Endogenous Growth and Stochastic Trends

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Abstract: This paper shows that there exists a strong positive correlation between long-term growth rates and the persistence of output fluctuations in a cross section of countries. We argue that the traditional explanation of persistence, an RBC model with exogenous productivity shocks, cannot produce this correlation. We propose an explanation based on an endogenous growth model with exogenous cyclical shocks. We find that, despite the cyclical nature of the shocks, output fluctuations are persistent and the degree of persistence is an increasing function of long-term growth rates. Growth dynamics become an important component of the transmission of business cycles. We conclude that the analysis of economic fluctuations in models where technological progress is assumed to be exogenous can be misleading.

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1. INTRODUCTION

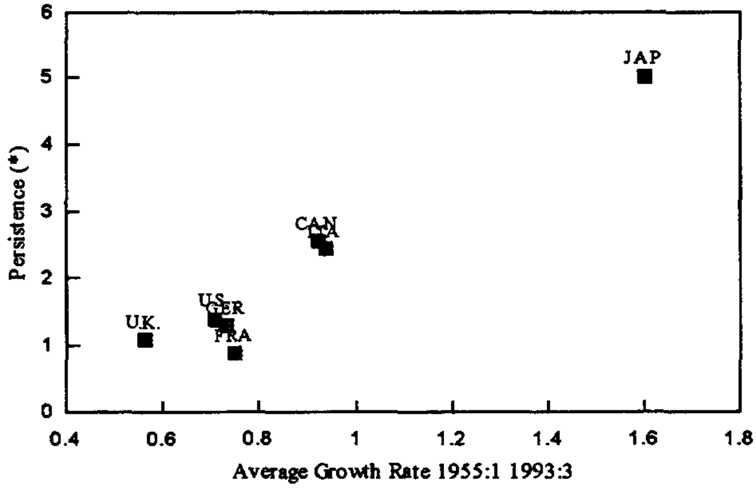
A common question in the literature on economic fluctuations is whether all business cycles are alike. Differences in business cycles across countries provide a good benchmark to test the predictions of alternative theories of the cause and transmission of economic fluctuations.

An important stylized fact of business cycles is the persistence of output fluctuations. By persistent fluctuations we mean output fluctuations that are long lasting. More precisely, a shock to GNP is persistent when its effects do not dissipate in the near future and GNP does not show a significant tendency to return to its trend level. After the work of Nelson and Plosser (1982), the prevailing view on persistence is that the effects of a shock last forever and, thus, GNP has a unit root. As a result, after a decline in GNP today, forecasts of GNP are lowered over any possible horizon. These results have been widely confirmed using data from different countries. For example, Campbell and Mankiw (1989) show that quarterly GNP, for the group of G-7 countries, is highly persistent. For all these countries, a 1% decline in output today, lowers the long-run forecast of output by even more than 1%. Their estimates of persistence display, however, large differences across countries and all countries, with the exception of the United Kingdom, exhibit fluctuations that last longer than in the US. For example, persistence in Japan is between 2 and 5 times larger than in the US. These results are further confirmed by Cogley (1989) who shows, for a similar sample of countries, considerable differences in the variability of the permanent component of output.

This paper analyzes cross-country differences in persistence and provides an explanation which has interesting implications for the interactions between growth and business cycles. We argue that an explanation to the observed differences in persistence requires a model where fluctuations are not just deviations around a trend determined by technological progress. A distinctive element of the explanation must be that the stochastic properties of the trend are not exogenous (as in the standard real business cycle model) but that they are the result of the endogenous response of technology to business cycles.

Figure 1 plots the degree of persistence (as measured by Cochrane's variance ratio with a window of 20 quarters) of quarterly GNP against its long-term (38 years) average growth rate for the group of G-7 countries. There is an almost perfect correlation between both variables. The standard explanation of persistence, an RBC model where fluctuations are driven by exogenous permanent

Figure 1. Persistence and Growth. G-7 Countries



(*) Variance ratio, window=20 quarters

productivity shocks, predicts no correlation between them.[1] More precisely, if in all these countries GNP followed a random walk with a drift we would expect all data points in Figure 1 to be lying on a horizontal line around $V = 1$. In fact, the X-axis variable of Figure 1 is generally absent in business cycle models. It represents the long-run trend of output which is commonly captured by a technological progress drift that is treated as exogenous. The positive correlation of Figure 1 indicates that analyzing significant features of the business cycle, such as persistence, in models where growth is treated as exogenous can be misleading.

In this paper we show that the positive correlation of Figure 1 is a natural outcome of a model where growth is endogenous and responds to business cycle fluctuations. The intuition is simple, as long as the amount of resources allocated to growth is procyclical, we expect transitory changes in the growth rate of trend GNP to produce long-lasting (permanent) effects in the level of output. Under this view, persistence represents a measure of the effects that fluctuations have on technological progress, which establishes a link between long-term growth and short-term persistence. Furthermore, as long as growth reacts *proportionally* to cyclical shocks, a fast-growing economy will have a larger degree of short-run persistence than a slow-growing one.[2] This is true even if the stochastic properties of the underlying shocks are the same for both economies. In other words, similar

[1] Unless one is willing to assume that a larger growth rate implies more persistent disturbances to the production function.

[2] The notion of *proportionality* will be precisely defined later in the paper.

shocks have larger permanent effects in a fast-growing economy.

Our explanation, by making explicit the interactions of economic fluctuations and growth, sheds light on the interpretation of the decompositions of output into a permanent and a transitory component. Under the traditional assumption that persistence originates in exogenous permanent productivity shocks, these decompositions have been extensively used to distinguish between supply (permanent) and demand (transitory) shocks. However, when persistence is interpreted as a measure of the permanent effects that fluctuations have on growth, then the degree of persistence of a shock does not contain any information about its origin. This result, previously explored in King, Plosser and Rebelo (1988) and Stadler (1989 and 1990), implies that, as long as technological progress is procyclical, temporary shocks (productivity, taste or aggregate demand shocks) can generate persistent fluctuations.

The type of model we propose also has more general implications about the measurement and identification of business cycles. The trend and cycle components of output are correlated with each other and fluctuations cannot be simply measured as deviations from a steady state. At the same time, this interaction between economic fluctuations and growth can possibly lead to larger measures of the welfare costs of business cycles.

The paper is organized as follows. Section 2 presents the concept of persistence and reviews the previous literature. Section 3 develops an endogenous growth model with exogenous cyclical fluctuations that is able to produce the correlation of Figure 1. Section 4 adds further evidence on the robustness of this result by looking at different frequencies and at a larger sample of countries. Section 5 concludes.

2. STOCHASTIC TRENDS

Fluctuations were traditionally measured as temporary deviations of output from a deterministic trend. This view was challenged by the work of Nelson and Plosser (1982) who showed the existence of a unit root in most U.S. macroeconomic series. Although there is still an open debate about the existence of an exact unit root in output, there is a broad agreement that fluctuations are highly persistent; GNP shows practically no tendency to revert to its trend level after a disturbance. Thus, the concept of a deterministic trend has been generally abandoned in favor of the notion of a stochastic trend and, as an example, filtering out cyclical components by using log differences, which presumes the existence of an stochastic trend, is standard practice today.

What are the implications of the presence of a stochastic trend in GNP for economic modeling? The presence of a stochastic trend is linked to the notion of stochastic growth. Models that fall into this label can be grouped into two different categories. First, there are models where the source of dynamics (exogenous permanent productivity shock) is the sole responsible of the existence of a stochastic trend. Second, there are models where the stochastic nature of growth is the result of the effects that fluctuations have on growth. Both of these types of models have quite similar empirical predictions but their assumptions about the cause of output fluctuations and their welfare implications can be fundamentally different. We next review the basic assumptions behind each of these types of models.

Exogenous Permanent Productivity Shocks

The initial explanation to the persistence of GNP fluctuations was provided by the real business cycle literature. In the standard RBC model, fluctuations are deviations from a steady-state solution to a neoclassical growth model. In its simplest form, GNP per capita follows a random walk with a drift, where the drift is exogenously determined by the rate of labor-augmenting technological progress. Furthermore, only small deviations around a steady state are analyzed and transitional growth dynamics are practically ignored. In this setup, permanent exogenous shifts in the production function are the only possible source of the persistence of output fluctuations.[3]

This model was the benchmark to interpret empirical decompositions of GNP into a permanent and a transitory component. These decompositions identify low-frequency output variability with exogenous technology shocks, while high-frequency movements are considered as demand shocks. As a result, the empirical contribution of the permanent component of output is regarded as a measure of the size and frequency of technology shocks relative to demand shocks.[4] All these papers confirm the significant contribution of the permanent component of GNP but they pay very little attention to the observed cross-country differences. For example, Cogley (1990) studies the variability of the low-frequency component of output in a sample of 9 countries and shows that there are significant differences among them, the US having the most stable low-frequency component of the sample. He concludes that output fluctuations, at least within his sample, are not all alike. If we were to use the standard RBC model as a benchmark to explain these

[3] See, for example, Kydland and Prescott (1982) or King, Plosser, Stock and Watson (1991).

[4] See Blanchard and Quah (1989), Campbell and Mankiw (1987), King, Plosser, Stock and Watson (1991) or Shapiro and Watson (1988)

cross-country differences, we would need to assume differences in the underlying stochastic process that drives technology shocks. The correlation of Figure 1 suggests, however, that average growth rates contain valuable information about the observed differences in the behavior of the long-term component of output. For this reason, we propose to incorporate in the analysis the endogeneity of productivity growth in order to understand its possible interactions with the persistence of economic fluctuations.

Endogenous Growth and Stochastic Trends

When productivity growth or technological progress are the outcome of conscious decisions made by economic agents, the notion of persistent fluctuations might have very different implications. King, Plosser and Rebelo (1988) and Stadler (1989) noticed that, within the context of an endogenous growth model, there are many types of disturbances, different from permanent shifts in the production function, that can produce persistent fluctuations. More specifically, any temporary disturbance causes permanent effects on the level of output as long as it produces temporary changes in the amount of resources allocated to growth. The growth mechanism adds up the transitory deviations to lead to a unit-root series of output. In this case, persistence cannot be used anymore to identify shocks and, as a consequence, the emphasis of the analysis shifts from the origin to the transmission of the shocks. A corollary is that disturbances such as aggregate demand shocks, traditionally considered as temporary, can have permanent effects on the level of economic activity.

The two previous sets of models share the presence of an stochastic trend and, as a result, they are able to produce persistent output fluctuations. However, the stochastic properties of the trend are exogenously assumed in the first case while they are the result of the response of productivity to cyclical fluctuations in the second one. Furthermore, their implications in terms of the cause and welfare costs of economic fluctuations can be quite different. It is nevertheless difficult to design tests to empirically distinguish between both explanations as their predictions are, in most cases, almost identical. In this paper, we propose to use the correlation of Figure 1 as a test for both explanations. Next section presents an endogenous growth model that produces this correlation and makes explicit the interaction between growth and persistence.

3. PERSISTENCE AND ENDOGENOUS GROWTH

This section presents a model where growth is the outcome of investment in physical capital. We add uncertainty to the model by assuming transitory productivity shocks and we look at the persistence of output and how it changes as we change the parameters of the model. We then compare how the degree of persistence relates to the steady-state growth rate.

3.1 CERTAINTY: STEADY-STATE GROWTH

We use a simple endogenous growth model where capital accumulation is the only source of output growth.[5] The production function is

$$Y_t = AK_t \tag{1}$$

where Y represents output, K is the aggregate capital stock and A is a country-specific parameter that will generate differences in long-term growth rates.[6] The single representative consumer maximizes the utility function

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \tag{2}$$

subject to the budget constraint

$$K_{t+1} = K_t(1-\delta) + (Y_t - C_t) \tag{3}$$

where δ is the depreciation rate.

The solution to this model is characterized by a constant saving rate and a balanced-growth path. Thus, we rewrite the maximization problem in terms of the saving rate, S, defined as the proportion of income that is not being consumed. Consumption can be written as $C_t = AK_t(1 - S_t)$ and the budget constraint is just

$$\frac{K_{t+1}}{K_t} = (1-\delta) + AS_t \tag{4}$$

Steady-state solution

In the absence of uncertainty, maximizing the utility function (1) subject to the constraint (4) leads to a balanced-growth path solution with a constant

[5] We adopt this model for its simplicity and similarity to RBC models used to analyze business cycles. The assumption that physical capital is the only source of long-term growth is not relevant for any of our arguments, other endogenous growth models would generate comparable output dynamics.

[6] To concentrate on productivity growth dynamics we have assumed that labor is supplied inelastically and normalize the labor supply to 1.

saving rate where Y, C and K all grow at the same rate. The equilibrium value for the saving rate and the growth rate are

$$S^* = \frac{\beta(A+(1-\delta))^{\frac{1}{\gamma}} - (1-\delta)}{A} \tag{5}$$

$$G^* = \frac{1}{\gamma}\ln\left[\beta(A+(1-\delta))\right] \tag{6}$$

We will refer to these values as steady-state values.

Cross-country differences in growth rates

We assume that countries differ in the parameter A and that this is the source of differences in steady-state growth rates. All other parameters in the model are assumed to be equal for all countries. From (6), the steady-state growth rate (G^*) is an increasing function of A. Therefore, our assumption implies that fast-growing economies are characterized by a larger A. This identification of the source of long-term growth differences will be important when we look at how persistence depends on G^*.

3.2 UNCERTAINTY: CYCLICAL SHOCKS

We now introduce uncertainty to the model by assuming transitory exogenous shocks and analyze the dynamic behavior of output around the steady state. We introduce an additional variable in the production function that captures the state of technology, Z_t. The production function is now

$$Y_t = Z_t A K_t \tag*{(1)$'$}$$

Uncertainty originates in Z_t which is assumed to follow an stochastic process with the Wold representation

$$\hat{z}_t = C(L)\epsilon_t \tag{7}$$

where small letters represent logarithms and a circumflex on top of the variable denotes deviations from its steady-state value. For simplicity we will assume that the steady-state value of Z_t is 1. $C(L)$ is a lag polynomial, $C(L) = 1 + c_1L + c_2L^2 + c_3L^3 + ...$ where all roots are assumed to be less than one so that the stochastic process is stationary. The stationarity of $\hat{z}$ allows us to refer to this process as cyclical. We need to emphasize that our results are not dependent on the specific origin of the shock (in this case technology). We look, for simplicity, at temporary technology shocks but other types of cyclical shocks (e.g. aggregate demand shocks) would lead to similar output responses.[7]

Maximization of the expected utility function (2) leads to the following first-order condition

$$\left[\frac{(1-\delta)+AS_tZ_t}{AZ_t(1-S_t)}\right]^{\gamma} = \beta E_t\left[\frac{AZ_{t+1}+(1-\delta)}{[AZ_{t+1}(1-S_{t+1})]^{\gamma}}\right] \tag{8}$$

Equations (4) and (8) define the equilibrium dynamics of the model. As a general closed-form solution to the equilibrium does not exist, our strategy is to approximate the equilibrium solution by linearizing both equations around the steady-state values (S^* and G^*). From the linearization of the first order condition we obtain an expression like

$$\kappa_1\,\hat{S}_t + \kappa_2\,\hat{z}_t = \kappa_3\,E_t(\hat{S}_{t+1}) + \kappa_4\,E_t(\hat{z}_{t+1}) \tag{9}$$

Where all κ_i's are functions of the parameters of the model. This is a linear first-order stochastic difference equation. We assume, for simplicity, that $\hat{z}_t$ follows an AR(1) process

$$\hat{z}_t = \rho\,\hat{z}_{t-1} + \epsilon_t \tag{10}$$

Using this assumption, (9) can be rewritten as

$$\kappa_1\,\hat{S}_t = \kappa_3\,E_t(\hat{S}_{t+1}) + (\rho\kappa_4 - \kappa_2)\,\hat{z}_t \tag*{(9)$'$}$$

and the solution takes the form

$$\hat{S}_t = \kappa\,\hat{z}_t$$

where

$$\kappa = \frac{\rho\,\kappa_4 - \kappa_2}{\kappa_1 - \rho\kappa_3} \tag{11}$$

We can now plug this expression into the budget constraint to obtain the equilibrium value for the growth rate. We linearize the resulting expression around the steady state to obtain a solution for the growth rate of capital which is linear in the productivity parameter. Let θ be the coefficient on that linearization so that[8]

$$\widehat{\Delta k_t} = \theta\,\hat{z}_t \tag{12}$$

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[7] For example, Stadler (1989) and Fatás (1994) present endogenous growth models with aggregate demand shocks resulting in persistent fluctuations.

[8] Where capital growth is measured as deviations from its steady state value G^*.

where

$$\theta = \frac{\kappa A + AS^*}{(1-\delta) + AS^*}$$

Using the production function (1) together with (10) and (12), we obtain an expression for the deviations of output growth from its steady state value (G^*)

$$\widehat{\Delta y_t} = (1-L)\,\hat{z}_t + \theta\, L\hat{z}_t = (1-(1-\theta)L)\ C(L)\,\epsilon_t \tag{13}$$

We can now use (13) to evaluate the stochastic properties of output. We start by discussing two different measures of persistence and then apply them to equation (13).

3.3 Measuring Persistence

Let y_t be the log of output and assume that it has the following Wold representation

$$\Delta y_t = D(L)\,\epsilon_t$$

Where $D(L) = d_0 + d_1 L + d_2 L^2 + d_3 L^3 + \ldots$ is a lag polynomial. Then, the coefficients d_j measure the impact of a shock ϵ_t on the growth rate of GNP in period $t+j$. If we add up these coefficients we can find the impact of a given shock on the level of GNP. In general,

$$P^J = \sum_{j=0}^{j=J} d_j$$

represents the impact of a shock ϵ_t on the level of GNP at $t+J$. The infinite sum of all d_j coefficients, measures the permanent impact of a given shock on the level of GNP, let P be this sum,

$$P = \lim_{J\to\infty} P^J = D(1)$$

A second measure of persistence proposed by Cochrane (1988), is a ratio of variances that can also be written as a weighted sum of autocorrelations

$$V^J = \frac{(1/J)\,\mathrm{var}(y_t - y_{t-J})}{\mathrm{var}(y_t - y_{t-1})} = 1 + 2\sum_{j=1}^{J-1}(1 - j/J)\,\rho_j$$

where ρ_j is the $j-$th autocorrelation of the growth rate of output. Taking the limit of this expression as J tends to infinity, we obtain a measure of long-run persistence,

$$V = \lim_{J\to\infty} V^J$$

Both V and P take value 0 for a trend-stationary series and value 1 for a random walk. For any other series,

$$V = |P|^2 \frac{\mathrm{var}(\epsilon)}{\mathrm{var}(\Delta y)}$$

3.4 PERSISTENCE AND GROWTH

The previous measures of persistence can be computed for the model of Section 3.2 by using (13). One should note that, although the type of disturbances that we are considering are cyclical, the model generates a unit-root series of output, both V and P are greater than zero. The reason for having a non-stationary output series is that shocks have an effect on capital accumulation. After the cyclical effects vanish, output does not return to its trend level and, thus, the shock has permanent effects. One can verify this property in equation (13) by showing that, as long as $\theta \neq 0$, output has a unit root.

The second issue of interest is how the measures of persistence vary with the parameter θ, which represents the contemporaneous impact that shocks have on capital accumulation. P and V can be written as

$$P = \frac{\theta}{1-\rho}$$

$$V = \frac{\theta^2\,(1-\rho^2)}{(1-\rho)^2(\theta^2 + 2(1-\rho)(1-\theta))}$$

which are both increasing in θ.[9]

Once we have established a connection between persistence and the parameter θ, now we want to use our previous assumption about cross-country differences in the technological parameter A, to obtain an expression that relates persistence and G^*. For simplicity, we restrict our attention to the case of a logarithmic utility function ($\gamma = 1$). In this case, θ takes the value

$$\theta = \frac{A}{A + (1-\delta)}$$

[9] See the appendix for the calculations that lead to these two expressions.

which is increasing in A (as long as the depreciation rate, δ, is less than one). Therefore, countries with a higher A will have both a higher steady-state growth rate and a higher θ. This, together with the fact that persistence is increasing in θ, predicts a positive correlation between persistence and long-term growth rates, the result of Figure 1.[10]

We can summarize all the previous arguments by making explicit the relation between persistence and the steady-state growth rate of output (G^*). To do so, we calculate the required value for A in equation (5) and plug it in the expression for P to obtain

$$P = \left[1 - \frac{(1-\delta)\beta}{\exp(G^*)}\right]\frac{1}{1-\rho}$$

which shows that persistence is increasing in the steady-state growth rate (G^*).

As a summary, we have found that the larger the growth rate is, the larger the *absolute* response of capital accumulation to a shock is (θ). This leads to a positive correlation between persistence and growth which mimics the result of Figure 1.

The intuition behind the result can be illustrated with a simple example: suppose two countries which suffer a common temporary shock that is supposed to affect them equally. Assume that trend GNP is growing at a 6% rate in country X, and at a smaller rate, say 2%, in country Y. The fact that θ is larger in country X implies that the *absolute* response of its long-term growth rate is going to be larger than in country Y. For example, while in country Y the rate might drop, as a consequence of the shock, to 1% (a drop of one percentage point), in X the rate drops to, say, 3% (a drop of three percentage points).[11] These numbers imply that the permanent effects of the shock on the level of GNP will be larger in country X than in country Y, exactly what the measures of persistence would say.

Dynamic Response of Output

The previous result can also be represented by drawing the impulse response function for different steady-state growth rates. Figure 2 shows the response of

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[10] When $\gamma \neq 1$, the solution for θ turns out to be much more complex and analytically intractable. We have numerically explored the solution by calibrating the model to fit differences in growth rates. We found that, for a plausible range of values of γ, θ is always positively related to the steady-state growth rate. The same result is obtained when we assume that differences in long-term growth rates are originated in differences in γ and not in A.

[11] In this example, the response of the growth rate of the trend is proportional; in both cases the rate drops by one third. This is, of course, not necessary. What is needed is that the absolute fall in the rate is larger in the country with a higher steady-state growth rate.

output to a transitory productivity shock for two different steady-state growth rates: 2% and 6%.[12]

Figure 2. Impulse-Response Function

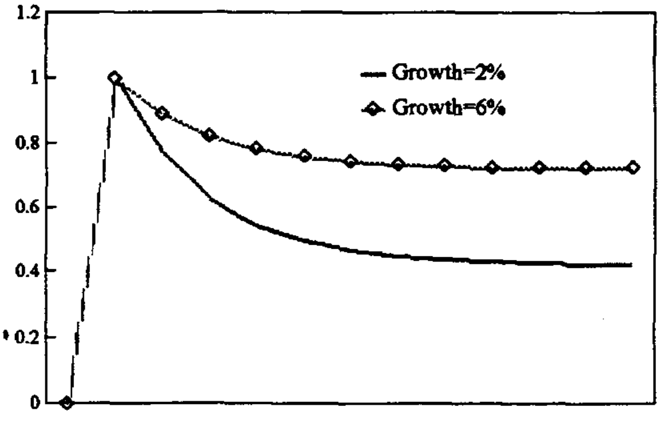
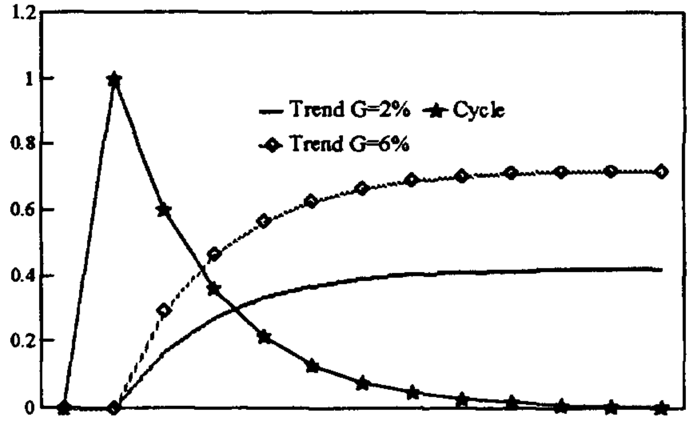


Figure 3. Cyclical and Permanent Components



In the case where $G^* = 6\%$, the response of output is more persistent and the permanent effects are larger than in the 2% case. Figure 3 decomposes the response of output into a cyclical component (the productivity shock) and a trend component (the capital stock). It shows that the difference in the response of output of Figure 2 is entirely caused by differences in the response of the permanent component. The cyclical shock is identical in both cases, but the

[12] We calculate these responses by setting $\beta = 0.96$, $\delta = 0.07$, $\rho = 0.5$ and $\gamma = 3$. Each of the steady-state growth rates is associated with a different value of the parameter A.

response of the permanent component is more pronounced for $G^* = 6\%$ than for $G^* = 2\%$.

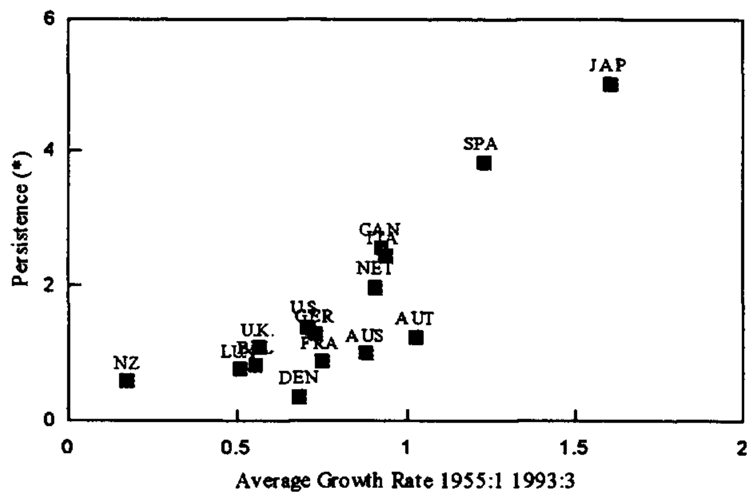
4. EMPIRICAL EVIDENCE

In this section we study the robustness of the correlation of Figure 1 by looking at a larger sample of countries. We also check whether the result holds for annual frequency data.

4.1 CROSS-COUNTRY EVIDENCE

Figure 1 showed a positive and significant correlation between the degree of persistence of short-term fluctuations and long-term average growth rates for the group of G-7 countries. Figure 4 enlarges the sample including eight additional OECD countries for which quarterly data on production is available.[13] Table 1 shows the regression results. The fit is still very good (the explained variance is 75%) and the coefficient is positive and significant. The size of the coefficient is indeed very similar in both samples.

Figure 4. Persistence and Growth. Quarterly Data



(*) Variance ratio, window=20 quarters

Figure 5 uses annual data from the Summers-Heston data set for the same group of countries. The result is confirmed and both the fit and the significance of the parameter are similar to the ones found above. For the group of G-7 countries, one even finds a stronger relation. Table 2 shows these results.

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[13] Production is measured as GNP or GDP, when available, or Industrial Production.

Table 1. Regression Results. Quarterly Data

$\text{Persistence}_i = \beta_0 + \beta_1 \text{ Avg.Growth}_i + \nu_i$

Sample	β_0	β_1	R^2
G-7	-1.55	4.12	0.95
	(0.41)	(0.43)	
Full (15 countries)	-1.05	3.38	0.75
	(0.47)	(0.54)	

Sample: 1950-1994
Standard errors in parentheses.

Figure 5. Persistence and Growth. Annual Data

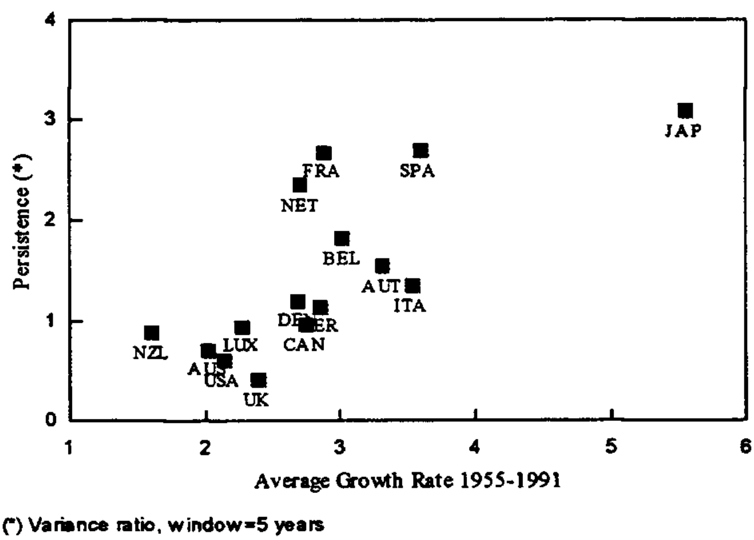


Table 2. Regression Results. Annual Data

$\text{Persistence}_i = \beta_0 + \beta_1 \text{ Avg.Growth}_i + \nu_i$

Sample	β_0	β_1	R^2
G-7	-0.74	0.70	0.61
	(0.84)	(0.25)	
Full (15 countries)	-0.44	0.67	0.54
	(0.52)	(0.17)	

Sample: 1955-1991
Standard errors in parentheses.

We have also performed the regressions of Tables 1 and 2 using different measures of persistence. We estimated univariate autoregressive processes for output growth and computed the parametric measure of persistence P, as defined in Section 3.2. The results were confirmed in all cases, the coefficient was always positive and significant and the fit of the regression was similar to the ones above.[14]

4.2.- CONSTRUCTING A SERIES OF TREND GNP

Tables 1 and 2 presented evidence about the positive correlation between short-term persistence and long-term growth rates. The explanation proposed in Section 3 relied on the procyclical behavior of capital accumulation or, more generally, technological progress. However, the measures of persistence were computed using GNP (or GNP per capita) and not a direct measure of the trend. In this section we construct an approximation to trend GNP and check whether the correlation still holds.

Ideally, one would like to list all factors that drive growth in the long run and measure their procyclicality. There is, however, no agreement on the main sources of long-term growth and their relative contribution. Nevertheless, there is a widespread consensus on the empirical relevance of the saving or investment rate as a driving force of output growth. The good empirical performance of the saving rate as a explanatory variable of cross-country differences in long-term growth rates, has been shown in the context of a Solow growth model and also in the context of endogenous growth models. In both cases, higher saving rates lead to higher capital accumulation and larger growth rates.

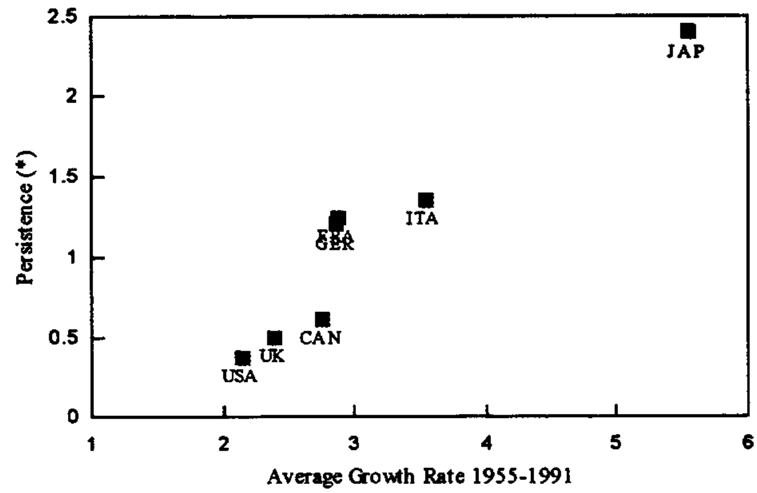
We use the information that the saving rate provides to construct a measure of trend GNP. For each country in our sample, we regress output growth into the contemporaneous and lagged values of the saving rate. We therefore run regressions of the type

$$\Delta y_t = \alpha_0 + \alpha_1 S_t + \alpha_2 S_{t-1} + \alpha_3 S_{t-2} + \mu_t$$

for each country in the sample. Where S_t is the saving ratio from the Summers-Heston data set and Δy_t is the growth rate of GNP per capita. We then use the fitted values of these regressions, as a measure of the long-run trend of output, and replicate the correlation of Figures 1, 4 and 5. Figures 6 and 7 show the outcome of this second step and Table 3 presents the regression results. In both cases, and remarkably so in the G-7 sample, the significance and size of the coefficient is practically unchanged with respect to our previous regressions.

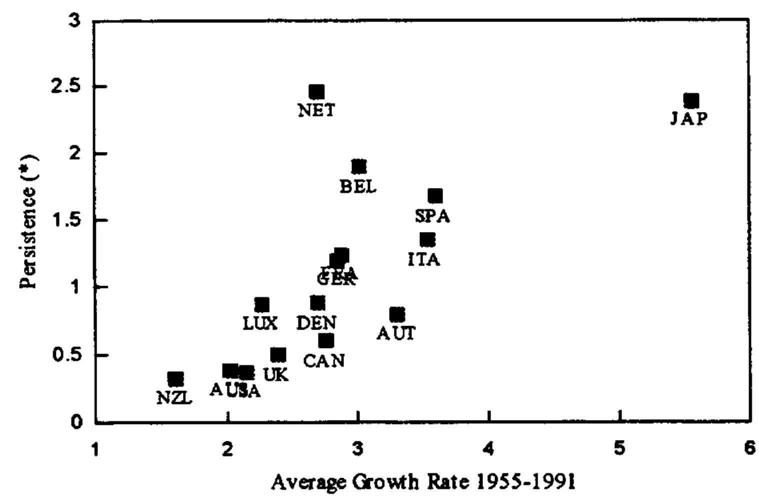
[14] For example, computing P from fitting an AR(2) process to GNP per capita growth, the 15 countries sample results in an R^2 of 0.69 and a slope of 0.37 (with a standard error of 0.07). In the G-7 sample the R^2 is 0.65 and the slope is 0.37 (with a standard error of 0.12).

Figure 6. Persistence and Growth. G-7 Countries. Constructed Trend



(*) Variance ratio, window=5 years

Figure 7. Persistence and Growth. 15 Countries. Constructed Trend



(*) Variance ratio, window=5 years

Therefore, we find that after constructing an approximation to trend GNP, the positive correlation between long-term growth rates and short-term persistence is still valid which seems to confirm our interpretation based on the endogenous response of technology to cyclical shocks.[15]

[15] We must admit, however, that our constructed measure of the trend is only a rough approximation to the true trend and further work on the construction of more elaborated estimates of the trend of GNP would be interesting.

Table 3. Regression Results. Constructed Trend

$$\text{Persistence}_i = \beta_0 + \beta_1 \text{ Avg.Growth}_i + \nu_i$$

Sample	β_0	β_1	R^2
G-7	-0.73	0.58	0.90
	(0.28)	(0.08)	
All	-0.43	0.54	0.45
	(0.46)	(0.15)	

Sample: 1955-1991
Standard errors in parentheses.

5. CONCLUSIONS

There is a consensus among macroeconomists that output fluctuations are very persistent and that long-term growth is not as stable as a deterministic trend would suggest. Empirical estimates of persistence display, however, large and significant differences across countries. We have shown that these differences can be almost fully explained by differences in long-term growth rates. Countries that grow faster have more persistent business cycles. In general, current business cycle models do not predict any correlation between these two variables as they treat one of the two, long-term growth rates, as an exogenous variable. For example, if all countries followed a random walk with a drift, then the degree of persistence would be the same in all countries even if the size of the drift varied.

We have looked at the concept of persistence within the context of an endogenous growth model and showed that, in this setting, transitory disturbances become persistent as they have effects in the amount of resources allocated to growth. Furthermore, the observed positive correlation of persistence and growth is a natural outcome in these models.

Our results suggest that stochastic growth is necessary to understand important features of the transmission of business cycles. But stochastic growth cannot be simply reduced to the presence of exogenous permanent shifts in the production function, as it is usually assumed in RBC models, the cyclical behavior of the resources allocated to growth needs to be taken into account. Indeed, if our goal is to analyze features of the business cycle such as persistence, the use of models where fluctuations are deviations around a steady-state solution to a neoclassical (Solow-type) growth model can be misleading.

Understanding all the implications that the endogeneity of growth has for business cycles is an open area for future research. One could imagine that, in these models, the transmission of shocks can lead to economic fluctuations that are quite different from the ones generated by a model where growth is treated as exogenous. The dynamics are possibly richer and could account for some of the empirical observations that are currently unexplained by business cycle models.

6. APPENDIX

Persistence and θ

Persistence can be calculated from expression (13)

$$\Delta y_t = (1 - (1 - \theta)L)\, C(L)\, \epsilon_t \tag{A.1}$$

Using the assumption that $\hat{z}$ follows an $AR(1)$ process, this equation can be rewritten as

$$\Delta y_t = \Big(1 + (\rho - (1 - \theta))L + (\rho^2 - (1 - \theta)\rho)L^2 + (\rho^3 - (1 - \theta)\rho^2)L^3 + ...\Big)\, \epsilon_t \tag{A.2}$$

By adding up the coefficients from the lag polynomial we obtain

$$P^J = \rho^J + \theta(1 + \sum_{j=1}^{J-1} \rho^j) \tag{A.3}$$

which is increasing in θ for any value of J. Long-run persistence is just

$$P = \frac{\theta}{1 - \rho} \tag{A.4}$$

The second measure of persistence V^J can be written as

$$V = |P|^2 \frac{\text{var}(\epsilon)}{\text{var}(\Delta y)} \tag{A.5}$$

From $(A.2)$, the variance of Δy is equal to

$$\text{var}(\Delta y_t) = \Big[1 + (\rho - (1 - \theta))^2 + \rho^2(\rho - (1 - \theta))^2 + \rho^4(\rho - (1 - \theta))^2 + ...\Big]\sigma_\epsilon^2 \tag{A.6}$$

which can be simplified into

$$\text{var}(\Delta y_t) = \frac{2(\theta - 1)(\rho - 1) + \theta^2}{1 - \rho^2}\sigma_\epsilon^2$$

Using $(A.4)$, $(A.5)$ and $(A.6)$,

$$V = \frac{\theta^2\,(1 - \rho^2)}{(1 - \rho)^2(\theta^2 + 2(1 - \rho)(1 - \theta))}$$

This expression is increasing in θ as long as $\theta < 2$, which will be satisfied as long as Δy is a stationary series (as long as y is an I(1) process).

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