ON SOME PROPOSITIONS REGARDING THE BEHAVIOR OF THE LABOR-MANAGED FIRM UNDER UNCERTAINTY

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ABSTRACT

In this paper we examine the proposition according to which the risk-averse labor-managed firm facing demand uncertainty employs more workers than the same firm operating under certainty and show that this proposition may not be valid in a multiple-period framework. We also demonstrate that the proposition according to which a capitalistic firm prefers price instability to price stability is also valid for the labor-managed firm.

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I INTRODUCTION

Recent contributions to the theory of the labor-managed firm under uncertainty were made by Ramachandran, Russell and Seo (1979), Paroush and Kahana (1980), Hawawini and Michel (1980), and Hawawini (1980). These authors have shown that the risk-averse labor-managed firm producing its output with labor as its only variable input and selling this output in a competitive market will employ more workers and produce a greater output than the labor-managed firm operating under certainty. This result is of particular interest because Sandmo (1971) has shown that the risk-averse capitalistic firm behaves in the opposite manner: it hires less workers and produces a smaller output under uncertainty than under certainty. It seems then that the employment and production behavior of the risk-averse labor-managed firm contradict what intuition would suggest regarding economic behavior under uncertainty: one would expect a reduction in employment and production by a risk-averse firm operating in an environment of price uncertainty.

In this paper we show that the somewhat "paradoxical" proposition according to which the risk-averse labor-managed firm hires and produces more under uncertainty than under certainty

may not be valid in a multiple-period framework. In this new framework, we give to "production under certainty" a meaning that differs from the standard approach used by Sandmo (1971) and others and which, we believe, is more realistic. Specifically, we demonstrate that the risk-averse labor-managed firm facing an uncertain demand for its output will hire and produce less than the same firm operating under certainty if the marginal product of labor is a linear or a concave function of workers. This conclusion is appealing because it has the advantage of reconciling theoretical behavior with what intuition would suggest regarding the behavior of risk-averse firms facing demand uncertainty.

In the next section we present alternative models of the labor-managed firm under uncertainty. The third section compares the employment and production decisions of the labor-managed firm under uncertainty to those under certainty, within a multi-period framework. In the fourth section we demonstrate that Oi's proposition (1961) according to which a risk-neutral capitalistic firm prefers price instability to price stability is also valid for the case of the labor-managed firm.

II MODELS OF THE LABOR-MANAGED FIRM

Following Vanek (1970), Ramachandran et al. (1979), and Paroush et al. (1980), we assume that the labor-managed firm makes employment and production decisions so as to maximize the expected utility of the income per member associated with the firm. Given that output (Q) is sold in a competitive market at a price (p) - which may be a random variable - and that this output is produced with labor (L) as the only variable input, we can write the following objective function:

$$\underset{Max}{E}\left[U(Y)\right] = \underset{Max}{E}\left[U\left(\frac{pQ(L) - F}{L}\right)\right] \tag{1}$$

where E is the expectation operator, Y the income per worker, U(•) a von Newmann-Morgenstern (953) utility function for money which summarizes the firm's attitude toward risk, and F are the fixed costs incurred in the short run.

The production function Q(L) is well-behaved, that is, the marginal product of labor ($\frac{dQ}{dL} = Q_L$) is positive and declining ($Q_{LL} < 0$) and production takes place in the range within which the marginal product of labor Q_L is smaller than the average product of labor ($\frac{Q}{L} = a$). The first order condition for an extremum is:

$$\frac{dE}{dL} = \frac{1}{L} E\left[U'(Y) \cdot (pQ_L - Y)\right] = 0 \tag{2}$$

and the second order condition for a maximum is assumed to be satisfied for the problem at hand.

Uncertainty: Single-period horizon

Consider first the labor-managed firm facing an uncertain demand $\tilde{p}$, where $\tilde{p}$ is a parametrically given random price with a known density function. Using the relationship E(AB) = E(AB) + cov(A,B), and noting that $p(Q_L - Y) = p(Q_L - a) - F/L$, condition (2) can be rewritten as:

$$E\left[pQ_L\right] = E\left[Y\right] - (Q_L - a)\,\frac{\mathrm{cov}\,(U'(Y),\, p)}{E\left[U'(Y)\right]}$$

Since the term $(Q_L - a)$ is negative by assumption $\left((Q_L < a)\right)$ and since the sign of the covariance is that of $U''(Y)$[1] it follows that the optimal level of employment under uncertainty and single-period horizon (L_u) must satisfy the conditions:

$$E\left[pQ_L\,(L_u)\right] \lesseqgtr E\left[Y\,(L_u)\right] \text{ if } U''(Y) \left\{ \lesseqgtr \right\} 0 \qquad (3)$$

where $U'' < 0$, $U'' = 0$ and $U'' > 0$ indicate risk-aversion, risk-neutrality and risk-seeking, respectively. This is the result established by Ramachandran et al. (1979) and Paroush et al. (1980): the risk-averse labor-managed firm ($U'' < 0$) employs and produces <u>more</u> ($E(pQ_L) < E(Y)$) than the risk-neutral firm ($U'' = 0$). This result is then extended to a comparison between the risk-averse firm and the "certainty" firm. Certainty, in this context, means a situation in which the firm faces the <u>average</u> value of $\tilde{p}$ with certainty. Thus, in this approach, certainty is analogous to risk-neutrality. According to this model, the labor-managed firm faces an end of period probability distribution of output prices and makes its employment and production decisions <u>before</u> the actual price is revealed at the

end of the period. Once the price is revealed, the firm sells all its output at that price. We have a case of ex-ante employment and production decisions.

Uncertainty: Multiple-period horizon

Suppose now that the firm has a multiple-period horizon. With the model just described, this means that the firm faces the same probability distribution of output prices period after period. Clearly, in this context, the levels of the firm's ex-ante employment and output are identical for each period. The average level of employment and output (computed as the arithmetic average for all periods) is simply the single period ex-ante level which maximizes $E[U(Y)]$. That is:

$$\bar{L}_u = L_u$$

where $\bar{L}_u$ is the average level of employment under uncertainty in a multiple-period framework and L_u is the single-period optimal employment level.

Certainty: Single-period horizon

Consider the case where the firm operates under condition of certainty. Certainty, in a single-period context, is given a meaning that differs from the standard approach to the theory of the firm under uncertainty. By certainty we mean a situation in which the firm waits until the end-of-period price is revealed and then makes its employment and production decisions, that is,

the firm makes ex-post decisions after it has observed the actual price at which it can sell its output. Ex-post production is, in this case, instantaneous. Note the difference with the traditional approach which assumes that under certainty the firm faces the average price for certain.

Suppose that p_r is the revealed price known with certainty by the labor-managed firm. Optimal employment and production decisions are found at the point where:

$$E\left[U'(Y)\cdot(p_r Q_L - Y)\right] \equiv U'(Y)\cdot(p_r Q_L - Y) = 0$$

and since $U'(Y) \neq 0$, it follows that the optimal level of employment under certainty and a single-period horizon, L_c, must satisfy the equilibrium conditions:

$$p_r Q_L(L_c) = Y\ (L_c) \tag{4}$$

Certainty: Multiple-period horizon

Suppose that the labor-managed firm has a multiple-period horizon. For the model just described, this means that the firm faces the same probability distribution of prices period after period. Over each period, the firm waits for the actual price to be revealed and then produces the corresponding optimal level of output for that price according to condition (4). The price the firm will observe at the end of each period will be drawn from the same distribution of prices and will generally not be the same period after period. Although the firm operates under

certainty, it nevertheless faces price instability. No decision, however, is made prior to the revelation of the output price.

The <u>average</u> level of employment and output (computed as the expected value of employment and output) in this multiple-period context must satisfy the condition:

$$E\left[p_r Q_L(L_c)\right] = E\left[Y\ (L_c)\right]$$

or

$$E\left[p_r Q_L(L_c)\right] = \bar{Y}\ (L_c) \qquad (5)$$

Note that L_c can be considered as a random variable in a multiple-period framework. Although its values are known with certainty at the end of each period, they are not identical. The framework, however, is one of certainty. Applying Jensen's inequality [2] to the expectation of the value of marginal product of labor $E\ p_r Q_L(L_c)$ we get:

$$E\left[p_r Q_L(L_c)\right] \quad \left\{ \begin{matrix} \leq \\ = \\ > \end{matrix} \right\} \quad \bar{p}\ Q_L\ (\bar{L}_c) \quad \text{if} \quad Q_{LLL} \left\{ \begin{matrix} \geq \\ = \\ < \end{matrix} \right\} 0$$

which, using (5), can be rewritten as:

$$\bar{Y}\ (L_c) \left\{ \begin{matrix} \leq \\ = \\ > \end{matrix} \right\} \bar{p}\ Q_L\ (\bar{L}_c) \quad \text{if} \quad Q_{LLL} \left\{ \begin{matrix} \geq \\ = \\ < \end{matrix} \right\} 0 \qquad (6)$$

III COMPARATIVE EMPLOYMENT AND PRODUCTION DECISIONS: UNCERTAINTY VS CERTAINTY

Within the multiple-period framework described above, we can compare the level of employment and production of the labor-managed firm operating under uncertainty to the level of employment and production of the labor-managed firm operating under certainty with price instability. This is done by comparing the average level of employment in the case of uncertainty ($\bar{L}_u$) to the average level of employment in the case of certainty ($\bar{L}_c$). Under uncertainty we have:

$$E\left[p\, Q_L\, (\bar{L}_u)\right] \lesseqgtr \bar{Y} \text{ if } U''(Y) \left\{\lesseqgtr\right\} 0$$

This is condition (3) in which $\bar{L}_u$ is substituted for L_u since $\bar{L}_u = L_u$. The left-hand term is equal to $\bar{p}Q_L(\bar{L}_u)$ and thus:

$$Q_L\, (L_u) \lesseqgtr \bar{Y}/\bar{p} \quad \text{if} \quad U''\,(Y) \left\{\lesseqgtr\right\} 0 \qquad (7)$$

Under uncertainty and price instability we have - see (6):

$$Q_L\, (\bar{L}_c) \lesseqgtr \bar{Y}/\bar{p} \qquad \text{if} \qquad Q_{LLL} \left\{\lesseqgtr\right\} 0 \qquad (8)$$

With conditions (7) and (8) we can now compare the behavior of the labor-managed firm under uncertainty to the behavior of the labor-managed firm under certainty and price instability.

Comparative employment and output under risk-aversion

Under risk-aversion $U''(Y) < 0$ and from (7) and (8) it follows that:

$$\bar{L}_u < \bar{L}_c \quad \text{if} \quad Q_L \text{ is concave} \quad (Q_{LLL} > 0)$$

$$\bar{L}_u < \bar{L}_c \quad \text{if} \quad Q_L \text{ is linear} \quad (Q_{LLL} = 0)$$

$$\bar{L}_u \lesseqgtr \bar{L}_c \quad \text{if} \quad Q_L \text{ is convex} \quad (Q_{LLL} < 0)$$

If the marginal product of labor curve is not convex, that is, when the production function is well-behaved, the average employment and production under risk-aversion and uncertainty are smaller than the average employment and production under certainty. Thus, we have established - using a multiple-period framework - a result that is the opposite of the conclusion obtained from the single-period, traditional model of the firm under uncertainty.[3] Note that if the marginal product of labor curve is convex, comparative employment and output are ambiguous.

Comparative employment and output under risk-neutrality

Under risk-neutrality $U''(Y) = 0$ and from (7) and (8) it follows that;

$$\bar{L}_u \left\{ \lesseqgtr \right\} \bar{L}_c \quad \text{if} \quad Q_L \text{ is } \left\{ \begin{array}{l} \text{concave} \\ \text{linear} \\ \text{convex} \end{array} \right\}$$

When the marginal product of labor curve is not convex and the firm is not risk-seeking ($U'' \leqq 0$) average employment and output under uncertainty are always smaller than in the case of certainty. Note that in our multiple-period framework, employment and production decisions under risk-neutrality are not similar to those under certainty as in the traditional approach.

Comparative employment and output under risk-seeking

When the firm is risk-seeker we have:

$$\bar{L}_u \begin{Bmatrix} ? \\ > \\ > \end{Bmatrix} \bar{L}_c \quad \text{if} \quad Q_{LLL} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$$

Results are ambiguous when the marginal product of labor curve is concave. Otherwise, average employment and production under uncertainty exceed those under certainty.

IV PRICE INSTABILITY VS PRICE STABILIZATION

Oi (1961) has shown that the risk-neutral competitive capitalistic firm capable of adjusting its production instantaneously prefers to sell its output in a market characterized by price instability to a market in which prices are stabilized at their average value. We can now demonstrate that this preference for price instability is also displayed by the labor-managed firm.

To establish this result, one must first prove that the income per worker $Y(L) = L^{-1}\, p(Q(L) - F)$ is a convex function of price. Then, by application of Jensen's inequality, the desired conclusion is reached.

The first derivative of Y(L) with respect to price, evaluated at the point of optimal employment, is:

$$\frac{dy}{dp} = a + \frac{1}{L}\,(pQ_L - Y) \cdot \frac{dL}{dp} = a > 0$$

since at the point of optimal employment $pQ_L = Y$. Recall that "a" is the average product of labor. The second derivative of Y(L) with respect to price, evaluated at the point of optimal employment, is:

$$\frac{d^2Y}{dp^2} = \frac{1}{L}\,(Q_L - a) \cdot \frac{dL}{dp}$$

By implicit differentiation of the equilibrium condition $(dY/dL)=0$ with respect to price we get:

$$\frac{dL}{dp} = -\frac{d}{dp}\left(\frac{dY}{dL}\right)\Big/\frac{d^2Y}{dL^2} = -\frac{Q_L - a}{pQ_{LL}} < 0$$

which is negative given the set of assumptions in the second section. Thus $d\,Y/dp$ is positive. The second derivative of income per worker with respect to output price being positive, it follows that income per worker is a convex function of price. Applying Jensen's inequality to the function Y(p) we get:

$$E\left[Y(p)\right] > Y(\bar{p})$$

Expected income per worker under price instability is greater than income per worker when prices are stabilized at their average value. Hence, the risk-neutral labor-managed firm prefers price instability to price stability.

V CONCLUSION

In this paper we have compared the employment and production decisions of a labor-managed firm operating under uncertainty to the employment and production decisions of a labor-managed firm facing an uncertain demand. We have shown that a multiple-period framework where certainty means that the firm makes ex-post employment and production decisions the risk-averse labor-managed firm will hire less workers and produce a smaller output than the same firm operating under conditions of certainty if the marginal product of labor curve is not convex. We have also shown that the behavior of the risk-neutral firm in a multiple-period framework differs from that of the "certainty" firm. Finally, we have demonstrated that Oi's proposition derived for a capitalistic firm is also valid for the labor-managed firm: under risk-neutrality, price instability is preferred to price stability.

FOOTNOTES

1. The sign of covariance is the same as the sign of the derivative $(dU'(Y)/dp)$. This derivative equals $U''.(dY/dp)$. Since (dY/dp) is positive, the sign of the covariance is that of $U''(Y)$. Now, $U''(Y) < 0$ under risk-aversion, $U'' = 0$ under risk-neutrality, and $U''(Y) > 0$ under risk-seeking. Note that $E\left[U'(Y)\right] > 0$.

2. According to Jensen's inequality, the expected value of a convex (concave) function of a random variable is greater (smaller) than the value of that function evaluated at the expected value of the random variable.

3. Note that the production and employment behavior of the risk-averse capitalistic firm is also modified when examined in a multiple-period framework. We do not, however, observe a reversal as in the case of the risk-averse labor-managed firm. Specifically, if the objective function of the risk-averse capitalistic firm is the maximization of $E\left[U(pQ-wL-F)\right]$, where w is the competitive wage rate, then by following a similar approach to that presented in the paper for the case of the labor-managed firm we get for the risk-averse capitalistic firm facing an uncertain demand:

$$\bar{p}Q_L(\bar{L}_u) > w$$

and for the same firm facing an unstable demand under certainty and having a concave marginal product of labor curve we get

$$\bar{p}Q_L(\bar{L}_c) > w$$

and, hence, the employment and production behavior of the capitalistic firm is ambigious since both $Q(\bar{L}_u)$ and $Q(\bar{L}_c)$ exceed the competitive wage rate w.

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