Marketing Information: A Competitive Analysis

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Abstract

Selling information that is later used in decision making constitutes an increasingly important business in modern economies (Jensen 1991). Information is sold under a large variety of forms: industry reports, consulting services, database access, and/or professional opinions given by medical, engineering, accounting/financial and legal professionals, among others.

This paper is the first attempt in marketing to study competition in the rapidly emerging information sector. Specifically, we are interested in answering the following questions: (1) Is competition fundamentally different when competing firms sell information rather than traditional goods and services, and - if yes - why? (2) What are the implications of such differences for decision makers (marketers and regulators)? (3) Can we explain some of the observed marketing strategies in the information industry? As such, the audience of the paper includes academics as well as professionals who are interested in understanding what is specific about competition in information markets. Familiarity with the practical implications of such differences and understanding of the mechanisms that drive them is essential for those who are faced with the problem of marketing information.

To answer the above research questions we build a simple game-theoretic model that consists of two firms selling information to a population of consumers who are heterogenous in their willingness to pay for the *quality* of information. The most important features of the

model are the following. Information products sold by the two firms are modeled as random draws from two normal distributions having equal mean. The variances of these distributions and their correlatedness constitutes the product-attribute space, which is assumed to be common knowledge. Consumers are interested in assessing the mean of the distributions and to do so they can buy the sample from any of the firms or they can buy both samples and combine them to obtain a more accurate estimate. *Quality* of information is linked to the accuracy of consumers' estimate of the mean which in turn is influenced by the accuracy of each sample as well as by their correlatedness. Consumers' utility depends on the quality of information they purchased, on their inherent utility for quality (taste) and on the total price they payed to acquire information. Knowing consumer preferences, firms simultaneously price their information products.

The main finding of the paper is that information markets face unique competitive structures. In particular, the qualitative nature of competition changes depending on basic product characteristics. While traditional products and services compete either as substitutes or as complements in the relevant product-attribute space, information may be one or the other depending on its position within the *same* product-attribute space. Said differently, the nature of competition changes *qualitatively* with a continuous change in basic product-attribute levels. The intuition behind this finding is the following. When purchasing information, consumers facing important decisions may find it beneficial to purchase from several information sellers. This is more likely to happen when the reliability of information is low and the sources of information are independent. Under such conditions information products tend to be complements and, as a result, competition between sellers is relatively mild. In the opposite case, when information is reliable and/or sellers' sources are highly correlated, consumers are satisfied after consulting a single source. In this case, information products are substitutes and sellers tend to undercut each other's prices to induce consumers to choose their brand. Understanding this discontinuity in competitive structures has important implications for decision makers as very different strategies are optimal under different product characteristics. Under substitution, traditional strategies to avoid competition (e.g. differentiation) are recommended. When the competing products' reliability is generally

low (they are complements) firms are better off accomodating competition. In fact, we find that a firm may benefit from "inviting" a competitor. Finally, our findings are also important for regulators of information markets. As the literature on complementarity suggests price fixing agreements between firms offering complementary products may benefit firms as well as consumers.

Keywords: Information sales; Competitive strategy; Complements; Substitutes.

"..[the new statistical system designed for the Information Age].. would consist of three economic sectors: goods-producing, services and information."
"The Spawning of a Third Sector: Information," *Business Week*, Nov. 7, 1994, pp 48.

1 Introduction

Recently, Portugal Telecom has simultaneously hired three independent investment banks to value the firm before its privatization. There are two decision problems underlying this story. From the Telecom's point of view the question is: how many information sources to consult in order to obtain a fair estimate for the value of the firm? On the other hand, the investment banks' problem is: how to price the consulting service given the uncertainty of the Telecom's value and given the presence of competing investment banks providing valuation services? In essence, what is being exchanged in this example is *information* - in this specific case, information about the value of a firm. This paper seeks to explore similar situations faced by marketing managers who sell information products or services in a competitive setting.

Information products can take on a variety of forms: industry reports, consulting services, database access, and professional opinions given by medical, engineering, accounting/financial, and legal professionals, among others. The term "information product" is used in a broader sense in the literature, including information technology, advertising or the media. In contrast, our definition follows Jensen (1991, p. 424), by referring to *information that is (1) paid for, and (2) valuable for making decisions* (e.g. expert advice). Our discussion, therefore, excludes advertising since it is not paid for. Likewise, information which is used to entertain (e.g. movies) is also not explored. Jensen (1991) reports detailed statistics on the business-to-business sector of the information industry in the U.S.A. for the period 1982-1988. Table 1 shows that this $17.7 billion industry is especially relevant for marketing where most of the revenues are generated.[1]

[1] In 1988 the U.S. information services industry included some 1,500 - mostly small - companies. Examples of large information services providers include Dun & Bradstreet Business Information Reports, F. W. Dodge Construction Project News, IMS International Sales Territory Reports and Consumer Reports among others.

INSERT TABLE 1 ABOUT HERE

Despite the dynamic development of the "information sector" (12% average annual growth over the 1980s) there is relatively little academic work devoted to the subject. The objective of this paper is to fill this gap by developing a simple game-theoretic framework to understand competition in information markets. In particular, we are interested in answering the following questions: (1) Is competition fundamentally different when competing firms sell information rather than traditional goods and services, and - if yes - why? (2) What are the implications of such differences for decision makers (marketers and regulators)? (3) Can we explain some of the observed marketing strategies in information markets?

The next section provides an intuitive description of the problem and relates our approach to the relevant literature. The model is formally developed in section 3. In this section, care is taken to relate the analytic findings to observed business strategies. Before concluding, section 4 briefly discusses potential limitations. To improve readability, details of the derivations as well as some extensions are reported in an appendix that is available from the authors upon request.

2 The Problem

2.1 Intuition

Traditionally, when we talk about "competing" products (brands) we think of differentiated substitutes. Most consumers choose one coffee brand over another or one car from a set of alternatives. When purchasing information, consumers facing important decisions may find it beneficial to buy from several information sellers. This is more likely to happen when the reliability of information is low and the sources of information are independent. Under such conditions, information sellers anticipate the consumers' problem and tend to charge higher prices. In the opposite case, when information is reliable and/or sellers' sources are highly correlated, consumers are satisfied after consulting a single source. In this case, sellers tend to decrease prices to induce consumers to choose their brand. In other words, higher correlation between information sources makes information products substitutes. Uncertainty and independence among information sources, however, make them complements.[2] What makes information different from other goods and services is that the latter are either substitutes or complements in the relevant product-attribute space, whereas information may be one or the other depending on its position within the *same* product-attribute space. Said differently, the nature of competition changes *qualitatively* with a (continuous) change in basic product-attribute levels.[3]

To better illustrate this difference consider the following example.[4] Imagine consumers of brass - a metal composed of zinc and copper - who are served by two brass sellers. Irrespective of product characteristics (qualities), their products will always be substitutes because one seller - at least partially - replaces the other. Firms are interested in decreasing prices to capture a larger part of the total demand. With identical firms one will observe marginal cost pricing. Consider now the case of two firms producing copper and zinc respectively for the same consumers. This time, irrespective of their characteristics (qualities), the two products are perfect complements. Purchasing from only one firm is pointless without purchasing from the other. In such a setting, firms do not try to undercut the competitor's price but rather try to increase prices to extract as much surplus as possible from the total price that consumers are willing to pay for the *composite good.* The prices set by a monopolist producing both components is lower than the equilibrium prices under competition, and the monopolist largely makes up for the lost margins

[2]In this paper we define substitutability and complementarity from the firms' point of view by referring to the sign of the cross-price elasticity of demand. If it is positive products are substitutes, in the opposite case they are complements. Had we taken the consumer's viewpoint our conclusions would not differ from those of standard sampling theory with fixed sampling cost. The innovation in our model is that the "cost of sampling" is *endogenous.*

[3]Technically speaking, for traditional goods the value of the cross-price elasticity may change with a change of basic product attributes but its sign does not. In the case of information products, changing the products' attributes can result in a change of the sign of the cross-price elasticity of demand.

[4]This example has been inspired by the earliest study on perfect complements by Cournot (1838). Economides and Salop (1992) provide a formal description of this Cournot model and a discussion on complements. Other important papers on complementarity include Matutes and Regibeau (1988), Economides (1989) and Gilbert and Riordan (1995).

through a substantial increase in volume.

In competition, traditional goods behave either like brass (they are substitutes) or like copper and zinc (they are complements). What is interesting with information products is that - unlike most other goods - they are substitutes or complements depending on the levels of basic product characteristics, attributes which are inherent to information: reliability (variance) and similarity (correlation). As a result, competitive behavior is very different in different regions of the product-attribute space. When information products are reliable or highly correlated, products are substitutes and competition between information suppliers is intense. When product reliability is low products are complements and competition between sellers is relatively mild (if reliability is a measure of quality, this may also mean that profits are higher when competitors have lower quality products). These findings are consistent with the general theory on substitutes and complements (see the references in footnote 4). In particular, we get the standard result that collusion or merger of firms producing perfect complements raises profits as well as consumer welfare (Allen 1938).

2.2 Related Literature

Our work is related to three important streams of literature: normative decision theory, industrial organization and information economics. The focus of normative decision theory is the problem of a decision maker who has to acquire information before making decision(s). The goal is to assess the value of information in order to trade it off against its cost. In this context, the "cost" of information is exogenous or it is under the control of the decision maker. In modern economies, however, the price of information is endogenously determined in competitive markets. In this paper we focus on the supply-side of information markets and allow competing information sellers to set the price of information. In this respect our work is closer in spirit to the second stream, the literature on oligopolistic competition. In particular, our model is related to the literature on complementarity (see references in footnote 4) and the literature on vertical differentiation (Mussa and Rosen 1978, Gabszewicz and Thisse 1979, Shaked and Sutton 1982, Moorthy 1988) which assumes that consumers differ in their willingness to pay for higher levels of product quality.

We depart from the third stream of literature, the information economics tradition, in a number of ways. Economic theory considers information to be a "public good", that is, a good available for free. The argument is either that information does not perish when used, and therefore it can be resold after "consumption", or that it gets revealed in the actions of the information users, so there is no incentive to purchase information. We depart from this tradition by realizing that, in modern economies, legal constraints (e.g. copyright laws) prevent the resale of information. We also assume that information is not revealed in the actions of the information users, this being an issue only when consumers can observe a large number of simple transactions (e.g. stock markets).[5] Second, we do not consider the effects of information asymmetry due to opportunistic behavior of the firms (e.g. signaling or screening). In our discussion, information asymmetry exists to the extent that sellers hold tradable information of value to consumers.

3 The model

To illustrate the model we will use the example of competing consulting companies (firms) selling reports on the status of an industry and its future prospects to their clients (the consumers) who buy these report(s) to use them when planning their business strategies. A good example of this scenario is EMCI Inc., a U.S.-based company specialized in selling reports on the mobile telecommunications industry. There are, of course, many other types of consulting services. Often consultants provide diagnosis for clients or recommendations on how to implement a given marketing strategy. For the sake of illustration, we have picked one type of consulting firm that nicely captures the essence of information selling. Later we show that a very similar formulation would apply to other types of consulting services that involve diagnoses or recommendations.[6]

[5] Important papers on this problem include Grossman and Stiglitz (1980), Admati and Pfleiderer (1986, 1988, 1990) and Sunder (1992).

[6] Our discussion excludes situations where the consultant is also contracted to implement the recommended strategy. We restrict our attention to cases where only information is delivered to the consumer. A recent paper by Wolinsky (1993) looks at competition in a market for services offered by informed experts who also *diagnose* how serious the con-

Besides consulting, this illustration is also analogous to a variety of other situations such as the competition between cardiologists providing medical diagnosis, lawyers giving legal advice, or stock brokers giving buy/sell recommendations. In some of the cases (e.g. industry reports) prices are set once the product exists. In other cases (e.g. diagnosis or recommendation) the report is commissioned to the seller. Both situations are captured by our model as long as the parties can commit to the agreed price (i.e. if a "contract" is enforceable).[7]

It is not rare in practice that a firm buys and compares several reports, each from different consultants, to get a better picture of the industry. The number of reports that the client buys depends on the perceived reliability of each report (e.g. its perceived quality), on their perceived similarity (e.g. their perceived substitutability) and their prices. Knowing these perceptions, consultants simultaneously set their prices while forming expectations about consumers' information acquisition strategies. We will require these expectations to be rational, i.e. they must be fulfilled in equilibrium. In this setting, product perceptions are exogenous and fixed. In the appendix, we briefly discuss the case of a two-stage game where firms first choose their positions in the perceptual space and then, set prices for their products.

3.1 The players and the product space

3.1.1 Firms

Assume a duopoly that consists of two consultants selling reports. They have similar cost structures and for simplicity we assume that the marginal, as well as fixed cost of producing a report are 0. In the appendix, we show that our key findings are unchanged if we relax this assumption. Finally, we also assume that entry in the industry is not possible.

sumer's problem is. His paper focuses on opportunistic behavior when the expert has an incentive to overstate the seriousness of the problem.

[7]Whether price is set before or after the production of the information product, the seller may enter in price-discrimination by offering a menu of price-quality options to consumers. We would like to thank the Area Editor for drawing our attention to this important topic left for future research. The present model becomes intractable assuming both price-discrimination and competition.

Suppose further that the information content of consultant i's report consists of a number, x_i, that can be thought of as the predicted dollar value of total business opportunities in the industry. We assume that x_i is a random draw from a normal distribution with mean m, the true value of business opportunities and variance σ_i^2, which represents the inverse of the "reliability" of firm i's report. Had we used another scenario, x_i and m would mean different things. In the case of EMCI Inc., for instance, m is the true demand for cellular services and x_i is a demand forecast. Alternatively, if the report would be about the implementation of a marketing strategy (say the recommended size of the salesforce), m would be the "best" implemented solution (the optimal size).[8] The reports issued by different firms do not have to be independent and we suppose that the x_i-s are correlated with correlation coefficient ρ.[9] Thus, the product space can be described with a bivariate normal distribution with mean vector (m, m) and covariance matrix

$$\Xi = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_2\sigma_1\rho & \sigma_2^2 \end{pmatrix},$$

where all parameters are exogenous. At the time of the purchase decision, the value of m is unknown to all players and the σ_i^2-s and ρ are common knowledge.[10] In marketing terms, the $(\sigma_1^2, \sigma_2^2, \rho)$ space should be thought of as the relevant attribute space for information products. There are several ways to interpret this space. For example, comparing two firms, the σ^2-s may be associated with firms' reputations. In another scenario, say, if the same firm produces two reports on two countries/industries, then the σ^2-s could be associated with the business uncertainty of the respective countries/industries. Similarly, ρ reflects consumers' assessment of the similarity between the firms (the extent to which firms use similar data sources or methodologies, represent similar "schools of thought", etc.).[11]

[8] We would like to thank the Editor for this insight.

[9] Positive correlation of expert opinion is supported by empirical evidence. It can result from the fact that experts share some information sources or that they have similar priors due to common education, for instance. Negative correlation is unlikely but theoretically possible.

[10] Later, the value of m may be revealed but this is irrelevant for the firms' current purchase decision. As it will become clear later, 1 is a natural limit for σ_i^2, above which no consumer would purchase product i alone. Thus we will assume that $\sigma_i^2 < 1$ $(i = 1, 2)$.

[11] Most consumers would expect, for instance, that a homeopath's perspective is different

3.1.2 Consumers

Consumers can choose to buy the report of firm 1, the report of firm 2 or both reports. For simplicity we will refer to these three alternatives as product configurations, indexed 1, 2, and Σ for the composite good. If the consumer buys one report, say from firm i, her estimate for m will be x_i with variance σ_i^2. Upon buying two reports the consumer weights their contents using Winkler's (1981) weights which provide least-squared error forecasts.[12] Thus her estimate of m will be the composite product, $\overline{x}$ with variance Σ^2, where

$$\overline{x} = \frac{x_1(\sigma_2^2 - \sigma_1\sigma_2\rho) + x_2(\sigma_1^2 - \sigma_1\sigma_2\rho)}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho}$$

and

$$\Sigma^2 = \frac{\sigma_1^2\sigma_2^2(1-\rho^2)}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho}.$$

Adopting the framework of the vertical differentiation literature, we assume that consumers are heterogeneous in their willingness to pay for the "quality" of the reports. The advantage of this approach is that we do not need to have specific assumptions on the way consumers use the purchased information.[13] We need to define, however, what we mean by the "quality" of information. We will define *quality* of the purchased product configuration as a linearly decreasing function of the product configuration's variance:[14]

$$\begin{array}{ll} s_i = 1 - \sigma_i^2 & \text{if only firm } i\text{'s report is bought,} \\ s_\Sigma = 1 - \Sigma^2 & \text{if both firms' reports are bought.} \end{array}$$

from a traditional MD's but they may assume that two MDs have quite similar views.

[12]The literature on expert resolution is divided on which weighting scheme should be used to combine overlapping expert opinion. In the appendix, we discuss the practical problems associated with the use of Winkler's (1981) weights and argue that for empirically plausible ranges of the parameters the weighting scheme does not alter the substantive findings of the paper.

[13]Notice that firms do not know the details of a consumer's decision problem which may in fact be specific to each consumer. Firms only have an aggregate view of the market; i.e. they only perceive how consumers are distributed in their willingness to pay for the reliability of information (see also footnote 15 on this issue).

[14]We could have defined quality as $s = 1/\sigma$, but this would have lead to huge values of s in the case of reliable reports. Another advantage of our definition is that, in this way, the parameter space is finite and allows graphical illustration of the results (see later the argument on the uniqueness of the equilibria, for instance).

Given the constraints on the σ^2-s, quality is allowed to vary between 0 and 1. Following the literature on vertical differentiation we will assume that the expected surplus of a consumer with type θ, for buying a configuration, given that firms charge prices p_i for their reports is:

$$U = \begin{cases} \theta(1-\Sigma^2) - p_1 - p_2 & \text{if both reports are bought,} \\ \theta(1-\sigma_i^2) - p_i & \text{if only firm } i\text{'s report is bought,} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Here, θ is a positive taste parameter (see Tirole 1990, Chapter 2, pp 96) with a higher θ meaning that the consumer values the quality of information more or, equivalently, that she is more sensitive to potential losses from an inaccurate prediction of m.[15] Another interpretation of θ is that it is the inverse of the marginal rate of substitution between income and quality (Tirole 1990), i.e. wealthier consumers have a higher θ. We assume that θ is known to be distributed across consumers between 0 and 1 according to the cumulative density function $\mathcal{F}$. Thus, there is a continuum of consumers and without loss of generality we normalize their total number to 1. Finally, note that utility does not depend on the content of the reports. As mentioned before, consumers do not know this value before making the purchase decision, so their *ex ante* utility for information should not depend on it.

3.2 Demand schedules

If a single report is offered by a firm expression (1) becomes $U = \theta s - p = \theta(1-\sigma^2) - p$. The demand consists of those consumers whose net surplus is higher than 0:

$$D(p) = \Pr(U > 0) = \Pr(\theta > p/s) = 1 - \mathcal{F}(p/s). \tag{2}$$

With more than one product the demand conditions are defined by the relative values of the "quality per dollar," (s/p), of the different product configurations. One can show (see Tirole, 1990, pp 97. for a formal proof) that

[15]For example, in EMCI's context a consumer may only be interested in the "range" of m (i.e. if it is "large" or "small") to perform a rough break-even analysis. As σ^2 is bounded, this consumer is not really interested in further improving the accuracy of the forecast and has therefore a small θ. Another consumer may try to fine-tune a marketing strategy and is very sensitive to forecast variance. Consequently it has a higher θ. Consumer heterogeneity (including consumer's *ex post* valuation for m) is entirely captured by the distribution of the θ parameter. We would like to thank the Editor for these examples.

in the above vertical differentiation model, if $s_2 > s_1$ and $s_2/p_2 \geq s_1/p_1$ then product 1 gets no demand, since all consumers who decide to purchase will end-up buying product 2. Said differently, product 2 dominates product 1. When $s_2/p_2 < s_1/p_1$, however, some consumers purchase the lower quality product. Thus, with more than one product the demand function is piece-wise continuous: there is a different demand function depending on the quality/price ratios of the products.

In our case, there are three alternative product configurations: buying the higher quality report, the lower quality report or both reports. In the last case the price is the sum of the prices of the individual reports and the quality is always higher than any of the individual report's. We need to analyze all possible situations (a total of five) defined by the relevant relations between the quality/price ratios of *each* product configuration. In what follows, we will analyze in detail only two situations, because only these two lead to a pure-strategy equilibrium. In Section 1 of the appendix we develop the demand for the remaining cases and show that under these, no equilibria exist.

First, we will look at a situation where each product configuration is bought by some consumers. Without loss of generality, suppose that $\sigma_1 > \sigma_2$, i.e. $s_\Sigma > s_2 > s_1$. For the proposed demand structure to exist the following condition has to hold:

$$\frac{s_\Sigma}{p_1 + p_2} < \frac{s_2}{p_2} < \frac{s_1}{p_1}, \tag{3}$$

in other words, no product configuration dominates others for all consumers. Given prices p_1 and p_2, we have for the marginal consumer who is indifferent between purchasing both reports and the higher quality report alone: $\overline{\overline{\theta}} s_\Sigma - p_1 - p_2 = \overline{\overline{\theta}} s_2 - p_2$. Thus, $\overline{\overline{\theta}} = p_1/(\sigma_2^2 - \Sigma^2)$. Similarly, for the consumer who is indifferent between buying the higher quality report and the lower quality one we have: $\overline{\theta} = (p_2 - p_1)/(\sigma_1^2 - \sigma_2^2)$. Finally, the consumer who is indifferent between buying the lower quality report and not buying at all has taste parameter $\underline{\theta} = p_1/(1 - \sigma_1^2)$. Thus, the demand for the three configurations respectively can be written as:

$$D_\Sigma = 1 - \mathcal{F}(\frac{p_1}{\sigma_2^2 - \Sigma^2})$$

$$D_2 = \mathcal{F}(\frac{p_1}{\sigma_2^2 - \Sigma^2}) - \mathcal{F}(\frac{p_2 - p_1}{\sigma_1^2 - \sigma_2^2})$$

$$D_1 = \mathcal{F}(\frac{p_2 - p_1}{\sigma_1^2 - \sigma_2^2}) - \mathcal{F}(\frac{p_1}{1 - \sigma_1^2}).$$

Therefore, under condition (3) the demand faced by the two firms is:

$$D_I = D_\Sigma + D_1 = 1 - \mathcal{F}(\frac{p_1}{\sigma_2^2 - \Sigma^2}) + \mathcal{F}(\frac{p_2 - p_1}{\sigma_1^2 - \sigma_2^2}) - \mathcal{F}(\frac{p_1}{1 - \sigma_1^2})$$

$$D_{II} = D_\Sigma + D_2 = 1 - \mathcal{F}(\frac{p_2 - p_1}{\sigma_1^2 - \sigma_2^2}). \tag{4}$$

The subscripts I and II are used to denote firm 1 and 2 respectively, to differentiate from the individual product configurations' demand. It is easy to see that $\partial D_{II}/\partial p_1 \geq 0$ and $\partial D_I/\partial p_2 \geq 0$, i.e. *under this scenario - when* (3) *is true - the two products are substitutes.*

Next, consider the scenario when the product configuration including both reports dominates all other product configurations. We have:

$$\frac{s_\Sigma}{p_1 + p_2} \geq \frac{s_2}{p_2} \quad \text{and} \quad \frac{s_1}{p_1}. \tag{5}$$

Then all consumers who decide to purchase at all will buy both products. No consumer will buy from only one firm. Following the same arguments as above, by finding the consumer who is indifferent between buying both reports or nothing, the demand for each configuration is:

$$D_\Sigma = 1 - \mathcal{F}(\frac{p_1 + p_2}{1 - \Sigma^2}) \quad \text{and} \quad D_2 = D_1 = 0.$$

Therefore, under condition (5) the demand faced by the two firms is:

$$D_I = D_{II} = D_\Sigma = 1 - \mathcal{F}(\frac{p_1 + p_2}{1 - \Sigma^2}). \tag{6}$$

Note, that now $\partial D_\Sigma/\partial p_i \leq 0 \;\; (i = 1, 2)$, i.e. *under this scenario - when* (5) *holds - the two products are complements.*

The three additional scenarios are: (1) when the configuration that consists of product 1 alone is dominated, (2) when the configuration consisting of product 2 alone is dominated and, (3) when no consumer purchases the joint product configuration. The conditions for these demand schedules are, respectively: (1) $s_2/p_2 > s_\Sigma/(p_1+p_2)$ and $s_2/p_2 \geq s_1/p_1$, (2) $s_1/p_1 > s_\Sigma/(p_1+p_2) \geq s_2/p_2$, and (3) $s_1/p_1 > s_2/p_2$ and $s_\Sigma \leq p_1 + p_2$. The derivations of the demand functions follow the spirit of the analysis above (see the appendix).

3.3 Solutions of the game

In what follows, we will analyze two cases. First, to set a benchmark we explore the model with a monopolist. Next, we take the case of two competing consulting firms. We compare the monopoly outcome to the equilibria under competition. For the remaining discussion we will assume that $\mathcal{F}$ is the c.d.f. of the uniform distribution between 0 and 1.[16] We also assume that $\sigma_i^2 < 1 \; \forall i$.

3.3.1 Monopoly

The monopolist sells a maximum of one report to each client. Given our assumption on $\mathcal{F}$, (2) becomes $D(p) = 1 - p/(1-\sigma^2)$. The optimal price charged by the monopolist is $p^M = (1-\sigma^2)/2$. Output is $q^M = 1/2$ and the monopolist's profit is $\pi^M = (1-\sigma^2)/4 = s/4$. Given p^M and the uniform distribution of θ, total consumer surplus is:

$$S^M = \int_{1/2}^{1} [\theta(1-\sigma^2) - p^M] d\theta = (1-\sigma^2)/8 = s/8.$$

3.3.2 Duopoly

In what follows we will show that in a duopoly there are two types of pure-strategy equilibria: one in which products act as substitutes (Proposition 1) and another in which products are complements (Proposition 3). For the case of symmetric competitors we will show that there is only a mixed-strategy substitute equilibrium (Proposition 2). The propositions tell us under what conditions these equilibria exist. We will analyze these conditions

[16]This assumption greatly simplifies the analysis by leading to linear demand functions. As long as $\mathcal{F}$ is a unimodal distribution, the results are similar to the ones reported below.

and relate them to some observed market outcomes. Finally, we compare each equilibrium to the monopoly case. We begin by exploring an equilibrium where all three alternatives are chosen by some consumers and argue that - as in this case, products act as substitutes - there will be "intense" competition among information sellers.

Proposition 1 (Substitutes) *Assume that $\sigma_2 < \sigma_1$, i.e. $s_2 > s_1$ and consider the following prices:*

$$p_1^* = \frac{3\delta d\Delta}{4\delta(\Delta + d) + 3d\Delta}$$

and

$$p_2^* = \frac{3\delta d\Delta + 2\delta^2(\Delta + d)}{4\delta(\Delta + d) + 3d\Delta}, \tag{7}$$

where $\delta = \sigma_1^2 - \sigma_2^2$, $d = 1 - \sigma_1^2$ and $\Delta = \sigma_2^2 - \Sigma^2$. Under conditions identified in the appendix and represented on Figure 1 these prices constitute a unique Nash equilibrium in pure strategies in which each product configuration is bought by some consumers and products compete as substitutes.

INSERT FIGURE 1 ABOUT HERE

Proof of existence (sketch):
The logic of the proof is the following. First, we need to identify best response pairs within a particular demand schedule. For example, in our case, we need to consider the demand schedule that is implied by condition (3), where no product configuration dominates any other for all consumers. Next, we need to verify if these pairs constitute a global equilibrium, i.e. if firms have no incentive to choose prices that change the demand schedule.

As shown before, under (3) firms face demand functions (4) and the products are substitutes. Using these demand functions, our assumption on $\mathcal{F}$, and supposing that firms choose prices in such a way that the demand schedule remains unchanged, we can calculate the candidate equilibrium prices which are equal to (7) (details are in the appendix). These prices need to fulfill condition (3) which results in the necessary condition for the existence of a substitute equilibrium: $d(3d+\delta)/(3d+2\delta) > \Delta$ (see the appendix for an illustration and discussion of this condition).

Next, given the above necessary condition, we need to make sure that prices (7) constitute a global equilibrium. For example, we know that as long as $p_2^*(s_\Sigma - s_2)/s_2 < p_1 < p_2^* s_1/s_2$ holds p_1^* is a best response to p_2^*, but what happens if firm 1 chooses a price outside this range? Similarly, we need to check under what conditions p_2^* remains a global best response to p_1^*. This tedious analysis, the result of which leads to the sufficient conditions for the existence of the equilibrium (Figure 1), is detailed in the appendix. □

Figure 1 shows only two cross-sections of the parameter space, one at $\rho = 0$ and another at $\rho = 1/2$. Other cross sections follow the same pattern. First, notice that products tend to compete as substitutes in the lower left half of the parameter space, i.e. when reliability (quality) is high. The figure also shows that the higher the correlation between the products the more likely they will compete as substitutes. Finally, when the products are undifferentiated (firms are symmetric) there is no pure strategy substitute equilibrium. This is because such an equilibrium would leave firms with 0 profits: prices (7) above become 0 when firms are symmetric ($\delta = 0$). Thus, it always pays to deviate and choose a price outside the range of the demand schedule implied by (3). Although there is not always a pure strategy equilibrium in the parameter region where we would expect products to be substitutes there may be a mixed strategy equilibrium. The next proposition shows for symmetric competitors that in this region, there is a mixed strategy equilibrium that also results in tough competition between firms.

Proposition 2 *Suppose that firms are symmetric ($\sigma_1 = \sigma_2$). If $2s > s_\Sigma$, i.e. $\rho > 3 - 2/\sigma^2$, then there is a mixed strategy equilibrium in which firms choose prices over the support $[(s_\Sigma - s)/2;\ \ s/2]$ with cumulative probability distribution*

$$F(p) = \begin{cases} \frac{s_\Sigma - s}{2s - s_\Sigma}\left(\frac{s-p}{p} - \frac{(s_\Sigma - s)(3s - s_\Sigma)}{4p^2}\right) & \text{if } (s_\Sigma - s)/2 \le p < s/2 \\ 1 & \text{if } p = s/2. \end{cases}$$

In this equilibrium, profits are smaller than monopoly profits.

Proof (sketch):
Given the condition $2s > s_\Sigma$ consider the demand of firm 2 in the (p_1, p_2) space (see Figure 2 for an outline of the relevant cases). When $p_2 > p_1$ no consumer buys from firm 2 alone. When $p_1 + p_2 > s_\Sigma$, no consumer buys both reports. Finally, when both firms' prices are higher than s, there is no demand at all. Figure 2 shows the profit of firm 2 under these different scenarios. First, one can show that any price outside the $[(s_\Sigma - s)/2, \;\; s/2]$ interval is strictly dominated. Next, in a mixed strategy equilibrium, firm 2 has to be indifferent between any of the prices within this interval. This analysis yields the cumulative distribution over the prices in the proposition. To calculate the expected profit of the equilibrium, assume that firm 2 chooses price $(s_\Sigma - s)/2$. Then, it faces a demand $(1 - p_2/s) = (3s - s_\Sigma)/(2s)$ as all consumers buying a single report purchase from firm 2. Its profit, i.e. the equilibrium profit is $(3s - s_\Sigma)(s_\Sigma - s)/(4s)$ which is always less than the monopoly profit $s/4$. □

INSERT FIGURE 2 ABOUT HERE

What is the meaning of Propositions 1 and 2? The propositions identify the condition under which products clearly act as substitutes. Whether or not the equilibrium is in pure strategies (differentiated products) or mixed strategies (undifferentiated products) firms have an incentive to undercut each other's prices. The conditions say that for this to happen information products need to be perceived reasonably reliable and/or correlated enough. In other words, in information markets with such product characteristics we can expect intense competition among sellers.

Now let us consider an equilibrium in which all consumers who decide to purchase at all buy both reports. We will argue that, in this equilibrium, products act as complements. Proposition 3 states the condition for such an equilibrium to exist.

Proposition 3 (Complements) *Suppose that $\sigma_2 \leq \sigma_1$ (i.e. $s_2 \geq s_1$) and consider the prices:*

$$p_1^* = p_2^* = (1 - \Sigma^2)/3. \tag{8}$$

These prices constitute a unique, symmetric Nash equilibrium in pure strategies in which consumers buy zero or two reports if

$$\rho \leq \frac{B - [B^2 - 4(B(\sigma_1^2 + \sigma_2^2) - 4\sigma_1^2\sigma_2^2)]^{1/2}}{4\sigma_1\sigma_2}, \tag{9}$$

where $B = 4 - 9(1 - \sigma_2^2)$. *Under* (9) *competing products are complements.*

Proof of existence (sketch): Following the same logic as before, we first need to identify a candidate equilibrium within the demand schedule implied by condition (5). In other words, now the product configuration including both reports dominates all other product configurations. We have seen that in this case firms face demand functions (6) and the two products are complements. Using this demand, our assumption on $\mathcal{F}$, and supposing that firms do not choose prices that change the demand schedule prices (8) are best responses to each other. Note that these prices are symmetric even if the qualities of the reports are different, so the seller with the lower quality report "free rides" on the other. Feeding these prices back in condition (5) and taking into account that only the first inequality is binding we get the necessary condition $s_\Sigma \geq 2s_2$ or $\Delta \geq d + \delta$, (see discussion in the appendix).

Again, this condition is only a necessary condition. Next, we need to make sure that the candidate equilibrium prices constitute a global equilibrium. This calculation is similar in spirit to the one in Proposition 1 and can be found in the appendix. It leads to the condition: $s_\Sigma \geq 9s_2/4$ which, after substitution, is equivalent to condition (9) of the proposition. □

In this equilibrium, consumers perceive information as complementary. Anticipating this, firms price their reports in such a way that, in equilibrium, buying a single report makes no sense to anyone. In this case, each firm tries to extract as much surplus as possible from the total price that the consumers are willing to pay for the composite product, i.e. firms tend to increase their prices rather than undercut competition. This results in relatively mild competition. Complementarity also drives the free-riding effect in this equilibrium. If both products are bought by all consumers irrespective of qualities, there is no reason for consumers to pay a higher price for any of them.[17] Proposition 3 says that for such an equilibrium to exist the reports have to be unreliable enough and not too correlated.

Summarizing, when the reports are reliable and/or highly correlated products compete as substitutes which results in intense competition between information sellers. This is true irrespective whether the products are differentiated (there is a pure strategy equilibrium) or undifferentiated (there is only a mixed strategy equilibrium). On the other hand, when the products are unreliable or highly correlated, products tend to compete as complements leading to mild competition between information sellers. To illustrate this finding we have plotted condition (9) on Figure 1. Also, Figure 3 shows the conditions of the different types of equilibria when $\sigma_1 = \sigma_2$. Observing these figures helps to understand the main message of the paper, namely that, unlike other goods, information products face very different competitive structures in different regions of the same product attribute space. This is best seen on Figure 3 i.e. when competitors are symmetric. Above the left curve there is only a mixed strategy equilibrium in which expected profits are smaller than the profit of a monopolist. Below the right curve there is an equilibrium in pure strategies where products compete as complements. In what follows, we will also see that in the latter case equilibrium profits are higher than the monopolist's (Proposition 4). It is this discontinuity in competitive structures that makes information products interesting.

INSERT FIGURE 3 ABOUT HERE

How can we relate this finding to observed market outcomes? The model predicts that the more information products are correlated and the more reliable they are the more they become substitutes and the more competitors have an incentive to decrease prices. In the opposite case, products become complements and equilibrium prices tend to increase. This may explain the differences in competitive structures for different types of consulting services. Contrast valuation services (the case of Portugal Telecom) with market research for instance (IRI or AC Nielsen). Techniques for company valuation

[17]The free-riding effect holds when firms are not too different. If perceived quality differences are very large then the firm with the higher quality may drive the other firm out of the market. Assuming the existence of marginal costs leads to different equilibrium prices but the free-riding effect persists in the sense that firms' profits are equal.

involve a fair amount of subjective assessment especially when intangible assets need to be taken into account. They also differ substantially in their approach. These methodologies are typically perceived to be unreliable - not because of a lack of expert competence but rather because of the nature of the problem. As a result, in most IPO-s (Initial Private Offerings), or take-overs, multiple consultants are hired to assess the value of the target firm. In contrast, market research techniques used by IRI or AC Nielsen tend to provide similar and quite accurate results. It is rare that a firm would hire both companies simultaneously[18] and, as a result, competition is fierce in this business. The model also helps us understand the contrast between competition of consultants in Eastern Europe and western countries. It is not rare to observe multiple consultations in East European countries where the economy is undergoing fundamental changes. Despite the fact that import costs are lower (firms hire locals) and consulting advice is less reliable due to the general lack of experience in economic environments under transformation, consultants are able to maintain fee structures comparable to western countries'. The reason is that in risky environments the combination of different opinions may lead to a better decision, i.e. information products are complements.

Based on these examples, one might naturally ask: are there conditions under which a monopolist is better off facing a competitor? We can address this question by comparing profits and consumer surplus under competition and monopoly. For symmetric competitors, we have already seen that profits are lower under competition than under a monopoly regime. When no consumer buys a single report (reports are complements) this is not the case. The following Proposition establishes that under the conditions of Proposition 3 a monopolist is better off encouraging the entry of a competitor.

Proposition 4 *Suppose that the condition of Proposition 3 holds. Then profits and consumer surplus are higher under competition than under the monopoly regime of any of the two firms.*

Proof:
To prove this proposition we need to compute firms' profits and consumer

[18] According to a recent survey by Mercer Management Consulting, only about 17% of consumer packaged goods manufacturers simultaneously purchase information from both IRI and AC Nielsen. We would like to thank Alan Montgomery for this example.

welfare. In a complements equilibrium, the demand for the configuration that includes both products (that is the sales of both firms) is $D^C = 1/3$, thus, firms' profits are $\pi^C = (1-\Sigma^2)/9 = s_\Sigma/9$ and consumer surplus is:

$$S^C = \int_{2/3}^{1} [\theta(1-\Sigma^2) - \frac{2}{3}(1-\Sigma^2)]d\theta = (1-\Sigma^2)/18 = s_\Sigma/18.$$

When $s_\Sigma \geq 9s_2/4$, both profits and consumer welfare are higher in this equilibrium than under monopoly irrespective of which firm is the monopolist. □

The intuition behind Proposition 4 is the following. When products are complements the quality and the price of the composite good alone determine the demand. When the individual reports' perceived reliability and correlation is very low, the increase in quality is very high after the combination of the reports. In other words, the base demand increases significantly. At the same time prices do not fall as shown in Proposition 3.

This finding may explain an apparently irrational behavior of EMCI Inc. (the publisher of industry reports, discussed earlier). EMCI seems to *encourage* the entry of competition by regularly swapping its client mailing lists with competitors.[19] In this way the firm increases the chance that his client base will overlap with competitors'. Table 2 sheds some light on why such a strategy may make sense. It shows industry forecasts for cellular services by several well-known consultants. The data illustrate that forecasts in the cellular industry are very uncertain: forecasts (even by the same firm) show very high variance. While there may be a number of reasons for this to happen, a buyer relying on this data will assume that these information products are typically unreliable. Realizing that their products are complements, EMCI does not expect that clients will purchase from competitors instead of EMCI. It expects them to purchase from both.

INSERT TABLE 2 ABOUT HERE

Finally, we would like to explore, what happens if competitors collude when setting the price of information products? The discontinuity in competitive

[19]Based on a personal interview with the company chairman, September 20, 1993.

structures suggests very different outcomes. When products are substitutes, collusion leads to increased prices, higher profits and lower consumer welfare than under competition. When the products are complements, this is no longer the case. It is a classic result that under complementarity, collusion while increasing profits leads to lower prices and higher consumer surplus (Allen 1938). Thus, the analysis above suggests that when information is unreliable and the sellers' sources are independent, competition may not necessarily lead to the increase of social welfare. Said differently, mergers or price fixing agreements may be socially beneficial under these circumstances.[20]

4 Model limitations

The proposed model has made several simplifications. First, the present paper studies competition in a duopoly. Assuming more firms may be interesting because it is possible that competition of three firms providing complementary information may be socially more efficient than a duopoly (i.e. it pareto dominates a duopoly). In the present model this is not the case. More precisely, an equilibrium where each consumer buys from all three information sellers does not exist. The intuition behind this finding is that combining expert information has decreasing returns in terms of the accuracy of the final forecast, especially if expert opinions are correlated.[21]

[20]The trade press has recently discussed a number of interesting cases related to the regulation of the information industry. Examples include the Federal Court's opposition to the merger of IRI and AC Nielsen (*Business Week*, Dec. 7, 1987), the successful acquisition of Arbitron's SAMI service by IRI from Control Data Corporation, (*Advertising Age*, Oct. 8., 1990, p.82), the merger of Dialog and Data-Star (*Online*, July, 1993; *Information World Review*, March, 1993; *Information Today*, April, 1993), and that of two other information providers, Dow Jones News/Retrieval and Data Times (*Information Today*, Sept. 1990; *Link-Up*, May/April, 1992). See also recent discussions on the regulation of referral firms (e.g. Olson, 1989).

[21]In our model the quality of information is linearly decreasing in variance. It is possible that under a different specification a market with more than two competitors can dominate a duopoly. Even in this case the size of the market would be limited to a few competitors due to decreasing returns to combining information. In reality, we would expect consumers to consider only a subset of potential information providers, those that are minimally correlated. However, we do not explicitly model such sequential search by consumers and this is clearly a limitation of the paper.

Second, our model uses a specific definition of information. In the appendix, we examine the case where information has a somewhat different meaning. In this extension, information suppliers have different information structures (in the sense that their partitions of the state space are different) and consumers are interested in having the finest possible partition of the state space (see Milgrom, 1981 for details). The informal analysis indicates that the results are similar to the ones found here.

Finally, our model considers a simultaneous one-shot game. This is justified in many situations. It is not uncommon that managers need to make one-shot decisions to buy from one, two or no information sources due to the lead time required to collect the data and the lack of time post delivery to order additional opinions before decision making. In reality, the purchased information is rarely a simple value as modeled here. Thus, the evaluation of the reports may be lengthy, and consumers have to gather information before full evaluation is possible.[22] Often, purchasing information means subscription to an on-line database. This decision may be independent from a specific application of the database.

From a modeling point of view we have chosen a static game for two reasons: First, our utility function depends only on the reliability and dependence of the reports and this is assumed to be common knowledge. A dynamic model, therefore, would provide the same results. One could consider the case where the utility for additional information depends on the value of previously purchased information (i.e. the purchase of the second report is contingent on the content of the first). In this case firms would make their prices contingent on the value of their own reports. This would allow rational consumers to extract the information in the reports from the observed prices without buying the reports, i.e. the market would break down. Our model preserves the rationality of consumers and the existence of the market by assuming a simultaneous game.

Another way to think of a dynamic model is to assume an endogenous percep-

[22]When evaluating marketing effectiveness using scanner data, for example, clients of market research companies, such as AC Nielsen or IRI, have to order all marketing research well in advance, before seeing the result(s) of any reports.

tual space. This has been partially done in the appendix, where we consider a two-stage game where firms first choose their positions in the perceptual space at some fixed cost and then compete in price according to the present model. This setting assumes that firms can easily communicate their product's quality to consumers (i.e. they can easily change consumers' quality perceptions). It could be interesting to model how consumers learn about firms' qualities in a context where communicating quality is only possible through repeated confrontation of the report with the "truth" revealed *ex post*. Finally, the explicit consideration of entry and exit may also lead to interesting insights. Addressing these questions through dynamic models of information markets is a challenging task left for future research.

5 Concluding remarks

This paper proposes a game-theoretic model to explain competitive structures in markets for information. Such markets become more and more common with the development of information technology and the services industry. Our approach concentrates on modeling competition, given the interdependencies among information products.

Our investigation suggests that these externalities lead to counter-intuitive results regarding competitive structures that require radically different managerial actions. Specifically, we find that the qualitative nature of competition changes with the variation of the basic product-attribute levels (reliability and correlation). When information products are reliable or correlated they tend to compete as substitutes which results in fierce competition between firms. On the other hand, when information products are unreliable and uncorrelated products tend to be complements leading to relatively mild competition. Some examples in the information services, medical and legal professions are provided to illustrate these conclusions.

It is important for managers to understand this feature of information markets as such radical differences in the nature of competition suggest very different strategies. Under substitution, traditional strategies to avoid competition (e.g. differentiation) are recommended. In contrast, we have seen that when the products' reliability is generally low (they are complements), firms are better off accomodating competition. In fact, under certain conditions a firm may benefit from "inviting" a competitor. Finally, our findings are also important for regulators of the information market. As the literature on complementarity suggests collusion between firms offering complementary products may benefit firms as well as consumers.

Information is often only one aspect of the total product/service sold which suggests that the results reported here may have wider applicability than "information-only" products. It is important to realize that in these cases our results can not explain *alone* the observed market outcomes but only provide insight with respect to information-related aspects. In the previous section, we have also highlighted a number of interesting - mostly theoretical - research directions. Beyond theoretical development there is potential for empirical research to test the proposed theory.

References

Admati, Anat R. and Paul Pfleiderer (1986), "A Monopolistic Market for Information", *Journal of Economic Theory*, 39, 400-438.

——— and ——— (1988), "Selling and Trading on Information on Financial Markets", *American Economic Review, Papers and Proceedings*, 78, 96-103.

——— and ——— (1990), "Direct and Indirect Sale of Information", *Econometrica*, 58, 4, 901-928.

Allen, R. G. D. (1938), *Mathematical Analysis for Economists*, St. Martin's Press, New York.

Cournot, A. (1927), *Researches into the Mathematical Principles of the Theory of Wealth*, originally published in French (1838), translated by Nathaniel Bacon, New York: Macmillan.

Economides, Nicholas (1989), "Desirability of Compatibility in the Absence of Network Externalities," *American Economic Review*, 79, 5, (December), 1165-81.

——— and Steven C. Salop (1992), "Competition and Integration Among Complements, and Network Market Structure," *Journal of Industrial Economics*, 40, 1, (March), 105-123.

Gabszewicz, J. Jaskold and J.-F. Thisse (1979), "Price Competition, Quality and Income Disparities," *Journal of Economic Theory*, 20, 340-59.

Gilbert, Richard J. and Michael H. Riordan (1995), "Regulating complementary products: a comparative institutional analysis", *RAND Journal of Economics*, 26, 2, (Summer), 243-256.

Grossman, Sanford, J. and Joseph E. Stiglitz (1980), "On the Impossibility of Informationally Efficient Markets", *American Economic Review*, 70, 393-408.

Hilton, Ronald W. (1981) "The Determinants of Information Value: Synthesizing Some General Results", *Management Science*, 27. (January) 54-64.

Jensen, Fred O. (1991), "Information Services", in *The AMA Handbook of Marketing for the Service Industries*, eds. Congram and Friedman, AMACOM, New York, Chapter 22, p. 423-443.

Matutes, Carmen and P. Regibeau (1988), "Mix and Match: product compatibility without network externalities", *RAND Journal of Economics*, 19. No.2 221-234.

Milgrom, Paul (1981), "An Axiomatic Characterization of Common Knowledge", *Econometrica*, 49, 219-222.

Moorthy, K. S. (1988), "Product and Price Competition in a Duopoly," *Marketing Science*, 7 (Spring), 141-168.

Mussa, Michael and Sherwin Rosen (1978), "Monopoly and Product Quality," *Journal of Economic Theory*, 18, 301-317.

Olson, Walter (1989), "The Case Against Expert Witnesses," *Fortune*, 120, 7 (Sept 25.), 133-138.

Shaked, Avner and John Sutton (1982), "Relaxing Price Competition Through Product Differentiation," *Review of Economic Studies*, 49, 3-13.

Sunder, Shyam (1992), "Market for Information: Experimental Evidence", *Econometrica*, 60. (May) 667-695.

Tirole, Jean (1990), "The Theory of Industrial Organization", MIT Press. Cambridge, MA.

Winkler, Robert L. (1981), "Combining Probability Distributions From Dependent Information Sources," *Management Science*, 27, 4, (April), 479-488.

Wolinsky, Asher (1993), "Competition in a Market for Informed Experts' Services", *RAND Journal of Economics*, 24. No.3 (Autumn), 380-398.

TABLE 1: THE US MARKET FOR INFORMATION SERVICES IN 1988

Information Subject Area	Estimated 1988 Revenue ($ billion)	Estimated 1988 Share (%)	Average Annual Increase in Revenue From 1982-1988 (%)
Market/Marketing/Media Research:	**6.6**	**37.3**	**13**
Economic/Financial/Securities:	3.9	22	16
Credit/Check authorization:	2.4	13.6	11
Product/Price:	1.5	8.5	9
Legal/Regulatory:	1.4	7.9	10
Medical/Scientific/Technological:	0.8	4.5	12
All others:	1.1	6.2	8
Total industry:	**17.7**	100	**12**

Note: Adopted from Jensen (1991)

TABLE 2: CELLULAR SUBSCRIBER PROJECTIONS IN THE 1980S

Source	Date of Projection	Population Included	Date Projected For	Number of Subscribers (Millions)
Yankee Group	1985	Total Market	1990	0.43
Shosteck Associates	1983	Urban Pop.	Potential	0.53
Shosteck Associates (a)	1987	n/a	1995	9-12
A. D. Little	1980	Total Market	1990	1
A. D. Little (b)	1985	n/a	1994	3
Cellular Business Systems (c)	1985	n/a	1993	3.8
BCG	1985	Total Likely	1990	1.2
Link Resources	1984	Total Market	1990	1.4
EMG	1985	Total Market	1990	1.8
Business Comm. Co. (d)	1985	n/a	1993	1.3
Lehman Brothers	1982	Top 90 Markets	1989	2
Dean Witter	1982	Total Market	1990	2.1
IRD	1980	Total Market	Cellular	2.5
RRNA	1985	Total Market	1990	2.6
Goldman Group (e)	1988	n/a	2000	9
DLJ	1985	Top 90 Markets	1990	2.6
Leigh	1982	Urban Pop.	1990	3
Arthur Andersen	1984	Total Market	1990	7
AT & T (f)	1985	n/a	2000	30-40
		Actual Market	1990	5.2

Sources: Telocator, February 1986, pp. 22-27. (a) Telephone Engineer and Management, July, 1987. (b) Washington Business Journal, April 1, 1985. (c) Charlotte NC News, June 17, 1985. (d) New York Times, June 23, 1985. (e) Cellular Business, January, 1988. (f) Peoria Illinois Journal Star, May 26, 1985.

Figure 1: Conditions for equilibria

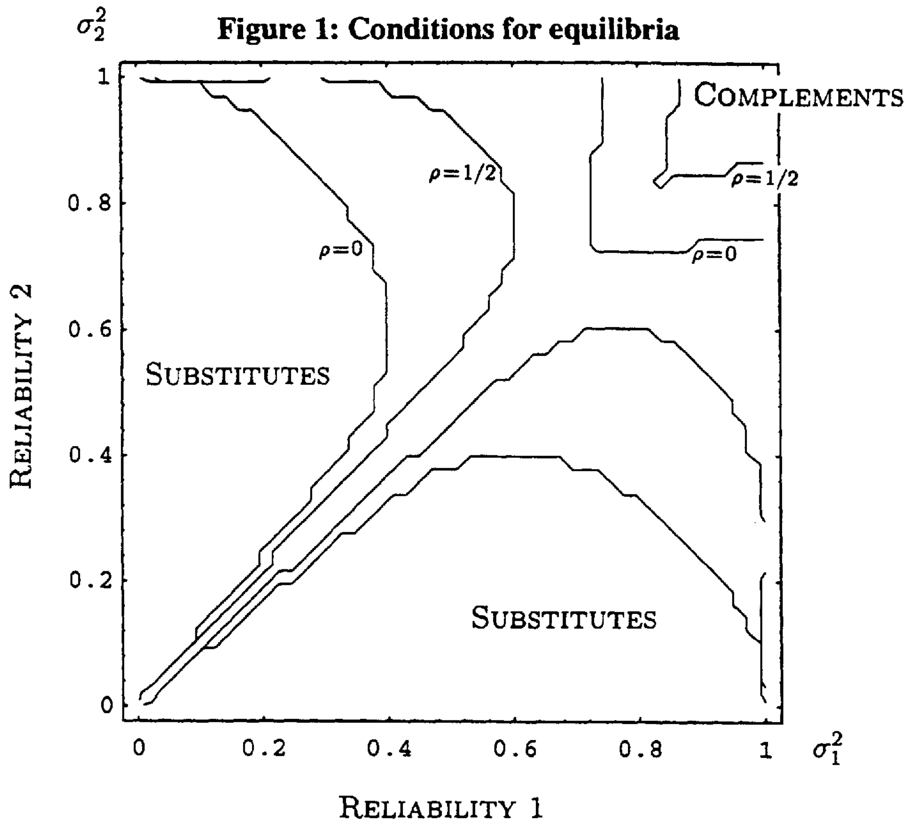


Figure 2: Mixed-strategy equilibrium with symmetric competitors

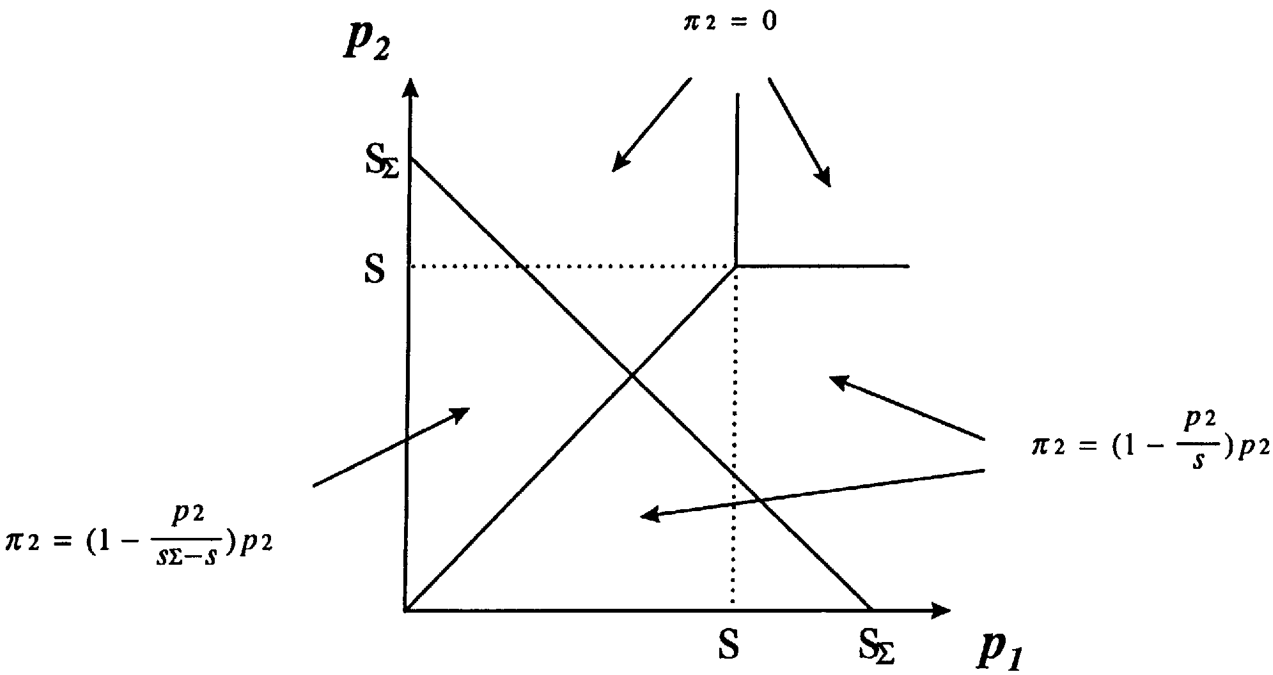
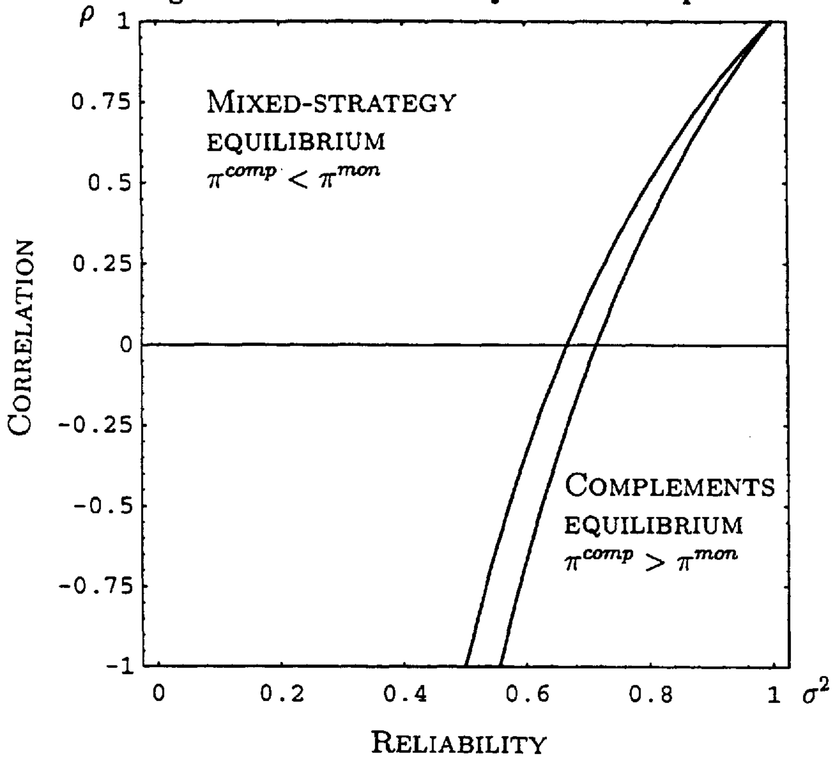


Figure 3: Conditions for symmetric competitors



Appendix to Marketing Information: A Competitive Analysis

September, 1996

The first section of the appendix proves that pure-strategy equilibria can only exist in the demand schedules outlined in section 3.2 of the paper. Section 2 provides detailed derivations of the proofs of Propositions (1)-(4) Section 3 explores the implications of introducing fixed/variable costs in the model. Section 4 discusses practical issues related to using Winkler's (1981) weights to aggregate information. Finally, Section 5 briefly discusses an alternative definition of information products. The equation numbering in the appendix is independent from the paper's and consistent within the appendix (i.e. (2) refers to equation (2) in the appendix *not* in the paper).

1 Necessary conditions for the existence of pure-strategy equilibria

Consumers have the option to choose from 3 product configurations: buying the higher quality product only, the lower quality product only or buying both products. They can also decide not to purchase anything. The demand conditions depend on the quality/price ratios of the 3 product configurations. This results in 5 possible scenarios each leading to a different demand schedule. In each, the necessary condition for the existence of a pure-strategy equilibrium is that the proposed equilibrium prices satisfy the pre-supposed relations between the quality/price ratios. In what follows, we will derive these necessary conditions and show that they can only be fulfilled for the two demand schedules outlined in section 3.2 of the paper. This is summarized in Lemma 1.

Lemma 1 *Assume that $\sigma_1 > \sigma_2$. The necessary condition for the existence of a pure-strategy equilibrium in which products compete as substitutes is $d(3d+\delta)/(3d+2\delta) > \Delta$ or:*

$$\rho^2 - \rho\frac{2D}{\sigma_1\sigma_2} + \frac{(\sigma_1^2+\sigma_2^2)D}{\sigma_1^2\sigma_2^2} - 1 < 0, \qquad (1)$$

where

$$D = \sigma_2^2 - \frac{(1-\sigma_1^2)[3(1-\sigma_1^2)+\sigma_1^2-\sigma_2^2]}{3(1-\sigma_1^2)+2(\sigma_1^2-\sigma_2^2)}$$

The necessary condition for the existence of a pure-strategy equilibrium in which products compete as complements is $\Delta > d + \delta$ or:

$$\rho^2 - \rho\frac{2(2\sigma_2^2 - 1)}{\sigma_1\sigma_2} + \frac{(\sigma_1^2 + \sigma_2^2)(2\sigma_2^2 - 1)}{\sigma_1^2\sigma_2^2} - 1 \geq 0. \tag{2}$$

If the above pure strategy equilibria exist they are unique.

Proof:
First, consider the demand schedule in which no product configuration is dominated. For this we need:

$$\frac{s_\Sigma}{p_1 + p_2} < \frac{s_2}{p_2} < \frac{s_1}{p_1}, \tag{3}$$

Given prices p_1 and p_2, we have for the marginal consumer who is indifferent between purchasing both reports and the higher quality report alone: $\overline{\overline{\theta}} s_\Sigma - p_1 - p_2 = \overline{\overline{\theta}} s_2 - p_2$. Thus, $\overline{\overline{\theta}} = p_1/(\sigma_2^2 - \Sigma^2)$. The same way, for the consumer indifferent between buying the higher quality report and the lower quality one we have: $\overline{\theta} = (p_2 - p_1)/(\sigma_1^2 - \sigma_2^2)$. Finally, the consumer who is indifferent between buying the lower quality report and not buying at all has taste parameter $\underline{\theta} = p_1/(1 - \sigma_1^2)$. Thus, the demand for the three configurations can be written as:

$$D_\Sigma = 1 - \mathcal{F}(\frac{p_1}{\sigma_2^2 - \Sigma^2})$$

$$D_2 = \mathcal{F}(\frac{p_1}{\sigma_2^2 - \Sigma^2}) - \mathcal{F}(\frac{p_2 - p_1}{\sigma_1^2 - \sigma_2^2})$$

$$D_1 = \mathcal{F}(\frac{p_2 - p_1}{\sigma_1^2 - \sigma_2^2}) - \mathcal{F}(\frac{p_1}{1 - \sigma_1^2}).$$

Therefore, the demand of firm 2 is $D_{II} = D_\Sigma + D_2$ and that of firm 1 is $D_I = D_\Sigma + D_1$. Note that,

$$\frac{\partial D_{II}}{\partial p_1} = \frac{\partial (D_\Sigma + D_2)}{\partial p_1} = f(\frac{p_2 - p_1}{\sigma_1^2 - \sigma_2^2})\frac{1}{\sigma_1^2 - \sigma_2^2} \geq 0$$

and

$$\frac{\partial D_I}{\partial p_2} = \frac{\partial (D_\Sigma + D_1)}{\partial p_2} = f(\frac{p_2 - p_1}{\sigma_1^2 - \sigma_2^2})\frac{1}{\sigma_1^2 - \sigma_2^2} \geq 0,$$

i.e. the two products are substitutes.

Let $\delta = \sigma_1^2 - \sigma_2^2$, $d = 1 - \sigma_1^2$ and $\Delta = \sigma_2^2 - \Sigma^2$. Using this notation, and assuming that firms choose prices within this demand schedule, firm 2 maximizes:

$$\pi_2 = [1 - \mathcal{F}(\frac{p_2 - p_1}{\delta})]p_2.$$

Differentiating we have:

$$\frac{\partial \pi_2}{\partial p_2} = 1 - \mathcal{F}(\frac{p_2 - p_1}{\delta}) - p_2 f(\frac{p_2 - p_1}{\delta})\frac{1}{\delta} = 0.$$

Using the uniform distribution[1] over the support [0, 1], we have:

$$1 - \frac{p_2 - p_1}{\delta} - \frac{p_2}{\delta} = 0.$$

The best response of firm 2 to p_1 within the demand schedule is therefore:

$$p_2(p_1) = (\delta + p_1)/2. \tag{4}$$

Similarly, firm 1 maximizes:

$$\pi_1 = [1 - \mathcal{F}(\frac{p_1}{\Delta}) + \mathcal{F}(\frac{p_2 - p_1}{\delta}) - \mathcal{F}(\frac{p_1}{d})]p_1.$$

Differentiating, setting the derivative to 0 and using the uniform distribution we obtain:

$$1 + \frac{p_2}{\delta} - 2p_1(\frac{1}{\delta} + \frac{1}{d} + \frac{1}{\Delta}) = 0.$$

Thus, the best response of firm 1 is:

$$p_1(p_2) = \frac{(\delta + p_2)d\Delta}{2(d\Delta + \delta\Delta + d\delta)}. \tag{5}$$

[1] Assuming an interior solution this means that $\mathcal{F}(x) = x$ and $f(x) = 1$.

Solving the system (4)-(5), we get the candidate equilibrium prices:

$$p_1^* = \frac{3\delta d\Delta}{4\delta(\Delta + d) + 3d\Delta}$$

and

$$p_2^* = \frac{3\delta d\Delta + 2\delta^2(\Delta + d)}{4\delta(\Delta + d) + 3d\Delta}. \tag{6}$$

We will also need the price of the configuration with both reports:

$$p_1^* + p_2^* = \frac{6\delta d\Delta + 2\delta^2(\Delta + d)}{4\delta(\Delta + d) + 3d\Delta}.$$

Now we have to put these prices back in conditions (3). To do this, it is helpful to use the following notation for the quality of the configurations: $s_1 = 1 - \sigma_1^2 = d$, $s_2 = 1 - \sigma_2^2 = d + \delta$, and $s_\Sigma = 1 - \Sigma^2 = d + \delta + \Delta$. Then, the second inequality gives $2d > \Delta$, while for the first we obtain:

$$\frac{d(3d + \delta)}{3d + 2\delta} > \Delta. \tag{7}$$

Since $2d \geq d(3d+\delta)/(3d+2\delta)$, (7) is the relevant condition. After substituting the definitions of δ, d, Δ and Σ in (7) and solving for ρ we get inequality (1) of Lemma 1.

Next, consider the demand schedule in which the configuration that includes both reports dominates all other configurations. For this we need:

$$\frac{s_\Sigma}{p_1 + p_2} \geq \frac{s_2}{p_2} \quad \text{and} \quad \frac{s_1}{p_1} \tag{8}$$

Notice that the relation between s_2/p_2 and s_1/p_1 is irrelevant for the existence of this demand schedule. The consumer who is indifferent between buying both reports or nothing has taste parameter $\underline{\theta} = (p_1 + p_2)/(1 - \Sigma^2)$. Thus, the demand for each configuration is

$$D_\Sigma = 1 - \mathcal{F}(\frac{p_1 + p_2}{1 - \Sigma^2})$$

and

$$D_2 = D_1 = 0.$$

Note that now $D_{II} = D_I = D_\Sigma$ and

$$\frac{\partial D_\Sigma}{\partial p_i} = -f(\frac{p_1 + p_2}{1 - \Sigma^2})\frac{1}{1 - \Sigma^2} \leq 0 \quad \forall i,$$

i.e. the two products are complements.

Assuming that prices are chosen in such a way that the demand schedule remains the same, firm 1 maximizes:

$$\pi_1 = [1 - \mathcal{F}(\frac{p_1 + p_2}{1 - \Sigma^2})]p_1$$

in p_1 which - given the uniform distribution - provides the following best response to p_2:

$$p_1(p_2) = \frac{1 - \Sigma^2 - p_2}{2}.$$

Note that the problem of firm 2 is identical, i.e. the best response of firm 2 is the same as that of firm 1. Therefore, the candidate equilibrium prices are equal:

$$p_1^* = p_2^* = \frac{1 - \Sigma^2}{3}. \tag{9}$$

This is true even if the qualities of the reports are different ("free riding effect"). Feeding these prices back in condition (8) and taking into account that only the first inequality is binding (since $s_2 > s_1$) we get $\Delta > d + \delta$. After substitution we obtain condition (2) of the lemma.

Next we need to consider the rest of the possible demand schedules and show that there is no pure-strategy equilibrium in them. Consider the scenario where nobody buys the report with the lowest quality but some consumers buy every other product configuration:

$$\frac{s_\Sigma}{p_1 + p_2} < \frac{s_2}{p_2} \tag{10}$$

and

$$\frac{s_2}{p_2} \geq \frac{s_1}{p_1}. \tag{11}$$

Here again, the relation between $s_\Sigma/(p_1+p_2)$ and s_1/p_1 is not relevant. Then the demand for each product configuration is:

$$D_\Sigma = 1 - \mathcal{F}(\frac{p_1}{\sigma_2^2 - \Sigma^2})$$

$$D_2 = \mathcal{F}(\frac{p_1}{\sigma_2^2 - \Sigma^2}) - \mathcal{F}(\frac{p_2}{1 - \sigma_2^2})$$

$$D_1 = 0.$$

Following the previous arguments, we obtain for the best responses of the firms:

$$p_1 = (\sigma_2^2 - \Sigma^2)/2$$

and

$$p_2 = (1 - \sigma_2^2)/2.$$

Note that these best responses do not depend on the other firm's strategies, therefore, the above expressions represent "equilibrium" strategies. Feeding these prices back into conditions (10)-(11), it is easy to see that the first condition can never be satisfied. Thus, this scenario can not lead to an equilibrium. A similar argument holds for the case when *ex ante* nobody is willing to buy the configuration that consists of the higher quality report alone (i.e. when $s_1/p_1 > s_\Sigma/(p_1+p_2) \geq s_2/p_2$).

The last possible scenario is when prices are such that there is no demand for the product configuration in which both reports are purchased. For this to be true we need:

$$s_\Sigma \leq p_1 + p_2 \tag{12}$$

and

$$\frac{s_1}{p_1} > \frac{s_2}{p_2}. \tag{13}$$

Then the demand for each product configuration is:

$$D_\Sigma = 0$$

$$D_2 = 1 - \mathcal{F}(\frac{p_2 - p_1}{\sigma_1^2 - \sigma_2^2})$$

$$D_1 = \mathcal{F}(\frac{p_2 - p_1}{\sigma_1^2 - \sigma_2^2}) - \mathcal{F}(\frac{p_1}{1 - \sigma_1^2}).$$

Solving the system (using the uniform distribution) we get the following prices:

$$p_1^* = \frac{(\sigma_1^2 - \sigma_2^2)(1 - \sigma_1^2)}{4(1 - \sigma_2^2) - (1 - \sigma_1^2)}$$

and

$$p_2^* = \frac{2(\sigma_1^2 - \sigma_2^2)(1 - \sigma_2^2)}{4(1 - \sigma_2^2) - (1 - \sigma_1^2)}.$$

Feeding these prices back into conditions (12)-(13) we observe that, although the second condition is always fulfilled, the first can never be fulfilled. In other words this scenario can not lead to an equilibrium.

Finally, to prove that *if they exist*, the proposed equilibria are unique consider Figures 4 and 5. They show the necessary conditions (1) and (2) in the parameter space.[2] For the existence of a "substitutes equilibrium" products have to be located between the upper- and lower-boundary surfaces in the

[2]The left hand side of conditions (1) and (2) is a quadratic form. Thus (1) is equivalent to $\rho_1 < \rho < \rho_2$ where ρ_1 and ρ_2 are respectively the smaller and larger roots of the quadratic form. Similarly (2) is equivalent to $\rho \leq \rho_1$ or $\rho \geq \rho_2$. However, in the latter case, the second inequality is never fulfilled, hence the existence of a single surface in the case of a complements equilibrium.

parameter space. On the other hand, for the existence of a "complements equilibrium" products have to be below the surface on Figure 5. As these two regions do not intersect and no other pure strategy equilibrium exists, there are no regions in the parameter space with multiple pure-strategy equilibria. □

INSERT FIGURES 4 AND 5 ABOUT HERE

Figures 4 and 5 are quite intuitive and they illustrate the paper's main message. Essentially, they say that a "substitute equilibrium" can only exist if the products are reliable enough or highly correlated. Similarly, a "complements equilibrium" is only possible if the products are unreliable and uncorrelated. For an interpretation of why can't there be a substitutes equilibrium when the products are asymmetric and highly correlated see the discussion on the Winkler weights in section 4.

2 Proofs of Propositions 1-5

Proposition 1 *Suppose that $\sigma_2 < \sigma_1$ (i.e. $s_\Sigma > s_2 > s_1$) and consider prices (6). Under conditions identified in the appendix and represented on Figure 1, these prices constitute the unique pure-strategy Nash equilibrium. In this equilibrium, products compete as substitutes.*

Proof:
In Lemma 1 we have already identified the necessary condition for prices (6) to constitute an equilibrium. We have also seen that if this equilibrium exists then it is unique and that in it, products compete as substitutes. We are left to find the sufficient condition for the existence of the equilibrium. To do this we need to verify if prices (6) are global best responses to each other, i.e. that they remain best responses even if one of the firms is allowed to choose price in a way that changes the demand schedule.

Consider Figure 6 which has been plotted assuming $\sigma_1 = \sigma_2$ and necessary condition (7). This ensures that the left line ($p_2 = s_2 p_1/(s_\Sigma - s_2)$ is always above the right line ($p_2 = s_2 p_1/s_1$). The substitute demand schedule is valid for all price pairs that are located between the two curves. Here p_1^* and p_2^*

are best responses to each other leading to the following profits:

$$\pi_2^* = (1 - \frac{p_2^* - p_1^*}{\delta})p_2^*,$$

$$\pi_1^* = (1 - \frac{p_1^*}{\Delta} + \frac{p_2^* - p_1^*}{\delta} - \frac{p_1^*}{d})p_1^*.$$

After substitution and simplification:

$$\pi_1^* = \frac{9d\delta\Delta}{E^2}(\delta\Delta + d\delta + d\Delta), \tag{14}$$

$$\pi_2^* = \frac{\delta}{E^2}[2\delta(\Delta + d) + 3d\Delta]^2, \tag{15}$$

where $E = 4\delta(\Delta + d) + 3d\Delta$.

INSERT FIGURE 6 ABOUT HERE

In what follows, we will fix the proposed equilibrium price of firm i, ($i = 1, 2$) and consider all possible deviations from the equilibrium by firm j, ($j \neq i$). First, we will verify if it pays to firm 1 to choose a price beyond $p_{1h} = s_1 p_2^*/s_2 = dp_2^*/(\delta + d)$. If such price is chosen the demand schedule changes. Since $p_2^* < s_2 p_1/(s_\Sigma - s_2)$ (we are below the left line) we still have $s_\Sigma/(p_1 + p_2) < s_2/p_2$, i.e. product 2 alone claims some demand. But now, we are below the right line, thus $s_2/p_2 > s_1/p_1$ i.e. the configuration that consists of product 1 alone is dominated by product 2. Therefore, the demand of firm 1 consists only of the demand for the joint-product configuration: $1 - p_1/(s_\Sigma - s_2) = 1 - p_1/\Delta$. Firm 1's profit is $\pi_1 = (1 - p_1/\Delta)p_1$. The optimal price that firm 1 can choose by deviating above p_{1h} is p_{1h} if $\partial\pi_1/\partial p_1 \leq 0$ at $p_1 = p_{1h}$ or $\Delta/2$ otherwise. Thus, the highest profit from choosing a price above p_{1h} is:

$$\pi_1^h = \begin{cases} (1 - p_{1h}/\Delta)p_{1h} & \text{if } 1 - 2p_{1h}/\Delta \leq 0 \\ \Delta/4 & \text{if } 1 - 2p_{1h}/\Delta > 0. \end{cases} \tag{16}$$

For prices (6) to constitute an equilibrium we need $\pi_1^* \geq \pi_1^h$.

Next, we need to verify, when does it pay for firm 1 to choose a price below $p_{1l} = (s_\Sigma - s_2)p_2^*/s_2 = \Delta p_2^*/(\delta + d)$. In this case, product 2 alone is dominated by the joint-products configuration. There are two cases to explore: (1) If $s_\Sigma/(p_1 + p_2) \geq s_1/p_1$, then product 1 alone is also dominated by the joint-product configuration and (2) if $s_\Sigma/(p_1 + p_2) < s_1/p_1$, then firm 1's product is bought alone by some consumers. It is easy to show that the first case can never occur in the region where the necessary condition (7) holds. $s_\Sigma/(p_1 + p_2) \geq s_1/p_1$ implies that $p_1 \geq s_1 p_2^*/(s_\Sigma - s_1) = dp_2^*/(\delta + \Delta) = \hat{p_1}$. Thus, we need prices p_1 such that $\hat{p_1} < p_1 < p_{1l}$. But given the necessary condition, $d > \Delta$, thus $d/(\delta + \Delta) > \Delta/(d + \delta)$, i.e. $\hat{p_1} > p_{1l}$. In sum, p_1 such that $\hat{p_1} < p_1 < p_{1l}$ cannot exist. So we only need to consider case (2). Then the demand of firm 1 is $1 - p_1/s_1 = 1 - p_1/d$ and its profit is $(1 - p_1/d)p_1$. Following the same arguments as above the optimal profit from choosing a price below p_{1l} is:

$$\pi_1^l = \begin{cases} (1 - p_{1l}/d)p_{1l} & \text{if } 1 - 2p_{1l}/d > 0 \\ \\ d/4 & \text{if } 1 - 2p_{1l}/d \leq 0. \end{cases} \tag{17}$$

For prices (6) to constitute an equilibrium we need $\pi_1^* \geq \pi_1^l$.

Next we need to repeat the same analysis for firm 2. This analysis follows the same logic. When firm 2 chooses a price below $p_{2l} = (\delta + d)p_1^*/d$ then product 1 alone is dominated by product 2 but product 2 is not dominated by the joint product configuration. Thus, firm 2's demand is $1 - p_2/s_2 = 1 - p_2/(\delta + d)$ and its profit is $(1 - p_2/(\delta + d))p_2$. As before, the optimal profit from a deviation below $p_{2l} = (\delta + d)p_1^*/d$ is:

$$\pi_2^l = \begin{cases} (1 - p_{2l}/(\delta + d))p_{2l} & \text{if } 1 - 2p_{2l}/(\delta + d) > 0 \\ \\ (\delta + d)/4 & \text{if } 1 - 2p_{2l}/(\delta + d) \leq 0. \end{cases} \tag{18}$$

π_2^* needs to excede π_2^l.

If firm 2 chooses prices above $p_{2h} = (\delta + d)p_1^*/\Delta$ then the joint product configuration dominates product 2 alone. Again, two cases are to be considered: (1) if $s_\Sigma/(p_1 + p_2) \geq s_1/p_1$, i.e. product 1 alone is also dominated by the joint-product configuration and (2) if $s_\Sigma/(p_1 + p_2) < s_1/p_1$, then product 1 alone is bought by some consumers. As before, it is easy to show that given the necessary condition the first case can never occur. In the second case firm 2's demand consists only of the demand for the joint product configuration: $1-p_2/(s_\Sigma-s_1) = 1-p_2/(\delta+\Delta)$ and its profit is $(1-p_2/(\delta+\Delta))p_2$. Following the same arguments as before, if firm 2 chooses prices above $p_{2h} = (\delta+d)p_1^*/\Delta$ then its optimal profit is:

$$\pi_2^h = \begin{cases} (1-p_{2h}/(\delta+\Delta))p_{2h} & \text{if } 1-2p_{2h}/(\delta+\Delta) \leq 0 \\ \\ (\delta+\Delta)/4 & \text{if } 1-2p_{2h}/(\delta+\Delta) > 0. \end{cases} \tag{19}$$

π_2^* needs to excede π_2^h.

Taking into account conditions (16)-(19) and necessary condition (1) over the entire parameter space it truns out that the only ones that are binding are: $\pi_1^* \geq \pi_1^h$ and $\pi_2^* \geq \pi_2^h$. These conditions are represented on Figure 1 and constitute the sufficient condition for the existence of the proposed equilibrium. □

Figure 1 shows that when $\sigma_1 = \sigma_2$ a pure-strategy substitute equilibrium does not exist. Here we would like to provide a formal argument that shows this result without going through the above proof. First, we show that for symmetric firms choosing a price above 0 cannot constitute an equilibrium. To see this assume that $p_1^* = p_2^* = p^* > 0$. Then, some consumers, namely those whose valuation for quality is very low, would purchase only one report (say they would purchase randomly from one of the firms). By offering a price slightly below p^* a firm could attract all these consumers without loosing profit from those who purchase both reports. In other words p^* cannot be an equilibrium. But $p_1^* = p_2^* = 0$ can not be an equilibrium either as it always pays to deviate and choose a higher price. This way some consumers will still purchase both reports and the firm's profit is positive.

Proposition 2 *Suppose that firms are symmetric ($\sigma_1 = \sigma_2$). If $2s > s_\Sigma$, i.e. $\rho > 3 - 2/\sigma^2$, then there is a mixed strategy equilibrium in which firms choose prices over the support $[(s_\Sigma - s)/2;\ \ s/2]$ with cumulative probability* *distribution*

$$F(p) = \begin{cases} \frac{s_\Sigma - s}{2s - s_\Sigma}\left(\frac{s-p}{p} - \frac{(s_\Sigma - s)(3s - s_\Sigma)}{4p^2}\right) & \text{if } (s_\Sigma - s)/2 \le p < s/2 \\ 1 & \text{if } p = s/2. \end{cases}$$

In this equilibrium, profits are smaller than monopoly profits.

Proof:
Given the condition $2s > s_\Sigma$ consider the demand of firm 2 in the (p_1, p_2) space (see Figure 2 for an outline of the relevant cases). When $p_2 > p_1$ no consumer buys from firm 2 alone. When $p_1 + p_2 > s_\Sigma$, i.e. when $p_2 > s_\Sigma - p_1$ no consumer buys both reports. Finally, when both firms' prices are higher than s, there is no demand at all. Figure 2 shows the profit of firm 2 under these different scenarios. $\pi_2 = (1 - p_2/s)p_2$ when all consumers who purchase the product configuration with a single product buy firm 2's (i.e. when $p_2 < p_1$). This is irrespective whether some consumers also purchase the joint-product configuration or not. Naturally when only firm 1's product is bought $\pi_2 = 0$. Finally, when some consumers buy both products but $p_2 > p_1$, then firm 2's demand consist of these consumers and its profit is $\pi_2 = (1 - p_2/(s_\Sigma - s))p_2$.

It is easy to see that choosing a price below $(s_\Sigma - s)/2$ is dominated. $\partial\pi_2/\partial p_2$ is always positive in this region irrespective of the price of firm 1. The same way decreasing price till $s/2$ is always profitable for firm 2. Thus firms will choose prices in the $[(s_\Sigma - s)/2, \;\; s/2]$ interval, where no price is dominated. Note that since $2s > s_\Sigma$, $(s_\Sigma - s)/2 < s/2$.

Next, assume that firm 2 chooses p_2 within this range and firm 1 chooses price with cumulative density function F. Firm 2's expected profit is:

$$\pi_2(p_2) = F(p_2)[(1 - \frac{p_2}{s_\Sigma - s})p_2] + (1 - F(p_2))[(1 - \frac{p_2}{s})p_2] \tag{20}$$

When firm 2 chooses price $(s_\Sigma - s)/2$, then it faces a demand $(1 - p_2/s) = (3s - s_\Sigma)/(2s)$ as all consumers buying a single report purchase from firm 2.[3] Thus, its profit, i.e. the equilibrium profit is $(3s - s_\Sigma)(s_\Sigma - s)/(4s)$. In

[3]Note that the distribution has no atome at $p = (s_\Sigma - s)/2$. If this were the case, a firm choosing a price slightly below $(s_\Sigma - s)/2$ could claim all the demand for the single product configuration.

a mixed strategy equilibrium, firm 2 has to be indifferent between this price and any of the prices within the interval. Equating the above profit with (20) we obtain the cumulative distribution over the prices in the proposition. Finally, note that expected profit of this equilibrium is always positive and less than the monopoly profit $s/4$. □

Proposition 3 (Complements) *Suppose that $\sigma_2 \leq \sigma_1$ (i.e. $s_2 \geq s_1$) and consider the prices:*

$$p_1^* = p_2^* = (1 - \Sigma^2)/3. \tag{21}$$

These prices constitute a unique, symmetric Nash equilibrium in pure strategies in which consumers buy zero or two reports if

$$\rho \leq \frac{B - [B^2 - 4(B(\sigma_1^2 + \sigma_2^2) - 4\sigma_1^2\sigma_2^2)]^{1/2}}{4\sigma_1\sigma_2}, \tag{22}$$

where $B = 4 - 9(1 - \sigma_2^2)$. Under (22) competing products are complements.

Proof: Lemma 1 has already identified the candidate equilibrium prices (9). If this equilibrium exists then it is unique, the products compete as complements and condition (2) holds. Furthermore, the profit of both firms is $\pi_1^* = \pi_2^* = s_\Sigma/9$. We need to make sure that the candidate equilibrium prices constitute a global equilibrium. Consider Figure 7 that takes into account the necessary condition (2): $\Delta > d + \delta$. The left curve ($p_2 = (s_\Sigma - s_1)p_1/s_1$) is above, and the right curve ($p_2 = s_2p_1/(s_\Sigma - s_2)$ is below the 45^o line.

INSERT FIGURE 7 ABOUT HERE

As before, fixing one firm's price to the proposed equilibrium price we need to check all possible deviations by the other firm. First, we check if p_1^* is a best response to p_2^* then we see when does it pay for firm 2 to deviate from the equilibrium. The same analysis as for Proposition 1 shows that it never pays to choose a price above $p_{1h} = (s_\Sigma - s_2)p_2^*/s_2$. In this case, product 1 is still dominated by the joint product configuration (we are below the left curve) but product 2 alone gets some demand. Product 1's demand still consists only of those consumers who buy both products: $1 - p_1/(s_\Sigma - s_2)$ and its profit is $(1 - p_1/(s_\Sigma - s_2))p_1$. As $\partial\pi_1(p_{1h})/\partial p_1 < 0$, the optimal deviation is p_{1h}. However the profit corresponding to this deviation is always less than π_1^*.

Next, we need to see if it makes sense for firm 1 to go below $p_{1l} = s_1 p_2^*/(s_\Sigma - s_1)$. In this case, product 2 is still dominated by the joint configuration (we are above the right curve), but product 1 is not dominated any more (we are above the left curve and also $s_1/p_1 > s_2/p_2$ bacause $p_1 < p_{1l} = s_1 p_2^*/(s_\Sigma - s_1) < s_1 p_2^*/s_2$; notice that $s_\Sigma > 2s_2 > s_2 + s_1$). Thus, firm 1's demand is $1 - p_1/s_1$ and its profit is $(1 - p_1/s_1)p_1$. If $\partial\pi_1(p_{1l})/\partial p_1 \leq 0$, then the optimal deviation is $s_1/2$ leading to a profit $s_1/4$. This leads to a condition $\pi_1^* = s_\Sigma/9 > s_1/4$. The condition for the derivative to be positive is harsher than this condition, and we know for sure that if the derivative is positive then π_1^* is larger than the resulting profit whatever it is. Therefore the only condition that needs to be satisfied is $s_\Sigma > 9s_1/4$.

The exact same arguments can be made for the deviations by firm 2: it never pays for firm 2 to choose a price above $p_{2h} = (s_\Sigma - s_1)p_1^*/s_1$ but it pays to go below $p_{2l} = s_2 p_1^*/(s_\Sigma - s_2)$ when $s_\Sigma/9 < s_2/4$. As $s_2 > s_1$ the relevant condition for the existence of the complements equilibrium is $s_\Sigma \geq 9s_2/4$. Having a better product, firm 2 has more incentive to deviate from the equilibrium than firm 1. $s_\Sigma \geq 9s_2/4$ is more restrictive than the necessary condition (2). After substitution, the condition leads to an inequality with a quadratic form which can only be fulfilled if $\rho < \rho_1$ where ρ_1 is the smaller root of the quadratic form. This leads to condition (22) of the proposition. $\square$

Proposition 4 *Suppose that the condition of Proposition 3 holds. Then profits and consumer surplus are higher under competition than under the monopoly regime of any of the two firms.*

The proof is detailed in the paper.

3 Including costs in the model

Fixed costs

In the model of the paper the parameters, σ_1, σ_2 and ρ cannot be influenced by firms. One could think of an extension in which, in the first period, producers choose their positions in the perceptual space at some fixed costs, $K_i(\sigma_i)$ (see Hauser and Shugan 1983), and compete according to our analysis in the second stage (ρ can stay exogenous but may also be influenced by firms at some cost). This case would be analogous to situations where the experts first need to establish *reputations* and then compete accordingly. The cost of establishing a reputation can come from the costs of a "good" education, the purchase of expensive equipment or from a costly marketing campaign.

With fixed costs in the first period, producer i needs to maximize:

$$\pi_i(\sigma_i,\ \sigma_j) - K_i(\sigma_i)$$

with respect to σ_i. In the above expression, $\pi_i(\sigma_i,\ \sigma_j)$ is the second period equilibrium profit given σ_i and σ_j. Note, that this profit function is not continuous in σ_i and we can expect the equilibrium positions in the parameter space to be very sensitive to the cost function, possibly leading to corner solutions.

Variable costs:

It is possible that higher perceived reliability is acquired with higher fixed costs as well as higher variable costs. The firm may use higher quality paper in producing its reports, for instance. In this section we would like to show that the main result of the paper does not change when, in the second stage, the marginal cost of producing a report with higher perceived reliability is also higher. Specifically, we will consider the case of symmetric competitors and show that the existence a complements equilibrium does not hinge on any specific cost function.

Assume that producing each report with quality s costs $c(s)$, where c is assumed to be increasing. First we need to identify the candidate equilibrium in which products are complements. The conditions to be fulfilled are: $s_\Sigma/(p_1+p_2) > s/p_1$ and s/p_2. Following the same arguments as in the proof of Lemma 1, we obtain equilibrium prices ($i = 1,\ 2$):

$$p_i^* = \frac{s_\Sigma + c}{3}$$

and the equilibrium profit is

$$\pi_i^* = \frac{(s_\Sigma - 2c)^2}{9s_\Sigma}$$

Next we need to fulfill the conditions that imply this demand schedule. The two inequalities collapse to 1 and we get:

$$(\Sigma^2)^2 - \Sigma^2(2\sigma^2 + c) + (1 + c)(2\sigma^2 - 1) > 0,$$

which means that

$$\Sigma^2 < 2\sigma^2 - 1 \quad \text{or} \quad \Sigma^2 > 1 + c.$$

The second inequality is never verified (by assumption $\Sigma^2 \leq 1$). Note that the first inequality does not contain the cost of the products (i.e. it is identical under any cost function). Finally, note that it is identical to condition (2) of Lemma 1.

Next we need to identify the sufficient conditions. Following the same arguments as before, this condition is:

$$\frac{(s_\Sigma - 2c)^2}{9s_\Sigma} > \frac{(s + c)(s - 3c)}{4s}$$

One can see that the higher c the more likely that this condition is fulfilled thus the existence of a complements equilibrium does not depend on our assumption of 0 marginal costs.

4 Practical problems associated with using Winkler's weights to aggregate information

What can be considered to be a "good weighting scheme" is not a trivial question. The literature on expert resolution is divided on the subject.[4] Besides statistical criteria (e.g. the weights should minimize the squared error

[4]See, for example Winkler 1981, Makridakis and Winkler 1983, Ashton 1986, Clemen 1987, Gupta and Wilton 1987, Morrison and Schmittlein 1989, Schmittlein et. al. 1990, Batchelor and Dua 1995. A good review can be found in Clemen (1989).

of the forecast) weighting schemes should have some other, seemingly *ad hoc* properties such as convexity, continuity or robustness.

To illustrate the importance of these last two properties consider two random draws coming from distributions with known variances. If the draws are perfectly correlated the ratio of their deviations from the mean has to be equal to the ratio of their distributions' standard deviations. Only a single mean value makes these two ratios equal (i.e. one can unambiguously recover the true value of the mean from two perfectly correlated draws having different variances). What this means in our context is that if the reports are perfectly correlated then an additional report with the same reliability does not provide any new information. However, if the second report has a slightly different reliability from the first one's, then with the second report we can perfectly recover the truth even if the reports are perfectly correlated![5] Said differently, a weighting scheme that minimizes least squared error is not continuous and as a result it is not robust. Making a slight mistake in evaluating the variances leads to dramatically different conclusions on the value of an additional report. Such a weighting scheme is too "risky" in an applied setting. Fortunately this situation only occurs when the correlation between the reports is very high, yet the reports' reliability is clearly different. This is unlikely to happen in practice.

5 An alternative model for information products

Until now the information product was a random draw from a given distribution. In this section, we briefly outline the case when information has a somewhat different meaning. This outline does not present a fully developed model and should be seen as an illustration of the concept.

Suppose that Ω represents all possible states of the world. Consumers can not distinguish between these states, i.e. their information partition is Ω

[5]This explains the upper-boundary of the condition on the existence of a substitutes equilibrium. Interestingly this effect does not play a role for the complements equilibrium, i.e. two reports that are highly correlated but have different variances can never be complements from the firms' point of view.

itself. Suppose also that they have a diffused prior over Ω. Consultants, have more refined partitions that do not have to be identical. For simplicity, assume that the resolution of their partitions is the same (in the paper this would correspond to the assumption that the σ_i-s are identical). The information structure of all players is known by all other players. The information product in this context is the element of the partition in which the current state is. Consumers can buy information from one or two sellers and thus get a more refined partition of Ω, i.e. a better identification of the true state of the world. Mathematically this problem is equivalent to the problem in the paper because the σ-algebra generated by a random variable provides a mapping between the random variable and the underlying state-space.

For an illustration (see Figure 8) suppose that Ω is a circle of unit diameter in the two dimensional space (two traders) and the partitions are represented by parallel lines whose distance, a, is equal ($a < 1$ is the inverse of the resolution and corresponds to σ^2 in the paper).[6] The angle between the two partitions, α, represents the "dissimilarity" of the consultants' partitions. The smaller this angle, the more similar the two partitions. In the limit, when $\alpha = 0$, the partitions are equivalent (this corresponds to $\rho = 1$ in the previous model). When $\alpha = \pi/2$ the partitions can be said to be "orthogonal" (this is equivalent to the $\rho = 0$ case in the paper).

INSERT FIGURE 8 ABOUT HERE

We will suppose that the consumers' expected loss is proportional to the measure, T, of the sets in the coarsest common refinement defined by the partitions of the sellers they have purchased from. Following the spirit of the paper we will define the quality of the information configuration as:

$$s_n = 1 - T_n,$$

where n refers to the number of sellers the customer purchased from. It is easy to see that s has similar properties to those in the previous model.

[6]This illustration does not completely conform the model in the paper because here, the state-space is bounded but it helps to convey the main idea. Suppose also that the rotation is perfectly symmetric to the center of the circle, and irregularities at the borders can be ignored.

Under such conditions we expect that the competition in such information markets will have similar characteristics than in the model developed in the paper. The lower the resolution of the sellers' information structures and the "more orthogonal" they are, the more the products become complements, resulting in higher equilibrium prices. In the opposite case they become substitutes and prices decrease with competition. Here, however, the value of buying from different sellers comes from the fact that they see different *aspects* of the decision problem (they have different expertise for instance). The externalities now come from the properties and interdependence of the information structures of the sellers.

Additional References

Ashton, Robert H. (1986), "Combining the Judgment of Experts: How Many and Which Ones?", *Organizational Behavior and Human Decision Processes*, 38, 405-414.

Batchelor, Roy and Pami Dua (1995), "Forecaster Diversity and the Benefits of Combining Forecasts", *Management Science*, 41, 1, (January), 68-75.

Clemen, Robert T. (1987), "Combining Overlapping Information", *Management Science*, 33, 3, (March), 373-380.

——— (1989), "Combining forecasts: A review and annotated bibliography", *International Journal of Forecasting*, 5, 559-583.

Gupta, Sunil and Peter C. Wilton (1987), "Combination of Forecasts: An Extension," *Management Science*, 33, 3, (March), 356-372.

Hauser, John R. and Steven M. Shugan (1983), "Defensive Marketing Strategy," *Marketing Science*, 2, 4, (Fall), 319-360.

Makridakis, Spiros and Robert L. Winkler (1983), "Averages of Forecasts: Some Empirical Results", *Management Science*, 29, 9, (September), 987-996.

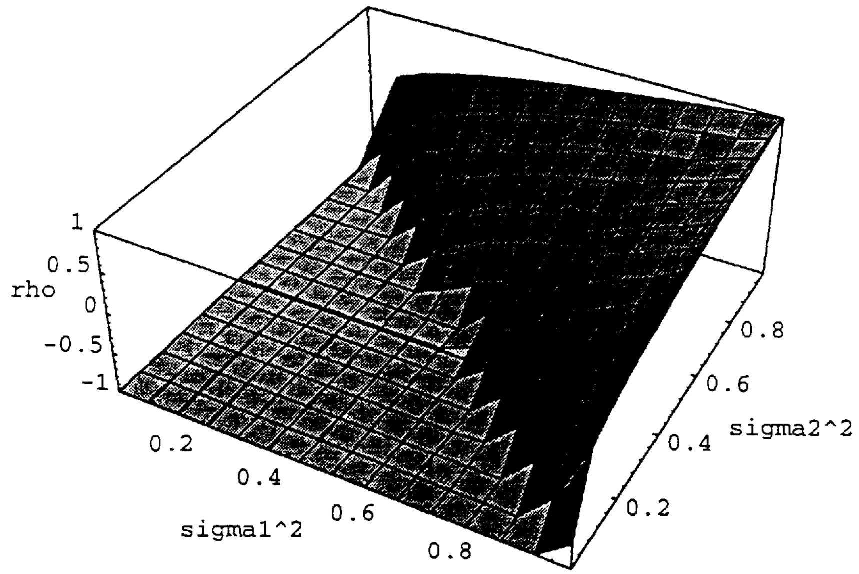
Morrison, Donald G. and David C. Schmittlein (1989), "How Many Forecasters Do You Really Have?: Mahalanobis Provides The Intuition for the Surprising Clemen and Winkler Result", *Working Paper*, No 195, UCLA.

Schmittlein, David C., Jinho Kim and Donald G. Morrison (1990), "Combining Forecasts: Operational Adjustments to Theoretically Optimal Rules", *Management Science*, 36, 9, (September), p. 1044-1056.

Winkler, Robert L. (1981), "Combining Probability Distributions From Dependent Information Sources," *Management Science*, 27, 4, (April), 479-488.

Figure 4: Necessary conditions for a substitutes equilibrium

Lower boundary



Upper boundary

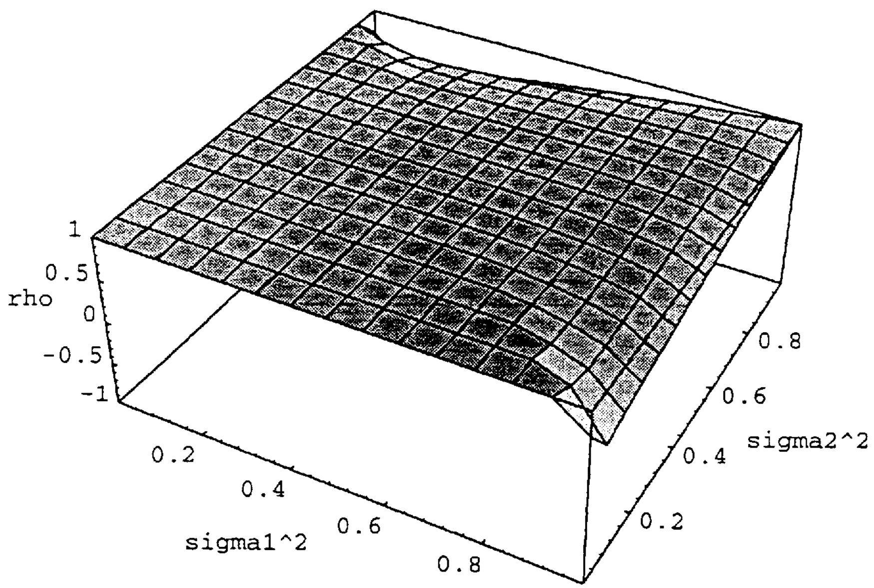


Figure 5: Necessary condition for a complements equilibrium

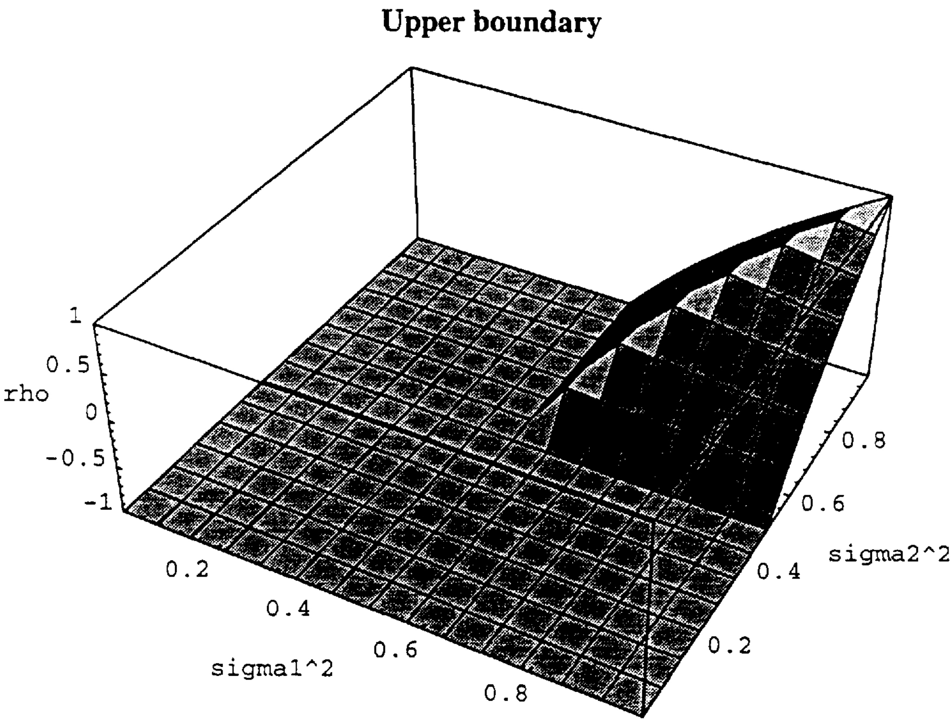


Figure 6: Pure-strategy substitutes equilibrium

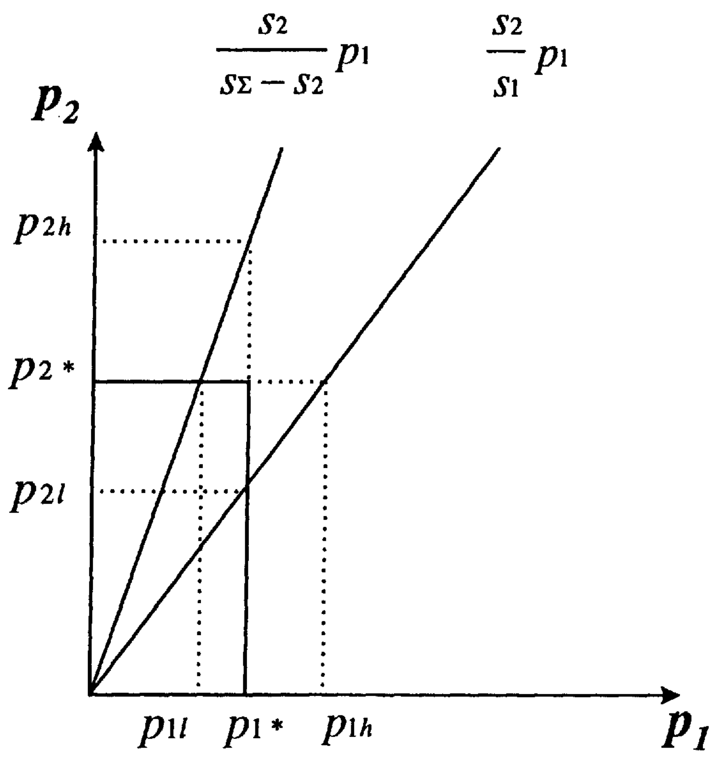


Figure 7: Pure-strategy complements equilibrium

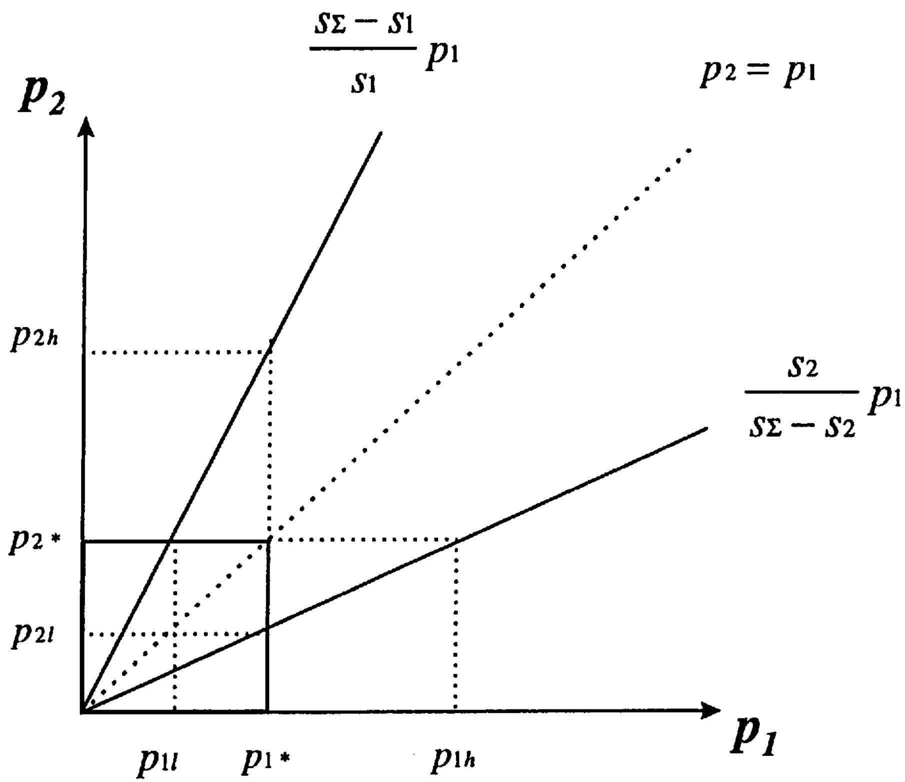


Figure 8: Information Structures of the Players