

**"THE EVALUATION OF INTEREST RATE RISK.  
SOME WARNINGS ABOUT  
THE BASLE PROPOSAL"**

by

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**Abstract**

The Basle Committee on Banking Supervision and the Federal Reserve have published recently papers on the measurement of bank's exposure to interest rate risk. The purpose of the paper is to warn that these proposals address one source of interest rate risk only. Senior bank managers and board of directors must design multiple (vector-type) indicators to capture the various sources of interest rate risk.

Banking supervisors have issued recently papers on banks' interest rate risk. In the United States, Section 305 of the Deposit Insurance Corporation Improvement Act requires Federal banking agencies to publish final regulations by June 19, 1993 revising risk-based capital standards to take into account interest rate risk. An advance notice of proposed rulemaking has been circulated in the industry (Federal Reserve, 1992). More recently, the Basle Committee on Banking Supervision has issued a consultative paper "*The Measurement of Banks' Exposure to Interest Rate Risk*" (BIS, 1993), updating an earlier paper (BIS, 1990). The Federal Reserve proposal outlines the measurement system, data requirement and capital charge methodology. The objective is to arrive at a 'single number' measure of interest rate risk which will be used to compute a capital requirement if the risk exceeds a threshold level. The Basle paper is a consultative paper sent to the banking industry, aiming at an agreement on measurement methodology by December 1993. For the moment, interpretation of the figures and regulatory responses will be left to each national authorities.

The purpose of the paper is to show why senior management and board of directors of banks should not base their risk management system on the 'single number' computed by the regulators. The purpose of the Federal Reserve proposal is quite explicit in this respect : "The proposed measurement system is designed to minimize reporting burdens while meeting regulatory needs. In view of the number of simplifying assumptions the system employs, the agencies do not intend for it to replace other more sophisticated procedures that banks may use in their asset and liability management procedures" (Federal Reserve, 1992, p.10). In our views, and although regulators are very explicit on the limitations of their proposal, there are two dangers : The first is that senior management design their information system to meet the regulation, instead of developing a more comprehensive one. The second danger is that management fails to recognize that the proposal by the Federal Reserve and the Basle

Committee intends to capture one source of interest rate risk only (parallel shift in the yield curve), and that it ignores for the moment more intricate sources of risk.

A brief description of the proposals by the Basle Committee and the Federal Reserve is followed by a discussion of twelve common pitfalls in interest rate risk measurement and by a call for the use of *multiple indicators* to recognize explicitly the *complexity* of interest rate risk.

## 1. THE MEASUREMENT OF INTEREST RATE RISK

A first methodological issue in the measurement of interest rate risk concerns the definition of risk. A first approach emphasizes the impact of interest rates on earnings (*'earnings at risk'* method) ; a second approach evaluates the effect of a movement of interest rates on the economic value (net worth) of a bank (*'value at risk'* method). With respect to this issue, one observes a remarkable convergence of views between the Federal Reserve and the Basle Committee. For instance, the last document (BIS, 1993, p.25) states explicitly that "The measurement system favoured by a sizeable majority of Basle Committee members is designed to look at the risk of future changes in interest rates to the economic value of a bank by using a duration-weighted gap methodology". However, while the Federal Reserve regulatory proposal is solely concerned with market value at risk (VAR), the Basle Committee recognizes that banks could also be concerned with the impact of interest rates on yearly reported earnings and that they should also monitor their earnings at risk (EAR). On a technical basis, it must be emphasized that the EAR and VAR methodologies are fully compatible, and are really two different ways to look at an identical problem. Quite rightly, the Basle Committee has argued in the 1990 paper that "the two approaches are reconcilable because if all future cash flows are considered and discounted to their present value, the change in the present value of future cash flows under the current earnings perspective will equal the change in market value under the market value approach"<sup>1</sup>. One should add a caveat that this statement is true if the 'maturity ladder' system is properly designed, which, we argue, is rarely the case.

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<sup>1</sup>A similar view is expressed in Houpt and Embersit (1991).

The proposals by the Basle Committee and by the Federal Reserve run as follows. The documents suggest the construction of 'duration-based' ladders (thirteen in the proposal of the Basle Committee) showing the duration of all on- and off-balance sheet activities. According to the Basle proposal, Trading Portfolio activities would be distinguished from Other Business (standard commercial banking activities). Duration-based ladders would be constructed for each currency.

In formulating the measurement system, the Basle Committee (1990) mentions several features that ideally should be present in a interest rate risk measurement system :

- The measurement system should provide a reasonably accurate reading of an institution's interest rate risk exposure with a single-number-risk indicator.
- The system should be as simple as possible, intuitively understandable, and not rely on complicated mathematical formulas.
- All on-balance and off- balance sheet transactions that have a direct bearing on interest rate exposure should be captured.
- The system should have sufficient flexibility to adapt to the institutional, market and legal conventions in each of the member countries so as to achieve a high degree of comparability and consistency across banks, and between banks and securities firms.
- The system should not impose an excessive burden on the banks.

To obtain a single-number risk indicator, the various gaps would be multiplied by a weighing factor which would reflect the amount of exposure inherent in a particular position or gap. For instance, in a market value framework, a fixed rate asset with a long duration would have a larger weight because its price sensitivity is higher. Finally, one would take the weighted sum of all gaps and arrive at a single number indicator representing the measurement of interest rate risk. According to the proposal by the Federal Reserve (1992), this indicator will evaluate the change in net value of a bank as a percentage of total assets for a 100 basis points change in interest rates. Capital (Tier 1) would be required whenever the interest rate risk exceeds some industry norm, which the Federal Reserve estimates at about one percent of

total assets<sup>2</sup>. This capital requirement will come in addition to the current capital ratio. As stated earlier, the Basle proposal is exclusively concerned with measurement methodology, stopping short of a capital adequacy proposal.

Although the methodology seems straightforward, there are twelve pitfalls quite common in reporting systems, many of them present in the current proposals. In line with the supervisors' approach, we adopt the 'value at risk' perspective. This is motivated by three reasons. Firstly, the market value is the present value of current and future interest margins and gives a long term view on risk. Secondly, in an efficient financial market, the stock market value of a bank will represent the discounted value of future net earnings, that is the market value of assets net of the liabilities. The third reason is that securitization and the sale of assets will increase managers' concern for the current market value of assets or liabilities.

## **2. TWELVE PITFALLS IN THE MEASUREMENT OF INTEREST RATE RISK<sup>3</sup>**

The first three pitfalls concern the treatment of assets and liabilities which are apparently non-interest sensitive : Reserves with Central Bank, Fixed Assets and Equity.

### **1. Reserves with Central Bank**

Since reserves earn no interest in most countries, they are being put by many banks and the Basle Committee in the long term 'indefinite' period of the ladder. This approach is misleading for the following reason. When interest rates on the market increase by one percent, the cost of deposits should increase by less than one per cent less because the implicit cost of

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<sup>2</sup>Denoting total assets and deposits by A and D, and the durations of total assets and deposits by  $DU_A$  and  $DU_D$ , one ends up that the change in net value (NV) as a percent of total asset is equal to the 'modified duration gap' times the change in interest rate :  
 $\Delta NV / A = -[(DU_A - (DU_D \times D/A))/(1+R)] \times \Delta R$ . The one percent change of the Fed implies a 'modified duration gap' of 1 year.

<sup>3</sup>A preliminary analysis of potential pitfalls in the measurement of interest rate risk has been published in Dermine (1991<sub>a</sub>).

the reserve requirement increases<sup>4</sup>. To reflect the reserve requirement-driven inelasticity of deposit rates, one should include the reserves in the same time band as the deposits. The argument runs as follows. Mathematically, the inelasticity of the deposit rate is equivalent to a complete sensitivity (one percent rate change) of the deposit rate, accompanied by an additional revenue of one percent on the reserves<sup>5</sup>. Let us emphasize that the inclusion of reserves in the ladder is only a mathematical trick to facilitate the interpretation of the figures. Instead of being forced to recognize the reserve-driven inelasticity of the deposit rate, one can assume, because of mathematical equivalence, full sensitivity of rates on deposits and reserves<sup>6</sup>.

One could argue that the logic of the deposit pricing rule may not apply in all cases. Quite true, but the point of the argument is that it would be wrong to apply the 'indefinite' time band for reserves to all banks, in all countries.

## 2.Fixed Assets

As was the case for reserves with central bank, fixed assets are included by the Basle Committee and the Fed in the long term 'indefinite' time band because they do not generate interest revenues. This view is misleading for similar reasons than for reserves. It is likely that margins on deposits and loans should increase when interest rate rises to take into account the higher implicit opportunity cost of fixed assets. That is, interest rate on loans will rise by more than one percent, and interest on deposits should increase by less than one percent. Again, this is mathematically equivalent to saying that interest rate on loans and deposits adjust by one

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<sup>4</sup>For instance, with an interbank rate of 10 % and a reserve requirement of 10 %, the 'zero profit' deposit rate would be 9 %. If market rate moves by one percent to reach 11 %, the deposit rate would increase by 0.9 % to reach 9.9 %.

<sup>5</sup>With reference to footnote 4, the cost of deposits of 9 % is equivalent to a cost of deposits of 10 % and a return of 10 % on the 10 % reserves, implying a net cost of 9 %.

<sup>6</sup>Consider 100 of three-month deposit invested for 90 in a three-month T-Bill and 10 in Reserves. Although there is no interest rate risk as footnote 3 shows, the Basle proposal would show an exposure, while our system would rightly ignore it.

percent and that there is an interest revenue on fixed assets. A case is made, under these conditions, to include fixed assets in the same time band as the maturity of loans or deposits to which they are linked.

An alternative view is that fixed assets are not costly because their value increases with inflation. If, as is expected, there is a close relationship between inflation and interest rates, there is a good reason to put fixed assets in the short time band. As was the case for reserves, the thrust of the argument is that there is little reason to include fixed assets in the 'indefinite' time band for all banks, in all countries.

### 3. Equity

Equity is also included in the long term 'indefinite' category because it does not generate any interest expense. This is of course the result of an accounting-based fallacy. Financial analysts evaluating bank profitability measure it against the current market interest rates available to shareholders. That is, the opportunity cost of equity is based on the current rate and equity should be included in the short term time band of the ladder<sup>7</sup>. This is particularly important in the measurement of earnings at risk, an exposure recognized explicitly by the Basle Committee. As an extreme example to illustrate the point, consider the case of an all equity- financed bank. The value of its shares would be immunized only if its assets were invested in short term maturity assets.

One could question the relevance of the first three pitfalls because reserves, fixed assets or equity represent, after all, a small percentage of the balance sheet. The argument is wrong for two reasons. The first is that their inclusions can change substantially the size of an individual time gap and its derived exposure. Secondly, given the high leverage of banks, it is important to calculate very precisely their interest rate exposure.

A second category of pitfalls concern the treatment of items such as savings deposits,

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<sup>7</sup>From an earnings at risk perspective, the adjustment in opportunity cost will be taken into account. From a market value basis, a weight (duration) of zero will be applied to equity.

pre-payable mortgages, demand deposits, and other items whose maturities (and repricing dates) tend to be unknown or consistently vary from contractual terms.

#### **4. Demand and Savings Deposits**

Demand, savings deposits and pre-payable mortgages present a difficulty because their effective maturity is uncertain. The committee recommends to slot them according to some standard assumptions regarding maturity or repricing dates based on historical behavior. As there will be some discretion across countries, the document suggests to apply identical rules to all banks in one country, and that the rules used should be acceptable to all members of the Basle committee. There are two considerations here. A first one concerns the assets with embedded options such as pre-payable mortgages. A discussion will be deferred to the 'option' pitfall. A second concern is that some categories of deposits such as demand or savings deposits do not pay market rates. Two problems arise. The first is that their volume can vary through time as interest rates change and the second is that their interest rate can be relatively inelastic because of regulation or lack of competition. A proper solution to these problems is to use a simulation tool which takes into account the elasticity of rates and the changes of volumes, or to use an adjusted duration which captures explicitly the regulatory and competitive environment (Dermine, 1985). In Appendix One, we develop a simple model of the market value of a bank to show that its interest rate sensitivity depends on the degree of competition. As this last factor is very subjective and can be bank-specific, it would be unwise to force all banks in a country to put demand deposits in the same time band. Proper risk measurement requires the risk evaluation under alternative scenarios of competition and deregulation. As an example, consider the case of non interest-bearing demand deposits. Should they be invested in long or short term securities ? If depositors are non-responsive to market forces, they could be invested in long term fixed rate assets, but if markets are responsive, they should go into short term securities.

The key-consideration regarding demand or savings deposits is that the measurement of interest rate risk is not a pure mathematical exercise but depends on subjective elements such as the current and future levels of deregulation and competition. The interest rate

exposure should be evaluated in different competition scenarios and it is unlikely that a single-number indicator can represent the interest rate exposure of a bank.

## **5. Floating Rate Asset**

Interest revenue or cost on floating rate instruments adjust to new interest rates at the time of reset. The Federal Reserve and Basle proposal recommend to slot these instruments according to their repricing dates. This approach can be misleading for an institution that measures its exposure on a market value basis. Because a floating interest rate is the sum of a 'pure' floating rate and a fixed spread, it has been shown that the duration can differ quite substantially from the time to repricing (Yawitz et al. 1987). Some floating rate assets may even have a negative duration. Consider the case of floating rate junk bonds which have a negative spread<sup>8</sup>. Their value can increase when interest rates go up, because the present value of the negative spreads falls.

The case of floating rate asset is a clear example where the uniform application of a simple rule to all banks (floating rate asset in the 'repricing' time band) could give a misleading view on interest rate risk.

The next two pitfalls concern the treatment of cash flows not reported in the Basle document : Interest payments and taxes.

## **6. Interest Payments**

As is well known from the duration literature, a fixed income asset is equivalent to a portfolio of zero coupon securities paying one single cash flow at maturity. Therefore, one should slot all cash flow payments (interest or principal) in bands corresponding to the time of payment. The additional reporting burden should be nil since the computation of duration weight does require a knowledge of the timing of cash flows.

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<sup>8</sup>Their spread is negative when the floating rate is lower than the discount rate used by the market for valuation. This can happen when there is a change in the credit risk of the borrower.

## **7. Taxes**

As is the case for interest payments, it can be shown that the timing of taxes changes the duration of an asset and its price sensitivity (Dermine, 1991<sub>b</sub>). In Appendix One, we show that the non-taxation of unrealized capital gains (losses) is a determinant of the market value of a bank, and therefore of its sensitivity to interest rate change. Therefore, a proper risk measurement system demands to take into account the timing of tax payments. Taxes should be included in the repricing bucket.

## **8. Options**

The standard approach is to report options at their delta value. This is simple, but could be misleading for large change of interest rates and non-parallel shifts in the yield curve. Ho (1990) suggests to work out a portfolio of zero coupon assets equivalent to the option. That is, for the one-quarter time band for instance, one computes the change in price of an option when the one-quarter rate changes. A zero coupon equivalent asset is calculated and included in the three-month time band. A similar exercise is repeated for all the time bands. The treatment of options requires a quite complex mathematical treatment which does not meet the Basle 'simplicity' criteria mentioned earlier. But one can wonder about the relevance of this criteria for activities such as options which create a complex risk exposure.

The next two pitfalls concern the aggregation of gaps over time bands and/or across currency ladders.

## **9. Aggregation across Time Bands**

This issue involves the aggregation of gaps over different time bands (referred to in the Basle document as '*horizontal*' aggregation). For instance a one-week positive gap could be hedged by a two-week negative gap. This issue is fully recognized in the Basle proposal which acknowledges that the risk of aggregation is related to non-parallel shifts in the yield curve. For instance, it could happen that the 1-week rate does not move in parallel with the 2-week rate, so that an interest rate exposure does exist. A very conservative approach would be to

forbid aggregation across time bands and consider, as worst case scenarios, adverse movements in interest rates of different maturities. A second approach is to allow some aggregation when the correlation across rates is fairly strong<sup>9</sup>. In Appendix Two, we discuss the statistical conditions required for the aggregation of positions. Technically, aggregation of two positions is warranted when the correlation between interest rates is high and when the standard deviation of the statistical distribution of interest rates are equal. It should be recognized that the single number measure of risk proposed by the Federal Reserve refers to only one source of risk, a parallel shift in the yield curve. Senior management must recognize non-parallel shifts as well. A repricing table based on the timing of the cash flows of assets and liabilities is necessary to identify the sources of risks linked to non-parallel shifts in the yield curve.

## **10. Aggregation across Currencies**

The question raised here is similar to the previous one and concerns the correlation of rates of identical maturities in different currencies (referred to as '*vertical*' aggregation). If two interest rates in different currencies are highly correlated, there is a case to aggregate the related gaps in the maturity ladders. In this case too, the document makes the appropriate recommendation to study the types of exchange rate regimes likely to prevail and the permanence of correlations<sup>10</sup>.

## **11. Interest Rate and Credit Risk**

An additional source of interest rate risk is derived from the impact of interest rate changes on loan default and credit risk. The case is exemplified by the effect of interest rate increases on floating rate loan delinquencies. If such a relation exists, it would be unwise to run a matched book, but it would be appropriate to run positive gaps (Santomero,1983) to hedge potential loan losses with increased interest revenues. It must be noted that a bank

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<sup>9</sup>Litterman and Scheinkman (1988) observe that movements in the yield curve can be represented by a few independent factors, so that partial aggregation is warranted.

<sup>10</sup>Saunders-Grammatikos and Swary (1986) report little evidence of stability in currency-interest rate correlation matrices.

hedging its credit risk with an open interest rate position would be penalized twice under the current Basle regulations : A first time through the BIS credit risk capital ratio, and a second time through a reported interest rate exposure.

## **12.Inflation**

It has been shown that the market value of banks can be affected by inflation because financial intermediaries are net holders of financial assets and have their nominal income being taxed (Dermine and Hillion, 1992). To the extent that inflation and interest rates are correlated, it would be unwise to hedge against a single source of risk. As was the case with credit risk, positive gaps could be run to hedge an other source of risk, inflation.

## **4. CONCLUSIONS**

The object of the article has been to question the efficiency of the regulatory proposal for the measurement of interest rate risk. The main conclusion is to recognize that the proposal is based on a set of very simplifying assumptions which recognize one source of interest rate risk only, a parallel shift in the yield curve in a market in which the sensitivity of demand and savings deposits is known. It is absolutely necessary that bank managers and board of directors design **multiple measure** of risks. A vector-type indicator which recognizes non-parallel shifts in interest rates and uncertainty about volume and rate sensitivity is a far superior approach which should be encouraged by regulators.

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### Appendix One : The Market Value of a Bank

The purpose of of Appendix One is to dissociate the main determinants of the market value of a commercial bank. This exercise is necessary to understand the sources of the interest rate sensitivity of the market value. Let us assume for expository convenience that a bank has two more years before it is liquidiated. The bank has on its asset side a portfolio of two year-to-maturity loans (L) and bonds (B) which have been acquired in the past and which carry fixed interest rates, respectively p and b. These assets are financed with deposits (D) and equity (E). Deposits have a maturity of two years and offer a fixed interest rate d. Loans, bonds and deposits are recorded at their historical book value. The current return on similar assets and liabilities are respectively  $p^*$ ,  $b^*$  and  $d^*$ . The bonds and equity markets are asumed to be very competitive, and the (certainty equivalent) cost of equity is the current market rate  $b^*$ . The balance sheet of the bank is as follows :

Loans L(p)	Deposits D(d)
Bonds B (b)	Equity E( $b^*$ ).

Denoting by t the corporate tax rate, the market value of the bank is the discounted value of future dividends, that is :

$$MV = \frac{(1-t)(pL+bB-dD)}{1+b^*} + \frac{(1-t)(pL+bB-dD)+(L+B-D)}{(1+b^*)^2}$$

Following Dermine (1987), this relation can be expressed as :

$$\begin{aligned}
 MV = & (L_1^* + B_1^* - D_1^*) \\
 & + \frac{(1-t)(p^* - b^*)L_1^*}{1+b^*} + \frac{(1-t)(p^* - b^*)L_2^*}{(1+b^*)^2} + \frac{(1-t)(b^* - d^*)D_1^*}{1+b^*} + \frac{(1-t)(b^* - d^*)D_2^*}{(1+b^*)^2} \\
 & - \frac{tb^*E_1^*}{1+b^*} - \frac{tb^*E_2^*}{(1+b^*)^2} \\
 & + \frac{t(L_2^* - L_1^*)}{1+b^*} + \frac{t(L_3^* - L_2^*)}{(1+b^*)^2} + \frac{t(B_2^* - B_1^*)}{1+b^*} + \frac{t(B_3^* - B_2^*)}{(1+b^*)^2} - \frac{t(D_2^* - D_1^*)}{1+b^*} - \frac{t(D_3^* - D_2^*)}{(1+b^*)^2}
 \end{aligned}$$

where  $L_i^*$ ,  $B_i^*$  and  $D_i^*$  are the current values in year  $i$  of the loans, bonds, and deposits evaluated at their loan, bond and deposit current interest rates (respectively  $p^*$ ,  $b^*$ ,  $d^*$ ).

The market value is the sum of four terms : 1) The current value of assets net of the liabilities, 2) the after-tax value of the franchise value (the ability to raise money at below market rate, and to price loan at above security rate), 3) the present value of the non-taxdeductibility of equity costs, and 4) the present value of the tax savings due to the non-taxation of unrealized capital gains (losses) on assets and liabilities. This formula is useful to illustrate that the interest rate sensitivity of the market value of a bank does not depend solely on the duration between assets and liabilities (the first term), but also on the interest rate sensitivity of the franchise value (the second term), and of the tax shelter (the fourth term).

## Appendix Two : Interest Rate Risk and Correlation

Define  $R_1$  and  $R_2$  as interest rates of different maturities (currencies),  $\sigma_{R_1}$  and  $\sigma_{R_2}$  the standard deviation of these rates,  $\rho$  the correlation between the interest rates, and  $a$  and  $b$  as the changes in value of assets for a one percent increase in interest rates. If interest rates  $R_1$  and  $R_2$  move, the loss (gain) on the portfolio will be given by the following relation :

$$\text{LOSS} = ( a \times \Delta R_1 ) + ( b \times \Delta R_2 ).$$

The risk of such a position can be expressed by the standard deviation  $\sigma_{aR_1 + bR_2}$ , the squared root of the variance (Var) calculated as follows :

$$\text{Var} (aR_1 + bR_2) = a^2 \text{Var} (R_1) + b^2 \text{Var}(R_2) + 2ab \rho \sigma_1 \sigma_2$$

$$= (a^2 \text{Var}(R_1) + b^2 \text{Var}(R_2)) + 2ab \rho \sigma_1 \sigma_2 - 2ab (1 - \rho) \sigma_1 \sigma_2$$

$$= (a\sigma_1 + b\sigma_2)^2 - 2(1 - \rho) ab \sigma_1 \sigma_2$$

Therefore, risk (the standard deviation) will be the sum of the risks on each single component  $(a\sigma_1 + b\sigma_2)$  in the only case where the two interest rates are perfectly correlated (case of a parallel shift in the yield curve) :

$$\text{Risk} = \sigma_{aR_1 + bR_2} = a\sigma_1 + b\sigma_2 \text{ if } \rho = 1.$$

Moreover, a condition for aggregation of positions is that the standard deviations of interest rates be equal :  $\sigma_1 = \sigma_2 = \sigma$  :

$$\text{Risk} = (a + b) \sigma.$$

In the other cases ( $\rho \neq 1$ ), the sum of the two components could over- or under-represent the

true measure of risk, depending on the signs of a, b and the correlation  $\rho$  :

$$\sigma_{aR_1 + bR_2} < a \sigma_1 + b \sigma_2 \text{ if } \rho < 1 \text{ and } a, b > 0$$

$$> a \sigma_1 + b \sigma_2 \text{ if } \rho < 1 \text{ and } a \text{ or } b < 0.$$