THE EXPECTED FUTURE SPOT EXCHANGE RATE,
THE FORWARD RATE, AND THE TRADE BALANCE

by

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ABSTRACT

It is well known that the forward exchange rate is an unbiased estimator of the expected future spot rate when (1) the market is efficient, (2) there exist no transaction costs and (3) no risk aversion. The present paper accepts the two first conditions and focuses on the effects of risk aversion. As with previous results (Solnik, Grauer et. al.), a risk related bias is established and linked to asymmetries in exposures to risk (this is the usual case of backwardation). However, this paper points to a source of asymmetry, overlooked in capital asset pricing models and important for macroeconomic purposes: the trade balance. The model attempts to point toward empirically measurable sources of a systematic bias between the forward and the expected future spot exchange rate.

It is well known that the forward exchange rate must be an unbiased predictor of the corresponding future spot rate if (1) the market is efficient, (2) there are no transaction costs and (3) no risk aversion. This is so because, should a bias appear under these conditions, arbitrage would immediately eliminate it. For example, if the forward rate were set below the commonly expected future spot rate, arbitragers would immediately purchase the foreign currency forward to sell it spot at the time of delivery.

It is not quite clear whether this property is met on actual exchange markets and, if not, which of the above three conditions is violated. Recent empirical investigations have been surveyed in Isard (1978), Kohlhagen (1978) and Levich (1979). They seem highly unconclusive.[1] The present paper, accepting conditions (1) and (2), focuses on the role of risk aversion and attempts to show that an identifiable measure of the trade balance can be a source of a systematic bias.

Empirically relevant results concerning the existence of a risk-related bias between forward and expected spot rates are interesting for several purposes. First, the forward rate is frequently used to forecast the future spot rate, with results at the same time unsatisfactory but hard to beat consistently (Levich (1978)). Uncovering a pattern of systematic bias might improve the accuracy of the predictions. Second, a structual relation linking the forward and the expected spot rate is required in order to build a model determining jointly the spot and the forward rates which is not yet available. Third, tests of the efficient market and rational expectation hypotheses will remain unconvincing as long as they do not deal directly with the potential bias induced by risk aversion (see footnote 1). Lastly, in order to approach meaningfully the issue of destabilizing speculation on exchange markets, one needs to have a risk-related description of how forward markets serve as a major channel for speculative activities.

It has been clearly established by Solnik (1974), Grauer, Litzenberger and Stehle (1976), and Frankel (1979) that risk aversion in asset markets leads to a bias: when there is net positive (negative) holding of foreign assets, then the forward rate must be lower (higher) than the expected spot rate, although Frankel shows that this general result may be reversed under plausible conditions. These papers all follow from the capital asset pricing model, thus ensuring general equilibrium properties and concentrating on the portfolio balance approach to exchange rate determination. Admittedly, the present model lapses into partial equilibrium, although it can be reinterpreted adequately. The benefits are those of reintroducing the current account, following recent work in exchange rate theory (Dornbusch and Fischer (1979)).

In a sense, the Solnik-Frankel result extends to exchange rate markets some previous seminal results on backwardation established for future markets. The fundamental thrust of this line of approach is akin to musical chair games: if some people, by virtue of their activity or taste, start from a position where they face uncertainty on future purchases or deliveries, all the others being initially immune to that risk, the formers will convince the latters to share the risk only if they offer a compensation. In short, those who stand have to bribe those who sit. In the Solnik-Frankel case, if some domestic net assets are not matched by net foreign liabilities, then someone somewhere cannot find a counterpart willing to diversify away the risk involved by exchange rate changes and will have to accept a forward contract at a rate different from the expected future spot level.[2]

Essentially the present paper extends this result by noting that each international trader, by definition, faces exchange risk if his purchases/sales are denominated in foreign currency, and for each deal, there must be such a trader.[3] This establishes a basic asymmetry which should normally lead to some bias. As will be shown, only if the trade account is perfectly balanced, and if traders exhibit the same risk aversion, will this assymetry be removed. Otherwise, and in addition to the Solnik-Frankel result, trade will be a source of bias between the forward and the expected future spot rate. An approach, similar in spirit to the present one, can be found in Fama and Farber (1979): they note that money holders are stuck, at the outset, with purchasing power risk which they will try to diversify away through nominal bonds but the forward rate (the real return on bonds) will differ from the risk-free real interest rate by a "risk adjustment". Fama and Farber extend their result to international capital markets to re-establish Grauer, Litzenberger and Stehle's result, insisting on the role of money supply as a source of asymmetric risk.

Where does the present paper differ from capital asset pricing models so as to re-introduce the trade balance? Essentially, these models assume that all markets are complete and well functioning. The specific features of the present model imply that traders accept delayed payments on delivery of their sales, thus implicitly granting a loan. It is precisely these loans which, in case of a trade imbalance, amount to a non-negative net liability of one country vis-à-vis the other one, as in the Solnik-Frankel models.

This is a two country model with one traded good, with a discrete time set-up. It focuses on the participants in the forward market: "traders", who buy or sell forward only to hedge their foreign currency positions, and "investors" who diversify their portfolios, and may hedge some of their exchange risks, and also intervene on the forward market as profit-seeking arbitragers. This distinction is for analysis purposes and does not imply that actual "traders" cannot adopt the behavior assigned to "investors". Section 1 presents the behavior of a forward market where the only actors would be the traders. Section 2 does the same with investors only. In Section 3 the complete model is presented, with a simplified version outlined in Section 4. Conclusions are offered in Section 5.

1. Traders Only

To start out, we assume that the only participants in the forward exchange market are international traders. There are two countries and one good, the price of which is assumed throughout to be constant.[4] All sales agreed upon in period t are payable in period t + 1, the length of the period thus being defined. There are no transaction costs or liquidity constraints. We sucessively consider the case of domestic and foreign exporters and importers, and distinguish according to currency of invoicing.

a - Domestic Exporters Paid for in Foreign Currency

The domestic exporter i expects to receive next period a payment of P^*X_i corresponding to the sales of X_i goods paid for in foreign currency at the foreign price P^*. He does not know yet the future spot exchange rate $\tilde{e}$, but he can hedge a fraction $(1-\alpha_i)$ of these receipts at the known forward rate of f.[5] (A tilde denotes a random variable). His next period return is:

$$\tilde{R}_i = \alpha_i \tilde{e} P^* X_i + (1-\alpha_i) f P^* X_i \quad . \qquad 0 \leq \alpha_i \leq 1$$

Writing $R_{oi} = fP^{*}X_i$ and $\tilde{\lambda} = (\tilde{e}-f)/f$ we have:

$$\tilde{R}_i = R_{oi} + \alpha_i \tilde{\lambda} R_{oi}$$

The restriction on α_i amounts to forbid short or long positions by traders. Such a pure speculative activity does not seem to be frequently observed (see Fieleke (1979) for data on U.S. multinationals) and is anyway entrusted here onto the "investors". Traders are assumed to have Von Neumann - Morgenstern utility function $u_i(\tilde{R}_i)$. To simplify the algebra, it is further assumed that it is of a mean variance type as follows:

$$E\left[u_i(\tilde{R}_i)\right] = U_i\left[E(\tilde{R}_i),\ V(\tilde{R}_i)\right], \qquad U_{i_1}>0, \qquad U_{i_2}<0$$

where E and V are, respectively the expectation and variance operators. The trader's problem consists in choosing the α_i that maximizes his expected utility, given the pre-determined volume of sales or purchases.[6] If we define exporter i's degree of relative risk aversion as:[7]

$$\rho_i^T = - fP^{*}X_i \frac{2U_{i2}}{U_{i1}}$$

then the first order condition implies the interior solution:

$$(1) \qquad \alpha_i = \frac{E(\tilde{\lambda})}{\rho_i^T V(\tilde{\lambda})} \qquad \text{when} \quad E(\tilde{\lambda})>0$$

The second order condition is guaranteed by $U_{i2}<0$, and it is assumed, for convenience only, that ρ_i^T is large enough for the restriction $\alpha_i \leq 1$ not to be binding.

When $E(\tilde{\lambda}) \geq 0$, we reach the interior solution:

$$\alpha_i = 0$$

Quite simply, (1) tells us that, if they expect the future spot rate to rise above the forward rate, exporters are willing to take a speculative position, i.e. $\alpha_i > 0$. Conversely, if $E(\tilde{e}) \leq f$, they fully hedge ($\alpha_i = 0$).

b - <u>Domestic Importers Paying in Foreign Currency</u>

Importer j buys a volume M_j of foreign goods with which he will obtain a non random value added V_j. He hedges a portion $(1-\beta_j)$ of his bill $P^* M_j$ so that his return will be:

$$\tilde{R}_j = V_j - \beta_j \tilde{e} P^* M_j - (1-\beta_j) f P^* M_j \quad \text{with} \quad 0 \leq \beta \leq 1$$

$$= R_{oj} - \beta_j f P^* M_j \tilde{\lambda} \text{ with } R_{oj} = V_j - f P^* M_j$$

With a mean variance expected utility function $U_j\left[E(\tilde{R}_j), V(\tilde{R}_j)\right]$ as above, his optimum choice is:

$$(2) \qquad \beta_j = -\frac{1}{\rho_j^T} \cdot \frac{E(\tilde{\lambda})}{V(\tilde{\lambda})} \qquad \text{when} \qquad E(\tilde{\lambda}) < 0$$

$$\beta_j = 0 \qquad \text{when} \qquad E(\tilde{\lambda}) \geq 0$$

With again $\rho_j^T = - f P^* M_j \dfrac{2U_{j2}}{U_{j1}}$ a measure of his relative risk aversion.

Clearly, (2) is the converse of (1), so that exporters and importers have opposed incentives to hedge, depending upon the sign of $E(\tilde{\lambda})$.

c - Trade Paid for in Domestic Currency

For domestic exports X_k invoiced in the domestic currency, the exchange risk is borne by the foreign importer k. He is expected to pay next period PX_k, where P is the domestic price. This is exactly the problem studied in section b above. With $t=1/f$ and $\tilde{s}=1/\tilde{e}$ the return to foreign importer k next period is (an asterisk denotes foreign agents):

$$R^{\star}_{k} = V^{\star}_{k} - \beta^{\star}_{k}\,\tilde{s}PX_k - (1-\beta^{\star}_{k})\,tPX_k$$

With $\tilde{\mu} = (t-\tilde{s})/t = \tilde{\lambda}(f/\tilde{e})$, the optimal choice is:

$$\text{(3)} \quad \begin{cases} \beta^{\star}_{k} = \dfrac{1}{\rho^{\star T}_{k}} \cdot \dfrac{E(\tilde{\mu})}{V(\tilde{\mu})} & \text{when } E(\tilde{\mu})>0 \\[2ex] \beta^{\star}_{k} = 0 & \text{when } E(\tilde{\mu})\leq 0 \end{cases}$$

where $\rho^{\star T}_{k} = -tPX_k \cdot \dfrac{2Uk2}{Uk_1}$

Finally, for domestic imports M_ℓ billed by foreign exporter ℓ, the exchange risk will be hedged for a portion $(1-\alpha^{\star}_{\ell})$ such that:

$$\text{(4)} \quad \begin{cases} \alpha^{\star}_{\ell} = -\dfrac{1}{\rho^{\star T}_{\ell}} \cdot \dfrac{E(\tilde{\mu})}{V(\tilde{\mu})} & \text{when } E(\tilde{\mu})<0 \\[2ex] \alpha^{\star}_{\ell} = 0 & \text{when } E(\tilde{\mu})\geq 0 \end{cases}$$

and where $\rho^{\star T}_{\ell} = -tPM_\ell \cdot \dfrac{2U_2}{U_1}$

d - Forward Market Equilibrium

When $E(\tilde{\lambda})>0$ (and $E(\tilde{\mu})>0$), from (1) to (4) we have:

- $\alpha^*_\ell = 0$ and $\beta_j = 0$, so that all import related payments are fully hedged. This is optimal from the point of view of both domestic importers paying in foreign currency when they expect the exchange rate to depreciate beyond the forward rate $(E(\tilde{e})>f)$, and foreign exporters being paid in domestic currency.

- $\alpha_i>0$ and $\beta^*_k>0$ so that a part of export related payments are not hedged since both domestic exporters and foreign importers see an opportunity of earning some profits.

All the supply of forward foreign currency is linked to domestic exports:

$$S_+ = \sum_i (1-\alpha_i)P^*X_i + \sum_k (1-\beta^*_k)tPX_k$$

while all the demand is generated by domestic imports:

$$D_+ = \sum_j P^*M_j + \sum_\ell tPM_\ell$$

Together with (1) and (3) we obtain the market clearing condition:

$$\left[\sum_i P^*X_i + \sum_k tPX_k\right] - \left[\sum_j P^*M_j + \sum_\ell tPM_\ell\right] = \frac{E(\tilde{\lambda})}{E(\tilde{\lambda})}\left[\sum_i \frac{P^*X_i}{\rho_i^T} + \sum_k \frac{tPX_k}{\rho_k^{*T}}\right]$$

where it is made use of the following approximations:[8]

$$E(\tilde{\lambda}) \sim E(\tilde{\mu}) \quad \text{and} \quad V(\tilde{\lambda}) \sim V(\tilde{\mu})$$

The first bracket of the left hand side is the foreign currency valuation[9] of domestic exports X, the imports M being measured by the second bracket. Thus we rewrite the equilibrium condition as:

$$(5) \qquad X - M = \frac{E(\tilde{\lambda})}{V(\tilde{\lambda})}\left[\sum_i \frac{P^{\star}X_i}{\rho_i^T} + \sum_k \frac{tPX_k}{\rho_k^{\star T}}\right] \qquad \text{when } E(\tilde{\lambda})\geq 0$$

Similarly when $E(\tilde{\lambda})\leq 0$ (and $E(\mu)\leq 0$) we have $\alpha_i = \beta_k^{\star} = 0$, with $\alpha_{\ell}^{\star}>0$ and $\beta_j \geq 0$. The corresponding condition is:

$$(6) \qquad X - M = \frac{E(\tilde{\lambda})}{V(\tilde{\lambda})}\left[\sum_j \frac{P^{\star}M_j}{\rho_j^T} + \sum_{\ell} \frac{tPM_{\ell}}{\rho_{\ell}^{\star T}}\right] \qquad \text{when } E(\tilde{\lambda})\leq 0$$

Equations (5) and (6) can be made somewhat more appealing by assuming that all traders have the same degree of risk aversion so that we have:

$$(5'),(6') \qquad E(\tilde{\lambda}) = \frac{X-M}{T} \cdot \rho^T V(\tilde{\lambda}) \qquad \text{with} \begin{cases} T=X \text{ when } E(\tilde{\lambda})\geq 0 \\ T=M \text{ when } E(\tilde{\lambda})\leq 0 \end{cases}$$

The results are conveniently summarized in the following proposition: Proposition 1: When only traders intervene on the forward exchange market, and do not buy or sell on the forward market more than their trade-related exchange risk:

1.1 - the sign of $E(\tilde{e})-f$ is determined by that of the trade balance and $f=E(\tilde{e})$ only when X=M.

1.2 - the bias $|f-E(\tilde{e})|$ increases with the size of the trade imbalance, the degree of traders risk aversion and the variation of $\tilde{e}$.

The essential intuition behind proposition 1 is that a trade surplus (deficit) leads to an excess supply (demand) of forward foreign exchange. The neatness of the link with the trade balance relies however on the assumption that all traders are risk averse.

Allowing for risk neutral traders can be done by defining the portion x (respectively m) of domestic exports (respectively imports) handled by risk neutral traders.[10] In the simplest case when all traders have the same degree of risk aversion ρ^T, (5') and (6') now become:

$$(5") \quad E(\tilde{\lambda}) = \frac{(1-x)X-M}{(1-x)X} \cdot \rho^T V(\tilde{\lambda}) \text{ when } E(\tilde{\lambda}) \geq 0, \text{ i.e. when } (1-x)X \geq M$$

$$(6") \quad E(\tilde{\lambda}) = \frac{X-(1-m)M}{(1-m)M} \cdot \rho^T V(\tilde{\lambda}) \text{ when } E(\tilde{\lambda}) \leq 0, \text{ i.e. when } X \leq (1-m)M$$

Proposition 2: When a fraction of trade is handled by risk neutral agents and under the other assumptions of proposition 1:

2.1 - The sign of $E(\tilde{e})-f$ is not governed anymore by the trade balance itself but rather:
when $(1-x)X-M>0$ $\quad E(\tilde{e})>f$
when $X-(1-m)M \; 0$ $\quad E(\tilde{e})<f$
elsewhere $E(\tilde{e})=f$, but the market is unstable

2.2 - When $E(\tilde{e})\neq f$, the results of 1.2 apply.

The presence of risk neutrality has two effects. First it reduces the likelihood of observing a bias and it breaks down the direct relationship with the trade balance. As can be seen on figure 1, when x and/or m increase, the area where $E(\tilde{e})=f$ spreads. To get no bias at all, we need all traders to be risk neutral. Second, it introduces instability in the forward market. This instability is best understood by considering the limiting case where x=m=1. When, $E(\tilde{e})\neq f$, one side of the market will fully hedge, the other one will fully speculate, so that the excess supply of foreign currency is X when $E(\tilde{e})<f$, and -M when $E(\tilde{e})>f$: No equilibrium is possible when $E(\tilde{e})\neq f$. However, with $E(\tilde{e})=f$, risk neutral traders are indifferent between hedging or not, so that their net demand for forward cover is undetermined: only by chance would they coordinate themselves to clear the market. In the case where x and m are between 0 and 1, figures 2a and 2b picture the excess supply function and 2b confirms that the case when x=m=1 is readily generalized. Once again, the larger x and/or m, the more likely the probability on such unstable cases. It should be realized that this instability obtains only because of our previous restrictions that all α_i's and β_i's be bound between zero and unity. Truely risk neutral traders might not wish to restrict their behavior this way.[11]

Conversely, we note that if we lift the restriction that the α_i's and β_i's be bound, we get the following equilibrium condition (given here in the simple case when all traders share the same degree of relative risk aversion ρ^T):

$$(5''') \qquad (X-M) = \frac{E(\tilde{\lambda})}{V(\tilde{\lambda})} \cdot \rho^T \cdot (X+M)$$

so that, while the sign of $E(\tilde{\lambda})$ behaves as in proposition 1, its absolute size is reduced (actually about halved). This is because traders intervene now as pure speculators, willing to risk exchange gains above and beyond trade related sums. However, note now, it is enough that there exists <u>one</u> risk neutral trader for the bias to be eliminated. This confirms that the instability previously observed under risk neutrality is directly related to the bounds set on α_i's and β_i's.

2. Investors Only

This section will assume that only investors intervene on the forward exchange market, thus being a counterpart to section 1. The results developped herein are not new (they have been derived in a more appealing general equilibrium framework e.g. in Solnik (1974)) and are only derived as a prelude to the complete market model of section 3 (as well as to establish consistency with earlier papers). In keeping with the general

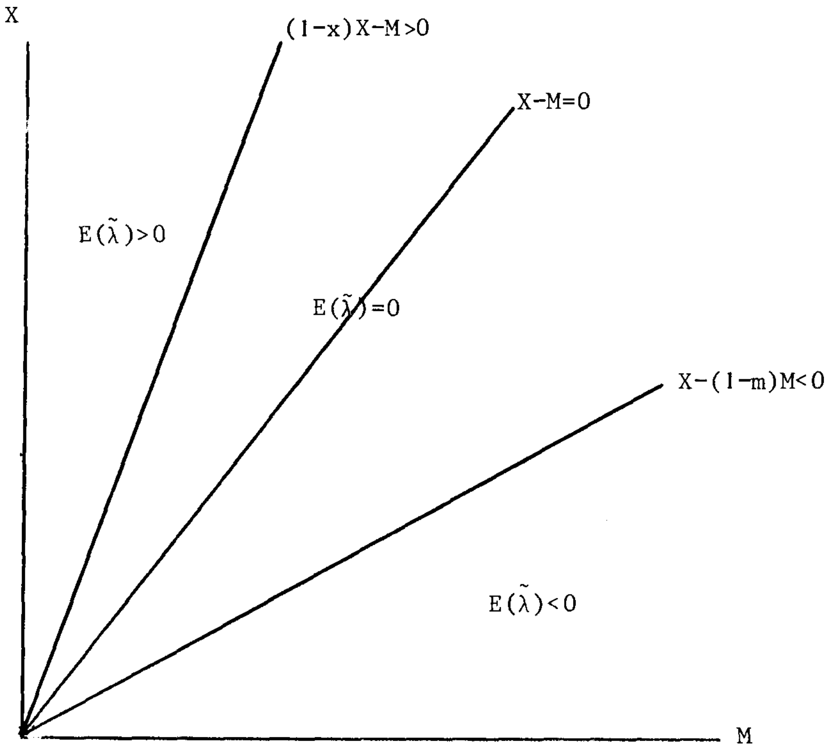

Figure 1

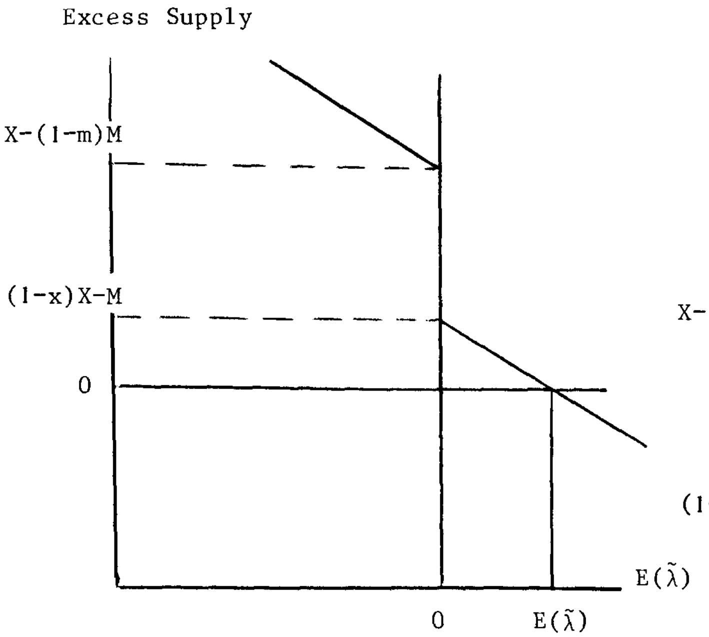

Figure 2a

Case E($\tilde{\lambda}$)>0

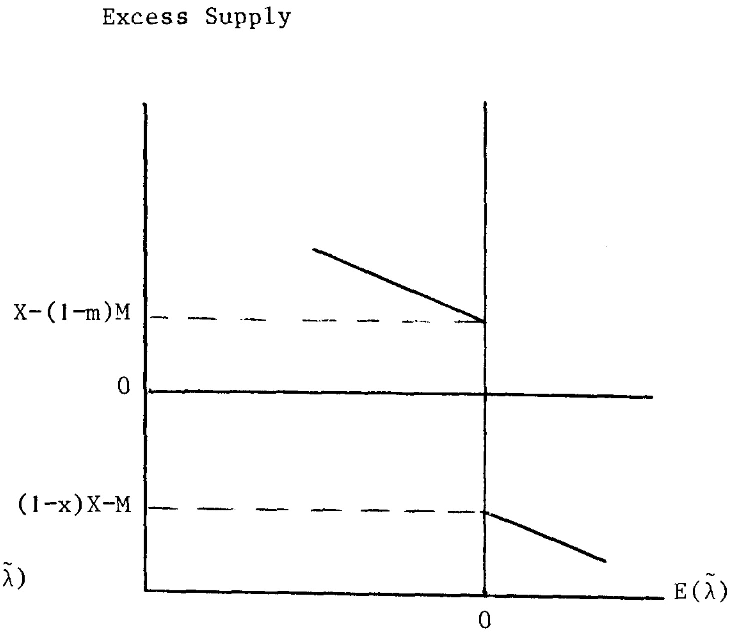

Figure 2b

Case E($\tilde{\lambda}$)=0

set-up of the paper, we separate out domestic and foreign investors and consider a two-country, two-period framework. In addition to the future spot exchange rate $\tilde{e}$, investors face uncertainty on returns, $\tilde{R}$ and $\tilde{R}^*$ (capital and interest), on domestic and foreign assets. These assets may be defined as nominal bonds with a real return subject to purchasing power uncertainty. Alternatively they can be viewed as stocks with an unknown yield.

a - Domestic Investors

Domestic investor i starts the present period with a wealth level W_{oi}. A share $(1-\gamma_i)$ of it is invested at home, being worth $(1-\gamma_i)W_{oi}\tilde{R}$ next period. The remaining is invested abroad. If $\bar{R}^* = E(\tilde{R}^*)$, the investor expects to hold in foreign currency $\gamma_i(W_{oi}/e_o)\bar{R}^*$ next period. He can decide to cover some portion ω_i of this exchange risk on the forward market. Finally, his actual return on foreign investment will turn out to be $\tilde{R}^* \neq \bar{R}^*$, leaving him with an unhedgedable position. All this is summarized in the definition of his wealth next period:

$$\tilde{W}_i = (1-\gamma_i)W_{oi}\tilde{R} + \gamma_i\omega_i \frac{W_{oi}}{e_o} \bar{R}^* f + \gamma_i(1-\omega_i)\frac{W_{oi}}{e_o} \bar{R}^*\tilde{e} + \gamma_i \frac{W_{oi}}{e_o}(\tilde{R}^* - \bar{R}^*)\tilde{e}$$

No restriction is imposed on γ_i or ω_i so that the investor is free to take any position he wishes. The above definition of $\tilde{W}_i$ can be transformed into:

$$\tilde{W}_i = W_{oi}\tilde{R} - \gamma_i\omega_i \frac{W_{oi}}{e_o} \bar{R}^*(\tilde{e}-f) + \gamma_i W_{oi}\left(\frac{\tilde{R}^*\tilde{e}}{e_o} - \tilde{R}\right)$$

It appears that what the investor will have to decide upon is: 1) γ_i, i.e. the degree of international diversification, and 2) $\gamma_i\omega_i$, i.e. the intervention on the forward market. As in the case of traders, we assume a mean-variance expected utility function:

$$E\left[u_i(\tilde{W}_i)\right] = U_i\left[E(W_i),V(W_i)\right] \quad , \quad U_{i1}>0,\ U_{i2}<0$$

Finally we simplify the notations by introducing:

$$\theta_i = \gamma_i\omega_i \frac{\bar{R}^{*}f}{e_o}$$

as the forward market decision variable so that:

$$\tilde{W}_i = W_{oi}\tilde{R} - \theta_i W_{oi}\ \tilde{\lambda} + \gamma_i W_{oi}\left(\frac{\tilde{R}^{*}\tilde{e}}{e_o} - \tilde{R}\right)$$

with $\tilde{\lambda} = (\tilde{e}-f)/f$ as before. Note that i's supply of foreign currency forward will be $S_i = \theta_i W_{oi}/f$.

Maximizing $E\left[u_i(\tilde{W}_i)\right]$ with respect to γ_i and θ_i gives two first-order conditions which can be solved for γ_i and θ_i, the second order ones being satisfied when $U_{i2}<0$. To simplify the economic interpretation, we take the optimal value of γ_i as $\hat{\gamma}_i$ and solve for θ_i.

$$(7) \qquad \theta_i = (1-\hat{\gamma}_i)\frac{\mathrm{cov}(\tilde{\lambda},\tilde{R})}{V(\tilde{\lambda})} + \hat{\gamma}_i \frac{f}{e_o}\frac{\mathrm{cov}(\tilde{\lambda},\tilde{R}^{*}) + \mathrm{cov}(\tilde{\lambda},\tilde{\lambda}\tilde{R}^{*})}{V(\tilde{\lambda})} - \frac{f}{\rho_i}\frac{E(\tilde{\lambda})}{V(\tilde{\lambda})}$$

where $\rho_i = -(2U_{i2}W_{oi}f)/U_{i1}$ is a measure of i's relative risk aversion.

The last term in (7) is the speculative term, similar to that obtained for the traders. When $E(\tilde{\lambda})>0$, investors can earn a profit by buying foreign currency forward (contributing to $\theta_i<0$), and selling it spot next period at the expected price $E(\tilde{e})>f$. As is customary, this speculative behavior is limited by their aversion to risk and the variability of $\tilde{e}$. The first term in (7) is the "domestic investment adjustment" component: when, for example, $\text{cov}(\tilde{\lambda},\tilde{R})>0$, high returns on domestic assets are frequently associated with positive values of $\tilde{\lambda}$, the latter allowing speculative profits on the forward purchases of foreign currency (i.e. $\theta_i<0$). Conversely, low $\tilde{R}$'s are associated with negative values of $\tilde{\lambda}$ which can be turned into speculative gains through the forward sales of foreign currency ($\theta_i>0$). A risk averse investor will be more sensitive to low than to high returns on $\tilde{R}$ and will, *ceteris paribus*, stand ready to offset such disappointing returns by potential gains on $\tilde{\lambda}<0$, thus the positive contribution of $\text{cov}(\tilde{\lambda},\tilde{R})$ to θ_i. Similarly, the second term of (7) is the "foreign investment adjustment" term, witnessing the tendency to offset through speculative gains low returns on foreign assets as measured by $\tilde{R}^*$ and $\tilde{\lambda}\tilde{R}^*$.

b - Foreign Investors

Foreign investor j's problem is exactly the same as that of his domestic counterpart.[12] He will supply forward an amount of foreign currency $S_j = -\theta_j^* W_{oj}^*$ where W_{oj}^* is his initial wealth and where θ_j^* is the optimal parameter computed as a function of $\hat{\gamma}_j^*$, his optimal degree of asset diversification:

$$(8) \qquad \theta_j^* = -(1-\hat{\gamma}_j^*)\frac{\text{cov}(\tilde{\mu},\tilde{R}^*)}{V(\tilde{\mu})} - \hat{\gamma}_j^* \frac{t}{s_o}\frac{\text{cov}(\tilde{\mu},\tilde{R}) + \text{cov}(\tilde{\mu},\tilde{R}\tilde{\mu})}{V(\tilde{\mu})} - \frac{1}{\rho_j^*}\frac{E(\tilde{\mu})}{V(\tilde{\mu})}$$

with the degree of risk aversion $\rho_j^* = -(2U_{j2}^* W_{oj}^*)/U_{j1}^*$.

The interpretation of (8) is exactly as for (7).

c - Forward Market Equilibrium

Summing over all investors' supplies of forward foreign currency, we obtain the equilibrium condition:

$$\sum_i S_i + \sum_j S_j^{*} = \sum_i \theta_i W_{oi}/f - \sum_j \theta_j^{*} W_{oj}^{*} = 0$$

The domestic and foreign investment adjustment components in θ_i and θ_j^{*} depend upon some covariance terms which are called here DA, FA, DA* and FA*.[13] With the further approximation already made in Section 1 that $E(\tilde{\lambda}) \sim E(\tilde{\mu})$ and $V(\tilde{\lambda}) \sim V(\tilde{\mu})$, the equilibrium condition reduces to:

$$(9) \qquad \frac{W_o}{f}\left[(1-\hat{\gamma})DA + \hat{\gamma}FA\right] - W_o^{*}\left[(1-\hat{\gamma}^{*})DA^{*} + \hat{\gamma}^{*}FA^{*}\right] = \frac{E(\tilde{\lambda})}{V(\tilde{\lambda})}\left[\sum_i \frac{W_{oi}}{\rho_i} + \sum_j \frac{W_{oj}^{*}}{\rho_j^{*}}\right]$$

where $(1-\hat{\gamma})W_o = \sum_i (1-\hat{\gamma}_i)W_{oi}$, $\hat{\gamma}W_o = \sum_i \hat{\gamma}_i W_{oi}$, etc.....

Equation (9) brings here the results of Solnik, and Grauer, Litzenberger and Stehle. It shows that if <u>all</u> investors are risk averse, there will still exist a bias between the expected future spot rate $E(\tilde{e})$ and the corresponding forward rate f as long as the left handside of (9) is nonzero. Conditions for this to happen are specified elsewhere and need not be worked out here. Briefly stated, this represents the net pressure for forward coverage from investors, both domestic and foreign.

Thus if we denote this hedging pressure term (the left hand side of equation (9)) by H, and if Φ measures the total amount of risk aversion present in the market (i.e. $1/\Phi = \sum_i W_{oi}/\rho_i + \sum_j W_{oj}^{*}/\rho_j^{*}$), (9) is rewritten as:

$$(9') \qquad E(\tilde{\lambda}) = H.\Phi.V(\tilde{\lambda})$$

A simple version in Section 4 will illustrate a case where one would expect $E(\tilde{\lambda})\neq 0$.

It is worth noting that should one investor be risk neutral, with free access to all markets and no bounds set on γ_i and θ_j, then the first order conditions of optimum would enforce the two relations:

$$E(\tilde{\lambda}) = 0$$

$$E\left(\frac{\tilde{R}^{*}\tilde{e}}{e_o} - \tilde{R}\right) = 0$$

There would be no bias between $E(\tilde{e})$ and f, and the uncovered interest rate parity condition would hold <u>ex-ante</u>. While there is some discussion that $\tilde{\lambda}$ is null on average, there seems to be considerable agreement that the uncovered interest rate parity condition is not actually met (although <u>ex-post</u>

3. <u>The Complete Model: Investors and Traders</u>

The analyses of Sections 1 and 2 can be brought together in the following market equilibrium condition:

$$(10) \qquad E(\tilde{\lambda}) = \left[H + (X-M)\right].\Phi'.V(\tilde{\lambda})$$

with H being the "hedging pressure" term defined as the left hand side of equation (9). Φ' is a measure of the total amount of absolute risk aversion present in the market.[14]

$$(11) \qquad 1/\Phi' = \sum_i W_{oi}/\rho_i + \sum_j W^{*}_{oj}/\rho^{*}_j + T/\rho^T = 1/\Phi + 1/\Phi^T$$

with T=X when $E(\tilde{\lambda})>0$, T=M when $E(\tilde{\lambda})<0$, T=0 when $E(\tilde{\lambda})=0$ and $\Phi^T=T/\rho^T$ (Φ has been defined in section 2).

An interesting special case arises when the investors do not intervene for themselves, but only at the request of those traders who want to reduce the amount of exchange risk imposed upon thme by their activity. Then the equilibrium condition becomes:

$$(12)\ E(\tilde{\lambda}) = (X-M).\Phi'.V(\tilde{\lambda})$$

Comparison of (10) and (12) show that, as long as the forward market interventions are only related to trade, what matters for the existence and sign of the bias is the trade balance itself, whether investors join in or not. However, if we rewrite (5') and (6') as:

$$(5'),(6')\ E(\tilde{\lambda}) = (X-M).\ \Phi^T.V(\tilde{\lambda})$$

comparison of (12) and (5') - (6') show that, since $\Phi'<\Phi^T$, the size of $|E(\tilde{e})-f|$ is reduced when investors stand ready to accept at cost some of the risk borne by the traders.

These results are summarized in the following proposition.

Proposition 3: When all traders and investors are risk averse,

3.1 - the sign of the bias $E(\tilde{e})-f$ is determined by the sum of two components: 1) the trade balance (X-M)

2) a "hedging pressure" term H resulting from assymetries in portfolio diversification.

3.2 - the size of the bias $|E(\tilde{e})-f|$ increases with this sum of asymmetries, the degrees of risk aversion of all market participants and the variability of $\tilde{e}$.

3.3 - ceteris paribus, investors' willingness to share in the risk inherent in trade relations reduces the size of the bias but cannot reverse its sign.

Finally it is worthwhile noting that if only one investor is risk neutral with free access to asset markets the bias is always wiped out. If some traders are risk neutral and refrain to act as investors (i.e. all α_i's and β_i's are constrained to remain between zero and one), the bias remains but the term (X-M) in (10) is replaced as in (5") and (6").

Proposition 4: When all investors are risk averse and some traders are risk neutral.

4.1 - the sign of the bias is determined as follows:

- $E(\tilde{e})>f$ when $H + \left[(1-x)X-M\right]>0$
- $E(e)<f$ when $H + \left[X-(1-m)M\right]<0$
- $E(\tilde{e})=f$, but the market is unstable when $H + \left[(1-x)X-M\right]<0$ and $H + \left[X-(1-m)M\right]>0$.

4.2 - when $E(\tilde{e})\neq f$, the results of 3.2 and 3.3 hold.

The unstability case has been already discussed in section 1. The presence of risk-averse investors does not alter its previous characterization.

4. A Simplified Case

The interpretation of (10) is quite complicated because of the cross-covariances of the DA and FA terms. In this section, a simpler version is presented which has more empirical potential. It is doubtful that the domestic investment terms should be of any significant magnitude in practice. The theoretical basis is that investors want to diversify their portfolios and in this two assets model use the forward market as one channel. In the real world, with a very large number of domestic and foreign assets available, the role of the forward market for assets diversification purposes should, if anything, be small. It might seem therefore reasonable to simply disregard the DA and DA^* terms in (10).

Another possibility, with less theoretical appeal but more illustrative, consists in assuming that both the domestic and foreign interest factors (R and R^*) are not random. Then, for domestic investor i, next period's wealth is:

$$\tilde{W}_i = (1-\gamma_i)W_{oi}R + \gamma_i\omega_i \frac{W_{oi}}{e_o} R^* f + \gamma_i(1-\omega_i)\frac{W_{oi}}{e_o} R^*\tilde{e}$$

which can be rewritten as:

$$\tilde{W}_i = W_{oi}R + \gamma_i(R^* f - R_o)\frac{W_{oi}}{e_o} + \theta_i\frac{W_{oi}}{e_o}\tilde{\lambda}$$

with θ_i now redefined as $\theta_i = \gamma_i(1-\omega_i)R^* f$. The problem with this case is that if $R^* f \neq R_{eo}$, investor i will set $\gamma_i = \pm\infty$ as $R^* f \gtrless R_{eo}$, because he can thus increase indefinitely his wealth with no risk involved. An obvious consequence is that arbitrage should enforce:

$$R^* f = Re_o$$

This is the covered interest rate parity condition. But then $\tilde{W}_i = W_{oi}R + \theta_i\frac{W_{oi}}{e_o}\tilde{\lambda}$ and γ_i is not determined, since all investors are strictly indifferent between domestic and foreign assets fully hedged. It is well known that the interest parity condition is met in practice between "risk free" bonds, which does not prevent portfolio diversification between risky bonds, as in Section 2. It will be assumed here that γ_i is otherwise determined and $\gamma_i = \hat{\gamma}_i$. Then, in a mean-variance framework:

$$\theta_i = \frac{1}{\rho_i}\frac{E(\tilde{\lambda})}{V(\tilde{\lambda})} \qquad \text{with } \rho_i = -\frac{2U_{i2}}{U_{i1}} \cdot \frac{W_{oi}}{e_o}$$

which implies a supply of forward foreign currency:

$$S_i = \left(\hat{\gamma}_i - \frac{1}{\rho_i}\frac{E(\tilde{\lambda})}{V(\tilde{\lambda})}\right).\frac{W_{oi}R}{f}$$

Through similar reasoning for foreign investor j and with the approximations $E(\tilde{\mu}) \sim E(\tilde{\lambda})$ and $V(\tilde{\mu}) \sim V(\tilde{\lambda})$ as before, we get his supply:

$$S_j^{\star} = -\left(\hat{\gamma}_j + \frac{1}{\rho_j^{\star}}.\frac{E(\tilde{\lambda})}{V(\tilde{\lambda})}\right).W_{oj}^{\star}R^{\star}$$

One can now write the forward market equilibrium condition to obtain:

$$\sum_i \hat{\gamma}_i (W_{oi}/f)R - \sum_j \hat{\gamma}_j^{\star} W_{oj}^{\star} R^{\star} + (X-M) = \frac{E(\tilde{\lambda})}{V(\tilde{\lambda})}\left[\frac{T}{\rho^T} + \sum_i \frac{(W_{oi}/f)R}{\rho_i} + \sum_j \frac{W_{oj}^{\star}R^{\star}}{\rho_j^{\star}}\right]$$

where T=X when $E(\tilde{\lambda}) \geq 0$ and T=M when $E(\tilde{\lambda}) \leq 0$.

The right handside is as in (10), but the left handside is more easily interpreted: it is the sum of the trade balance and of total foreign asset holdings by domestic investors net of domestic asset holdings by foreign residents. Not only is this a more tractable formula, but if we remember that the net asset position here refers to those assets and liabilities for which international investors acquire hedging protections, then $\sum_i \hat{\gamma}_i (W_{oi}/f)R - \sum_j \hat{\gamma}_j^{\star} W_{oj}^{\star} R^{\star}$ resembles the short term capital account.

Then the whole left handside is likely to be approximated by the sum of the current and the short term capital accounts.

5. Conclusion

The main purpose of this paper is to show that the trade balance, and possibly the short term capital account added to it, may explain the existence of a systematic bias between the expected future spot rate and the forward rate of same maturity. The bias had already been theoretically identified by various authors working with international capital assets pricing models. The contribution of this paper is to point to measurable sources of the bias. Although the model used here suffers from being a partial equilibrium one, it can be shown that it is in tune with those defined in a general equilibrium context. This is readily done when it is recognized that sales contracts with deferred payments of the kind portrayed here are nothing more than loans by the sellers to the buyers. The new assets and liabilities thus created can be re-inserted in a capital assets pricing model to yield similar conclusions. Therefore the results obtained in the present paper should prove to be robust.

This paper suggests why many recent empirical studies, while failing to reject the no-bias hypothesis, still come up with mixed results.[15] Cornell (1977), for example, finds a zero average bias for the seven currencies in his sample, yet the forecast errors seem to exhibit some serial correlation. He comments that "it is hard to believe that the premium could change sign over a period as short as four years. Furthermore it is difficult to understand why such variation in the premium would occur for some currencies and not others" (p.59). Clearly, if the model presented here is true so that:

$$\frac{E(e_{t+1}) - f_t}{f_t} = H_t + B_t$$

wher H_t and B_t are, respectively, the hedging pressure and trade balance terms, we have:

$$(13)\quad e_{t+1} = (H_t + B_t)f_t + f_t + \varepsilon_t$$

with ε_t a white noise.

It now appears reasonable that $(e_{t+1}) - f_t)$ should behave as found by Cornell. Similarly, most empirical studies, as revised in Levich (1979) or Kohlhagen (1978) test a=0 and b=1 in:

$$(14) \quad e_{t+1} = a + bf_t + \varepsilon_t$$

which in terms of (13) amounts to testing whether $(H_t+D_t)f_t$ is white noise. While failures of tests based on (14) do not tell us much about the reasons for a bias, tests based on (13) should be more informative, especially in separating out market efficiency (which is consistent with (13)) from risk-related biases.

Several difficulties stand out before actually performing tests with (13). First, this is a two country model. Second, it rests on a discrete lag description with fixed horizon. Obviously a richer term structure is necessary. Third, it is a one good model with a purchasing power parity condition. Introducing more goods and doing away with purchasing power parity might change the results obtained.[16] Finally, inasmuch as central banks intervene on the forward markets, one should specifically model their reaction functions.

Finally, when linked with a model of exchange rate determination of the type developped by Dornbusch and Fischer (1979) or Rochiguez (1978), an equation like (13) provides systematic patterns of both the spot and forward rates in response to disturbances. These fluctuations have been called by Isard (1978, p.13) an "unexplained puzzle".

Footnotes

1. As Levich notes, all of them test jointly the efficient market hypothesis together with (usually) hidden exchange rate models. The problem is that these tests rely on arbitrary reduced forms. As shown by Tryon (1979), alternative equations lead to opposite conclusions with the same data. Also at issue is the use of OLS techniques when both spot and forward rates are presumably jointly determined.

2. Then the direction of the bias will be, in the conventional case, dictated by where the excess supply appears. But Frankel rightly shows that what matters is not the excess supply per se, rather the correlation of the risk involved with the return of "the market", thus allowing for reversed results.

3. One might argue that for each sale, the two traders can agree to denominate the bill half in each currency, thus removing all uncertainty. As will be seen later, this is not an optimal behavior for both, since one of them thus generally foregoes an expected gain. Optimal behavior would suggest that this trader would only hedge part of his exposure, depending upon his degree of risk aversion.

4. This is not an inocuous assumption since uncertainty on prices can be put at the center of exchange rate uncertainty as in Frankel (1979), Stockman (1978) or Fama and Farber (1979). However, this price randomness usually ends up in uncertainty on the real return on assets, which is precisely what is done here.

5. One limitation of this model is that hedging is achieved only on the forward market. Some different means do exist to cover exchange risk, such as simultaneous borrowing abroad, matching of sales and purchaseswithin different sections of a multinational company. The former role is assigned here to investors, while the second one is achieved by importers. Other means are excluded from the analysis, for example though currency diversification as in Makin (1978).

6. The volume of sales and purchases could be made a choice parameter under full expected profit maximization. It is assumed here that traders first decide upon the volume of their shipments at the given world price and then deal with the exchange risk. For a justification see the separation theorems in Ethier (1973) and Danthine (1978).

7. The function ρ_i^T is defined around R_{oi} and thus is not a truely conventional measure of risk aversion

8. Overlooking Jensen's inequality, i.e. assuming $E(1/\tilde{e}) = 1/E(\tilde{e})$,

we have $E(\tilde{\lambda}) - E(\tilde{\mu}) = \dfrac{[E(\tilde{e}) - f]^2}{f.E(\tilde{e})}$ so that $E(\tilde{\lambda}) \sim E(\tilde{\mu})$

is valid as long as $|E(e)-f|$ remains small with respect to $f.E(\tilde{e})$, which seems reasonable. Also, with $V(1/\tilde{e}) \sim V(\tilde{e})/E^4(\tilde{e})$ (through Taylor approximation), we have $V(\tilde{\lambda}) - V(\tilde{\mu}) = V(e).\left[1-f^4/E^4(\tilde{e})\right]/$

9. The exchange rate used for translating the domestic currency price of traded goods P into foreign currency is the forward rate $t=1/f$, whereas accuracy would imply using the currently known spote rate e_o. Under purchasing power parity, for example, when $P = e_o P^*$, the left hand side of (5) and (6) would be $(X-M)+\left(\dfrac{e_o - f}{f}\right)\left[\sum_k P^* X_k - \sum_\ell P^* M_\ell\right]$ instead of X-M, which gives a measure of the approximation. Note that $(e_o - f)/f$ is the forward premium.

10. For example $x = \dfrac{\sum_{i\in RN} X_i + \sum_{k\in RN^*} X_k}{X}$ where RN and RN^* are, respectively, the sets of risk neutral domestic exporters and foreign importers.

11. However, this unstability result can be generalized to any financial market provided some agents are "risk neutral within bounds", i.e. they are not willing to, or allowed to, or able to commit enough funds to perform full arbitraging. This then calls either for government intervention or for some collusive behavior.

12. His next period wealth will be:

$$\tilde{W}_j^* = (1-\gamma_j^*)W_{oj}^*\tilde{R}^* + \gamma_j^*\omega_j^* \frac{W_{oj}^*}{s_o}\bar{R}_t + \gamma_j^*(1-\omega_j^*)\frac{W_{oj}^*}{s_o}\bar{R}s + \gamma_j^*\frac{W_{oj}^*}{s_o}(\tilde{R} - \bar{R})s$$

$$= W_{oj}^*\tilde{R}^* + \theta_j^* W_{oj}^*\tilde{\mu} + \gamma_j^* W_{oj}^*\left(\frac{R\tilde{s}}{\bar{s}_o} - R^*\right)$$

with $\theta_j^* = \gamma_j^*\omega_j^*\bar{R}^* t/s_o$ and $\tilde{s}$, t, s_o and $\tilde{\mu}$ as defined in Section 1.

13. For domestic investors, the domestic and foreign investment adjustment coefficients are respectively:

$$DA = \frac{cov(\tilde{\lambda},\tilde{R})}{V(\tilde{\lambda})} \quad , \quad FA = \frac{f}{e_o}\,\frac{cov(\tilde{\lambda},\tilde{R}^*) + cov(\tilde{\lambda},\tilde{\lambda}\tilde{R}^*)}{V(\tilde{\lambda})}$$

For the foreign investors we have:

$$DA^* = \frac{cov(\tilde{\mu},\tilde{R}^*)}{V(\tilde{\mu})} \quad , \quad FA^* = \frac{t}{S_o}\,\frac{cov(\tilde{\mu},\tilde{R}) + cov(\tilde{\mu},\tilde{\mu}\tilde{R})}{V(\tilde{\mu})}$$

14. It is assumed for simplicity that all traders exhibit the same degree of relative risk aversion. Definition of ϕ^T in the general case is straight-forward.

15. In addition to some works alluded to in the introductory part, it is worth mentioning that of Geweke and Feige (1979) because they attempt to explain the sources of observed biases. They conclude that it might be transaction costs over the fixed regime period (1962:3 - 1967:2) and risk-aversion over the flexible rate period (1972:3 - 1977:1).

16. Fama and Farber discuss this problem and conclude that allowing for more than one good does not change much if P.P.P. holds. But that clearly is not true as soon as 1) there exist nontraded goods or 2) consumption patterns differ among countries so that the CPI is not the same weighted average of the same prices.

REFERENCES

Cornell, Bradford, "Spot Rates, Forward Rates and Exchange Market Efficiency", Journal of Financial Economics 5, 55-66, August 1977.

Danthine, Jean-Pierre, "Information, Future Prices, and Stabilizing Speculation", Journal of Economic Theory 17, 79-98, February 1978.

Dornbusch, Rudi and Stanley Fischer, "Exchange Rates and the Current Account", unpublished paper, M.I.T., 1979.

Ethier, Wilfred, "International Trade and the Forward Exchange Market", American Economic Review 63, 494-503, June 1973.

Fama, Eugene F., and André Farber, "Money, Bonds, and Foreign Exchange", American Economic Review 69, 639-649, September 1979.

Fieleke, Norman S., "Foreign Exchange Speculation by US Firms: Some New Evidence", New England Economic Review, 5-17, March-April 1979.

Frankel, Jeffrey A., "The Diversifiability of Exchange Risk", Journal of International Economics 9, 379-394, August 1979.

Geweke, John and Edgar Feige, "Some Joint Tests of the Efficiency of Markets for Forward Foreign Exchange", Review of Economics and Statistics 61, 334-341, August 1979.

Grauer, Frederic, L. A., Robert H. Litzenberger and Richard E. Stehle, "Sharing Rules and Equilibrium in an Internation Capital Market under Uncertainty", Journal of Financial Economics 3, 233-256, June 1976.

Isard, Peter, "Exchange Rate Determination: A Survey of Popular View and Recent Models", Princeton Studies in International Finance No. 42, May 1978

Kohlhagen, Steven W., The Behaviour of Foreign Exchange Markets - A Critical Survey of the Empirical Litterature, New York University, 1978.

Levich, Richard M., "On the Efficiency of Markets for Foreign Exchange", Chap. 8 in: R. Dornsbusch and J. A. Frenkel (Eds), International Economic Policy, Baltimore, The Johns Hopkins University Press, 1979.

Rodriguez, Carlos A., "The Role of Trade Flows in Exchange Rate Determination: A Rational Expectations Approach", Unpublished paper, Columbia University, 1978.

Solnik, Bruno H., "An Equilibrium Model of the International Capital Market", Journal of Economic Theory 8, 500-524, 1974.

Stockman, Alan C., "Risk, Information, and Forward Exchange Rates", Chap. 9 in: J. A. Frenkel and H. G. Johnson (Eds), The Economics of Exchange Rates, leading, Mass.: Addison-Wesley Publishing Co., 1978.

Tryon, Ralph, "Testing for Rational Expectations in Foreign Exchange Markets", unpublished paper, Federal Reserve Board, International Finance Discussion Paper No. 139, May 1979.