THE FREQUENCY AND PRICING OF PRODUCT INNOVATIONS

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Abstract

This paper constructs and analyzes an economic model of repeated product innovations in which the introduction of one product forms the base from which the development of the next product will start. The innovation process depends both on the flow of R&D expenditures and on the length of time over which they accrue. The products are durable, so although products become technologically obsolete due to the introduction of new and superior products, consumers need not replace the products they already possess. We determine rational expectations equilibria under the monopoly and the duopoly regimes, and use them to interpret a vast body of empirical literature which documents the frequency of innovations of various durable goods, changes in their quality adjusted prices, and how these variables vary in a cross section of products and over time. The model shows that the decentralized market equilibria are inefficient. We also analyze the effect of various policy instruments, including output taxes or subsidies and patent limitations, which can be used to remedy the inefficiencies.

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1. Introduction

The words technological progress usually conjure up a Schumpeterian image of a small number of technologically spectacular inventions, such as the cotton gin, radio, computers, transistors, lasers, silicon chips and so on. However, the history of technology (see Rosenberg (1982)) reveals that a large portion of total productivity growth takes the form of a slow and almost invisible accretion of individually small improvements. Substantial improvements in productivity often continue to come long after the initial innovation, as the product goes through innumerable minor modifications and alterations in design.

This cumulative aspect of technical progress derives from a variety of sources. Among these are learning by doing and learning by using. The former involves learning that requires intimate familiarity with, and ongoing participation in, the details of the product-manufacturing process. The latter refers to learning that is generated by *use* of the product, as significant product characteristics are revealed only after intensive or prolonged use by consumers. This creates a feedback loop from final consumption into product development. A third source behind the cumulative process involves the role of complementarities in innovation: Exogenous new developments in other products and processes which enable further progress in already extant innovations. For example, in agriculture, the introduction of techniques for the mechanical harvesting of crops has been sharply accelerated by advances in genetic knowledge that permit changes in the physical characteristics of agricultural products[1].

This paper constructs and analyzes a dynamic model of technical progress designed to capture this cumulative and ongoing character of inventive activity. Each innovation in the model is based on previous innovations and, at the same time, contains within itself the seeds of future innovations.

The setting is a market for durable products, e.g., automobiles, industrial equipment or minicomputers, where new models of the product appear every so often and replace old models. The extent to which a new model improves upon its predecessor depends on the amount of R&D expenditures invested in developing the new model, and on the amount of time that is allowed to elapse between successive introductions.

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[1] For example, making the skin of tomatoes thicker, which allows them to be handled by mechanical equipment. Likewise, the development of sophisticated software has been accelerated by the introduction of faster and more powerful PC's.

At the same time, while the introduction of a new model renders previous models of the same product *technologically* obsolete, consumers have the option of continuing to use their old models. Their decision to adopt the state of the art model therefore depends on the incremental utility that the latter delivers, on its price, *and* on consumers' expectations about the new model's longevity—the duration for which the new model will be on the technological frontier before being rendered obsolete by a still better model. Successful innovation must balance all these factors, technological factors and development costs, on the one hand, and consumers' willingness to adopt on the other hand.

Against this background, we derive a rational expectations equilibrium path of innovation and technical progress under alternative market structures. The equilibrium enables us to identify the way technological and other determinants govern the path of technical progress and to make empirically testable predictions. In particular, we are able to show how the frequency of product introductions and the quality-adjusted prices are determined, and how they vary over time and in a cross-section of products. In doing so, we relate to and shed light upon the rich empirical literature on these topics (see Chow (1967), Mansfield (1968), Griliches (1971), Brendt and Griliches (1993), Greenstein (1994), Gandal (1994), and Roff and Trajtenberg (1995).)

A major objective of the analysis is to analyze the efficiency properties of the equilibrium innovation path. This efficiency question arises quite naturally in our dynamic context, because the market structure we examine is incomplete, consumers only being able to buy the present generation of a product, and not to contract on upgrading it in the future when better products appear on the scene. Hence the first fundamental theorem of welfare economics need not (and, in fact, does not) hold in this context.

In the case of a monopoly inventor, it turns out that efficiency hinges on whether the monopolist is allowed to rent its products or whether it must sell. In the former case, the decentralized outcome is fully efficient. In the latter case, the decentralized outcome is inefficient, the rate of product introductions being *too slow*. The essential reason for this is that when only spot markets operate, the value that consumers attach to a new product reflects its incremental contribution over and above pre-existing products, but not the contribution it makes to the quality of *future* products. This undervaluation dilutes the incentives of product developers to invest in R&D, which generates the inefficiency.

The duopoly outcome is also generally inefficient, although it is impossible to say in general whether the introduction of products is too fast or too slow under duopoly (by comparison with the social optimum), or whether the duopoly outcome is more or less efficient

than the monopoly outcome. In addition to the externality which applies in the case of the monopoly, and which continues to operate under duopoly, there is a second force, namely, the attempt by each firm to pre-empt the other. This force hastens the introduction of new products, which may generate a more or less efficient outcome, depending on the *degree* to which product introductions is hastened. Therefore under the presence of two opposing forces (pre-emption and consumers' undervaluation of product introductions) we may end up with too frequent or too infrequent introductions. But either way, the decentralized duopoly outcome is inefficient.

Given the inefficiency of market outcomes (under monopoly or duopoly), we also explore the role of different policy instruments, for example output taxes or subsidies and patent limitations, which can be used to remedy the inefficiency.

The remainder of the paper is organized as follows. The next section sets up the model. Section 3 characterizes the social optimum, and derives its comparative statics properties. Section 4 sets up the monopoly program, characterizes a solution to it, derives the comparative statics properties of the solution, and compares the solution to the social optimum. Section 5 sets up the duopoly scenario, determines an equilibrium to it and compares the equilibrium to the social optimum and to the monopoly outcome. Section 6 suggests how the inefficiencies that occur under the different market structures can be rectified using various policy instruments.

2. The Model

Consider a perfectly durable product which, for concreteness, will be referred to as a computer. There is a continuum of infinitely-lived consumers of measure 1. The utility a consumer derives from a computer depends on its quality, denoted by q. A computer of quality q delivers $q worth of utility per period. Consumers' demand for computers is discrete: they buy either zero or one unit. Time is continuous and indexed by $t \in [0,\infty)$. The interest rate, $r>0$, is constant and the same for consumers and producers.

New and improved computers can be introduced over time. Each introduction enables the production of still better models in the future. Hence, the process of improvement is *ongoing*. The extent to which a new computer is better than its predecessor depends on two things. First it depends on the flow of R&D expenditures; call this variable x. Second it depends on the length of time over which these expenditures accrue; call this variable t. The expenditure of $x over t units of time makes it possible to produce a new computer which improves upon its predecessor by $g(x,t)$. Thus, if the preceding computer was of quality q, the new computer is of quality $q+g(x,t)$. The g function is called the "improvement (production) function", and x and t are considered as inputs into it.[2] $g(\bullet,\bullet)$ satisfies the following restrictions.

Assumption A: $g(x,t)$ is monotonically increasing in each variable, twice continuously differentiable, strictly concave and $g(0,t)=g(x,0)=0$.

Assumption A says that x and t have positive marginal productivity, that they are subject to diminishing returns and that no input gives no output.

An important consequence of the concavity of g is that the potential value of future introductions is greater, the more frequently innovation has taken place in the past. For example, holding the stream of R&D expenditures constant, the value of the next computer is higher if it was preceded by yearly introductions as opposed to bi-yearly introductions. This captures the idea that the quality of new products depends not only on the amount of *laboratory* research which led to their development, but also on the number of earlier versions that have been tried and, hence, on the extent of "learning by using".

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[2] A special case of g is when it depends on total R&D expenditures only, i.e., on xt. However, the model allows more general scenarios, e.g., when stretching the same total expenditures over a longer period gives a better end-product.

The partial derivatives of g are denoted $g_x=\partial g/\partial x$ and $g_t=\partial g/\partial t$, and the second derivatives g_{ij}, for i, j=x,t (for example, $g_{xt}=\frac{\partial^2 g}{\partial x \partial t}$, etc.). We shall sometimes use further properties of g, for example the distinction between x and t being substitutes or complements, $g_{xt}(x,t)<$ or >0, or the assumption that $g_x(x,t)-tg_{xt}(x,t)>0$. The last assumption is implied by x and t being substitutes, but it is weaker than that. For example, if $g(x,t)=\sqrt{xt}$, x and t are complements but we still have $g_x-tg_{xt}>0$.

In addition to R&D expenditures, a fixed cost, F, has to be paid each time a new computer is introduced. F is paid in a lump (unlike x which is spread over time), and is called the "implementation cost". We can interpret F in two ways. First, it can represent the cost of harnessing new knowledge into the present product. Thus, we can consider x and t as leading to basic knowledge and F is the cost of translating the new knowledge into commercial applications. Second, F can represent the cost of embarking upon the development of the *next* product. For example, F might be the cost of building a new plant or acquiring new equipment needed for starting work on the next product[3].

Variable costs of production are assumed to be constant and equal across different generation products. The idea is that a large chunk of the cost of a new computer is the cost of *developing* it. Once it exists, its "physical" cost, e.g., labor and materials, is not much different from the cost of its predecessor. For example, it costs hundreds of millions of dollars to develop a new micro-chip, but approximately $150 to physically produce it, a cost which has not changed appreciably across different generations of micro-chips[4].

Since consumers are identical and since the marginal cost of production is constant, all consumers acquire a new computer as soon as one becomes available[5]. Thus, the volume of production is one (equal to the measure of consumers). This allows us to include the variable cost of production in F. Therefore, from this point onward, F will be interpreted as implementation cost *plus* variable production cost.

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[3]A third interpretation of F is that it represents the cost to *consumers* of learning how to use a new product and its range of applications. This interpretation is consistent with the analysis of the social optimum. However, when we analyze the market equilibrium, we assume that *producers* bear the cost F. Hence, we shall stick to the first two interpretations which apply throughout the paper.

[4]This is also consistent with the fact that the nominal price of computers have not changed much over time, only that their characteristics have improved. The equilibrium we construct below has this feature as well.

[5]If marginal cost was increasing, it would be productively more efficient to spread production over time. If consumers demand was decreasing it would pay the monopolist to spread sales over time, in an attempt to price discriminate.

This defines two problems. First is the problem of the social planner: the introduction of new products raises consumers' utility, but is costly. Hence, the planner balances these two effects, choosing the optimal R&D expenditures and the optimal frequency of introductions. Second is the problem of the monopolist: how much to spend on R&D, how frequently to introduce new products *and* how to price them. The monopolist takes consumers' demand (which is shaped by the fact that the product is durable and by expectations about the future path of introductions) as given and maximizes discounted profits. We start with the planner's problem.

3. The Social Optimum

The "social planner" takes the initial quality, q, as given and chooses R&D expenditures, x_1, x_2, ..., and introduction intervals, t_1, t_2, ..., so that new products are introduced at $T_1=t_1$, $T_2=t_1+t_2$, etc. Thus, T_i's are the calendar dates at which new products are introduced while t_i's are the lengths of time between introductions. The computer introduced at T_i is of quality $q+\sum_{j=1}^{i} g(x_j,t_j)$, and will be "consumed" over $[T_i, T_{i+1})$. Thus it generates total benefit of $\left[q+\sum_{j=1}^{i} g(x_j,t_j)\right]\frac{1-e^{-rt_{i+1}}}{r}$ (as of date T_i). The value of the welfare program given a list of R&D expenditures (x_1, x_2,...) and introduction intervals (t_1, t_2, ...) is:

$$W(x_1,t_1,x_2,t_2,...)=(q-x_1)\frac{1-e^{-rt_1}}{r}+e^{-rT_1}\left\{-F+[q+g(x_1,t_1)-x_2]\frac{1-e^{-rt_2}}{r}\right\}$$

$$+e^{-rT_2}\left\{-F+[q+g(x_1,t_1)+g(x_2,t_2)-x_3]\frac{1-e^{-rt_3}}{r}\right\}+...$$

$$=\frac{q}{r}+\sum_{i=1}^{\infty}e^{-rT_i}\left\{-F+[\sum_{j=1}^{i}g(x_j,t_j)-x_{i+1}]\frac{1-e^{-rt_{i+1}}}{r}\right\}-x_1\frac{1-e^{-rt_1}}{r}.$$

Consider the auxiliary optimization program of a social planner who is constrained to choose a constant level of x and t. That is, it must choose an (x,t) to maximize

$$G(x,t)\equiv W(x,t,x,t,...)=\frac{e^{-rt}}{1-e^{-rt}}\left[-F+\frac{g(x,t)}{r}\right]-\frac{x}{r}. \qquad (1)$$

Proposition 1, below, argues that the solution of the planner's unconstrained optimization problem (the maximization of W( )) is equivalent to the maximization of the constrained function, G(•,•), provided that the latter obeys the following restrictions.

Assumption B: G is strictly concave and attains an interior maximum.

Remark: Assumption B could be replaced by requirements on the primitive g. For instance, requiring the marginal productivity of x and t to be infinite at x=0 and t=0, respectively, and to be zero as $x \to \infty$ and $t \to \infty$ which ensures an interior maximum. However, such requirements are stronger than assumption B and are more cumbersome to work with.

Proposition 1: (i) Optimal introductions are at equally spaced intervals, $t_1=t_2=...=t^o$ and optimum R&D expenditures are constant over time, $x_1=x_2=....=x^o$. The optimum is independent of the present q.

(ii) The optimum is interior (x^o, $t^o \in (0,\infty)$) and is characterized by the following conditions.

$$g(x,t) = rF + g_t(x,t)\frac{1-e^{-rt}}{r} \quad , \tag{2a}$$

$$\frac{1-e^{-rt}}{r} = e^{-rt}\frac{g_x(x,t)}{r}. \tag{2b}$$

Condition (2a) brings into equality the marginal cost and the marginal benefit of the research period, t. The left-hand-side is the marginal cost of postponing innovations, which is due to the fact that consumers are denied the utility increment g(x,t) for Δt units of time. The right-hand-side is the marginal benefit, which is due to the fact that the fixed cost, F, is incurred later (saving $rF\Delta t$), and to the quality increment, $g_t(x,t)\frac{1-e^{-rt}}{r}\Delta t$, which a longer research period brings about. Likewise condition (2b) equates the marginal cost of an extra $\$\Delta x$ of research expenditures with the corresponding marginal benefit.

Proof of Proposition 1: (i) In maximizing W we can ignore the q/r term which does not depend on the decision variables (x_1, t_1, x_2, t_2, ...). Therefore, we maximize $\sum_{i=1}^{\infty} e^{-rT_i}\left\{-F + \frac{1-e^{-rt_{i+1}}}{r}[\sum_{j=1}^{i} g(x_j,t_j) - x_{i+1}]\right\} - x_1\frac{1-e^{-rt_1}}{r}$, which we continue to call W. After some manipulations we can write (the new) W as :

$$W(x_1,t_1,x_2,t_2,\ldots)=e^{-rt_1}\left[-F+\frac{g(x_1,t_1)}{r}+W(x_2,t_2,x_3,t_3,\ldots)\right]-x_1\frac{1-e^{-rt_1}}{r}. \quad (3)$$

From this expression we see that maximization with respect to (x_2, t_2, x_3, t_3,...) is independent of how (x_1, t_1) is chosen. Also, since only the names of variables are different, we have $\underset{x_1,t_1,\ldots}{Max}W(x_1,t_1,x_2,t_2,\ldots)=\underset{x_2,t_2,\ldots}{Max}W(x_2,t_2,x_3,t_3,\ldots)\equiv W^*$. Therefore, if we take the maximum on both sides of (3), we see that the maximum is characterized by the following two conditions:

$$(x_1^*, t_1^*) \text{ Maximizes } e^{-rt_1}\left[-F+\frac{g(x_1,t_1)}{r}+W^*\right]-x_1\frac{1-e^{-rt_1}}{r} \qquad \text{(M1)}$$

and

$$W^*=\frac{e^{-rt_1^*}}{1-e^{-rt_1^*}}\left[-F+\frac{g(x_1^*,t_1^*)}{r}\right]-\frac{x_1^*}{r}. \qquad \text{(M2)}$$

Now let (x^o, t^o) be the unique maximizer of the G function defined in (1) ((x^o, t^o) is unique because G is strictly concave), and let W^o=G(x^o, t^o). Then (x^o,t^o,W^o) satisfies (M2). Also, if there was an (x',t') so that $e^{-rt'}\left[-F+\frac{g(x',t')}{r}+W^o\right]-x'\frac{1-e^{-rt'}}{r}>W^o$, we would have

$\frac{e^{-rt'}}{1-e^{-rt'}}\left[-F+\frac{g(x',t')}{r}\right]-\frac{x'}{r}>W^o$. But this would contradict the choice of W^o. Hence, (M1) is satisfied too.

Assume there was another triple (x', t', W') satisfying (M1) and (M2) with (x', t')≠(x^o, t^o). Then, we must have $W'<W^o$ and this implies:

$$e^{-rt^o}\left[-F+\frac{g(x^o,t^o)}{r}+W'\right]-x^o\frac{1-e^{-rt^o}}{r}=\left(1-e^{-rt^o}\right)W^o+e^{-rt^o}W'>W'.$$

But this contradicts (M1). Therefore, we have shown that (M1) and (M2) are uniquely satisfied by (x^o, t^o). Now, going back to (3), it remains to maximize $(x_2,t_2,x_3,t_3,\ldots)$. But, by the same token, the maximizing value of (x_2, t_2) must be (x^o, t^o), and likewise for all other (x_i, t_i)'s. So the optimal program is indeed constant.

(ii) We have now shown that maximizing W is equivalent to maximizing G. The first-order conditions for maximizing G are (2a) and (2b), and they are uniquely satisfied at an interior point because of assumption B. ■

Next we determine the comparative statics properties of the optimum with respect to r and F. The appendix shows that the effect of changes in these parameters is as follows.

$$\frac{\partial t^o}{\partial F} > 0, \quad \frac{\partial x^0}{\partial F} \gtrless 0. \tag{4a}$$

When implementation costs increase we want to economize on them which results in a longer waiting time between implementations. On the other hand, the effect on R&D expenditures is ambiguous. If x and t were substitutes (g_{xt}<0) the increase in t would imply a decrease in x, meaning less R&D in response to higher implementation cost. However, if x and t are complements, the increase in t implies nothing about x[6].

$$\frac{\partial x^o}{\partial r} \gtrless 0, \quad \frac{\partial t^0}{\partial r} \gtrless 0. \tag{4b}$$

That is, the effect of changes in the interest rate are ambiguous. An increase in the interest rate works in two opposite directions: On the one hand, it raises the benefit to waiting because it raises the cost savings, rF. On the other hand, it reduces the benefit to waiting because it decreases the incremental productivity, $g_t(x,t)\frac{1-e^{-rt}}{r}$. The net effect is ambiguous.

By way of numerically illustrating the results let F=20 and r=0.05. Then if g(t)=t, t^o=6.676, while if g(t)=√t, t^o=3.424.

4. Monopoly Equilibrium

Suppose computers are produced by a monopolist which cannot commit in advance to a sequence of introduction dates and/or R&D expenditures. If the monopolist rents computers

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[6] The effect of substitutes vs. complements is not symmetrical. As per equation (1) we maximize $\frac{e^{-rt}}{1-e^{-rt}}g(x,t)-\frac{x}{r}$ with respect to x. When g_{xt}<0 and we increase t, the marginal productivity of x decreases on account of $\frac{e^{-rt}}{1-e^{-rt}}$ being smaller and on account of g_{xt} being negative. Hence, $\frac{\partial x^o}{\partial F}$ must be negative. However, when g_{xt}>0 and we increase t, we have two opposing effects, so it is not clear whether the marginal productivity of x goes up or down.

(which is the case with some mainframes), it will charge the full value, which is $q per unit time for a quality q computer. In that case, maximizing discounted profits is the same as maximizing social welfare. Therefore, under rentals, the monopoly equilibrium implements the social optimum (and the monopoly realizes the full social surplus).

In contrast, suppose the monopolist *sells* computers to consumers. (Because of legal limitations, which preclude exclusive rentals. Or, because the cost of computers is sufficiently low that consumers find it financially feasible to buy them outright.) Then, since computers are durable, their owners have the option to keep using them, even though technologically more advanced computers are available for sale. Therefore, to induce an owner to upgrade, new computers must be offered at sufficiently attractive prices.

In particular, suppose a consumer currently owns a computer of quality q_{-1}, a new computer of quality $q_0=q_{-1}+g(x,t)$ is offered for sale, and she expects the next introduction to occur t^e periods hence (at which point she expects to upgrade to the next generation computer, say q_{+1}). Consult figure 1 below for illustration of this timing sequence.

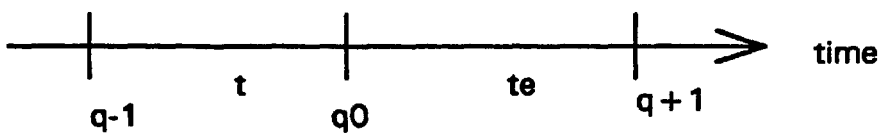

FIGURE 1

By buying the new model of quality q_0, she expects to get a flow utility increment of g(x,t) over t^e periods, yielding a discounted utility increment of $g(x,t)\frac{1-e^{-rt^e}}{r}$. This is the maximum that the consumer is willing to pay for the q_0 model. The monopolist, in turn, takes this willingness to pay as given when determining its timing and R&D expenditures decisions.

We shall derive a rational expectations equilibrium in which the monopolist innovates optimally, taking consumers' expectations as given, and in which consumers' expectations are fulfilled. We restrict attention to equilibria in which consumers' expectations are *constant*, i.e., in which t^e is the same for each generation of technology. Then, $g(x,t)\frac{1-e^{-rt^e}}{r}$ expresses the

consumers' willingness to pay[7] for *any* innovation. Let $\pi(x_1, t_1,...)$ be the monopolist's profit function when the time between the $(i-1)^{th}$ and i^{th} introduction is t_i, and the R&D expenditure level associated with the ith introduction is x_i. Then

$$\pi(x_1,t_1,\ldots)=e^{-rt_1}\left[-F+g(x_1,t_1)\frac{1-e^{-rt^e}}{r}+\pi(x_2,t_2,\ldots)\right]-x_1\frac{1-e^{-rt_1}}{r}. \qquad (5)$$

Consider the auxiliary maximization program in which the monopolist is constrained to choose a constant level of x and t (analogous to the planner's auxiliary program, (1)). That is, the monopolist must choose an (x,t) to maximize

$$H(x,t;t^e)\equiv\frac{e^{-rt}}{1-e^{-rt}}\left[-F+g(x,t)\frac{1-e^{-rt^e}}{r}\right]-\frac{x}{r}.$$

We now add the following restriction.

Assumption C: H is strictly concave and attains an interior maximum for every $t^e\in(0, \infty)$.

This assumption coincides with assumption B when $t^e=\infty$ and is automatically satisfied when $t^e=0$. Therefore, it strengthens assumption B.

Proposition 2: (i) There exists a rational expectations monopoly equilibrium in which computers are introduced at equally spaced intervals. The monopolist's profit and the R&D expenditures remain constant over time. (ii) The equilibrium strategies, call them (x^m, t^m), are determined from the following conditions.

$$g(x,t)=\frac{rF}{1-e^{-rt}}+g_t(x,t)\frac{1-e^{-rt}}{r}, \qquad (6a)$$

$$\frac{1-e^{-rt}}{r}=e^{-rt}\frac{g_x(x,t)}{r}(1-e^{-rt}). \qquad (6b)$$

(iii) If $g_x(x,t)-tg_{xt}(x,t)>0$ the equilibrium is unique.

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[7]Therefore, consumers' willingness to pay depends on the incremental quality of the product they are being offered, which is determined by the R&D expenditures, x, and the length of the development period, t. However, the length of time they expect the next product to appear (rendering the product they now buy obsolete) is held constant by assumption. Therefore their willingness to pay is only a function of x and t, and is the same function at any point in time.

Conditions (6a)-(6b) are analogous to (2a)-(2b): They bring into equality the marginal cost and the marginal benefit of the research period, t, and the R&D budget, x.

Proof of Proposition 2: (I) Fix t^e, which the monopolist takes as given. Because of the consumers' *constant* willingness to pay function, the future looks the same to the monopolist following any introduction. Consequently, analogous arguments to those in the proof of proposition 1 establish that the optimal program for the monopoly involves a *constant* $x^*(t^e)$ and $t^*(t^e)$, which maximize $H(x,t;t^e)$.

Since H is concave, the optimal solutions, $x^*(t^e)$ and $t^*(t^e)$, are continuous in t^e. Also, for $t^e=0$, the optimum t^* is ∞ while for $t^e=\infty$, the optimum t^* is 0. Therefore, there must be a t^e for which $t^*(t^e)=t^e$. This t^e constitutes a rational-expectations equilibrium since the monopolist is maximizing against consumers' demands, consumers are maximizing given their expectations, and consumers' expectations are fulfilled.

(ii) Differentiating H gives the following F.O.C.'s:

$$\frac{e^{-rt}}{1-e^{-rt}} g_x(x,t)\frac{1-e^{-rt}}{r} - \frac{1}{r} = 0, \tag{7a}$$

$$\frac{-r}{1-e^{-rt}}\left[-F + g(x,t)\frac{1-e^{-rt}}{r}\right] + g_t(x,t)\frac{1-e^{-rt}}{r} = 0. \tag{7b}$$

If we let $t=t^e$ in those conditions we obtain (6a) and (6b)

(iii) In appendix II.1 we verify, under the stated assumption, that $t^*(t^e)$ is strictly decreasing. Therefore, there is a unique t^e so that $t^*(t^e)=t^e$.∎

The comparative statics properties of the monopoly equilibrium are shown in the appendix to be similar to the social optimum's. Namely,

$$\frac{\partial x^m}{\partial F} \gtrless 0, \quad \frac{\partial t^m}{\partial F} > 0. \tag{8a}$$

$$\frac{\partial x^m}{\partial r} \gtrless 0, \quad \frac{\partial t^m}{\partial r} \gtrless 0. \tag{8b}$$

Results (8a-8b) are relevant to the empirical evidence on the rate of technological progress for various durable products. A variety of studies, see Chow (1967), Mansfield (1968), Berndt and Griliches (1993) and Greenstein (1994), indicate a high degree of heterogeneity with respect to the pace of technological progress in a cross-section of products. For example, the rate of technological progress of PC's is significantly faster than that of mainframe computers[8]. Our model enables one to make empirically testable predictions about what drives the rate of technological progress, and to interpret heterogeneity across different products as coming from different values of the model's underlying parameters. For instance, comparing PC's and mainframes, we might interpret faster technological progress for PC's as coming from smaller implementation costs or from a larger market size (which makes the effective F smaller.) Likewise, the model would predict (other things being the same) that software programs are upgraded more frequently than the PC's on which they are run.

Another interesting feature of the monopoly equilibrium is that the nominal price of each generation of technology remains constant over time, $g(x^m,t^m)\frac{1-e^{-rt^m}}{r}$, although each generation is of higher quality than its predecessor. Hence, the real, or the *quality-adjusted*, price of computers decreases over time.

This situation has been extensively studied in the empirical literature, which devised ways of measuring the quality-adjusted price (QAP, henceforth) of products which undergo technological improvment. The QAP accounts for changes in the characteristics of products by estimating the hedonic prices of characteristics. This enables one to compute the quality of products containing different sets of characteristics, which enables the measurement of, and the comparison between, their QAP's. Examination of the time series of QAP's reveals several features. First, the QAP decreases over time as products with the same characteristics sell for a lower nominal price and/or as the nominal price remains constant but as products contain more characteristics (or both). Second, the rate of decrease of the QAP is typically not constant over time; it decreases rapidly at early phases of the product and then the rate of decrease tapers off. Third, as reported by Chow (1967), Berndt and Griliches (1993) and Greenstein (1994), the QAP of mainframe computers had decreased at an average rate of 20% per year, while the QAP of PC's had decreased at an average rate of 30% per year. Thus, the rate of decrease of the QAP is

[8]Berndt and Griliches (1993) report an average of 2-3 years between new PC model introductions, while Greenstein reports an average of 6 years for mainframes.

positively correlated with the rate of technological progress: the faster new products are introduced, the faster is the rate of decline in their QAP's[9].

Our model is consistent with these relationships. Define the quality-adjusted price as the ratio of the nominal price, p, to the product quality, q[10]. Then we have.

Proposition 3: (i)The faster is the equilibrium rate of product introductions, $1/t^m$, the faster will be the rate of decrease in the quality-adjusted price.

(ii) The rate of decrease in the QAP is monotonically decreasing as the product ages.

Proof of Proposition 3: First, the nominal price of products remains constant over time: $g(x^m,t^m)\frac{1-e^{-rt^m}}{r}$. Second, if we start with initial quality of 0 at time 0, and consider a date $T=nt^m$, then the quality of the product introduced at T is $ng(x^m,t^m)$. Therefore, the QAP at T is $\frac{1-e^{-rt^m}}{nr}$. Third, consider two distinct dates $T_1=n_1t^m$ and $T_2=n_2t^m$ with $n_2>n_1$ and with QAP's of $\frac{1-e^{-rt^m}}{n_ir}$, i=1,2. Then the rate of decrease in the QAP between T_1 and T_2 is determined as the solution, λ, to the equation $\frac{1-e^{-rt^m}}{n_1r}e^{-\lambda(T_2-T_1)}=\frac{1-e^{-rt^m}}{n_2r}$. Solving this equation we obtain $\lambda=\frac{1}{(n_2-n_1)t^m}\log\frac{n_2}{n_1}$. From this expression we can see two things: (i) λ is proportional to $1/t^m$. Therefore the more frequently products are introduced (the smaller is t^m), the larger is the rate of decrease, λ, in their QAP. (ii) Fixing the difference n_2-n_1, the ratio $\frac{n_2}{n_1}$ is decreasing as n_1 increases. Therefore as the product ages, λ decreases.∎

Numerically illustrating the results in this section, we let again F=20, r=0.05 (see end of last section). Then if g(t)=t we have $t^m=10.701$, while if $g(t)=\sqrt{t}$, $t^m=12.218$. Therefore, in this

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[9] It should be noted, however, that this is not a *causality* relationship. Rather, the hedonic price and the rate of product introductions are endogenous variables and are determined (in equilibrium) by another set of variables, namely, the "fundamentals", r, F and g(x,t). As the fundamentals change, the endogenous variables change as well, which generates the positive correlation.

[10] In our model, "quality" is a uni-dimensional concept. Hence, the quality-adjusted price is naturally defined as the ratio of nominal price to quality. In empirical studies one has to account for the fact that quality corresponds to a combination of various characteristics, for instance to the size of hard-drive memory, measured by MB's, or to the clock speed as measured by Mhz. Accordingly, the procedure is to generate hedonic prices of the various characteristics, which enables one to attach a uni-dimensional number (the "quality") to any combination of characteristics. Then this number can be used to deflated the nominal price which gives the QAP.

numerical example, introductions occur less frequently under the monopoly than under the social optimum. The following proposition shows that this is true in general.

Proposition 4: (i) The monopolist introduces new products less frequently than the social planner. (ii) If x and t are substitutes, the monopolist invests less in R&D than the social planner, $x^m<x^o$. If x and t are complements, x^m could be either smaller or larger than x^o.

Proof of Proposition 4: The proof is based on comparison between the first-order conditions to the planner's program, (2a) and (2b), and the equilibrium conditions, (6a) and (6b). Consider an equilibrium, (x^m, t^m), at which (6a) and (6b) are satisfied. Then, since $rF/(1-e^{-rt})>rF$, we must have $G_t(x^m, t^m)<0$ and since $1-e^{-rt}<1$ we must have $G_x(x^m, t^m)>0$, where $G(x,t)$ is the social welfare function. Now consider the geometric representation of the first order conditions to the planner's program, $G_t=0$ and $G_x=0$. Then, as shown in the appendix, there are two possibilities which are depicted in figures 2A and 2B: Either both $G_t=0$ and $G_x=0$ are upward sloping, or both are downward sloping. In the first instance, (x^m, t^m) must lie to the north east of (x^o, t^o) while in the second instance it must lie to the north west (because of $G_x>0>G_t$). Either way, $t^o<t^m$, i.e., the social planner introduces new products more frequently than the monopolist.

(ii) If $g_{xt}(x,t)<0$, i.e., x and t are substitutes, the $G_x=0$ and $G_t=0$ curves are downward sloping, so x^m must be less than x^o. Otherwise, the curves might be downwards or upwards sloping, so $x^m \lessgtr x^o$.[11] ■

[11] We can further elucidate on the intuition behind the proof as follows. First, holding the R&D expenditures constant, the monopolist has to wait longer before consumers' valuation justifies introduction of a new product: $t^m>t^o$. Second, if time and R&D expenditures are substitutes more development time implies less R&D expenditures. However, if they are complements we *may* increase R&D (see footnote 6 about the asymmetric effect of complements vs. substitutes)

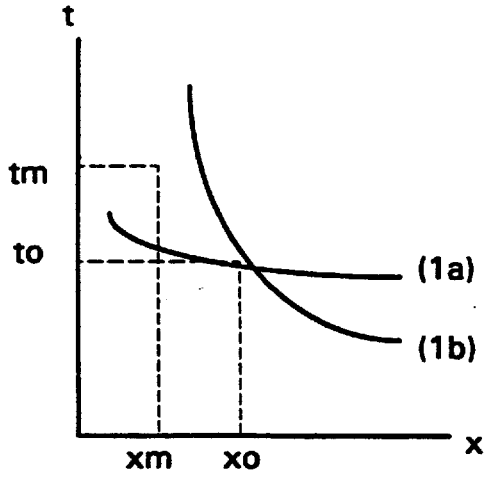

FIGURE 2A

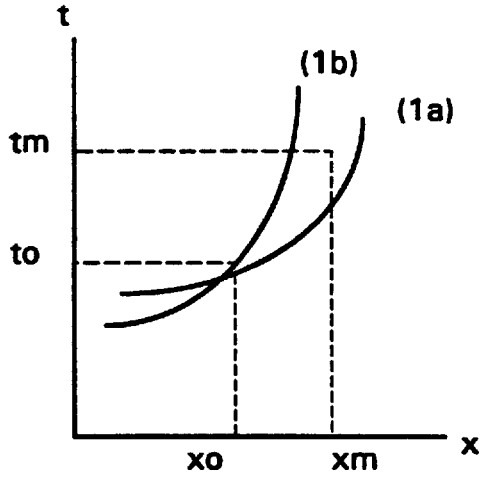

FIGURE 2B

The economic reason behind this is that each product introduction is socially valuable not only because it allows consumers to upgrade to a better product, but also because it raises the base from which the development of future products will start, resulting in a quality increase for *all* subsequent products. However, in the decentralized setting only the market for one product is operating, and consumers' willingness to pay for it reflects its incremental quality over and above the quality of the previous product. Consumers are not taking into account the contribution that each new product introduction makes to the quality of future products. This drives a wedge between the private value and the social value of product introductions, which leads to a discrepancy between the social optimum and the monopoly equilibrium.[12]

Therefore, comparing the efficiency of market outcomes, the present formulation provides an argument in favor of rentals as opposed to sales: The monopoly equilibrium under rentals replicates the social optimum, while under sales it does not (either way, the monopoly extracts the full social surplus, so in terms of equity there is no difference between the two arrangements). In the more general case where demand exhibits some elasticity, there would be a countervailing force, namely, the static monopoly distortion. However, it is still possible (even with demand elasticity) that social welfare under rentals exceeds social welfare under sales. This result should be contrasted with the Coase conjecture literature (Coase (1972), Bulow (1982), and many others), where product development is not an issue and, hence, where sales always result in more

[12]This result is similar in spirit to the one in the learning-by-doing literature (see Arrow (1962)), or in the more recent R&D literature (see Schotchmer (1991)).

efficient outcomes than rentals (without product development, sales have the sole effect of eliminating monopoly distortion without distorting the incentive to introduce new products).

5. The Duopoly Scenario

Next we consider the duopoly case. There are two firms in the market, playing a noncooperative repeated game with two strategic variables: When to introduce a new product (given how much time has elapsed since the last introduction), and how to price it. To simplify the analysis we consider the case where g depends on t only[13]. That is the technology is assumed to progress exogenously and firms only decide when to implement it, but not how much to spend on R&D. There are two ways to interpret this. First it represents the situation where research results are in the public domain, as is the case with some research done at the University or the Government level, but where it is still costly (F) to put research results into commercial use. Then firms decide at which point research results have been sufficiently significant that it pays to put them into commercial use. In that case our formulation is in the spirit of the technology adoption literature--see Reinganum (1981), Fudenberg and Tirole (1985), or Riordan and Salant (1994). A second interpretation is that another firm (Intel, say) specializes in R&D and sells its results, at a price F, to the product-market implementor. This interpretation, however, is somewhat more tenable since the vertical structure, including the pricing decision of the upstream firm, is not modeled here.

The following restriction, which is a special case of assumptions A and B, is imposed on g:

Assumption C: g is strictly increasing, strictly concave and there exists a t for which $g(t)>rF$.

Price competition in the product market is modeled as follows. If two firms sell a product at the same date, the products they offer for sale are identical, Bertrand competition ensues and both firms make zero profits (until the next introduction). If only one firm introduces a product, it acquires an infinite patent on it, and is therefore a monopolist. As before, its monopoly power is

[13] The inclusion of x and t in the g function would lead to technical complications, including the non-existence of pure strategy equilibria. To get around that problem we could either allow mixed strategy equilibria, although mixed strategies are hard to interpret in this context. Or, we might allow for stochastic improvement functions. However, since the state of technology is continuous, one would need to use fairly advanced stochastic process theory, for example let the improvement follow a Brownian motion. A more appealing alternative and one which allows us to compare the planner's-monopoly-and duopoly cases (see proposition 5) is to suppress the R&D variable.

constrained by consumers' expectations about the next product introduction (which may be by the same firm, cannibalizing its own product, or by the other firm). There are no breadth limitations[14], so that even small product improvements are patentable[15].

As in the monopoly section, we consider only rational expectations equilibria of this game in which, following any past history, consumers expect future products to be introduced at equally spaced intervals. Let t^e denote consumers' expectations (the time between introductions). Then, consumers' willingness to pay is $g(t)\frac{1-e^{-rt}}{r}$ if t periods have elapsed since the last introduction.

We restrict firms' strategies to be *Markovian*. These are strategies which depend only on state variables which affect profits directly, and are independent of 'irrelevant' past actions[16]. In this context, the natural state variable is the time elapsed since the last introduction, t, since only this determines profits from further innovation. Let V(t) be the value to a firm of participating in the market when t periods have elapsed since the last introduction. The Markovian assumption implies that V must be independent of anything but t and, in particular, is independent of firms' identity, implying that V is common to both firms. Since a firm can always drop out (not innovate), any equilibrium V must be non-negative.

Proposition 5: There exists a rational expectations duopoly equilibrium at which a lone firm introduces each new product, products are introduced at equally spaced intervals and firms make zero profits. The equilibrium spacing, t^d, is unique and is characterized by the condition:

$$g(t)\frac{1-e^{-rt}}{r} = F. \tag{9}$$

Proof of Proposition 5: Assume there was an equilibrium of the type described in the proposition and let t* be the equilibrium spacing. Let V*=V(t*). Consider a date at which firm i (say) is scheduled to introduce the next technology. Its discounted profit from the product introduced at

[14]See Klemperer (1990) and Gilbert and Shapiro (1990) for discussion and analysis of "breadth limitations"

[15]Although the lack of breadth limitations allow firms to introduce at "every instant", the presence of implementation cost will cause them to wait a positive amount of time between introductions.

[16]Otherwise, as is well known from the theory of infinitely repeated games, there are multiple equilibria which, as the interest rate converges to zero, can implement any individually rational payoff.

that date is $-F+g(t^*)\frac{1-e^{-rt^*}}{r}$ (using the rational expectations requirement that $t^*=t^e$), so its total discounted profit from that date and onwards is $-F+g(t^*)\frac{1-e^{-rt^*}}{r}+e^{-rt^*}V^*$. Thus,

$$V^* = -F+g(t^*)\frac{1-e^{-rt^*}}{r}+e^{-rt^*}V^*, \text{ which implies } V^* = \frac{-F+g(t^*)\frac{1-e^{-rt^*}}{r}}{1-e^{-rt^*}}.$$

Suppose $V^*>0$. This implies $g(t^*)\frac{1-e^{-rt^*}}{r}>F$. But then there must be a $t'<t^*$ so that $g(t')\frac{1-e^{-rt'}}{r}>F$. Consider firm j which strictly prefers not to introduce a new technology at t' (according to the postulated equilibrium). If firm j follows its equilibrium strategy its total discounted profit is $e^{-r(t^*-t'+t')}V^*$. On the other hand, if it pre-empts firm i and innovates only t' after the previous introduction, its total discounted profit from that date and onwards is $-F+g(t')\frac{1-e^{-rt'}}{r}+e^{-rt'}V^*$. This is because its pre-emptive introduction has no effect on the path of future introductions, by the Markovian assumption. But $-F+g(t')\frac{1-e^{-rt'}}{r}+e^{-rt'}V^*>e^{-r(t^*-t'+t')}V^*$. Therefore, firm j can increase its discounted profit , which contradicts the supposition that $V^*>0$. This shows that $V^*=0$ and that $t^*=t^d$, the t which uniquely solves equation (9) (unique because the left-hand side of (9) is strictly increasing). This shows that the equilibrium described in the proposition is the only possible stationary one. It remains to show that such an equilibrium exists.

Let the firms strategies be as follows. For $t<t^d$, firms innovate with probability zero. For $t=t^d$ one firm, say firm i, innovates with probability 1 and the other innovates with probability 0. For $t>t^d$ (which is off the equilibrium path), each firm innovates with probability $\frac{g(t)(1-e^{-rt^d})-rF}{g(t)(1-e^{-rt^d})}$. When firm i innovates with this probability, firm j's expected profit from innovating at the same date is zero. These strategies implement the equilibrium outcome described in the proposition. ∎

<u>Remark:</u> Fudenberg and Tirole (1985) show how to represent discrete probability distributions (the mixed strategies) in a continuous time framework, a concept we use in constructing the off-equilibrium path. Relatedly, they also show that there exists a *symmetric* Markovian equilibrium in which each firm introduces each new technology with probability 1/2, and firms never duplicate

each other. This formalization can be replicated here. However, it would add a considerable amount of notation without a compensating increase in results.

Now we can compare the duopoly outcome with the monopoly outcome and the social optimum when the g function depends on t only (in all three cases). First, comparing (9) with (6), we see that the duopoly introduces products more frequently than the monopoly: $t^d<t^m$. The intuition for this is that each firm tries to seize the monopoly-rents from the other firm which, in the end, leads to more frequent introductions and to the dissipation of rents. The comparison to the social planner is less clear. We have either $t^d<t^o$ or $t^d>t^o$, depending on parameters of the model. There are two opposing forces here: On the one hand, consumers still ignore the contribution of the present introduction to the quality of future products, which delays the introduction of products (same force as under the monopoly market structure). On the other hand, firms compete, which hastens the introduction process. Hence, the net result can go either way.

By way of numerically illustrating these possibilities, let again F=20, r=0.05. Then for $g(t)=t$ we have $t^d=4.739$ which indicates more frequent introductions than the social optimum. On the other hand, for $g(t)=\sqrt{t}$ we have $t^d = 8.439$ which indicates less frequent introductions.

In terms of efficiency, the duopoly outcome is more efficient than the monopoly outcome if $t^d>t^o$, because the welfare function is concave and t^d is closer to the optimum. In case $t^d<t^o$ we cannot compare the efficiency of the two outcomes.

In terms of equity, the duopoly and monopoly outcomes represent two extremes: in the monopoly case the social surplus is fully extracted by the monopoly; in the duopoly case it is fully extracted by the consumers.

6. Policy instruments

Continuing with the case of exogenous technological progress (g depending on t only), we now examine the working of different policy instruments (other than the choice between rentals and sales which has already been discussed) to remedy the inefficiency created under the duopoly or under the monopoly outcome. There are two types of instruments to consider. First, we can influence costs and benefits in the product market, e.g., by taxing or subsidizing sales of the product. Second, we can use patent policy, e.g., by limiting the life of patents on existing products, or instituting breadth limitations on future patents.

When introductions are too infrequent ($t^d > t^o$), we can induce a faster introduction process by making the private F smaller. For instance, the government can give firms an investment tax credit which induces an $F' < F$. The F' is chosen so that the solution to $g(t)\frac{1-e^{-rt}}{r} = F'$ is t^o. (this can be done since the LHS is zero at t=0 and is increasing in t). Alternatively, product subsidies can be used to achieve the same outcome. If we let σ be the subsidy rate, then σ should be chosen so that $(1+\sigma)g(t^o)\frac{1-e^{-rt^o}}{r} = F$. Either way, product subsidy or investment tax credit, the introduction of new products becomes more profitable, which speeds up the innovation process. The same idea applies to the monopoly solution, which also exhibits too infrequent introductions.

When introductions under duopoly are too frequent ($t^d < t^o$), there are two possible remedies. First of all, a sales tax can be imposed on the product which will lower the return to introductions and, hence, will delay their frequency. Let τ be the tax rate. Then τ should be chosen so that $(1-\tau)g(t^o)\frac{1-e^{-rt^o}}{r} = F$.

Alternatively, we can shorten the patent life. Then, once a patent expires and competitors are able to costlessly imitate the product design, the price of computers drops to zero (consequence of Bertrand competition). Consequently, given that consumers anticipate the price decrease, the maximum price a computer can sell for when it is first introduced is lowered. This lowers the return to new products, which induces the duopolists to wait longer so they can recover their fixed cost. Let t^P denote the length of patents. Then the maximum that a consumer is willing to pay is $g(t)\frac{1-e^{-rt^P}}{r}$ (assuming $t^d > t^P$). So, again, we can find a value, t^P, so that the solution to $g(t)\frac{1-e^{-rt^P}}{r} = F$ is t^o.

However, while there may be more than one policy instrument to remedy the duopoly distortion, not all instruments are equally effective. Consider again the case of a duopoly who introduces too frequently ($t^d<t^o$). Then a "natural" solution is to institute a breadth limitation, t^o, so that a new product which improves upon an existing product by less than $g(t^o)$ is not recognized as sufficiently novel. Then, the duopoly payoff, (9), becomes discontinuous at $t=t^o$: For $t<t^o$ the payoff is negative, since F is paid but there is no corresponding revenue; for $t>t^o$ the payoff is positive and coincides with the original (9). But, then, both firms try to introduce at t^o which, at a symmetric equilibrium, implies mixed strategies, the duplication of F and zero revenues[17]. Therefore, breadth limitation in this instance results in less social welfare than length limitation.

[17]In this situation there is also an asymmetric equilibrium, with one firm implementing at t^o, and the other not implementing. However, the implementing firm makes higher profits than the non-implementing firm which raises the question how firms would coordinate on such equilibrium. Therefore, this equilibrium seems less reasonable than the symmetric, mixed strategy equilibrium, where both firms make the same (zero) profits.

APPENDIX

I. Maximization of G and its comparative statics properties.

I.1 We want to maximize the differentiable function

$$G(x,t) \equiv \frac{e^{-rt}}{1-e^{-rt}}\left[-F+\frac{g(x,t)}{r}\right]-\frac{x}{r},$$

with respect to x,t≥0. First, compute its partial derivatives, G_x and G_t.

$$G_x = \frac{e^{-rt}}{1-e^{-rt}}\frac{g_x(x,t)}{r}-\frac{1}{r}, \qquad \text{(A1)}$$

$$G_t = \frac{e^{-rt}}{1-e^{-rt}}\left\{\frac{1}{1-e^{-rt}}\left[rF-g(x,t)\right]+\frac{g_t(x,t)}{r}\right\}. \qquad \text{(A2)}$$

The first order conditions to the maximization of G are G_x=0 and G_t=0. Differentiating these conditions, we see that they are represented in (x,t)-space by 2 curves whose respective slopes are $\frac{dx}{dt}=-\frac{G_{xt}}{G_{xx}}$ for G_x=0 and $\frac{dt}{dx}=-\frac{G_{xt}}{G_{tt}}$ for G_t=0. But

$$G_{xt} = \frac{-e^{-rt}}{(1-e^{-rt})^2}g_x(x,t)+\frac{e^{-rt}}{1-e^{-rt}}\frac{g_{xt}(x,t)}{r}, \qquad \text{(A3)}$$

which is unsigned (unless we assume $g_x(x,t)-tg_{xt}(x,t)>0$).

Therefore, there are two possibilities: (i) either $G_{xt}>0$, in which case the two curves are upward sloping; (ii) or $G_{xt}<0$, in which case the two curves are downward sloping. Either way, since an interior maximum obtains and since G is strictly concave (assumption B) the two curves intersect uniquely and divide the (x,t)-space into 4 regions. Also, since G is strictly concave we have $\left|\frac{G_{xx}}{G_{xt}}\right|>\left|\frac{G_{xt}}{G_{tt}}\right|$. Thus, the G_x=0 curve is always steeper than the G_t=0 curve. The figure below shows the two possibilities and the the signs of G_x and G_t in the four regions.

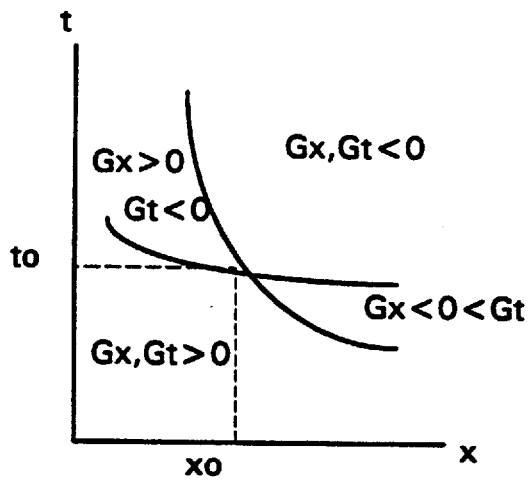

FIGURE A1

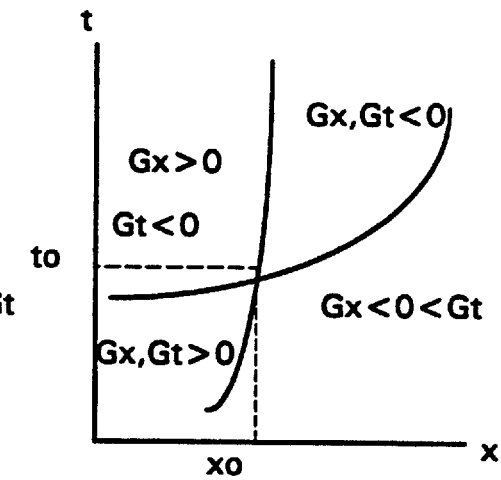

FIGURE A2

I.2. Now let us consider the comparative statics properties of the maximum with respect to F and r.

First, totally differentiate the F.O.C.'s, $G_x=0$, $G_t=0$, with respect to F which gives:

$$G_{xx}\frac{dx}{dF}+G_{xt}\frac{dt}{dF}=0,$$

$$G_{tx}\frac{dx}{dF}+G_{tt}\frac{dt}{dF}+\frac{re^{-rt}}{(1-e^{-rt})^2}=0.$$

Thus,

$$\frac{dx^o}{dF}=\frac{\begin{vmatrix} 0 & G_{xt} \\ -\dfrac{re^{-rt}}{(1-e^{-rt})^2} & G_{tt} \end{vmatrix}}{\Delta}=\frac{G_{xt}\dfrac{re^{-rt}}{(1-e^{-rt})^2}}{\Delta},$$

and

$$\frac{dt^o}{dF}=\frac{\begin{vmatrix} G_{xx} & 0 \\ G_{tx} & -\dfrac{re^{-rt}}{(1-e^{-rt})^2} \end{vmatrix}}{\Delta}=\frac{-G_{xx}\dfrac{re^{-rt}}{(1-e^{-rt})^2}}{\Delta},$$

where $\Delta \equiv G_{xx}G_{tt} - (G_{xt})^2$. But $\Delta>0$ by the strict concavity of G, so the signs of $\frac{dx^o}{dF}$ and $\frac{dt^o}{dF}$ are determined by the signs of the numerators of the above expresions. Therefore $\frac{dt^o}{dF}>0$ and $\frac{dx^o}{dF} \gtrless 0$ (depending on whether the sign of G_{xt}, which is unrestricted by our assumptions, is + or -).

Similarly, totally differentiate the F.O.C.'s, $G_x=0$, $G_t=0$, with respect to r which gives:

$$G_{xx}\frac{dx}{dr} + G_{xt}\frac{dt}{dr} + G_{xr} = 0,$$

$$G_{tx}\frac{dx}{dr} + G_{tt}\frac{dt}{dr} + G_{tr} = 0.$$

Thus,

$$\frac{dx^o}{dr} = \frac{\begin{vmatrix} -G_{xr} & G_{xt} \\ -G_{tr} & G_{tt} \end{vmatrix}}{\Delta}, \qquad \frac{dt^o}{dr} = \frac{\begin{vmatrix} G_{xx} & -G_{xr} \\ G_{tx} & -G_{tr} \end{vmatrix}}{\Delta}. \qquad \text{(A4)}$$

Since the signs of G_{xt} and G_{tr} are undetermined we cannot sign either derivative, so we have dx^O/dr and dt^O/dr >0 or <0.

II. The Monopoly Solution.

II.1. The F.O.C. to the maximization of H are:

$$H_x = \frac{e^{-rt}}{1-e^{-rt}} g_x(x,t)\frac{1-e^{-rt'}}{r} - \frac{1}{r} = 0,$$

$$H_t = \frac{e^{-rt}}{1-e^{-rt}}\left\{\frac{1}{1-e^{-rt}}\left[rF - g(x,t)(1-e^{-rt'})\right] + g_t(x,t)\frac{1-e^{-rt'}}{r}\right\} = 0.$$

Then, totally differentiating these conditions with respect to r^e, we obtain:

$$H_{xx}\frac{dx}{dt^e} + H_{xt}\frac{dt}{dt^e} + H_{xt'} = 0,$$

$$H_{tx}\frac{dx}{dt^e} + H_{tt}\frac{dt}{dt^e} + H_{tt'} = 0.$$

And this implies:

$$\frac{dx}{dt^e}=\frac{\begin{vmatrix}-H_{xt^e} & H_{xt}\\ -H_{tt^e} & H_{tt}\end{vmatrix}}{\Delta_H},\quad \frac{dt}{dt^e}=\frac{\begin{vmatrix}H_{xx} & -H_{xt^e}\\ H_{tx} & -H_{tt^e}\end{vmatrix}}{\Delta_H}, \qquad \text{(A5)}$$

where $\Delta_H \equiv H_{xx}H_{tt}-(H_{xt})^2$ which is positive by the concavity of H.

Also,

$$H_{xt}=\frac{-re^{-rt}}{(1-e^{-rt})^2}g_x(x,t)\frac{1-e^{-rt^e}}{r}+\frac{e^{-rt}}{1-e^{-rt}}g_{xt}(x,t)\frac{1-e^{-rt^e}}{r},$$

$$H_{xt^e}=\frac{e^{-rt}}{1-e^{-rt}}g_x(x,t)e^{-rt^e}>0, \text{ and } H_{tt^e}=e^{-rt^e}\frac{r}{1-e^{-rt}}\left[-g(x,t)+g_t(x,t)\frac{1-e^{-rt}}{r}\right]<0,$$

where the last inequality follows from the concavity of g, from g(x,0)=0 and from $\frac{1-e^{-rt}}{r}<t$.

Therefore, plugging this into (A5) it is seen that $dx/dt^e>0$ and that $dt/dt^e<0$, provided $g_x(x,t)-tg_{xt}(x,t)>0$

II.2. The comparative Statics properties of the Monopoly Equilibrium:

The monopoly equilibrium is characterized by conditions (6a) and (6b). We re-write them here, defining the functions $\hat{H}_x(x,t)$ and $\hat{H}_t(x,t)$:

$$\hat{H}_x(x,t)\equiv e^{-rt}g_x(x,t)-1=0,$$

$$\hat{H}_t(x,t)\equiv\frac{rF}{1-e^{-rt}}+g_t(x,t)\frac{1-e^{-rt}}{r}-g(x,t)=0. \qquad \text{(A6)}$$

The comparative statics with respect to F are determined by total differentiation of (A6):

$$\hat{H}_{xx}\frac{dx^m}{dF}+\hat{H}_{xt}\frac{dt^m}{dF}=0,$$

$$\hat{H}_{tx}\frac{dx^m}{dF}+\hat{H}_{tt}\frac{dt^m}{dF}+\hat{H}_{tF}=0,$$

where $\hat{H}_{tF} = \frac{r}{1-e^{-rt}} > 0$. Therefore, we have:

$$\frac{dx^m}{dF} = \frac{\begin{vmatrix} 0 & \hat{H}_{xt} \\ -\hat{H}_{tF} & \hat{H}_{tt} \end{vmatrix}}{\hat{\Delta}}, \quad \frac{dt^m}{dF} = \frac{\begin{vmatrix} \hat{H}_{xx} & 0 \\ \hat{H}_{tx} & -\hat{H}_{tF} \end{vmatrix}}{\hat{\Delta}},$$

where $\hat{\Delta} \equiv \hat{H}_{xx}\hat{H}_{tt} - \hat{H}_{xt}\hat{H}_{tx}$. Since $\hat{H}$ is concave we have $\hat{H}_{xx}$, $\hat{H}_{tt} < 0$ and $\hat{\Delta} > 0$. However, $\hat{H}_{xt}$ and $\hat{H}_{tx}$ are unsigned and therefore $\frac{dx^m}{dF} \gtrless 0$, $\frac{dt^m}{dF} > 0$.

Similarly, the comparative statics with respect to r are determined by total differentiation of (A6):

$$\hat{H}_{xx}\frac{dx^m}{dr} + \hat{H}_{xt}\frac{dt^m}{dr} + \hat{H}_{xr} = 0,$$

$$\hat{H}_{tx}\frac{dx^m}{dr} + \hat{H}_{tt}\frac{dt^m}{dr} + \hat{H}_{tr} = 0,$$

where $\hat{H}_{xr} = -te^{-rt}g_x < 0$ and $\hat{H}_{tr} = F\frac{1-e^{-rt}-rte^{-rt}}{(1-e^{-rt})^2} - g_t\frac{1-e^{-rt}-rte^{-rt}}{r^2}$ is unsigned. Therefore, we have:

$$\frac{dx^m}{dr} = \frac{\begin{vmatrix} -\hat{H}_{xr} & \hat{H}_{xt} \\ -\hat{H}_{tr} & \hat{H}_{tt} \end{vmatrix}}{\hat{\Delta}}, \quad \frac{dt^m}{dr} = \frac{\begin{vmatrix} \hat{H}_{xx} & -\hat{H}_{xr} \\ \hat{H}_{tx} & -\hat{H}_{tr} \end{vmatrix}}{\hat{\Delta}}.$$

Since $\hat{H}_{tr}$, $\hat{H}_{xt}$ and $\hat{H}_{tx}$ are unsigned, dx^m/dr and dt^m/dr could have either sign.

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