"NONSTATIONARY CONDITIONAL TREND ANALYSIS: AN APPLICATION TO SCANNER PANEL DATA".

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Nonstationary Conditional Trend Analysis: An Application to Scanner Panel Data

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Abstract

Conditional trend analysis (CTA) predicts the number of purchases in a test period by all households that purchase a given number of items in a base period. The underlying model assumes that households' purchases follow stationary Poisson processes with rate parameters that vary across the households in a market. However, stationarity is often an unrealistic assumption because of marketing variables and seasonal effects. This paper extends CTA to the nonstationary setting and compares the stationary and nonstationary models. Falsely assuming stationarity systematically biases forecasts. Although modeling nonstationarity reduces bias, under–prediction, especially of the zero class, persists. We show that this under–prediction is, in part, a mathematical artifact due to the skewness of the negative binomial distribution. The methodology is applied to scanner panel data.

KEY WORDS: Forecasting; Hierarchical Bayes; Negative binomial distribution; Poisson process.

1 Introduction

In 1959 Andrew Ehrenberg introduced the negative binomial distribution (NBD) as a model of consumer purchase behavior. The NBD is based on the following assumptions:

- A household's purchase occasions follow a stationary Poisson process with constant rate λ.
- Purchase rates λ are heterogeneous across the households in a market, and this heterogeneity is characterized by a gamma distribution.

Morrison and Schmittlein (1988) point out in their review paper that the NBD model has worked well in a large variety of situations over the last 30 years, both as a de-

scription of customer purchasing behavior and as a benchmark from which the impact of marketing variables can be measured. In spite of this success, many efforts have been made to generalize the NBD model by relaxing the assumptions and including marketing and other predictor variables. See, for example, Morrison and Schmittlein (1981 & 1988), Wagner and Taudes (1986), and the references in those papers.

Frequently, these extensions obtain their generality at the expense of the simplicity and intuitive appeal of the NBD. In this paper, we retain the structure of the NBD yet give the model greater flexibility by introducing predictor variables through the rate parameter or intensity function of the Poisson processes. That is, a household's purchase occasions follow a nonstationary Poisson process. This approach maintains the probabilistic structure of the NBD while providing a method to incorporate predictor variables.

Other researchers have proposed nonstationary stochastic models. For example, Vilcassim and Jain (1991) describe the times between a household's purchases. Their analysis provides a detailed picture of individual households. Vilcassim and Jain's model and the one in this paper are complimentary to each other. Their unit of analysis is a household, and their data are inter–purchase times. The unit of analysis of this paper is market segment, and our data are total number of purchase occasions by the segment during a period of time.

Among the most important questions to which the NBD provides an answer is: If a customer makes x purchases in a base period, what is the expected number of purchases he or she will make in a following test period? These conditional expectations can be used as baselines for the measurement of deviations caused by marketing effort during the test period. This application, called conditional trend analysis (CTA), was introduced by Goodhardt and Ehrenberg (1967) and Morrison (1968).

Applications of CTA highlight some problems with the NBD model, as noted by Morrison and Schmittlein (1988):

1. The NBD tends to under-predict the zero class, that is, the conditional expectations for the test period of those customers who bought zero in the base period appear to be systematically biased downwards in empirical experience; see Morrison and Schmittlein (1981) for examples. The zero class has a special role in CTA because an unexpectedly large number of purchases by the zero class in a forecast period in response to marketing effort usually is interpreted as a net gain for the brand. Consequently, under-predicting the zero class can be a serious problem as it will lead to an overstatement of the impact of marketing effort.

2. In today's marketing environment of heavy dealing it is almost impossible to find a "clean" base period, that is, a base period without promotional activity. In the absence of such clean base periods, using the CTA's baselines to assess marketing impact will result in biased estimates.

Morrison and Schmittlein (1988) go on to suggest that the stationarity assumption likely causes (1) and the explicit introduction of marketing variables in the NBD framework is required to resolve (2). Since the levels of marketing variables change over time, the recommended introduction of these variables is also a call to relax the stationarity assumption. They go on to hypothesize that nonstationarity causes a much greater regression to the mean, i. e. customers who have few purchase occasions in the base period are expected to purchase more frequently in the test period, while customers who buy frequently in the base period are expected to buy less frequently in the test period, than that implied by the stationary NBD model.

This paper models nonstationarity in conditional trend analysis and shows that systematic errors are introduced by falsely assuming stationary Poisson processes. These errors can be serious when the base and forecast periods are short, which is often the case in practical applications, as for example when evaluating the impact of a promotion or advertising campaign. However, the variation due to nonstationarity

usually averages–out over longer periods, resulting in reduced forecasting bias. This result formalizes Ehrenberg's (1972) view of the domain of application for the negative binomial model.

Nonstationarity can be quite severe over short periods of time. This is most easily demonstrated graphically by means of a control chart for detecting nonconforming observations. If the individual household purchase processes are stationary Poisson processes, then the total number of purchases by a panel of households — the result of the superposition of Poisson processes — is also a stationary Poisson process. Thus, the appropriate control chart is the c–chart (Ryan 1989 pp. 196–201). Figure 1 graphs the number of weekly purchase occasions for two series: powder laundry detergents[1] and Tide.[2] Powder detergent is a product category, and Tide is the leading brand within the powdered detergent category. Superimposed on each series is a c–chart. The center line is the mean weekly number of purchase occasions, and the control limits are plus and minus three times the square root of the weekly mean. Under the hypothesis of a stationary Poisson process and using a normal approximation, approximately 99.7% of the weekly purchases should be within the control limits. The c–charts indicate considerable nonstationarity in the series. Hence, ignoring nonstationarity in consumer purchases when the observation periods are one week will seriously affect forecasting performance. Part of the variation in purchase occasions may be explained by marketing variables and seasonal effects. Figure 2 shows time plots of an aggregate measure of weekly advertising by the stores where the purchases were made. The plots demonstrate that weekly advertising by the stores is not constant over time. Moreover, the correlation of the logarithm of number of

[1]The source of the data is the MSI Library supplied by A. C. Nielsen. The data consists of the purchase history for a panel of households in Springfield, Missouri for 138 weeks from the beginning of 1986 to the 34th week of 1988.

[2]The purchase occasions for Tide are for 42 ounce packages, which is the leading universal product code (UPC), accounting for 17.6% of total purchase occasions.

weekly purchase occasions and weekly advertising is 0.37 for powder detergents and 0.56 for Tide. Hence, weekly advertising is a potential predictor variable.

The nonstationarity we consider is of a specific type. Each household's purchase occasions are described by a Poisson process with proportional intensity, $\lambda\psi(t)$, at time t where λ is a household specific component, and $\psi(\cdot)$ is a component common to all households. The specific component, λ, is a household's propensity to purchase the product and varies across the population, while the common component, $\psi(t)$, is due to a common environment at time t. The common component, $\psi(t)$, changes over time for all members of the population and can include marketing variables and seasonal effects. Examples of proportional intensity models are in Hausman, Hall, and Griliches (1984), Lenk and Rao (1990 b), and Wagner and Taudes (1986).

Traditionally, applications of CTA using a stationary model proceed in two stages: first, forecasts are made using the stationary NBD model, and second, these forecasts are adjusted by multiplying them by the ratio of the number of purchases in the test period to that in the base period (Morrison and Schmittlein 1981). This procedure can only be done *ex post*. Modeling nonstationarity using a proportional intensity allows us to make a similar adjustment *ex ante*. For example, if the advertising and promotional schedules in the base and test periods are known, then they can be included in ψ and used in CTA.

Although introducing predictor variables improves the accuracy of the forecasts, under–prediction persists in the nonstationary NBD. We show that under–prediction is due, in part, to a mathematical artifact stemming from the skewness of the NBD. For distributions that are symmetric about the mean, such as normal distributions, the expected positive forecast errors are equal to the expected negative forecast errors. However, for positively skewed distributions, such as the negative binomial distribution, the expected positive forecast errors are greater than the expected negative forecast errors. Thus, the observed under–prediction may not imply model

inadequacy as it would for a normal model.

The other major assumption of the NBD model is the gamma mixing distribution. Robbins (1977) relaxes the gamma assumption and analyzes the problem without specifying the distribution of the λ's while maintaining the assumption that events follow a stationary Poisson process. His approach leads to a remarkably simple estimation and forecasting procedure. We show that nonstationarity affects Robbins' model in the same way that it affects the NBD model. This result is intuitive after realizing that the stationarity assumption applies to the underlying Poisson process, which is a common element to both models, and not to the distributional assumptions for λ, which differentiates the two models.

The remainder of this paper is structured as follow. Section 2 defines notation and describes nonstationary Poisson processes and negative binomial distributions. Section 3 evaluates the forecasting bias associated with incorrectly assuming a stationary model. Section 4 develops the model for conditional trend analysis and describes the relation of the forecasts between the stationary and nonstationary models. Section 5 presents an empirical study using scanner panel data. Section 6 analyzes under-prediction for skewed distributions. Section 7 discusses the results.

2 Nonstationary Models

This section defines notation and presents relevant facts about nonstationary Poisson processes that are used in the negative binomial distribution (NBD) and Robbins model.

1. Z is a nonstationary Poisson process with proportional intensity $\lambda\psi(t)$ where λ is constant over time, and $\psi(\cdot)$ can depend on time and marketing variables.

2. $(s_k, t_k]$ for $s_k < t_k$ is a time interval with length $\Delta_k = t_k - s_k$. The intervals are disjoint for different values of k.

3. $N_k = Z(t_k) - Z(s_k)$ is the number of purchase occasions during $(s_k, t_k]$.

4. $\theta_k = \int_{s_k}^{t_k} \psi(u)du$.

5. In the stationary case, $\psi \equiv 1$, and $\theta_k = \Delta_k$.

6. $\{N_k\}$ are mutually independent Poisson random variables with means $\{\lambda\theta_k\}$.

The NBD is derived by integrating λ by a gamma distribution with shape parameter α and scale parameter β. The density for the gamma distribution is

$$g(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} \exp(-\beta\lambda) \text{ for } \lambda \geq 0, \alpha > 0, \beta > 0 \tag{1}$$

with mean and variance

$$E(\lambda) = \alpha/\beta \text{ and } Var(\lambda) = \alpha/\beta^2.$$

After mixing over the gamma distribution, N_k has a negative binomial distribution:

$$\begin{aligned} Pr(N_k = n_k) &= \frac{\Gamma(\alpha + n_k)}{\Gamma(\alpha)\, n_k!} \left(\frac{\beta}{\beta + \theta_k}\right)^\alpha \left(\frac{\theta_k}{\beta + \theta_k}\right)^{n_k} \\ &\quad \text{for } n_k = 0, 1, \ldots. \end{aligned}$$

Its mean and variance are

$$\begin{aligned} E(N_k) &= \theta_k \alpha \beta^{-1} \\ Var(N_k) &= E(N_k)\left[1 + \alpha^{-1} E(N_k)\right]. \end{aligned}$$

Following the analysis of Morrison (1968), after observing the number, N_1, of purchases in the base period, $(s_1, t_1]$, the distribution of λ is updated by Bayes theorem. The updated distribution is also a gamma distribution: α is updated to $\alpha + N_1$, and β is updated to $\beta + \theta_1$. Then the number, N_2, of purchases in the second interval given N_1 also has a negative binomial distribution with the updated parameters. The conditional mean of N_2 given N_1 is

$$E(N_2|N_1) = \theta_2 \frac{\alpha + N_1}{\beta + \theta_1}. \tag{2}$$

In Robbins' model the distribution of λ is not constrained to be from the gamma family. The next theorem presents Robbins' result, as adjusted for nonstationary Poisson processes.

Theorem 1 *Robbins. Suppose that the distribution of λ is G, and let $f(x)$ be the marginal distribution of N_1 evaluated at x:*

$$\begin{aligned} f(x) &= \int_0^\infty Pr(N_1 = x|\lambda)dG(\lambda) \\ &= (x!)^{-1} \int_0^\infty \exp(-\lambda\theta_1)(\lambda\theta_1)^x dG(\lambda). \end{aligned}$$

Then the conditional mean of N_2 given N_1 is

$$E(N_2|N_1 = n_1) = (n_1 + 1)\,\frac{\theta_2}{\theta_1}\,\frac{f(n_1+1)}{f(n_1)}. \tag{3}$$

The stationary case is Robbins' (1977) result.

The proofs of this and the other theorems are in Appendix B.

3 Forecasting Bias

This section analyzes the impact on forecasts from incorrectly assuming a stationary process, regardless of the assumptions about the mixing distribution for λ. Assume that the parameters are known. Let Z be the true nonstationary process with intensity $\lambda\psi(\cdot)$, and let Z^* be the assumed stationary process with constant intensity $\lambda\xi$. Appendix A shows that the value of ξ which makes Z^* as close as possible to Z, in a probabilistic sense, over K disjoint intervals is the time average of $\psi(\cdot)$ over the intervals:

$$\xi^* = \left(\sum_{k=1}^{K} \theta_k\right) / \left(\sum_{k=1}^{K} \Delta_k\right).$$

CTA uses a base period $(s_1, t_1]$ and a forecast period $(s_2, t_2]$ so that K is two and

$$\xi^* = (\theta_1 + \theta_2) / (\Delta_1 + \Delta_2). \tag{4}$$

The forecasting bias due to falsely assuming stationary Poisson processes is evaluated by combining this result with the conditional expectations in equations (2) and (3).

Theorem 2 *Let N_k and N_k^* be the number of events in the kth interval for $k = 1$ and 2 for the processes Z and Z^*. Given λ, N_k has a Poisson distribution with mean $\lambda\theta_k$, and N_k^* has a Poisson distribution with mean $\lambda\xi^*\Delta_k$ where ξ^* is defined in equation (4). Under both the negative binomial and Robbins' models,*

$$E(N_2^*|N_1^* = n) < E(N_2|N_1 = n) \quad \textit{if and only if} \quad \theta_1/\Delta_1 < \theta_2/\Delta_2.$$

In other words, the forecast using the optimal, stationary Z^* under-estimates if the average rate in the first period is less that the average rate in the second period.

The amount of bias in the forecasts by using the stationary models tends to decrease as the time intervals increase. In many situations, the time averages, θ_k/Δ_k, of ψ over the two intervals become approximately equal. This situation implies that the nonstationarity "washes-out" or "averages-out" over the long-run.

An example of this behavior may obtain in mature markets for frequently purchased nondurables, where the household's purchases are sensitive to promotions. At any given time, one or more manufacturers have promotional campaigns. Although there may be seasonal effects, over longer periods of time, the average level of promotional activity and the total number of purchase occasions remain fairly steady. It is this feature that probably accounts for the robustness of the NBD model; in most applications of CTA the intervals are long — six months or more.

4 Nonstationary Conditional Trend Analysis

This section applies nonstationary Poisson processes to conditional trend analysis (CTA). The first subsection develops the model for CTA, and the second describes

the relation of the estimates and forecasts between the stationary and nonstationary models.

4.1 Model Development

This section formulates the NBD for purchase occasions with an hierarchical Bayes model (Berger 1985 pp. 180–195; Lenk & Rao 1990 a; and Lenk 1991). An attractive feature of the model for purchase occasions is that it provides a coherent framework for analyzing various levels of aggregation with regard to households, products, and time periods. For example, models of purchase occasions at the brand level will be consistent with those at the category level. Additionally, CTA forecasts the total number of purchases in a test period by aggregating households according to their purchases in a base period. Both the base and test periods could change in different CTAs.

Let Z_i be the purchase process for the ith household in a panel of h households. The hierarchical Bayes model is:

1. Within Household Variation:

   Given $\{\lambda_i\}$ and $\psi(\cdot)$, the household purchase processes, $\{Z_i\}$, for the households in the panel are mutually independent Poisson processes where the intensity of household i is $\lambda_i\psi(\cdot)$. Assume that $\psi(\cdot)$ depends on a vector, ϕ, of parameters.

2. Between Household Variation:

   $\{\lambda_i\}$ is a random sample from a gamma distribution (equation 1) with shape parameter α and scale parameter β.

3. Prior Distribution:

   The unknown parameters, α, β, and ϕ, have locally uniform prior distributions.

Conditional trend analysis aggregates purchases over households and over time periods. The aggregated purchase process, Z, for h households in a market segment

is the sum of the individual households' purchase processes:

$$Z = \sum_{i=1}^{h} Z_i.$$

Given $\psi(\cdot)$ and $\{\lambda_i\}$, the aggregated process is the superposition of independent Poisson processes. Hence, Z is a Poisson process with intensity $\lambda\psi(\cdot)$ where

$$\lambda = \sum_{i=1}^{h} \lambda_i,$$

which has a gamma distribution with shape parameter $h\alpha$ and scale parameter β.

A further simplification in the analysis results if purchase occasions are aggregated by time intervals instead of recording the exact time of each purchase. The NBD, which has one time period, is extended easily to multiple, disjoint time periods (Hausman, Hall, and Griliches 1984). Appendix C describes this multivariate generalization, known as the "negative multinomial distribution." The joint distribution of the total number, $\{N_k\}$, of purchases in K intervals after mixing over λ is given by equation (11) in Appendix C with α replaced by $h\alpha$. Here, h, the number of households, is known.

4.2 Estimation and Prediction

CTA stratifies a panel of M households by the number of purchases, x, in a base period and forecasts the total number, T_x, of purchases in a test period by the M_x households that purchased x items in the base period. As mentioned in Section 4.1, households buy according to independent Poisson processes with intensities $\{\lambda_i\psi(\cdot)\}$. $\{\lambda_i\}$ is a random sample from a mixing distribution, G, which is a gamma distribution in the NBD model and any distribution with positive support in Robbins' model. Let $N_{i,1}$ and $N_{i,2}$ be the number of purchases by the ith household in a base and a forecast period, respectively. Thus,

$$T_x = \sum_{\{i:N_{i,1}=x\}} N_{i,2}.$$

The next theorem states the joint distribution of $\{T_x\}$ given the purchases in the base period.

Theorem 3 *Assume that $\{\lambda_i\}$ is a random sample from a gamma distribution with shape parameter α and scale parameter β. Given the parameters, α, β, and ϕ and the purchases $\{M_x\}$ in the base period, $\{T_x\}$ are mutually independent negative binomial random variables with probability mass function:*

$$\begin{aligned} P(T_x|M_x) &= \frac{\Gamma(\alpha M_x + xM_x + T_x)}{\Gamma(\alpha M_x + xM_x)T_x!}\left(\frac{\beta+\theta_1}{\beta+\theta_1+\theta_2}\right)^{\alpha M_x + xM_x}\left(\frac{\theta_2}{\beta+\theta_1+\theta_2}\right)^{T_x} \quad (5) \\ &= \textit{for } x = 0, 1, \ldots \textit{ and } T_x = 0, 1 \ldots. \end{aligned}$$

CTA forecasts T_x by its conditional mean:

$$\begin{aligned} E(T_x|M_x, x) &= \sum_{\{i:N_{i,1}=x\}} E(N_{i,2}|N_{i,1} = x) \\ &= \sum_{\{i:N_{i,1}=x\}} \theta_2 E(\lambda_i|N_{i,1} = x) \\ &= \begin{cases} M_x\theta_2\frac{\alpha+x}{\beta+\theta_1} & \text{Nonstationary Negative Binomial} \\ M_x(x+1)\frac{\theta_2 f(x+1)}{\theta_1 f(x)} & \text{Nonstationary Robbins' Model} \end{cases} \end{aligned}$$

The conditional expectations follow from equation (2) or Theorem 3 for the NBD model and from equation (3) for Robbins model.

Usually, ψ is unknown. If so, suppose that it is modeled and estimated over a calibration period which precedes the base period of the CTA. Next, θ's, which are integrals of ψ, are computed for the base and forecast periods. For a brand, this computation would be done using planned marketing activity for the forecast period. This leaves estimating the parameters α and β for the negative binomial distribution and $f(x)$ for Robbins' model.

For both models, there is a simple relationship between the stationary and nonstationary forecasts. The forecasts for the nonstationary model adjust the forecasts from the stationary model by the nonstationarity in the test period relative to that

of the base period. This factor is the ratio of the average purchase rate in the forecast period divided by that in the base period. Consequently, the forecasts of the nonstationary model is easily obtained from those of the stationary model.

Theorem 4 *Let $\hat{\alpha}_N$ and $\hat{\beta}_N$ be the maximum likelihood estimators for the nonstationary negative binomial model, and let $\hat{\alpha}_S$ and $\hat{\beta}_S$ be the maximum likelihood estimators for the stationary negative binomial model. The two sets of estimators are related by*

$$\hat{\alpha}_N = \hat{\alpha}_S \text{ and } \hat{\beta}_N = \left(\frac{\theta_1}{\Delta_1}\right) \hat{\beta}_S.$$

The forecast of T_x for the nonstationary negative binomial model is

$$NNB_x = \left(\frac{\theta_2/\Delta_2}{\theta_1/\Delta_1}\right) SNB_x$$

where

$$SNB_x = M_x \Delta_2 \left(\frac{\hat{\alpha}_S + x}{\hat{\beta}_S + \Delta_1}\right)$$

is the forecast of T_x for the stationary negative binomial model.

A similar result holds for the Robbins' model.

Theorem 5 *The forecast of T_x for the nonstationary Robbins' model is*

$$NRM_x = \left(\frac{\theta_2/\Delta_2}{\theta_1/\Delta_1}\right) SRM_x.$$

where

$$SRM_x = \begin{cases} (x+1)M_{x+1}\Delta_2/\Delta_1 & \text{if } M_x > 0 \\ 0 & \text{if } M_x = 0 \end{cases}$$

is the forecast of T_x for the stationary Robbins' model.

Forecasts for the nonstationary models simply multiply the forecasts for the stationary models by the ratio of the average rates in the test to the base periods. This procedure is the *ex ante* version of the *ex post* adjustment of the stationary forecasts by the ratio of total purchases in the test to the base periods.

5 Scanner Panel Data

This section analyzes scanner panel data[3] with four models: stationary and nonstationary NBD and Robbins model. An outline of the data analysis follows. First, we develop predictor variables and identify potential models for the intensity function by using standard data analytic techniques. Next, we estimate the nonstationary negative multinomial model by maximum likelihood with the first two years (104 weeks) of data and forecast the total number of weekly purchase occasions by the panel for the last 34 weeks. Then, using the estimated intensity function from the calibration period, we perform "rolling" CTA, week-by-week, on the remaining 34 weeks. The base period for the first CTA is week 105 with test period of week 106, and the last CTA has base period of week 137 and test period of week 138, resulting in 33 CTAs. The first subsection describes fitting the nonstationary negative multinomial model, and the second reports the CTA.

5.1 Nonstationary Negative Multinomial Model

We consider five aggregate marketing variables as potential predictor variables. These variables are derived from marketing activity at the retail level (File 9, Retail Tracking of the MSI Library provided by A.C. Nielsen). When a product is purchased, the Retail Tracking file records the marketing activity of the store where the product was purchased. Thus, for a given week and store we have information on whether a given universal product code (UPC) was displayed in the store, was featured in a store advertisement, had an associated coupon, had a point-of-purchase display, or had a special price code. A promotion by a leading brand in a large store would have a greater impact on purchase occasions than a similar promotion by a smaller brand or store. In order to account for this differential impact of promotions, the marketing

[3]Supplied by A. C. Nielsen, and described by the footnote in the Introduction.

activities were weighted by the share of the UPC/store combination.

The predictor variables enter the model through the intensity function of the nonstationary Poisson process. The form of the intensity function is

$$\lambda\psi(k) = \lambda \exp(X_k'\phi)$$

where X_k is a vector of independent variables for week k, and ϕ is the vector of unknown regression parameters. The measurement of time is weekly, and ψ is constant over one week. Hence,

$$\theta_k = \int_{k-1}^{k} \psi(t)dt = \exp(X_k'\phi).$$

Model specification is facilitated by an observation by Hausman, Hall and Griliches (1984). Let N_k be the number of purchases in week k, and define

$$Y_k = \log(N_k).$$

They note that the maximum likelihood estimator of ϕ is close to its ordinary least square estimator based on the log-linear model

$$Y_k = \phi_0 + X_k'\phi + \epsilon. \tag{6}$$

Our experience confirms their observation.

Correlation and regression analysis indicates that only advertising is useful in predicting the number of weekly purchase occasions. Table I summarizes the OLS estimated coefficients for models using advertising and/or price as independent variables. The coefficients for advertising and price should be positive, because they are weighted sums of zero/one variables which indicate the absence or presence of the marketing activity. In a simple linear regression model for powder detergent the special price variable was not significantly different from zero, while advertising was. Multicollinearity between advertising and price is indicated by their large correlation (0.659) and the negative coefficient for price when both variables are used. Consequently, price was eliminated from the model. For Tide, both advertising and price

are significantly different from zero in simple linear regression models, but price is not significantly different from zero given that advertising is in the model. The correlation between price and advertising for Tide is 0.589. These large correlations between advertising and special price may be due to the fact that when stores offer a special price, they frequently advertise it.

A residual analysis indicates systematic large positive and negative residuals which seem to indicate the presence of holiday and seasonal effects. For example, purchases of detergents tend to be unusually low during the week of Valentines Day with surges before and after the holiday. This pattern also holds for holidays such as Thanksgiving and the Christmas – Hanukkah – New Year season. Also, there seems to be an unusually high number of purchases during the week of the Super Bowl. With only two years of data for estimation, it is not possible to estimate separate effects for each holiday and season. Instead, we define two dummy variables which indicate negative and positive seasonal – holiday effects for the two years of data in the fit period. Although we view these dummy variables to be less satisfactory than separate variables for different seasonal and holiday effects, their use demonstrates incorporating seasonal effects into the model.

We use the negative multinomial distribution (12) of Appendix C to estimate the parameters from the first 104 weeks of data. We replace the shape parameter, α, in equation (12) by

$$\alpha = 2620 \exp(\phi_0)$$

where 2620 is the number of households in the panel. The expected number of purchases by the entire panel for a week in the fit period is

$$E(N_k) = 2620 \exp(\phi_0 + X_{k,1}\phi_1 + X_{k,2}\phi_2 + X_{k,3}\phi_3)/\beta,$$

where

$$X_{k,1} \quad = \quad \text{aggregate advertising for week } k$$

$$X_{k,2} = \begin{cases} 1 & \text{if negative seasonal or holiday effect for week } k \\ 0 & \text{otherwise} \end{cases}$$

$$X_{k,3} = \begin{cases} 1 & \text{if positive seasonal or holiday effect for week } k \\ 0 & \text{otherwise} \end{cases}$$

The first part of Table II reports the maximum likelihood estimators and their asymptotic standard errors for the two series. The effect of advertising, ϕ_1, is greater for Tide than for powder detergents. Powder detergents includes various brands, such as Tide and Cheer, which are almost never advertised simultaneously by a particular store. Rather, stores tend to rotate the brands that they advertise. Not infrequently, different stores advertise different brands in any week so that aggregate advertising at the category level exhibits proportionally less variation than advertising at the brand level. To obtain a quantitative measure, suppose that the weekly advertising aggregated across stores for b brands are independent with common mean μ and standard deviation σ. Then the coefficient of variation for the weekly, aggregated advertising is $\sigma/(\sqrt{b}\mu)$ which is less than the coefficient of variation, σ/μ, for any individual brand. Thus, the systematic variation in purchases due to advertising is partially masked at the category level. In fact, the coefficient of variation of weekly advertising is 0.84 for powder detergents compared to 1.53 for Tide.

The coefficients for negative and positive seasonal effects have practical significance, despite their relatively small magnitude. For example, fix the level of advertising and compare a week with positive seasonal effects to a week without seasonal effects for Tide. Define N^+ to be the number of purchases in the week with positive seasonal effects and N^o to be the number of purchases in the week without seasonal effects. The expected weekly sales for positive seasonal effects is 49% greater than that without seasonal effects:

$$\frac{E(N^+) - E(N^o)}{E(N^o)} = \exp(\phi_3) - 1 = 0.49$$

The estimated household shape parameters, $\alpha = \exp(\phi_0)$, for the two series are fairly close. However, the estimated scale parameters, β, differ considerably. In the absence of advertising and seasonal effects, that is, $\psi(k) = 1$, the expected number of weeks between purchase occasions for household i is

$$E(\lambda_i^{-1}) = \frac{\beta}{\exp(\phi_0) - 1}.$$

Thus, the expected time between purchases for a randomly selected household is seven weeks for powder laundry detergents and 41 weeks for Tide. As anticipated, time between purchases is shorter at the product category level than at the brand level. The expected time between purchase occasions for Tide is large because many households infrequently or never buy Tide during the observational period.

The second part of Table II reports the fit and prediction statistics — root mean squared errors (RMSE) and mean absolute percent errors (MAPE) - for the stationary and nonstationary negative multinomial models. The nonstationary model substantially improves on the stationary model. The predictor variables account for 34.6% of the total variation in the purchase occasions for powder detergents and 57.8% for Tide. Purchase forecasts are given by Equation (14) of Appendix C where α is replaced by $2620\exp(\phi_0)$. The nonstationary model improves the forecast MAPE by 19.1% for powder detergent and 50.5% for Tide.

5.2 Conditional Trend Analysis

A goal of CTA is to predict the total number of purchases, T_x, in the test period for the M_x households that purchased x items in the base period. We perform "rolling" CTAs from week 105 to week 138 for the stationary and nonstationary NBD and Robbins models. The nonstationary models use the estimated values of ϕ_1, ϕ_2, and ϕ_3 from the calibration period (weeks 1–104). One difficulty of the negative binomial models is that the zero purchase class includes households that will at some point purchase

the product and households that may never purchase the product. To compensate for the resulting "spike at zero," we use the model of Morrison (1969). The forecasts are computed by equation (15) of Appendix D.

Table III reports the MAPE for CTAs where the base and test periods are either one or four weeks. Purchase classes are reported in the Table III only if M_x is greater than zero for every CTA. The MAPEs are computed across the CTAs for each purchase class. As is usually the case, we estimate the shape and scale parameters from the base period. The nonstationary forecasts are obtained from the stationary forecasts using Theorems 4 and 5. With one week base and test periods, the CTAs have large MAPEs when x is greater than zero because T_x is small, so small absolute forecast errors have large MAPEs. The main message of Table III is that the nonstationary models generally perform better than the stationary models, and the MAPEs tend to be smaller for four week periods than for one week periods. However, the magnitude of improvement is modest. On the other hand, once ψ is estimated, nonstationary forecasts are obtained from the stationary ones without much additional effort by using Theorems 4 and 5.

A different perspective is obtained in Figure 3, which combines MAPEs across purchase class x within each CTA with one week periods. For each CTA we compute the ratio, θ_2/θ_1, of the average intensity in the test to base periods. This ratio is a measure of nonstationarity of the test period relative to the base period and is the adjustment factor of the stationary forecasts to obtain the nonstationary ones. The percent improvement, Y, of the nonstationary to stationary forecasts is:

$$Y = \frac{\text{MAPE Stationary} - \text{MAPE Nonstationary}}{\text{MAPE Stationary}} \times 100.$$

When θ_2/θ_1 is one, the stationary and nonstationary forecasts are identical, so Y is zero. Weeks with negative values of Y resulted when sales headed in the wrong direction: θ_2/θ_1 is greater (less) than one, while N_2/N_1 is less (greater) than one. This fact indicates that additional predictor variables may improve the model. Figure

3 graphs the percent improvement, Y, versus the ratio, θ_2/θ_1, for the stationary and nonstationary NBD and Robbins models. The graphs for Tide best illustrates the effects of nonstationarity. As the ratio, θ_2/θ_1, is farther from one, the percent improvement increases considerably. Powder detergents have a similar pattern as Tide, except there is less variability in the ratio θ_2/θ_1, due to the coefficients of variation for advertising as discussed earlier.

6 Under–Prediction

Morrison and Schmittlein (1981 and 1988) note that CTA tends to under–predict T_0, the total purchase in the test period by those households that make no purchases in the test period. They hypothesize that nonstationarity is a possible cause. In this section we show that although the forecasts from the nonstationary models are more accurate, under–prediction is a persistent phenomenon due to the skewness of the NBD. First, we examine the empirical evidence from our study. Next, we intuitively explain the cause of the under–prediction. Finally, we show that under–prediction is expected for the NBD model.

Table IV summarizes predicting the zero class, T_0, for the two series when the base and test periods are one week. The forecast residuals, which are defined as the actual number of purchase occasions minus the predicted number, are stratified according to their sign. A negative residual indicates over–prediction, while a positive residual indicates under–prediction. Table IV reports the mean of the negative and positive residuals for each model and the improvement of the nonstationary model over the corresponding stationary model. The nonstationary models provide more accurate forecasts than the corresponding stationary models. For powder detergents the NBD nonstationary model improves the mean negative residuals by 77% and the mean positive residuals by 23%. Similarly, for Tide the nonstationary NBD improves the mean negative residuals by 52% and the mean positive residuals by 50%. On

the other hand, all of the models tend to under-predict because the mean positive residuals are greater than the absolute value of the mean negative residuals. Thus, the nonstationary model improves prediction accuracy, but under-prediction is a persistent phenomenon.

One possible explanation of the under-prediction for the NBD is the choice of measurement scale. In Section 5.1 we used the heuristic that the least squares estimates when fitting the logarithm of the number of purchase occasions are approximately the same as the maximum likelihood estimates from the NBD. If the log-linear model is correctly specified on the logarithmic scale, then the forecasts will tend to under-predict on the original scale. To see this, define the forecast residuals on the logarithmic scale as

$$r_k = \log(N_k) - \log(\hat{N}_k) \text{ for } J+1 \leq k \leq K.$$

If the linear model for $\log(N_k)$ is correctly specified, then the mean forecast errors should be zero:

$$\bar{r} = (K-J)^{-1} \sum_{k=J+1}^{K} r_k = 0.$$

Then by Jensen's inequality we have

$$\begin{aligned} 1 &= \exp(\bar{r}) \\ &\leq (K-J)^{-1} \sum_{k=J+1}^{K} \exp(r_k) \\ &\leq (K-J)^{-1} \sum_{k=J+1}^{K} N_k/\hat{N}_k \end{aligned}$$

Consequently, we should expect the forecasts $\{\hat{N}_k\}$ to under-predict $\{N_k\}$ on the original scale of the data even though the forecasts on the logarithmic scale are not biased. Thus, under-prediction does not necessarily invalidate NBD in the same manner that it indicates an incorrectly specified linear model.

In CTA, N_k is replaced by T_0 which also has a negative binomial distribution by Theorem 3. Thus, the above reasoning applies.

A more detailed account of under-prediction follows. The mean μ of T is its forecast of purchases in the test period. In Table IV the mean, negative residuals estimates the expected over-prediction, $E(T-\mu|T \leq \mu)$, and the mean, positive residuals estimates the expected under-prediction, $E(T-\mu|T > \mu)$. The next theorem describes the relationship between these two quantities for varying types of skewness.

Theorem 6 *Let T be a random variable with support on the non-negative integers and distribution F. Suppose that the mean, μ, of T is finite, and define $\lfloor\mu\rfloor$ to the largest integer less than or equal to μ. Then*

$$E(T-\mu|T > \mu) = \frac{F(\lfloor\mu\rfloor)}{1-F(\lfloor\mu\rfloor)}\{-E(T-\mu|T \leq \mu)\}$$

In particular,

$$E(T-\mu|T > \mu)\begin{cases} > & -E(T-\mu|T \leq \mu) \text{ if } F(\lfloor\mu\rfloor) > 0.5 \\ = & -E(T-\mu|T \leq \mu) \text{ if } F(\lfloor\mu\rfloor) = 0.5 \\ < & -E(T-\mu|T \leq \mu) \text{ if } F(\lfloor\mu\rfloor) < 0.5. \end{cases}$$

The distribution of T is positively (negatively) skewed if $F(\lfloor\mu\rfloor)$ is greater than (less than) 0.5. In other words, one would expect under-prediction if the distribution of T is skewed to the right and over-prediction if the distribution of T is skewed to the left. If the distribution of T is symmetric about $\lfloor\mu\rfloor$, the expected amount of under- and over-prediction are equal. Because the third central moment is positive for NBD, T_x is skewed to the right. Thus, under-prediction is expected.

A final caveat about biased forecasts. The preceding analysis is conditional on the sign of the forecast error and knowing the mean, μ. Unconditionally, μ is an unbiased predictor of T if the model is correctly specified. Empirically, the average forecast residuals may not sum to zero because the estimate of μ may be biased. Maximum likelihood estimators of the parameters are consistent but need not result in unbiased estimates of the mean.

7 Discussion

This paper proposes a simple modification of the standard NBD model by considering a specific type of nonstationarity, the **proportional intensity** model. The nonstationary model retains the simple and intuitive structure of the NBD model, while allowing the introduction of prediction variables. Our findings are:

- Modeling nonstationarity in conditional trend analysis quantifies the effects of independent variables and improves forecasts.
- If stationarity is falsely ignored, then the forecasts will under-estimate if the average intensity in the base period is less than the average intensity in the forecast period, and conversely.
- Mild nonstationarity "averages-out" over long base and forecast periods.
- While explicitly accounting for nonstationarity results in more accurate forecasts, under–prediction is a persistent phenomenon.

A Nonstationary Poisson Processes

Suppose that, in reality, Z is a nonstationary Poisson process with intensity $\lambda\psi(t)$, and that we incorrectly assume that the process is stationary with intensity $\lambda\xi$. Z^* is our incorrect choice for the stationary process. A natural question is to find ξ such that our process Z^* is as "close" as possible to the true process Z on disjoint interval $(s_k, t_k]$ for $k = 1$ to K. Intuitively, the optimal choice of ξ is the time average of ψ over the intervals. This choice is optimal with respect to minimizing the Kullback–Leiber information (Bickel and Doksum 1977 p. 226) which is the expected log-likelihood ratio. The Kullback–Leiber information is a measure of the "probabilistic closeness" of two distributions: small measures indicate similar distributions.

Consider K disjoint time interval $(s_k, t_k]$ for $k = 1, \ldots, K$. For each interval, define

$$\begin{aligned} N_k &= Z(t_k) - Z(s_k) \\ \theta_k &= \int_{s_k}^{t_k} \psi(u)du \\ N_k^* &= Z^*(t_k) - Z^*(s_k) \\ \Delta_k &= t_k - s_k. \end{aligned}$$

Theorem 7 *The ξ that minimizes the Kullback-Leiber information*

$$KL(\xi) = E_{N_1,\ldots,N_K}\left[\log\left\{\frac{Pr(N_1,\ldots,N_K)}{Pr(N_1^*,\ldots,N_K^*)}\right\}\right].$$

is the time average of ψ over the intervals:

$$\xi^* = \frac{\sum_{k=1}^K \theta_k}{\sum_{k=1}^K \Delta_k}.$$

PROOF:

The N_k's are independent Poisson random variables, and the mean of N_k is $\lambda\theta_k$. Also, the N_k^*'s are independent Poisson random variables, and the mean of N_k^* is $\lambda\xi\Delta_k$. Consequently, their log-likelihood ratio is

$$\log\left\{\frac{Pr(N_1 = n_1,\ldots,N_K = n_K)}{Pr(N_1^* = n_1,\ldots,N_K^* = n_K)}\right\} = \lambda\left(\sum_{k=1}^K \xi\Delta_k - \theta_k\right) + \sum_{k=1}^K n_k \log\left(\frac{\theta_k}{\xi\Delta_k}\right).$$

The Kullback-Leiber information is

$$\begin{aligned} KL(\xi) &= \sum_{n_1=0}^{\infty}\cdots\sum_{n_K=0}^{\infty}\log\left\{\frac{Pr(N_1 = n_1,\ldots,N_K = n_K)}{Pr(N_1^* = n_1,\ldots,N_K^* = n_K)}\right\} \\ &\quad \times Pr(N_1 = n_1,\ldots,N_K = n_k) \\ &= \lambda\left(\sum_{k=1}^K \xi\Delta_k - \theta_k\right) + \sum_{k=1}^K \lambda\theta_k \log\left(\frac{\theta_k}{\xi\Delta_k}\right). \end{aligned}$$

Setting the first derivative of $KL(\xi)$ with respect to ξ equal to zero, and solving for ξ results in ξ^*. Because the second derivative is positive, ξ^* minimizes $KL(\xi)$.

B Proofs

B.1 Theorem 1

After observing $N_1 = n_1$, the posterior distribution of λ is

$$dG(\lambda|n_1) = \exp(-\lambda\theta_1)(\lambda\theta_1)^{n_1}(n_1!)^{-1}dG(\lambda)/f(n_1).$$

The conditional mean is

$$\begin{aligned} E(N_2|N_1 = n_1) &= E_{\lambda|N_1}[E(N_2|\lambda)] \\ &= E_{\lambda|N_1}(\lambda\theta_2) \\ &= \theta_2 \int_0^\infty \lambda dG(\lambda|n_1) \\ &= (n_1+1)\,\frac{\theta_2}{\theta_1}\,\frac{f(n_1+1)}{f(n_1)}. \end{aligned}$$

The last line follows by

$$\begin{aligned} &(n_1!)^{-1}\int_0^\infty \lambda\exp(-\lambda\theta_1)(\lambda\theta_1)^{n_1}dG(\lambda) = \\ &= \frac{n_1+1}{\theta_1}\,[(n_1+1)!]^{-1}\int_0^\infty \exp(-\lambda\theta_1)(\lambda\theta_1)^{n_1+1}dG(\lambda) \\ &= \{(n_1+1)/\theta_1\}\,f(n_1+1). \end{aligned}$$

B.2 Theorem 2

ξ^* can be expressed as a convex combination of θ_1/Δ_1 and θ_2/Δ_2, i.e.

$$\xi^* = w\,\theta_1/\Delta_1 + (1-w)\,\theta_2/\Delta_2$$

where $w = \Delta_1/(\Delta_1+\Delta_2)$. Therefore, the inequality $\theta_1/\Delta_1 < \theta_1/\Delta_2$ implies the further inequalities

$$\theta_1/\Delta_1 < \xi^* < \theta_2/\Delta_2.$$

Although the negative binomial model, which assumes that λ has a Gamma distribution, is a special case of Robbins' model, which does not specify λ's distribution, we will prove the result for the two models separately.

B.2.1 Negative Binomial Model

The result for the negative binomial model follows from simple algebra. The forecasts are

$$\begin{aligned} E(N_2|N_1 = n) &= \theta_2(\alpha + n)/(\beta + \theta_1) \\ E(N_2^*|N_1^* = n) &= \xi^* \Delta_2(\alpha + n)/(\beta + \xi^* \Delta_1). \end{aligned}$$

The bias is

$$\begin{aligned} &E(N_2|N_1 = n) - E(N_2^*|N_1^* = n) = \\ &\quad (\alpha + n)(\beta + \theta_1 + \theta_2)\left(\frac{\Delta_1 \Delta_2}{\Delta_1 + \Delta_2}\right)\left(\frac{\theta_2}{\Delta_2} - \frac{\theta_1}{\Delta_1}\right). \end{aligned}$$

Consequently,

$$E(N_2|N_1 = n) > E(N_2^*|N_1^* = n) \text{ if and only if } \theta_2/\Delta_2 > \theta_1/\Delta_1.$$

B.2.2 Robbins' Model

Let $G(\lambda)$ be the distribution of λ. The conditional expectations are

$$\begin{aligned} E(N_2|N_1 = n) &= \theta_2 E(\lambda|\theta_1, n) \\ E(N_2^*|N_1^* = n) &= \Delta_2 \xi^* E(\lambda|\Delta_1 \xi^*, n). \end{aligned}$$

Since $\theta_2 > \Delta_2 \xi^*$, the proof will follow if we can show that for fixed n, $E(\lambda|\theta, n)$ is a decreasing function of θ, which implies that

$$E(\lambda|\theta_1, n) > E(\lambda|\Delta_1 \xi^*, n)$$

because $\theta_1 < \Delta_1 \xi^*$ by hypothesis.

To proceed,

$$\begin{aligned} E(\lambda|\theta, n) &= H(\theta, n+1)/H(\theta, n) \text{ where} \\ H(\theta, n) &= \int_0^\infty \exp(-\lambda\theta)\lambda^n dG(\lambda). \end{aligned}$$

Since $H(\theta, n)$ is the Laplace transform for $\lambda^n dG(\lambda)$, it is finite, and we are able to interchange integrals and derivatives with respect to θ. We will show that the derivative of $\log\{E(\lambda|\theta, n)\}$ is negative, so that it is a decreasing function of θ.

$$\begin{aligned}
\frac{d}{d\theta}\log\{E(\lambda|\theta,n)\} &= \frac{d}{d\theta}\log\{H(\theta,n+1)\} - \frac{d}{d\theta}\log\{H(\theta,n)\} \\
&= -\frac{H(\theta,n+2)}{H(\theta,n+1)} + \frac{H(\theta,n+1)}{H(\theta,n)} \\
&< 0
\end{aligned}$$

if and only if

$$\{H(\theta,n+1)\}^2 < H(\theta,n+2)\,H(\theta,n).$$

The last line is a consequence of the Cauchy–Schwartz inequality and the following identifications:

$$\begin{aligned}
f(\lambda) &= \lambda; \quad g(\lambda) = 1; \text{ and} \\
d\mu(\lambda) &= \exp(-\lambda\theta)\lambda^n dG(\lambda).
\end{aligned}$$

Then

$$\begin{aligned}
H(\theta,n+1) &= \int_0^\infty f(\lambda)g(\lambda)d\mu(\lambda) \\
H(\theta,n+2) &= \int_0^\infty f(\lambda)^2\,d\mu(\lambda) \\
H(\theta,n) &= \int_0^\infty g(\lambda)^2\,d\mu(\lambda).
\end{aligned}$$

The Cauchy–Schwarz inequality is

$$\left\{\int_0^\infty f(\lambda)g(\lambda)d\mu(\lambda)\right\}^2 \le \int_0^\infty f(\lambda)^2\,d\mu(\lambda)\int_0^\infty g(\lambda)^2\,d\mu(\lambda)$$

with strict equality if and only if f and g are linearly related. Consequently, we have a strict inequality in our application. The proof can be reversed so that the "if and only if" statement of the result is true.

B.3 Theorem 3

T_x given $\{\lambda_i\}$ is a sum of independent Poisson random variables. Thus, T_x is a Poisson random variable with rate

$$\lambda(x)\theta_2 = \theta_2 \sum_{\{i:N_{i,1}=x\}} \lambda_i,$$

and $\{T_x\}$ are mutually independent. Since $\{\lambda_i\}$ is a random sample from a gamma distribution, $\{\lambda(x)\}$ are mutually independent, and $\lambda(x)$ has a gamma distribution with shape αM_x and scale β. After observing purchases in the base period, $\{\lambda(x)\}$ given $\{M_x\}$ are mutually independent gamma random variables where $\lambda(x)$ has shape parameter $\alpha M_x + xM_x$ and scale parameter $\beta + \theta_1$. Thus, $\{T_x\}$ are mutually independent negative binomial random variables with probability mass functions given by Equation 5.

B.4 Theorem 4

After observing that M_x households had x purchase occasions in the base period, the likelihood for α and β under nonstationarity is

$$\prod_{x=0}^{\infty} \left[\frac{\Gamma(\alpha + x)}{\Gamma(\alpha)\, x!} \left(\frac{\beta}{\beta + \theta_1} \right)^{\alpha} \left(\frac{\theta_1}{\beta + \theta_1} \right)^{x} \right]^{M_x}, \tag{7}$$

while under stationarity it is

$$\prod_{x=0}^{\infty} \left[\frac{\Gamma(\alpha + x)}{\Gamma(\alpha)\, x!} \left(\frac{\beta}{\beta + \Delta_1} \right)^{\alpha} \left(\frac{\Delta_1}{\beta + \Delta_1} \right)^{x} \right]^{M_x}.$$

Comparing the two likelihoods, we see they have the same functional form for α and that replacing β by $\beta\theta_1/\Delta_1$ in the nonstationary likelihood results in the stationary likelihood. The first result follows since the maximum likelihood estimator of a function of a parameter is the function of maximum likelihood estimator of the parameter.

The estimated forecasts are found by substituting the unknown parameters by their maximum likelihood estimators:

$$\begin{aligned} NNB_x &= M_x\theta_2 \left(\frac{\hat{\alpha}_N + x}{\hat{\beta}_N + \theta_1}\right) \\ &= \left(\frac{\theta_2/\Delta_2}{\theta_1/\Delta_1}\right) M_x\Delta_2 \left(\frac{\hat{\alpha}_S + x}{\hat{\beta}_S + \Delta_1}\right) \\ &= \left(\frac{\theta_2/\Delta_2}{\theta_1/\Delta_1}\right) SNB_x. \end{aligned}$$

B.5 Theorem 5

In Robbins' model $f(x)$ is unknown and is estimated by M_x/M, which is consistent. Substituting this estimate of $f(x)$ into the forecasts results in

$$\begin{aligned} NRM_x &= \left(\frac{\theta_2}{\theta_1}\right) (x+1)\, M_{x+1} \\ &= \left(\frac{\theta_2/\Delta_2}{\theta_1/\Delta_1}\right) SRM_x. \end{aligned}$$

B.6 Theorem 6

Define $[\mu]$ to be the greatest integer less than or equal to μ, and

$$\begin{aligned} p(k) &= P(T = k) \text{ for } k = 0, 1, \ldots \\ F(j) &= \sum_{k=0}^{j} p(k). \end{aligned}$$

We will use two identities which results from rearranging terms:

$$\sum_{k=1}^{[\mu]} kp(k) = [\mu]F([\mu]) - \sum_{j=1}^{[\mu]} F(j-1) \tag{8}$$

$$\sum_{k=[\mu]+1}^{\infty} kp(k) = [\mu]\{1 - F([\mu])\} + \sum_{j=[\mu]+1}^{\infty} \{1 - F(j-1)\}. \tag{9}$$

Applying identities (8) and (9), we obtain

$$\mu = \sum_{k=1}^{[\mu]} kp(k) + \sum_{k=[\mu]+1}^{\infty} kp(k)$$

$$= \lfloor\mu\rfloor - \sum_{j=1}^{\lfloor\mu\rfloor} F(j-1) + \sum_{j=\lfloor\mu\rfloor+1}^{\infty} \{1 - F(j-1)\}.$$

Rearranging terms results in the identity:

$$\sum_{j=\lfloor\mu\rfloor+1}^{\infty} \{1 - F(j-1)\} = \mu - \lfloor\mu\rfloor + \sum_{j=1}^{\lfloor\mu\rfloor} F(j-1). \tag{10}$$

From indentity (8) we can express the expected amount of over-prediction as

$$\begin{aligned} E(T-\mu|T \le \mu) &= \sum_{k=1}^{\lfloor\mu\rfloor} kp(k)/F(\lfloor\mu\rfloor) - \mu \\ &= -\sum_{j=1}^{\lfloor\mu\rfloor} F(j-1)/F(\lfloor\mu\rfloor) - (\mu - \lfloor\mu\rfloor). \end{aligned}$$

From identities (8), (9), and (10) the relation between the expected amount of under- and over-prediction is

$$\begin{aligned} E(T-\mu|T > \mu) &= \{1 - F(\lfloor\mu\rfloor)\}^{-1} \left(\sum_{k=\lfloor\mu\rfloor+1}^{\infty} kp(k) \right) - \mu \\ &= \frac{F(\lfloor\mu\rfloor)}{1 - F(\lfloor\mu\rfloor)} \{-E(T-\mu|T \le \mu)\}. \end{aligned}$$

C Negative Multinomial Distribution

In this Appendix, we define the negative multinomial distribution, derive its conditional distribution and expectations which are used in forecasting, and compute the normal equations for maximum likelihood estimation.

The negative multinomial distribution is the multivariate generalization of the negative binomial distribution, much as the multinomial distribution is the multivariate generalization of the binomial distribution. The negative multinomial distribution is the joint probability of the number of events in K, disjoint intervals.

The computations for the negative multinomial distribution are identical to those for the negative binomial distribution; however, the notation becomes somewhat cumbersome. The negative multinomial distribution is derived from the number of events

from the Poisson process over K disjoint time intervals, $(s_k, t_k]$. Define N_k to be the number of events in $(s_k, t_k]$ and $\theta_k = \int_{s_k}^{t_k} \psi(u)du$. Also, we will use the following notation

$$\Theta(I,J) = \sum_{k=I}^{J} \theta_k \text{ and } N(I,J) = \sum_{k=I}^{J} N_k \text{ for } 1 \leq I < J \leq K.$$

Given λ, each N_k has a Poisson distribution with rate $\lambda\theta_k$, and they are mutually independent. Their joint distribution is

$$Pr(N_1, \ldots, N_K|\lambda) = \exp\{-\lambda\Theta(1,K)\}\lambda^{N(1,K)} \prod_{k=1}^{K} \frac{\theta_k^{N_k}}{N_k!}.$$

Mixing λ by a gamma distribution with shape parameter α and scale parameter β ($E(\lambda) = \alpha/\beta$), results in the negative multinomial distribution:

$$Pr(N_1, \ldots, N_K) = \frac{\Gamma\{\alpha + N(1,K)\}}{\Gamma(\alpha)\prod_{k=1}^{K} N_k!} \left\{ \frac{\beta}{\beta + \Theta(1,K)} \right\}^{\alpha} \prod_{k=1}^{K} \left\{ \frac{\theta_k}{\beta + \Theta(1,K)} \right\}^{N_k}. \quad (11)$$

The mean, variance and covariances are

$$\begin{aligned} E(N_k) &= \alpha\beta^{-1}\,\theta_k \\ Var(N_k) &= E(N_k)\left\{1 + \alpha^{-1}\,E(N_k)\right\} \\ Cov(N_j, N_k) &= \alpha^{-1}\,E(N_j)\,E(N_k). \end{aligned}$$

The marginal and conditional probability distributions are easily derived from Equation (11). For any $J < K$ the marginal distribution of $(N_1, \ldots, N_J)$ has a negative multinomial distribution:

$$Pr(N_1, \ldots, N_J) = \frac{\Gamma\{\alpha + N(1,J)\}}{\Gamma(\alpha)\prod_{j=1}^{J} N_j!} \left\{ \frac{\beta}{\beta + \Theta(1,J)} \right\}^{\alpha} \prod_{j=1}^{J} \left\{ \frac{\theta_j}{\beta + \Theta(1,J)} \right\}^{N_j}. \quad (12)$$

In the data analysis of Section 5.1, K is the 138 weeks, and J is 104 weeks of the calibration period. The joint distribution of $N_{J+1}, \ldots, N_K$ is

$$Pr(N_{J+1}, \ldots, N_k) = \quad (13)$$
$$\frac{\Gamma\{\alpha + N(J+1,K)\}}{\Gamma(\alpha)\prod_{j=J+1}^{K} N_j!} \left\{ \frac{\beta}{\beta + \Theta(J+1,K)} \right\}^{\alpha} \prod_{j=J+1}^{K} \left\{ \frac{\theta_j}{\beta + \Theta(J+1,K)} \right\}^{N_j}$$

Given the number of event in the first J periods, λ has a gamma distribution with updated parameters: α is updated to $\alpha + N(1, J)$, and β is updated to $\beta + \Theta(1, J)$. Thus conditional distribution of $(N_{J+1}, \ldots, N_K)$ given $(N_1, \ldots, N_J)$ is obtained by substituting the updated values of α and β into Equation (13). The conditional expectation of N_{J+k} given $(N_1, \ldots, N_J)$ is:

$$E(N_{J+k}|N_1, \ldots, N_J) = \theta_{J+k} \frac{\alpha + N(1, J)}{\beta + \Theta(1, J)}. \tag{14}$$

The normal equations and Hessian for estimating the parameters are computed in the usual manner except that the maximum likelihood estimators obey a functional constraint. To see this, compute the normal equations. Suppose that the number of events, $N_1, \ldots, N_J$, in the first J time periods have been observed, and that ψ is a function of the parameters $\phi_1, \ldots, \phi_I$. The log-likelihood of α, β, $\phi_1, \ldots, \phi_I$ is defined by equation (12):

$$L = \log[Pr(N_1, \ldots, N_J)].$$

Set

$$N \equiv N(1, J) \text{ and } \theta \equiv \theta(1, J),$$

and define

$$\Psi(x) = \frac{d}{dx} \log[\Gamma(x)].$$

The normal equations are:

$$\begin{aligned}
\frac{\partial L}{\partial \alpha} &= \Psi(\alpha + N) - \Psi(\alpha) + \log(\beta) - \log(\beta + \theta) \\
\frac{\partial L}{\partial \beta} &= \frac{\alpha}{\beta} - \frac{\alpha + N}{\beta + \theta} \\
\frac{\partial L}{\partial \phi_i} &= \sum_{k=1}^{J} \left(\frac{N_k}{\theta_k} - \frac{\alpha + N}{\beta + \theta} \right) \left(\frac{\partial \theta_k}{\partial \phi_i} \right).
\end{aligned}$$

By setting the partial derivative of L with respect to β to zero and solving for β, we find that the maximum likelihood estimator, $\hat{\beta}$, is a function of the maximum likelihood estimators of the other parameters:

$$\hat{\beta} = \hat{\alpha}\hat{\theta}N^{-1}.$$

Therefore, there is a restriction on the solution to the normal equations which reduces the dimension of the problem from $I+2$ to $I+1$ parameters. Hausman, Hall, and Griliches (1984 p. 916) derive the normal equations for the negative multinomial distribution by assuming that $\alpha = \beta$. This assumption is unnecessarily restrictive.

D Morrison's Fix for the Zero Class

Frequently, the NBD model performs poorly in estimating T_0 because the M_0 households that did not purchase in the base period consists of households that buy the product but did not in the base period and households that do not buy the product. Morrison (1969) suggested a modification of the NBD model to adjust for non-buyers. Let B be the set of buyers, and let $\bar{B}$ be the set of non-buyers. Define $p = P(B)$, and let $f(x)$ be the negative binomial probability of purchasing x items in the base period. If a household is in B, then the number, N_1, of purchases in the base period has a negative binomial distribution. For the zero class,

$$\begin{aligned} g(0) = P(N_1 = 0) &= P(N_1 = 0|B)P(B) + P(N_1 = 0)|\bar{B})P(\bar{B}) \\ &= f(0)p + 1 - p. \end{aligned}$$

For positive x,

$$\begin{aligned} g(x) = P(N_1 = x) &= P(N_1 = x|B)P(B) + P(N_1 = x)|\bar{B})P(\bar{B}) \\ &= f(x)p. \end{aligned}$$

Note that $g(\cdot)$ is a probability distribution.

The likelihood function (7) in Appendix B becomes

$$\prod_{x=0}^{\infty} g(x)^{M_x}.$$

If $f(0)$ is known and is less than one, then the maximum likelihood estimate of p is

$$\hat{p} = \min\left(1, \frac{1 - M_0/M}{1 - f(0)}\right)$$

where M is the total number of households in the panel. The normal equations and the Hessian are easily computed with sufficient perseverance.

The forecasts of T_x are

$$E(T_x|x, M_x) = \begin{cases} pM_0\theta_2\frac{\alpha}{\beta+\theta_1} & \text{for } x = 0 \\ M_x\theta_2\frac{\alpha+x}{\beta+\theta_1} & \text{for } x > 0 \end{cases} \tag{15}$$

It is easily seen that the results of Theorem 4 hold for Morrison's fix if we redefine

$$SNB_x = \begin{cases} \hat{p}M_0\Delta_2\frac{\hat{\alpha}s}{\hat{\beta}s+\Delta_1} & \text{for } x = 0. \\ M_x\Delta_2\frac{\hat{\alpha}s+x}{\hat{\beta}s+\Delta_1} & \text{for } x \geq 1. \end{cases}$$

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FIGURE 1: Weekly Purchase Occasions for Powder Detergents and Tide. Solid horizontal line is the center line, and dashed lines are upper and lower limits for c-charts to detect nonconforming observations in Poisson data.

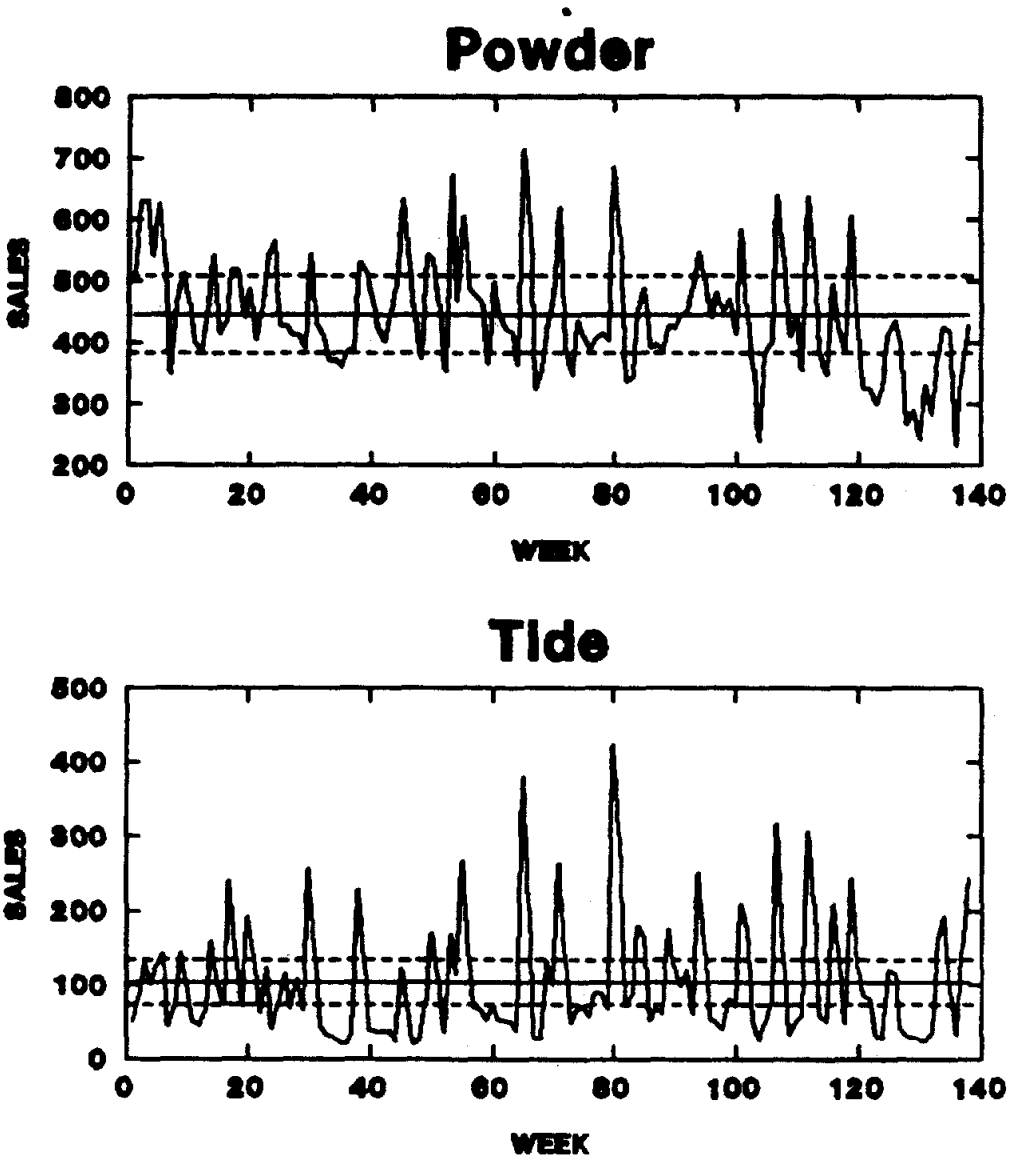

FIGURE 2: Weekly Aggregate Store Advertising for Powder Detergents and Tide. Solid horizontal line is the median. The median for Tide is zero.

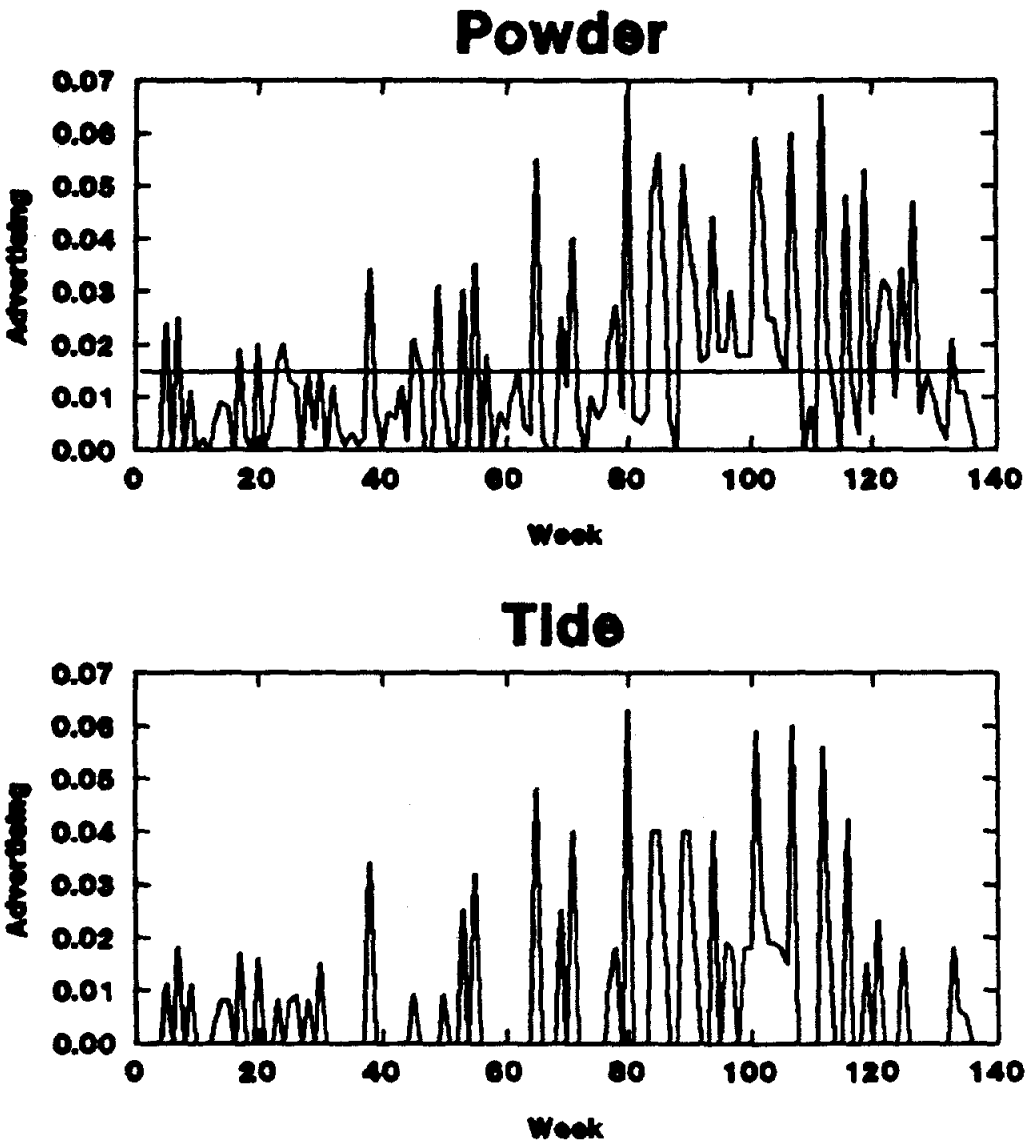

FIGURE 3: Effect of Nonstationarity on Conditional Trend Analysis. The horizontal axis is the ratio of the nonstationarity in the test period to that in the base period. The verticle axis is the improvement, as a percentage, of the mean absolute forecast errors by using the nonstationary model relative to the stationary model.

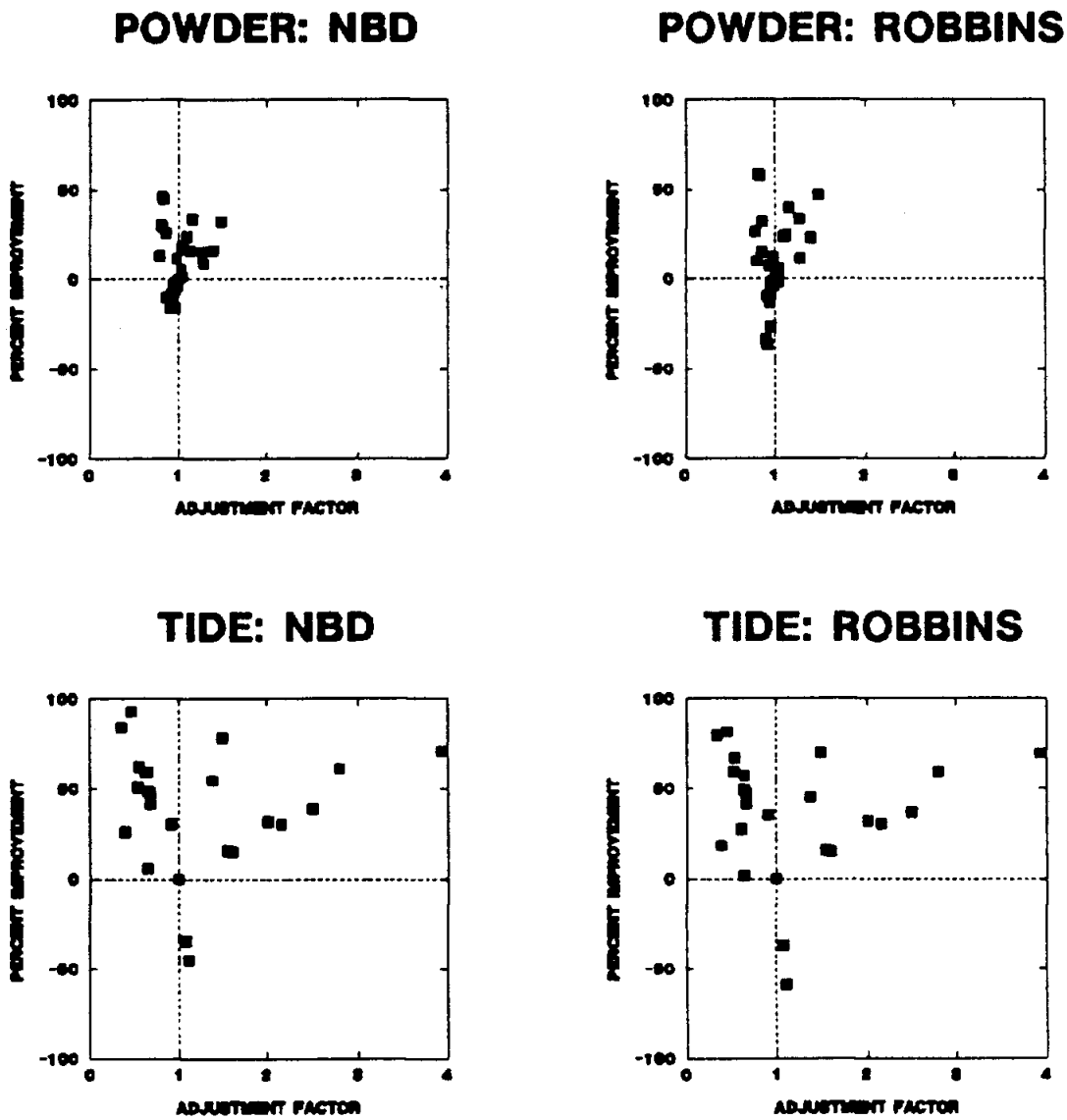

TABLE I: Log–linear Models of the Number of Weekly Purchase Occasions Estimated by Ordinary Least Squares. The dependent variable is the natural logarithm of the number of weekly purchase occasions. The independent variables are aggregate measures of weekly store advertising and weekly special prices, which is positive if one or more stores are offering a discount. Standard errors are in parenthesis below the estimated coefficients.

	CONSTANT	ADVERTISING	PRICE	R^2
POWDER DETERGENTS	6.01	4.72		0.14
	(0.02)	(1.03)		
	6.06		1.28	0.01
	(0.03)		(1.08)	
	6.03	6.82	-3.12	0.17
	(0.03)	(1.35)	(1.32)	
TIDE	4.15	28.50		0.34
	(0.06)	(3.44)		
	4.27		22.49	0.12
	(0.07)		(5.23)	
	4.15	28.23	0.52	0.34
	(0.06)	(4.27)	(5.64)	

TABLE II: Negative Multinomial Distribution fitted to Panel Scanner Data.

ESTIMATED PARAMETERS FROM WEEKS 1–104					
				Tide	
Variable	Parameter	MLE	Std.Err.	MLE	Std.Err.
log(Shape)	ϕ_0	2.50	0.108	2.35	0.186
Advertising	ϕ_1	5.01	0.109	24.48	0.196
Negative Seasonal	ϕ_2	-0.19	0.004	-0.48	0.021
Positive Seasonal	ϕ_3	0.17	0.012	0.40	0.006
Scale	β	79.58	0.562	387.38	4.362

FIT STATISTICS FOR WEEKS 1–104				
	Powder		Tide	
Statistic	STAT	NONS	STAT	NONS
RMSE	85.6	69.2	75.8	49.2
MAPE	14.6	11.8	88.3	54.5

PREDICTION STATISTICS FOR WEEKS 105–138				
	Powder		Tide	
Statistic	STAT	NONS	STAT	NONS
RMSE	117.9	95.6	83.4	46.2
MAPE	29.3	23.7	109.6	54.3

RMSE is root mean squared error.
MAPE is mean absolute percent error.
STAT indicates stationary model.
NONS indicates nonstationary model.

TABLE III: Conditional Trend Analysis for Powder Detergents and Tide.
Mean Absolute Percent Errors within Purchase Class and Across Periods.

	POWDER DETERGENTS ONE WEEK TIME PERIODS				TIDE ONE WEEK TIME PERIODS			
	NBD		Robbins		NBD		Robbins	
CLASS	STAT	NONS	STAT	NONS	STAT	NONS	STAT	NONS
0	21.6	18.6	21.0	16.8	75.6	38.4	73.2	38.6
1	56.1	52.0	37.9	32.2	75.3	65.7	76.9	67.8
2	73.9	68.3	101.5	92.2	103.5	102.4	151.9	146.1
3	105.3	102.4	195.4	190.0				

	POWDER DETERGENTS FOUR WEEK TIME PERIODS				TIDE FOUR WEEK TIME PERIODS			
	NBD		Robbins		NBD		Robbins	
CLASS	STAT	NONS	STAT	NONS	STAT	NONS	STAT	NONS
0	17.4	13.5	19.1	14.6	68.2	42.5	58.2	35.4
1	13.9	10.9	14.8	12.3	33.8	30.4	35.9	31.9
2	14.2	13.5	18.7	20.0	40.9	38.1	41.9	47.4
3	12.5	12.0	21.9	21.6	47.9	55.0	61.5	65.1
4	14.9	16.2	24.9	24.2				
5	28.2	24.7	53.2	49.9				
6	32.9	31.6	55.2	48.1				

NBD is negative binomial distribution.
STAT indicates stationary models.
NONS indicates nonstationary models.

TABLE IV: Residuals from Predicting the Total Purchases of the Zero Class. Lengths of base and test periods are one week.

		POWDER DETERGENTS			
		NBD		Robbins	
		STAT	NONS	STAT	NONS
Negative Residuals	Mean	-25.7	-6.0	-25.7	-7.3
	Improvement	77%		72%	
Positive Residuals	Mean	77.2	59.0	81.6	56.1
	Improvement	23%		31%	

		TIDE			
		NBD		Robbins	
		STAT	NONS	STAT	NONS
Negative Residual	Mean	-62.8	-30.2	-55.3	-26.3
	Improvement	52%		52%	
Positive Residual	Mean	78.8	39.6	85.0	43.7
	Improvement	50%		49%	

NBD is negative binomial distribution.
STAT indicates stationary models.
NONS indicates nonstationary models.
"Improvement" measures the improvement of the nonstationary model over the corresponding stationary model.

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89/65 (TM, AC, FIN)	Soumitra DUTTA and Piero BONISSONE	"MARS: A mergers and acquisitions reasoning system", November 1989.
89/66 (TM,EP)	B. SINCLAIR-DESGAGNÉ	"On the regulation of procurement bids", November 1989.
89/67 (FIN)	Peter BOSSAERTS and Pierre HILLION	"Market microstructure effects of government intervention in the foreign exchange market", December 1989.

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90/01 TM/EP/AC	B. SINCLAIR-DESGAGNÉ	"Unavoidable Mechanisms", January 1990.
90/02 EP	Michael BURDA	"Monopolistic Competition, Costs of Adjustment, and the Behaviour of European Manufacturing Employment", January 1990.
90/03 TM	Arnoud DE MEYER	"Management of Communication in International Research and Development", January 1990.
90/04 FIN/EP	Gabriel HAWAWINI and Eric RAJENDRA	"The Transformation of the European Financial Services Industry: From Fragmentation to Integration", January 1990.
90/05 FIN/EP	Gabriel HAWAWINI and Bertrand JACQUILLAT	"European Equity Markets: Toward 1992 and Beyond", January 1990.

90/06 FIN/EP	Gabriel HAWAWINI and Eric RAJENDRA	"Integration of European Equity Markets: Implications of Structural Change for Key Market Participants to and Beyond 1992", January 1990.
90/07 FIN/EP	Gabriel HAWAWINI	"Stock Market Anomalies and the Pricing of Equity on the Tokyo Stock Exchange", January 1990.
90/08 TM/EP	Tawfik JELASSI and B. SINCLAIR-DESGAGNÉ	"Modelling with MCDSS: What about Ethics?", January 1990.
90/09 EP/FIN	Alberto GIOVANNINI and Jae WON PARK	"Capital Controls and International Trade Finance", January 1990.
90/10 TM	Joyce BRYER and Tawfik JELASSI	"The Impact of Language Theories on DSS Dialog", January 1990.
90/11 TM	Enver YUCESAN	"An Overview of Frequency Domain Methodology for Simulation Sensitivity Analysis", January 1990.
90/12 EP	Michael BURDA	"Structural Change, Unemployment Benefits and High Unemployment: A U.S.-European Comparison", January 1990.
90/13 TM	Soumitra DUTTA and Shashi SHEKHAR	"Approximate Reasoning about Temporal Constraints in Real Time Planning and Search", January 1990.
90/14 TM	Albert ANGEHRN and Hans-Jakob LÜTHI	"Visual Interactive Modelling and Intelligent DSS: Putting Theory Into Practice", January 1990.
90/15 TM	Arnoud DE MEYER, Dirk DESCHOOLMEESTER, Rudy MOENAERT and Jan BARBE	"The Internal Technological Renewal of a Business Unit with a Mature Technology", January 1990.
90/16 FIN	Richard LEVICH and Ingo WALTER	"Tax-Driven Regulatory Drag: European Financial Centers in the 1990's", January 1990.
90/17 FIN	Nathalie DIERKENS	"Information Asymmetry and Equity Issues", Revised January 1990.
90/18 MKT	Wilfried VANHONACKER	"Managerial Decision Rules and the Estimation of Dynamic Sales Response Models", Revised January 1990.
90/19 TM	Beth JONES and Tawfik JELASSI	"The Effect of Computer Intervention and Task Structure on Bargaining Outcome", February 1990.
90/20 TM	Tawfik JELASSI, Gregory KERSTEN and Stanley ZIONTS	"An Introduction to Group Decision and Negotiation Support", February 1990.
90/21 FIN	Roy SMITH and Ingo WALTER	"Reconfiguration of the Global Securities Industry in the 1990's", February 1990.
90/22 FIN	Ingo WALTER	"European Financial Integration and Its Implications for the United States", February 1990.
90/23 EP/SM	Damien NEVEN	"EEC Integration towards 1992: Some Distributional Aspects", Revised December 1989
90/24 FIN/EP	Lars Tyge NIELSEN	"Positive Prices in CAPM", January 1990.
90/25 FIN/EP	Lars Tyge NIELSEN	"Existence of Equilibrium in CAPM", January 1990.
90/26 OB/BP	Charles KADUSHIN and Michael BRIMM	"Why networking Fails: Double Binds and the Limitations of Shadow Networks", February 1990.
90/27 TM	Abbas FOROUGHI and Tawfik JELASSI	"NSS Solutions to Major Negotiation Stumbling Blocks", February 1990.
90/28 TM	Arnoud DE MEYER	"The Manufacturing Contribution to Innovation", February 1990.

90/29 FIN/AC	Nathalie DIERKENS	"A Discussion of Correct Measures of Information Asymmetry", January 1990.
90/30 FIN/EP	Lars Tyge NIELSEN	"The Expected Utility of Portfolios of Assets", March 1990.
90/31 MKT/EP	David GAUTSCHI and Roger BETANCOURT	"What Determines U.S. Retail Margins?", February 1990.
90/32 SM	Srinivasan BALAK-RISHNAN and Mitchell KOZA	"Information Asymmetry, Adverse Selection and Joint-Ventures: Theory and Evidence", Revised, January 1990.
90/33 OB	Caren SIEHL, David BOWEN and Christine PEARSON	"The Role of Rites of Integration in Service Delivery", March 1990.
90/34 FIN/EP	Jean DERMINE	"The Gains from European Banking Integration, a Call for a Pro-Active Competition Policy", April 1990.
90/35 EP	Jae Won PARK	"Changing Uncertainty and the Time-Varying Risk Premia in the Term Structure of Nominal Interest Rates", December 1988, Revised March 1990.
90/36 TM	Arnoud DE MEYER	"An Empirical Investigation of Manufacturing Strategies in European Industry", April 1990.
90/37 TM/OB/SM	William CATS-BARIL	"Executive Information Systems: Developing an Approach to Open the Possibles", April 1990.
90/38 MKT	Wilfried VANHONACKER	"Managerial Decision Behaviour and the Estimation of Dynamic Sales Response Models", (Revised February 1990).
90/39 TM	Louis LE BLANC and Tawfik JELASSI	"An Evaluation and Selection Methodology for Expert System Shells", May 1990.
90/40 OB	Manfred KETS DE VRIES	"Leaders on the Couch: The case of Roberto Calvi", April 1990.
90/41 FIN/EP	Gabriel HAWAWINI, Itzhak SWARY and Ik HWAN JANG	"Capital Market Reaction to the Announcement of Interstate Banking Legislation", March 1990.
90/42 MKT	Joel STECKEL and Wilfried VANHONACKER	"Cross-Validating Regression Models in Marketing Research", (Revised April 1990).
90/43 FIN	Robert KORAJCZYK and Claude VIALLET	"Equity Risk Premia and the Pricing of Foreign Exchange Risk", May 1990.
90/44 OB	Gilles AMADO, Claude FAUCHEUX and André LAURENT	"Organisational Change and Cultural Realities: Franco-American Contrasts", April 1990.
90/45 TM	Soumitra DUTTA and Piero BONISSONE	"Integrating Case Based and Rule Based Reasoning: The Possibilistic Connection", May 1990.
90/46 TM	Spyros MAKRIDAKIS and Michèle HIBON	"Exponential Smoothing: The Effect of Initial Values and Loss Functions on Post-Sample Forecasting Accuracy".
90/47 MKT	Lydia PRICE and Wilfried VANHONACKER	"Improper Sampling in Natural Experiments: Limitations on the Use of Meta-Analysis Results in Bayesian Updating", Revised May 1990.
90/48 EP	Jae WON PARK	"The Information in the Term Structure of Interest Rates: Out-of-Sample Forecasting Performance", June 1990.
90/49 TM	Soumitra DUTTA	"Approximate Reasoning by Analogy to Answer Null Queries", June 1990.
90/50 EP	Daniel COHEN and Charles WYPLOSZ	"Price and Trade Effects of Exchange Rates Fluctuations and the Design of Policy Coordination", April 1990.

90/51 EP	Michael BURDA and Charles WYPLOSZ	"Gross Labour Market Flows in Europe: Some Stylized Facts", June 1990.
90/52 FIN	Lars Tyge NIELSEN	"The Utility of Infinite Menus", June 1990.
90/53 EP	Michael Burda	"The Consequences of German Economic and Monetary Union", June 1990.
90/54 EP	Damien NEVEN and Colin MEYER	"European Financial Regulation: A Framework for Policy Analysis", (Revised May 1990).
90/55 EP	Michael BURDA and Stefan GERLACH	"Intertemporal Prices and the US Trade Balance", (Revised July 1990).
90/56 EP	Damien NEVEN and Lars-Hendrik RÖLLER	"The Structure and Determinants of East-West Trade: A Preliminary Analysis of the Manufacturing Sector", July 1990
90/57 FIN/EP/ TM	Lars Tyge NIELSEN	Common Knowledge of a Multivariate Aggregate Statistic", July 1990
90/58 FIN/EP/TM	Lars Tyge NIELSEN	"Common Knowledge of Price and Expected Cost in an Oligopolistic Market", August 1990
90/59 FIN	Jean DERMINE and Lars-Hendrik RÖLLER	"Economies of Scale and Scope in the French Mutual Funds (SICAV) Industry", August 1990
90/60 TM	Peri IZ and Tawfik JELASSI	"An Interactive Group Decision Aid for Multiobjective Problems: An Empirical Assessment", September 1990
90/61 TM	Pankaj CHANDRA and Mihkel TOMBAK	"Models for the Evlauation of Manufacturing Flexibility", August 1990
90/62 EP	Damien NEVEN and Menno VAN DIJK	"Public Policy Towards TV Broadcasting in the Netherlands", August 1990
90/63 SM	Sumantra GHOSHAL and Eleanor WESTNEY	"Organising Competitor Analysis Systems", August 1990
90/64 SM	Sumantra GHOSHAL	"Internal Differentiation and Corporate Performance: Case of the Multinational Corporation", August 1990
90/65 EP	Charles WYPLOSZ	"A Note on the Real Exchange Rate Effect of German Unification", August 1990
90/66 TM/SE/FIN	Soumitra DUTTA and Piero BONISSONE	"Computer Support for Strategic and Tactical Planning in Mergers and Acquisitions", September 1990
90/67 TM/SE/FIN	Soumitra DUTTA and Piero BONISSONE	"Integrating Prior Cases and Expert Knowledge In a Mergers and Acquisitions Reasoning System", September 1990
90/68 TM/SE	Soumitra DUTTA	"A Framework and Methodology for Enhancing the Business Impact of Artificial Intelligence Applications", September 1990
90/69 TM	Soumitra DUTTA	"A Model for Temporal Reasoning in Medical Expert Systems", September 1990
90/70 TM	Albert ANGEHRN	"'Triple C': A Visual Interactive MCDSS", September 1990
90/71 MKT	Philip PARKER and Hubert GATIGNON	"Competitive Effects in Diffusion Models: An Empirical Analysis", September 1990
90/72 TM	Enver YÜCESAN	"Analysis of Markov Chains Using Simulation Graph Models", October 1990
90/73 TM	Arnoud DE MEYER and Kasra FERDOWS	"Removing the Barriers in Manufacturing", October 1990
90/74 SM	Sumantra GHOSHAL and Nitin NOHRIA	"Requisite Complexity: Organising Headquarters-Subsidiary Relations in MNCs", October 1990

90/75 MKT	Roger BETANCOURT and David GAUTSCHI	"The Outputs of Retail Activities: Concepts, Measurement and Evidence", October 1990
90/76 MKT	Wilfried VANHONACKER	"Managerial Decision Behaviour and the Estimation of Dynamic Sales Response Models", Revised October 1990
90/77 MKT	Wilfried VANHONACKER	"Testing the Koyck Scheme of Sales Response to Advertising: An Aggregation-Independent Autocorrelation Test", October 1990
90/78 EP	Michael BURDA and Stefan GERLACH	"Exchange Rate Dynamics and Currency Unification: The Ostmark - DM Rate", October 1990
90/79 TM	Anil GABA	"Inferences with an Unknown Noise Level in a Bernoulli Process", October 1990
90/80 TM	Anil GABA and Robert WINKLER	"Using Survey Data in Inferences about Purchase Behaviour", October 1990
90/81 TM	Tawfik JELASSI	"Du Présent au Futur: Bilan et Orientations des *Systèmes Interactifs* d'Aide à la Décision," October 1990
90/82 EP	Charles WYPLOSZ	"Monetary Union and Fiscal Policy Discipline," November 1990
90/83 FIN/TM	Nathalie DIERKENS and Bernard SINCLAIR-DESGAGNE	"Information Asymmetry and Corporate Communication: Results of a Pilot Study", November 1990
90/84 MKT	Philip M. PARKER	"The Effect of Advertising on Price and Quality: The Optometric Industry Revisited," December 1990
90/85 MKT	Avijit GHOSH and Vikas TIBREWALA	"Optimal Timing and Location in Competitive Markets," November 1990
90/86 EP/TM	Olivier CADOT and Bernard SINCLAIR-DESGAGNE	"Prudence and Success in Politics," November 1990
90/87 FIN/EP	Lars Tyge NIELSEN	"Existence of Equilibrium in CAPM: Further Results", December 1990
90/88 OB/MKT	Susan C. SCHNEIDER and Reinhard ANGELMAR	"Cognition in Organisational Analysis: Who's Minding the Store?" Revised, December 1990
90/89 OB	Manfred F.R. KETS DE VRIES	"The CEO Who Couldn't Talk Straight and Other Tales from the Board Room," December 1990
90/90 MKT	Philip PARKER	"Price Elasticity Dynamics over the Adoption Lifecycle: An Empirical Study," December 1990

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91/01 TM/SM	Luk VAN WASSENHOVE, Leonard FORTUIN and Paul VAN BEEK	"Operational Research Can Do More for Managers Than They Think!," January 1991
91/02 TM/SM	Luk VAN WASSENHOVE, Leonard FORTUIN and Paul VAN BEEK	"Operational Research and Environment," January 1991
91/03 FIN	Pekka HIETALA and Timo LÖYTTYNIEMI	"An Implicit Dividend Increase in Rights Issues: Theory and Evidence," January 1991
91/04 FIN	Lars Tyge NIELSEN	"Two-Fund Separation, Factor Structure and Robustness," January 1991
91/05 OB	Susan SCHNEIDER	"Managing Boundaries in Organisations," January 1991
91/06 OB	Manfred KETS DE VRIES, Danny MILLER and Alain NOEL	"Understanding the Leader-Strategy Interface: Application of the Strategic Relationship Interview Method," January 1990 (89/11, revised April 1990)

91/07 EP	Olivier CADOT	"Lending to Insolvent Countries: A Paradoxical Story," January 1991
91/08 EP	Charles WYPLOSZ	"Post-Reform East and West: Capital Accumulation and the Labour Mobility Constraint," January 1991
91/09 TM	Spyros MAKRIDAKIS	"What can we Learn from Failure?", February 1991
91/10 TM	Luc Van WASSENHOVE and C. N. POTTS	"Integrating Scheduling with Batching and Lot-Sizing: A Review of Algorithms and Complexity", February 1991
91/11 TM	Luc VAN WASSENHOVE et al.	"Multi-Item Lotsizing in Capacitated Multi-Stage Serial Systems", February 1991
91/12 TM	Albert ANGEHRN	"Interpretative Computer Intelligence: A Link between Users, Models and Methods in DSS", February 1991
91/13 EP	Michael BURDA	"Labor and Product Markets in Czechoslovakia and the Ex-GDR: A Twin Study", February 1991
91/14 MKT	Roger BETANCOURT and David GAUTSCHI	"The Output of Retail Activities: French Evidence", February 1991
91/15 OB	Manfred F.R. KETS DE VRIES	"Exploding the Myth about Rational Organisations and Executives", March 1991
91/16 TM	Arnoud DE MEYER and Kasra FERDOWS et.al.	"Factories of the Future: Executive Summary of the 1990 International Manufacturing Futures Survey", March 1991
91/17 TM	Dirk CATTRYSSE, Roelof KUIK, Marc SALOMON and Luk VAN WASSENHOVE	"Heuristics for the Discrete Lotsizing and Scheduling Problem with Setup Times", March 1991
91/18 TM.	C.N. POTTS and Luk VAN WASSENHOVE	"Approximation Algorithms for Scheduling a Single Machine to Minimize Total Late Work", March 1991
91/19 MKT	Vikas TIBREWALA and Bruce BUCHANAN	"An Aggregate Test of Purchase Regularity", March 1991
91/20 MKT	Darius SABAVALA and Vikas TIBREWALA	"Monitoring Short-Run Changes in Purchasing Behaviour", March 1991
91/21 SM	Sumantra GHOSHAL, Harry KORINE and Gabriel SZULANSKI	"Intersuit Communication within MNCs: The Influence of Formal Structure Versus Integrative Processes", April 1991
91/22 EP	David GOOD, Lars-Hendrik RÖLLER and Robin SICKLES	"EC Integration and the Structure of the Franco-American Airline Industries: Implications for Efficiency and Welfare", April 1991
91/23 TM	Spyros MAKRIDAKIS and Michèle HIBON	"Exponential Smoothing: The Effect of Initial Values and Loss Functions on Post-Sample Forecasting Accuracy", April 1991 (Revision of 90/46)
91/24 TM	Louis LE BLANC and Tawfik JELASSI	"An Empirical Assessment of Choice Models for Software Evaluation and Selection", May 1991
91/25 SM/TM	Luk N. VAN WASSENHOVE and Charles J. CORBETT	"Trade-Offs? What Trade-Offs?" April 1991
91/26 TM	Luk N. VAN WASSENHOVE and C.N. POTTS	"Single Machine Scheduling to Minimize Total Late Work", April 1991
91/27 FIN	Nathalie DIERKENS	"A Discussion of Correct Measures of Information Asymmetry: The Example of Myers and Majluf's Model or the Importance of the Asset Structure of the Firm", May 1991
91/28 MKT	Philip M. PARKER	"A Note on: 'Advertising and the Price and Quality of Optometric Services', June 1991
91/29 TM	Tawfik JELASSI and Abbas FOROUGHI	"An Empirical Study of an Interactive, Session-Oriented Computerised Negotiation Support System (NSS)", June 1991

91/30 MKT	Wilfried R. VANHONACKER and Lydia J. PRICE	"Using Meta-Analysis Results in Bayesian Updating: The Empty Cell Problem", June 1991
91/31 FIN	Rezaul KABIR and Theo VERMAELEN	"Insider Trading Restrictions and the Stock Market", June 1991
91/32 OB	Susan C. SCHNEIDER	"Organisational Sensemaking: 1992", June 1991
91/33 EP	Michael C. BURDA and Michael FUNKE	"German Trade Unions after Unification - Third Degree Wage Discriminating Monopolists?", June 1991
91/34 FIN	Jean DERMINE	"The BIS Proposal for the Measurement of Interest Rate Risk, Some Pitfalls", June 1991
91/35 FIN	Jean DERMINE	"The Regulation of Financial Services in the EC, Centralization or National Autonomy?" June 1991
91/36 TM	Albert ANGEHRN	"Supporting Multicriteria Decision Making: New Perspectives and New Systems", August 1991
91/37 EP	Ingo WALTER and Hugh THOMAS	"The Introduction of Universal Banking in Canada: An Event Study", August 1991
91/38 EP	Ingo WALTER and Anthony SAUNDERS	"National and Global Competitiveness of New York City as a Financial Center", August 1991
91/39 EP	Ingo WALTER and Anthony SAUNDERS	"Reconfiguration of Banking and Capital Markets in Eastern Europe", August 1991
91/40 TM	Luk VAN WASSENHOVE, Dirk CATTRYSSE and Marc SALOMON	"A Set Partitioning Heuristic for the Generalized Assignment Problem", August 1991
91/41 TM	Luk VAN WASSENHOVE, M.Y. KOVALYOU and C.N. POTTS	"A Fully Polynomial Approximation Scheme for Scheduling a Single Machine to Minimize Total Weighted Late Work", August 1991
91/42 TM	Rob R. WEITZ and Tawfik JELASSI	"Solving A Multi-Criteria Allocation Problem: A Decision Support System Approach", August 1991
91/43 SM	Sumantra GHOSHAL and Christopher BARTLETT	"Building Transnational Capabilities: The Management Challenge", September 1991
91/44 SM	Sumantra GHOSHAL and Nitin NOHRIA	"Distributed Innovation in the 'Differentiated Network' Multinational", September 1991
91/45 MKT	Philip M. PARKER	"The Effect of Advertising on Price and Quality: An Empirical Study of Eye Examinations, Sweet Lemons and Self-Deceivers", September 1991
91/46 MKT	Philip M. PARKER	"Pricing Strategies in Markets with Dynamic Elasticities", October 1991
91/47 MKT	Philip M. PARKER	"A Study of Price Elasticity Dynamics Using Parsimonious Replacement/Multiple Purchase Diffusion Models", October 1991
91/48 EP/TM	H. Landis GABEL and Bernard SINCLAIR-DESGAGNE	"Managerial Incentives and Environmental Compliance", October 1991
91/49 TM	Bernard SINCLAIR-DESGAGNE	"The First-Order Approach to Multi-Task Principal-Agent Problems", October 1991
91/50 SM/TM	Luk VAN WASSENHOVE and Charles CORBETT	"How Green is Your Manufacturing Strategy?" October 1991
91/51 MKT	Philip M. PARKER	"Choosing Among Diffusion Models: Some Empirical Guidelines", October 1991
91/52 EP	Michael BURDA and Charles WYPLOSZ	"Human Capital, Investment and Migration in an Integrated Europe", October 1991
91/53 EP	Michael BURDA and Charles WYPLOSZ	"Labour Mobility and German Integration: Some Vignettes", October 1991
91/54 TM	Albert ANGEHRN	"Stimulus Agents: An Alternative Framework for Computer-Aided Decision Making", October 1991

91/55 EP/SM	Robin HOGARTH, Claude MICHAUD, Yves DOZ and Ludo VAN DER HEYDEN	"Longevity of Business Firms: A Four-Stage Framework for Analysis", November 1991
91/56 TM/EP	Bernard SINCLAIR-DESGAGNE	"Aspirations and Economic Development", November 1991
91/57 MKT	Lydia J. PRICE	"The Indirect Effects of Negative Information on Attitude Change", November 1991
91/58 OB	Manfred F. R. KETS DE VRIES	"Leaders Who Go Crazy", November 1991
91/59 OB	Paul A. L. EVANS	"Management Development as Glue Technology", November 1991
91/60 TM	Xavier DE GROOTE	"Flexibility and Marketing/Manufacturing Coordination", November 1991 (revised)
91/61 TM	Arnoud DE MEYER	"Product Development in the Textile Machinery Industry", November 1991
91/62 MKT	Philip PARKER and Hubert GATIGNON	"Specifying Competitive Effects in Diffusion Models: An Empirical Analysis", November 1991
91/63 EP	Michael BURDA	"Some New Insights on the Interindustry Wage Structure from the German Socioeconomic Panel", December 1991
91/64 FIN	Jean DERMINE	"Internationalisation of Financial Markets, Efficiency and Stability", December 1991

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92/01 MKT/EP/TM	Wilfried VANHONACKER	"CONPRO*DOGIT: A New Brand Choice Model Incorporating a Consideration Set Formation Process", January 1992
92/02 MKT/EP/TM	Wilfried VANHONACKER	"The Dynamics of the Consideration Set Formation Process: A Rational Modelling Perspective and Some Numerical Results", January 1992
92/03 OB	Manfred F.R. KETS DE VRIES	"The Family Firm: An Owner's Manual", January 1992
92/04 SM	Philippe HASPESLAGH and David JEMISON	"Making Acquisitions Work", January 1992
92/05 TM	Xavier DE GROOTE	"Flexibility and Product Diversity in Lot-Sizing Models", January 1992 (revised)
92/06 FIN	Theo VERMAELEN and Kees COOLS	"Financial Innovation: Self Tender Offers in the U.K.", January 1992
92/07 TM	Xavier DE GROOTE	"The Flexibility of Production Processes: A General Framework", January 1992 (revised)
92/08 TM	Luk VAN WASSENHOVE, Leo KROON and Marc SALOMON	"Exact and Approximation Algorithms for the Operational Fixed Interval Scheduling Problem", January 1992
92/09 TM	Luk VAN WASSENHOVE, Roelof KUIK and Marc SALOMON	"Statistical Search Methods for Lotsizing Problems", January 1992
92/10 SM	Yves DOZ and Heinz THANHEISER	"Regaining Competitiveness: A Process of Organisational Renewal", January 1992
92/11 TM	Enver YUCESAN and Sheldon JACOBSON	"On the Intractability of Verifying Structural Properties of Discrete Event Simulation Models", February 1992
92/12 FIN	Gabriel HAWAWINI	"Valuation of Cross-Border Mergers and Acquisitions", February 1992
92/13 TM	Spyros MAKRIDAKIS and Michèle HIBON et.al.	"The M2-Competition: A Budget Related Empirical Forecasting Study", February 1992
92/14 MKT	Lydia PRICE	"Identifying Cluster Overlap with NORMIX Population Membership Probabilities", February 1992