"'TRIPLE C': A VISUAL INTERACTIVE MCDSS"

by

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A visual interactive MCDSS

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Abstract

"Triple C" is a new visual interactive system which supports the exploration of multicriteria decisions. The novelty of this system consists in an overall utilization of Visual Interaction for supporting all the information processing activities related to the task of incrementally defining and analyzing a multicriteria decision situation.

Departing from traditional approaches in the MCDM field, the presented Decision Support System is neither an automation of the decision process nor an attempt to capture the decision-makers' preferences and goals into a formal model. Rather than providing objective "solutions", this system aims at supporting end-users to progressively gain insights into a decision situation, providing different tools to enable them

(1) to better *understand* the decision situation at hand by expressing and analyzing their *own* preference structures, testing different alternatives and comparing them interactively,

(2) to *support* and *verify* their *individual* judgement by performing different types of sensitivity analysis, and

(3) to *justify* their *subjective* choices and *communicate* them easily - a crucial factor in group decisions.

Keywords

Multicriteria Decision Making, Decision Support Systems, Human-Computer and Visual Interaction.

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(1) Introduction

Multicriteria decision making (MCDM) is a classical research field in Management Science/Operation Research ([Zeleny 1982], [Edwards, Newman 1982], [Saaty 1986], [Roy 1971] and [Keeney, Raiffa 1976]). The theories and methods proposed in this context have stimulated the development of Decision Support Systems (DSS; cf. [Keen, Scott Morton 1978], [Sprague, Watson 1989]) aiming at helping decision makers deal with multicriteria real-world situations with the aid of interactive computer-based systems, so-called Multi Criteria Decision Support Systems (MCDSS; cf. a survey in [Eom 1989]). As a matter of fact, MCDM is probably the best example for the complexity of human decision making and of the attempts to support this process in an effective way, or in other words: "The multicriteria problem is at the core of Decision support" [Keen 1987].

On the other hand, the existing approaches to understand, model and support MCDM do not seem to have the expected impact on real-world decision making and they have been more and more criticized. One of the main arguments of criticisms refers to the fact that researchers in this area have mostly ignored the cognitive aspects of human decision making, and have "imposed a mathematical artifact, both simple and simple-minded in its design, on the rich, natural, self-organizing and knowledge-producing processes of individual and social decision making, without even attempting for its deeper understanding" [Zeleny 1989].

As discussed in [Angehrn 1990] , even the development of interactive systems supporting multicriteria decision making needs to be "humanized": MCDM needs to be reinterpreted in terms of an incremental, individual *learning* process instead of a mechanical solving procedure leading to the identification of an illusive "optimal" solution. This different perspective also requires a different approach to system development, the concrete application of which is discussed below.
The guidelines described in [Angehrn 1990] have served as a theoretical basis for the design of "Triple C", a new MCDSS whose aim is not to "solve" any predefined problem, but to offer a flexible working environment in which individual learning about a decision situation can take place. Accordingly, the two main roles (support dimensions) the system plays in order to support the decision making process, are (see Figure 1):

(1) A "Facilitator" role,
i.e. it supports decision makers in easily *expressing/representing* their views interactively without constraining their creativity and intuition during the phases of problem formulation and generation and exploration of satisfactory solutions.

(2) A "Stimulator" role,
i.e. it supports an incremental reviewing/learning process (analysis & synthesis cycle) by stimulating decision-makers to *process* the information at hand and to continuously question the solutions they generate during the decision making process.

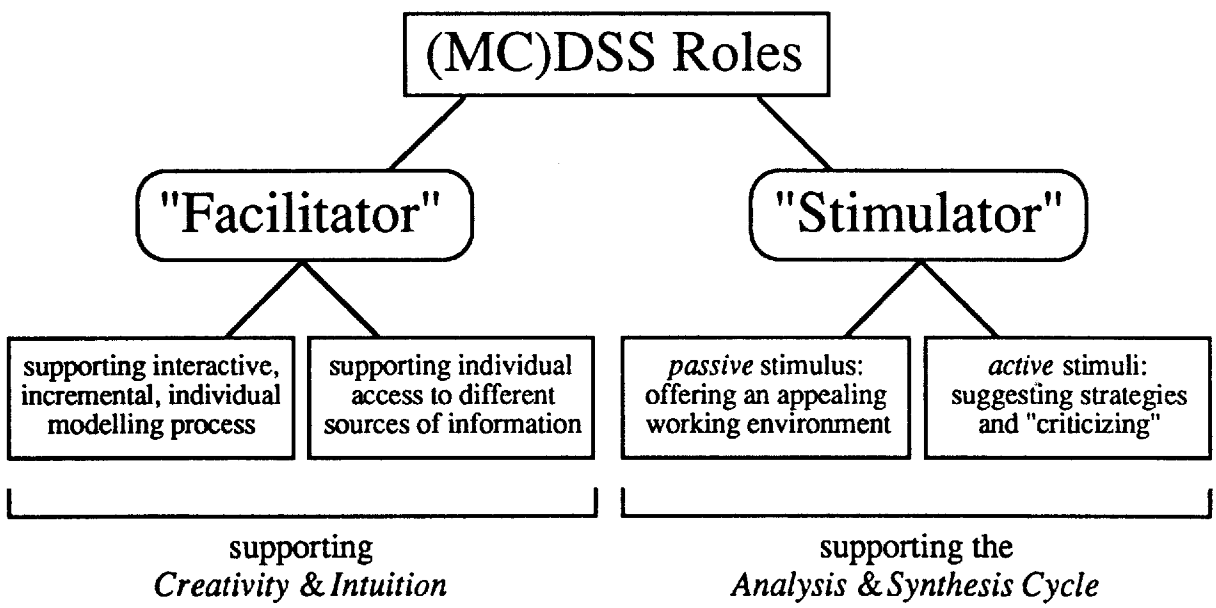

Figure 1: Generic (MC)DSS support dimensions [Angehrn 1990].

In the following sections the "Triple C" system is described in more detail, offering a concrete example of how Visual Interaction [Turban, Carlson 1989] can be used for realizing "humanized" Decision Support Systems with the characteristics described above. In section 2 the general characteristics of the system are discussed, followed by an illustration of the main tools "Triple C" supplies to a decision maker. Finally, the mathematical methods, which have been developed and integrated into the systems are described in section 4.

(2) "Triple C": Objectives and Application Fields

'Triple C' is a visual interactive environment supporting decision-making processes. Figure 2 illustrates the objectives and the resulting application fields of the system.

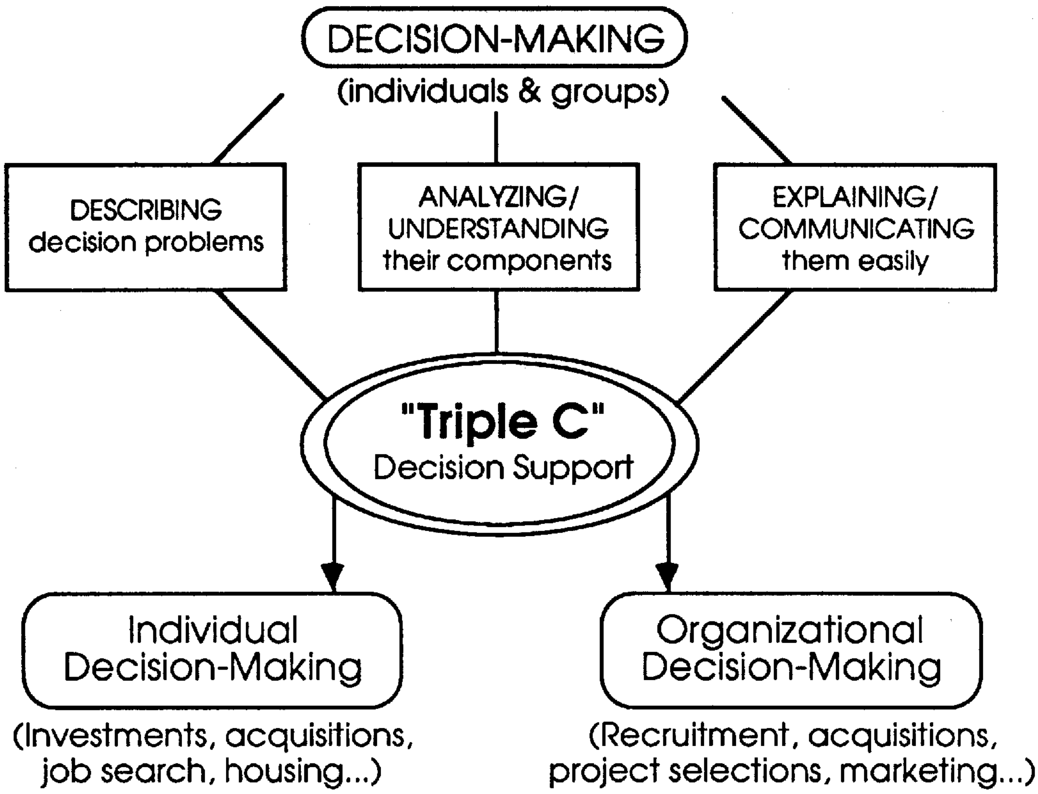

Figure 2: Objectives and application fields.

As discussed in part 1, human decision-making requires modern instruments which can support individuals and groups in:

- flexibly and clearly **describing** their decision problems,
- **analyzing** and **understanding** their different components, and
- easily **communicating** and **explaining** them.

Accordingly, 'Triple C' aims at supplying decision-makers with:

- a flexible environment for representing their decision problems in an individual way;
- tools supporting the analysis of different decision situations and the exploration of their 'objective' as well as 'subjective' components.
- a user-centered software which extensively uses graphical visualization for enhancing communication and explanation of decisions.

(2.1) Which kind of Decisions?

The decision situations addressed 'Triple C' are characterized (like in other MCDSSs) by two main components:

A set of alternatives and a set of criteria, by which the different alternatives can be compared and evaluated. Both sets can be defined, updated and manipulated interactively during the decision process.
Thus, 'Triple C' is a multi-purpose program easily applicable to:

- **Individual** decision-making
  Examples are important individual choices and decisions, such as the acquisition of assets like cars or houses and the choice of a suitable job or a good investment.
- **Organizational** decision-making
  Such as recruitment, project selection, logistic choices, acquisition of important organizational resources (like a computer network), marketing and others.

(2.2) Which kind of Support?

The main activities supported by 'Triple C' encompass:

- the flexible definition of alternatives
- the introduction of relevant criteria
- the specification of the user's subjective views
- the interactive comparison and analysis of alternatives.

For supporting these activities in a suitable way, 'Triple C' offers the user a highly **visual** environment conceptually based on the so-called 'Triple C'-Model (**C**ircular **C**riteria **C**omparative Model), a new framework for representing and dealing with 'multi-attribute' decision situations.

Compared to other tools, 'Triple C' has some peculiarities, briefly summarized in the following list:

- The main goal of 'Triple C' is not the automation of the decision process. On the contrary the system has been designed for emphasizing **man-machine cooperation**. Thus, it helps the user in identifying 'good' alternatives but it also supports him in better understanding - and communicating - 'why' a certain decision has been taken.
- The interaction between the user (decision-maker) and the system is strongly based on **visualization** and **direct manipulation** [Shneiderman 1987; Norman, Draper 1986], thus avoiding the use of an abstract command language and emphasizing the user's view of the decision at hand.
- The control on the decision process always remains in the user's hands. In this way the decision-maker is supported in developing **individual strategies**, analyzing different preference structures, testing different alternatives and comparing them interactively.
- Taking into consideration the **visual skills** of the user as a crucial component of the decision process, 'Triple C' supplies a problem view which strongly supports the recognition of critical patterns and the identification of conflict resolution strategies.
- Different techniques are offered to the decision-maker for supporting the choice of alternatives globally (ranking of the alternatives) as well as locally (analysis of single components). The application of these techniques is **fully integrated** within the user-centered working environment, which is a necessary condition for warranting user acceptance [Angehrn, Lüthi 1990].

- The program is easy to learn and to use. The functionality is supplied in a **transparent** way and can be learned by the user gradually ("learning by using & by trying").

(2.3) Which kind of System?

Figure 3 shows a typical "Triple C" screen. The three windows displayed on the screen represent the three main components of a decision (in this specific example, the choice of a car):

- In a first window the system displays a visual representation of the **criteria** the user wants to consider in selecting a car (speed, comfort, power, fuel consumption, price...).
  The criteria are represented according to their relative importance (for example the user gives more importance to "speed" than to "comfort").

- A second window shows a list of **alternatives** the user wants to consider and analyze. In addition, this window offers a global view on the data relevant to the decision at hand.

- A third window (only partially displayed in figure 1) shows a possible **"ranking"** of the alternatives. This ranking is interactively calculated by the system. It reflects the importance of the criteria (and other subjective elements) introduced by the user during the decision process and can be modified by the user according to his/her preferences.

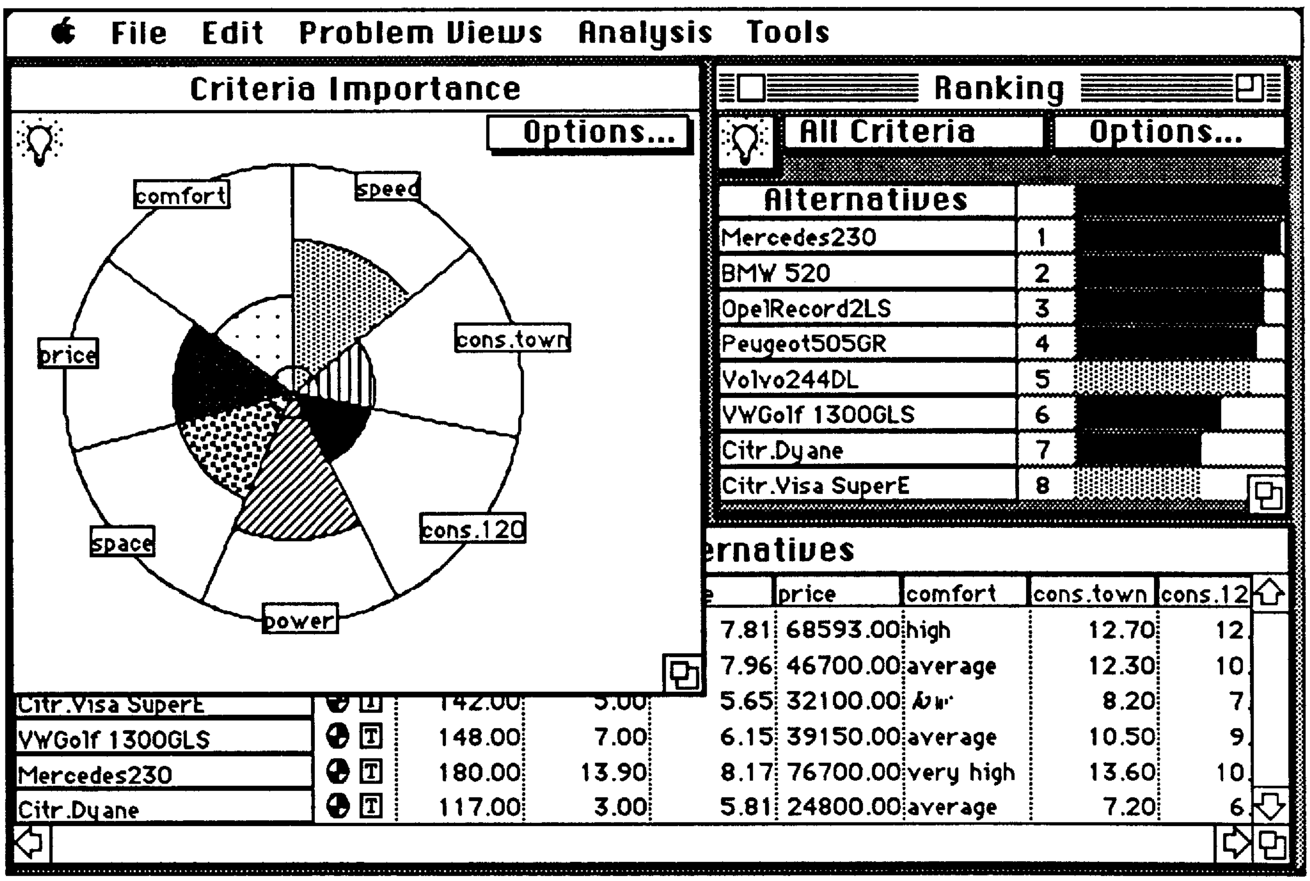

Figure 3: A "Triple C" screen.

(3) "Triple C": An Overview

(3.1) Working with "Triple C" : General features

A basic characteristic of 'Triple C' is its genuine realization of the so-called *desktop metaphor* [Smith et al. 1982].
This means that the user is supported in using the screen as his own desk, putting on it and manipulating all different elements (sheets of paper, graphics...) he needs to consider during a decision process.
In 'Triple C', all these components (working tools) can be used by the decision-maker for visualizing different information sources, like the criteria relevant to his decisions, his subjective view of the problem, the different alternatives to be considered, sheets reporting on a direct comparison of alternatives and so on.
By this means, i.e. opening, moving, changing or closing different sources of information, the user can tailor the screen to his *individual* needs, creating his own problem *views* without being obliged to follow fixed sequences or to 'navigate' through a predefined menu structure.
Furthermore the system permanently guarantees *consistency*, i.e. it monitors the user's activities (cf. [Angehrn, Lüthi 1990]): Any change carried out by the user on one component is automatically 'spread' to all the other components which are directly or indirectly affected by this change. This updating process is accompanied by a visual feedback allowing the user to always have an updated view of his work (e.g. modifying the textual data of the alternatives will cause an automatic update of the graphic representation of the affected alternatives which are visible on the screen).

(3.2) Interacting with the main "Triple C" - tools

In this part the main decision support tools offered by 'Triple C' are described in more detail. Note that every tool can be called and used at any time during the decision process. The program does not set any constraint on the individual working style of the user (unless his actions could create inconsistency), i.e. with 'Triple C' it is the user, who *drives*! The system merely puts at his disposal tools and instruments facilitating his ride, making it more efficient and more effective.
Sections (3.2.1) and (3.2.2) report on the two main decision components, criteria and alternatives. Section (3.2.3) describes the analysis tools supplied by 'Triple C' from the user's point of view. The same analysis tools and the underlying mathematical models embedded in 'Triple C' will be described formally in part (4).

(3.2.1) Criteria: A picture of Subjectivity

The way criteria are represented in 'Triple C' is shown in figure 4. This graphic representation is a first example of application of the **C**ircular **C**riteria **C**omparison Model mentioned in the introduction, which supports:

- The visual representation of criteria and criteria-related information as well as the visual comparison of alternatives.
- A visual-based interaction style for acting on the different decision components dynamically, structuring and manipulating them interactively.

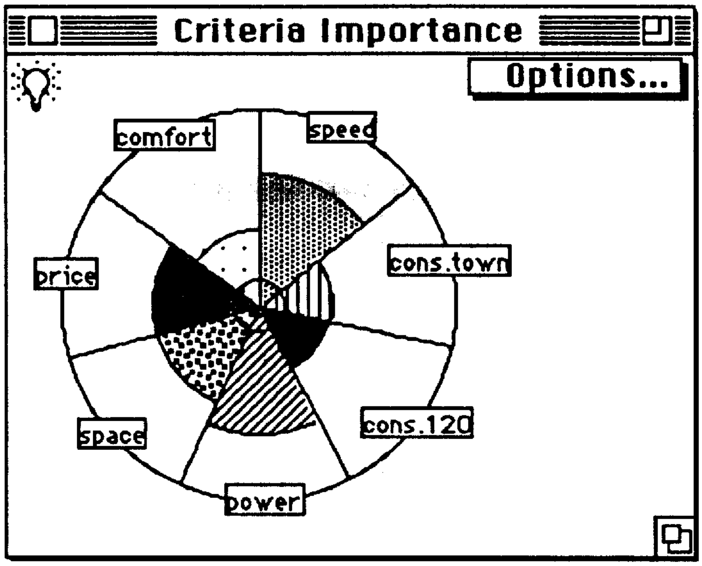

Figure 4: Criteria representation.

In this special kind of representation, criteria are displayed in a circular framework (cf. the 7 criteria "speed", "consumption town",..., "comfort" for the example of a car choice in figure 4). The size (the radius) of each sector visually indicates the importance of a criterion in respect to the others. By this means we can easily *see*, that e.g. the criterion "speed" is approximately twice as important as the criterion "comfort" and almost as important as the criterion "space".
Within the "Triple C" visual framework it is quite simple to assign weights to the criteria and adapt their relative importance to the user's own subjective view. The user only has to select the border of one of the sectors to modify its size. A circle will appear and help the user in evaluating the relative importance of the selected criterion compared to the importance of all the others ("One-to-All" comparison). Besides the visual-based specification of weights, the user has also the possibility to see, edit or modify the (normalized) relative importance of each criterion numerically (e.g. to set the weight of the criterion "speed" to 50%, the one of "comfort" to 25% etc.)

In "Triple C", criteria can belong to four different types:

- logical criteria, having only the values YES or NO, as is the case for criteria like "European", "water-proof" or "compatible".
- numerical criteria, whose values can be described within a real interval, like in the case of the criteria "speed", "power" or "age".
- visual criteria, with a visually represented value corresponding to a degree of preference between 0 and 100%, and
- self-defined criteria, whose discrete values are defined by the user such as in the case of the criterion "comfort", whose values have been specified with the adjectives "very high", "high", "average", "low" and "very low".

Furthermore, the user can indicate his own preference for each criterion, specifying if he prefers a higher value or a lower one, if he prefers the value YES to the value NO (for logical criteria) or if he considers the evaluation as indifferent (e.g. for "nationality" the user could introduce a new "self-defined" criterion with values like "French", "German", "Italian", etc. and declare that this value is indifferent to him because he does not want or cannot express his preference between an 'Italian' alternative and a 'German' one).

Besides the specification of individual weights, the decision-maker can express his subjective preferences using the two other tools discussed in the following: 'Subjective Intervals' and 'Aspiration Levels/Ideal Values'.

Let us take again the example of choosing a car. One relevant criterion for this decision could be 'speed' (cf. figure 4). 'speed' can be evaluated numerically, let's say with values between 0 and 200 km/h. But let us assume that the user is only interested in cars whose speed lies in a range of (90,180).
By defining such a 'subjective interval' he can communicate to the system (and make clear to himself) that he is not willing to consider every car as a possible alternative but only those fulfilling certain conditions: The conditions he can express specifying his own 'subjective intervals' for each criterion.
Another example is the logical criterion 'European', which only assumes the values YES or NO. Defining the subjective interval as the single value YES, the user can express a preference for European cars and declare all the others as 'unfeasible' alternatives.
Even for supporting the definition and the interactive modification of such intervals, the system exploits the 'Triple C' Model.

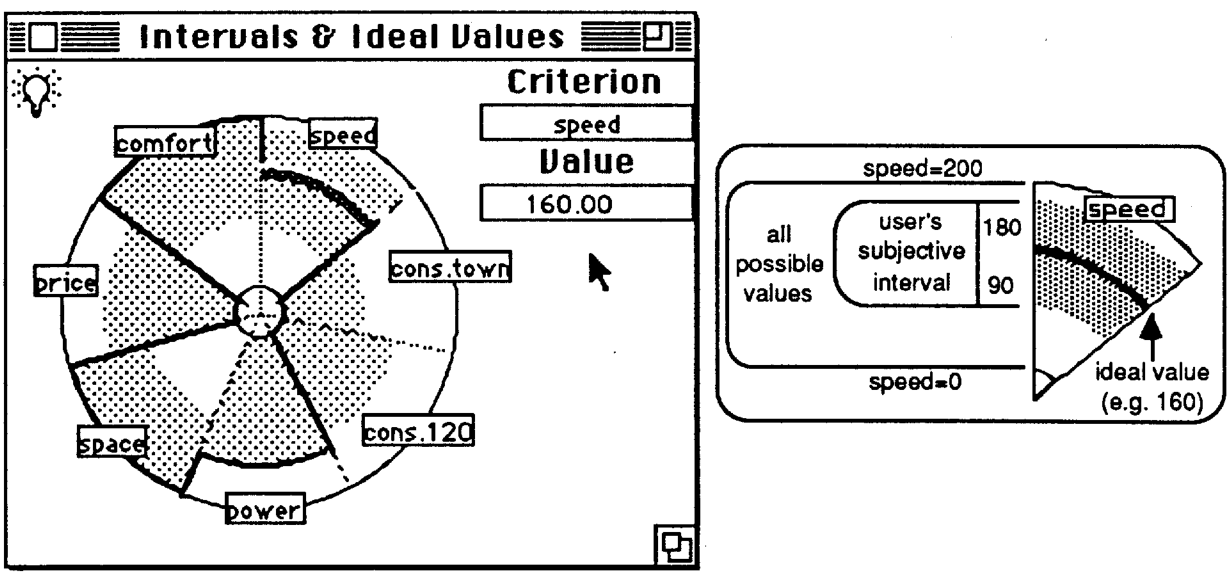

Fig. 5 : 'Subjective Intervals' and 'Ideal Values'

Figure 5 illustrates the tool the user can employ for specifying 'Subjective Intervals' visually. We have here the same circular representation as for the tool depicted in figure 4, but this time the sectors represent the possible values which a criterion can assume. This tool allows the user to define and modify subjective intervals simply by moving the borders of the shadowed area representing the range of values he is willing to accept for a specific criterion.
Beside the specification of subjective intervals, 'Triple C' gives the user another possibility for expressing his preferences: Using the same tool displayed in figure 5, the user can design the shape of his 'ideal' alternative.

If the user has previously specified that a specific criterion should be preferably as high (or as low) as possible, 'Triple C' will automatically put the 'ideal' value at the corresponding border of the subjective range (for instance the 'ideal' value for speed would be set at 180, the maximum value of the range). But again the user has the possibility to easily adapt the ideal values of a criterion to his own preference, setting it to an intermediate value (for instance setting the aspiration level for speed to 160 km/h as in figure 5).

(3.2.2) Alternatives: Handling data easily

Whatever kind of decision situation the user is facing, the alternatives he considers always have the same basic structure:

- They have a unique identifier (name)
- They have a set of 'attributes' (alternative data)

Accordingly, alternatives can be described, visualized and updated using a tabular form such as the one displayed in figure 6.

Car Choice : Alternatives

Alternatives		speed	power	comfort	price	space	cons.town
Mercedes230	T	180.00	13.90	very high	76700.00	8.17	13.60
BMW 520	T	182.00	11.00	high	68593.00	7.81	12.70
OpelRecord2LS	T	176.00	11.00	average	[illegible]	7.96	12.30
Volvo244DL	T	145.00	11.00	high	56000.00	8.38	14.30
Peugeot505GR	T	173.00	10.00	average	49500.00	7.88	11.40

Fig. 6: Overview on alternative attributes.

Besides this textual overview - which allows an easy handling of the alternatives - the user can generate visual representations of single alternatives within the 'Triple C' Model by selecting the symbols which are associated with each alternative (the second symbol, (T), is used for associating 'free-text' information to an alternative). Such a visual representation of single alternatives is displayed in figure 7.

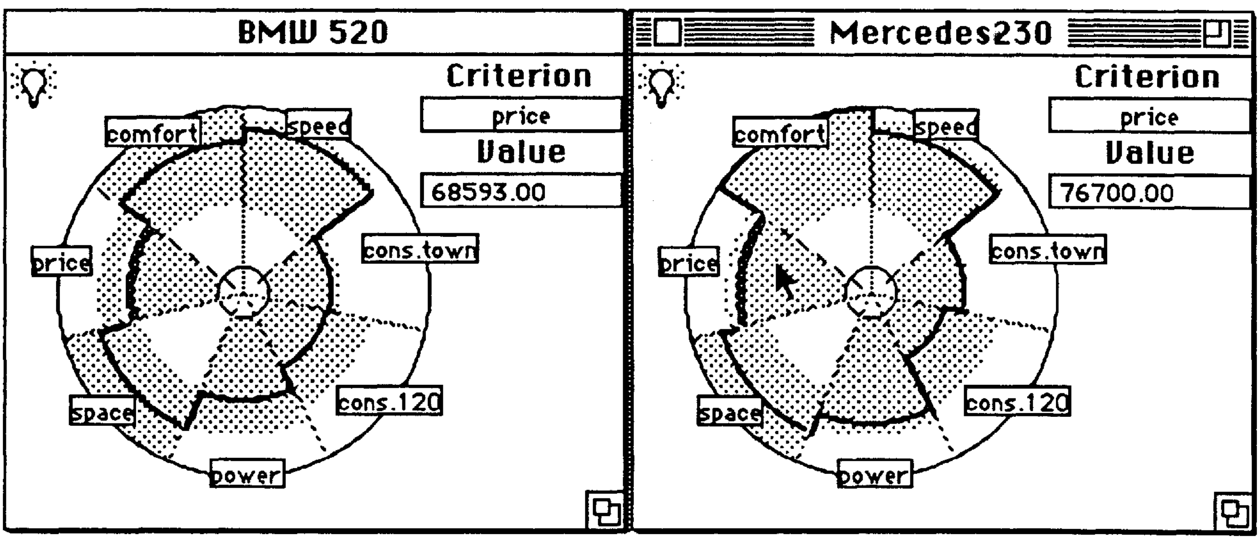

Figure 7: Alternatives representation within the 'Triple C' Model

These graphical representations contribute to give a visual dimension to the data displayed in figure 6. Furthermore, they help the user in comparing and analyzing the differences between the different alternatives.

Obviously, the management of the alternatives and of the criteria during the problem definition phase and along the whole decision making process has been made as comfortable as possible for the end-user: The definition, modification, updating, and elimination of all the involved data can be performed through a visual language, manipulating graphics and symbols without any use of abstract commands.

(3.2.2) Analysis Tools: Ranking & Analyzing Values

There are basically four important tools in "Triple C" supporting the analysis of a decision situation and the choice between alternatives.

The first tool is a so-called "Ranking" which always shows how the system would rank the alternatives using a mathematical method (see part 4 for a formal description) which tries to reflect the user's subjective view expressed through the weighting of the criteria , the subjective intervals and the ideal values .

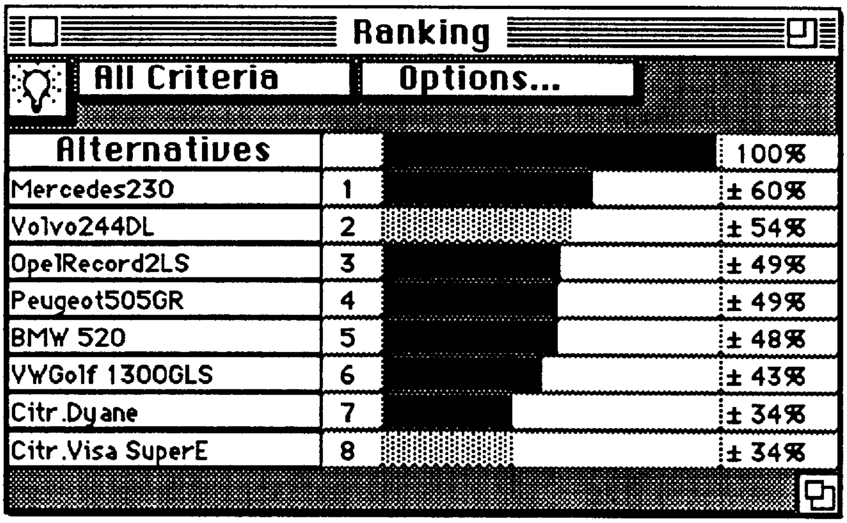

Figure 8: Example of a suggested ranking of the alternatives

The tool displayed in figure 8 contains following information:

- A ranking of the alternatives with the corresponding position (1, 2, 3,...).
- A percentage of "satisfaction". The "ideal" alternative - which corresponds to the user's aspiration levels - is supposed to give him 100% satisfaction. All the other alternatives are measured respectively.
- A graphic-based view on the alternatives fulfilling the conditions implicitly defined by the subjective ranges (The "unfeasible" alternatives - like e.g. "Volvo244DL" - are displayed with a gray pattern).

The user can generate different rankings based on an aggregation of all criteria (like in figure 8) or on single criteria (e.g. the criterion power), and can choose between different kinds of graphic representations (see figure 9).

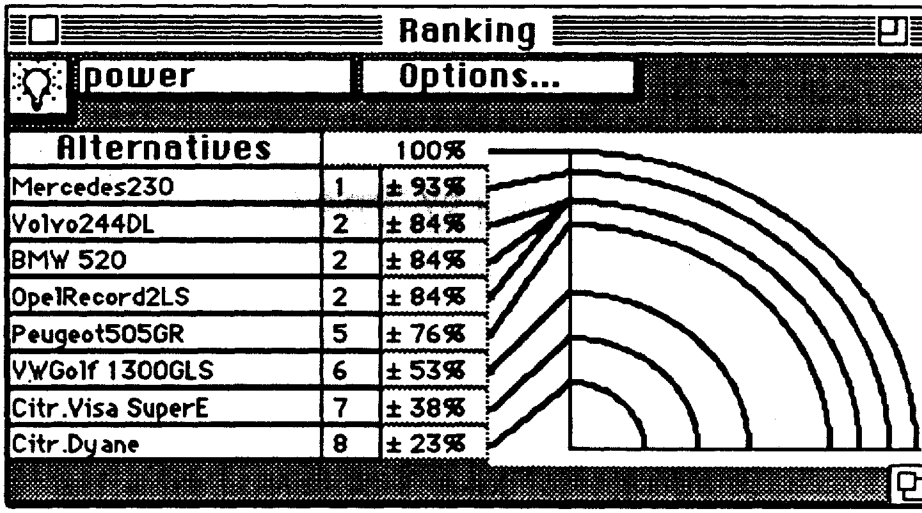

Figure 9: Example of a ranking based on one single criterion ("power")

These rankings are dynamically updated by the system each time the user changes one of the parameters (e.g. the weight of a criterion). They allow in this way the user to perform an interactive sensitivity analysis.

A second important use of the "Ranking" tool is the possibility for the decision-maker to modify the rankings computed and suggested by the system, e.g. by shifting an alternative from the i-th to the j-th place according to his own subjective preferences. In this case, a second mathematical method (described in part 4) is activated: The system will now interactively compute and suggest a new weighting of the criteria and display it in the tool illustrated in figure 4.

The third analysis tool supplied by "Triple C" allows the user to analyze the alternatives and their ranking using again the "Triple C" visual representation. This tool, displayed in figure 10, can now be used for analyzing the so-called "extrema" ("Which is the best alternative on each single criterion?") or the ranking of single alternatives.

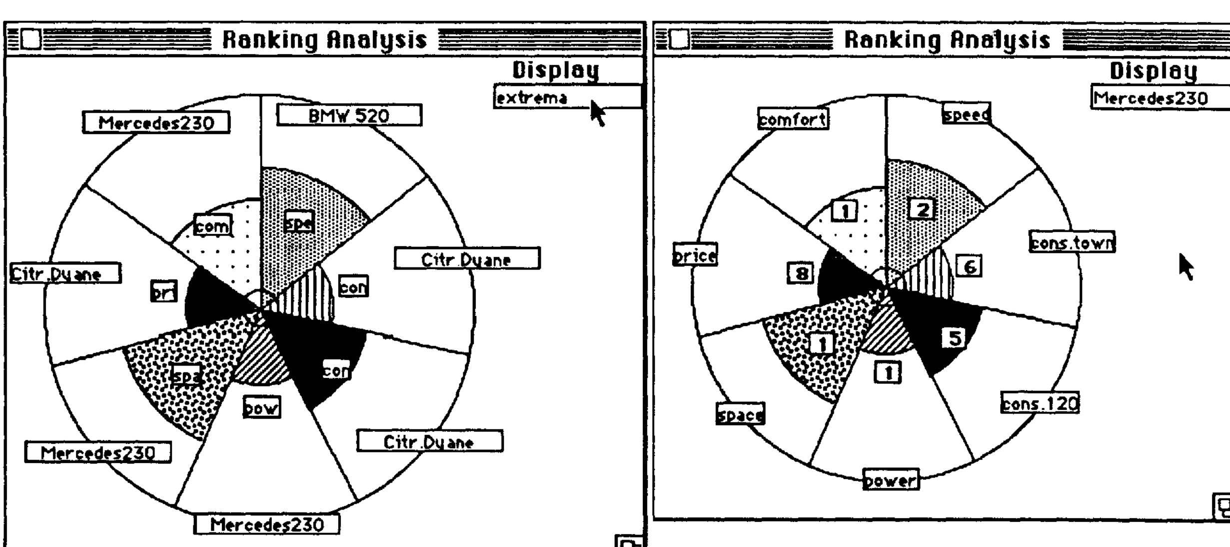

Figure 10: Visual-based analysis of the alternatives ranking

In the example of figure 10 (left) we can see, that the alternative 'Mercedes230' is the best one ('extremum') according to the three different criteria 'comfort', 'space' and 'power'. (The background remembers to the user the relative importance of the criteria).

The fourth analysis tool supplied by 'Triple C' allows the user to perform a kind of 'sensitivity analysis' of single values (alternative attributes) and therefore plays a very important role.

Using a magnifying glass, , the user can select any value displayed on the 'Overview' window (see figure 6). Doing this corresponds to asking the system to answer the following question 'Which value should this attribute have for making out of its alternative the best one?'
For example, selecting the value which represents the 'speed' of the alternative 'OpelRecord2SL' the user would implicitly ask 'Which 'speed' would make out of 'OpelRecord2SL' the best car?'
The system-generated answer to this question would then look as in figure 11:

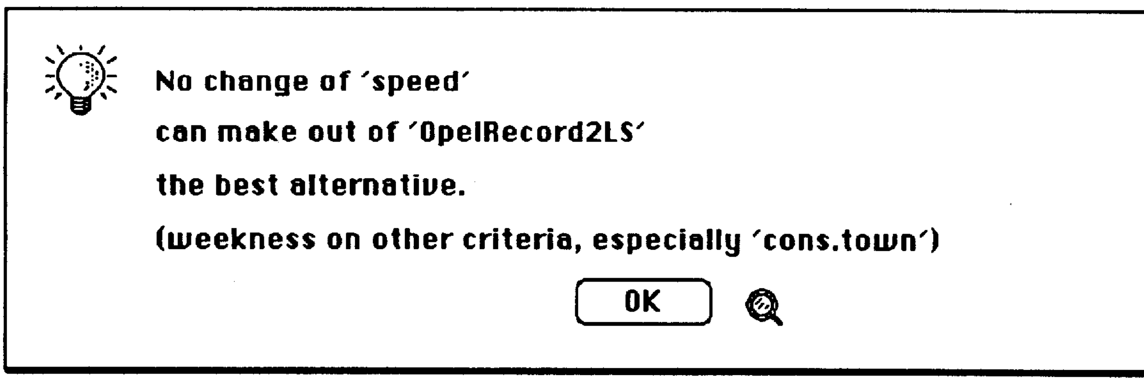

Figure 11: An example of the 'Triple C' value analysis tool

By selecting now the value of the suggested criterion 'consumption in town' for the same alternative, the user could receive the new relevant piece of information illustrated in figure 12.

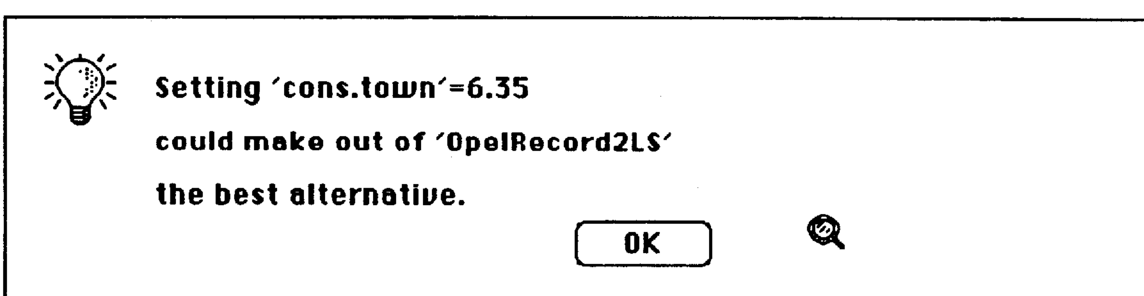

Fig. 12: Continuing the dialogue with the analysis tool

(4) Formal Description of the System's Functionality

In this section we describe in a formal way the different sources of information controlled by the decision-maker, as well as the mathematical methods used in 'Triple C'.

4.1) Basic Variables

Criteria-related Information

The criteria-related parameters controlled by the decision-maker are:

(a) n : the number of criteria $c_1, c_2, \dots, c_n$.
(b) $C_1, C_2, \dots, C_n$: the value ranges associated with each criterion.

Without restriction we can assume $C_j \subseteq \mathbf{R}, \forall j=1..n$ as every C_j can be defined as

- a compact interval on the real axis: $C_i = [a,b] \subseteq \mathbf{R}$, or
- an interval of discrete values, e.g. $C_j = [\mathit{low}, \mathit{average}, \mathit{high}]$ or

  $C_j = [\mathit{TRUE}, \mathit{FALSE}]$ which can be mapped into $C_j = [t, t+1, \dots, t+k] \subseteq \mathbf{N} \subseteq \mathbf{R}$

Alternatives-related Information

The basic alternatives-related parameters controlled by the decision-maker are

(a) m : the number of alternatives $a_1, a_2, \dots, a_m$.
(b) x_{ij}, $i=1..m, j=1..n$: attribute values for each pair (alternative i, criterion j).

Note that as a consistency constraint $x_{ij} \in C_j$ must hold.

Preference-related Information

For expressing and exploring his subjective preference structure, the decision-maker disposes of 4 further elements: criteria weights (CW), feasibility ranges (FR), ideal values (IV) and alternatives ranking (AR). A suitable notation for these elements is introduced in the following.

Criteria Weights (CW): The weights corresponding to the importance the decision-maker assigns to every single criterion can be modelled through

the vector $\mathbf{w} = (w_1, w_2, \dots, w_n)$, $w_j \in \mathbf{R}$, with $\sum_{j=1}^{n} w_j = 1$, and

$$(w_{j_1} > w_{j_2}) \Leftrightarrow (c_{j_1} \text{ more important than } c_{j_2})$$

Feasibility Ranges (FR): Feasibility ranges can be defined by the decision-maker for expressing further conditions on the attribute values x_{ij} by declaring them "feasible" (corresponding to his/her expectation) or "unfeasible". This can be formalized through n intervals S_j

$$\mathbf{S} = (S_1, S_2, \dots, S_n),\ S_j \subseteq C_j$$

$$\left(x_{ij} \in S_j\right) \Leftrightarrow x_{ij} \text{ is a } \textit{feasible} \text{ value for criterion } c_j$$

$$\left(x_{ij} \notin S_j\right) \Leftrightarrow x_{ij} \text{ is an } \textit{unfeasible} \text{ value for criterion } c_j$$

The concept of "feasibility" can be extended to the alternatives as follows:

$$\left(x_{ij} \in S_j\right) \forall j \Rightarrow a_i \text{ is a } \textit{feasible} \text{ alternative}$$
$$\exists j \left(x_{ij} \notin S_j\right) \Rightarrow a_i \text{ is an } \textit{unfeasible} \text{ alternative}$$

Ideal Values (IV): Ideal values can be defined for each criterion as those attribute values corresponding to the decision-maker's ideal alternative.

They are represented here through a vector $\mathbf{x^*} = (x_1^*, x_2^*, \dots, x_n^*),\ x_j^* \in S_j$.

Alternatives Ranking (AR): The next element the decision-maker has at his disposal for expressing and analyzing a decision situation, is a ranking of the alternatives represented here as a function $\mathbf{r} : \mathbf{N} \to 2^{\{a_1, \dots, a_m\}}$. This ranking function r can be formally interpreted as follows:

$$r(1) := \mathcal{R}_1 \subseteq \{a_1, \dots, a_m\}$$
Set of alternatives ranked at the first place
$$r(2) := \mathcal{R}_2 \subseteq \{a_1, \dots, a_m\} \setminus \mathcal{R}_1$$
Set of alternatives ranked at the second place
$$r(k) := \mathcal{R}_k \subseteq \{a_1, \dots, a_m\} \setminus \cup_{j=1}^{k-1} \mathcal{R}_j$$
Set of alternatives ranked at the k-th place

$$\text{and} \quad \cup_{j=1}^{m} \mathcal{R}_j = \{a_1, \dots, a_m\}$$

Such a ranking function partitions the set of alternatives in k ($m \geq k$) equivalence classes corresponding to the subsets $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_k$.
This partition can be interpreted as follows:

$$\left(a_p \in \mathcal{R}_i\right) \wedge \left(a_q \in \mathcal{R}_j\right) \wedge (i < j) \Rightarrow a_p \text{ is } \textit{better than } a_q$$
$$\left(a_p \in \mathcal{R}_i\right) \wedge \left(a_q \in \mathcal{R}_j\right) \wedge (i = j) \Leftrightarrow a_p \text{ is } \textit{as good as } a_q$$

Important: Note that the ranking r as well as the implicitly assumed full comparability between the m alternatives do not pretend to be considered as a "mirror" of the decision-maker's preference structure. In "Triple C", the ranking merely plays the role of a "rational myth", i.e. an additional tool supporting the user in better understanding and stepwise refining his own, partially unexpressed or inexpressible preference structure.

(4.2) Mathematical Models and Techniques

In "Triple C", the main role of mathematical models is to support the decision-maker in performing sensitivity analysis and gaining insights into the relationships between the alternatives and into the relative importance associated with the criteria.
Basically, there are 2 mathematical models used by the system in supporting these analytical activities:
A first model (see part 4.2.1) generates and suggests a possible ranking of the alternatives based on the parameters described by the decision-maker (criteria weights, ideal values,...).

A second model - using quadratic optimization - performs the inverse operation, recomputing the criteria weights each time the decision-maker modifies the ranking according to his subjective view (see part 4.2.2).

(4.2.1) System-suggested Rankings

The first model developed and implemented in 'Triple C' serves for generating a ranking r

$$r = f^{[1]}(\mathbf{w}, \mathbf{S}, \mathbf{x}^*)$$

through the aggregation of the criteria- and preference-related information (see 3.1.2 and 3) described by the decision-maker.
This system-generated ranking of the m alternatives builds a basis for analyzing the impact of changes in the 3 parameters **w**, **S** and **x***.
The function r is calculated starting from the computation of a 'loss' function $\mathcal{L}$ defined on the alternatives $a_1, \ldots a_m$ as follows:

$$\mathcal{L}(a_i) = \sum_{j=1}^{n} w_j\, l_{ij}$$

$$l_{ij} = \begin{cases} \dfrac{\left|x_{ij}-x_j^*\right|}{\max\{y \mid y \in S_j\}-\min\{y \mid y \in S_j\}} & \text{if } \exists\, x_j^* \\ 0 & \text{else} \end{cases}$$

The induced ranking r is then determined by ordering the alternatives according to the increasing value of their loss function $\mathcal{L}$, or more formally through the construction:

$$r(1) = \mathcal{R}_1 := \{a_i \mid \mathcal{L}(a_i) \le \mathcal{L}(a_j),\ a_j \in \{a_1,\ldots,a_m\}\}$$
$$r(2) = \mathcal{R}_2 := \{a_i \mid \mathcal{L}(a_i) \le \mathcal{L}(a_j),\ a_j \in \{a_1,\ldots,a_m\} \setminus \mathcal{R}_1\}$$
$$\ldots\ldots$$
$$r(k) = \mathcal{R}_k := \{a_i \mid \mathcal{L}(a_i) \le \mathcal{L}(a_j),\ a_j \in \{a_1,\ldots,a_m\} \setminus \cup_{l=1}^{k-1}\mathcal{R}_l\}$$

(4.2.2) System-suggested Criteria Weights

The second model developed and implemented in 'Triple C' allows the decision-maker to manipulate a given ranking r by replacing step by step the position of different alternatives according to his subjective view.
As a response to each of these replacements the system will generate a suggestion for updating the set of criteria weights:

$$\mathbf{w}^{new} = f^{[2]}(\mathbf{w}^{old}, \mathbf{S}, \mathbf{x}^*, \mathbf{r}^{new}, \mathbf{r}^{old})$$

As the new ranking r^{new} always results from the replacement of a single alternative a_i. in the original ranking r^{old}, the problem of generating a new set of criteria weights $\mathbf{w}^{new}$ can be stated as follows:

Given an alternative $a_{i^\bullet}$ with the original position p_1 $(a_{i^\bullet} \in \mathcal{R}_{p_1}^{old} = r^{old}(p_1))$ and given a new, different position p_2 $(a_{i^\bullet} \in \mathcal{R}_{p_2}^{new} = r^{new}(p_2))$, find a new set of weights

$$w^{new} = (w_1^{new}, w_2, \dots, w_n^{new}),\ w_j^{new} \in \mathbf{R}, \sum_{j=1}^{n} w_j^{new} = 1$$

so that the new ranking $r^{new} = f^{[1]}(w^{new}, S, x^*)$ reflects the change ($p_1 \rightarrow p_2$) for alternative $a_{i^\bullet}$ and maintains unchanged the relation between the other alternatives in the original ranking r^{old}.

The first step in solving the above problem consists in computing the new ranking r^{new}. This operation can be performed in the following two steps of deletion (of the alternative in the old position p_1) and of reinsertion (of the moved alternative in the new position p_2):

Let k be defined as: $k := \min\{j \mid \mathcal{R}_j^{old} \neq \Omega\}$

deletion step

For $j = 1, \dots, p_1-1$ $\qquad \mathcal{R}_j^{del} := \mathcal{R}_j^{old}$

For $j = p_1-1, \dots, k$ $\qquad$ IF $(\mathcal{R}_j^{old} \setminus \{a_{i^\bullet}\} = \Omega)$ THEN $\mathcal{R}_j^{del} := \mathcal{R}_{j+1}^{old}, \quad j = p_1, \dots, k-1$

$\mathcal{R}_k^{del} := \Omega$

ELSE $\qquad \mathcal{R}_{p_1}^{del} := \mathcal{R}_{p_1}^{old} \setminus \{a_{i^\bullet}\}$

$\mathcal{R}_j^{del} := \mathcal{R}_j^{old}, \quad j = p_1+1, \dots, k$

reinsertion step

For $j = 1, \dots, p_2-1$ $\qquad \mathcal{R}_j^{new} := \mathcal{R}_j^{del}$

For $j = p_2$ $\qquad \mathcal{R}_{p_2}^{new} := \{a_{i^\bullet}\}$

For $j = p_2+1, \dots, k+1$ $\qquad \mathcal{R}_j^{new} := \mathcal{R}_{j-1}^{del}$

Given the new ranking r^{new}, the second step consists in identifying a new set of weights $\mathbf{w}^{new}$ which generates this modified ranking.
According to the description of part 4.2.1, such a set of weights must fulfill the conditions

$$r(k) = \mathcal{R}_k := \{a_i \mid \mathcal{L}(a_i) \leq \mathcal{L}(a_j),\ a_j \in \{a_1, \dots, a_m\} \setminus \cup_{l=1}^{k-1} \mathcal{R}_l\} \quad k = 1..m$$

Defining k_{max} through $k_{max} := \min\{j \mid \mathcal{R}_j^{new} \neq \Omega\}$ these conditions can be reduced to a system of m-1 constraints, more precisely:

$$\sum_{j=1}^{k_{max}} \left(|\mathcal{R}_j^{new}| - 1\right) \text{ equations and } (k_{max} - 1) \text{ inequalities}$$

Note that $\sum_{j=1}^{k_{max}} \left(|\mathcal{R}_j^{new}| - 1\right) + (k_{max} - 1) = \sum_{j=1}^{k_{max}} |\mathcal{R}_j^{new}| - k_{max} + k_{max} - 1 = m - 1.$

Equations:
For each $\mathfrak{R}_j^{new}$ with $|\mathfrak{R}_j^{new}| > 1$, the ranking condition within each equivalence class can be expressed through the $(|\mathfrak{R}_j^{new}| - 1)$ equations

$$\boxed{\mathcal{L}(a_1^R) = \mathcal{L}(a_j),\ \ a_j \in \mathfrak{R}_j^{new} \setminus \{a_1^R\},}$$

where $a_1^R \in \mathfrak{R}_j^{new}$ is a fixed representative of the equivalence class.

Inequalities:
Additional $(k_{max} - 1)$ inequalities are needed for expressing the ranking condition between the different equivalence classes:

$$\boxed{\begin{array}{c}\mathcal{L}(a_i) < \mathcal{L}(a_j),\ \ a_i \in \mathfrak{R}_t^{new},\ \ a_j \in \mathfrak{R}_{t+1}^{new} \\ t = 1, \ldots, k_{max} - 1\end{array}}$$

Applying the definitions of section 4.2.1, the m-1 conditions can be expressed in function of $\mathbf{w}^{new}$ as follows:

$$\begin{aligned}\mathcal{L}(a_i) \ \substack{<\\=}\ \mathcal{L}(a_j) &\Leftrightarrow \sum_{s=1}^{n} w_s^{new}\, l_{is} \ \substack{<\\=}\ \sum_{s=1}^{n} w_s^{new}\, l_{js} \\ &\Leftrightarrow \sum_{s=1}^{n} w_s^{new}\,(l_{is} - l_{js}) \ \substack{<\\=}\ 0 \\ &\Leftrightarrow \sum_{s=1}^{n} w_s^{new}\left(|x_{is} - x_s^*| - |x_{js} - x_s^*|\right) \ \substack{<\\=}\ 0\end{aligned}$$

As a result, adding the constraint $\sum_{s=1}^{n} w_s^{new} = 1$, we obtain a system of m linear constraints in $\mathbf{w}^{new}$, whose solution corresponds to a weighting as formulated in the problem statement at the beginning of this section.
The new criteria weights suggested by "Triple C" result from solving the optimization problem:

$$\boxed{\begin{array}{c}\mathbf{A}\ \mathbf{w}^{new} \leq \mathbf{b} \\ w_s^{new} \geq 0, \quad s=1,\ldots,n \\ \sum_{s=1}^{n} (w_s^{new} - w_s^{old})^2 = \min.\end{array}}$$

where the first line represents the m linear constraints described below.

The solution of this quadratic program guarantees - through the choice of the objective function - that if a solution exists, then the system will suggest the one whose "distance" from the previous weighting of the criteria will be as small as possible.
In this way, the tool described in this section helps the user to appreciate more easily the impact resulting from a change in the alternative ranking and hence contributes to the main goal of "Triple C", i.e. to support the decision-maker's incremental learning process.

(5) Conclusions

The visual interactive approach underlying "Triple C", as well as the flexible analysis tools described in part 3 and 4, contribute in supporting the user to easily gain insights into a decision situation. With the resulting system decision making is transformed into an incremental learning process. Step by step, decision makers can define their problems, compare and analyze alternatives, and communicate their results using the visual representations presented in this paper.

This corresponds to the basic idea of "humanized" systems, whose aim is not to substitute the decision makers by computing "optimal" solutions (a pure illusion in the domain of human multicriteria decision making), but to accompany them through an individual process towards the identification of decisions they can feel comfortable and responsible with, and that they can easily justify and explain.

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