PROPORTIONAL VS. LOGARITHMIC MODELS OF ASSET PRICING

by

GABRIEL HAWAWINI

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Gabriel Hawawini
INSEAD, Fontainebleau, France
Ashok Vora
Baruch College, CUNY

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1. INTRODUCTION

1.1 Objectives of the study

Empirical tests of models such as the Sharpe (1964)-Lintner (1965) capital asset pricing model (CAPM) and the market model of Sharpe (1963) require measuring securities' returns over differencing intervals of given length. The usual choice, often dictated by data availability, is a month. One can calculate either a proportional return (percentage change in prices, including any dividend) or a logarithmic return (natural logarithm of monthly wealth relatives). Tests of the market model and the CAPM have been performed using either one of these two definitions of securities' returns with practically no attempt to justify the choice or examine the effect of the return specification on the estimated statistics.[1]

As early as 1970, Sharpe (1970), in a comment on Fama's (1970) review, pointed out that "The crucial question is not what to name risk, but how to measure it---- Should one use return or the logarithm of the value relative? -----". Over the last decade, researchers seem to have opted for the logarithmic specification without attempting to examine (either analytically or empirically) the differential effect, if any, that the definition of returns may have on the test-results of asset pricing models. As we will show in this paper, the effect is substantial with significant implications for the estimation of required returns on common stocks. Recently, Rosenberg and Marathe (1979) have extensively tested the CAPM. They addressed the question of whether returns should be measured in the proportional or the logarithmic form. After surveying the advantages of the linear-proportional model over the linear-logarithmic model they concluded that: "... from a theoretical point of view, neither model is satisfactory..... Since no form of the model is dominant in theoretical tractability one might ask whether goodness of fit to the data can be used to determine the choice. To our knowledge, this question is not fully resolved. ..... Clearly, further work on the interface between proportional and logarithmic models is necessary."

This paper is a systematic attempt in this direction. We seek to provide analytical and empirical answers to the following questions:
(1) What is the relationship between the distributions of proportional and logarithmic returns? (2) How well do proportional returns approximate logarithmic returns? (3) What is the differential effect of proportional and logarithmic returns on the slope (beta) of Sharpe's (1963) single-index market model? (4) What is the differential effect of proportional and logarithmic returns on the empirical security market line (SML), the slope of which is an estimate of the market price of risk (MPR). Does one model exhibit consistently better fit to the data and hence, could be selected as the "superior" model? The practical usefulness of the answers to these questions resides in the increasing use of beta and the empirical SML to calculate firms' cost of equity capital. (5) Finally, what is the <u>joint</u> effect of the return definition and the length of the investment horizon (monthly vs. quarterly vs. semi-annual vs. annual returns) on estimates of systematic risk and the MPR? That is, does the definition of returns matter more than the length of the differencing interval over which these returns are measured?

The rest of the paper is organized as follows. The next subsection is a summary of our findings and their implications. Section 1.3 summarizes the advantages of the linear-logarithmic model of asset pricing over the linear-proportional model. Section 2 describes the sample used in this study. Section 3 examines the relationship between the distribution of proportional and logarithmic returns. The differential effect of the definition of returns and the length of the investment horizon on estimates of systematic risk and the coefficients of the empirical SML are examined in sections 4 and 5, respectively. Concluding remarks make up the last section of the paper.

1.2 Summary of findings and implications

Following is a summary of our findings and some of their implications:

(1) For a given security, the time-series average of proportional returns is always greater than the time-series average of logarithmic returns for a given length of the return interval.

(2) The cross-sectional mean of the time-series average of proportional returns is always greater than the corresponding average of logarithmic returns for a given length of the return interval.

(3) For a given security, in an up-market the time-series variance of proportional returns is greater than the variance of logarithmic returns. In a down-market the relationship between the two variances is not predictable.

(4) In an up-market, the cross-sectional variance of the time-series averages of proportional returns will be greater than the corresponding variance of logarithmic returns. In a down-market the relationship between the two cross-sectional variances is not predictable. This holds irrespective of the length of the return interval.

(5) The coefficients of determination for the market model based on proportional and logarithmic returns though not equal, are very close but the sign of the difference between them is not predictable.

(6) Aggressive securities (those with a beta coefficient greater than one) will have larger proportional betas than logarithmic betas. Defensive securities will have smaller proportional betas than logarithmic betas. Neutral securities will have same betas regardless of whether they are computed with proportional or logarithmic returns. In other words, the cross-sectional distribution of proportional betas is significantly more dispersed than that of logarithmic betas.

(7) In a down-market the proportional MPR is always greater than the logarithmic MPR (the slope of the estimated SML). In an up-market, under most

circumstances, the proportional MPR is greater than the logarithmic MPR but the possibility exists for the inequality to be in the other direction.

(8) The comparative behavior of the coefficients of determination of the proportional and logarithmic SMLs will be unpredictable.

Some of the implications of the results reported in this paper are that (1) an item as innocuous as the definition of returns may, in the final analysis, affect the estimated statistics of a pricing model more than the model itself, (2) goodness-of-fit to the data cannot be used to determine the choice between the proportional and logarithmic models since no systematic "superiority" of one over the other exists. The behavior of the general market movement (up or down market) affect the goodness-of-fit of the data differently for different return specification, (3) with MPR exogenously given, a logarithmic beta from short return intervals (say monthly) will yield higher estimates of a firm's cost of equity if its beta is below one (say a public utility) than a proportional beta from lengthy return intervals (say quarterly). The opposite is true if the firm has a beta coefficient above one.

1.3 Some advantages of the linear-logarithmic model

There are several standard arguments in favor of the linear-logarithmic model of asset pricing. First, the distribution of proportional returns has been observed to be positively skewed with a non-normal kurtosis whereas the distribution of logarithmic returns are almost symmetric and closely approximates the normal distribution. The fact that logarithmic returns are practically normal is an advantage for at least two reasons. Theoretically, the CAPM is based on the normality assumption, and Merton (1973) has developed an asset pricing model which assumes that continuously compounded returns are normally distributed. Statistically, tests of asset pricing models are usually based on least-square estimates, and this estimator is "best" only for normally distributed error terms.

Secondly, the linear-logarithmic model is fully preserved under compounding whereas the linear-proportional model is not. This raises the ancillary issue of the impact of the return interval on estimates of systematic risk and the MPR.[2] Some of these problems have been examined in the literature with the following conclusion: estimates of systematic risk based on logarithmic returns are somewhat less sensitive to the return interval than those based on proportional returns, but the use of logarithmic returns will not eliminate the intervalling effect on beta. For this reason we have also elected to examine in this paper the joint effect of return-length and return-definition on models of asset pricing.

2. SAMPLE

The sample consists of 1,115 securities - 828 from the NYSE and 287 from the AMEX - from the Center for Research in Security Prices of the University of Chicago. This is the set of securities on the tape which traded continuously from July 1962 to June 1977. Thus each security in our sample has 180 observations of monthly returns. The proxy for the market portfolio is a value-weighted index of 3950 securities appearing on the CRSP tape.

In performing our empirical work we have generated returns in the proportional and logarithmic forms as specified in (1a) and (1b). All returns were adjusted for cash dividends and changes in capitalization.

In order to test the effect of changes in the return interval we have performed our empirical work with monthly, quarterly, semi-annual and annual returns. To examine the stationarity of our estimated statistics we have divided the sample of 180 monthly observations into two 90-month nonoverlapping subperiods over which all our tests were replicated.

3. RELATIONSHIP BETWEEN PROPORTIONAL AND LOGARITHMIC RETURNS

The purpose of this section is to examine analytically and empirically the time-series and the cross-sectional properties of proportional and logarithmic returns. In subsection 3.1 expected returns (means) are compared. In subsection 3.2 variances are compared. Finally, in subsection 3.3 the sample cross-sectional averages and variances are compared. The results established in this section provide the necessary elements for the comparative analysis of the proportional and logarithmic forms of the market model (section 4) and the empirical SML (section 5).

3.1 Expected returns

We define proportional return R_t as:

$$R_t = (P_t + D_t - P_{t-1})/P_{t-1} = [(P_t + D_t)/P_{t-1}] - 1 \tag{1a}$$

where $(P_t + D_t)/P_{t-1}$ is the wealth relative adjusted for any cash dividend (D_t) and P_t is the asset's market price at time (t). Since logarithmic returns (r_t) are defined as the natural logarithm of wealth relative we get:

$$r_t = \ln (1 + R_t) \tag{1b}$$

For sake of clarity and without loss of generality we drop the time subscript in the subsequent discussion. We will also adopt the following convention: upper case letters denote parameters related to the proportional returns whereas lower case letters denote parameters related to the logarithmic returns.

For the theoretical analysis that follows, the relationship between proportional returns (R) and logarithmic returns (r) can only be approximated. We have from (1b):

$$1 + R = \exp(r)$$

and

$$R = r + \frac{1}{2!}r^2 + \frac{1}{3!}r^3 + \text{-----}$$

Taking expectations on both sides of the above equality we get:

$$E(R) = E(r) + \frac{1}{2!} E(r^2) + \frac{1}{3!} E(r^3) + \text{-----} \tag{2a}$$

Let $M = E(R)$, $m = E(r)$, $S^2 = E(R-M)^2$, and $s^2 = E(r-m)^2$; where M, m, S and s are the expected value of $\tilde{R}$, the expected value of $\tilde{r}$, the standard deviation of $\tilde{R}$, and the standard deviation of $\tilde{r}$, respectively. A summary of our notation is in Table 1. Given the notation, (2a) can be rewritten as:

$$M = m + \frac{1}{2}(m^2 + s^2) + \frac{1}{6}(m^3 + 3ms^2) + \text{----} \tag{2b}$$

As a first-order approximation we get:

$$R \simeq r \tag{3a}$$

and

$$M \simeq m \tag{3b}$$

As a second-order approximation we have:

$$R \simeq r + \frac{1}{2} r^2 \tag{4a}$$

and

$$M \simeq m + \frac{1}{2}(m^2 + s^2) \tag{4b}$$

Finally, as a third-order approximation we get:[3]

$$R \simeq r + \frac{1}{2} r^2 + \frac{1}{6} r^3 \tag{5a}$$

and

$$M \simeq m + \frac{1}{2}(m^2 + s^2) + \frac{1}{6}(m^3 + 3ms^2) \tag{5b}$$

Relations (3b), (4b) and (5b) are expressions of the expected proportional return as a function of the first two moment of the distribution of logarithmic returns. The "best" approximation can only be determined by empirical examination of the data. To check for the "best" approximation among the three presented above, we specify the following regression equations:

$$M = c + d\{m\} \tag{6a}$$

$$M = c + d\{m + \frac{1}{2}(m^2 + s^2)\} \tag{6b}$$

$$M = c + d\{m + \frac{1}{2}(m^2 + s^2) + \frac{1}{6}(m^3 + 3ms^2)\} \tag{6c}$$

Whichever equation gives us a coefficient of determination closest to one, estimated slope coefficient "d" closest to one, and estimated intercept "c" closest to zero will be used exclusively in all our subsequent analyses.

Empirical results are presented in Table 2(a) for the complete 180 - month period and in Table 2(b) for two nonoverlapping 90 - month subperiods. In order to examine the sensitivity of the results to the investment horizon, statistics are given for monthly, quarterly, semi-annual and annual returns. Results reported in Tables 2 reveal that the second-order approximation gives the best goodness-of-fit regardless of the length of the investment horizon. This leads us to the selection of the second-order approximation for the remaining analysis.

3.2 Variances

From (4a), the selected second-order approximation, we have:

$$R^2 = (r + \frac{1}{2}r^2)^2 = r^2 + r^3 + \frac{1}{4}r^4$$

and

$$E(R^2) = E(r^2) + E(r^3) + \frac{1}{4}E(r^4) \tag{7}$$

We also have:

$$E(r^2) = m^2 + s^2, \tag{8a}$$

$$E(r^3) = m^3 + 3ms^2, \tag{8b}$$

$$E(r^4) = m^4 + 6m^2s^2 + 3s^4, \tag{8c}$$

assuming, in the case of (8c), that r is normally distributed.[4] Using (8a), (8b) and (8c), equation (7) can be rewritten as:

$$E(R^2) = m^2 + s^2 + m^3 + 3ms^2 + \frac{1}{4}(m^4 + 6m^2s^2 + 3s^4) \qquad (9)$$

Furthermore, we have:

$$E(R) = M = m + \frac{1}{2}m^2 + \frac{1}{2}s^2 \text{ and hence}$$

$$M^2 = m^2 + \frac{1}{4}m^4 + \frac{1}{4}s^4 + m^3 + ms^2 + \frac{1}{2}m^2s^2. \qquad (10)$$

It follows that the variance of $\tilde{R}$ which is expressed as $S^2 = E(R-M)^2 = E(R^2) - M^2$, becomes

$$S^2 = s^2\{(1+m)^2 + \frac{1}{2}s^2\} \qquad (11)$$

by simply substituting $E(R^2)$ and M^2 for their respective values given in (9) and (10). Equation (11) expresses the variance of the proportional returns as a function of the first two moments (m and s) of the distribution of logarithmic returns.

3.3 Cross-sectional averages and variances

In this subsection we examine the relationships between the cross-sectional averages and variances. The results established in this subsection will be used in section 5 when we examine the comparative properties of logarithmic and proportional SML (the cross-sectional relationships between securities mean-returns and their corresponding level of systematic risk). The subscript i (i = 1, 2, ---, N) will designate securities. We will use the symbols "avg" and "var" to denote cross-sectional average and variance, respectively. We have from (4b):

$$M_i = m_i + \frac{1}{2}m_i^2 + \frac{1}{2}s_i^2 \qquad (12)$$

and therefore:

$$Avg(M_i) = Avg(m_i) + \frac{1}{2}Avg(m_i^2) + \frac{1}{2}Avg(s_i^2) \qquad (13)$$

which is the relationship between the cross-sectional average of securities' proportional (M_i) and logarithmic (m_i) mean-returns.

It should be noted that we cannot use the logarithmic mean-returns (m_i) in cross-sectional tests of CAPM. We must, instead, use the antilog of m_i in cross-sectional tests. If we use m_i rather than the antilog of m_i, then the "market" portfolio with (1/n) "wealth" in each security would imply a diversification through time. That is, total wealth is invested in security-1 for (1/n)-th of a year, total wealth is invested in security-2 for (1/n)-th of the year, and so on. Using antilog (m_i) will imply a cross-sectional diversification. For details, see the analysis of Elton and Gruber (1974).

$$\text{Let } (1 + g_i) = \exp(m_i)$$

$$= 1 + m_i + \frac{1}{2} m_i^2 + \frac{1}{6} m_i^3 + \text{------}$$

then, a second order approximation will yield:

$$g_i = m_i + \frac{1}{2} m_i^2 \tag{14}$$

and

$$\text{Avg } (g_i) = \text{Avg}(m_i) + \frac{1}{2} \text{Avg } (m_i^2) \tag{15}$$

where Avg (g_i) is the cross-sectional average of securities' geometric mean-return. From (13) and (14) it follows that:

$$\text{Avg } (M_i) - \text{Avg } (g_i) = \frac{1}{2} \text{Avg } (s_i^2) > 0 \tag{16}$$

That is, the cross-sectional average of the proportional returns is strictly greater than the cross-sectional average of geometric returns. This analytical result is supported by the data. Referring to Table 3 we can see that the cross-sectional proportional average, Avg (M_i), is roughly twice the size of the cross-sectional geometric average, Avg(g_i). Note that the average values reported in Table 3 are independent of the return interval. We have "annualized" the mean-return of all securities before calculating their cross-sectional average values.

This was achieved by multiplying monthly means by 12, quarterly means by 4, and semiannual means by 2. Our objective is to investigate the non-trivial effect of changes in the length of the return interval after neutralizing the obvious effect of lengthening the return interval.[5]

We now turn to the relationship between the cross-sectional variances of the proportional and geometric mean returns. Using (12) and (13) we can easily derive the following expression:

$$\mathrm{Var}\ (M_i) = E\ [\ M_i - \mathrm{Avg}\ (M_i)\ ]^2$$

$$\mathrm{Var}(M_i) = \mathrm{Var}\ (m_i) + \frac{1}{4}\ \mathrm{var}\ (m_i^2) + \frac{1}{4}\ \mathrm{var}\ (s_i^2) + \mathrm{cov}\ (m_i, m_i^2) + \mathrm{cov}\ (m_i^2, s_i^2)$$

$$+ \frac{1}{2}\ \mathrm{cov}\ (m_i^2,\ s_i^2) \tag{17}$$

Likewise, using (14) and (15) we get:

$$\mathrm{Var}(g_i) = E\ [g_i - \mathrm{Avg}\ (g_i)\ ]^2$$

$$= \mathrm{Var}\ (m_i) + \frac{1}{4}\ \mathrm{var}\ (m_i^2) + \mathrm{cov}\ (m_i,\ m_i^2) \tag{18}$$

Now, taking the difference between (17) and (18) yields:

$$\mathrm{Var}(M_i) - \mathrm{Var}(g_i) = \frac{1}{4}\ \mathrm{var}(s_i^2) + \mathrm{cov}(m_i, s_i^2) + \frac{1}{2}\ \mathrm{cov}(m_i^2, s_i^2) \tag{19a}$$

which is the difference between the cross-sectional variances of arithmetic (proportional) and geometric (logarithmic) averages. The sign of (19a) will depend on the two covariance terms. Two cases must be considered. Assume first that all $m_i > 0$; and that the higher the risk of a security the higher will be the average realized return. In this case, $\mathrm{cov}(m_i, s_i^2)$ and $\mathrm{cov}(m_i^2, s_i^2)$ will both be positive.[6] Hence, we have:

$$\mathrm{Var}\ (M_i) > \mathrm{Var}\ (g_i)\ \text{if}\ m_i > 0 \tag{19b}$$

In an upmarket the cross-sectional variance of proportional mean-returns exceeds that of geometric mean-returns.

Consider now the case of a downward trend in the prices of securities. The mean-return m_i will be negative, and the first covariance term in (19a) will be negative because the higher risk security will have higher negative average realized return. The second covariance term in (19a) will be positive because the higher the negative return, the higher will be the value of squared-return. In this case the sign of the LHS of (19a) is indeterminate.

$$\text{Var } (M_i) \gtreqless \text{Var } (g_i) \text{ if } m_i < 0 \qquad (19c)$$

Referring to Table 4, again we find support for our analytical results. For the entire sample period of 180 months the market was rising and (19b) holds. However, results for the first subperiod are more pronounced since the market was clearly rising over the first 90 months of our sample period. Over the second 90-month subperiod we have $\text{Avg}(g_i) < 0$ (that is, m_i generally negative) and $\text{Var } (M_i) < \text{Var } (g_i)$.

4. EFFECTS ON SYSTEMATIC RISK AND ITS IMPLICATIONS

In this section we compare securities' systematic risks based on the proportional and logarithmic models. An estimate of a security's systematic risk is obtained by using Sharpe's (1963) single-index market model. According to this model, the returns of security i are related to the returns of a market index (which we identify by a subscript I) such as:

$$\tilde{R}_{i,t} = A_i + B_i\tilde{R}_{I,t} + \tilde{U}_i \qquad (20)$$

where $\hat{A}_i$, $\hat{B}_i$ are the estimated intercept and slope coefficients, respectively and $\tilde{U}_i$ is a normally distributed random error with zero expected value and constant variance. The slope coefficient $\hat{B}_i$ is security i's estimated systematic risk.

Alternatively, one can specify the market model in its logarithmic form:

$$\tilde{r}_{i,t} = a_i + b_i \tilde{r}_{I,t} + \tilde{u}_i \tag{21}$$

with the same definitions as those spelled out for the proportional form of the model. The estimated beta coefficients for the two forms of the model can be expressed as:

$$\hat{B}_i = \frac{cov_t(R_i, R_I)}{S_I^2} = \frac{cor_t(R_i, R_I) \cdot S_i}{S_I} \tag{22a}$$

$$\hat{b}_i = \frac{cov_t(r_i, r_I)}{s_I^2} = \frac{cor_t(r_i, r_I) \cdot s_i}{s_I} \tag{22b}$$

where cov_t and cor_t are the (time-series) covariance and correlation between security i's returns and those of the market index I. For the sake of simplicity we drop the subscript t from the returns (R and r).

The relationship between $\hat{B}_i$ and $\hat{b}_i$ will depend on the relationships between $cor_t(R_i, R_I)$ and $cor_t(r_i, r_I)$ and between S and s. the latter has been established in subsection 3.2, equation (11). The former is developed in Appendix 1 in which we show that:

$$cor_t(R_i, R_I) \simeq cor_t(r_i, r_I) \tag{23a}$$

and $$cor_t(R_i, R_I) - cor_t(r_i, r_I) \gtreqless 0 \tag{23b}$$

That is, the correlation coefficient between proportional security and market returns is approximately equal to that between logarithmic security and market returns with an ambiguous sign for their difference. This result is supported by the evidence presented in Table 5. Substituting (11) in (22a) and assuming that (23a) holds exactly it follows that:

$$\hat{B}_i = \hat{b}_i \left[\frac{(1+m_i)^2 + \frac{1}{2} s_i^2}{(1+m_I)^2 + \frac{1}{2} s_I^2} \right]^{1/2} \qquad (24)$$

Assuming equilibrium in the capital markets, $b_i \lesseqgtr 1$ implies $m_i \lesseqgtr m_I$. If we further assume that $s_i \lesseqgtr s_I$ according as $b_i \lesseqgtr 1$, then we have:[7]

$$\hat{B}_i \gtreqless \hat{b}_i \gtreqless 1$$

<u>Aggressive securities (those with a beta coefficient greater than one) will have proportional betas ($\hat{B}_i$) larger than logarithmic betas ($\hat{b}_i$). Defensive securities will have proportional betas ($\hat{B}_i$) small than logarithmic betas ($\hat{b}_i$). Neutral securities will have same betas whether they were computed with proportional or logarithmic returns.</u>

Finally, if our assumption that $s_i \lesseqgtr s_I$ according as $b_i \lesseqgtr 1$ does not hold strictly (i.e., for each security in our sample), our conclusion should hold on average. In order to verify if the behavior of our sample conforms to our analytical results we run the following regression:

$$\tilde{\hat{B}}_i = c + d\, \tilde{\hat{b}}_i + \tilde{e} \qquad (24a)$$

According to our results expressed in (24) we should observe: (1) a wider dispersion of proportional betas than logarithmic betas (Var $(\hat{B}_i)$ > Var $(\hat{b}_i)$), (2) a slope coefficient d larger than one and (3) a negative intercept coefficient c.[8]

Empirical results are reported in Tables 6 and 7. Table 6 shows that the average proportional betas are slightly higher than the average logarithmic betas and both averages are higher than one.[9] As expected, the dispersion of proportional betas is greater than the dispersion of logarithmic betas, with the difference between the two being more pronounced as the investment horizon is lengthened. Table 7 gives the regression results for equation (24a). As expected, the intercepts are significantly negative and the slopes significantly greater than one, particularly for investment horizons of more than a month. The relationship between $\hat{B}_i$ and $\hat{b}_i$ appears to be linear (high values of R-square) despite the slight nonlinearity of the theoretical relationship expressed in (24).

It should be pointed out that evidence of the effect of the return definition on estimates of systematic risk have been indirectly presented by Hamada as early as 1971. In his investigation of the effect of the firm's capital structure on the systematic risk of common stocks, Hamada calculates beta coefficients based on proportional returns and logarithmic returns. Results presented in his Table 4 show that the dispersion of proportional betas is larger than that of logarithmic betas.

The above analytical and empirical results have implication for the determination of firms' cost of equity capital. According to the CAPM, that cost is proportional to the firm's estimated systematic risk. With exogenously given MPR, proportional betas will provide lower cost of equity than logarithmic betas for those firms which are less risky than the market (betas smaller than one) and higher cost of equity for those firms which are more risky than the market.

5. EFFECTS ON THE EMPIRICAL SECURITY MARKET LINE AND IMPLICATIONS

In this section we examine the impact of return definition on the coefficients of the "empirical" SML. The SML depicts the relationship between the average returns of securities and their estimated systematic risk. The systematic risk is obtained from the model discussed in the previous section. The theoretical underpinning for the empirical SML is provided by the CAPM of Sharpe (1964) and Lintner (1965).

If returns are defined in the logarithmic form then the empirical SML can be stated as:

$$\tilde{g}_i = h_o + h_1 \tilde{\hat{b}}_i + \tilde{e}_i \tag{25a}$$

where g_i is the geometric average return of security i and $\hat{b}_i$ is its logarithmic beta. The slope coefficient $\hat{h}_1$ is then an estimate of the MPR, a crucial variable in financial markets, which gives the "average" compensation received by market participants for the risk they bear.

If returns are defined in the proportional form then the empirical SML can be stated as:

$$\tilde{M}_i = H_o + H_1 \tilde{\hat{B}}_i + \tilde{E}_i \tag{25b}$$

where M_i is the proportional average return of security i and $\hat{B}_i$ is its proportional beta. Again, $\hat{H}_1$ is an estimate of the MPR.

In this section we show, analytically and empirically, that the estimated MPR is substantially affected by the model used to estimate it. More than the case of estimated betas, for which we have shown that the effect of the return definition is significant, in the case of the MPR there is substantial difference between estimates from the logarithmic and proportional models. A quick look at Table 8(b) shows that the estimated MPR can vary, for example, from roughly 5 percent for the logarithmic model to about 10 percent for the proportional model (monthly interval, first subperiod).

First, note that for the proxy-market portfolio, based on (25a) and (25b), respectively:

$$\hat{g}_I = \hat{h}_o + \hat{h}_1 \tag{26a}$$

and

$$\hat{M}_I = \hat{H}_o + \hat{H}_1 \tag{26b}$$

Since we have shown in (16) that $M_I = g_I + \frac{1}{2} s_I^2 > g_I$ it follows that:

$$\hat{H}_o + \hat{H}_1 > \hat{h}_o + \hat{h}_1 \tag{27}$$

That is, the sum of the estimated proportional coefficients exceeds the sum of the estimated logarithmic coefficients. This is also what we observe in Tables 8(a) and 8(b).

Turning now to the relationship between $\hat{H}_1$ and $\hat{h}_1$ we have:

$$\hat{H}_1 = \frac{\text{cov}(M_i, \hat{B}_i)}{\text{var }(\hat{B}_i)} = \frac{\text{cov}(g_i + \frac{1}{2} s_i^2, \hat{B}_i)}{\text{var }(\hat{B}_i)}$$

$$= \frac{\text{cov }(g_i, \hat{B}_i)}{\text{var }(\hat{B}_i)} + \frac{\text{cov }(s_i^2, \hat{B}_i)}{2 \text{ var }(B_i)}$$

$$\hat{H}_1 = \frac{\text{cor}(g_i, \hat{B}_i) \cdot \text{dev }(g_i)}{\text{dev}(B_i)} + \frac{\text{cov }(s_i^2, \hat{B}_i)}{2 \text{ var }(B_i)}$$

where dev(·) signifies standard deviation.

Assuming that the proportional and logarithmic betas are positively related (as indicated by our analytical and empirical results in the previous section) then:

$$\text{cor }(g_i, \hat{B}_i) = \text{cor }(g_i, \hat{b}_i)$$

and since we have shown that:

$$\text{dev }(\hat{B}_i) = \frac{1}{k} \cdot \text{dev }(\hat{b}_i), \text{ where } 0 < k < 1$$

it follows that:

$$\hat{H}_1 = \frac{\text{cov }(s_i^2, \hat{B}_i)}{2 \text{ var }(\hat{B}_i)} + k \cdot \hat{h}_1 \tag{28}$$

Given that the first term in (28) above is positive (recall that we can and do assume that risk-measures are positively correlated) we can show that:

$$\underline{\text{Case (i):}} \quad \text{If } \hat{h}_1 \leqq 0 \text{ then } \hat{H}_1 > \hat{h}_1 \tag{29a}$$

$$\underline{\text{Case (ii):}} \quad \text{If } \hat{h}_1 > 0 \text{ then } \hat{H}_1 \gtreqless \hat{h}_1 \tag{29b}$$

depending on the value of $\text{cor}(g_i, \hat{b}_i)$.

The case where $\hat{h}_1$ is nonpositive follows directly from (28): <u>If the logarithmic estimate of the MPR is nonpositive then it is smaller than the proportional estimate of the same MPR.</u> The case where $\hat{h}_1$ is strictly positive is slightly more complex to analyze. In appendix 2 we show that the sign of the difference between $\hat{h}_1$ and $\hat{H}_1$ will, in this case, depend on the value of correlation coefficient between the geometric means g_i and the estimated logarithmic betas $\hat{b}_i$. The following result is demonstrated: when $\hat{h}_1$ is strictly positive, then for relatively small (large) values of $\text{cor}(g_i, \hat{b}_i)$ it is more likely that $\hat{H}_1$ is greater (smaller) than $\hat{h}_1$.

Turning to the empirical results summarized in Tables 8(a) and 8(b) we see that for the 180 - month period (July 1962 - June 1977) $\hat{h}_1 < 0$ and, as expected, $\hat{H}_1 > \hat{h}_1$. This result is confirmed over the second 90 - month subperiod. Over the first 90 - month subperiod $\hat{h}_1 > 0$ and, again, $\hat{H}_1 > \hat{h}_1$ confirming that $\hat{h}_1 > 0$ implies that the proportional MPR will exceed the logarithmic MPR when $\text{cor}(g_i, \hat{b}_i)$ is relatively low.

Referring to Tables 8 we note that the coefficient of determination of the proportional and logarithmic SML exhibit substantially different behavior. In general, the proportional model has relatively higher R-square compared to the logarithmic model. In Appendix 3 we have shown that it is not possible to establish unambiguous analytical results that can describe and predict the differential behavior of the proportional and logarithmic R-squares. The implication should be clear: goodness of fit statistics cannot be used to discriminate between the two models of asset pricing. As shown in Appendix 3, goodness-of-fit will depend on whether the market is rising or falling or whether the logarithmic MPR is greater or smaller than the proportional MPR. In Table 9 we present the comparative behavior of the variances of the error terms of the logarithmic ($Var(e_i)$) and the proportional SML ($Var(E_i)$). As can be seen, $Var(e_i)$ may be greater or smaller than $Var(E_i)$ depending on the length of the return interval and the choice of the testing period.

5. CONCLUDING REMARKS

In this paper we examine, analytically and empirically, the differential characteristics of the proportional and logarithmic models of asset pricing. We have shown that significant differences exist between the estimated statistics obtained from the two models. Our major results are summarized in section 1.2. Our concluding remarks are: first, replying to the question raised by Rosenberg and Marathe (1979) who asked whether goodness-of-fit of the data can be used to determine the choice between the two competing models, the answer is resounding (and unfortunate) no! Second, researchers who study the empirical behavior of asset prices and practioners who use asset pricing models to estimate, for example, firms' cost of capital must display extreme care before drawing definitive conclusions from their empirical observations. An item as innocuous as the definition of securities' returns may, in the final analysis, affect the estimated statistics as much or more than the choice of the theoretical model upon which the estimation procedure is based.

FOOTNOTES

1. The study by Rogalski and Tinic (1978) is one of the few papers that allude to this issue. In estimating the SML they use the average return calculated as an "appropriately" weighted average of the ex post geometric and proportional averages.

2. The impact of the differencing interval length on estimates of systematic risk and the market price of risk has been examined by Hawawini and Vora (1980) under the assumption of intertemporal cross-correlation between securities' returns. The impact of the differencing interval length on estimates of systematic risk has been examined by Levhari and Levi (1977) under the assumption of return independence. For empirical evidence see Hawawini and Vora (1980, 1981) and Levhari and Levi (1977).

3. For the third-order approximation we assume that $\tilde{r}$ is normally distributed.

4. The data have justified this normality assumption, see Fama (1970). If $\tilde{r}$ were not normal, $3s^4$ in (8c) will be replaced by a term between $2s^4$ and $4s^4$ - and this will not affect the generality of subsequent conclusions.

5. If someone held a portfolio of 1,115 securities, with an equal amount invested in each security in July 1962 and rebalanced monthly, and with all the dividends reinvested for 15 years the ending value of the portfolio would indicate an annual compounded return of 6.15% and <u>not</u> 13.36%, 13.73%, 14.39%, or 13.88%. The value 6.15% is the average <u>geometric</u> (logarithmic) return of Table 3.

6. This assumption implies that either well-diversified portfolios are used instead of securities or that the <u>ranking</u> of securities according to their systematic risk is highly correlated to their ranking according to their total risk.

7. See footnote - 6.

8. Although equation (24) indicates a slight nonlinearity, we are imposing a linear relationship between $\hat{b}_i$ and B_i in equation (24a). This assumption is justified by later empirical results.

9. Both the averages are higher than one for the following two reasons: First, while the market index comprises 3,950 securities our analysis is limited to 1,115 securities because of the full data requirement. Second, while the market index is a value-weighted average of the constituent securities the averages presented by us are equally-weighted.

TABLE 1

NOTATION

	MODEL USED		
STATISTICS	PROPORTIONAL	LOGARITHMIC	RELATIONSHIP
Periodic return	R	r=log(1+R)	$R=r+\frac{1}{2}r^2$
Expected return (mean)	M	m	$M=m+\frac{1}{2}(m^2+s^2)$
Standard deviation	S	s	$S=s\{(1+m)^2+\frac{1}{2}s^2\}^{1/2}$
Systematic risk (beta)	B	b	see eq. (24)
Coefficients of the SML	H_0, H_1	h_0, h_1	see eq. (28)

Upper case letters are used exclusively to designate statistics based on the proportional model and lower case letters to designate statistics based on the logarithmic model.

TABLE 2(a)

Proportional mean vs. logarithmic mean

Period: July 1962 - June 1977

Interval	Specification	$\hat{c}$	$t(\hat{c})$	$\hat{d}$	$t(\hat{d}-1)$	R-square
Month	eq. (6a)	.0882	47.6	.7396	-12.8	.5434
	eq. (6b)	.0024	7.2	.9672	-15.3	.9945
	eq. (6c)	.0094	22.5	.8931	-40.2	.9902
Quarter	eq. (6a)	.0916	43.1	.7426	-11.1	.4772
	eq. (6b)	.0008	1.6	.9999	- 0.0	.9870
	eq. (6c)	.0084	13.1	.9217	-19.6	.9794
Semi-annual	eq. (6a)	.0962	37.9	.7752	- 8.1	.4097
	eq. (6b)	-.0039	-5.4	1.0598	13.4	.9805
	eq. (6c)	.0043	5.2	.9763	- 4.8	.9718
Annual	eq. (6a)	.0816	31.8	.9313	- 2.4	.4954
	eq. (6b)	-.0089	-10.0	1.1158	20.0	.9710
	eq. (6c)	-.0016	- 1.9	1.0344	6.2	.9688

t-values have 1,113 degrees of freedom

TABLE 2(b)

Proportional mean vs. logarithmic mean

Subperiods results

Interval	Specification	$\hat{c}$	$t(\hat{c})$	$\hat{d}$	$t(\hat{d}-1)$	R-square
		July 1962 - December 1969				
Month	eq. (6a)	.0401	16.7	1.1413	9.1	.8295
	eq. (6b)	.0075	15.8	.9155	-41.0	.9944
	eq. (6c)	.0146	31.0	.8405	-81.9	.9940
Quarter	eq. (6a)	.0342	12.9	1.2009	11.8	.8162
	eq. (6b)	.0035	6.1	.9558	-17.8	.9925
	eq. (6c)	.0110	21.0	.8769	-56.8	.9932
Semi-annual	eq. (6a)	.0386	10.7	1.2987	12.8	.7348
	eq. (6b)	-.0047	-5.4	1.0332	9.1	.9863
	eq. (6c)	.0049	6.3	.9387	-20.0	.9883
		January 1970 - June 1977				
Month	eq. (6a)	.0852	63.2	.6853	-26.6	.7499
	eq. (6b)	-.0075	-19.7	1.0328	10.6	.9900
	eq. (6c)	.0075	19.7	.9075	-31.5	.9881
Quarter	eq. (6a)	.0907	54.8	.6501	-24.1	.6418
	eq. (6b)	-.0046	-9.7	1.0399	10.5	.9853
	eq. (6c)	.0112	15.8	.9103	-16.6	.9621
Semi-annual	eq. (6a)	.0873	44.7	.7005	-17.4	.5985
	eq. (6b)	-.0050	-8.7	1.0799	17.6	.9807
	eq. (6c)	.0083	10.1	.9686	-5.0	.9552

TABLE 3

Cross-sectional proportional average ($Avg(M_i)$) vs. Cross-sectional geometric average ($Avg(g_i)$)

Interval	July 1962-June 1977		July 1962-Dec. 1969		January 1970-June 1977	
	$Avg(g_i)$	$Avg(M_i)$	$Avg(g_i)$	$Avg(M_i)$	$Avg(g_i)$	$Avg(M_i)$
Month	.0615	.1336	.1266	.1846	-.0037	.0827
Quarter	.0615	.1373	.1266	.1862	-.0037	.0883
Semi-annual	.0615	.1439	.1266	.2030	-.0037	.0848
Annual	.0615	.1388	(1)	(1)	(1)	(1)

(1) Insufficient data to compute significant statistics.

deviation of geometric averages ($dev(g_i)$)

Interval	July 1962-June 1977		July 1962-Dec.1969		January 1970-June 1977	
	$dev(g_i)$	$dev(M_i)$	$dev(g_i)$	$dev(M_i)$	$dev(g_i)$	$dev(M_i)$
Month	.0674	.0676	.0893	.1119	.1137	.0900
Quarter	.0674	.0724	.0893	.1187	.1137	.0923
Semi-Annual	.0674	.0816	.0893	.1352	.1137	.1030
Annual	.0674	.0891	(1)	(1)	(1)	(1)

(1)
Insufficient data to compute significant statistics.

TABLE 5

Cross-sectional averages of
logarithmic vs. proportional coefficients of determination

Interval	July 1962-June 1977		July 1962-Dec. 1969		January 1970-June 1977	
	$cor^2(r_i,r_I)$	$cor^2(R_i,R_I)$	$cor^2(r_i,r_I)$	$cor^2(R_i,R_I)$	$cor^2(r_i,r_I)$	$cor^2(R_i,R_I)$
Month	.2892	.2794	.2483	.2392	.3237	.3162
Quarter	.3733	.3704	.2951	.2790	.4322	.4427
Semi-Annual	.4300	.4260	.4118	.3782	.4786	.5087
Annual	.3961	.3495	(1)	(1)	(1)	(1)

(1)
Insufficient data to compute significant statistics.

TABLE 6

Cross-sectional average betas and cross-sectional deviation in betas

b = logarithmic betas and B = proportional betas

Interval	Avg(b)	Avg(B)	dev(b)	dev(B)
	July 1962 - June 1977			
Month	1.1682	1.1741	.4248	.4283
Quarter	1.2442	1.3287	.4849	.5837
Semi-annual	1.3131	1.3810	.5747	.7092
Annual	1.1996	1.1966	.6443	.6851
	July 1962 - December 1969			
Month	1.2064	1.2008	.4862	.4875
Quarter	1.3303	1.3472	.6413	.6812
Semi-annual	1.3753	1.4047	.6819	.7743
	January 1970 - June 1977			
Month	1.1491	1.1612	.4452	.4529
Quarter	1.2105	1.3200	.4984	.6287
Semi-annual	1.2679	1.3615	.6494	.8404

TABLE 7

Regressions $\hat{B}_i = c + d \cdot \hat{b}_i$

Interval	$\hat{c}$	$t(\hat{c})$	$\hat{d}$	$t(\hat{d}-1)$	R-square
			July 1962 - June 1977		
Month	-.0007	-0.2	1.0056	2.6	.9947
Quarter	-.1334	-12.8	1.1750	22.4	.9528
Semi-annual	-.1841	-13.4	1.1919	20.0	.9327
Annual	.0025	0.2	0.9954	-0.4	.8762
			July 1962 - December 1969		
Month	-.0046	-1.4	0.9992	-0.3	.9930
Quarter	-.0412	-4.7	1.0436	7.4	.9653
Semi-annual	-.0793	-4.9	1.0790	7.5	.9029
			January 1970 - June 1977		
Month	-.0031	-0.9	1.0132	4.8	.9919
Quarter	-.1522	-11.6	1.2161	21.6	.9296
Semi-annual	-.2044	-12.4	1.2350	20.3	.9107

t-values have 1,113 degrees of freedom

TABLE 8(a)

Estimated Security Market Lines

July 1962 - June 1977

Interval	Logarithmic SML: $g_i = \hat{h}_0 + \hat{h}_1 \cdot \hat{b}_i$					
	$\hat{h}_0$	$t(\hat{h}_0)$	$\hat{h}_1$	$t(\hat{h}_1)$	$\hat{h}_0+\hat{h}_1$	R-square
Month	.0901	14.6	-.0208	-4.2	.0693	.0154
Quarter	.1033	18.0	-.0301	-7.0	.0732	.0421
Semi-annual	.1077	21.0	-.0319	-8.9	.0758	.0666
Annual	.0880	19.8	-.0185	-5.7	.0695	.0282

Interval	Proportional SML: $M_i = \hat{H}_0 + \hat{H}_1 \cdot \hat{B}_i$					
	$\hat{H}_0$	$t(\hat{H}_0)$	$\hat{H}_1$	$t(\hat{H}_1)$	$\hat{H}_0+\hat{H}_1$	R-square
Month	.0629	11.5	.0602	13.8	.1231	.1457
Quarter	.0734	14.8	.0480	14.0	.1214	.1501
Semi-annual	.0711	15.0	.0527	17.2	.1238	.2100
Annual	.0860	17.0	.0441	12.0	.1301	.1150

t-values have 1,113 degrees of freedom.

TABLE 8(b)

Estimated Security Market Lines

Subperiods

Logarithmic SML: $g_i = \hat{h}_0 + \hat{h}_1 \cdot \hat{B}_i$

Interval	$\hat{h}_0$	$t(\hat{h}_0)$	$\hat{h}_1$	$t(\hat{h}_1)$	$\hat{h}_0+\hat{h}_1$	R-square
			July 1962 - December 1969			
Month	.0771	9.4	.0518	8.2	.1289	.0574
Quarter	.0926	13.1	.0353	7.4	.1279	.0465
Semi-annual	.1086	15.5	.0226	4.9	.1312	.0214
			January 1970 - June 1977			
Month	.1183	14.2	-.1007	-14.9	.0176	.1668
Quarter	.1379	18.5	-.1117	-19.7	.0262	.2576
Semi-annual	.1096	17.5	-.0844	-19.2	.0252	.2492

Proportional SML: $M_i = \hat{H}_0 + \hat{H}_1 \cdot \hat{B}_i$

Interval	$\hat{H}_0$	$t(\hat{H}_0)$	$\hat{H}_1$	$t(\hat{H}_1)$	$\hat{H}_0+\hat{H}_1$	R-square
			July 1962 - December 1969			
Month	.0623	7.8	.1019	16.5	.1642	.1971
Quarter	.0771	11.1	.0810	17.5	.1581	.2163
Semi-annual	.0879	11.9	.0820	17.7	.1699	.2201
			January 1970 - June 1977			
Month	.0931	12.6	-.0090	-1.5	.0841	.0020
Quarter	.0928	14.4	-.0034	-0.8	.0894	.0005
Semi-annual	.0567	9.8	.0206	5.7	.0773	.0285

t-values have 1,113 degrees of freedom.

TABLE 9

Logarithmic vs. proportional variance of regression error terms of SML

Interval	$Var(e_i)\cdot 10^3$	$Var(E_i)\cdot 10^3$	$Var(E_i)/Var(e_i)$
		July 1962 - June 1977	
Month	4.9842	3.9021	0.7829
Quarter	4.8490	4.4558	0.9189
Semi-annual	4.7253	5.2583	1.1128
Annual	4.9196	7.0305	1.4291
		July 1962 - December 1969	
Month	10.4037	10.0470	0.9657
Quarter	10.5249	11.0356	1.0485
Semi-annual	10.8011	14.2656	1.3208
		January 1970 - June 1977	
Month	10.0325	8.0831	0.8057
Quarter	8.9396	8.5129	0.9523
Semi-annual	9.0400	10.3038	1.1398

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Appendix 1

RELATIONSHIP BETWEEN $cor(R_i, R_I)$ AND $cor(r_i, r_I)$

We have,

$$cor(R_i, R_I) = cov(R_i, R_I)/S_i S_I \qquad (A1.1)$$

Substituting the approximations for R_i and R_I from section 3.1 we get;

$$cov(R_i, R_I) = cov(r_i + 1/2\ r_i^2,\ r_I + 1/2\ r_I^2)$$

$$= cov(r_i, r_I) + 1/2\ cov(r_i, r_I^2) + 1/2\ cov\ (r_i^2,\ r_I)$$

$$+ 1/4\ cov(r_i^2, r_I^2) \qquad (A1.2)$$

If $\tilde{r}_i$ and $\tilde{r}_I$ have the same sign, then:

$cor(r_i, r_I^2) \simeq cor(r_i, r_I)$ and

$$cor(r_i, r_I^2) < cor(r_i, r_I) \qquad (A1.3a)$$

$cor(r_i^2, r_I) \simeq cor(r_i, r_I)$ and

$$cor(r_i^2, r_I) < cor(r_i, r_I) \qquad (A1.3b)$$

$cor(r_i^2, r_I^2) \simeq cor(r_i, r_I)$ and

$$cor(r_i^2, r_I^2) < cor(r_i, r_I) \qquad (A1.3c)$$

Assume that $\tilde{r}_i$ and $\tilde{r}_I$ have normal distribution, we get:

$$dev(r_i^2) = 2s_i(m_i + 1/2\ s_i^2)^{1/2} = 2s_i D_i \qquad (A1.4a)$$

$$dev(r_I^2) = 2s_I(m_I + 1/2\ s_I^2)^{1/2} = 2s_I D_I \qquad (A1.4b)$$

Substituting (A1.3) and (A1.4) in (A1.2), we get:

$$cov(R_i, R_I) = (C)\ [cov(r_i, r_I)](1 + D_I + D_i + D_I D_i) \qquad (A1.5)$$

$$\text{where } C < 1 \text{ but } C \simeq 1 \qquad (A1.6)$$

Substituting (A1.5) and (11) in (A1.1), we get:

$$\mathrm{cor}(R_i, R_I) = (C.F)\ [\mathrm{cor}(r_i, r_I)] \qquad \text{(A1.7)}$$

where
$$F = \frac{(1 + D_I + D_i + D_I D_i)}{(1 + D_I^2 + 2m_I)^{1/2}\ (1 + D_i^2 + 2m_i)^{1/2}} \qquad \text{(A1.8a)}$$

and
$$F > 1 \text{ but } F \simeq 1 \qquad \text{(A1.8b)}$$

Therefore, from (A1.6) and (A1.8b) $CF \simeq 1$, but $CF \gtrless 1$ depending on securities in the sample and up- or down-movement of the market during the period.

Hence,
$$\mathrm{cor}(R_i, R_I) \simeq \mathrm{cor}(r_i, r_I) \qquad \text{(A1.9a)}$$

and
$$\mathrm{cor}(R_i, R_I) - \mathrm{cor}(r_i, r_I) \gtrless 0. \qquad \text{(A1.9b)}$$

Appendix 2

RELATIONSHIP BETWEEN $\hat{H}_1$ AND $\hat{h}_1$ WHEN $\hat{h}_1 > 0$

From eqn(28) we get:

$$\hat{H}_1 - \hat{h}_1 = \frac{\mathrm{cov}(s_i^2, \hat{B}_i)}{2\ \mathrm{var}(\hat{B}_i)} + (k - 1)\ \hat{h}_1 \qquad \text{(A2.1)}$$

Substituting the expressions for $\mathrm{var}(\hat{B}_i)$ and $\hat{h}_1$ from section 5, we get:

$$\hat{H}_1 - \hat{h}_1 = \frac{k \cdot \mathrm{cor}(s_i^2, \hat{B}_i) \cdot \mathrm{dev}(s_i^2)}{2\ \mathrm{dev}(\hat{b}_i)} + \frac{(k - 1)\mathrm{cor}(g_i, \hat{b}_i)\mathrm{dev}(g_i)}{\mathrm{dev}(\hat{b}_i)} \qquad \text{(A2.2)}$$

Assuming that, <u>either</u> securities have little or no unsystematic risk <u>or</u> the rankings of securities according to their total risk and their systematic risk are the same, we get:

$$\mathrm{cor}(s_i^2, \hat{B}_i) \doteq 1 \qquad \text{(A2.3)}$$

Substituting (A2.3) in (A2.2), we get:

$$\mathrm{sign}\ (\hat{H}_1 - \hat{h}_1) = \mathrm{sign}\ [\mathrm{dev}(s_i^2) + 2(1-1/k)\mathrm{dev}(g_i)\mathrm{cor}(g_i, \hat{b}_i)] \qquad \text{(A2.4)}$$

We have $(1 - 1/k) < 0$, and $\mathrm{cor}(g_i, \hat{b}_i) > 0$ (i.e. $\hat{h}_1 > 0$),

therefore at given $\mathrm{dev}(s_i^2)$ and $\mathrm{dev}(g_i)$:

:for relatively small values of $\mathrm{cor}(g_i, b_i)$, $\hat{H}_1 - \hat{h}_1 > 0$. (A2.5a)

:for relatively large values of $\mathrm{cor}(g_i, b_i)$, $\hat{H}_1 - \hat{h}_1 < 0$. (A2.5b)

RELATIONSHIP BETWEEN R-SQUARE AND r-square

we have,

$$\text{R-SQUARE} = (\hat{H}_1)^2 \ (\text{var}(\hat{B}_i)/\text{var}(M_i)) \qquad \text{(A3.1a)}$$

$$\text{r-square} = (\hat{h}_1)^2 \ (\text{var}(\hat{b}_i)/\text{var}(g_i)) \qquad \text{(A3.1b)}$$

Substituting the expression for $\text{var}(\hat{B}_i)$ from section 5, we get:

$$\frac{\text{r-square}}{\text{R-SQUARE}} = k^2\left(\frac{\hat{h}_1}{\hat{H}_1}\right)^2 \left(\frac{\text{var}(M_i)}{\text{var}(g_i)}\right) \qquad \text{(A3.2)}$$

where $0< k <1$

Case (i) In an up-market

From (19b): $\dfrac{\text{var}(M_i)}{\text{var}(g_i)} > 1$

From appendix 2: $\left(\dfrac{\hat{h}_1}{\hat{H}_1}\right)^2 \lesseqgtr 1$

and: $k^2 < 1$

therefore, $\dfrac{\text{r-square}}{\text{R-SQUARE}} \gtreqless 1$ (A3.3a)

Case (ii) In a down-market

From (19c): $$\frac{\mathrm{var}(M_i)}{\mathrm{var}(g_i)} \gtrless 1$$

From (28) and (29a):

if $\hat{h}_1 < \hat{H}_1 < 0$, then $(\hat{h}_1/\hat{H}_1)^2 > 1$

if $\hat{h}_1 < 0 < \hat{H}_1$, then $(\hat{h}_1/\hat{H}_1) \gtrless 1$

and: $k^2 < 1$

therefore, $$\frac{\text{r-square}}{\text{R-SQUARE}} \gtrless 1 \qquad \text{(A3.3b)}$$

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77305 Fontainebleau Cedex, France
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