

**"ECONOMIC MODELS FOR VENDOR EVALUATION  
WITH QUALITY COST ANALYSIS"**

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# ECONOMIC MODELS FOR VENDOR EVALUATION

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## **ABSTRACT**

Successful vendor-vendee partnership is viewed as an important ingredient for maintaining competitiveness in the current market place. This calls for a careful and comprehensive approach in selecting vendors. The cost of quality (or better phrased as the cost of "unquality") resulted from imperfections of a vendor's incoming input materials is one component of the total costs in the evaluation of vendors. In this paper, we explore the relationship between the vendor's quality cost, the vendor's input quality, and the imperfections of the manufacturing process. We analyze the properties of the resulting quality cost model, and draw managerial implications in the selection of vendors.

# ECONOMIC MODELS FOR VENDOR EVALUATION

## WITH QUALITY COST ANALYSIS

### 1. INTRODUCTION

Selection of vendors and the monitoring of vendor performance are becoming increasingly major challenges facing manufacturing managers. This issue is particularly important in light of the current trend of manufacturing practices, which emphasize concepts like just-in-time and long term strategic alliances/partnerships of vendors and vendees (see, for example, Hayes and Wheelwright, 1984). Activities such as vendor certification programs, vendor visits, supplier/customer data integration through electronic data interchanges, supplier education programs, participation by suppliers in product development, and other exchange programs or projects can also be commonly observed. Companies are increasingly dependent on suppliers for both the quality and timeliness of delivery of their products, in order to maintain their own manufacturing competitiveness. Suppliers are viewed as the "extended factory" (Suzaki, 1987). The manufacturing efficiency and resulting product quality of a manufacturing enterprise depend heavily on the quality of the incoming materials. Hence, vendor selection criteria should, besides costs and delivery performance, include the quality of the incoming materials (see ASQC, 1981).

Traditional wisdom says that a vendor who can quote a lower price but has less reliable quality products could be inferior to a vendor who charges more but has highly reliable quality products (Mihalasky, 1982). The question, of course, is how to perform the economic tradeoffs of these two possibly opposing dimensions. In this paper, we consider the problem of assessing the "quality" costs of vendors. These quality costs must be based on the economic impact of the "unquality" of the vendor's product on the manufacturing enterprise itself. Such an approach provides a more comprehensive way of evaluating vendors. The concept of quality costs, of course, is not new (see

Juran, 1951, for example). Nevertheless, research works that incorporate such quality costs explicitly in vendor selection were only emerging (e.g., Bisgaard et al., 1984, Pettit, 1984, and Tang, 1988). Tang's (1988) model represents the most recent model that bears some similarity to our current work. His model considers the interaction of the vendor's input quality and the vendee's quality control (incoming inspection) system. However, his model did not include the imperfections of the manufacturing enterprise's internal manufacturing process. In our work here, we found that the interactions of the imperfections of incoming materials and the manufacturing process can have a profound impact on how the quality costs of the incoming materials are assessed. Specifically, the same vendor with the same quality of incoming materials can result in different quality costs to different manufacturing enterprises with different degrees of imperfections in the manufacturing processes. In fact, the interaction of all three major factors, vendor's input quality, vendee's quality control system, and the reliability of the vendee's manufacturing process, need to be included in evaluating vendors.

In what follows, we will concentrate on measuring the quality of incoming materials by means of the percentage of items that conform to some given standards. The paper is organized in the following way. In the next section, we present a simple model that shows how the two types of imperfections (materials from vendor and manufacturing process) interact, and draw managerial insights from the model. Section 3 extends the simple model to assess vendor costs as a combination of acquisition costs and quality costs. Section 4 incorporates the effect of incoming inspection. Section 5 considers the case when the reliability of the manufacturing process depends on the input quality. We conclude the paper with a summary of results in Section 6.

## **2. A SIMPLE MODEL**

Consider a manufacturing enterprise that receives a single type of input materials from a particular vendor. The manufacturing process makes use of these input materials, and transforms

them into final products. Incoming materials and the manufacturing process may not be perfect. A simple way to characterize the imperfections of the manufacturing process is to assume that there are two states of the process (e.g., see Porteus, 1986 and Lee, 1992). In one state, the output of the process would be conforming if the input is. In another state, the process would be non-conforming (or, simply, defective) regardless of whether the input is or is not. Hence, we assume that the final product would be non-conforming if either the input material is defective, or the manufacturing process is imperfect, or both.

We will use the following notation:

- $p$  = probability that a unit of incoming material is defective;
- $q$  = probability that the manufacturing process is in a state to produce defective output;
- $r_1$  = unit cost of a defective output if the defect was due to defective input while the manufacturing process was perfect;
- $r_2$  = unit cost of a defective output if the defect was due to an imperfect manufacturing process while the input was nondefective;
- $r_{12}$  = unit cost of a defective output if the defect was produced while both the input and the manufacturing process were imperfect.

In this simple model, we assume the following:

1.  $p$  and  $q$  are independent.
2. All costs are linear.
3. No incoming inspection is performed.

In later sections, the above assumptions will be relaxed. Note that we also implicitly assume that the cost of a defective output can be measured. This cost can be in the form of rework, scrap, and external failure costs, etc. It is reasonable to assume that  $r_{12} \geq r_1$  and  $r_{12} \geq r_2$ . Our analysis focuses on the expected cost incurred for every unit processed by the manufacturing process.

The respective probabilities for the incurrence of costs  $r_1$ ,  $r_2$ , and  $r_{12}$  are  $p(1-q)$ ,  $q(1-p)$ , and  $pq$ . Hence, for a given  $p$ , the expected failure cost per unit processed is:

$$\phi(p) = p(1-q)r_1 + q(1-p)r_2 + pq r_{12}$$

If the input material has been perfect, then the expected failure cost per unit would have been  $\phi(0) = qr_2$ . Consequently, one can view  $Q(p) = \phi(p) - \phi(0)$  as the "incremental" quality cost due

to the imperfections of the vendor's input materials. We shall henceforth term  $Q(p)$  as the average cost of vendor quality per unit. Note that:

$$Q(p) = p[r_1 + q(r_{12} - r_1 - r_2)]. \quad (1)$$

Given that  $r_{12} \geq r_2$ , we first observe that  $Q(p)$  is increasing in  $p$ . As expected, the quality cost of a vendor increases as the vendor's input quality deteriorates.

Consider the simplest case of  $r_1 = r_2 = r_{12} = r$ , i.e., the failure cost is independent of the source of imperfections that lead to a defective product. Then  $Q(p) = p(1-q)r$ . Some interesting managerial implications can be drawn. For a given  $p$ ,  $Q(p)$  is linearly decreasing in  $q$ . Hence, if the manufacturing process is perfect ( $q=0$ ), then the vendor's quality cost is  $pr$ . If the manufacturing process is completely imperfect ( $q=1$ ), then the vendor's quality cost is 0! In general, as the manufacturing process deteriorates in reliability, i.e., higher value of  $q$ , the vendor's quality cost decreases. When the manufacturing process is less reliable, then the cost consequence of the vendor's imperfections is smaller in magnitude. The implication is that when a manufacturer's own process is not well maintained (or, in statistical process control terms, always out of control), then it does not help as much to work on the vendor to improve its quality. One needs to clean up one's own household before asking one's vendor to do so.

It is easy to see that the above qualitative results hold when  $r_{12} < r_1 + r_2$ . When  $r_{12} = r_1 + r_2$ , then  $Q(p) = pr$ , and is thus independent of the reliability of the manufacturing process. One interpretation of  $r_{12} = r_1 + r_2$  is that the cost consequences of the two sources of defect are completely separable and additive. For example, if the failure costs represent costs of rework, then the two types of imperfections can be isolated and separate reworks need be applied to each. When these two costs are completely separable, then obviously the vendor's quality cost is independent of the imperfections of the manufacturing process.

Finally, when  $r_{12} > r_1 + r_2$ , then there exist interactive effects of the two sources of imperfections so that the damages compound one another. In this case, the vendor's quality cost would be an increasing function of  $q$ . This time, the "poorer" is the manufacturing process, the higher is the quality cost of the vendor.

The above model, although simple, illustrates the interactive effects of the two sources of imperfection on the assessment of the vendor's quality cost. It also shows the importance of

incorporating these two sources in models that evaluate the total costs of vendors.

### 3. TOTAL ACQUISITION AND QUALITY COSTS

The eventual decision on vendor selection should be based on economic grounds. Hence, in order to evaluate a particular vendor, both the purchase and quality costs must be taken into account. In general, it is reasonable to assume that the unit purchase cost depends on the average product quality, i.e., on the probability  $p$  that a random unit purchased from the vendor is defective. Then, the total acquisition and quality costs that are *relevant* to the vendor evaluation can be written as:

$$K(p) = C(p) + Q(p)$$

$C(p)$  is the unit purchase cost,  $Q(p)$  is the average cost of vendor quality per unit, which was computed in the previous section, and  $K(p)$  is the total relevant cost per unit, excluding the constant  $r_2q$  from  $Q(p)$  as has already been explained. While  $Q(p)$  is increasing in  $p$ , the purchase cost  $C(p)$  is expected to be decreasing (or, at least, non-increasing) in  $p$ , since a lower quality (high  $p$ ) product is usually less expensive (low  $C(p)$ ).

If  $n$  vendors are available with respective quality characteristic  $p_i$ ,  $i = 1, \dots, n$ , then  $K(p_i)$  should be evaluated for each vendor and the vendor with the lowest  $K(p_i)$  should be chosen as the most preferred supplier.

If the relationship between  $p$  and  $C(p)$  can be expressed in a functional form so that  $C(p)$  becomes a continuous function of  $p$ , then in theory the best vendor should be chosen as the one whose  $p$  minimizes  $K(p)$ . Analytically, the optimal  $p$  (vendor) will satisfy the first and second order conditions

$$\frac{dK(p)}{dp} = 0 \quad \text{and} \quad \frac{d^2K(p)}{dp^2} \geq 0.$$

Although such analysis is based on the assumption that the optimal  $p$  can be chosen from a continuum of values in the interval  $[0,1]$ , which is almost never the case in practice, examination of

the mathematical properties of the analytically derived solution provides useful managerial insights. The following examples illustrate the use of mathematical analysis in the simple cases of unit purchase costs that are linear and quadratic functions of the nonconformance probability  $p$ .

### Example 1: Linear acquisition cost

Let  $c$  be the unit cost of guaranteed conforming product ( $p = 0$ ). Suppose that the unit purchase cost is  $C(p) = c(1 - p)$ , for  $p \in [p_l, p_u]$ ,  $0 \leq p_l \leq p_u \leq 1$ .  $p_l$  and  $p_u$  are the effective lower and upper bounds on  $p$ , respectively. Note that if  $C(p=1) = c_1 > 0$ , then we can write

$$C(p) = (c - c_1)(1 - p) + c_1$$

and the following analysis will remain unchanged.

The total acquisition and quality cost is given by

$$K(p) = c(1 - p) + p[r_1 + q(r_{12} - r_1 - r_2)].$$

The first derivative of the total cost is

$$\frac{dK(p)}{dp} = -c + [r_1 + q(r_{12} - r_1 - r_2)], \text{ which is independent of } p.$$

If  $c < r_1 + q(r_{12} - r_1 - r_2)$ , then  $\frac{dK(p)}{dp} > 0$  and the optimal  $p$  is  $p^* = p_l$ .

If  $c > r_1 + q(r_{12} - r_1 - r_2)$ , then  $\frac{dK(p)}{dp} < 0$  and the optimal  $p$  is  $p^* = p_u$ .

If  $c = r_1 + q(r_{12} - r_1 - r_2)$ , then  $\frac{dK(p)}{dp} = 0$  and any  $p \in [p_l, p_u]$  is optimal.

Thus, when the acquisition cost decreases linearly with  $p$ , the optimal vendor quality is either the best possible, or the worst possible (but least expensive) depending on the relationship between  $c$ , the costs of defects and the probability  $q$  of a nonconformance introduced by the process. If the two defect sources are completely separable and independent ( $r_{12} = r_1 + r_2$ ), then the choice of vendor quality depends simply on the relationship between  $c$  and  $r_1$ ; if the purchase cost  $c$  of perfect inputs to the process is lower than the cost of a defective input then the best possible quality is preferred; otherwise, the least expensive-lowest quality vendor should be selected, because the

savings in purchase cost,  $cp$ , always exceed the expected quality cost,  $r_1p$ .

### Example 2: Quadratic acquisition cost

Again, if  $c$  is the unit cost of guaranteed conforming product ( $p = 0$ ) and  $p_l, p_u$  are the effective lower and upper bounds on possible values of  $p$ , the unit purchase cost is

$$C(p) = c(1 - p)^2, \text{ for } p \in [p_l, p_u], \quad 0 \leq p_l \leq p_u \leq 1.$$

This case, characterized by substantial increases in the price discount as the quality deteriorates, is more common in practice, especially in the present time of increased quality awareness.

The total acquisition and quality cost is

$$K(p) = c(1 - p)^2 + p[r_1 + q(r_{12} - r_1 - r_2)]$$

Differentiating  $K(p)$  with respect to  $p$  yields

$$\frac{dK(p)}{dp} = -2c(1 - p) + r_1 + q(r_{12} - r_1 - r_2)$$

Since  $d^2K(p) / dp^2 = 2c > 0$ , the optimal  $p$  is the solution to  $dK(p) / dp = 0$ , namely,

$$p_o = 1 - [r_1 + q(r_{12} - r_1 - r_2)] / 2c.$$

If  $p_o \in [p_l, p_u]$ , then  $p^* = p_o$ ; if  $p_o < p_l$  or  $p_o > p_u$ , the optimal vendor quality is  $p^* = p_l$  or  $p^* = p_u$  respectively. If the two defects sources are independent ( $r_{12} = r_1 + r_2$ ),  $p_o$  is reduced to  $p_o = 1 - r_1/2c$ .

The most important implication in the above analysis is common to both Examples 1 and 2 and applies to the situation where the two defect sources (vendor and internal process) are not independent. Then, there are two possibilities:

a)  $r_{12} < r_1 + r_2$

The optimal  $p$  is increasing in  $q$ . Hence, if the internal process is reliable (low  $q$ ) the vendor should also provide high quality (low  $p$ ) inputs to the process, because input failure is costly ( $r_1 > r_{12} - r_2$ ). However, if the internal operation has a high defective rate  $q$ , it is not worthwhile to choose a high quality vendor. Instead, an inexpensive vendor should be chosen as the

savings in the purchase cost will generally offset the additional quality cost, since there are economies of scale in simultaneously correcting both defects ( $r_{12}$ ).

$$b) \quad r_{12} > r_1 + r_2$$

The optimal  $p$  is decreasing in  $q$ . Thus, if the internal operation is of high quality (low  $q$ ), a low-cost vendor should be chosen, because there is only a small likelihood that the high failure cost for both defects will be incurred. If the internal operation has a high defective rate  $q$ , then a high quality (albeit expensive) supplier should be selected, in order to avoid incurring the high cost  $r_{12}$ , characteristic of compounding defect sources.

Thus, the conclusion of the simple model of Section 2 is confirmed by the above analysis : in addition to vendor's prices and quality, the economic attractiveness of a vendor also depends on the quality and cost characteristics of the purchaser's own operations.

#### 4. INCORPORATING THE INPUT INSPECTION DECISION IN VENDOR EVALUATION

A manufacturing operation can be buffered against poor input quality through the application of inspection of incoming materials. The option to inspect the vendor's products provides an additional degree of freedom in the decision process of vendor selection. Therefore, when such an inspection operation is possible, it should be considered and analyzed as another alternative.

Let  $a$  be the unit inspection cost for an item externally purchased, that will be the input to the production process. Assume that defective inputs can be repaired at a cost  $r_1$ . Assume also that both inspection and repair are always perfect. If all incoming units are inspected, then the expected quality cost for a conforming unit is  $a + r_2q$ . If the incoming unit is nonconforming, the expected quality cost is  $a + r_1 + r_2q$ , since the defective unit will be identified and repaired at the inspection station with probability 1. Therefore, the total expected quality assurance cost is

$$(a + r_2q)(1 - p) + (a + r_1 + r_2q)p = a + r_1p + r_2q.$$

If the vendor's quality is always perfect ( $p = 0$ ), then no inspection is needed and the expected

quality assurance cost is  $r_2q$ . Thus, the average cost of vendor quality per unit is

$$Q_i(p) = a + r_1p \quad (2)$$

and the total cost becomes  $K_i(p) = C(p) + Q_i(p)$ .

If no inspection is performed, the respective vendor quality cost has already been computed as

$$Q(p) = r_1p + pq(r_{12} - r_1 - r_2).$$

Note that we assume that it also costs  $r_1$  to fix a defective product that was attributed to a defective input. While the failure costs could be general, for the sake of clarity of exposition in this and the following section we are dealing with repair costs for nonconforming items (inputs or outputs), as in the case of output screening.

The following lemma shows that partial inspection-correction never dominates both extreme policies of 100% inspection or zero inspection.

**LEMMA:** *The optimal input inspection policy is either to perform 100% inspection or not to inspect at all.*

**PROOF:** Let  $\beta$  be the probability that an incoming unit is inspected, i.e.,  $\beta$  is the fraction of units inspected in a partial inspection-correction scheme. The total expected cost of quality assurance is

$$\begin{aligned} QA(\beta) &= \beta(a + r_1p) + (1 - \beta)[r_1p + pq(r_{12} - r_1 - r_2)] \\ &= \beta a + r_1p + (1 - \beta)pq(r_{12} - r_1 - r_2). \end{aligned}$$

The partial derivative of  $QA(\beta)$  with respect to  $\beta$  is

$$\frac{QA(\beta)}{\beta} = a - pq(r_{12} - r_1 - r_2).$$

If  $a < pq(r_{12} - r_1 - r_2)$  then  $\beta^* = 1$  (100 % inspection).

If  $a > pq(r_{12} - r_1 - r_2)$  then  $\beta^* = 0$  (no inspection).

If  $a = pq(r_{12} - r_1 - r_2)$  then any  $\beta$  in  $[0,1]$  is optimal. ■

A comparison between  $Q_i(p)$  and  $Q(p)$  verifies that inspection is preferred if and only if  $a < pq(r_{12} - r_1 - r_2)$ . Inspection is obviously unprofitable if  $r_{12} \leq r_1 + r_2$ . In fact, inspection should only be performed if the defect sources are compounding so that  $pq[r_{12} - (r_1 + r_2)] > a$ .

The last inequality has a very intuitive interpretation. Inspection should be performed if the additional cost of repairing both defects  $r_{12} - (r_1 + r_2)$  multiplied by the probability that an item will have both types of nonconformances, exceeds the unit inspection cost.

Vendor evaluation is again based on the total cost  $K_t(p) = C(p) + Q_t(p)$ , where  $Q_t(p)$  and  $K_t(p)$  now denote the quality and the total cost respectively, generalizing  $Q(p)$ ,  $Q_i(p)$  and  $K(p)$ ,  $K_i(p)$ . If  $r_{12} \leq r_1 + r_2$ , then no inspection should be performed and the vendor is evaluated according to the analysis of the previous section. For example, in the case of two vendors with  $p_1 > p_2$  and  $C(p_1) < C(p_2)$ , either the lower quality - lower cost vendor 1 is dominating with lower total cost for any value of  $q$  (when  $C(p_2) - C(p_1) \geq r_1(p_1 - p_2)$ ), or, if  $C(p_2) - C(p_1) < r_1(p_1 - p_2)$  and  $r_{12} < r_1 + r_2$ , then

$$K_t(p_2) \leq K_t(p_1) \text{ for } q \leq \min \left\{ \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)]}{(p_2 - p_1)(r_{12} - r_1 - r_2)}, 1 \right\}$$

$$K_t(p_1) < K_t(p_2) \text{ for } q > \min \left\{ \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)]}{(p_2 - p_1)(r_{12} - r_1 - r_2)}, 1 \right\}.$$

Thus, when the expected savings in input repair  $r_1(p_1 - p_2)$  exceed the price differential  $C(p_2) - C(p_1)$ , the higher quality - higher cost vendor 2 is overall more economical when the purchaser's process quality is very high, because of the relatively expensive repair of incoming nonconforming items ( $r_1 > r_{12} - r_2$ ). When the internal process quality deteriorates, then the lower quality - lower cost vendor 1 becomes preferable since repair of both types of nonconformances is relatively inexpensive ( $r_{12} < r_1 + r_2$ ). This example again shows that when assessing a vendor in terms of acquisition and quality costs, one must take into account one's own internal manufacturing reliability.

If  $r_{12} > r_1 + r_2$  inspection becomes an option and the quality cost  $Q_t(p)$  depends on whether or not inspection is performed. In the remainder of this section, it is assumed that  $r_{12} > r_1 + r_2$  and that any  $p$  in the interval  $[0,1]$  is possible. The latter assumption can be easily relaxed, but it is retained here for clarity of exposition.

Since inspection is preferred for  $p > a / [q(r_{12} - r_1 - r_2)]$ , the quality cost can be written as

$$Q_t(p) = \begin{cases} Q(p) = p[r_1 + q(r_{12} - r_1 - r_2)] & \text{for } p \leq p' \\ Q_i(p) = a + r_1 p & \text{for } p > p' \end{cases}$$

where

$$p' = \min \{ a / [q(r_{12} - r_1 - r_2)], 1 \} .$$

The total cost  $K_t(p)$ , the unit purchase cost  $C(p)$  and the quality cost  $Q_t(p)$  are shown in Figure 1 as functions of the vendor quality.

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Figure 1 around here

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It is clear that the total cost  $K_t(p)$  will in general have two local minima (unless  $p' = 1$ ) for a smooth strictly convex decreasing function  $C(p)$ . (If  $C(p)$  is linearly decreasing the optimal  $p$  will be either 0 or 1). The global optimum is the  $p$  that yields the minimum total cost  $K_t(p)$ , between the two local minima with and without inspection. It can be seen in Figure 1 that the local optimum without inspection is smaller than the local optimum with inspection, when  $C(p)$  is decreasing and strictly convex in  $p$ . Analytically, without inspection the local optimum satisfies

$$\frac{dK(p)}{dp} = \frac{dC(p)}{dp} + r_1 + q(r_{12} - r_1 - r_2) = 0$$

while with inspection

$$\frac{dK_i(p)}{dp} = \frac{dC(p)}{dp} + r_1 = 0$$

hence for any  $p \in [0,1]$ ,  $dK(p)/dp > dK_i(p)/dp$ . Since  $dC^2(p)/dp^2 > 0$ ,  $dC(p)/dp$  is increasing in  $p$ , therefore  $dK(p)/dp$  is also increasing in  $p$ , and

$$\arg \left\{ \frac{dK(p)}{dp} = 0 \right\} < \arg \left\{ \frac{dK_i(p)}{dp} = 0 \right\}.$$

The intuitive justification behind the above result is that better input quality (lower  $p$ ) is required when no input inspection is performed. This corroborates with the result of Rosenblatt et al., 1991, which shows that the process capability of a manufacturing operation is higher when no final inspection is performed.

Of particular significance is the sensitivity of the total acquisition and quality cost to the quality of the purchaser's operation. Figure 2 depicts the total cost as a function of  $q$  for two

vendors, one with high probability of nonconformance  $p_1$  but relatively inexpensive, and another one with better quality ( $p_2 < p_1$ ) but more expensive in terms of unit purchase cost:  $C(p_2) > C(p_1)$ . For each vendor  $i$ ,  $i = 1, 2$ , inspection is preferred if the internal process nonconformance rate  $q$  is higher than a certain level  $q_i' = \min \{ a / p_i (r_{12} - r_1 - r_2), 1 \}$ ,  $i = 1, 2$ . Note that  $q_1' < q_2'$  because of  $p_1 > p_2$ .

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Figure 2 around here

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The slope of  $K_i(p_1)$  in  $[0, q_1']$  is  $p_1(r_{12} - r_1 - r_2)$ , hence it is larger than the slope of  $K_i(p_2)$  in  $[0, q_2']$ , which is  $p_2(r_{12} - r_1 - r_2)$ . The slopes of  $K_i(p_1)$  and  $K_i(p_2)$  for  $q > q_1'$  and  $q > q_2'$  respectively are zero, because in that range inspection is performed, which "insulates" the process from incoming nonconforming items. The quality cost therefore is  $a + r_1 p_i$  in that range, independent of  $q$ . If the process quality is bad enough that inspection is required, the quality cost of the vendor is unaffected by the process quality, no matter how worse the latter becomes.

As Figure 2 indicates there are three possibilities depending on the cost and quality characteristics of the producer's process and the vendors. The following theorem analyzes those possibilities and provides a general result:

**THEOREM:** *If  $r_{12} > r_1 + r_2$ ,  $p_1 > p_2$  and  $C(p_1) < C(p_2)$ , there exist process nonconformance rates  $q_1$  and  $q_2$ ,  $0 \leq q_1 \leq q_2 \leq 1$  such that if  $q < q_1$  ( $q \leq q_1$  if  $q_1 = 1$ ) vendor 1 is overall more economical, if  $q_1 \leq q \leq q_2$  and  $q_1 \neq 1$  vendor 2 is overall more economical, and if  $q > q_2$  vendor 1 is again overall more economical.*

**PROOF:** *See Appendix, where the values of  $q_1$  and  $q_2$  are also provided as functions of  $p_1, p_2, C(p_1), C(p_2), r_{12}, r_1, r_2$ .* ■

The theorem states that in general the lower quality - lower cost vendor should be preferred if the purchaser's process quality is either very good ( $q < q_1$ ) or very poor ( $q > q_2$ ). If the former is

true, there is no need for inspection and the low purchase cost is the decisive factor; the good process quality will compensate for the poor input quality as the overall likelihood of a defect will still be very small. If the latter is true, inspection of the input will be necessary, especially for low quality inputs; thus the high process defective rate  $q$  will not affect the supplier's quality cost and vendor selection will be once again based on purchase cost.

In special cases one of the two vendors may be dominating over the whole range of purchaser's process quality. This would happen if the unit purchase cost for the lower quality vendor is relatively very low and the repair cost of incoming nonconforming items and unit inspection cost are also small; then that vendor is preferable on economic grounds. On the other hand, if there is a small difference between unit purchase costs, but relatively big difference in the input quality and if the unit repair cost of nonconforming inputs is high, the high quality vendor is superior no matter what the producer's process quality is.

We have thus repeatedly observed that vendor evaluation should take account of the quality of the purchaser's manufacturing operation. As Figure 2 attests, in all cases the lower envelope of the total supplier's cost is concave in  $q$ , because it is the minimum of concave functions. The significant implication is that the producer gets a bigger gain from the supplier if the producer's process quality is high. Double savings in costs are incurred when  $q$  is very small: inspection costs are avoided since inspection is not needed, and purchase costs can be reduced because it is not necessary to acquire process inputs of extremely high quality.

It is not to be concluded, though, that the producer should always prefer to have a perfect internal production operation, if it means to invest exorbitant amounts to improve the existing operation. If the choice of optimal process quality and vendor selection were to be made simultaneously, the cost of process quality should also be taken into account, so that the total supplier and process quality cost be minimized. Given that the additional marginal investment needed to improve the process quality generally increases as the process quality approaches perfection, it is not clear what the jointly optimal decision should be. The optimal combination of vendor and process quality can however be easily obtained through the use of standard analytical techniques (e.g. setting the partial derivatives of the total cost with respect to  $p$  and  $q$  equal to zero, and solving for the optimal  $p^*$ ,  $q^*$ ).

## 5. DEPENDENCE OF PROCESS QUALITY ON INPUT QUALITY

In some situations the reliability of a manufacturing operation is not independent of the quality of the input to that operation, but instead it deteriorates when the incoming material is nonconforming. Formally, if  $q$  is the nonconformance rate at the operation stage when the input is of acceptable quality, then a nonconforming input to the manufacturing operation will lead to an additional nonconformance at the operation stage with higher probability  $q_d > q$ . Thus, if no inspection of incoming materials is performed and the process input is conforming to the specifications ( $p=0$ ), the expected failure cost is  $r_2q$ ; if however the incoming item is defective with probability  $p$ , the expected failure cost is

$$r_2q(1-p) + [r_1(1-q_d) + r_{12}q_d]p.$$

Consequently, in the case of no inspection of incoming materials, the expected quality cost of a supplier characterized by a nonconformance rate  $p$  becomes:

$$\begin{aligned} Q^d(p) &= r_2q(1-p) + [r_1(1-q_d) + r_{12}q_d]p - r_2q \\ &= p[r_1 + q(r_{12} - r_1 - r_2) + (q_d - q)(r_{12} - r_1)] \\ &= Q(p) + p(q_d - q)(r_{12} - r_1), \end{aligned} \quad (3)$$

where  $Q(p)$  is the quality cost for  $q_d = q$  from (1).

From above it is concluded that there is an additional quality cost if the process quality depends on the input quality, unless  $r_{12} = r_1$ . This additional quality cost is proportional to the difference  $r_{12} - r_1$  in failure costs, to the input nonconformance rate  $p$  and to the increase  $q_d - q$  in process nonconformance rate because of defective input.

If the incoming product is inspected, the expected quality cost of the vendor is still  $a + r_1p$ , unaffected by and independent of the increased probability  $q_d > q$ , because inspection insulates the internal operation from nonconforming units.

It can be easily seen that inspection now will be economically justified if

$$a < p[q(r_{12} - r_1 - r_2) + (q_d - q)(r_{12} - r_1)]$$

or for  $p$  such that

$$p > a / [q(r_{12} - r_1 - r_2) + (q_d - q)(r_{12} - r_1)] \equiv p_d'$$

provided that the denominator in the calculation of  $p_d'$  is positive.

Comparison with the case of independent probabilities  $p$  and  $q = q_d$ , reveals that when  $q_d > q$  inspection is even more attractive in the sense that it is preferable for lower  $p$ , as should be expected. Contrary to the findings for  $q = q_d$ , inspection may now be more economical even when  $r_1 < r_{12} < r_1 + r_2$ , especially when either  $q_d - q$  or  $r_{12} - r_1$  or both are large, and  $q$  is small. This is explained by the relatively large savings in repair costs of additional nonconforming inputs that would acquire both types of nonconformances because of the deterioration in the process quality. These expected savings, which can be expressed as  $p(q_d - q)(r_{12} - r_1)$  per unit can now exceed the additional expected cost of repairing a nonconforming unit separately for both defects, which is  $pq[(r_1 + r_2) - r_{12}]$  per unit, by more than the inspection cost  $a$ , so that inspection of incoming items becomes a less costly alternative.

Vendor selection will now depend (among other parameters) not only on the quality of the process, but also on the effect of a nonconforming input on the process quality, i.e., the deterioration of the process quality as measured by  $q_d - q$ . Let

$$q_d - q = \alpha(1-q) \quad , \quad 0 < \alpha \leq 1$$

where  $\alpha$  is interpreted as an index of the process deterioration because of a nonconforming process input.  $\alpha$  is the process deterioration as a fraction of the maximum possible deterioration. Using

$$q_d = q + \alpha(1-q) = \alpha + q(1-\alpha),$$

in the case of no input inspection  $Q^d(p)$  of (3) can be written as

$$\begin{aligned} Q^d(p) &= p \{r_1(1-\alpha) + \alpha r_{12} + q[(1-\alpha)r_{12} - (1-\alpha)r_1 - r_2]\} \\ &= p \{r_1(1-\alpha) + \alpha r_{12} + q[(1-\alpha)(r_{12} - r_1 - r_2) - \alpha r_2]\}. \end{aligned}$$

Again, the optimal  $p$  (vendor) will satisfy

$$\frac{dK^d(p)}{dp} = 0 \quad \text{and} \quad \frac{d^2K^d(p)}{dp^2} \geq 0$$

with

$$K^d(p) = C(p) + Q^d(p).$$

### Example 3: Quadratic acquisition cost - No input inspection

Using the same notation as in section 3, the total acquisition and quality cost is

$$K^d(p) = c(1-p)^2 + p \{r_1(1-\alpha) + \alpha r_{12} + q[(1-\alpha)(r_{12} - r_1 - r_2) - \alpha r_2]\}.$$

Differentiating  $K^d(p)$  with respect to  $p$  yields

$$\frac{dK^d(p)}{dp} = -2c(1-p) + r_1(1-\alpha) + \alpha r_{12} + q[(1-\alpha)(r_{12} - r_1 - r_2) - \alpha r_2].$$

Since  $d^2K^d(p)/dp^2 = 2c > 0$ , the optimal  $p$  is the solution to  $dK^d(p)/dp = 0$ , namely

$$p_o^d = 1 - \{r_1(1-\alpha) + \alpha r_{12} + q[(1-\alpha)(r_{12} - r_1 - r_2) - \alpha r_2]\}/2c.$$

If  $p_o^d \in [p_l, p_u]$ , then  $p^* = p_o^d$ ; if  $p_o^d < p_l$  or  $p_o^d > p_u$ , the optimal vendor quality is  $p^* = p_l$  or  $p^* = p_u$ , respectively. ■

A comparison with Example 2 in Section 3 reveals that when  $q_d > q$ , the optimal  $p$  is smaller when  $r_{12} > r_1$  (there is no difference if  $r_{12} = r_1$ ). This result, which holds for general convex decreasing functions  $C(p)$ , implies that when the process input affects process quality, there is an incentive to acquire better quality input, albeit at a higher price, in order to avoid extra repair costs. As  $p_o - p_o^d = \alpha(1-q)(r_{12} - r_1)/2c$ , the above result becomes even more pronounced when the difference  $r_{12} - r_1$  is larger,  $1-q$  increases, and/or the unit price decreases.

When inspection is introduced as an additional tool for controlling input quality and total supplier costs, the analysis with dependent probabilities parallels that of the previous section where  $p$  and  $q$  were assumed independent. Interesting insights can be derived from comparisons between the two cases. For example, suppose again that there are two candidate vendors 1 and 2 with  $p_1 > p_2$  and  $C(p_1) < C(p_2)$ . Suppose also that there is a compounding effect in the repair cost of the two sources (input and process), i.e.,  $r_{12} > r_1 + r_2$ . It can be seen that, in the general case depicted in Figure 2(c), when  $q_d > q$  the slopes of the increasing portions of  $K_t(p_1)$  and  $K_t(p_2)$  as functions of  $q$  are smaller:

$$p[(1-\alpha)(r_{12} - r_1 - r_2) - \alpha r_2] < p[r_{12} - r_1 - r_2].$$

At the same time the intercepts are larger:

$$p r_1 + p \alpha (r_{12} - r_1) > p r_1.$$

The new  $q_1', q_2'$ , denoted by  $q_{1d}', q_{2d}'$  are smaller than the respective  $q_1'$  and  $q_2'$ , because inspection is more economical when  $q_d > q$  if

$$a < p[q(r_{12} - r_1 - r_2) + \alpha(1-q)(r_{12} - r_1)]$$

or

$$q > \frac{a - p\alpha(r_{12} - r_1)}{p(r_{12} - r_1 - r_2) - p\alpha(r_{12} - r_1)} = q_d',$$

but

$$q_d' = \frac{a - p\alpha(r_{12} - r_1)}{p(r_{12} - r_1 - r_2) - p\alpha(r_{12} - r_1)} < \frac{a}{p(r_{12} - r_1 - r_2)} = q', \text{ for } q' < 1.$$

(If  $q' = 1$ , then necessarily  $q_d' \leq q'$ ).

Those changes are depicted in Figure 3, for positive slopes of  $K_t(p_1)$  and  $K_t(p_2)$ . It can be shown that a theorem similar to that of Section 4 holds, with  $q_{1d}$  and  $q_{2d}$  instead of  $q_1$  and  $q_2$  respectively, and with

$$q_{1d} \leq q_1$$

$$q_{2d} \leq q_2.$$

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Figure 3 around here

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Specifically, the following expressions for  $q_1$ ,  $q_2$  in case (c) of the Theorem of Section 4:

$$q_1 = \min \left\{ \frac{C(p_2) - C(p_1) - r_1(p_1 - p_2)}{(p_1 - p_2)(r_{12} - r_1 - r_2)}, 1 \right\}$$

and

$$q_2 = \min \left\{ \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + a}{p_2(r_{12} - r_1 - r_2)}, 1 \right\}.$$

are replaced, respectively, by

$$q_{1d} = \min \left\{ \frac{C(p_2) - C(p_1) - r_1(p_1 - p_2) - \alpha(r_{12} - r_1)(p_1 - p_2)}{(p_1 - p_2)(r_{12} - r_1 - r_2) - \alpha(r_{12} - r_1)(p_1 - p_2)}, 1 \right\}$$

and

$$q_{2d} = \min \left\{ \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + a - p_2\alpha(r_{12} - r_1)}{p_2(r_{12} - r_1 - r_2) - p_2\alpha(r_{12} - r_1)}, 1 \right\}.$$

The implication here is that when  $q_d > q$  and the process quality is good, the range of  $q$  for which the lower cost - lower quality vendor is more economical shrinks; it may be preferable to use the more expensive - higher quality vendor for even lower  $q$  than before. If the process quality is poor, however, vendor 1 becomes more attractive for  $q \in (q_{2d}, q_2)$ , because inspection of vendor's 1 input is less costly than the incremental purchase and expected repair cost for the product of vendor 2.

If  $(1 - \alpha)(r_{12} - r_1 - r_2) - \alpha r_2 < 0$ , or equivalently  $r_2 > (1 - \alpha)(r_{12} - r_1)$  the slope of  $K_t(p)$  as a function of  $q$  becomes negative. Thus, when there is a strong dependence between the process nonconformance rates of the process input and the process itself (represented by a large  $\alpha$ ), the total purchase and quality cost of a supplier becomes decreasing in  $q$ , even if there is a compounding effect on the repair cost from the two sources ( $r_{12} > r_1 + r_2$ ). As the  $q_d > q$  effect is "stronger" than the  $r_{12} > r_1 + r_2$  effect, the result is analogous to that result in the case of  $r_{12} < r_1 + r_2$ . The explanation is that as  $q$  increases there is not much room for further process deterioration (recall that  $\alpha$  is constant in this analysis) and consequently the implications from poor input quality are weakened to the extent that the *vendor's* quality cost  $Q^d(p)$  is reduced. However, after the addition of the term  $r_2 q$  the *total* quality cost will still be increasing in  $q$ , as should be expected.

## 6. CONCLUSIONS

Several economic models for the evaluation of suppliers have been developed and analyzed in this paper. Whether incoming inspection is an option or not and whether the reliability of the purchaser's manufacturing process depends on the input quality or not, a recurring result always

emerged as the major conclusion of our analysis: vendors must not be evaluated only on the basis of their own quality and prices. The economic attractiveness of a vendor also depends on the quality and cost characteristics of the purchaser's own operations.

Although the above conclusion may already be qualitatively applied in practice, the formal analysis presented in this paper contributes to a concrete understanding of the different relationships and tradeoffs involved. It also facilitates the quantification of the costs of vendor's unquality and the comparison between different suppliers on economic grounds in simple contexts. A natural extension of this research would be in the direction of modelling more complex situations that usually arise in the business world.

## APPENDIX:

## PROOF OF THEOREM IN SECTION 4

Case (a):  $r_1 \geq [C(p_2) - C(p_1)]/(p_1 - p_2)$

i.e., the unit repair cost  $r_1$  of an incoming nonconforming item exceeds the rate of increase in purchase cost when the quality improves from  $p_1$  to  $p_2$ . Then

$$C(p_2) + p_2 r_1 \leq C(p_1) + p_1 r_1$$

so that  $K_t(p_2) \leq K_t(p_1)$  at  $q = 0$ .

Also, for  $q > q_1'$

$$K_t(p_1) = C(p_1) + p_1 r_1 + a \geq \max_{q \in [0, 1]} K_t(p_2) = C(p_2) + p_2 r_1 + a.$$

Since the slope of  $K_t(p_2)$  does not exceed the slope of  $K_t(p_1)$  for  $q < q_1'$  and  $\max K_t(p_2) \leq K_t(p_1)$  for  $q > q_1'$ , it is established that  $K_t(p_2) \leq K_t(p_1)$  for all  $q \in [0, 1]$  and vendor 2 dominates vendor 1. In this case which is shown in Figure 2(a),  $q_1 = 0$  and  $q_2 = 1$ .

In Cases (b) and (c), the inequality of Case (a) is reversed:

$$r_1 < [C(p_2) - C(p_1)]/(p_1 - p_2).$$

Then

$$C(p_1) + p_1 r_1 < C(p_2) + p_2 r_1 \tag{A.1}$$

so that

$$K_t(p_1) < K_t(p_2) \text{ at } q = 0.$$

Case (b):  $C(p_1) + p_1 r_1 - C(p_2) - p_2 r_1 + q_1'(r_{12} - r_1 - r_2)(p_1 - p_2) \leq 0.$  (A.2)

Since the left-hand side of (A.2) is  $K_t(p_1) - K_t(p_2)$  at  $q = q_1'$ , condition (A.2) is equivalent to  $K_t(p_1) \leq K_t(p_2)$  at  $q = q_1'$ . From (A.2) it is clear that for  $q \leq q_1'$

$$K_t(p_1) \leq K_t(p_2)$$

while for  $q > q_1'$

$$K_t(p_1) = \{K_t(p_1) \text{ at } q = q_1'\}$$

$$K_t(p_2) > \{K_t(p_2) \text{ at } q = q_1'\}$$

Therefore  $K_t(p_1) < K_t(p_2)$  for  $q > q_1'$ . Consequently  $K_t(p_1) \leq K_t(p_2)$  for every  $q \in [0, 1]$  and vendor 1 dominates vendor 2. In this case, which is shown in Figure 2(b),  $q_1 = q_2 = q_1'$  if (A.2) is satisfied as equality,  $q_1 = q_2 = 1$  otherwise.

$$\text{Case (c): } C(p_1) + p_1 r_1 - C(p_2) - p_2 r_1 + q_1'(r_{12} - r_1 - r_2)(p_1 - p_2) > 0, \quad (\text{A.3})$$

$$\text{and } C(p_1) + p_1 r_1 - C(p_2) - p_2 r_1 < 0.$$

We will first define  $q_1$  and  $q_2$  and then show that for those values the Theorem holds in this case as well.

$q_1$  is either the point  $q_1 < q_1'$  corresponding to the intersection of  $K_t(p_1)$  and  $K_t(p_2)$  or  $q_1 = 1$ . Hence,  $q_1$  is the solution to

$$C(p_1) + p_1 r_1 + p_1(r_{12} - r_1 - r_2)q_1 = C(p_2) + p_2 r_1 + p_2(r_{12} - r_1 - r_2)q_1,$$

provided this solution is not greater than 1. Therefore

$$q_1 = \min \left\{ \frac{[C(p_2) - C(p_1)] - r_1(p_1 - p_2)}{(p_1 - p_2)(r_{12} - r_1 - r_2)}, 1 \right\} > 0$$

Notice that if  $q_1 < 1$ , then  $q_1 < q_1'$  because by (A.3)

$$0 < \frac{[C(p_2) - C(p_1)] - r_1(p_1 - p_2)}{(p_1 - p_2)(r_{12} - r_1 - r_2)} < q_1'.$$

For the definition of  $q_2$  we consider two cases:  $q_1' = 1$  and  $q_1' < 1$ .

If  $q_1' = 1$ , then  $q_2$  is defined to be equal to 1:  $q_2 = 1$ .

If  $q_1' < 1$  then  $q_2$  is defined as the value of  $q$  that satisfies  $q_1' < q_2 \leq q_2'$  and corresponds to the intersection of  $K_t(p_1)$  and  $K_t(p_2)$ . Therefore  $q_2$  is the solution to

$$C(p_1) + p_1 r_1 + a = C(p_2) + p_2 r_1 + p_2(r_{12} - r_1 - r_2)q_2$$

provided this solution does not exceed 1. It follows that

$$q_2 = \min \left\{ \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + a}{p_2(r_{12} - r_1 - r_2)}, 1 \right\} > 0.$$

The nonnegativity follows from (A.3) and from  $a = p_1 q_1' (r_{12} - r_1 - r_2)$ , the latter being equivalent to  $q_1' < 1$ .

We will next show that  $q_1' < q_2 \leq q_2'$  by considering two subcases:  $q_2 = 1$  and  $q_2 < 1$ :

I:  $q_2 = 1$  is equivalent to

$$r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + a \geq p_2(r_{12} - r_1 - r_2)$$

or

$$a \geq p_2(r_{12} - r_1 - r_2) + C(p_2) + p_2 r_1 - C(p_1) - p_1 r_1.$$

Because of (A.1), the above inequality leads to  $a \geq p_2(r_{12} - r_1 - r_2)$  and therefore to  $q_2' = 1$ .

Thus, in this case  $q_1' < 1 = q_2 = q_2'$ .

II:  $q_2 < 1$  means that

$$\begin{aligned} q_2 &= \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + a}{p_2(r_{12} - r_1 - r_2)} \\ &= \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + p_1 q_1' (r_{12} - r_1 - r_2)}{p_2(r_{12} - r_1 - r_2)} \\ &= \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)]}{p_2(r_{12} - r_1 - r_2)} + \frac{p_1}{p_2} q_1' \end{aligned} \quad (\text{A.4})$$

From (A.3)

$$\frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)]}{p_2(r_{12} - r_1 - r_2)} > \frac{p_2 - p_1}{p_2} q_1' = q_1' - \frac{p_1}{p_2} q_1' \quad (\text{A.5})$$

From (A.4) and (A.5) it follows that  $q_1' < q_2 < 1$ . Also if  $q_2' = 1$ , then  $q_2 < q_2'$ . If  $q_2' < 1$  then  $a = p_2 q_2' (r_{12} - r_1 - r_2)$ , so that

$$\begin{aligned} q_2 &= \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + p_2 q_2' (r_{12} - r_1 - r_2)}{p_2 (r_{12} - r_1 - r_2)} \\ &= \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)]}{p_2 (r_{12} - r_1 - r_2)} + q_2' \end{aligned}$$

From this last expression and (A.1) it follows that  $q_2 < q_2' < 1$ . We have thus proved that  $q_1' < q_2 \leq q_2'$  when  $q_1' < 1$ .

We must now prove that in Case (c)

$$\begin{aligned} K_t(p_1) &< K_t(p_2) \quad \text{for } q < q_1 \quad \text{and for } q > q_2 \\ K_t(p_1) &\geq K_t(p_2) \quad \text{for } q_1 \leq q \leq q_2, \quad q_1 \neq 1. \end{aligned}$$

For  $q < q_1 \leq 1$ ,

$$\begin{aligned} K_t(p_1) &= C(p_1) + p_1 r_1 + p_1 (r_{12} - r_1 - r_2) q \\ K_t(p_2) &= C(p_2) + p_2 r_1 + p_2 (r_{12} - r_1 - r_2) q \end{aligned}$$

and

$$\frac{K_t(p_1) - K_t(p_2)}{(p_1 - p_2) (r_{12} - r_1 - r_2)} = q - \frac{[C(p_2) - C(p_1)] - r_1 (p_1 - p_2)}{(p_1 - p_2) (r_{12} - r_1 - r_2)} \leq q - q_1.$$

Since  $(p_1 - p_2) (r_{12} - r_1 - r_2) > 0$ , for  $q < q_1$ ,  $K(p_1) - K(p_2) < 0$ .

If  $q_1 < 1$  and  $q_1' = 1$ , then  $q_2 = 1$  and for  $q$  such that  $q_1 \leq q \leq 1 = q_2$ ,  $K_t(p_1)$  and  $K_t(p_2)$  are given by the same expressions as above. In this case

$$\frac{K_t(p_1) - K_t(p_2)}{(p_1 - p_2) (r_{12} - r_1 - r_2)} = q - q_1$$

since  $q_1 < 1$ . Therefore, for  $1 = q_2 \geq q \geq q_1 \neq 1$   $K_t(p_1) - K_t(p_2) \geq 0$ .

When  $q_1 < q_1' < 1 = q_2$ , for  $q$  such that  $q_1 \leq q \leq q_1'$  the same analysis as before applies and  $K_t(p_1) - K_t(p_2) \geq 0$ .

When  $q_1 < q_1' < 1 = q_2$ , for  $q$  such that  $q_1' \leq q \leq 1$ , since it has been shown that  $q_2 = 1$  also means  $q_2' = 1$ , we have:

$$K_t(p_1) = C(p_1) + p_1 r_1 + a \quad (\text{A.6})$$

$$K_t(p_2) = C(p_2) + p_2 r_1 + p_2(r_{12} - r_1 - r_2)q \quad (\text{A.7})$$

$q_2 = 1$  implies that

$$\frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + a}{p_2(r_{12} - r_1 - r_2)} \geq 1$$

therefore

$$q p_2(r_{12} - r_1 - r_2) \leq r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + a$$

or

$$C(p_1) + p_1 r_1 + a \geq C(p_2) + p_2 r_1 + p_2(r_{12} - r_1 - r_2)q$$

and finally  $K_t(p_1) \geq K_t(p_2)$ .

Thus again  $K_t(p_1) < K_t(p_2)$  for  $q < q_1$  and  $K_t(p_1) \geq K_t(p_2)$  for  $q_1 \leq q \leq q_2 = 1$ .

When  $q_1 < q_1' < q_2 < q_2' \leq 1$ , then for  $q \leq q_1'$  the same analysis as before applies.  $q_2 < 1$  implies

$$q_2 = \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + a}{p_2(r_{12} - r_1 - r_2)}$$

and again for  $q_1' \leq q \leq q_2$   $K_t(p_1)$  and  $K_t(p_2)$  are given by (A.6) and (A.7) and, since  $q \leq q_2$ ,  $K_t(p_1) \geq K_t(p_2)$  for all  $q$  such that  $q_1 \leq q \leq q_2$ .

For  $q > q_2$

$$K_t(p_1) = C(p_1) + p_1 r_1 + a$$

$$K_t(p_2) > C(p_2) + p_2 r_1 + p_2(r_{12} - r_1 - r_2)q_2$$

$$K_t(p_1) - K_t(p_2) < C(p_1) + p_1 r_1 + a - C(p_2) - p_2 r_1 - p_2(r_{12} - r_1 - r_2)q_2$$

$$= r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + a - p_2(r_{12} - r_1 - r_2)q_2$$

$$\frac{K_t(p_1) - K_t(p_2)}{p_2(r_{12} - r_1 - r_2)} < \frac{r_1(p_1 - p_2) - [C(p_2) - C(p_1)] + a}{p_2(r_{12} - r_1 - r_2)} - q_2 = 0$$

and finally  $K_t(p_1) < K_t(p_2)$ . This last, more general case with  $q_1 < q_1' < q_2 < q_2' < 1$  is shown graphically in Figure 2(c). ■

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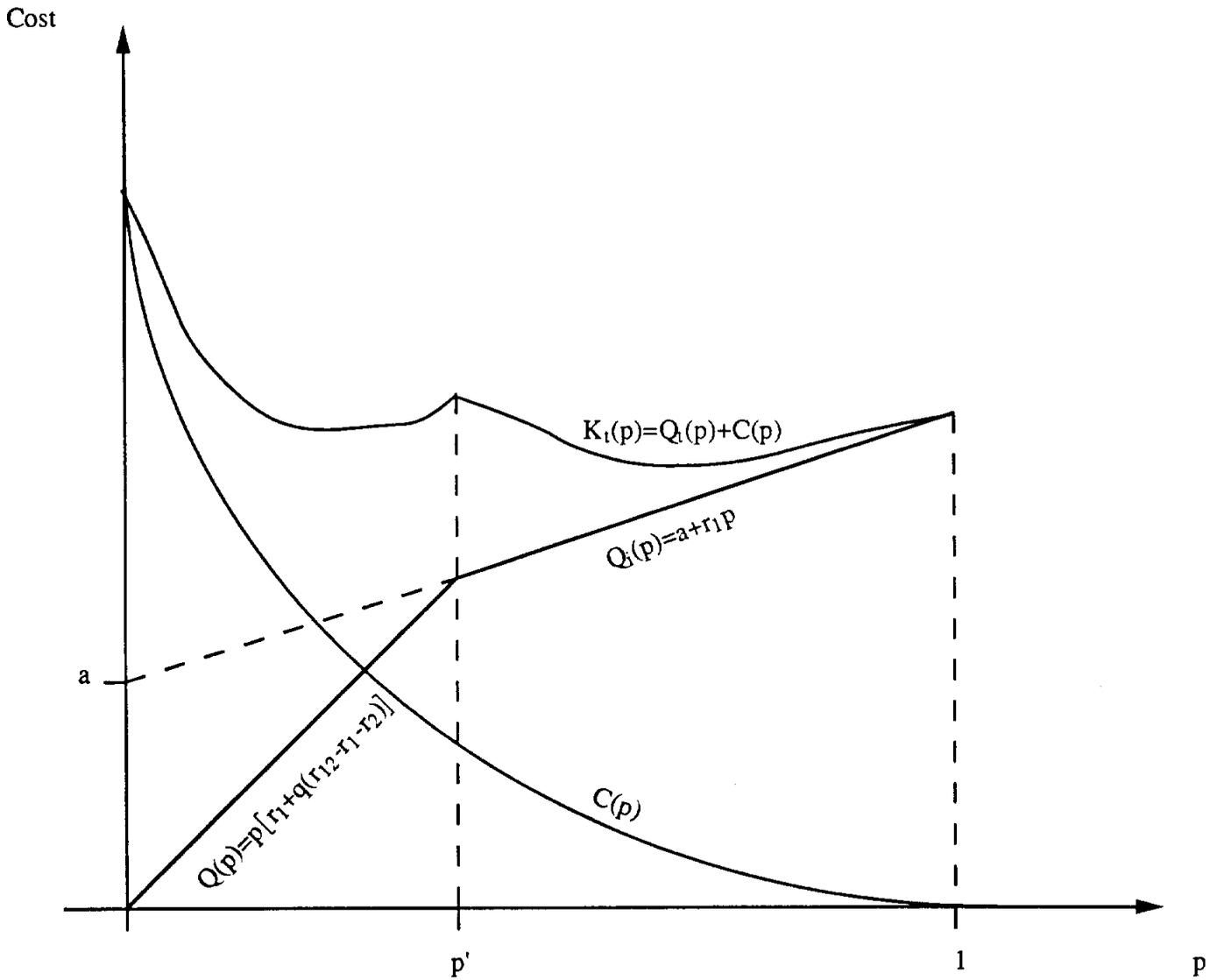
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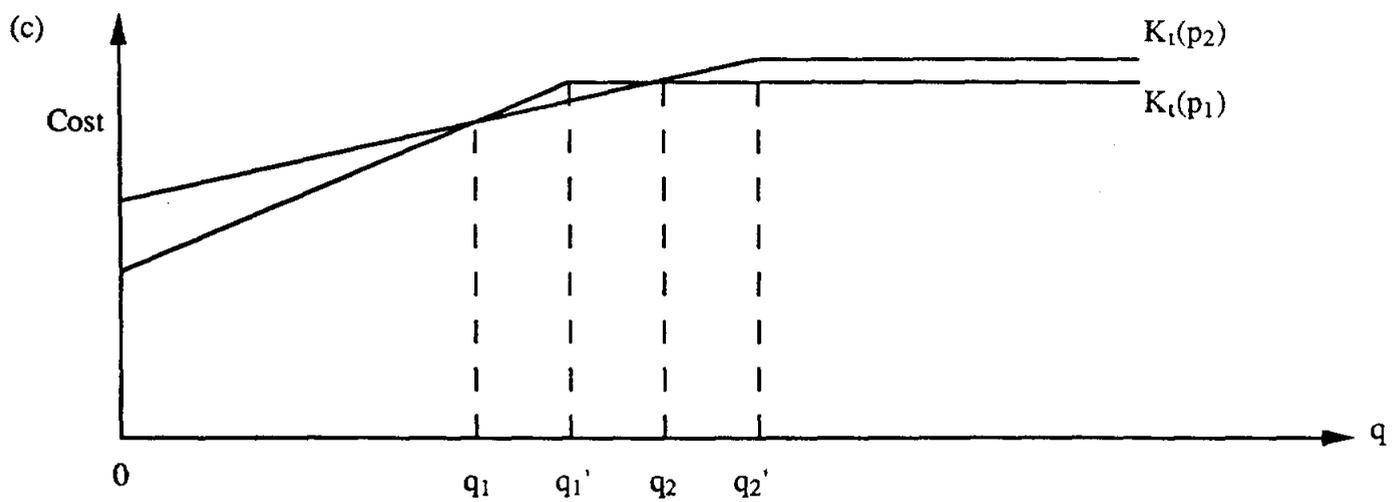
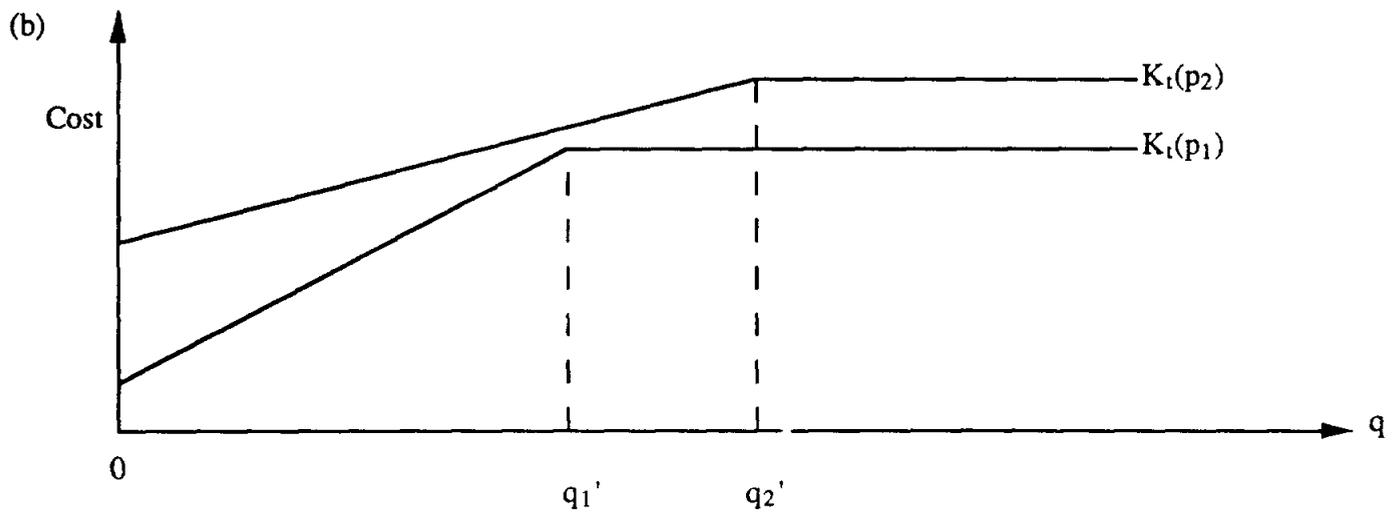
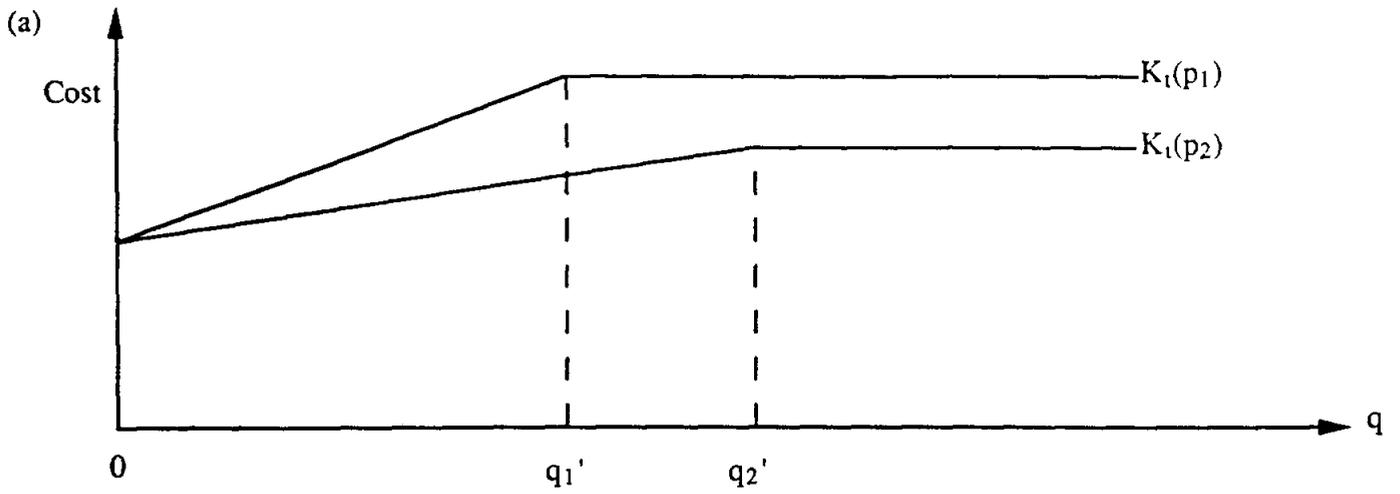
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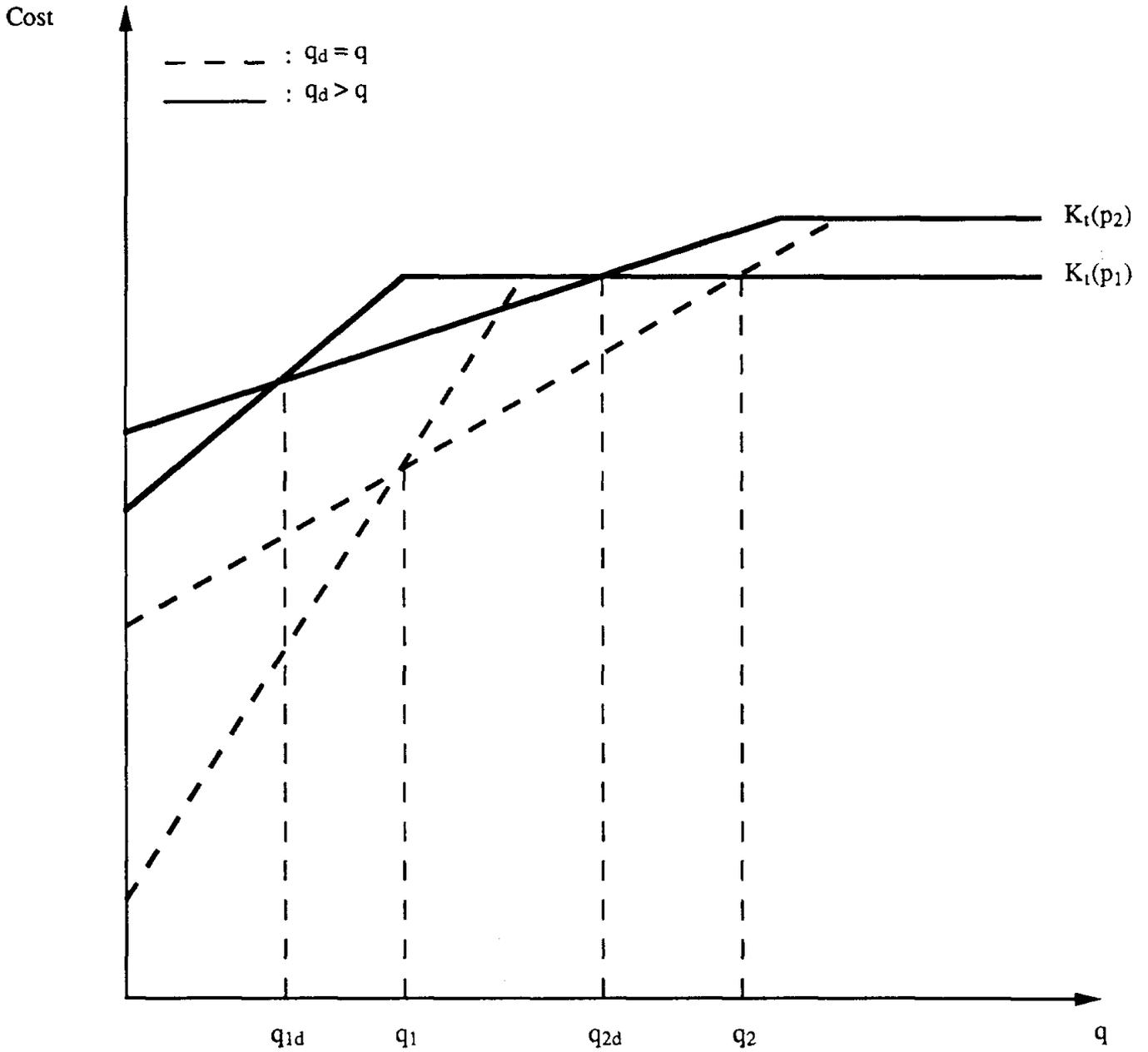
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**Figure 1: Total purchase and quality cost of vendor as a function of vendor's quality**



**Figure 2: Total purchase and quality cost of vendor as a function of purchaser's process quality**



**Figure 3: Total purchase and quality cost of vendor as a function of purchaser's process quality ( $r_{12} > r_1 + r_2$ )**