A TEST OF THE GENERALIZED CAPITAL ASSET PRICING MODEL

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Abstract

In this note we present an empirical test of one of the implications of Levy's Generalized Capital Asset Pricing Model, namely, that "the same investor may face two (or more) different prices of risk". Using a random-coefficients model of the security market line, we cannot reject the hypothesis that there exist multiple market prices of risk over a given calendar period regardless of the length of the investment horizon chosen to perform the empirical tests.

A TEST OF THE GENERALIZED CAPITAL ASSET PRICING MODEL

I. Introduction

In a recent contribution, Levy [7] proposes a Generalized Capital Asset Pricing Model (GCAPM) based on the assumption that, "as a result of transaction costs, indivisibility of investments, or even the cost of keeping track of the new financial development of all securities, the k-th investor decides to invest only in N_k securities" (p. 644). The major results of the GCAPM are: (1) the i-th security variance will play a central role in its equilibrium price determination, quite contrary to the result of the traditional CAPM" (p. 649), and (2) in equilibrium "the same investor may face two (or more) different prices of risk, one appropriate for security i and one for security j" (p. 648).

Levy tests only the first theoretical result with 101 stocks traded on the New York Stock Exchange (NYSE) during 1948-1968. His emphasis is toward theoretically interpreting, within a framework that still retains the basic CAPM structure, the result obtained by Lintner [9], Douglas [2], and Miller and Scholes [12] that the variance of a security is a major factor in the determination of its average return, whereas according to the traditional CAPM, only the systematic risk is compensated for in the capital market. In empirical work, the commonly used measures of risk are systematic risk (β: beta), total risk (σ^2: variance of return), and unsystematic risk (s^2: variance of time-series regression residuals). They are employed

in isolation or in combination to empirically determine the ex post price of each type of risk.[1] But, in all these studies, through the construction of the regression equation, there is an implicit assumption that there exists a unique market price of risk for any given measure of risk.

In this note, we empirically test Levy's GCAPM through its associated second theoretical result that multiple prices of risk may exist in an imperfect capital market. We analyze a large sample of 1,115 securities over a 15-year (1962-1976) period hypothesizing a security market line (SML) with random coefficients. The tests are performed over 4 non-overlapping subperiods of 900 days each using return intervals (investment horizons) of varying length (daily, weekly, biweekly, and monthly). We find that our empirical results support the hypothesis that there exist multiple market prices of risk prevailing simultaneously within the same calendar period irrespective of the length of the investment horizon chosen to perform the tests.

II. The Sample.

The data are from the Center for Research in Security Prices (CRSP). The sample consists of 828 common stocks traded on the NYSE and 287 on the American Stock Exchange. A security is included in the sample only if it has 3600 observations of daily returns beginning on July 1962 and ending in November 1976. Over this interval of 15 years there are 3,950 securities listed on these two exchanges. The proxy for the general market movement is the return on a value-weighted index (Rm) of all the securities listed on a particular day. The weight is equal to the total market value of an individual security divided by the total market value of all the securities generating that index.

III. The Methodology

Based on the GCAPM, the SML is:

$$\bar{R}_i = \gamma_{0i} + \gamma_{1i} b_i + \eta_i \tag{1}$$

where, for a given period, $\bar{R}_i$ is the average realized return and b_i is the estimated systematic risk of security -i[2], γ_{1i} is the market price of risk (MPR) and γ_{0i} is the return on a zero-beta portfolio appropriate for security-i, and η_i is the error term with $E(\eta_i)=0$. The usual cross-sectional regression of (1) will ignore the differences in γ_{1i} and γ_{0i}, and will generate biased estimates of cross-sectional averages $\bar{\gamma}_1$ and $\bar{\gamma}_0$.

To test for the cross-sectional instability of γ_0 and γ_1, we define,

$$\gamma_{0i} = \bar{\gamma}_0 + d_{0i} \text{ , and} \tag{2}$$

$$\gamma_{1i} = \bar{\gamma}_1 + d_{1i}. \tag{3}$$

Then, (1) can be rewritten as,

$$\bar{R}_i = \bar{\gamma}_0 + \bar{\gamma}_1 b_i + d_{0i} + d_{1i} b_i + \eta_i. \tag{4}$$

If the cross-sectional variance of d_1 is significantly different from zero it indicates that γ_{1i} are different across securities, i.e., there is no unique MPR.

To estimate the values of var(d_0) and var(d_1), we specify the following OLS regression:

$$\bar{R}_i = \bar{\gamma}_0 + \bar{\gamma}_1 b_i + \varepsilon_i. \tag{5}$$

from (4) and (5) together, we have,

$$\varepsilon_i = d_{0i} + d_{1i}b_i + \eta_i, \quad \text{and} \tag{6}$$

$$\varepsilon_i^2 = d_{0i}^2 + \eta_i^2 + d_{1i}^2 b_i^2 + 2(d_{0i}d_{1i}b_i + d_{0i}\eta_i + d_{1i}b_i\eta_i) \tag{6'}$$

Assuming that the cross-sectional average of each of the three cross-product terms is zero, we can specify the following regression equation:[3,4]

$$\varepsilon_i^2 = V_0 + V_1 b_i^2 + \varphi_i. \tag{7}$$

Then, from (6') and (7) we get,

$$\hat{V}_0 = \hat{var}(d_0) + \hat{var}(\eta_i) \leq \hat{var}(\varepsilon), \quad \text{and} \tag{8}$$

$$\hat{V}_1 = \hat{var}(d_1). \tag{9}$$

If $\hat{V}_1$ $(=\hat{var}(d_1))$ is significantly greater than zero it implies that γ_{1i} are not identical for the securities in the sample, i.e., there are multiple MPR. It also implies that the OLS estimates of (5), where the assumption is of a unique MPR, are inconsistent and the GLS estimation process should be used. Also, if $\hat{V}_1 > 0$, then $\hat{V}_0(=\hat{var}(d_0) + \hat{var}(\eta))$ will be less than $\hat{var}(\varepsilon)$ implying that $\hat{var}(\varepsilon)$ extremely overestimates, and $\hat{V}_0$ may overestimate, the variance $(var(\eta))$ of the true error, terms (η_i). But, given the methodology we will not be able to decompose $\hat{V}_0$ in its two components - the variance of differing intercepts γ_{0i} and the variance of the true error terms.

IV. The Empirical Findings

To test the validity of the results over different calendar periods we divide the 3600 days of data into four equal periods of 900 days each. These periods are: (1) July 1962 - February 1966, (2) February 1966 - September 1969, (3) September 1969 - May 1973, and (4) May 1973 - November 1976.

Within each of the four calendar periods the tests were performed over return intervals (investment horizons) of varying length. We use daily (1-day), weekly (5-day), biweekly (10-day), and monthly (20-day) returns. While estimating the MPR we neutralize the trivial effect of differing return intervals by "annualizing" all the data. This is achieved by multiplying the daily, weekly, biweekly, and monthly returns by 250, 50, 25, and 12.5, respectively. Thus, the annualized average returns ($\bar{R}_i$) will be identical for the four return intervals.[5]

The first and second stage estimates of the SML for the four periods are presented in the middle and last panels of Tables 1 through 4. In all periods, for all the return intervals, the t-values of γ_1 (MPR) indicate that the estimated MPR are significantly different from zero. The GLS estimates of γ_1 are higher than the corresponding OLS estimates in periods 1 and 4, while for the other two periods these two sets of estimates are not significantly different. The behavior of MPR with respect to <u>increasing</u> return interval can be summarized as follows: Period 1, MPR is positive and decreases steadily. Period 2, MPR is positive and increases steadily. Period 3, MPR is negative and stable.[6] Period 4, MPR is negative and decreases steadily.[7]

The second stage estimates of V_1 (var(d_1)) and V_0(var(d_0)+(var(η)) for the four periods are presented in the top panels of the tables. In periods 1, 2 and 4, $\hat{V}_1$ are highly significant indicating that there are multiple MPR in each of these three periods. As expected, based on the significance of $\hat{V}_1$, $\hat{V}_0$ is significantly lower than $\hat{var}(\varepsilon)$. This implies that the variance of true residuals is overestimated by $\hat{var}(\varepsilon)$ in the first stage of the SML estimation. In periods 2 and 4, $\hat{V}_1$ decreases as the return interval is increased implying that the randomness in the MPR is lower when a longer return interval is used. For period 3, $\hat{V}_1$ is not significant indicating that multiple MPR may not exist in this period. As a corollary, in period 3, $\hat{V}_0$ is not significantly different from $\hat{var}(\varepsilon)$.

V. Conclusion

In this comment we present an empirical test of one of the implications of Levy's GCAPM, namely, that "the same investor may face two (or more) different prices of risk". Using a random coefficients model of the SML we cannot reject the hypothesis that there exist multiple MPR over a given calendar period.

FOOTNOTES

1. A selective list of studies includes Lintner [9], Douglas [2], Friend and Blume [4], Miller and Scholes [12], Black, Jensen and Scholes [1], Fama and MacBeth [3], Levy [7], and Litzenberger and Ramaswamy [10].

2. The $\bar{R}_i$ and b_i for security-i are derived from the time-series data as under:

$$\bar{R}_i = \Sigma_{t=1}^{T} R_{it}/T$$

$$b_i = \hat{cov}(R_i, R_m)/\hat{var}(R_m)$$

3. This is a very simplified explanation of a linear model wwith random coefficients. In estimating (5), (7), and the GLS version of (5), we follow all the details specified in Thiel [15], Swamy [14], and Maddala [11].

4. Eq. (7) itself has heteroskedasticity, and to derive consistent estimators of V_0 and V_1 we use two-stage estimation. In this note, we present only the second stage results for (7).

5. Returns are measured as logarithms of price relativestare hence "annualized" average returns are invariant to the length of the return interval.

6. As these tests are besed on <u>ex post</u> returns, it is possible to have negative MPR.

7. The behavior of MPR with respect to changing return interval is examined in detail bt Hawawini and Vora [5].

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Table 1

Statistics for Daily Returns

Second Stage (GLS) Estimates of the Variances

Period	$\hat{V}_0$	$t(\hat{V}_0)$	$\hat{V}_1$	$t(\hat{V}_1)$
1	.01481	14.41	.01240	3.35
2	.01827	14.38	.00876	1.84
3	.02275	15.67	.00188	0.58
4	.01806	14.87	.01303	2.56

First Stage (OLS) Estimates of the SML

Period	$\hat{\gamma}_0$	$t(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$t(\hat{\gamma}_1)$	$\hat{var}(\varepsilon)$
1	.12202	15.94	.06302	8.62	.01908
2	.03398	3.77	.04046	5.30	.02106
3	.10055	10.71	-.13651	-16.09	.02326
4	.03516	3.86	-.00038	-0.04	.02047

Second Stage (GLS) Estimates of the SML

Period	$\hat{\gamma}_0$	$t(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$t(\hat{\gamma}_1)$
1	.11693	14.48	.07116	8.43
2	.02923	3.07	.04595	5.42
3	.10123	10.03	-.13693	-15.83
4	.02967	3.08	.00913	0.85

Table 2

Statistics for Weekly Returns

Period	$\hat{V}_0$	$t(\hat{V}_0)$	$\hat{V}_1$	$t(\hat{V}_1)$
1	.01581	13.68	.01448	3.45
2	.01827	14.68	.00876	1.84
3	.02433	16.30	.00040	0.12
4	.01834	15.15	.01291	2.23

Period	γ_0	$t(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$t(\hat{\gamma}_1)$	$\hat{var}(\varepsilon)$
1	.13998	15.43	.03742	4.68	.01996
2	.01111	1.07	.05794	6.81	.02073
3	.09485	8.71	-.12885	-13.94	.02441
4	.06175	5.72	-.02918	-2.72	.02033

Period	$\hat{\gamma}_0$	$t(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$t(\hat{\gamma}_1)$
1	.13018	13.30	.04898	5.38
2	.01207	1.10	.05714	6.28
3	.09518	8.72	-.12911	-13.91
4	.05757	5.03	-.02239	-1.93

Table 3

Statistics for Biweekly Returns

Period	$\hat{V}_0$	$t(\hat{V}_0)$	$\hat{V}_1$	$t(\hat{V}_1)$
1	.01563	13.57	.01491	3.62
2	.01827	14.38	.00876	1.84
3	.02258	15.96	.00129	0.43
4	.01830	15.06	.01076	2.04

Period	$\hat{\gamma}_0$	$t(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$t(\hat{\gamma}_1)$	$\hat{var}(\varepsilon)$
1	.14386	15.76	.03275	4.16	.02004
2	.00044	0.04	.06432	7.59	.02053
3	.11927	11.16	-.13805	-16.68	.02294
4	.07877	7.11	-.04371	-4.29	.02013

Period	$\hat{\gamma}_0$	$t(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$t(\hat{\gamma}_1)$
1	.11693	14.48	.07116	8.43
2	.00122	0.11	.06328	6.93
3	.11982	11.09	-.13833	-16.45
4	.07679	6.55	-.04018	-3.65

Table 4

Statistics for Monthly Returns

Period	$\hat{V}_0$	$t(\hat{V}_0)$	$\hat{V}_1$	$t(\hat{V}_1)$
1	.01636	13.18	.01200	3.62
2	.01800	13.94	.00559	1.72
3	.02162	16.21	.00129	0.43
4	.01792	15.20	.00914	1.78

Period	$\hat{\gamma}_0$	$t(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$t(\hat{\gamma}_1)$	$\hat{var}(\varepsilon)$
1	.15127	16.75	.02449	3.29	.02015
2	-.00518	-0.51	.06209	8.72	.02019
3	.12685	12.32	-.13775	-18.21	.02209
4	.09721	8.62	-.05626	-5.96	.01983

Period	$\hat{\gamma}_0$	$t(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$t(\hat{\gamma}_1)$
1	.14302	14.37	.03423	3.97
2	-.00594	-.56	.06224	8.21
3	.12782	12.22	-.13833	-17.84
4	.09499	7.99	-.05313	-5.22

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