PRODUCTION DECISIONS IN THE MIXED FIRM

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This paper develops a model for production decisions under uncertainty in a mixed firm, that is, a firm whose shares are jointly held by private investors and state institutions. It is shown that unanimity among all shareholders on the optimal production plan of the firm can be reached, provided the government shareholder is willing to allocate subsidies when its own optimal plan diverges from the private shareholders' one. Under this assumption, the model yields the firm's optimal production decisions and subsidies. An example is given. It is concluded that mixed firms should be run like any private firm facing subsidized input and output prices instead of market ones.

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I - Introduction

Active participation of the state in national economic activity is a common feature of most countries where central planning has not replaced market institutions. Beyond their traditional roles as regulators of the "free market" mechanism by means of instruments like tax, credit and monetary policies, governments in decentralized economies are engaging in more direct interventions at most levels of the economic process. This government activism covers a large and complex spectrum of interactions with the private economic sector depending upon the country's socio-cultural background, the ideology in place and the perceived ability of the existing private firms to preserve and develop welfare achievements through changes in the economic, social and political environment. Outright nationalisation of private firms, price and wage controls, protection from competition, investment incentives and direct subsidies are the most known forms of intervention, and have been the subject of numerous studies. Less familiar to the public, as well as to the academic community, is the state's participation in the country's economic activity through mixed firms, that is, firms whose shares are jointly held by the state and private shareholders.[1] Yet, the economic importance of those firms is quite substantial. For example, by the definition given above, nearly one out of three of the ten firms with the largest turnover in Germany, Italy, France and Great Britain is a mixed firm (Carson and Losco, 1976). In countries as diverse as Japan, India, Israel and Mexico, mixed firms have been and still are instrumental in economic development (Musolf, 1972; Friedman, 1974).

Among the few studies specific to mixed firms, it is worth mentioning the works of Spencer (1975) on mixed firm in India and those of Friedman (1974), Schonfield (1965) and Musolf (1972) on the role of mixed firms in the economic development of diverse countries. Unfortunately, none of these works address the important question of the impact of government partial ownership on the decision making process in the mixed firms. This paper attempts to overcome this deficiency by developing a model for production decisions of the mixed firm under uncertainty.[2]

The paper is organized as follows. Section 2 discusses the significance of mixed firms in decentralized economies. Section 3 describes the behaviorial assumptions of the model. In Section 4, the economic framework is presented. The equilibrium of the financial market where the mixed firm shares are traded, is characterized in Section 5. In Section 6, the optimal production plan of the mixed firm is developed together with the amount of subsidies that the government might have to allocate to the firm and an example is given. Section 7 summarizes the results obtained and discusses their implications.

II - Mixed Firms in Decentralized Economies

The objective of this section is to provide the reader, who is not familiar with the subject, with the origins and main characteristics of mixed firms as they are perceived from a study of this form of enterprise in diverse countries.[3]

Although mixed firms most often originate from government initiative, their creation is not always the expression of a deliberate policy. In many developed countries like France and Italy, where mixed firms occupy a prominent position in the national economy, more than a few have originated from the inability of existing private firms to avoid bankruptcy. This happened recently in France where the setting up of three mixed firms in the steel industry was initiated by the French government to prevent private firms, in an important economic sector, from defaulting.

However, in other circumstances, the creation of mixed firms is the expression of a more willful government policy. While there are differing motives for this policy, there is a general trend towards considering a mixed firm as one of the most efficient vehicles of government intervention in the economic process. There are, broadly, four reasons given to account for efficiency : (a) by engaging public funds in a joint venture, a government can directly channel private capital towards business activities in line with its economic policy or economic plan when one exists, (b) in setting up a jointly held firm it does not have to raise, through increased taxation or public debt, the entire capital which may be necessary to create a public firm, (c) the need to provide a return to private capital invested in mixed firms is often seen by governments as a means to reach more efficient management of public funds, (d) partial ownership of a firm allows the government to exercise more direct and effective control over the use of public funds (subsidies, tax credits, investment incentives) than it could in a private firm. It is also argued that mixed firms provide governments with a new source of revenue, i.e. the dividend income it is entitled to as shareholder.

Another aspect of mixed enterprise concerns the degree of permanence of the joint venture. Since mixed firms are most often created in order to initiate or support investment of private capital in the government's preferred business activities and given existing alternatives for the use of public funds, one may expect that the government will sell their stocks when mixed firms reach a certain level of growth and profitability. In fact, the experience shows that this has not been the prevalent behaviour of governments in most countries. The aversion of bureaucracy to changes, the fear of the political impact of selling government property as well as the feeling that private investors might be reluctant to buy, seem to explain the persistence of mixed enterprises in most countries.[4]

As for the performance of mixed firms, the desirability of making "some" profits is largely shared and often publicly claimed by most governments.[5] In a decentralized economy, private investors have alternative investment opportunities. Thus, for a mixed firm to be viable, it must provide its shareholders with a return on their stocks which is commensurate with that provided by private firms. It follows that when the government shareholder wants to impose some welfare constraints on mixed firms decisions which might impair the profitability of those firms, proper compensation is needed.[6] It is one of the objectives of this paper to determine the form and amount of compensation that mixed firms are entitled to claim for taking into account government preferences with regard to the production decisions of the firm.

III - Behavioral Assumptions

Decisions made by a firm are generally presumed to be made in the best interests of its shareholders. Applying this postulate to a private firm, it is well known that, under certainty, shareholders unanimously prefer the firm to act so as to maximize its market value (Hirschleifer, 1970 ; Fama and Miller, 1972). Under uncertainty, Stiglitz (1972), Jensen and Long, (1972) and Baron (1979) have shown that value maximizing criterion is not Pareto Optimal and, in some cases, not unanimously supported by the shareholders when applied in a mean variance framework. To provide for an alternative criterion, Ekern and Wilson (1974), Leland (1973), Radner (1974), and Baron (1979), have defined conditions under which the firm's decisions are Pareto Optimal and have the unanimous consent of all behaviour of governments in most countries. The aversion of bureaucracy to changes, the fear of the political impact of selling government property as well as the feeling that private investors might be reluctant to buy, seem to explain the persistence of mixed enterprises in most countries.[4]

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On the other hand, the state expects the mixed firm to make decisions according to the "public" interests. Since the state's activities are carried out by its government, it is assumed that these decisions should account for the government's trade-offs between the competing objectives it wants to achieve directly through the mixed firm.[7] This paper takes these trade-offs into account by means of a government multiattribute welfare function. The relevant attributes of this function include the revenue the government receives from the firm and any decision variable it is willing to trade off with this revenue.

Usually, the government holds a fixed proportion of a mixed firm's common stock and does not trade in the market for private shares except for the purpose of maintaining its proportion of ownership when the firm issues or repurchases shares. As a consequence, it is assumed that there is no capital market effect on the attributes of the government utility function.

In deriving the model it will be shown that, under some conditions, there exist decisions made by the mixed firm that are unanimously favored by its private shareholders. These decisions may not be optimal for the government, however. In this case, if the decisions actually made by the mixed firm are those favored by the government, the unanimity property that characterizes the private shareholders' behavior implies that <u>all</u> of them will be willing to divest themselves of their shares, thus threatening the very existence of the mixed firm as such. So, for the mixed firm to exist in a decentralized economy, it is necessary to consider some kind of compensation to be allocated by the government when the latter imposes some constraints on the firm. When the compensation takes the form of subsidies allocated to the mixed firm, it will be shown that there exists a production plan supported by all its shareholders, private ones and government alike. For this plan, the marginal rates of technical substitution for the mixed firm are not equal to those for private firms. In other words, unanimous decisions in a mixed firm are non Pareto Optimal except for those decisions which are optimal for both the government and the private shareholders. Given that a government in a mixed economy pursues social welfare goals, this result is not surprising. It follows that decisions in a mixed firm should obey a "second best" criterion that allows for non Pareto Optimal decisions provided the government shareholder allocates subsidies to the mixed firm in order to obtain the agreement of its private partners on its preferred decisions.[8]

IV - The Economic Framework

The model spans two periods with all decisions made at the beginning of the first period and uncertainty in the model is represented by a state θ that affects the output prices for the products of the firms. The set of states Ω is assumed to be finite and to take on discrete values. For each state θ, it is assumed that all investors ascribe the same price $P_{m\theta}$ to each of the mixed firm product m and that $P_{m\theta} = P_m\varphi_\theta$ where φ_θ is a stochastic component common to all m.

To simplify and focus the analysis, the model will include only one mixed firm in the economy with all other firms being privately owned. The input, output and financial markets will be assumed to be competitive.

At the beginning of the first period, all firms announce their production and financial decisions and then proceed to operate until the beginning of the second period at which time they sell their products, repay their debt and distribute their net income to their shareholders.

All private investors are assumed to follow the axioms of choice in the von Neumann-Morgenstern (1953) sense so that their preferences can be expressed in terms of their expected utility of consumption. Taxes on personal income are ignored.

The following notation is used :

r	gross rate of interest ($r > 1$).
$\bar{n}_j^i$	number of shares of firm j ($j = 0,1....,J$) currently held by investor i ($i = g,1,...,I$) where $j = 0$ refers to the mixed firm and $i = g$ to the government shareholder.
n_j^i	investor i optimal holding of firm j shares at the financial market equilibrium.
N_j	total number of shares issued by firm j.
B	market value of all riskless assets whose rate of return is r with B^i referring to the portion held by private investor i.
p_j	share price of firm j.
π_θ^i	investor i's probability belief with respect to the likelihood of the occurrence of state θ.
Ω	set of states θ.
$R_{j\theta}$	state dependent profit of firm j.
$R_{0\theta}^g$	income received by the government from the mixed firm.

$U^i_\theta = U^i_\theta\,(C^i_1, C^i_{2\theta})$	investor i ($i \neq g$) state dependent utility function for consumption in period 1 and in period 2. U^i_θ is assumed to be monotone increasing, concave and differentiable.
U^g_θ	government social welfare function whose relevant attributes are $R^g_{0\theta}$ and any production variable that it wishes to trade off with $R^g_{0\theta}$. U^g_θ is assumed to be differentiable.
$q_1 = q_1(q_m, x_k)$ $m \neq 1$	generalized production function of the mixed firm where q_m, $m = 1,\ldots,M_1,M_2,\ldots,M_M$ and x_k, $k = 1,\ldots,K_1,K_2,\ldots,K_K$ refer to output and input levels respectively.
q^*_m, x^*_k	level of product q_m (factor x_k) that the government wishes to trade off with its income $R^g_{0\theta}$ accrued from the mixed firm with $m = 2,\ldots,M_1$ and $k = 1,\ldots,K_1$. Thus $q^*_2,\ldots,q^*_{M_1}$ and $x^*_1,\ldots,x^*_{K_1}$ enter the government welfare function.
p^x_k, $p^q_{m\theta} = p^q_m\varphi_\theta$	input and state dependent output prices (respectively) of the mixed firm.
D_j, B_j	dividends distributed and amount of debt issued by firm j at the beginning of the first period.
T	uniform corporate tax rate (interest paid on debt is not taxed).

V - The Financial Market Equilibrium

In the traditional manner, a financial market equilibrium is derived by solving each private investor's expected utility maximization problem at the beginning of the first period given the announced plans of the firms. That is,

$$\underset{n^i_j,\ B^i,\ C^i_1}{\text{Max}}\ E^i \left\{U^i_\theta(C^i_1,\ C^i_{2\theta})\right\} \qquad i \neq g$$

under the budget constraint

$$\bar{b}^i + \sum_j \bar{n}^i_j(\frac{D_j}{N_j} + p_j) = C^i_1 + \sum_j n^i_j p_j + rB^i \qquad (1)$$

where $\bar{b}^i$ denotes the actual amount of cash held by investor i.

The second period consumption $C^i_{2\theta}$ is equal to the return of investor i's portfolio: [9]

$$C^i_{2\theta} = \sum_j \frac{n^i_j}{N_j} R_{j\theta} + rB^i \qquad (2)$$

To solve the maximization problem first eliminate rB^i by solving (1) and substituting into (2) to obtain:

$$C^i_{2\theta} = r\left\{\sum_j p_j(\bar{n}^i_j - n^i_j) + \sum_j \frac{\bar{n}^i_j}{N_j} D_j + B^i + (\bar{b}^i - C^i_1)\right\} + \sum_j \frac{n^i_j}{N_j} R_{j\theta} \qquad (3)$$

Then, substitute $C^i_{2\theta}$ for this expression in the objective function and set the partial derivatives of U^i_θ with respect to the independent variables equal to zero. Doing so, one obtains the first order necessary conditions:

$$E^i\left\{\frac{\partial U^i_\theta}{\partial C^i_1}\right\} - rE^i\left\{\frac{\partial U^i_\theta}{\partial C^i_{2\theta}}\right\} = 0 \qquad (4)$$

$$E^i\left\{\frac{\partial U^i_\theta}{\partial C^i_{2\theta}}\left(\frac{R_{j\theta}}{N_j} - rp_j\right)\right\} = 0 \qquad (5)$$

Equations (4) and (5) determine the optimal holdings B^i, n^i_j for each private investor and his level of consumption in period one for a given set of prices p_j and r. [10]

Now define ρ_θ^i such that

$$\rho_\theta^i = \frac{\pi_\theta^i \left\{\dfrac{\partial U_\theta^i}{\partial C_{2\theta}^i}\right\}}{E^i \left\{\dfrac{\partial U_\theta^i}{\partial C_{2\theta}^i}\right\}}$$

which implies

$$E^i \left\{\frac{\partial U_\theta^i}{\partial C_{2\theta}^i} z(\theta)\right\} = E^i \left\{\frac{\partial U_\theta^i}{\partial C_{2\theta}^i}\right\} \sum_\Omega \rho_\theta^i z(\theta)$$

where z (θ) is any state dependent variable.

Then, equation (5) may be written as

$$\frac{1}{N_0} \sum_\Omega \rho_\theta^i R_{0\theta} = rp_0$$

or

$$\sum_\Omega \rho_\theta^i R_{0\theta} = rV_0$$

where V_0 is the market value of the mixed firm.

It is shown in Appendix One that ρ_θ^i may be interpreted as the relative implicit price that investor i is willing to pay at the beginning of the second period for a lottery ticket that gives him the right to a unit of consumption in period two if state θ occurs and nothing if it does not. Thus equation (9) indicates that the market value of the mixed firm may be viewed as a certainty equivalent imputed to its profit by private shareholder i. Since

rV_0 is independent of any investor's i characteristic, this certainty equivalent is the same for all investors.

VI - The Mixed Firm's Optimal Production Plan

The production decisions of the mixed firm are analysed by characterizing first the decisions preferred by the private shareholders and those that are preferred by the government. Then, assuming that the government is willing to allocate subsidies to the firm, it is shown that an optimal production plan exists for the mixed firm that has the unanimous consent of all its shareholders. Moreover, the amount of subsidies can be characterized.

The Optimal Production Plan from the Private Shareholder's Point of View

Let X be the level of any production variable of the mixed firm. Private shareholder i will or will not prefer a change in X depending upon the sign of

$\dfrac{\partial E^i\left\{U_\theta^i\,(C_1^i,\ C_{2\theta}^i)\right\}}{\partial X}$, $i \neq g$, where U_θ^i is evaluated at the capital market equilibrium corresponding to the initially proposed production plan at which the capital market equilibrium is determined. Substituting $C_{2\theta}^i$ for the expression given by (3) and taking the partial derivative of U_θ^i with respect to X one obtains:

$$\frac{\partial E^i\left\{U_\theta^i\right\}}{\partial X} = E^i\left\{\frac{\partial U_\theta^i}{\partial C_1^i}\cdot\frac{\partial C_1^i}{\partial X}\right\}$$

$$+\ E^i\left\{\frac{\partial U_\theta^i}{\partial C_{2\theta}^i}\left(-r\,\frac{\partial C_1^i}{\partial X}+\frac{n_0^i}{N_0}\cdot\frac{\partial R_{0\theta}}{\partial X}-\sum_j r\left(n_j^i-\bar{n}_j^i\right)\left(\frac{\partial p_j}{\partial X}\right)^i\right)\right\} \qquad (10)$$

where $\left(\frac{\partial p_j}{\partial X}\right)^i$ is the investor's i forecasted change in the price of firm's j share.

At the market equilibrium $E^i\left\{\frac{\partial U_\theta^i}{\partial C_1^i}\right\}$ can be substituted for its value given by (4). Doing so, equation (10) reduces to:

$$\frac{\partial E^i\left\{U_\theta^i\right\}}{\partial X} = E^i\left\{\frac{\partial U_\theta^i}{\partial C_{2\theta}^i}\left(\frac{n_0^i}{N_0}\cdot\frac{\partial R_{0\theta}}{\partial X} - \sum_j r\left(n_j^i - \bar{n}_j^i\right)\left(\frac{\partial p_j}{\partial X}\right)^i\right)\right\} \tag{11}$$

or, making use of equation (7)

$$\frac{\partial E^i\left\{U_\theta^i\right\}}{\partial X} = E^i\left\{\frac{\partial U_\theta^i}{\partial C_{2\theta}^i}\right\}\left(\frac{n_0^i}{N_0}\sum_\Omega \rho_\theta^i\,\frac{\partial R_{0\theta}}{\partial X} - \sum_j r\left(n_j^i - \bar{n}_j^i\right)\left(\frac{\partial p_j}{\partial X}\right)^i\right) \tag{12}$$

As a capital market equilibrium exists for all values of X, one can differentiate equation (8) to obtain:

$$\frac{1}{N_0}\left(\sum_\Omega \rho_\theta^i\frac{\partial R_{0\theta}}{\partial X} + \sum_\Omega \frac{\partial \rho_\theta^i}{\partial X}R_{0\theta}\right) = r\left(\frac{\partial p_j}{\partial X}\right)^i \quad j = 0,\ldots, J \tag{13}$$

Equations (12) and (13) together determine the investor's i ($i \neq g$) attitude towards a change in X. Now if one assumes that in most circumstances an investor evaluates the effect of a change in X at his actual implicit price ρ_θ^i, one can reduce equations (12) and (13) to more workable forms.[11]

Thus, from equation (13)

$$\left(\frac{\partial p_j}{\partial X}\right)^i = 0\,, \quad j \neq 0$$

$$\frac{1}{N_0}\sum_\Omega \rho_\theta^i\,\frac{\partial R_{0\theta}}{\partial X} - r\left(\frac{\partial p_0}{\partial X}\right)^i = 0 \tag{14}$$

and equation (12) simplifies to

$$\frac{\partial E^i\{U^i_\theta\}}{\partial X} = E^i\left\{\frac{\partial U^i_\theta}{\partial C^i_{2\theta}}\right\} rn^i_0\left(\frac{\partial p_0}{\partial X}\right)^i \tag{15}$$

Since $E^i\left\{\frac{\partial U^i_\theta}{\partial C^i_{2\theta}}\right\} > 0$,

the sign of (14) is thus the same as the sign of $\left(\frac{\partial p_0}{\partial X}\right)^i$. From this it can be asserted 1) that investor i prefers X to increase if he perceives it to increase p_0, i.e. the market value of the mixed firm and 2) that the same investor agrees with the change in X proposed by the firm that increases p_0 until $\left(\frac{\partial p_0}{\partial X}\right)^i = 0$. Then, from (14), the optimum level of any production variable X favored by investor i must satisfy

$$\sum_{\Omega}\rho^i_\theta \frac{\partial R_{0\theta}}{\partial X} = 0 \tag{16}$$

In other words, private investor i is expecting the mixed firm to make decisions such that its marginal certainty equivalent return is zero.

Direct application of equation (16) to the return function of the mixed firm is the last step in the determination of the optimal values of the production variables favored by investor i. The mixed firm after tax return $R_{0\theta}$ is given by the firm cash flow equation at the beginning of the second period:

$$R_{0\theta} = \sum_m q_m p^q_{m\theta} - T\left(\sum_m q_m p_{m\theta} - \sum_k x_k p^x_k - (r-1)B_0\right) - rB_0 \tag{17}$$

with, as assumed in Section 3, $p^q_{m\theta} = p^q_m \varphi_\theta$

If $R_{0\theta}$ in equation (16) is replaced by its expression given by (17) and (16) solved for the production variables $q_m (m \neq 1)$ and x_k, one obtains the following set of equations, which define the optimal values of the production variables unanimously agreed upon by private shareholders:

$$\frac{p^q_m}{p^q_1} = - \frac{\partial q_1}{\partial q_m} \qquad m = 2, \ldots, M_M \tag{18}$$

$$\frac{r p^x_k}{p^q_{1} \sum_{\Omega} \rho^i_\theta \varphi_\theta} = \frac{\partial q_1}{\partial x_k} \qquad k = 1, \ldots, K_K \tag{19}$$

with p^q_1 referring to output q_1.

For the values of the production variables that satisfy (18) and (19), there exists a capital market equilibrium characterized by equation (9). Substitution of $R_{0\theta}$ given by (17) in equation (9) shows that $\sum_\Omega \rho^i_\theta \varphi_\theta$ does not depend on any characteristic of private investor i. From this, it results that there exists <u>complete unanimity</u> among all the <u>private</u> shareholders of the mixed firm on the optimal values of the production variables as given by equations (18) and (19). Moreover, as $\sum_\Omega \rho^i_\theta \varphi_\theta$ is the same for all these investors, so must be $\sum_\Omega \rho^i_\theta p^q_m \varphi_\theta = \sum_\Omega \rho^i_\theta p^q_{m\theta}$. In other words, the unanimity property implies that the certainty equivalent unit price of output is the same for all <u>private</u> investors.

<u>The Optimal Production Plan from the Government's Point of View</u>

Following the assumptions made in Section III, and using the notations defined in Section IV, the government multiattribute welfare function may be written as:

$$U^g_\theta = U^g_\theta(R^g_{0\theta}, q^*_m, x^*_k, \alpha) \qquad \begin{array}{l} m = 2, \ldots, M_1 \\ k = 1, \ldots, K_1 \end{array} \tag{20}$$

where α includes all the attributes of U^g_θ that are independent of the mixed firm decisions.

The government will favor decisions made by the mixed firm provided that they maximize its expected utility. If the mixed firm does so, any independent production variable X, at its desired value, must verify

$$E^g\left\{\frac{\partial U^g_\theta(R^g_{0\theta}, q^*_1, x^*_k, \alpha)}{\partial X}\right\} = 0 \tag{21}$$

To solve (21) one needs to express $R^g_{0\theta}$ as a function of the production variables. The government income from the mixed firm includes (1) the dividends $\frac{n^g_0}{N_0} D_0$ received at the beginning of the first period, (2) the share of the net income of the firm $\frac{n^g_0}{N_0} R_{0\theta}$ (3) the receipt of the mixed firm's corporate tax received as (2) at the beginning of the second period. Defining $R^g_{0\theta}$ as the present value of this income discounted at the rate r, one can write:

$$R^g_{0\theta} = \frac{n^g_0}{N_0} D + \frac{1}{r}\frac{n^g_0}{N_0} R_{0\theta} + \frac{T}{r}\left(\sum_m q_m p^q_{m\theta} - \sum_k x_k p^x_k - (r-1)B_0\right) \tag{22}$$

Substituting $R_{0\theta}$ with its expression given by (17) into (22) yields:

$$R^g_{0\theta} = \frac{1}{r}\left(\frac{n^g_0}{N_0}(1 - T) + T\right)\left(\varphi_\theta \sum_m q_m p_m - r\sum_k x_k p^x_k\right) + \frac{T}{r}(r-1)\left(\frac{n^g_\theta}{N_0} - 1\right) B_0$$

$$+ \frac{n^g_0}{N_0}(D_0 - B_0) \tag{23}$$

Now let us define a government implicit price ρ^g_θ similar to the private shareholders' ones ρ^i_θ:

$$\rho^g_\theta = \frac{\pi^g_\theta \left\{\dfrac{\partial U^g_\theta}{\partial R^g_{0\theta}}\right\}}{E^g\left\{\dfrac{\partial U^g_\theta}{\partial R^g_{0\theta}}\right\}} \tag{24}$$

so that one can write:

$$E^g\left\{\frac{\partial U^g_\theta}{\partial R^g_{0\theta}}\varphi_\theta\right\} = E^g\left\{\frac{\partial U^g}{\partial R^g_{0\theta}}\right\}\sum_\Omega \rho^g_\theta\varphi_\theta \tag{25}$$

To solve (21), replace $R^g_{0\theta}$ by its expression given by (23) and take the partial derivatives with respect to the independent variables $q_m, x_k (m \neq 1)$. Doing so, one obtains a set of optimality conditions which can be expressed in terms of ρ^g_θ using equation (25):

$$\frac{p^q_m}{p^q_1} = \frac{\partial q_1}{\partial q_m} \quad m = M_2, \ldots, M_M \tag{26}$$

$$\frac{1}{p^q_1\sum_\Omega\rho^g_\theta\varphi_\theta}\left(p^q_m\sum_\Omega\rho^g_\theta\varphi_\theta + \frac{E^g\left\{\dfrac{\partial U^g_\theta}{\partial q^*_m}\right\}}{AE^g\left\{\dfrac{\partial U^g_\theta}{\partial R^g_{0\theta}}\right\}}\right) = -\frac{\partial q_1}{\partial q^*_m} \quad m = 2, \ldots, M_1 \tag{27}$$

$$\frac{rp^x_k}{p^q_1\sum_\Omega\rho^g_\theta\varphi_\theta} = \frac{\partial q_1}{\partial x_k} \qquad k = K_2, \ldots, K_K \tag{28}$$

$$\frac{1}{p_1^q \sum_\Omega \rho_\theta^g \varphi_\theta} \left(r p_k^x + \frac{E^g\left\{\dfrac{\partial U_\theta^g}{\partial x_k^*}\right\}}{A E^g\left\{\dfrac{\partial U_\theta^g}{\partial R_{0\theta}^g}\right\}} \right) = \frac{\partial q_1}{\partial x_k^*} \quad k = 1, \ldots, K_1 \tag{29}$$

$$\text{with} \quad A = \frac{1}{r}\left(\frac{n_0^g}{N_0}(1 - T) + T\right) \tag{30}$$

representing the government share of the mixed firm operating income.

By comparing these equations with the ones obtained in determining the private shareholders' preferred production plan, one notices differences in the content of the equations that may be interpreted as follows.

First, the government which does not trade in the capital market may ascribe certainty equivalent $p_m \sum_\Omega \rho_\theta^g \varphi_\theta$ to output prices that are different from the market ones $p_m \sum_\Omega \rho_\theta^i \varphi_\theta$. It follows that, even if the government does not wish to trade off its revenue with preferred levels of production variables, it may favor rates of technical substitution that are different from those favored by its private partners. While the model generates identical rates for the production variables $m = M_2, \ldots, M_M$ (equation 18 and 26), this would not generally be the case since this result is only the consequence of the multiplicative form of uncertainty assumed in the model for output prices. Thus, a more general form of uncertainty would lead to different rates of substitution for those production variables too.

Second, the existence of government trade offs among its income $R^g_{0\theta}$, x^*_k and q^*_m brings another distortion to the optimal values of the production variables with respect to the values favored by the private shareholders. In fact, these trade offs affect the unit prices of the preferred production variables which are used in the derivation of the marginal rates of technical substitution. These production variables are valued at the market price for the factors of production (equation (29)) or at the government certainty equivalent price for the products (equation (27)), plus a premium. This premium is equal to the ratio of the government's marginal utilities for its preferred output(input) and for its revenue.

Thus, it appears that the government preferred production plan can be determined in the same manner as the private shareholders'. Differences originate only in the valuation of the mixed firm products and government preferred production variables which are used in setting the optimality condition.

The Mixed Firm Optimal Production Plan

In general, the solutions of the two systems of equations which give the optimal values of the production variables for the private shareholders (equations 18 and 19) and for the government (equations 26 to 30) will not be the same. However, it can be shown that there exists a unique set of solutions for both systems of equations when, according to the assumption made in Section III, the government is willing to allocate subsidies to the mixed firm in order to obtain the agreement of its private partners on its production plan.

The subsidies to be considered should account (1) for divergent opinions on the value of the certainty equivalent prices ascribed to product $p^q_{m\theta}$ and (2) for setting the variables q^*_1 and x^*_k at levels favored by the government given its desire to have its private partners in agreement with it. In general, the final production plan will not be the same as the one the government originally favored, since the net income that the government will receive from the mixed firm will be reduced by the amount of subsidies it will allocate to the firm.

Let X^g and X^i be the vectors of optimal decisions initially favored by the government and the private shareholders. The allocation of subsidies can be seen as a means to generate a new vector of decisions, say X^{ig}, such that the private shareholders are indifferent to the choice between X^{ig} on the one hand and X^i on the other. This will occur if the condition expressed by equation (16) is verified, that is, if the mixed firm certainty equivalent return is still 0 when X^{ig} is adopted. Since factor and product prices are the main determinants of returns, it follows that the most appropriate way to subsidize the mixed firm is to change the set of prices faced by the firm through allocation of subsidies per unit of input and output.[12] Therefore, the mixed firm is supposed to make its production decisions according to the following set of unit prices:

$$
\begin{array}{ll}
P^q_{m\theta} + S^q_m & m = 1, M_2, \ldots, M_M \\
P^q_{m\theta} + S^{q*}_m & m = 2, \ldots, M_1 \\
P^x_k & k = K_2, \ldots, K_K \\
P^x_k + S^{x*}_k & k = 1, \ldots, K_1
\end{array}
$$

with subsidies to input S^{x*}_k allocated at the beginning of the first period and subsidies to output S_m^q, S_m^{q*} at the beginning of the second period. From this set of subsidized prices, new optimal conditions are obtained for both the private shareholders and the government using the same procedure followed in the case of non-subsidized prices (see Appendix Two). These conditions yield the following system of equations whose solutions are the optimal levels of the production variables and the amounts of subsidies:

$$\frac{p_m^q \sum_\Omega \rho_\theta^i \varphi_\theta + S_m^q}{p_1^q \sum_\Omega \rho_\theta^i \varphi_\theta + S_1^q} = -\frac{\partial q_1}{\partial q_m} \qquad m = M_2, \ldots, M_M \tag{31}$$

$$\frac{p_m^q \sum_\Omega \rho_\theta^i \varphi_\theta + S_m^{q*}}{p_1^q \sum_\Omega \rho_\theta^i \varphi_\theta + S_1^q} = -\frac{\partial q_1}{\partial q_m^*} \qquad m = 2, \ldots, M_1 \tag{32}$$

$$\frac{r p_k^x}{p_1^q \sum_\Omega \rho_\theta^i \varphi_\theta + S_1^q} = \frac{\partial q_1}{\partial x_k} \qquad k = K_2, \ldots, K_K \tag{33}$$

$$\frac{r p_k^x - S_k^{x*}}{p_1^q \sum_\Omega \rho_\theta^i \varphi_\theta + S_1^q} = \frac{\partial q_1}{\partial x_k^*} \qquad k = 1, \ldots, K_1 \tag{34}$$

with

$$S_m^q = Ar\left\{\sum_\Omega \rho_\theta^g \, p_m^q \varphi_\theta - \sum_\Omega \rho_\theta^i \, p_m^q \varphi_\theta\right\} \quad m = 1, M_2, \ldots, M_M \tag{35}$$

and

$$A = \frac{1}{r}\left(\frac{n_0^g}{N_0}(1 - T) + T\right)$$

$$S_k^{x*} = \frac{E^g\left\{\dfrac{\partial U_\theta^g}{\partial x_k^*}\right\}}{E^g\left\{\dfrac{\partial U_\theta^g}{\partial R_{0\theta}^g}\right\}} \qquad k = 1, \ldots, K_1 \qquad (36)$$

$$S_m^{q*} = Ar\left(\sum_\Omega \rho_\theta^g \; p_m^q{}_\theta^\varphi - \sum_\Omega \rho_\theta^i \; p_m^q{}_\theta^\varphi\right) + r\,\frac{E^g\left\{\dfrac{\partial U_\theta^g}{\partial q_m^*}\right\}}{E^g\left\{\dfrac{\partial U_\theta^g}{\partial R_{0\theta}^g}\right\}} \qquad m = 2, \ldots, M_1 \qquad (37)$$

As in the case of non-subsidized prices, it can be shown that the certainty equivalent unit price of output $\sum_\Omega \rho_\theta^i \; p_m^q{}_\theta^\varphi$ is the same for all private investors so that unanimity among all private shareholders regarding the mixed firm's optimal production plan still holds. Thus, equations (31) to (37) show that there exists a production plan X^{ig} for the mixed firm that has the unanimous support of <u>all</u> its shareholders. As shown by equations (31) to (34), X^{ig} is the same vector of decisions as the one which would be obtained in the deterministic case (Henderson and Quandt, 1971) for a privately owned firm in which certain output prices are substituted for subsidized certainty equivalent product prices and input prices for subsidized ones.

Equations (35) to (37) formulate explicitly the amount of subsidies that must be allocated to the mixed firm for the unanimity property to hold. The structure of these equations shows two properties of the model. First, one can identify and isolate two types of subsidies. There are those subsidies, subsequently referred to as type I subsidies, which the government must allocate to each unit of output when it ascribes certainty equivalent prices $\sum_{\Omega} \rho^g_\theta p^q_m \varphi_\theta$ different from the one ascribed by its private partners $\sum_{\Omega} \rho^i_\theta p^q_m \varphi_\theta$. From equations (35) and (37), these subsidies are equal

to $Ar\left(\sum_{\Omega} \rho^g_\theta p^q_m \varphi_\theta - \sum_{\Omega} \rho^i_\theta p^q_m \varphi_\theta\right)$ which is the expected net

incremental income accrued to the government per unit of output. And there are those subsidies, denoted type II subsidies, to allocate to each unit of the government's favored output (q^*_m) or input (x^*_k). These subsidies, as shown by equations (36) and (37) are equal to the ratio of the government marginal utility for the favored level of factor (product) to its marginal utility for its income accrued from the mixed firm. They result from the government trade-offs between two attributes of its utility functions. Second, as shown by equation (37), the two types of subsidies are additive. However, since type II subsidies are determined by the government expected marginal utility for its income received from the mixed firm, they depend upon the government implicit prices ρ^g_θ, as do type I subsidies. In other words, both subsidies, although additive, are not independent.

When government and private shareholders ascribe the same certainty equivalent prices to the mixed firm's product, that is when $\sum_{\Omega} \rho^g_\theta p^q_{m\theta} = \sum_{\Omega} \rho^i_\theta p^q_{m\theta}$, only type II subsidies have to be considered. In that case, the amount of subsidies are only determined by the government trade-offs between its income accrued from the mixed firm and the level of the firm production variables.

An Example

Let us consider a mixed firm investment decision to produce a good denoted G.

Suppose that the production unit of food G could be built on two sites, say A and B, located in regions with high unemployment. While the firm would prefer the plant to be built on site B where distribution costs are the lowest, the government expressed its preference for site A where unemployment is higher. The government having agreed upon the principle of subsidies allocation if site A is chosen, the model can be used to determine the actual amounts of the subsidies to be allocated to the mixed firm. Since it would be practically impossible to determine the yearly government state preference among the attributes of its welfare utility function, it is assumed that the government exhibits only a state independent utility function $U_g(L^A,\tilde{R}_g) = U_g$ relevant to the decision to be made where:

- L^A, size of the labor force needed to operate the industrial plant in site A; $L^A = \bar{L}$ given.
- $\tilde{R}_g$ present values of the government's income accrued from the mixed firm during the investment lifetime. $\tilde{R}_g$ is a function of the random investment yearly revenues accrued from yearly sales of good G.

Furthermore, it is assumed that the government wishes the mixed firm to make decisions like any other private firm would make in the use of its factors of production (except labor) and in the production of its goods. This assumption implies that the government certainty equivalent output price is the same as the market's one. Consequently, only type II subsidies should be allocated.

Assuming further that U_g takes the form of a simple additive utility function, one can write

$$U_g = a_R\, u(\tilde{R}_g) + a_A\, u(L^A) \tag{38}$$

where $u(\tilde{R}_g)$ and $u(L^A)$ are conditional utility functions over $\tilde{R}_g$ and L^A scaled from zero to one and a_R, a_A are scaling constants.

From equation (36), the amount of subsidies S^A the government should allocate to the mixed firm if site A is chosen is the solution of

$$S^A = \left. \frac{E\left\{U_g'\ (L^A)\right\}}{E_g\left\{U_g'\ (\tilde{R}_g)\right\}} \right|_{L^A = \bar{L}} \tag{39}$$

Taking the partial derivatives of $E\left\{U_g\right\}$ with respect to L^A and $\tilde{R}_g$ respectively, one obtains directly the amount of subsidy to allocate per job created in site A:

$$S^A = \left. \frac{a_A\ u'(L^A)}{a_R\ E\left\{u'(\tilde{R}_g)\right\}} \right|_{L^A = \bar{L}} \tag{40}$$

The main difficulty in evaluating S^A lies in the assessment of the utility function U_g. Recent progress in the determination of the decision makers' preference structure shows that such a utility function can be assessed in a quantitative manner (Keeney abd Raiffa, 1976). Hence, there should be no practical obstacle in applying formulae like (39) or (40) to actual investment decisions in mixed firms. [13]

VII - Conclusion

This paper has developed a model which applies to the mixed firm's production decisions. Central to the model is the assumption, well accepted in most countries where mixed firms operate, that the government shareholder is willing to allocate subsidies to the mixed firm in order to obtain the agreement of private shareholders on the production plan. It is through this allocation of subsidies that unanimity among all the mixed firm's shareholders, public and private, on the optimal production plan can be reached. It was shown that the subsidies must account (1) for divergent opinions between the private investors and the government on the certainty equivalent prices ascribed to the firm's products and (2) for the government trade-offs between the revenue it receives from the mixed firm and its preferred input and output levels.

Usually, if not always, governments in decentralized economies do not get involved in the formidable task of evaluating their own certainty equivalent future prices. They, de facto, subscribe to market prices which reflect a consensus opinion amongst all private investors. Thus, in most cases, mixed firms are justified in evaluating their production plans on the basis of these market prices. The model then led to three important conclusions. First, subsidies should be allocated to those factors or products subject to governmental preferences on a per unit basis. Second, the amount of subsidies are determined only by the government trade-offs between its income accrued from the mixed firm and its preferred levels of input and output as shown by equations (36) and (39). Finally, the mixed firm should evaluate its optimal production plan like any other privately owned firm provided, however, that subsidized input and output prices are substituted for market prices.

The implications of these findings on the mixed firm decision strategy, as well as on the government intervention policy, are quite clear. The managers of a mixed firm should run it as if it were privately owned. Any alteration in the production plan which might be sought by the government shareholder should only be subjected to changes in input and output prices brought on by the allocation of subsidies. As for the government, leaving aside the argument as to the legitimacy of its intervention, the model indicates that the relevant decision factors are its trade-offs between its revenue accrued from the mixed firm and any production decision it would like the firm to make. Recent progress made in the area of applied multiattribute utility theory shows that the assessment of these government trade-offs is feasible, thus allowing for operational use of the model.

Finally, although this study is restricted to mixed firms, most of its results could be extended to privately owned firms which are subject to government intervention as is the case in most mixed economies. As corporate tax recipient, the government still draws some revenue from other firms that can be subjected to trade-offs with any production decision it would like to enforce. There are then no conceptual or practical difficulties in applying the model to the evaluation of the private firm's optimal production plan and the amount of subsidies it is entitled to claim in those circumstances.

NOTES

1. A broader definition of mixed firms would include those firms where the government participates by making appointments to the board of directors (Musolf 1972, p.3). While the model developed in this paper applies primarily to joint stock companies it could be easily extended to this other type of mixed firm.

2. This paper is not intended to develop a general mixed-economy model in which the contribution of mixed firms (in eliminating market imperfections) would be analyzed. Rather, it is an attempt to find how decisions can be made within mixed firms that have the support of all their shareholders given that there exists a capital market in which the mixed firms' shares are traded.

3. This section draws on the works of Musolf (1971), Friedman (1974), as well as on the author's on-going study of European mixed firms.

4. However, a few exceptions do exist. In the United Kingdom, for example, shares of the electronic company Ferranti which were purchased by the National Enterprise Board to prevent the company from defaulting, have recently been sold on the stock market as the company's profit performance has improved dramatically.

5. MUSOLF (1971), p. 132.

6. SARACENO (1962), NORA (1967).

7. Examples are: increasing the revenue accrued to the government as shareholder and tax recipient, implementing labor intensive capital investment projects in a period of high unemployment, producing goods of strategic value that otherwise could be controlled by foreign firms, etc ...

8. In countries like France and Italy where the mixed firms have been the object of repeated government interference, the compensation principle through subsidies is widely accepted. See Nora (1967) and Saraceno (1962). Divergence is usually confined to the amount of subsidies to be allocated. The model presented in this paper addresses these questions directly by providing a set of equations whose solutions directly by providing a set of equations whose solutions include the amount of subsidies that have the agreement of all parties involved.

9. Wage income are ignored in the model for reasons of simplicity. Insofar as the mixed firm decisions have no effect on investors' wage incomes, it can be shown that the optimal production plan of the firm is not altered if these incomes were accounted for.

10. The conditions for the existence of these optima are assumed to hold.

11. This assumption is equivalent to the competitivity assumption under which investors behave as price takers with respect to their implicit prices ρ_θ^i. See Baron (1979).

12 Alternatively, the government may buy private shareholders' proxies by making a lump sum payment to them. It can be shown that this type of compensation leads to the same unanimously supported production plan and the same amount of compensation as obtained through input and output subsidies.

13. One will find in Viallet (1977) an application to a real industrial plant location problem which involved the assessment of a government multiattribute utility function.

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APPENDIX ONE - Investor's i Implicit Price ρ_θ^i

Investor i's expected utility is defined as

$$E^i\{U_\theta^i\} \equiv \sum_\Omega \pi_\theta^i U_\theta^i (C_1^i, C_{2\theta}^i) \qquad (1\text{-}1)$$

From this expression, one can derive the marginal rates of consumption between consumption in period two (state θ) and consumption in period one to obtain:

$$\frac{\partial C_{2\theta}^i}{\partial C_1^i} = - \frac{\sum_\Omega \pi_\theta^i \frac{\partial U_\theta^i}{\partial C_1^i}}{\pi_\theta^i \frac{\partial U_\theta^i}{\partial C_{2\theta}^i}} \qquad (1\text{-}2)$$

If one assumes further that there exists complete and competitive markets for time-state claims, trading in C_1^i and $C_{2\theta}^i$ will result in a set of market prices ϕ_1^i, $\phi_{2\theta}^i$ such that the marginal rates of substitution between two state claims is equal to the ratio of their prices, that is:

$$\frac{\partial C_{2\theta}^i}{\partial C_1^i} = - \frac{\phi_1^i}{\phi_{2\theta}^i} \qquad (1\text{-}3)$$

One may interpret $\phi_{2\theta}^i$ as the price that investor i is willing to pay <u>now</u> for a lottery ticket whose outcomes are a unit of consumption in period two if state θ occurs and nothing if it does not.

If C_1^i is chosen as numéraire, $\phi_1^i = 1$ and equation (1-3) expressed in <u>relative</u> prices becomes:

$$\frac{\partial C_{2\theta}^i}{\partial C_1^i} = - \frac{1}{\phi_{2\theta}^i} \qquad (1\text{-}4)$$

Combining equations (1-2) and (1-4), one gets

$$\phi^i_{2\theta} = \frac{\pi^i_\theta \dfrac{\partial U^i_\theta}{\partial C^i_{2\theta}}}{\sum_\Omega \pi^i_\theta \dfrac{\partial U^i_\theta}{\partial C^i_1}} \tag{1-5}$$

But, at the capital market equilibrium, equation (5) in the text is verified so that

$$\sum_\Omega \pi^i_\theta \frac{\partial U^i_\theta}{\partial C^i_1} = rE^i\left\{\frac{\partial U^i_\theta}{\partial C^i_{2\theta}}\right\} \tag{1-6}$$

Consequently, equation (1-5) can be rewritten as

$$\phi^i_{2\theta} = \frac{1}{r}\,\frac{\pi^i_\theta \dfrac{\partial U^i_\theta}{\partial C^i_{2\theta}}}{E^i\left\{\dfrac{\partial U^i_\theta}{\partial C^i_{2\theta}}\right\}} \tag{1-7}$$

Adding all over the states of nature, one obtains

$$\sum_\Omega \phi^i_2 = \frac{1}{r}\,\frac{\sum_\Omega \pi^i_\theta \dfrac{\partial U^i_\theta}{\partial C^i_{2\theta}}}{E^i\left\{\dfrac{\partial U^i_\theta}{\partial C^i_{2\theta}}\right\}} = \frac{1}{r} \tag{1-8}$$

In other words, the relative price that investor i is willing to pay <u>now</u> for a unit of certain consumption in period two is equal to the discounted value of one dollar. Now, equation (1-6) together with equation (6) imply

$$\rho^i_\theta = r\phi^i_{2\theta}$$

Thus, ρ_θ^i may be interpreted as the relative implicit price that investor i is willing to pay at the <u>beginning of the second period</u> for a lottery ticket that gives him the right to a unit of consumption in period two if state θ occurs and nothing if it does not.

APPENDIX TWO - Derivation of the Mixed Firm Optimal Production Plan

Substituting subsidized factors and certainty equivalent products prices for non-subsidized ones in equations (18) and (19), one gets the following set of optimality conditions from the private shareholder's point of view.

$$\frac{p_m^q \sum_{\Omega} \rho_\theta^i \varphi_\theta + s_m^q}{p_1^q \sum_{\Omega} \rho_\theta^i \varphi_\theta + s_1^q} = - \frac{\partial q_1}{\partial q_m} \qquad m = M_2, \ldots, M_M \qquad (2\text{-}1)$$

$$\frac{p_m^q \sum_{\Omega} \rho_\theta^i \varphi_\theta + s_m^{q*}}{p_1^q \sum_{\Omega} \rho_\theta^i \varphi_\theta + s_1^q} = - \frac{\partial q_1}{\partial q_m} \qquad m = 2, \ldots, M_1 \qquad (2\text{-}2)$$

$$\frac{r p_k^x}{p_1^q \sum_{\Omega} \rho_\theta^i \varphi_\theta + s_1^q} = \frac{\partial q_1}{\partial x_k} \qquad k = K_2, \ldots, K_K \qquad (2\text{-}3)$$

$$\frac{r p_k - s_k^{x*}}{p_1^q \sum_{\Omega} \rho_\theta^i \varphi_\theta + s_1^q} = \frac{\partial q_1}{\partial x_k} \qquad k = 1, \ldots, K_1 \qquad (2\text{-}4)$$

This system of equations gives the new level of production variables favored by the private shareholders, provided the amount of subsidies are known. The latter is obtained by solving the government expected utility maximization problem as indicated in the text when the government optimal production plan was determined. Doing so, the following set of equations is obtained:

$$\frac{Ar\left(p_m^q \sum_{\Omega} \rho_\theta \varphi_\theta + s_m^q\right) - s_m^q}{V} = - \frac{\partial q_1}{\partial q_m} \qquad m = M_2, \ldots, M_M \quad (2\text{-}5)$$

$$\frac{Ar\left(p_m^q \sum_\Omega \rho_\theta^g \varphi_\theta + S_m^{q*}\right) - S_m^{q*}}{V} + r\frac{E^g\left\{\dfrac{\partial U_\theta^g}{\partial q_m^*}\right\}}{VE^g\left\{\dfrac{\partial U_\theta^g}{\partial R_{0\theta}^q}\right\}} = \frac{\partial q_1}{\partial q_m^*} \quad m = 2,\ldots, M_1 \tag{2-6}$$

$$\frac{Ar^2 p_k^x}{V} = \frac{\partial q_1}{\partial x_k} \qquad k = K_2,\ldots, K_K \tag{2-7}$$

$$\frac{AP_k^x + rS_k^{x*}}{V} - r\frac{E^g\left\{\dfrac{\partial U^g}{\partial x_k^*}\right\}}{VE^g\left\{\dfrac{\partial U^g}{\partial R_{0\theta}^g}\right\}} = \frac{\partial q_1}{\partial x_k^*} \qquad k = 1,\ldots, K_1 \tag{2-8}$$

with

$$A = \frac{1}{r}\left(\frac{n_0^g}{N_0}(1 - T) + T\right)$$

and

$$V = Ar\left(p_1^q \sum_\Omega \rho_\theta^g \varphi_\theta + S_1^q\right) - S_1^q$$

From equations (2-3) and (2-7), one gets

$$Ar^2\left(p_1^q \sum_\Omega \rho_\theta^g \varphi_\theta + S_1^q\right) - rS_1^q = Ar^2\left(p_1^q \sum_\Omega \rho_\theta^i \varphi_\theta + S_1^q\right)$$

or

$$S_1 = Ar\left(\sum_\Omega \rho_\theta^g\, p_1^q \varphi_\theta - \sum_\Omega \rho_\theta^i\, p_1^q \varphi_\theta\right) \tag{2-9}$$

Substituting S_1^q with this expression in (2-1) and (2-5) and solving for S_m^q, one obtains

$$S_m^q = Ar\left(\sum_{\Omega} \rho_\theta^g \, p_m^q \varphi_\theta - \sum_{\Omega} \rho_\theta^i \, p_m^q \varphi_\theta\right) \quad m = M_2, \ldots, M_M \tag{2-10}$$

The same substitution in (2-4) and (2-8) and then into (2-2) and (2-6) gives, after further rearrangement

$$S_k^{x*} = \frac{E^g\left\{\dfrac{\partial U_\theta^g}{\partial x_k^*}\right\}}{E^g\left\{\dfrac{\partial U_\theta^g}{\partial R_{0\theta}^g}\right\}} \qquad k = 1, \ldots, K_1 \tag{2-11}$$

$$S_m^{q*} = Ar\left(\sum_{\Omega} \rho_\theta^g \, p_m^q \varphi_\theta - \sum_{\Omega} \rho_\theta^i \, p_m^q \varphi_\theta\right) + r \frac{E^g\left\{\dfrac{\partial U_\theta^g}{\partial q_m^*}\right\}}{E^g\left\{\dfrac{\partial U_\theta^g}{\partial R_\theta^g}\right\}} \quad m = 2, \ldots, M_1 \tag{2-12}$$

The solutions of equations (2-1) to (2-4) and (2-9) to (2-12), which are identical to equations (31) to (37) in the text, are the optimal values of the production variables and subsidies which have the agreement of both the group of private shareholders and the government.

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