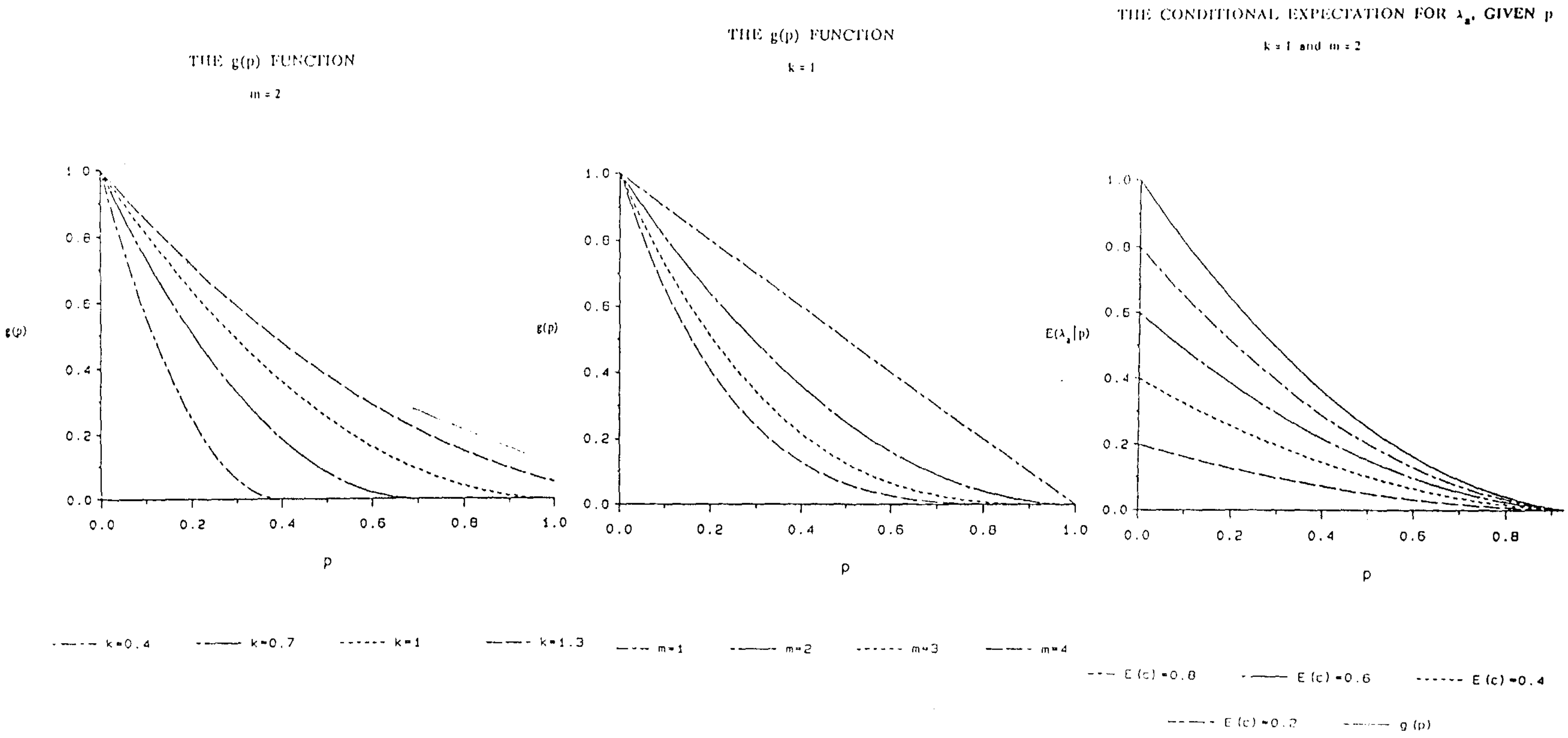
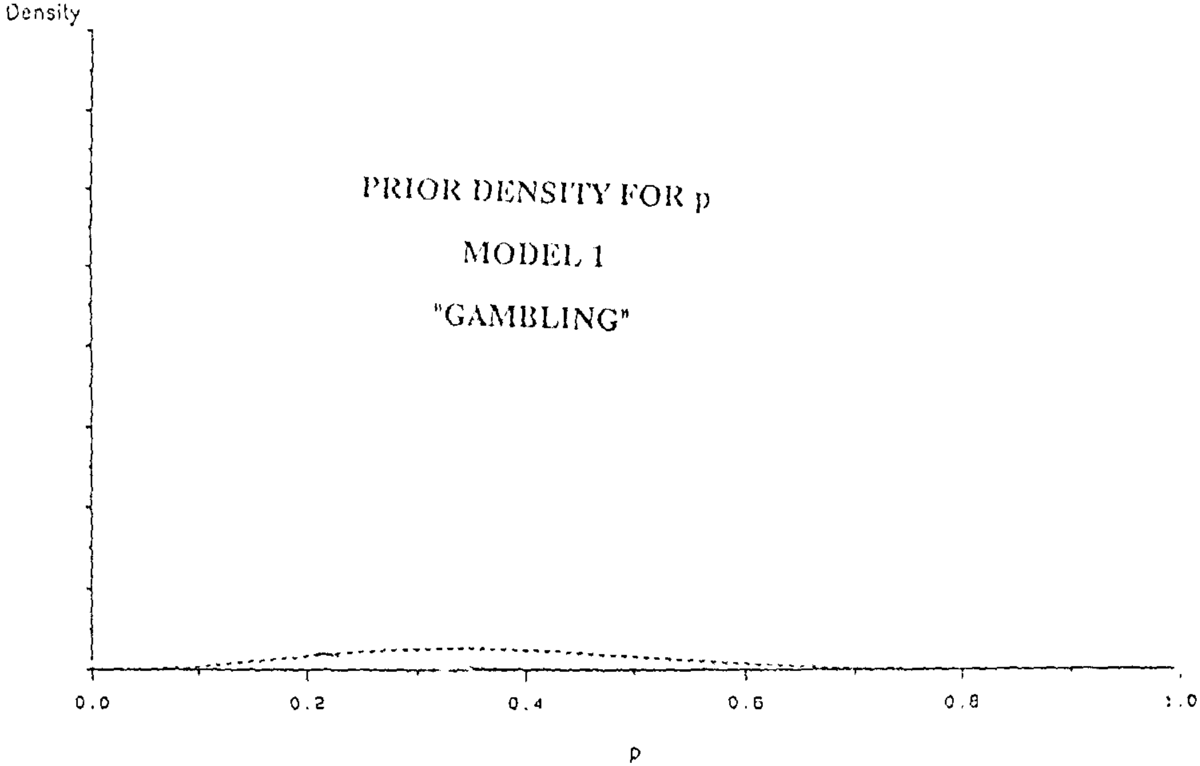

THE g(p) FUNCTION
m = 2
g(p)
p
- - - k=0.4
—— k=0.7
- - - - - k=1
— — k=1.3
THE g(p) FUNCTION
k = 1
g(p)
p
- - - m=1
—— m=2
- - - - - m=3
— — m=4
THE CONDITIONAL EXPECTATION FOR λ_a, GIVEN p
k = 1 and m = 2
$E(\lambda_a|p)$
p
- - - E(c)=0.8
—— E(c)=0.6
- - - - - E(c)=0.4
— — E(c)=0.2
—— g(p)

FIGURE 6


Density
PRIOR DENSITY FOR p
MODEL 1
"GAMBLING"
0.0
0.2
0.4
0.6
0.8
1.0
p

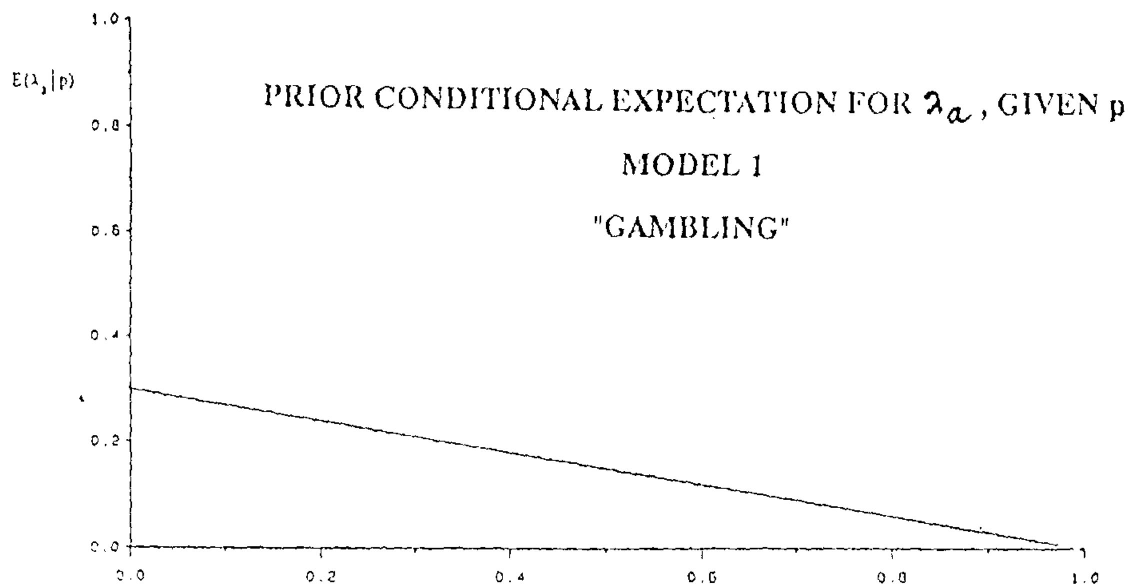

1.0
0.8
0.6
0.4
0.2
0.0
PRIOR CONDITIONAL EXPECTATION FOR λ_a, GIVEN p
MODEL 1
"GAMBLING"
0.0
0.2
0.4
0.6
0.8
1.0

FIGURE 7

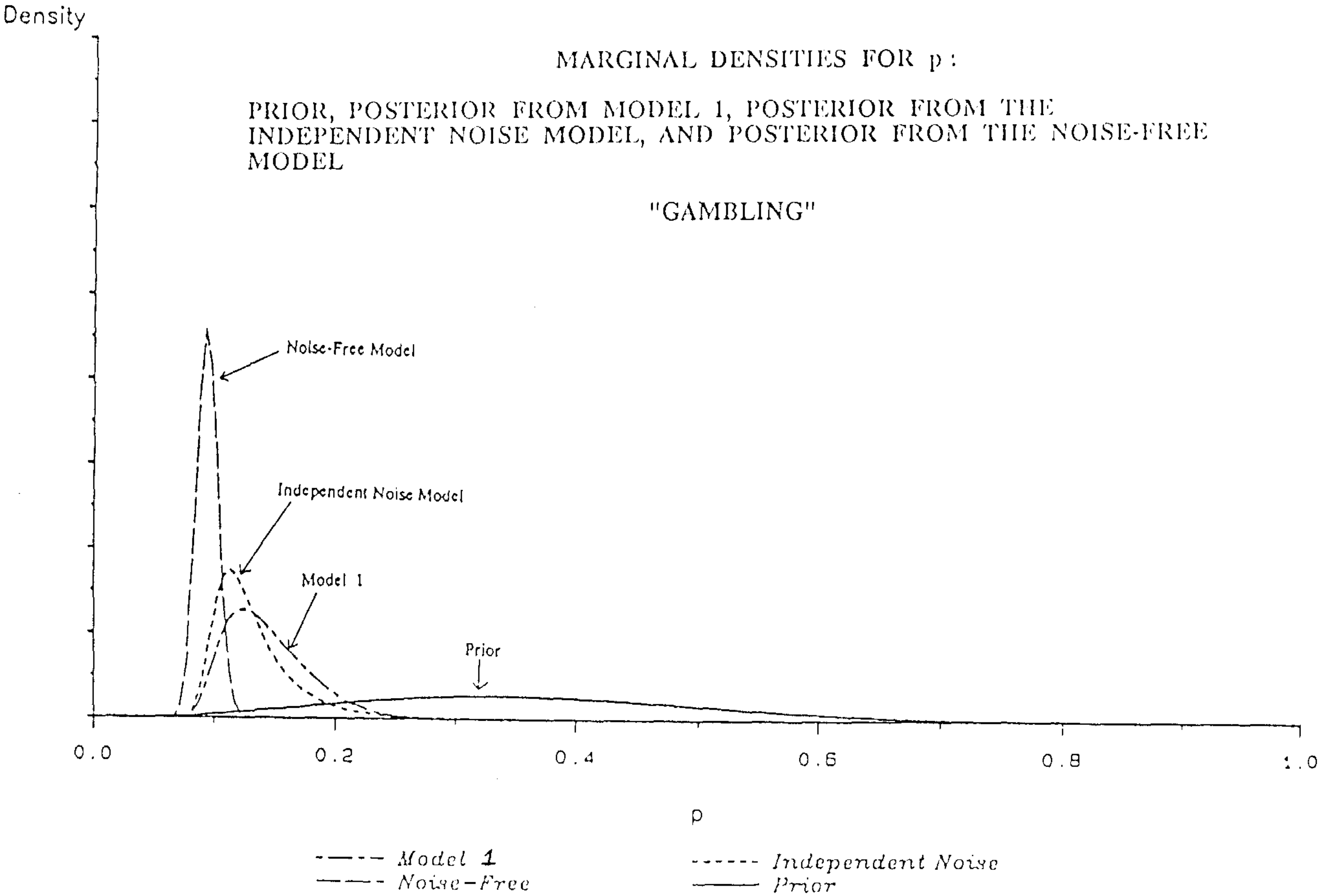
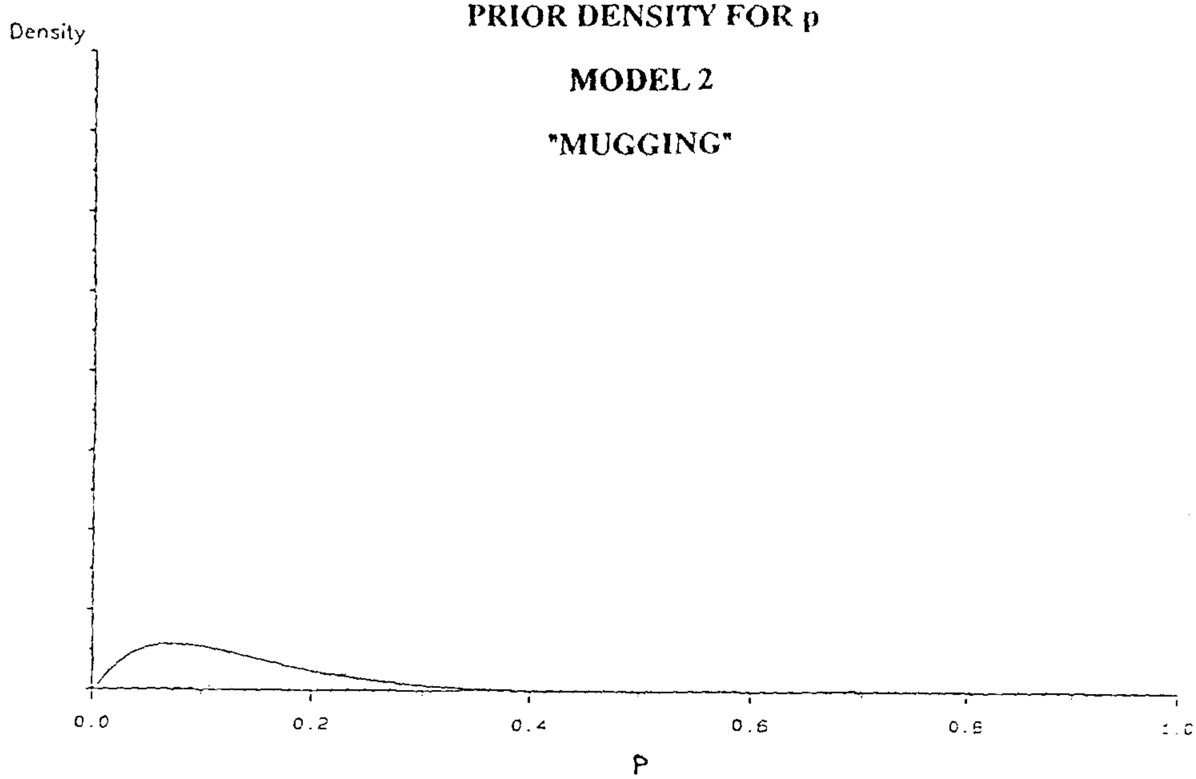

Density
MARGINAL DENSITIES FOR p :
PRIOR, POSTERIOR FROM MODEL 1, POSTERIOR FROM THE INDEPENDENT NOISE MODEL, AND POSTERIOR FROM THE NOISE-FREE MODEL
"GAMBLING"
Noise-Free Model
Independent Noise Model
Model 1
Prior
0.0
0.2
0.4
0.6
0.8
1.0
p
Model 1
Noise-Free
Independent Noise
Prior

FIGURE 8

PRIOR DENSITY FOR p

MODEL 2

"MUGGING"



PRIOR CONDITIONAL EXPECTATION FOR λ_a, GIVEN p

MODEL 2

"MUGGING"

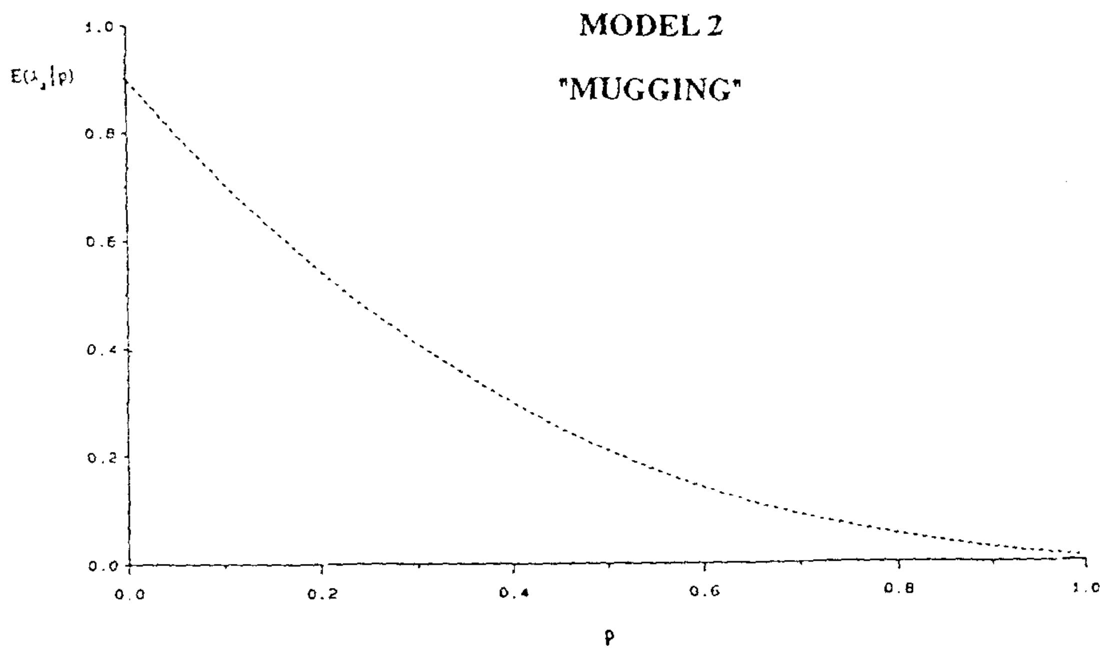
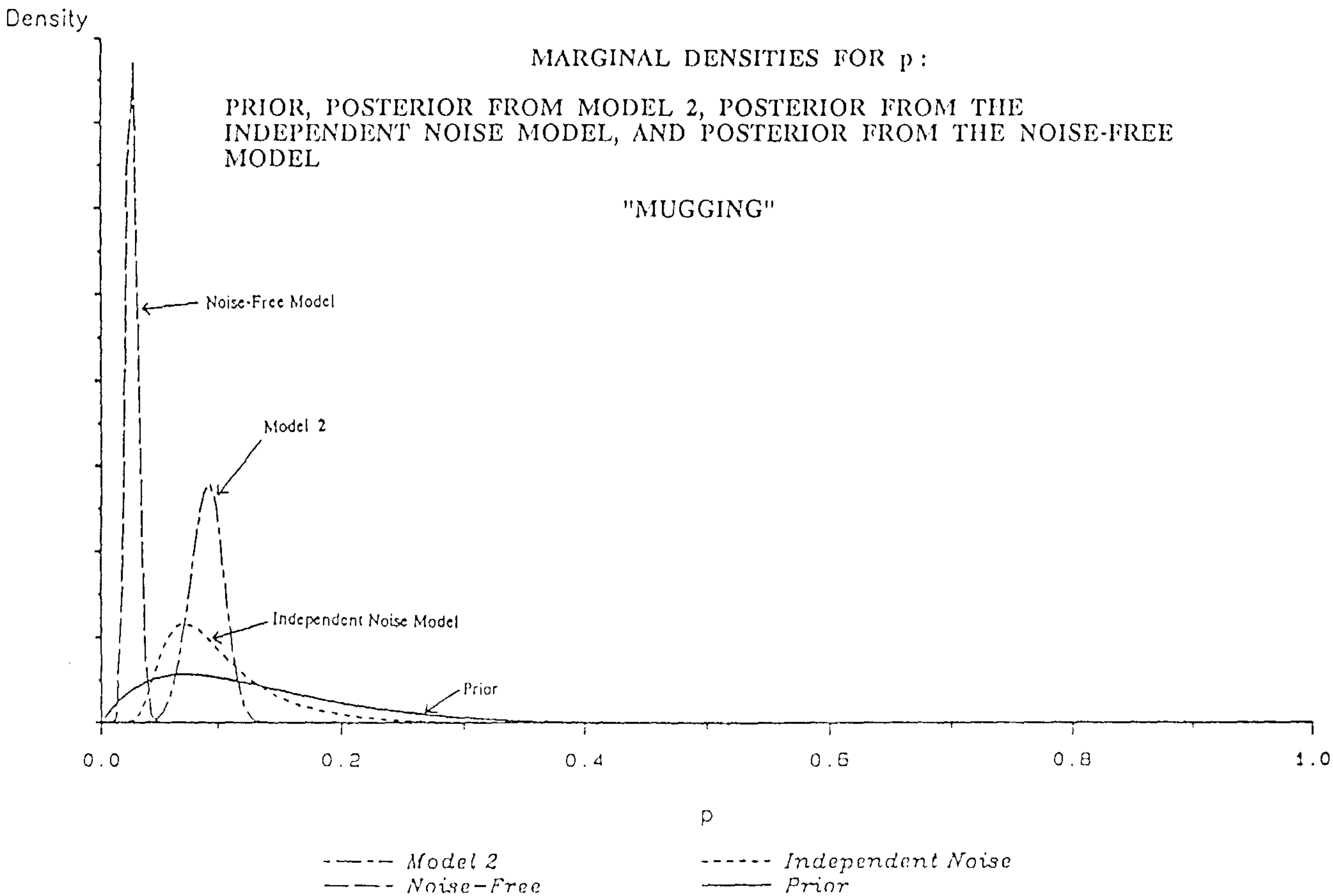


FIGURE 9



Density
MARGINAL DENSITIES FOR p:
PRIOR, POSTERIOR FROM MODEL 2, POSTERIOR FROM THE INDEPENDENT NOISE MODEL, AND POSTERIOR FROM THE NOISE-FREE MODEL
"MUGGING"
Noise-Free Model
Model 2
Independent Noise Model
Prior
0.0
0.2
0.4
0.6
0.8
1.0
p
Model 2
Noise-Free
Independent Noise
Prior

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<u>1988</u>

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<u>1989</u>

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1990

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