"FUZZY LOGIC APPLICATIONS: TECHNOLOGICAL AND STRATEGIC ISSUES"

by

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Fuzzy Logic Applications: Technological and Strategic Issues

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Abstract

Fuzzy logic was invented about 25 years ago. The past few years have seen an extra-ordinary spurt in the commercialization of the technology. Not only has fuzzy logic significantly enhanced knowledge-based/expert system technology, it has fundamentally altered the granularity of intelligence. With the help of fuzzy logic, manufacturers of home appliances are today embedding intelligence inside individual products. The application of fuzzy logic has also transformed industrial process control and enabled new product development strategies. Oddly enough, though theoretical developments in fuzzy logic have largely occurred in America and Europe, Japanese companies have spearheaded the commercialization of the technology. Aimed at an audience of business/industrial managers, this paper reviews the evolution of fuzzy logic technology and analyzes the strategic business impact of the technology.

1. Introduction

Fuzzy logic (FL) is a part of artificial intelligence (AI) which has rapidly gained in prominence and importance over the past few years. FL coupled with rule-based systems is enabling the modelling of the approximate and imprecise reasoning processes common in human problem solving. Within industry, FL is being used to simplify the modelling of complex systems and revolutionize product design, manufacturing and control. It is rapidly becoming one of the most commercially successful parts of AI. This section describes some introductory examples of the commercial applications of FL and outlines the focus and structure of the paper.

1.1 Hitachi: The Sendai Subway Control System

Sendai is a major city in the northern part of Japan's main island of Honshu with one of the highest ratios of car ownership in Japan. Road capacity reached its peak in 1965 with the average speed of a municipal bus during rush hour dropping to a snail's pace of 8 km/hr- within 10 km of the city's center [12]. Against this background, Sendai's city planners commenced designing a municipal subway system that would alleviate the city's subway snarls and also provide its citizenry with the highest levels of comfort, safety and efficiency.

While designing the subway system at Sendai, Hitachi engineers spent many years trying to automate the train operations based on conventional machine logic and industrial control methods. In the conventional approach, accurate operation is achieved by having a predetermined speed pattern and issuing (frequent) commands for acceleration or braking in order to adapt to changing running conditions such as gradient of track and braking force of rolling stock. The results obtained with the conventional methods were unsatisfactory. Next, the engineers looked at how a human driver controlled the train. While operating a train, a skilled driver made control decisions based upon his knowledge of conditions of the track and his understanding of the performance characteristics of the train. His decisions and actions were approximations of what is reasonable and necessary at that particular instant of time, rather than a series of detailed quantitative decisions. The engineers noted that the drivers "internal reasoning might be something like: 'have to watch the speed along that stretch' or 'easy on the brakes coming in'. These seemingly vague and imprecise decisions were actually the result of highly complex thought processes based on his knowledge and experience, and allowed constant readjustment of indices such as safety, speed, quality of ride, time table and so on" [12]. It is very difficult to process the "fuzzy" and "imprecise" information used by a skilled motorman by conventional Boolean logic or by traditional rule based technology [44]. The engineers next looked at FL for help with designing a radically different subway control system. FL allowed the Hitachi engineers to incorporate the knowledge of the experienced motorman and his ability to process fuzzy and imprecise information into the control procedures. The resulting system not only was simpler, but it has resulted in significant improvements in the degree of comfort and safety and has yielded cost

savings. Since July 15th 1987, a nearly fully automated "intelligent" subway system (designed by Hitachi) whisks passengers between stations with reputedly the highest degree of comfort and safety in the world - "straphangers don't need to grip hard because station stops are gentle as a kitten" [32].

1.2 Yamaichi Securities: Intelligent Trading Programs

Conventional trading programs for index-linked portfolios are common in Japanese financial companies. It is however widely recognized [33] that the Tokyo stock market is more faddish than fundamental and that standard computerized-program strategies, based on traditional portfolio theory, won't work perfectly there.

Experienced fund managers are adept at recognizing "fads" and trends. Much like the skilled motorman running the Sendai subway, these fund managers at Yamaichi securities are experts at understanding and reasoning with imprecise and ambiguous data. For example, a reasoning rule in the mind of such a manager might be: "If market sentiment is *strong* now and the previous session's market sentiment was *normal*, then sell futures in *large* volume". The concepts *strong*, *normal* and *large* in the above reasoning rule are fundamentally imprecise and vague. As explained later (Section 3), they are not easily quantified or expressed as symbols in rule premises. FL enables the representation of and reasoning with such vague concepts. Since August 1989, Yamaichi securities has been managing several billions of Yen with a computerized trading system which is based on FL and attempts to capture the reasoning processes of an experienced fund manager. The computerized trading model looks at basic financial data such as stock prices and trading volumes and integrates it with about 600 trading rules-of-thumb which represent the approximate reasoning processes of fund managers. The trading heuristics are represented by fuzzy rules and not conventional IF..THEN.. rules. Fuzzy rules have special advantages as compared to conventional rules and are described in more detail later (Sections 3.2 and 3.3). According to company sources, a two year trial period in futures arbitrage was excellent - "only one in 100 pros could have done better" [33].

1.3 Matshushita: Intelligent Washing Machines

The latest washing machine sold by Matshushita in Japan makes the chore of washing clothes much simpler for users. All that the user has to do is to insert laundry in the washing machine and press the start button. The machine takes care of the rest: it automatically selects the wash, rinse and spin cycles from among 600 possible combinations by analyzing the dirt in the wash, the size of the load and other relevant factors. The result: a cleaner and more efficient wash. The first washing machines sold out its initial production within weeks. It is not surprising that these latest models already (in early 1991) account for over half of Matshushita's washing machine sales in Japan [35].

Since 1989, FL processing has been available on a chip. This has dramatically changed the scope of applicability of FL technoiogy. It has now become feasible to insert "expertise on a chip" within individual products. The Matshushita washing machine is one example of an "intelligent" product resulting from the integration of "intelligence" into the product. Expert rules about the best combination of wash, rinse and spin cycles to use are stored in a FL chip inside the washing machine. Various sensors in the washing machine provide data about the wash load to the chip which selects the best combination. A lot of the information used in this decision is also imprecise and vague, such as "the degree of dirt". FL chips are being used to incorporate expert knowledge into many other products. For example, Toshiba Corp.'s cooking range incorporates the expertise of four professional chefs. Virtually every Japanese home appliance maker has integrated FL intelligence (on chips) into the current models of cameras, video camcorders, vacuum cleaners, air conditioners, rice cookers, microwave ovens, toasters and refrigerators. Major manufacturers such as Matsushita estimate that all of their products will have some form of embedded fuzzy intelligence within the next five years [28].

1.4 Rockwell: Modelling Stress on Wings

Engineers at Rockwell were facing the formidable task of modelling the stress on a wing while designing controls for the Air Force/NASA advanced technology wing. Stress on the wing is caused by many different variables - air speed, shape of the wing in flight, drag and aerodynamics - all of which

constantly change during flight. The domain is enormously complex to model precisely. After spending months using conventional modelling techniques, the engineers decided to use a novel method based on FL. The result: simpler rules, less math and a more accurate representation of a complicated process. According to sources within Rockwell, the adoption of FL allowed them to get better results, faster [35].

Precision is expensive and often not required as demonstrated by the above example. Using FL, it is possible to exploit the tolerance for imprecision in many engineering situations. The result need not necessarily be less useful or less realistic. On the contrary, the models are usually simpler and the results are often more accurate. Many real world situations are too complex to model precisely. Conventional approaches usually simplify the domain and develop precise models of the simplified situation. The results obtained using such models are precise but typically apply to the simplified domain modeled. FL, in contrast, allows an "approximate model" of the actual domain to be developed with comparatively less effort. Experience shows that the results obtained with such an "approximate model" are also usually very useful and usable.

1.5 Focus and Structure of Paper

As evident from the above examples, FL is an important technology for industry/business [44]. FL applications are transforming important industrial and business processes such as product design, manufacturing, and control. The primary aim of this paper is to provide the engineering/business manager with knowledge of the fundamentals of FL technology and the strategic business potential of its applications.

The structure of this paper is as follows. There are five additional sections. The next section describes the evolution of FL starting from its invention in 1965. The fundamentals of FL technology which are most relevant to its commercial applications are described in section three. Section four contains two case studies to illustrate the practical application of FL. World-wide commercial activities in FL are surveyed in section five. The last section concludes the paper by analyzing the strategic business impact of FL applications.

2. Development of Fuzzy Logic

FL was invented in the USA during the mid 60's by Lotfi Zadeh [42], a professor at the University of California at Berkeley. Zadeh's initial motivation to develop FL came from his observation that traditional mathematics with its emphasis on precision was ill-suited for modelling "soft, non-quantifiable" systems (such as those studied in the social sciences) [36]. The commercial successes of FL applications have however been in entirely different areas - industrial process control and knowledge based systems. FL began to find wide ranging commercial applications when it was coupled with expert systems technology [4]. The coupling of FL and expert system technologies allowed a better modelling of the imprecise reasoning processes of skilled human experts (see sections 1.1 and 1.2) and provided the bases for a new heuristic theory of industrial process control.

Reasoning with uncertain and imprecise information is of central concern in both FL and general AI. However, for a long time FL was not considered as being a part of mainstream AI. The reasons for this are many and complex. First, FL is "different" from conventional AI approaches. Most AI systems are based on classical predicate logic and are symbolic in nature. In contrast, FL systems are based on multi-valued logic and are numerical in nature [24]. Second, questions about the underlying theoretical bases of FL were raised by some researchers [5] (see section 3.1). Third, for most of the 1970's and the 1980's the dominant emphasis within AI was the building of knowledge-based systems using conventional rule-based technology [4].

Though largely ignored in the USA (both by industry and academia) for about two decades after its invention, FL was adopted and developed by researchers in Asian and European countries. Independent of mainstream AI research, FL acquired a large following of dedicated researchers around the world and developed into a major academic field with its own associations (such as the International Fuzzy Systems Association), journals (such as the International Journal of Fuzzy Sets and Systems and the International Journal of Approximate Reasoning), and conferences (organized by IEEE and the International Fuzzy Systems Association). FL has also been applied in many other fields including mathematics, operations research, medicine, and transportation [8,9,49]. Today, China has the largest number of FL researchers (more than 10,000 of them) with

Japan and Europe following in 2nd and 3rd places respectively [6]. While Zadeh's papers [41,42,45,48] have outlined fundamental FL principles, significant theoretical developments in FL have come from research groups in (both Western and Eastern) Europe. Of special note are FL research groups headed by Prof. H.J. Zimmerman in Aachen, Germany, and by D. Dubois and H. Prade in Toulouse, France. Capitalizing on the theoretical developments in the USA and Europe, Japan has spearheaded the commercialization of FL technology. The successes of Japanese companies with FL applications have revived interest within industry in the USA and Europe. Western industry has recently turned its attention to FL and its commercial potential. FL is also slowly gaining acceptance from the mainstream AI community.

The first practical application of FL to the control of industrial processes was pioneered in Europe by Mamdani and Assilian in 1974 in connection with the regulation of a steam engine [25]. In 1980, F. L. Smidth and Co. of Copenhagen [19] started marketing the first commercial fuzzy expert system to control the fuel-intake rate and gas flow of a rotating kiln used to make cement. While academic interest in FL in Japan started quite early (their first paper on FL appeared in 1968 [24]), it was only during the early 1980's that Japanese companies realized the commercial potential of FL and started investing in it seriously. The shot in the arm for the commercialization of FL came with the invention of the first FL chip by researchers in Bell Laboratories in 1985 [34] and which became commercially available in late 1988. A FL chip enabled the coding and execution of a set of fuzzy rules in hardware inside a chip. With the availability of the chip, it became possible to mimic human like intelligence in a small package, and to do it cheaply and quickly. FL chips can perform over 580,000 fuzzy inferences per second and can be purchased for about $70 a piece. This invention lead to the proliferation of a wide range of "fuzzy products" (primarily developed and marketed by Japanese companies), i.e., everyday appliances such as cameras, camcorders, rice cookers and air conditioners with an embedded fuzzy chip.

Closely following on the fuzzy chip, Prof. Yamakawa designed the first fuzzy computer in 1987 [36]. The logic inside a fuzzy computer, in contrast to conventional computers (utilizing a binary logic of 1/0), is based on FL (utilizing a range of degrees of truth ranging from 0 to 1). Such a computer is

not only able to perform fuzzy inferences at a very high speed (10,000,000 inferences per second), but more important, is looked upon as an important step towards the development of sixth-generation computers with a capability of processing uncertain and imprecise knowledge [22]. Recently announced in Japan, is the development of the first fuzzy computer by a commercial company (OMRON).

The importance that Japan attaches to these developments can be seen clearly from the fact that in 1989, Japan's Ministry of International Trade and Industry (MITI) setup a consortium of 48 top Japanese companies to develop fuzzy systems. The consortium is called the "Laboratory for International Fuzzy Engineering" and has a $70M starting budget (see section 5.2). Similar, but less ambitious, efforts are underway in Europe also. The Ministry of Research in France is coordinating research and investment in FL and taking steps to increase cooperation between universities and industry, such as aiding in the formation (in 1991) of the club CRIN "Logique Floue" within the Association ECRIN (Echange et Coordination Recherche Industrie). Also in 1991, the European foundation, ELITE (European Laboratory for Intelligent Techniques Engineering), has been started in Aachen, Germany with the aim of enhancing research in FL and stimulating FL applications in European industry.

Table 1 lists some important milestones in the commercial development of FL.

Table 1 about here

3. Fundamentals of Fuzzy Logic Technology

Fuzzy logic is today a vast field. However only a small part of FL technology - the integration of FL and rule-based reasoning - is being used at present in most commercial applications. This section describes the fundamentals of fuzzy rule-based reasoning. FL theory can get very mathematical. To keep the paper accessible to a large variety of readers, an intuitive graphical representation has been deliberately adopted in this section in preference over a more rigorous mathematical description. Readers interested in the mathematical details of fuzzy rule-based reasoning (or other aspects of FL

technology) may consult one or more of the following references [3,8,9,15,40,42,47,49].

3.1 Representational Concepts Using Fuzzy Sets

FL aims to model the imprecise modes of reasoning that play an important role in the remarkable human ability to make rational decisions in an environment of uncertainty and imprecision. Zadeh [38] has suggested that rather than regard human reasoning processes as themselves "approximating" to some more refined and exact logical processes that could be carried out with mathematical precision, the essence and power of human reasoning lies in its capability to grasp and use inexact concepts directly. For example, we are usually quite comfortable dealing with questions (like those give below) based on a store of knowledge that is inexact, incomplete, or not totally reliable:

> Most wealthy actors live in Beverly Hills. Mary is a rich actress. What can be said about the location of Mary's residence?
>
> Most players on the basketball team are quite tall. Gecrge is on the basketball team. How tall is George?

Classical logical systems are usually based on Boolean two valued logic, i.e., in it, any proposition can be either true or false. This imposes an inherent restriction in its ability to represent "vague" and "imprecise" concepts such as "wealthy" and "tall". To understand this better, consider the concept "tall". Most persons would agree that a person whose height is more than 7 feet is definitely tall while someone whose height is below 5 feet is surely short, i.e., not tall. How to then quantify the distinction between tall and short persons? A common approach is to use a threshold (above which everyone is tall and below which everyone is short). Let us choose the threshold height to be 6 feet. This threshold correctly classifies the height of 7' as tall and of 5' as short. Next consider two persons A and B, with heights of 5'11" and 6'1" respectively. Assuming the earlier threshold of 6', we see that A is classified as short and B as tall. Intuitively, this does not seem right. While A is indeed shorter than B, their heights are very similar. Note that this problem persists even if we move the threshold upwards or downwards (we can always find similar heights, each one on either side of the threshold).

This problem exists because we are using a two-valued logical system in which "middle-values" such as 5'11" and 6'1" are forced into one of the two sets "tall" or "not tall" by the choice of the threshold height.

FL, in contrast would assign a "degree of membership" to each height in the set of "tall persons". Thus, a person with a height of 6'1" would be tall to a certain degree while a person with a height of 5'8" would be tall to another degree. This allows the explicit representation of the fact that different heights can be "tall" to different degrees. Figure 1 illustrates how different heights can be assigned different degrees of membership in the representation of the imprecise concept "tall" using FL. The horizontal axis gives the height and the vertical axis specifies the corresponding degree of membership of the height in the set of tall persons. Persons with heights below 5' are tall to the degree 0 (i.e., they are not tall) while persons taller than 7' are tall to the degree 1 (i.e., definitely tall). All intermediate heights are tall to varying degrees (e.g., a person with a height of 6'5" is tall to the degree 0.7). Figure 1 can be said to represent the fuzzy set "tall". Note that fuzzy sets are a generalization of conventional Boolean sets. The degree of membership for domain elements in a Boolean set can be only either 0 (not a member of the set) or 1(a member of the set). In contrast, the degree of membership in a fuzzy set can be any value between 0 and 1. It thus follows that models developed using FL are also applicable for "crisp" data.

Figure 1 about here

To see the practical implications of such a representation, consider the example of an air conditioner. A conventional air conditioner, for example, only recognizes two states: too hot or too cold. Under normal thermostat control, the cooling system either operates at full blast or shuts off completely. A fuzzy air conditioner can, by contrast, recognize that some room temperatures are closer to the human comfort zone than others. Its cooling system would begin to slow down gradually as the room temperature approached the desired setting. The result of this is a more comfortable room and a smaller electric bill. These principles have been used in the fuzzy air conditioner marketed by Mitsubishi.

As another example, consider the formulation of a database queries. A typical query such as "get all companies with revenues above 400M$" would

not retrieve companies with revenues just below the threshold (such as 399M$). This problem can be solved by the use of FL. Assume that the concept "high revenue" is defined using an appropriate fuzzy set (analogous to that shown in Figure 1). Now a query such as "get all companies with high revenues" shall retrieve all companies whose revenues qualify as "high" to a certain degree. This facilitates intelligent information retrieval from databases [23].

In Figure 1, we note that the degree of membership of the height 6'5" is 0.7. This can be alternatively interpreted as the following: the *degree of possibility* that someone whose height is 6'5" is tall is 0.7. The graph of Figure 1 then defines the "*possibility distribution*" for the fuzzy set representing the concept "tall". These notions of "possibility" and "possibility distribution" are some of the central concepts within FL and are the bases of "possibility theory" as proposed by Zadeh [41]. The concept of possibility is often a source of confusion and controversy. There has been much research within FL and AI on the exact distinctions between probability and possibility. It is now accepted that they are indeed two different, complementary concepts. There are some obvious differences. The sum of the possibility distribution does not need to sum to 1, as is required for probability distributions. For example, the sum of the possibility distribution shown in Figure 1 is greater than 1 (note that the distribution is open ended - all heights greater than 7' have a degree of membership 1). In general, probability is concerned with the uncertainty in the outcome of clearly defined and randomly occurring events, while possibility theory is directed at the uncertainty in the description of the event itself. A simple intuitive description of the difference between possibility and probability can be given with the following example [47]. If a person walks into a room, possibility theory can be used to determine the degree to which that person satisfies the condition of being tall. In contrast, probability theory would predict the chances of that person being tall before s/he walked into the room or the chance that a random observer would classify her/im as tall. More sophisticated explanations of the differences between the concepts of probability and possibility can be given using mathematical interpretations of the concept of possibility. Several interpretations have been proposed in the literature [9,10,27,43], but they are beyond the scope of this paper.

Analogous to "tall", the concepts "short" and "average height" can also be represented using FL. Figure 2 depicts one possible representation of these concepts. Note that a particular height can "belong" to different fuzzy concepts (i.e., fuzzy sets) and can have different degrees of membership in each of them. For example, the height, 6'5" has a degree of membership 0.7 in the fuzzy concept "tall" and a degree of membership 0.49 in the fuzzy concept "very tall". Also, as would be intuitively expected, the degree of membership of the height 6'5" in the concept "tall" is more than that in the concept "very tall".

Figure 2 about here

In 1975, Zadeh [48] introduced the concept of a linguistic variable and it has played an important role in the commercial applications of FL. Zadeh's basic idea was to extend the possible values of a particular variable to include not only numerical values, but also linguistic values. For example, a variable such as height would typically have a particular number as its value. By treating it as a linguistic variable, it is possible to (also) assign as its value linguistic labels such as *short*, *tall*, *very tall*, *very short* and *average*. These linguistic labels are given a numerical interpretation by fuzzy sets similar to those shown in Figure 2. The use of linguistic variables makes it much easier for humans to express statements or conditions linguistically. (Recall the use of linguistic labels in the reasoning processes of the Sendai subway driver and the Yamaichi fund manager in sections 1.1 and 1.2.) The use of linguistic variables in fuzzy reasoning procedures is explained in sections 3 and 4. Note that a linguistic variable can be viewed as a generalization of a conventional variable.

How does one obtain the possibility distributions (such as those given in Figure 2) defining the fuzzy sets used in the representation of vague concepts? Though some formal procedures have been proposed in the literature for obtaining these distributions [20], no one theoretically correct method exists. These distributions are essentially context dependent and are usually defined by subjective estimates. For example, while a height of 6' would be considered average in Scandinavian countries, the same would be considered "very tall" among Indians. Thus the distributions defining the same concept "tall" would vary depending upon the context of problem solution. In most cases, it is the responsibility of the domain expert to define

the shapes of the distributions. This subjectivity in the procedure for obtaining the distributions has often been criticized by opponents of FL. Proponents of FL counter this criticism by noting that even probability theory admits subjective estimates of probability. Another criticism has been that a distribution such as that shown in Figure 1 can (possibly) be obtained within classical probability theory by taking a sample of respondents and noting what proportion of the chosen sample respond in the affirmative to a particular height being classified as 'tall". Proponents of FL counter this argument by mentioning the following two points: (a) first, the described procedure is elaborate and cumbersome (making it very difficult to follow in practice) and (b) second, forcing a person to categorize an intermediate height (such as 6') as either tall or not tall elicits an unnatural response (because the respondent is "more at ease" with an answer of the form "tall to a degree x"). The origins of these fuzzy distributions has been one of the controversial aspects of fuzzy set theory. Though the controversy has not ended, the large number of successful commercial applications of FL are making these debates less important from a practical perspective. It has also been observed from practice that the exact shapes of the various possibility distributions are not very critical to the success or failure of fuzzy systems. This is due to the special nature of the fuzzy inference procedure as explained below.

3.2 Reasoning Using Fuzzy Logic

The combination of FL and rule-based reasoning has found wide applications in the control of industrial processes, modelling of complex systems and the development of fuzzy expert systems. Fuzzy rules are like ordinary IF..THEN.. rules, except for two important differences:

- The premises and conclusions of fuzzy rules contain linguistic variables and
- The inference procedure with fuzzy rules is different from that with conventional IF..THEN... rules.

Consider a simple fuzzy reasoning problem. Assume that the following two fuzzy rules are given in a knowledge base:

IF Voltage is *small* **THEN** Current is *medium* ...(rule **a**)

IF Voltage is *large* **THEN** Current is *small* ...(rule **b**)

Here Voltage and Current are two linguistic variables representing particular values of voltage and current in some unspecified electrical system. *Small*, *medium* and *large* are fuzzy sets defining the different linguistic values that can be assigned to the two linguistic variables Voltage and Current. Rules **a** and **b** relate the output value of current to the input value of voltage. A typical fuzzy reasoning problem in this context would be

"determine the value of current given that Voltage is *medium*"

Note that this problem is non-trivial as the input value of voltage (= *medium*) does not match exactly with the premises of either rule **a** or rule **b** above. Thus neither rule can be directly applied to obtain the output value of the current. However, rules **a** and **b** specify the value of the current when the values of the voltage are *small* and *large*. The inference mechanism of FL allows us to compute the "partial match" of the input premise with the premises of the above two rules and obtain an "interpolated" current value an answer to the posed problem. To understand how these partial matches are computed and how the fuzzy inference procedure operates, it is necessary to know about some basic mathematical operations on fuzzy sets. These are described below. For ease of comprehension, a graphical explanation is adopted below in preference over a more rigorous mathematical exposition. Mathematical details are given in references [30,47,49].

Fuzzy sets were earlier described as extensions of classical sets. Thus it is possible to define the classical set theoretical operations of union, intersection and complementation for fuzzy sets. These operations are defined using the degree of memberships of the various elements in the concerned fuzzy sets.

- **Union**: The possibility distribution of the union, $A \cup B$, of two fuzzy sets A and B is given by taking the maximum of the the degrees of membership of the elements in A and B. This is graphically depicted in Figure 3 for two arbitrary fuzzy sets A and B.

Figure 3 about here

- **Intersection**: The possibility distribution of the intersection, $A \cap B$, of two fuzzy sets A and B is given by taking the minimum of the the degrees of membership of the elements in A and B. This is graphically depicted in Figure 4 for two arbitrary fuzzy sets A and B. Note that only elements common to both sets are retained in their intersection.

Figure 4 about here

- **Complement**: The possibility distribution of the complement of a fuzzy set A is obtained by subtracting from 1 the degree of membership (in the fuzzy set A) of the various elements in the domain. Figure 5 depicts a graphical interpretation of the process of obtaining the complement of a fuzzy set. Also note the fuzzy sets *short* and *tall* in Figure 2. The concept of *short* is the complement of the concept *tall* (and vice versa). Therefore the possibility distribution for *short* can be obtained by taking the complement of that for *tall* and vice versa.

Figure 5 about here

The operators min-max (*minimum* and *maximum*) used above for defining the fuzzy set theoretic operations were first proposed by Zadeh in his seminal paper on fuzzy sets [42]. There is nothing inherently unique about the min-max operators, and over the years several new operators for fuzzy sets have been proposed by different researchers (see references [9,49] for good overviews). A description of these other operator categories is beyond the scope of this paper. This paper focuses on the min-max operators as they remain the most popular operators used in commercial applications due to their simplicity and appealing properties [3].

For simplicity, let the fuzzy sets *small*, *medium* and *large* used in rules **a** and **b** have similar representations for both Voltage and Current as shown in Figure 6 (X-axis scales would be different for Voltage and Current). The appropriate value of current is to be determined by fuzzy reasoning. Figure 7 graphically depicts the two fuzzy rules and the fuzzy input. For ease of visualization, the distribution of the input fact is superimposed (as a lightly dotted region) on the premises of the two fuzzy rules in Figure 7. Though there is no direct match between the premises of the two fuzzy rules and the

input fact, note that a partial match with each rule does exist. Thus intuitively one can expect each of the two rules to contribute partially to the output answer. The desired value of Current is computed as follows. First, the intersection of the input fact with each of the fuzzy premises is computed (using the same operation as shown in Figure 4). The intersections are shown by the striped regions in the input premises in Figure 7. Second, the contribution of each rule to the output answer is determined by taking the portion of the output fuzzy distributions (shown shaded in the conclusions in Figure 7) which are contained within the intersections computed in the above step. Third, a union (as shown in Figure 3) is taken of the contributions of the conclusions of each rule to give the final desired output value of Current (shown by the shaded region in the bottom right hand corner of Figure 7).

Figures 6 & 7 about here

Sometimes, a linguistic answer may be desired, specially if the output answer is to be given to a human. In such situations, a process known as "linguistic approximation" is used. The output distribution of Figure 7 does not directly correspond to any distribution shown in Figure 6. Thus the fuzzy sets *small*, *medium* or *large* cannot be directly assigned to the output answer. Rather some combination of the fuzzy sets has to be determined whose distribution approximates the distribution of the output answer. The previously defined operations of union, intersection, complementation (and other operations not defined in this paper) are used to obtain the linguistic label combination (such as *not small and not large Current*) whose possibility distribution closely approximates the output distribution. This linguistic label combination is then output as the answer of the fuzzy inference procedure.

Note that the output value of current is in general a possibility distribution. Often the output fuzzy set has to be "defuzzified" to give a crisp number. Crisp numbers are important for controlling physical systems because possibility distributions may not make much sense by themselves. For example, if the value of current has to be set, a crisp number is required (the value of current cannot be set to a distribution). Different methods for defuzzification have been proposed in the literature [8,9,49]. A popular defuzzification procedure (which has been used successfully in many

different applications) is to take the center of mass of the output distribution. Figure 8 illustrates the defuzzification of the output of Figure 7. Point A is the center of mass of the output distribution. The coordinate, y, of A along the Current (Y) axis represents the defuzzified output value of Current. A more concrete example of the defuzzification procedure is given in section 4.1.

Figure 8 about here

3.3 Comparing Fuzzy And Conventional Rule-Based Inference

It is apparent that the inference procedure with fuzzy rules is significantly different from that with conventional rules. Some important differences are mentioned below.

In contrast to the usual IF..THEN.. rules which are based on Boolean logic, fuzzy rules can accommodate partial matches. Assume for a moment that the labels "small", "medium" and "large" are treated as symbols (not as fuzzy sets). Then as there is no rule (in the rule-base) for the value of Current when the value of Voltage is medium, the system will not be able to produce an answer for Current (as no rule will be triggered). A conventional rule-based system would fail under such circumstances. However, the fuzzy inference procedure described above computes partial matches and can thus produce a meaningful answer even when there is no direct match between rule premises and input facts. This feature causes the fuzzy rules to be more robust to variability in the input descriptions.

The number of fuzzy rules required for a particular inference is significantly less than that if conventional rules were used. This is primarily due to the ability of fuzzy rules to accommodate partial matches with inputs. The premises and conclusions of each fuzzy rule are possibility distributions. Each distribution typically covers a certain range of domain values. Thus a few rules are sufficient for covering the regions of interests in the domains. All possible values in each domain do not need separate rules (as would be required if conventional rules were being used). Even if a particular input does not match exactly with the rule premises, its partial match with a set of rules will provide an answer. While using fuzzy rules, it is only important to see that the regions of interest in the concerned domains are adequately

covered by appropriate knowledge spaces (generated by the fuzzy rules). Thus systems using fuzzy rules are often simpler to build and maintain.

The conventional inference procedures of backward or forward chaining are essentially sequential procedures, with each step in the process being represented by a single rule firing. A rule fires and that rule in turn triggers another rule. The fuzzy inference procedure described above is, in contrast, a parallel procedure. The match of each rule premise with the input fact is computed in parallel. This makes the fuzzy inference procedure easily amenable to implementations on parallel computers (an attractive feature given the predicted future importance of parallel computers).

The parallel nature of the rule firing makes the inference process robust to breakdowns and gives it the desirable property of graceful degradation. As each rule contributes (in proportion to its match with the input) to the final output, the removal of any one rule (due to mechanical breakdowns or other reasons) does not decapitate the system. The system usually still operates (due to the contribution of other rules), though maybe not as effectively as before. This is a very useful property. Compare this to a conventional rule based system. In case a critical rule does not fire (due to a software bug or other reasons), then the entire system can crash suddenly. This is not a very desirable characteristic. For example, if some error occurs in the control of an industrial process, a graceful degradation of the control procedures is much more desirable than a sudden crash. Graceful degradation provides advance warning of the error and allows more time for repairs or alternative actions.

The exact shapes of the distributions in the premises and conclusions of fuzzy rules are not critical. As various rules contribute in parallel when presented with a particular input, there is some in-built tolerance for (slight) errors in the shapes of the distributions. This is in contrast to conventional rules, where the premise and output labels (values) have to be chosen carefully. If the input data do not match exactly with the specified labels, the rule-based system cannot function accurately. This property of fuzzy rules also facilitates the engineering of fuzzy systems.

Expert systems and conventional rule-based systems are based on classical binary logic and are unable to represent vague concepts as effectively as

FL. A conventional expert system used to control the thermostat of an air conditioner would still be forced to choose a threshold temperature for deciding what is "too hot" or "too cold". In contrast, an expert system coupled with FL can avoid this trap by using a "fuzzy" representation for temperature and using fuzzy rules of the form: "If the difference of the current temperature and the desired temperature is *small* then open the valve a *little*", where the imprecise concepts *small* and *little* are represented using FL.

As a final point it must be emphasized that because fuzzy sets are a generalization of classical Boolean sets, the fuzzy inference procedure is also valid with crisp non-fuzzy data. Thus the fuzzy inference procedure can be considered as a generalization of conventional rule-based inference. Consequently it is easy to build systems which integrate fuzzy inference and conventional rule-based inference procedures.

4. Two Case Studies of Fuzzy Logic Applications

Two examples of real life FL applications are described below. The first is a simple, but nevertheless complex, inverted pendulum system. The inverted pendulum system is well suited for laboratory experiments and also helps to clearly illustrate the methodology of using fuzzy rules for heuristic control of physical systems. The second example is a limited description of a large real life FL application: the Sendai subway control system. Both examples aim to provide the reader with a better idea of how fuzzy rules are actually used in applications.

4.1 The Inverted Pendulum

The inverted pendulum is one of the classical systems used for studying control procedures. The problem consists of applying a suitable force to balance an inverted pendulum. In principle, it is very similar to balancing a stick on one's hand by moving the hand forward or backward as necessary. Figure 9 depicts a simple inverted pendulum in which a motor applies the required force for balancing the pendulum. In this simple system, there are two inputs and one output. The inputs are the angular displacement of the bob, θ, and the angular velocity of the bob, i.e., rate of change of θ with time ($d\theta/dt$).

Figure 9 about here

Despite its simplicity, the inverted pendulum is a non-linear system (i.e., the required current strength varies as a non-linear function of the angular displacement and the angular velocity). The system can be modelled accurately, but requires the solution of a second order differential equation. As is usual, some simplifications (such as the approximation of $\sin\theta$ by θ) are introduced for solving the equation quickly. These simplifications are not always valid and can be violated (e.g., the approximation of $\sin\theta$ by θ is not valid for large θ, i.e., for large displacements). A correct solution would require the use of special numerical methods (which are computationally intensive). Moreover, a solution of the equations to balance the pole is not easily possible as the solution point is unstable with respect to the system's initial conditions. The solutions are also sensitive to various parameters such as the mass of the bob, the length of the shaft and the strength of the motor.

Accurate and robust control of such a system can be obtained by using a set of fuzzy rules of the form:

IF Angular_displacement is *small* **AND** Angular_velocity is *small*
THEN Force_applied is *small*

Figure 10 depicts a set of 11 rules which can be used to control a simple inverted pendulum [14]. Theta and Dtheta are used to represent the angular displacement and the angular velocity of the pendulum respectively. It is also assumed that an electric motor is being used to apply the necessary force to balance the inverted pendulum. NM, NS, Z, PS and PM are used as acronyms for the fuzzy sets *negative_medium, negative_small, zero, positive_small* and *positive_medium* respectively. The shapes of these fuzzy sets are as shown in Figure 11 (not drawn to scale).

Figures 10 & 11 about here

Note that the input values of Theta and Dtheta are crisp. Also a crisp value has to be given to the motor as the recommended value of current to balance the inverted pendulum. The inference procedure with crisp input values is similar to that for fuzzy input values (recall that fuzzy sets are

generalizations of classical sets) and is graphically depicted in Figure 12. Two fuzzy rules from Figure 10 are shown in Figure 12. Note that now there are two premise clauses in the rules. The input values of theta and dtheta are 0 and an arbitrary value u respectively. The highest degree of match between the input values of theta and dtheta and the premises of rule 1 are given by r_{11}(=1) and r_{12} (a number around 0.5 as shown in Figure 12). Note that r_{11}=1 as the input value of theta can be considered as a fuzzy set of one element, 0, with a degree of membership of 1 for that element. The contribution of the rule conclusion to the output is the portion below the minimum of the two degrees of match, r_{11} and r_{12}. Note that there is no contribution from rule 2 to the output in Figure 12 for the shown input as r_{21} = 0 (the minimum of r_{21} and r_{22} is 0). The rest of the inference procedure is the same as shown in Figure 7.

Figure 12 about here

Note that in Figure 12, even though crisp values are used as inputs, the output of the fuzzy rules is a fuzzy set. However for specifying the current for the motor, the output fuzzy set has to be *defuzzified,* i.e., converted into a crisp number that can be specified as the desired value of the current. The center of mass defuzzification technique (of Figure 8) is used in Figure 12. Thus the value v (which is close to zero) in Figure 12 is the final output crisp value for the current.

The fuzzy rules used for controlling the inverted pendulum just use as inputs the angular position and the angular velocity of the bob. Thus even if the mass of the bob or the motor strength is varied dynamically, the same 11 rules can be used successfully for balancing the inverted pendulum. This is in contrast to conventional control methods which are very sensitive to changes in these parameters. Obtaining the correct fuzzy rules requires more of experience and engineering skills than sophisticated mathematics. It has been observed that the performance of the system is not very sensitive to the precise definitions of the various fuzzy distributions used. This makes the task much easier from an engineering viewpoint. Also, the system is very robust to noise and breakdowns. Even if one or more rules in Figure 10 are disabled (for example, by a hardware error), the remaining active fuzzy rules are still able to control the inverted pendulum, albeit not as steadily as

before. This graceful degradation is one of the attractive properties of fuzzy control.

Note how the inverted pendulum system is modelled not by a mathematical model but by a set of heuristic rules. These rules use linguistic variables defined on the input and output domains under consideration. The rules used in other fuzzy logic applications (such as the Yamaichi trading program) are analogous to these fuzzy rules in structure and function.

There are several complex variations on the basic inverted pendulum shown in Figure 9. These include:

• An pendulum with a soft, pliable rod. The pendulum of Figure 9 assumes that the connecting rod is rigid. Without the assumption of a rigid rod, the system becomes enormously more complex to model and solve precisely.

• A pendulum with 2 or 3 stages, i.e., in which the (rigid) rod holding the pendulum bob consists of one or two joints. Such a system can be modelled fairly accurately, but the resulting control equations are complex enough to demand the use of supercomputers to solve them in real time.

It is nearly impossible to build a real-time controller (with current hardware) to control the above "complex" variations of an inverted pendulum. However, researchers from Aptronix (China/USA) and many other companies in Japan have successfully used FL to successfully control these complex inverted pendulum structures. The fuzzy rules used are similar to those shown in Figure 10. The success of FL in controlling such complex systems (where conventional control techniques have failed) has endeared it to engineers who find it an extremely practical and reliable tool for controlling complex processes.

4.2 Sendai Subway Control

Research on the automation of train operations in Japan started around 1960 with the first automatic train operation (ATO) devices being used for diesel locomotives in 1968 and in the Shinkansen bullet trains during the 1970's [2,12].

A conventional ATO system is usually based on PID (Proportional, Integrated and Differential) control. PID control is a popular conventional control procedure used in many industrial control systems. Its essence consists of taking the difference of the desired and the actual values (of a certain variable), aggregating it over time and generating a proportional feedback corrective signal. The variations in the train speed with the use of an ATO system is depicted in Figure 13. There is usually a preset target speed (below the maximum speed limit) which the train ATO system tries to reach by generating appropriate powering and braking commands. In actual practice, there are many changes in the running conditions such as the gradient of track and the braking force of rolling stock. To follow the target speed, it then becomes necessary to send control commands frequently for acceleration and brake application. As a result, the smooth operation of the train becomes difficult and riding comfort degrades. Though PID control is very popular, one of its major weaknesses is that it works best for linear systems, i.e., systems in which the output variables vary with some linear combination of the input variables. A train is a complex non-linear system (note that even the simple inverted pendulum system described earlier is an example of a non-linear system) and all ATO systems use simplified linear representations (of the train's operations) to apply PID control. Needless to say, it is very difficult to construct linear models of a train which are also fairly accurate.

Figure 13 about here

Though a careful manipulation of numerical parameters can let the train follow the target speed limits fairly closely, the requirements on automatic train operations have diversified recently to include other factors such as energy savings and improved riding comfort. Satisfying these additional requirements with the use of conventional techniques is near impossible task. This was realized by Hitachi engineers working on the Sendai subway line who spent many years trying to improve in vain the existing PID control systems to meet the newer performance requirements. Towards the early 1980's they started looking into the use of fuzzy control for automatic train operation on the Sendai subway system. The results have been a dramatic success.

The fuzzy control procedures used in the Sendai subway system consist of a set of fuzzy rules quite similar in structure and function to those used for the control of the inverted pendulum. These rules attempt to emulate the reasoning and control decisions made by an experienced subway driver as he maneuvers the train down a certain track. The control rules not only attempt to minimize the difference between the desired speed and the actual speed but also satisfy other goals such as "good riding comfort" and "accurate stopping". For example, consider the choice of an optimum "notch" for the traction and/or braking. To enhance riding comfort, it would be desirable to minimize the changes in the selected notch. The fuzzy control rules thus also take into account factors like the predicted speed at some later time, the time elapsed after previously changing notch and the judged coasting point. Figure 14 gives a qualitative feel for the fuzzy distributions for some of the input variables used in the fuzzy control system.

Figure 14 about here

Consideration of such factors is difficult with conventional PID control. It has been observed that the use of fuzzy control results in about 3 times less changes in the notch settings as compared to PID control. The enhanced results obtained using fuzzy control are depicted in Figure 15.

Figure 15 about here

The use of fuzzy control in the Sendai subway system has also resulted in energy savings of about 10% and better stop accuracy, i.e., the accuracy with which the door markers on the station are followed. It has been seen that the standard deviation of the errors in stopping are also about 3 times less than that with PID control.

5. Commercial Activities in Fuzzy Logic

This section describes the commercialization of FL technology with particular emphasis on activities in Japan, Europe, and America.

5.1 Observed General Trends

FL is one of the fastest growing sectors of the commercial applications of AI technology. According to experts, FL is forecasted to become a 2-to-3 billion dollar business within a decade [32]. Already there are several hundred successful applications of FL. In Japan alone, there are more than 2000 FL patents. The success rates of FL applications have been phenomenally high. According to Prof. Sugeno, an expert on AI and FL [22], the success rates of FL projects (in Japan) has been around 80%.

There are two broad categories of commercial applications of FL:

- **Industrial process control:** FL is being used successfully for modelling and controlling complex industrial systems/processes [16,30,31,37]. Examples of such applications are the controls of the Sendai subway system and the cement kiln controllers marketed by F.L. Smidth & Co. of Copenhagen.

- **Modelling human intelligence**: The coupling of FL and expert system technology has lead to the development of fuzzy expert systems [38,39,44,49]. Using the representation and inference techniques of FL, these expert systems are adept at modelling the imprecise reasoning processes of human experts and dealing with an uncertain environment. Examples of such applications are the trading program of Yamaichi securities and the fuzzy expert system chip in Toshiba cookers.

Though the theoretical foundations of FL have largely been developed in the USA and Europe, Japan has until recently monopolized the commercialization of the technology. Most important Japanese companies are investing actively in FL applications and almost all "show-case" applications of FL are from Japan. The reasons for this lop-sided adoption and development of the technology are not entirely clear. Even within American academic circles, acceptance of FL and its successes have been slow in coming and there has been much debate about the benefits and limitations of fuzzy logic [5,17,29]. Some academics argue that there is "no need" for FL as everything that can be done by FL can be done by conventional approaches. Zadeh notes that while in principle this may be

true, using FL is much faster and cheaper [18]. Given the competitive pressures on costs and time to market, few companies can afford to neglect technology which promises to cut costs and development time. Others ascribe the cool US reception to FL to the name "fuzzy logic" , which tends to give naive readers a "fuzzy, uneasy and confused" feeling. Prof. B. Kosko of USC compares the name "fuzzy logic" to a "boy named Sue" [7]. Yet others ascribe it to deeper cultural factors [11]. They argue that Asian cultures, and in particular the Japanese, have shown little resistance to the adoption of the new approaches espoused by FL because their cultures are not so deeply rooted in scientific rationalism (in contrast to Western cultures).

Whatever the actual reasons for the enthusiasm of the Japanese or the circumspection of Americans, the fact remains that Japanese companies have been the most active and successful in the application of FL technology. Competitive pressures in the market are now forcing American industry into seriously looking at FL. Today, there is a renewed surge of interest in FL in the USA. The European response has been markedly more enthusiastic than that of the Americans. Major FL research establishments exist in most European countries and European industry adopted FL technology much before their American counterparts. The following sub-sections briefly summarize the status of commercial activities in FL in different parts of the world.

5.2 Asia

The two most important players within Asia are China and Japan, with India and Singapore playing minor roles. China boasts of the largest collection of researchers (over 10,000!) working on FL, and Japan has played the dominant role in the commercialization of the technology. All major Japanese companies are actively involved in the application of FL. According to a sources within Mitsubishi, all their products can potentially use FL to improve their performance [13]. FL has proven to be an extremely useful alternative methodology for industrial process control. However, the availability of the "fuzzy chip" with the resulting capability to embed an expert system cheaply and effectively inside each individual product has been the primary catalyst in the large scale commercialization of FL. Today, virtually all Japanese makers of home appliances have either already applied FL in many of their products or are planning to do so in the near future. Some

representative examples from the wide range of current applications of FL applications in Japan are given in Table 2.

Table 2 about here

Omron, a $2.6 Billion Kyoto based manufacturer, is perhaps the most active Japanese company in the development and sale of fuzzy products. Since early 1989, Omron Tateisi Electronics have been selling fuzzy processor chips for control and simple pattern recognition tasks. It also sells a fuzzy software tool designed to create and run fuzzy rules on the IBM PC and has developed hardware boards which can be plugged into the expansion slot of an IBM PC to enable faster execution of fuzzy rules. Omron has recently announced the development of a fuzzy computer, although it is not yet sold commercially. It has also developed the first industrial temperature controller using FL. It projects sales of $750M, or nearly 20% of total revenues by 1994 from the sale of fuzzy hardware, and hopes to include fuzzy functions in 20% of its manufactured end products (which range from ticket vending machines to health care diagnostic products). Omron has filed about 600 FL patents in Japan. In Aug. 91, Omron created a strategic alliance with NEC, the world's number one micro-controller supplier, for the development and marketing of FL micro-controller chips. The commercial importance of this development can be understood by noting that the current world market for micro-controller chips is over $ 5.2 Billions [21].

The strongest endorsement for FL and is successes has come from Japan's Ministry of International Trade and Industry (MITI) which is now throwing its weight behind the rapid development of FL in Japan. It has set up a consortium of 48 top Japanese companies to develop fuzzy systems with a five year initial life (till 1994). The consortium is called the Laboratory for International Fuzzy Engineering (LIFE) and has a $70M budget coming from member companies such as Cannon, Fuji, Fujitsu, Nissan, Honda, Minolta, NEC and Omron. Its board of directors is equally impressive, consisting of the Presidents of Hitachi, Toshiba, Nissan, Nippon Steel, Minolta, Matshushita, Fujitsu and others. LIFE is definitely intended to be the world's premier center for FL development. Its efforts are to be focussed in three areas: (a) fuzzy control of industrial processes (b) fuzzy information processing and (c) building fuzzy computer systems (hardware and software). In a separate effort, Prof. Yamakawa has set up (in March 1990)

the Fuzzy Logic Systems Institute (FLSI) in Fukuoka, Japan with the sponsorship of MITI, and in collaboration with various Japanese universities and companies. FLSI has an initial asset base of 100 M Yen and aims to further stimulate FL research and applications in Japan.

Japan' Ministry of International Trade and Industry (MITI) is clearly very impressed by the success rates of FL projects. Also, given the poor results of Japan's 5th Generation project, MITI is now turning to FL for providing a new base for artificial intelligence. The director of LIFE, Toshiro Terano, has been quoted [22] as saying that fuzzy logic "technology harmonizes computers with humans, making it perfect for forming a mathematical model of a human in his society, building intelligent machines and vastly improving the human/machine interface". The ultimate aim of LIFE is to build more "human" computers, ones based on fuzzy rather than binary logic and which can execute programs more closely resembling natural language. Long range plans for MITI call for the development of a prototype of such a computer by the end of 1994. The support of MITI to FL is being viewed with some concern in the USA. The memories of other successful MITI efforts jointly coordinated with academia and industry are fresh. Japan already has a preeminent position in FL research and development. LIFE and FLSI aim to consolidate and solidify this leadership.

5.3 America

Though FL technology was invented in the USA, it has until recently, been largely ignored by the US industry. Today, partly as a result of the increasing competitive pressure of Japanese products, American companies are taking a serious look at the technology to judge its usefulness and applicability. Despite the slow take-up of FL technology by American industry, a few American companies have been successful in building FL applications, and in producing leading edge fuzzy hardware and software.

The first US patents in FL have been awarded to Nissan for its fuzzy transmission and anti-braking systems. NASA has recently announced (in May 1992) the first successful application of FL technology in outer space. FL was used in the Commercial Refrigerator Incubation Module (CRIM) aboard the Space Shuttle Endeavor, STS-49, which was launched on May 7, 1992. The CRIM provides a regulated thermal environment for

experiments such as the growth of protein crystals. NASA has other FL applications in the prototype stages. One of the most advanced projects is a controller for maneuvering the space shuttle to keep it in position with respect to another object in space. In response to competitive pressures created by FL elevators marketed by Hitachi and Mitsubishi, Otis Elevators is now using FL controls to help elevators respond rapidly to changing demands. Southwestern Bell Corp. has used FL to build a sophisticated information retrieval system which reduces the problem of calling up unwanted citations. General Electric is considering using FL to regulate jet engines, save water in home appliances, and drive 200 ton rollers in steel mills [1]. Allen-Bradley (Milwaukee) plans to introduce FL controllers to replace its line of conventional PID control systems.

Similar to the alliance between Omron and NEC, Motorola has created (in Feb. 92) a strategic alliance with Aptronix Inc., a small San Jose company specializing in FL products. Steven C. Marsh, the director of strategic operations at Motorola's micro-controller division estimates that by 1995, about half of all Motorola micro-controllers will incorporate FL [1]. In another major development, MCC of Austin, Texas, has recently added FL as its fifth major research area (in addition to artificial intelligence, natural language, neural networks, and database technology) and has plans to spin off a new company, Pavillion Technologies, to exploit the commercial potential of FL. Note that MCC is an industry consortium whose members include Bell Communications Research Corp., Control Data, DEC, Kodak, and NCR.

Despite the slow commercialization of FL in America, there are a few companies in the USA which sell fuzzy hardware comparable to and sometimes better than those produced in Japan. The two most active companies are Togai Infralogic of Irvine and Aptronix of San Jose. The founder of Togai Infralogic, M. Togai, was the co-designer of the first fuzzy chip at Bell Laboratories. Though over 90% of their sales did come from Japan in the past, they are currently experiencing a strong spurt of interest in FL and fuzzy chips from the US market. They predict that by the mid-1990's, their sales will be evenly split between Japan, the USA and Europe. Togai Infralogic also markets a variety of software packages. For example, the TilShell is a object oriented software which allows the creation of stand-alone FL expert systems or the integration of FL inference capabilities into existing expert systems. Aptronix has a rather interesting background. It is

founded by Wei Xu and a group of Chinese researchers and was originally based in Tokyo, but it moved to San Jose in the wake of the Tiananmen Square episode. Aptronix markets Fuzzy Inference Development Environment (FIDE), a general purpose fuzzy software development environment and has recently entered into a potentially important strategic alliance with Motorola.

Fuzzy chips are also manufactured by the Microelectronics Center of North Carolina, a non-profit research consortium in Research Triangle Park, N.C. These chips have the distinction of being among the fastest today on the "fuzzy chip" market - being able to perform about 580,000 fuzzy inferences per second. Another small player is Micro-Devices of Orlando, Florida, which markets a version of a fuzzy chip. A variety of expert system vendors are also beginning to incorporate FL inference capabilities in their expert system development packages. Some examples are Micro Data Base Systems Inc. of Lafayette, Indiana (in their Guru system) and Knowledge Based Technologies of White Plains, N.Y. (in their Telus system).

American companies have yet to match the Japanese in unleashing a wide range of everyday appliances using FL. However, most major US companies including General Electric, General Motors, Hewlett-Packard, Honeywell and Rockwell are planning to use FL more actively in the future. More commercial products from these companies should hit the market within a few years.

5.4 Europe

European academia and industry have traditionally been more receptive to FL technology than their counterparts in America. A reason for this is perhaps the fact that conventional AI technology (with its accompanying emphasis on Boolean logic) is considerably more entrenched in the USA as compared to Europe. Over the years, significant new theoretical developments in FL technology have come from European researchers.

France and Germany are perhaps the two leading European countries in FL research and applications. Major FL research centers in France include IRIT (University Paul Sabatier, Toulouse), LAFORIA (University of Paris 6, Paris), NEURINFO (Institut Mediterraneen de Technologie, Marseille), CRAN-LEA

(Vandoeuvre) and the University of Valenciennes. In September 91, the club CRIN "Logique Floue" (supported by French academic and industry) was created with the encouragement of the French government to stimulate research in FL theory and applications. Another research group (formed in Jan. 91), GDR Automatique et Commande Floue, is composed of 20 laboratories and 30 companies, and aims to encourage FL developments within French industry. Also, in 1991, the European Laboratory for Intelligent Techniques Engineering (ELITE) was started in Aachen, Germany. The goal of ELITE is to coordinate and integrate scientific activities in different intelligent technologies including FL, neural networks, and expert systems. ELITE has research organizations and companies from across Europe (Germany, France, UK, Benelux, Italy, Finland, Austria, and Spain) as partners. Strong FL research interest groups also exist in most other (both Western and Eastern) European countries. European governments and the European Community are supporting and stimulating FL research and applications. For example, the French government has been sponsoring special missions to Japan for French scientists and industrialists to study the applications of FL in Japan.

While some of the earliest industrial applications of FL occurred in Europe [19,25], European industry is far behind the Japanese in exploiting the commercial potential of FL. One of the very first commercial applications of FL in France was in 1989, and it involved the control of the furnace in a cement factory in Rochefort (Jura). The group PSA (Peugeot-Citroen) is using FL for automatic braking in its vehicles equipped with the Interactive Road Sign System (ISIS). Researchers at the University of Valenciennes are researching the application of FL to the Paris metro. Dassault is considering using FL for guidance in its flight simulators. Moulinex plans to use FL in its next line of ovens and vacuum cleaners. Usinor Sacilor is using FL in its high temperature furnaces. Many other French/European companies such as EDF-GDF, Aerospatiale, BNP, and Credit Lyonnais are also considering using FL in their operations. SGS Thompson announced in Aug. 1991 that it is investing $ 30 Millions over the next five years to develop FL hardware and micro-controllers. Two French companies, ITMI and Tekelec, are actively working in the domain of fuzzy hardware and applications. Omron has also recently established a subsidiary in Paris to sell fuzzy products to French and European companies.

Similar FL applications and products can be found in other European countries also. A German company INFORM sells a variety of state-of-the-art fuzzy hardware and software products. These include FuzzyTech, a graphical fuzzy software development environment, and a fuzzy chip (developed in collaboration with Siemens). Various research prototypes of FL applications have been developed at the RWTH University of Aachen. Different German companies such as Siemens, Volkswagen, and Software AG. are considering using FL in their products.

On the whole, the adoption of FL within European industry started earlier than American companies, but they are still far behind the Japanese. In fact, because American companies have reacted very sharply in the recent months to the competitive onslaught of Japanese FL products, America is fast overtaking Europe in the successful commercialization of FL.

6. The Strategic Business Impact of Fuzzy Logic Applications

FL is proving to be an important source of competitive advantage in business. The impact of FL can be analyzed along several dimensions. The simplest of these is that it is adding to expert system technology by introducing new representational and modelling techniques. Of a more fundamental nature is the impact it is having on changing the granularity of intelligence. With the help of a fuzzy chip, it is possible to embed intelligence into individual products. Also very important is its role in transforming industrial process control and enabling new product development strategies. This section concludes the paper by discussing the major strategic impacts of fuzzy logic applications.

6.1 Enhanced Modelling of Intelligence

An important deficiency of conventional knowledge based systems is in dealing with vague data and the modelling of imprecise reasoning procedures. The integration of expert system concepts and FL technology has led to a more robust modelling of uncertainty and imprecision. The utility of this has been demonstrated by the successful integration of the intelligence of skilled personnel in the examples of the Sendai subway controls and Yamaichi's computerized trading program. This ability is also

being exploited in building more intelligent user interfaces to databases (which allow queries of the form "find companies with high revenues").

Another common limitation of conventional expert systems has been their "brittleness". Expert systems tend to embody a lot of detailed knowledge about a specific domain and can usually reproduce sophisticated reasoning procedures when given a problem that is contained within the rules in the knowledge base. However when presented with a minor variation of the same problem, they are usually unable to deal with this slight variability and tend to "crash" or give meaningless answers. This is because the rules in an expert system are based on Boolean logic and require a Boolean match (either true/false) on the input description. FL allows rules to have a partial match with the input description and produce meaningful answers under a broader range of inputs.

Conventional expert systems have also been plagued by their dependence on one (or a few) critical rule(s). As the rules fire sequentially, a missing match on the critical rule can lead to a breakdown in the inference process. FL has helped alleviate this limitation. An useful analogy is with autocracies and democracies. A conventional expert system can be compared to an autocracy, where all decisions rest with a single person, the autocrat (the critical rule). Taking the autocrat out of the system can lead to a break-down of the decision process. On the other hand, a fuzzy expert system can be compared to a democracy, where all members (rules) participate in the decision process. The process still functions (albeit not equally effectively) with one or more members missing.

6.2 Embedded Intelligence

FL has changed the granularity of intelligence. The invention of the fuzzy chip is doing for artificial intelligence what the microprocessor did for computing. It is useful to draw this analogy. The invention of the personal computers during the 1970's distributed the presence of computers within an organization, but it was only with the invention of the microprocessor that the applicability of computers to everyday products took off in a big way. A microprocessor is like a small computer on a chip and it could be easily (and cheaply) inserted into products to perform various functions. In enabled a whole generation of computerized products - much of what we use regularly

today. The history of artificial intelligence and expert system technology is quite similar. Though commercialized during the 1980's, expert systems technology has till now been software based requiring (at the least) fairly powerful personal computers to execute. This has limited the distribution and use of expert systems. With the fuzzy chip, it is now possible to distribute intelligence at the ultimate granularity - inside every individual product.

It is now possible not only to computerize products, but to do it smartly. Such "embedded intelligence" makes the products much easier to use for the end user and more efficient in its performance. It is thus not surprising that these new appliances are meeting with a huge success in the Japanese market. Distribution of intelligence at such a fine granularity is not possible with conventional software based expert system technology (which is too bulky and expensive to distribute within each product). Thus FL is spawning a whole new generation of products much like what the microprocessor did a few years ago. The business impact of this is obvious. Today customers are very demanding in their needs and companies face an increasingly competitive market world-wide. It is no longer sufficient to produce one good product. Innovation is critical to success. Making products more intelligent is one means of innovation. FL is proving to be invaluable in this process.

With this ability to "embed intelligence", more daring and innovative products shall hit the market in the future. Plans for the future include products with small diagnostic fuzzy expert systems for trouble shooting and trouble free maintenance. The impact of this ability to distribute the expertise and knowledge of experienced technicians inside products shall have far ranging impacts on the quality of products marketed. It shall at the least, extend the notion of "total quality".

6.3 Intelligent Process Control

Industrial process control saw the earliest commercial applications of FL. Today, it continues to be one of the most important application areas for FL [16,30,31,37]. Most industrial systems are complex and non-linear. Conventional control theory is most developed for controlling relatively simple, linear systems. The traditional approach to industrial process control has been to develop a linear model of the system to be controlled and then apply some conventional approaches. The success of these approaches

has been limited by fidelity of the linear model in simulating the behavior of the actual non-linear system.

FL has given rise to a new form of process control. The essence of this new approach is the modelling of the complex non-linear relationships between the various parameters in the system by a set of (heuristic) fuzzy rules. Such an approach has proven to be very successful in practice. The use of rules for process control also allows the integration of the expertise of skilled personnel into the control procedures, as in the example of the Sendai subway control system. Though not as visibly as in home appliances with "embedded intelligence", FL is playing an important role in reshaping industrial process control systems. All major suppliers of industrial micro-controllers estimate that nearly all their products will contain FL within the next decade.

6.4 New Product Development Strategies

FL saves time and money. Omron, Japan's leading maker of factory controllers, claims to have slashed development time by 75% by using FL in a system (designed for Komatsu) to check machine tools for worn-out gears or dirty oil filters. Similar experiences have been reported by other Japanese companies also. The reasons for this are as follows. Precision is in general very expensive. While a mandatory requirement for many situations, it is often not desired for many engineering problems. Consider for example, the problem of programming an automated vehicle to park itself within a certain marked slot. In principle, it is possible to park the vehicle with a fine tolerance of a few hundredth's of an inch within the allocated space. But such an vehicle would not only be very difficult to build, it would also be very expensive to program and operate. Is this precision really desired? It turns out that satisfactory performance can be obtained using FL but minus the high cost of precision [26]. The solution is simpler to operate and easier to build. This tolerance for imprecision is very common in humans. We rarely try to park cars with all four wheels perfectly aligned and equidistant from the curb.

By not requiring that certain modelling choices be made (e.g., threshold temperatures or heights), FL builds into the heuristic modelling process a greater tolerance for imprecision. By directly representing heuristic

relationships between important input and output parameters, fuzzy modelling avoids dependencies on a variety of intermediate system parameters which complicate the modelling process. For example, note that fuzzy rules of Figure 10 used to control the pendulum of Figure 9 are independent of several parameters such as the mass of the bob and the length of the rod. Thus the same 12 rules of Figure 10 can be used to control pendulums with different bob masses and rod lengths. Such a flexibility is impossible with conventional modelling approaches.

Precise modeling of real world situations is usually impossible (certain simplifications and assumptions often have to be made), and even when possible, it is normally very expensive. FL has proven to a very handy tool for modelling real world complex situations. The developed models are usually simpler and give more accurate results, as shown by the experience of the Rockwell engineers (Section 1.4). The fact that the use of FL can give better products while saving both time and money has important implications for product design and development procedures. Few companies can afford to ignore a technology which holds the potential to cut product development costs and times.

6.5 Conclusion

According to most indications, the full commercial impact of FL is only starting to be felt. The long term implications for industry and business are enormous. While FL does not provide a panacea for all problems facing industry, it does provide proven benefits (as described above). The need to market more innovative products, and to do it cheaply and quickly is being increasingly seen as an important determinant of a company's ability to survive in the global market. Few organizations can afford to ignore a technology which helps to further its competitive position. It is therefore necessary for managers to understand the fundamentals of FL technology and to see if it can be creatively used for strategically important business processes in their own organizations. It is hoped that this paper aids this process.

Acknowledgements

The author had the advantage to observe the rapid commercialization of FL while working as a graduate student with Prof. L.A. Zadeh from 1985-89. Prof. Zadeh has been a continuing source of help and guidance. The author would also like to acknowledge the help of Togai Infralogic of Irvine, California for some insights into the commercial applications of fuzzy logic. Thanks are also due to an anonymous reviewer for helpful suggestions to make this paper more complete and useful.

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Year	Development
1965	First paper on FL by Zadeh
1972	Theory of application of FL to industrial control is developed by Zadeh
1974	First implementation of fuzzy industrial control by Mamdani & Assilian
1975	Theory of merger of FL and expert system technology is developed by Zadeh
1980	First commercial fuzzy expert system is marketed by F.L.Smidth & Co. of Copenhagen
1985	First implementation of a fuzzy expert system on a chip by Togai and Watanabe at Bell Laboratories
1987	• Sendai's advanced subway system based on fuzzy control is opened • First fuzzy computer is designed by Yamakawa
1988	Fuzzy chips are available commercially from Omron
1989	• "Fuzzy" home appliances start being sold in Japan • LIFE, a consortium of 48 top Japanese companies is started in Japan by MITI

Table 1: Milestones in the development of FL

Company	Application Domain
Hitachi	Automatic control for subway trains (in Sendai) yielding a smoother, more efficient ride with a higher stopping precision.
Hitachi	Control of container cranes in Moji Harbor, Kitakyushu, to increase precision in loading/unloading
Nippon Electric	Used for controlling temperatures in glass fusion at the Notogawa and Takatsuki factories
Cannon	Used for stepper alignment in semi-conductor production
Mitsubishi	Regulation of surface temperatures in ethylene cracking furnaces at the Mizushima plant.
Nissan	Anti-skid brake systems and automatic transmissions for cars
Subaru	Automatic transmissions for cars
Cannon Ricoh Minolta	Auto-focussing in cameras by choosing the right subject in the picture frame to focus.
Panasonic	Jitter removal in video camcorders by distinguishing between jitters and actual movement of subjects.
Sanyo	Iris control in video camcorders by helping react subtly and automatically to changes in light conditions.
Matshushita Toshiba Sanyo Hitachi	Vacuum cleaners that use sensors to gather information about floor and dirt conditions and then use a fuzzy expert system to choose the right program.
Sanyo Toshiba	Ovens and cooking ranges which incorporate the culinary expertise of chefs.
Matshushita	Air conditioners that makes judgements based on factors such as number of persons in room and optimum degree of comfort
Yamaichi	Computerized trading programs which mimic the approximate reasoning processes of experienced fund managers
Sony	In palmtop computers for recognizing handwriting

Table 2: Representative applications of FL in Japan

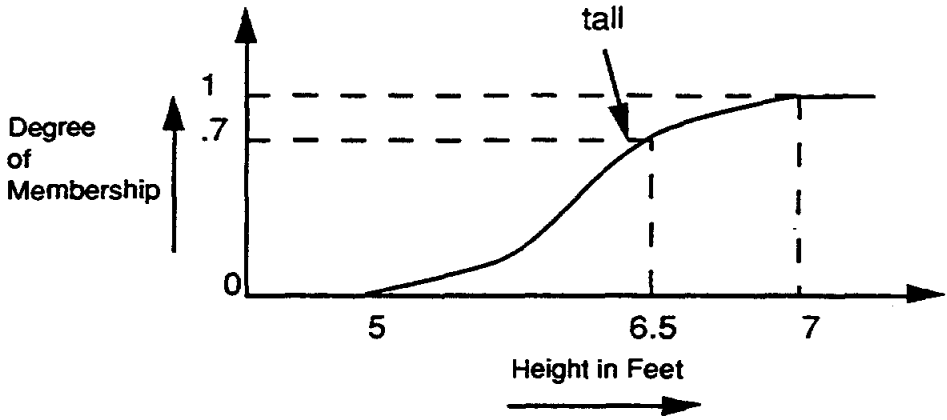

Figure 1: Representation of the concept "tall" using fuzzy sets

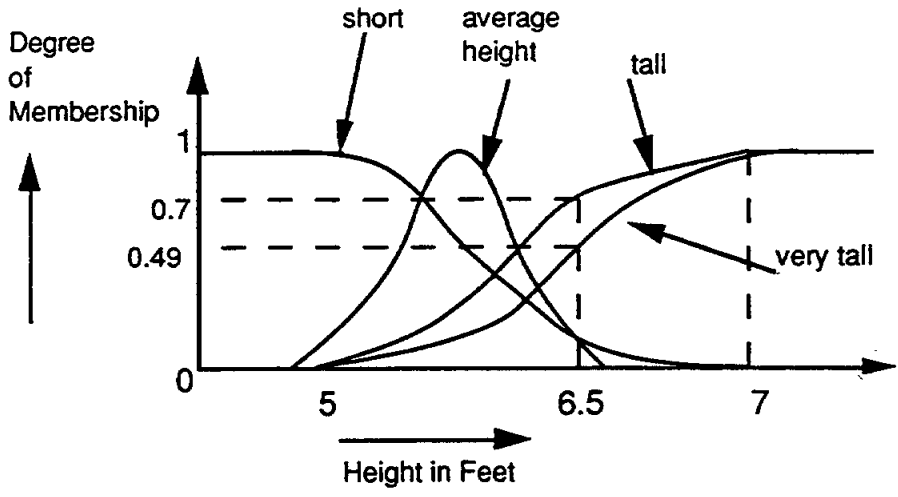

Figure 2: The fuzzy sets "tall", "short" and "average height"

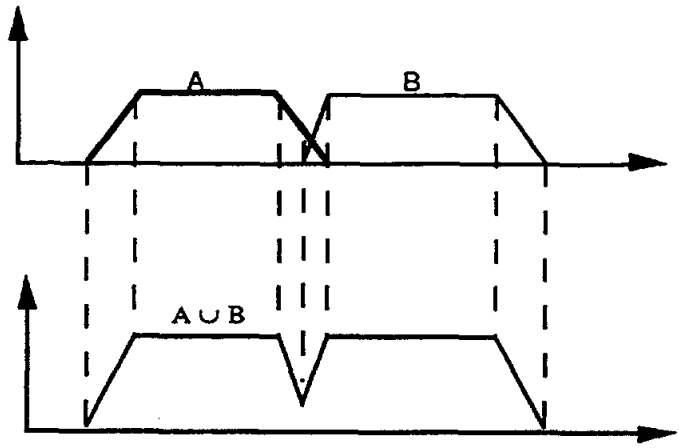

Figure 3: Union of fuzzy sets

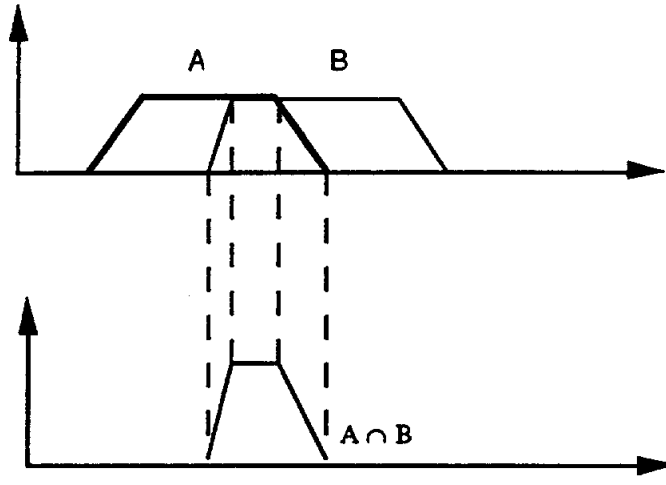

Figure 4: Intersection of fuzzy sets

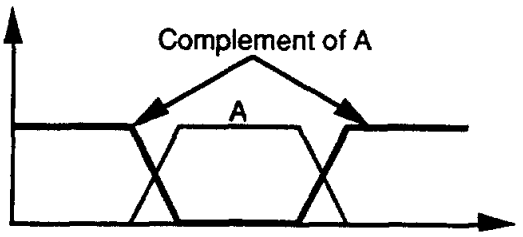

Figure 5: Complement of a fuzzy set

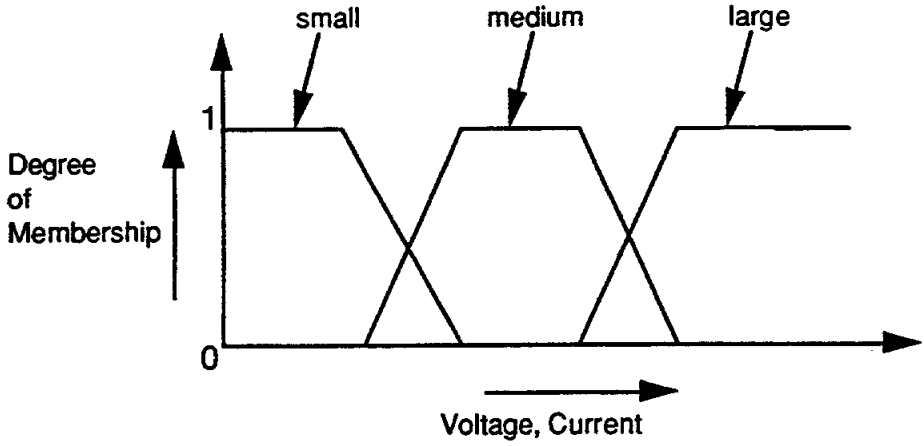

Figure 6: The fuzzy sets small, medium and large

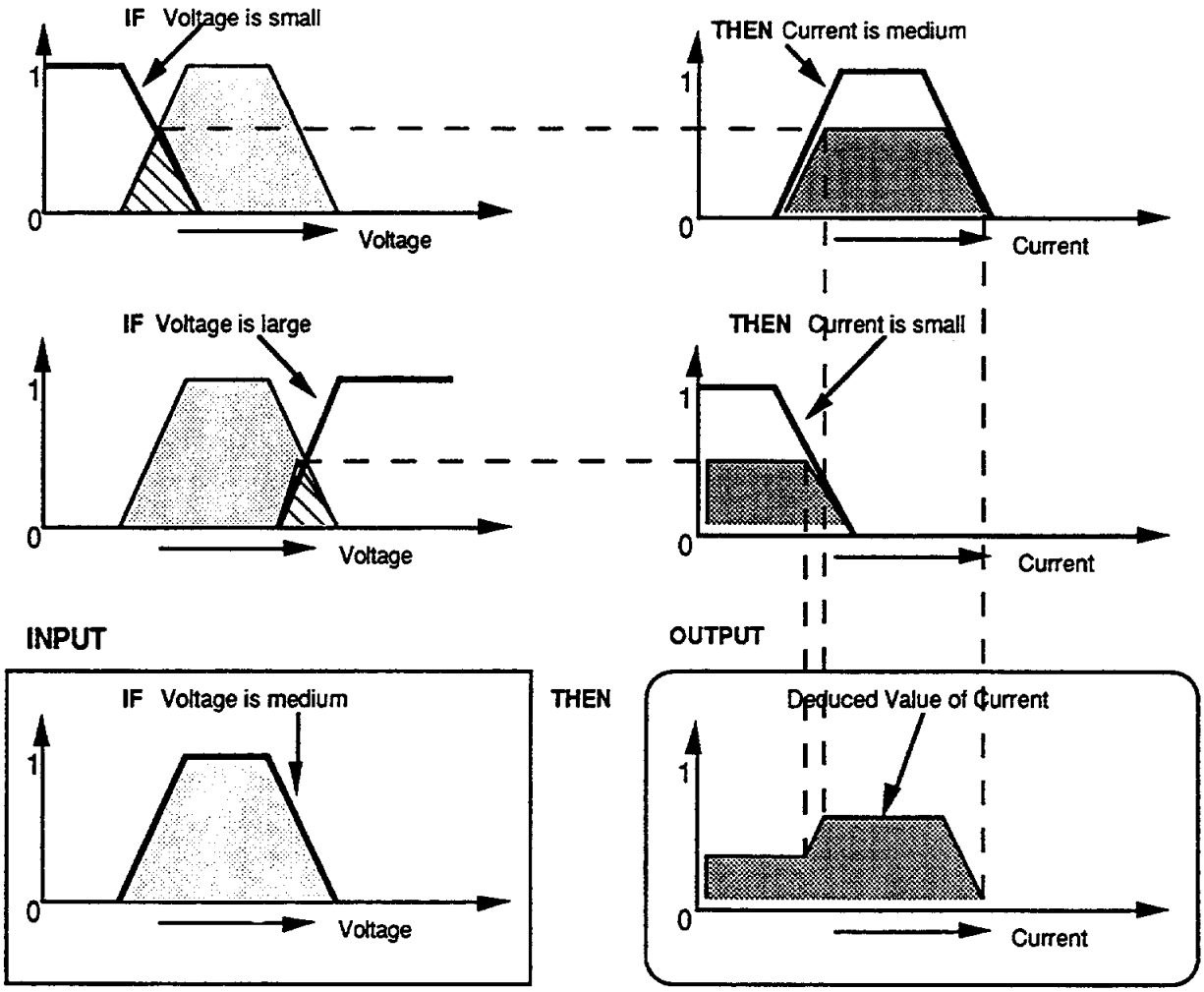

Figure 7: Approximate inference using fuzzy rules (fuzzy input)

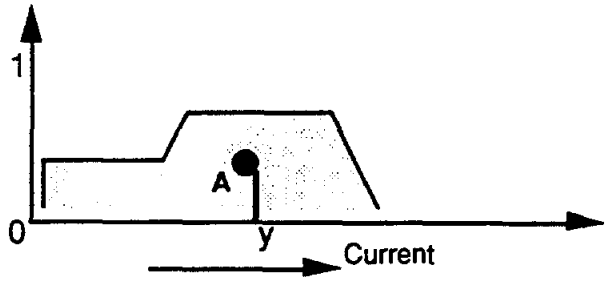

Figure 8: Defuzzification of a fuzzy set

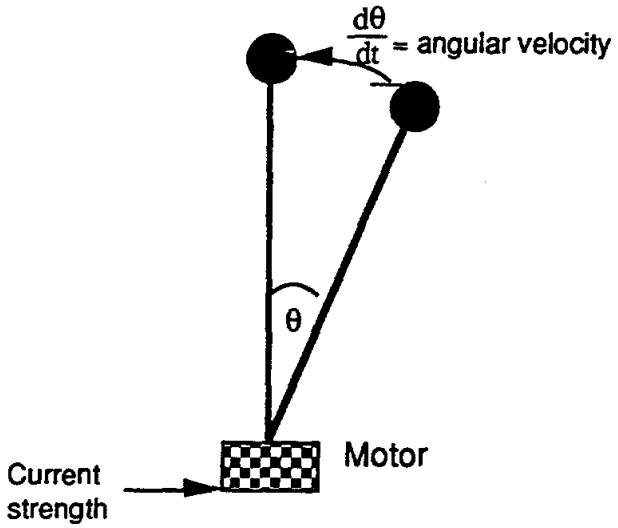

Figure 9: An inverted pendulum

IF	*Theta*	is NM	**AND**	*Dtheta*	is Z	**THEN**	*Current*	is PM
IF	*Theta*	is NS	**AND**	*Dtheta*	is Z	**THEN**	*Current*	is PS
IF	*Theta*	is Z	**AND**	*Dtheta*	is Z	**THEN**	*Current*	is Z
IF	*Theta*	is PS	**AND**	*Dtheta*	is Z	**THEN**	*Current*	is NS
IF	*Theta*	is PM	**AND**	*Dtheta*	is Z	**THEN**	*Current*	is NM
IF	*Theta*	is Z	**AND**	*Dtheta*	is NM	**THEN**	*Current*	is PM
IF	*Theta*	is Z	**AND**	*Dtheta*	is NS	**THEN**	*Current*	is PS
IF	*Theta*	is Z	**AND**	*Dtheta*	is Z	**THEN**	*Current*	is Z
IF	*Theta*	is Z	**AND**	*Dtheta*	is PS	**THEN**	*Current*	is NS
IF	*Theta*	is Z	**AND**	*Dtheta*	is PM	**THEN**	*Current*	is NM
IF	*Theta*	is PS	**AND**	*Dtheta*	is Z	**THEN**	*Current*	is Z
IF	*Theta*	is NS	**AND**	*Dtheta*	is PS	**THEN**	*Current*	is Z

Figure 10: Fuzzy rules used to control an inverted pendulum

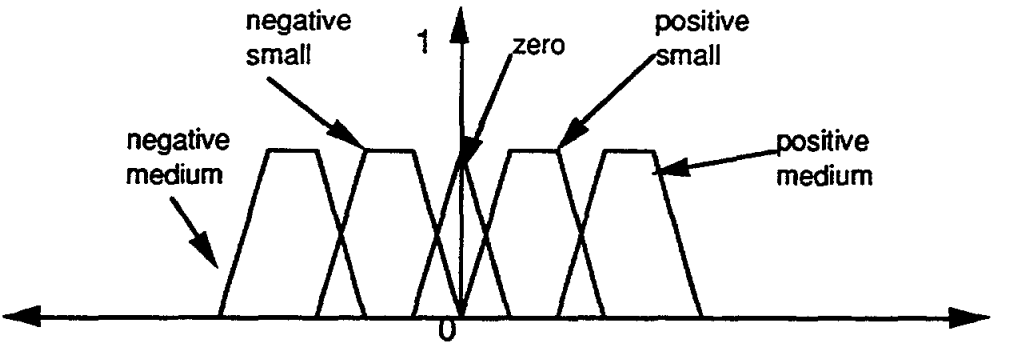

Figure 11: Fuzzy set shapes for the inverted pendulum

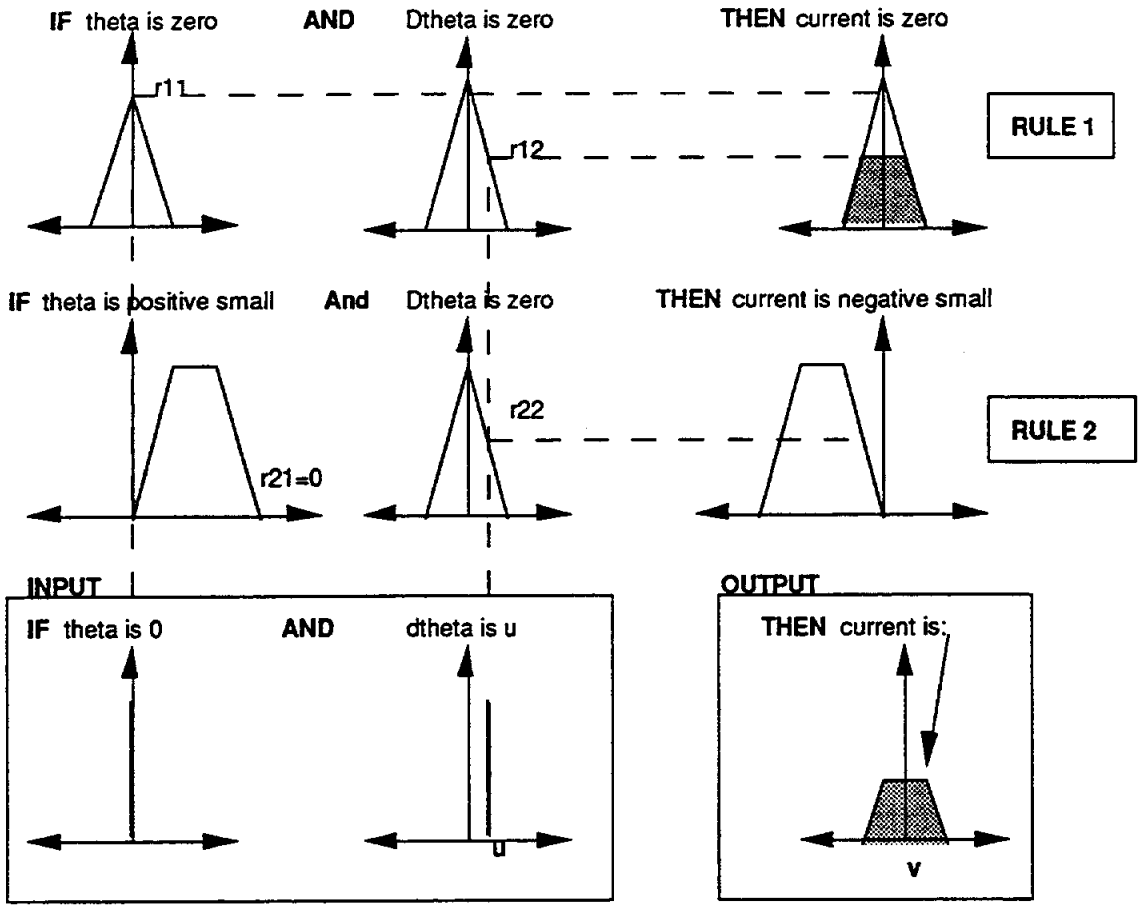

Figure 12: Fuzzy inference in the inverted pendulum

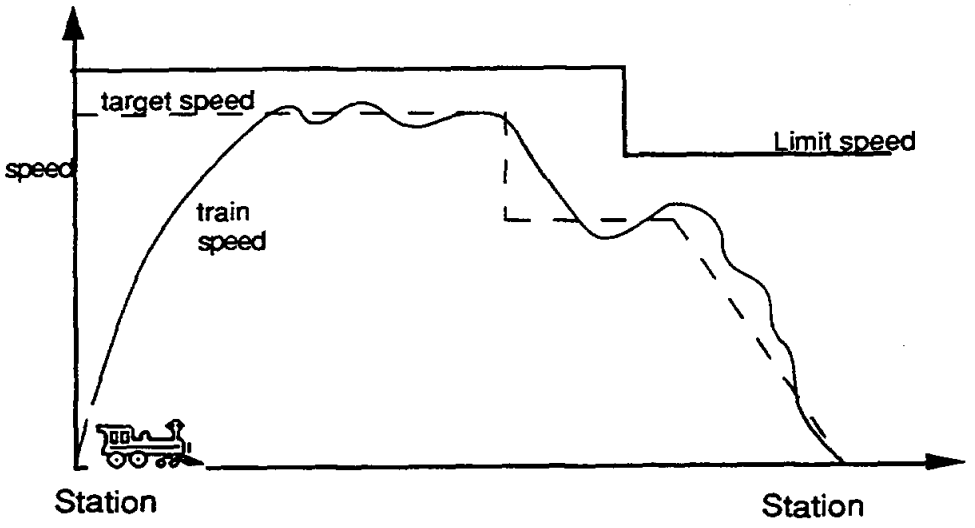

Figure 13: Automatic train operation by conventional control

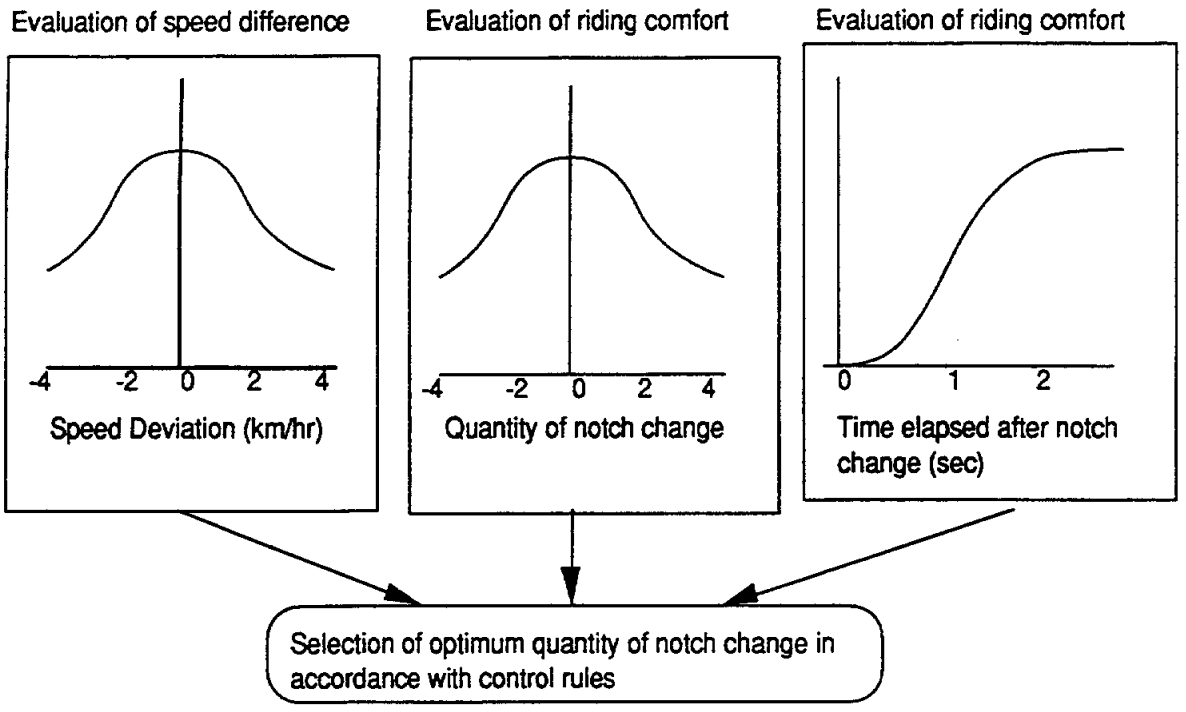

Figure 14: Example of fuzzy rule in automatic train operation

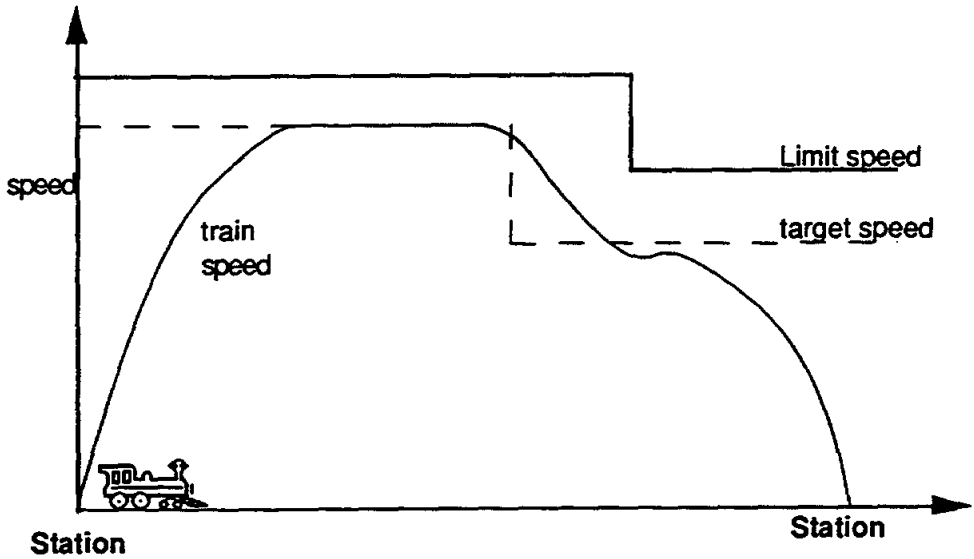

Figure 15: Automatic train operation with fuzzy control

Table & Figure Captions

Table 1: Milestones in the development of FL

Table 2: Representative applications of FL in Japan

Figure 1: Representation of the concept "tall" using fuzzy sets

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Figure 14: Example of fuzzy rule in automatic train operation[2]

Figure 15: Automatic train operation with fuzzy control[3]

Footnotes

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[1] Figure adapted from [2]
[2] Figure adapted from [2]
[3] Figure adapted from [2]