PARTS RECOVERY PROBLEM: THE VALUE OF INFORMATION IN REMANUFACTURING

by

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Parts Recovery Problem: The Value of Information in Remanufacturing

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Abstract

This paper deals with the uncertainty in the quality of the goods used as inputs in the remanufacturing shop. The manager in this job shop has to select an efficient procedure to determine whether a given part can be recovered and sent to the assembly line. Such procedure may identify bad parts earlier or later in the recovery process, as a function of the process design. Different scenarios are compared, where knowledge acquisition happens at different times in the recovery process, gaining insights about the importance of information systems in remanufacturing.

Keywords: *remanufacturing, job-shop, stochastic process yield*

1. Introduction

In several industries, remanufacturing has become an important complement to the production process. The remanufacturing process changes with the stage in the life cycle of the product. Early in the life cycle, the amount of returns available for disassembly and repair is not sufficient to feed the assembly line with all parts that are necessary, and the process requires the use of new parts. Later in the life cycle, there is a balance between the number of returns and the assembly line needs. When the product is closer to retirement, the supply of used machines for remanufacturing tends to be larger but the outcome of the repair process tends to be less favorable.

One of the constraints that differentiate remanufacturing from other types of production is the coordination between two supply functions: the supply of new parts, usually procured from an outside supplier, and the supply of used parts, repaired internally. In a typical remanufacturing site, a job shop is responsible for the disassembly of used machines to obtain parts that are repaired, cleaned, tuned, polished, painted, tested or whatever operation is necessary to make it perform and look like new. The coordination challenge increases with the yield uncertainty of the disassembled machines. The less we know about the outcome of the repair process, the harder it is to coordinate the procurement of new parts and the disassembly of used machines to reclaim some parts. Hence, the knowledge about the repair outcome is a valuable asset but, how much is it really worth? Intuitively, this value is higher when information is acquired early in the recovery process.

This paper deals with value of information in the remanufacturing site. Also, it discusses some alternatives to full information, and their trade-offs. Next section briefly describes the related literature. Section 3 describes the remanufacturing process and the scenarios considered in this paper. Sections 4 through 7 identifies the optimal inventory policy in each of the four scenarios, establishing the basis for the comparison that occurs in section 8. In section 9, I present some numerical illustrations showing the impact of the different process designs in the operating cost. The paper concludes with a summary.

2. Related Literature

There is a stream of literature in remanufacturing regarding its environmental relevance. Corbett and Van Wassenhove (2) discussed the corporate environmental responsibility suggesting how to analyze environmental programs from an operations management perspective. They bring about a set of analogies between environmental programs and existing operations management concepts shedding light over the contributions that operations management can bring to environmental research and management. Through their contribution, one can realize that remanufacturing can be an efficient environmental program not just as a cost-effective mean to reduce waste but as an integral part of the firm's manufacturing and marketing strategy. In the same lines, Bloemhof-Ruwaard et al. (1) developed an integration between the environmental chain and the supply chain. They suggest that operations research can provide with methods to evaluate and improve environmental management through appropriate internalization of environmental constraints and adaptation of its original models. In a preliminary study of product recovery management, Ferrer (3) discloses a number of industry practices, describing the remanufacturing and recycling efforts performed with a variety of products including automobiles, photocopiers, electronic goods and other items.

Several analytical works have dealt with reverse logistic issues, as encountered in a remanufacturing facility. Many of them are inspired by bottle refill plants or by planned repair instances. Salomon et al. (4) present what is probably the first analytical study specific for a remanufacturing facility. They develop two models: the first one is a steady-state analysis of a continuous review policy in a remanufacturing facility with no planned disposal, assuming Poisson arrival of used parts, exponential inspection and exponential repair times. The results are obtained with the use of simulation. Their second model allows for planned disposal, for which the continuous review parameters are obtained by approximation. Both models are based on a zero lead-time of procured parts for a single-part production process.

The problem discussed requires the determination of optimal lot-size policies for a variety of scenarios. The interested reader should check the literature review by Yano and Lee (5) about lot-size determination when yield is a random variable.

3. The Value of Perfect Information In Remanufacturing

The objective is to determine the value of perfect information about the yield in the parts' recovery process. The model describes a one-period decision for a single part that has to be supplied to the assembly line. The availability of information affects the decision making process in different ways, depending on when and how this information is acquired.

3.1 Process Description

There are two possible sources for the part: from an outside (perfectly reliable) supplier, and from the job shop that performs the part's recovery process. This job shop employs used machines returned to the plant as the main source of materials. The part's recovery process is subject to some yield whose probability distribution is perfectly known by the manager before he makes his decisions on disassembly and procurement quantities.

Each disassembly entails a fixed cost. Parts that are disassembled but not repaired and delivered to the assembly line incur a linear holding cost; the opportunity cost of the used resources plus the inventory management cost until the parts are used. Parts that are disassembled and repaired but not delivered to the

Table 1: Notation

Symbol	*Notation*
N	number of machines to be disassembled, a decision variable
x	number of parts to be procured from outside, another decision variable
Y	reclaim yield, a stochastic variable, a fraction between 0 and 1
y	the realization of the reclaim yield
C(N,x,y)	remanufacturing cost function per period
D	demand per period
F(y)	probability that $Y \leq y$, a distribution function
k	fixed cost per disassembled machine
r	repair cost per part
p	new part procurement cost
h	holding cost per part out of disassembly, before repairing takes place
h_r	holding cost per repaired part
s	shortage cost per part that is not delivered to the assembly line

assembly line incur holding cost, as well. It has the same interpretation as the previous holding cost, but it may be larger to account for the increased opportunity cost of used capacity.

Procurement lead-time is deterministic. It is longer than the time required for disassembly and repair. Likewise, the cost of procuring new parts outside is higher than the expected cost of remanufacturing. Demand that is not satisfied incurs a shortage cost corresponding to the profit forgone by the assembly line because of insufficient supply of parts. These assumptions make a stylized description of the remanufacturing site of a typical remanufacturer.

3.2 The Cost Function:

The cost function is composed of five terms: a purchase cost, a disassembly cost, a repair cost, a holding cost and a shortage cost. These components take different functional forms, depending on the organization of the remanufacturing job-shop, the alternatives available, and the relative timing between the information acquisition and the decision to disassemble, repair or procure parts. The cost is a function of two decision variables (the number of machines sent to disassembly and the number of parts procured from the outside supplier) and the realization of the stochastic variable (the parts' recovery yield). Table 1 shows the notation used in this paper:

If remanufacturing is economically viable, the parameters must satisfy the following conditions:

1 All costs involved are positive:

2 Being short is more expensive than supplying the part; obtaining the part from outside is more expensive than the expected cost of obtaining it by remanufacturing used machines:

$$s > p > \frac{k + r}{\bar{y}}$$

In other words, "if the sum of repaired and procured parts is insufficient to satisfy demand at any yield, cost can be reduced by disassembling and repairing more of the used machines".

3 Repairing an extra part (with uncertain yield) at the risk of increasing holding cost is less expensive than obtaining it from the outside supplier:

$$\frac{p}{r - h} > \int_0^1 \frac{dF(y)}{y}$$

This condition conveys a similar message. "If the number of parts that can be obtained from the disassembled machines is in excess of demand at any yield, cost can be reduced by purchasing fewer new parts." The analysis includes four scenarios, corresponding to different process capabilities or strategies. Let's look at a brief description of them.

3.3 Scenarios Analyzed:

1. The Hard Way: Information Comes Late: This is the base case, which corresponds to the reality of some existing remanufacturing plants. It provides an upper-bound of the cost function in remanufacturing environments. It is used against other remanufacturing strategies as a benchmark. I assume that the manager has to make all decisions without precise information about the parts' recovery yield; all he knows is the yield distribution. He procures x new parts, disassembles N used machines, repairs the used parts out of disassembly before he finally realizes the actual yield. The yield distribution is the only information that he can use to make his decision.

2. The Value of Learning while Working: Disassembly Builds Reparability Knowledge: This scenario is a relaxation of the base case. I assume that the disassembly operation is a source of information. The manager procures x parts and disassembles N machines based on the yield distribution, having no more information than he would have in the base case. However, during disassembly he builds perfect knowledge about the parts that can be repaired. Hence, he may stop repairing, once the lot is completed, if he wishes so.

3. The Value of Speed: Lead-Time of Procured Parts Is Short: Here, the base case is subjected to a different relaxation. No longer it is assumed that the lead-time of procured part is very long. The manager chooses the number of machines N to disassemble, the resulting parts are repaired and finally, if he is short, he places an order for the x parts still missing. Possible ways to implement this strategy include adopting more efficient order tracking, selecting suppliers geographically close or improving supplier coordination.

4. The Value of Information: Actual Yield Is Previously Known: Before making any decision, the manager knows precisely the proportion of the machines that have parts that can be successfully repaired. He decides the number of machines to disassemble (N) and the number of parts (x) to procure. This case provides a lower bound to remanufacturing costs. In practice, this scenario approximates the instances when there is a comprehensive information system in the field. Such system would track the quality of the machines that eventually return to the remanufacturing plant, when the user decides to dispose of it.

I use the optimal policies for each scenario above as their respective objectives. Drawing the yield realization from a probability distribution, I compare the operational cost in each of them. As it is usually true, having early information provides significant reduction in operational cost. Numerical examples show the magnitude of these gains.

4. The Hard Way: Information Comes Late

Under this scenario, once the machines are disassembled, the manager may decide to repair all parts or just the ones needed to satisfy the demand. This sequence of events assumes that the firm repairs up to demand:

1. Order x parts from the outside supplier.
2. Disassemble N machines.
3. Repair *min*{(D − x)/y, N} parts, while learning yield y (too late to adjust number of machines to disassemble).
4. Receive x parts from the outside supplier
5. Deliver *min*{D, x + Ny}
6. Incur holding cost $h(N-(D-x)/y)^+$ or shortage cost $s(D-x-Ny)^+$

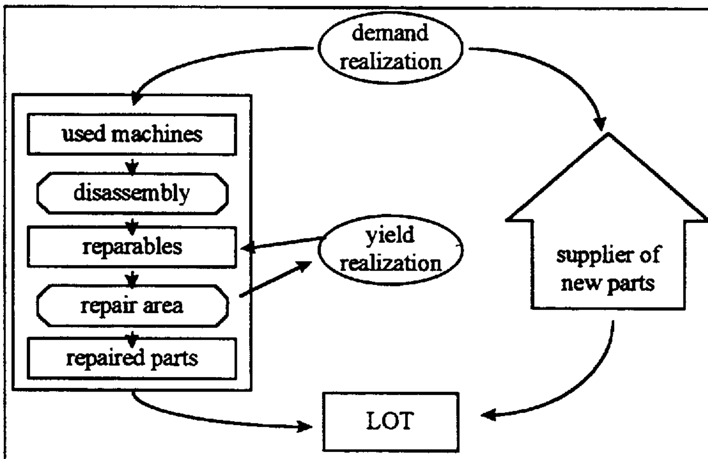

Figure 1: The Hard Way

For a given choice of N and x, and a realization of yield y, the manager faces this cost:

$$C_{1a}(N,x,y) = px + kN + r\min\left(N, \frac{D-x}{y}\right) + h\left(N - \frac{D-x}{y}\right)^+ + s(D-x-Ny)^+ \tag{1}$$

Alternatively, the manager may decide to repair all disassembled parts. Both repair and holding cost change, as represented by this sequence of events:

1. Order x parts from the outside supplier.
2. Disassemble N machines.
3. Repair all N parts, while learning yield y (too late to adjust number of machines to disassemble).
4. Receive x parts from the outside supplier.
5. Deliver *min*{D, x + Ny}.
6. Incur holding cost $h_r(Ny-(D-x))^+$ or shortage cost $s(D-x-Ny)^+$.

For a given choice of N and x, and a realization of yield y, the cost expression is slightly different:

$$C_{1b}(N,x,y) = px + kN + rN + h_r(Ny + x - D)^+ + s(D-x-Ny)^+ \tag{2}$$

Equations (1) and (2) differ just at the repair cost and the holding cost. So, one can make a direct comparison between the two cost functions to identify which policy is generally more efficient: *repair up to demand* or *repair all disassembled parts*. This is the first result:

PROPOSITION 1: *Given the option to repair all parts obtained from the disassembly of used machines, or just the parts needed to satisfy the demand for this period, the manager of the remanufacturing job shop should repair parts up-to demand.*

Proof: It suffices to compare the point-wise realizations of the cost functions in (1) and (2) writing their difference:

$$C_{1b} - C_{1a} = rN + h_r(Ny + x - D)^+ - r\min\left\{\frac{D-x}{y}, N\right\} - h\left(N - \frac{D-x}{y}\right)^+$$
$$= \left(r + \frac{h_r}{y} - h\right)\left(N - \frac{D-x}{y}\right)^+$$

The expression in the right parenthesis is non-negative. Also, the unit repair cost is larger than the holding cost of a part from disassembly (r > h). So, the expression in the left parenthesis is positive. Therefore, C_{1b} is point-wise larger than C_{1a}. That means that, at any yield, the operating cost of repairing up to demand is at most as large as the operating cost of repairing all disassembled parts. QED

4.1 Optimal Policy for Remanufacturing the Hard Way

Problem 1 identifies the optimal choice of N and x, given that repairing up to demand is more efficient:

$$P_1 \qquad \min_{N,x} E_Y C_1(N, x, Y)$$

The solution is found by defining the expected value of the cost function and identifying its minimum. Let's consider the first approach only, repair up to demand. In this case, the expected value of the cost function takes this form:

$$E_Y C_1(N,x,Y) = px + kN + r\left\{\int_0^{\frac{D-x}{N}} N dF(y) + \int_{\frac{D-x}{N}}^1 \frac{D-x}{y} dF(y)\right\}$$
$$+h\int_{\frac{D-x}{N}}^1 \left(N - \frac{D-x}{y}\right) dF(y) + s\int_0^{\frac{D-x}{N}} (D - x - Ny) dF(y)$$

The first derivatives take the forms:

$$\frac{\partial C_1}{\partial N} = k + r\int_0^{\frac{D-x}{N}} dF(y) + h\int_{\frac{D-x}{N}}^1 dF(y) - s\int_0^{\frac{D-x}{N}} y dF(y) \tag{3}$$

$$\frac{\partial C_1}{\partial x} = p - (r-h)\int_{\frac{D-x}{N}}^1 \frac{dF(y)}{y} - sF\left(\frac{D-x}{N}\right) \tag{4}$$

The function is convex in both variables (the Hessian is positive semidefinite). Here one cannot find the minimum by applying the first-order conditions automatically, since x and N are linked in both first derivative expressions. Let Y_{N1} and Y_{x1} be the ratios (D − x)/ N that solve the first-order condition in N and x, respectively. The following proposition provides the optimal policy in similar situations:

PROPOSITION 2: *The optimal policy for simultaneously deciding the number of machines to disassemble and the number of parts to procure is either to obtain all parts from the disassembly and repair process (remanufacturing) or to buy all parts from the outside (perfectly reliable) supplier.*

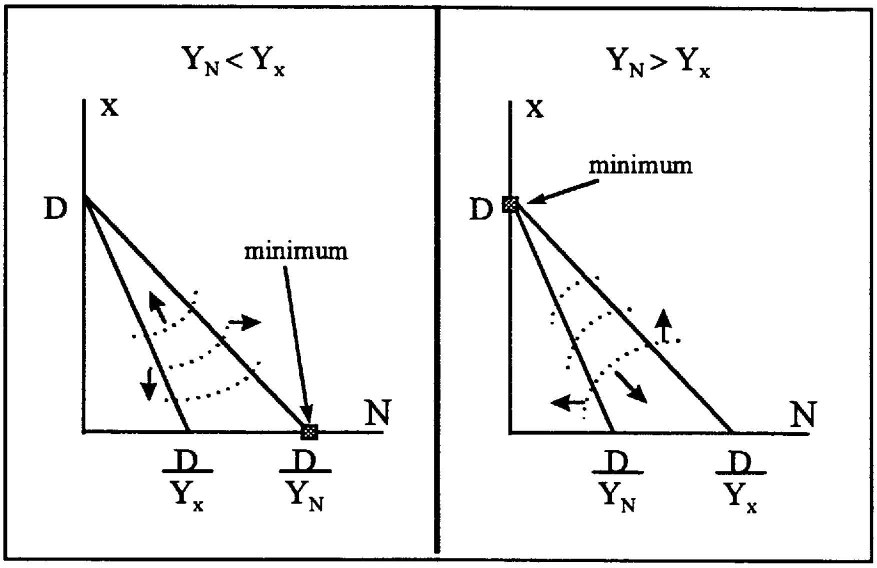

Figure 2: Location of minimizing pair (N*, x*). (Small arrows indicate gradient vectors. Level curves are dashed.)

Proof: In other words, a mixed policy is not optimal. It is shown just for this specific scenario; later I argue why this must be true in other scenarios where the decision for N and x must be made simultaneously.

The first order conditions of the minimization problem require that Y_{N1} and Y_{x1} satisfy:

$$F(Y_{N1}) = \frac{s\int_0^{Y_{N1}} y dF(y) - (k+h)}{r-h} \quad \text{and} \quad F(Y_{x1}) = \frac{p - (r-h)\int_{Y_{x1}}^{1} \frac{dF(y)}{y}}{s}$$

If Y_{N1} and Y_{x1} coincide, the optimal policy is to choose any pair (N, x) satisfying these ratios. However, Y_{N1} and Y_{x1} do not coincide in general. Figure 2 shows $(D - x) = NY_{N1}$ and $(D - x) = NY_{x1}$ graphed as two line segments intersecting at $N = 0$ and $x = D$. Obviously, the minimum must be within the triangle defined by these line segments and the N-axis, or on its borders. The gradient of the cost function at Y_N is perpendicular to the N-axis pointing away from Y_x. The gradient of the cost function at Y_x is perpendicular to the x-axis pointing away from Y_N. In the area bounded by the two lines, the gradient points towards the quadrant defined by the gradients of the boundary lines.

Notice that if $Y_{N1} < Y_{x1}$ the gradients point towards the second quadrant. Hence, the cost is highest at (0, D) and lowest at $(D/Y_{N1}, 0)$. However, if $Y_{N1} > Y_{x1}$ the gradient points to the fourth quadrant. Hence, the cost is highest at $(D/Y_{x1}, 0)$ and lowest at (0, D). This resumes the optimal policy:

$$\begin{cases} Y_{N1} < Y_{x1} \\ Y_{N1} > Y_{x1} \end{cases} \Rightarrow \begin{matrix} N^* = D/Y_{N1} & x^* = 0 \\ N^* = 0 & x^* = D \end{matrix}$$

It is proven that the mixed policy is not optimal in this special scenario. QED

Proposition 2 is true even when some assumptions in this scenario are not satisfied. Notice that in the beginning of the period, the manager has to decide N and x *simultaneously*, to satisfy a fixed demand D. Once x* is chosen, there remains a sure demand (D – x*) that has to be satisfied from the uncertain source, remanufacturing. The manager chooses a value N* corresponding to the critical fractile of the yield distribution that minimizes holding and shortage costs under yield uncertainty. The critical fractile minimizes

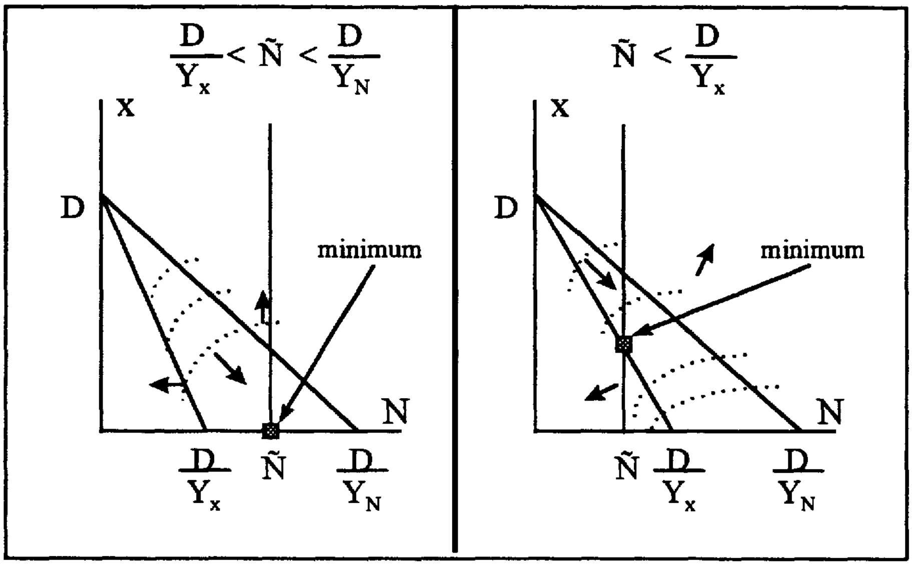

Figure 3: Location of minimizing pair (N^*, x^*) when availability of used machines is limited to Ñ. (Small arrows indicate gradient vectors. Level curves are dashed.)

the expected unit cost for each part produced, just like in the well-known newsvendor model. Not having setup costs, the unit cost is a function of the selected fractile, only. Now, if the unit cost under uncertainty (that is, remanufacturing) is lower than with certainty (that is, procuring), than the manager obtains all parts from his remanufacturing process. Hence, $x^* = 0$. Otherwise, the manager satisfies all of his demand buying parts from outside and forgo remanufacturing, setting $x^* = D$.

The following corollary gives the optimal policy when the assumption of sufficiently large stock of used machines is relaxed.

COROLLARY: *If the optimal policy (in proposition 2) determines to disassemble more machines than there is in stock, it is still not optimal to buy new parts from outside, unless the stock of machines available is very low.*

Proof: Notice Figure 3. In this case, the number of machines in stock is bounded at Ñ. Remember that the optimal solution cannot be outside the triangle defined by the two lines corresponding to the first-order conditions.

Now, the feasible region is cut by a vertical plane at $N = \tilde{N}$. This plane can be located in two regions: in the first one ($D/Y_x < \tilde{N} < D/Y_N$), there is a moderate constraint. Since the gradients are pointing towards the second quadrant, the lower cost is at the feasible point with largest N and smallest x: ($\tilde{N}$, 0).

In the second region, $0 < \tilde{N} < D/Y_x$, there is a hard constraint. Again, one should limit the analysis to the area within the triangle and bounded by $N \leq \tilde{N}$. With the gradients pointing to the second quadrant, the lowest cost is at ($\tilde{N}$, $D - \tilde{N}Y_x$), resulting in a mixed policy. QED

When Ñ is very small there is the special case of mixed policy. The number of used machines available to the remanufacturing process is so low that the shortage cost would be unbearable. This result is rather counterintuitive, because it says that one should not recourse from the outside supplier before the shortage is significantly large. Now, let's close this scenario with the expected value of the cost function when the optimal policy is observed. First, suppose that $Y_{N1} < Y_{x1}$. Hence,

$$E_Y C_1(N^*, x^*, Y) = \frac{kD}{Y_{N1}} + rD\left\{\int_0^{Y_{N1}} \frac{1}{Y_{N1}} dF(y) + \int_{Y_{N1}}^1 \frac{1}{y} dF(y)\right\}$$
$$+ hD\int_{Y_{N1}}^1 \left(\frac{1}{Y_{N1}} - \frac{1}{y}\right) dF(y) + sD\int_0^{Y_{N1}} \left(1 - \frac{y}{Y_{N1}}\right) dF(y)$$

After some manipulation, this simplifies to

$$E_Y C_1(N^*, x^*, Y) = D\left\{sF(Y_{N1}) + (r-h)\int_{Y_{N1}}^1 \frac{dF(y)}{y}\right\} \tag{5}$$

where Y_{N1} is the unique solution to the equation $F(Y_{N1}) = \dfrac{s\int_0^{Y_{N1}} y dF(y) - (k+h)}{r-h}$. However, if $Y_{N1} > Y_{x1}$, the expected cost takes the form:

$$E_Y C_1(N^*, x^*, Y) = pD \tag{6}$$

5. The Value of Learning while Working: operation builds reparability knowledge

Figure 4 depicts this scenario, which can also be approached in two ways. The manager can decide to repair all parts at once or just up-to demand. It is very simple to prove that repairing parts up to demand is more efficient than repairing all disassembled parts, just like in the previous case. Hence, only the former case is analyzed.

This case is a relaxation of the previous scenario. There is cost reduction because only parts with the potential to be successfully repaired is subjected to the complete remanufacturing process. The following sequence of events describes the process:

1. Order x new parts.
2. Disassemble N machines, while identifying parts that can be repaired.
3. Learn yield y.
4. Repair *min*{D – x, Ny} parts, pre-identified.
5. Receive x new parts.
6. Deliver *min*{D, x + Ny}.
7. Incur holding cost $h(Ny + x - D)^+$ or shortage cost $s(D - x - Ny)^+$.

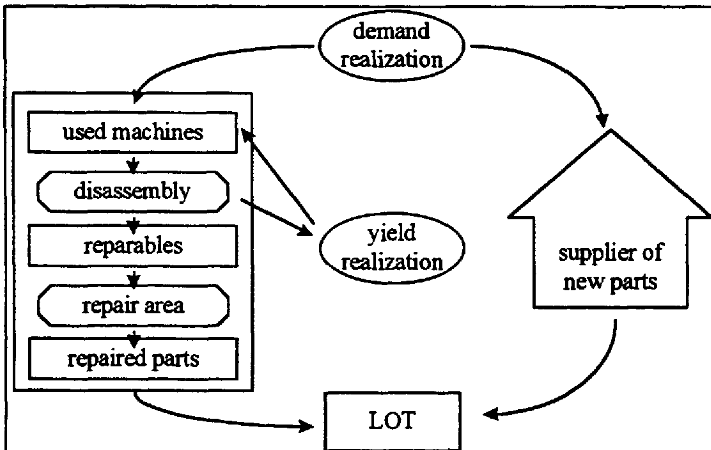

Figure 4: The Value of Learning while Working

For a given choice of N and x, and a realization of yield y, the manager faces this cost:

$$C_2(N, x, y) = px + kN + r\min\{D - x, Ny\} + h(Ny + x - D)^+ + s(D - x - Ny)^+ \tag{7}$$

Under this strategy, when one decides the number of machines to disassemble, or the number of new parts to procure from the new parts' supplier, the yield information is not available yet. Hence, there is the need for a policy that simultaneously chooses the number of machines (N) and the number of parts (x). The policy should minimize the expected value of the cost function with respect to the yield y. Problem P_2 looks for this policy:

$P_2 \qquad \min_{N,x} E_Y C_2(N,x,Y)$

$$\begin{aligned} E_Y C_2(N,x,Y) = px + kN + r\left\{ N\int_0^{\frac{D-x}{N}} y\,dF(y) + (D-x)\int_{\frac{D-x}{N}}^1 dF(y) \right\} \\ + h\int_{\frac{D-x}{N}}^1 (Ny + x - D)\,dF(y) + s\int_0^{\frac{D-x}{N}} (D - x - Ny)\,dF(y) \end{aligned} \tag{8}$$

Define $\tilde{y}(w)$, *bounded* expectation of the yield distribution, and call $\bar{y}$ the expected value of the yield:

$$\begin{cases} \tilde{y}(w) = \int_{-\infty}^{w} y\,dF(y) \\ \bar{y} = \int_{-\infty}^{+\infty} y\,dF(y) \end{cases} \tag{9}$$

Hence, the first derivatives become

$$\frac{\partial C_2}{\partial N} = k + (r-s)\int_0^{\frac{D-x}{N}} y\,dF(y) + h\int_{\frac{D-x}{N}}^1 y\,dF(y)$$

$$\frac{\partial C_2}{\partial x} = p - r\int_{\frac{D-x}{N}}^1 dF(y) + h\int_{\frac{D-x}{N}}^1 dF(y) - s\int_0^{\frac{D-x}{N}} dF(y)$$

The cost function is convex in both variables. Because of the relaxation over the hard case, the second condition reduces to $s > p > \frac{k}{\bar{y}} + r$. Again, one cannot find the minimum by applying the first-order conditions automatically, because each expression zeroes for different ratios (D – x)/N, as follows:

$$\left.\frac{\partial C_2}{\partial N}\right|_{(N^*,x^*)} = 0 \Rightarrow \tilde{y}\left(\frac{D-x^*}{N^*}\right) = \frac{k + h\bar{y}}{s+h-r}$$

$$\left.\frac{\partial C_2}{\partial x}\right|_{(N^*,x^*)} = 0 \Rightarrow F\left(\frac{D-x^*}{N^*}\right) = \frac{p+h-r}{s+h-r}$$

For the same reason as in Problem 1, the following policy is optimal (see Figure 3):

$$\begin{cases} Y_{N2} < Y_{x2} \\ Y_{N2} > Y_{x2} \end{cases} \Rightarrow \begin{matrix} N^* = D/Y_{N2} & x^* = 0 \\ N^* = 0 & x^* = D \end{matrix}$$

where Y_{N2} and Y_{x2} are the ratios (D – x)/N that satisfy the first-order condition in N and in x, respectively. This policy is not surprising because, as seen in Proposition 2, as long as the manager is obliged to decide N and x simultaneously, he chooses one alternative or the other, not a mix. If $Y_{N2} < Y_{x2}$, the expected value of the cost function becomes:

$$\begin{aligned} E_Y C_2(N^*,x^*,Y) = \frac{kD}{Y_{N2}} + r\left\{ \int_0^{Y_{N2}} \frac{Dy}{Y_{N2}}\,dF(y) + D\int_{Y_{N2}}^1 dF(y) \right\} \\ + h\int_{Y_{N2}}^1 \left(\frac{Dy}{Y_{N2}} - D\right) dF(y) + s\int_0^{Y_{N2}} \left(D - \frac{Dy}{Y_{N2}}\right) dF(y) \end{aligned}$$

Which simplifies to

$$E_Y C_2(N^*, x^*, Y) = D\{sF(Y_{N2}) + (r-h)(1-F(Y_{N2}))\} \quad (10)$$

where Y_{N2} satisfies $\int_0^{Y_{N2}} y dF(y) = \frac{k + h\bar{y}}{s+h-r}$. However, if $Y_{N2} > Y_{x2}$, the expected cost becomes:

$$E_Y C_2(N^*, x^*, Y) = pD \quad (11)$$

6. The Value of Speed: Lead-Time of Outside Supplier Is Short

Figure 5 shows this scenario. Again, it can be operated under two approaches: repair all disassembled parts or up-to demand. It is different relaxation of the first scenario, where the decision on N and x is not simultaneous. Since Proposition 2 is not applicable under this scenario, the mixed policy is likely to occur. It is simple to show that repairing up to demand is less costly than repairing all parts out from disassembly: it suffices to proceed a point-wise comparison between their cost functions. The following sequence of events describes the process:

1. Disassemble N machines.
2. Repair $min\{D/y, N\}$ parts, while learning yield y (too late to adjust number of machines to disassemble).
3. Order $x = (D - Ny)^+$ parts from the outside supplier.
4. Receive x parts from the outside supplier.
5. Deliver all D parts required.
6. Incur holding cost $h(N - D/y)^+$.

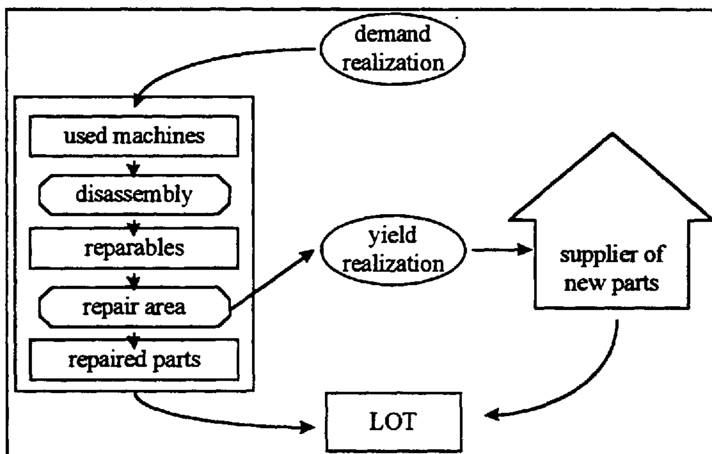

Figure 5: The Value of Speed

Given the "reasonable" conditions, the number of parts procured x is exactly the number required to avoid shortage costs. Therefore, the cost function takes the form:

$$C_3(N, x, y) = px + kN + r\min\left\{\frac{D}{y}, N\right\} + h\left(N - \frac{D}{y}\right)^+$$

6.1 Optimal Policy when Lead-Time of Outside Supplier Is Short

The optimal policy is found solving problem P_3':

P_3' $\quad \min_N E_Y \min_x C_3(N, x, Y)$

We already know that $x^* = (D - Ny)^+$ solves P_3'. Hence, there is a need for a policy to select the value of N that minimizes the expected value of this cost function with respect to y, given x*. Problem P_3 represents this problem:

P_3 $\quad \min_N E_Y C_3(N, x^*, Y)$

Here, the expected cost (and its first derivative) takes the form:

$$E_Y C_3(N,x^*,Y) = p\int_0^{\frac{D}{N}}(D-Ny)dF(y) + kN$$

$$+r\left[D\int_{\frac{D}{N}}^{1}\frac{dF(y)}{y} + N\int_0^{\frac{D}{N}}dF(y)\right] + h\int_{\frac{D}{N}}^{1}\left(N-\frac{D}{y}\right)dF(y)$$

$$\frac{\partial C_3}{\partial N} = -p\int_0^{\frac{D}{N}}ydF(y) + k + r\int_0^{\frac{D}{N}}dF(y) + h\int_{\frac{D}{N}}^{1}dF(y)$$

Again, the cost function is convex. The first-order condition says:

$$-p\tilde{y}\left(\frac{D}{N^*}\right) + k + rF\left(\frac{D}{N^*}\right) + h\left[1-F\left(\frac{D}{N^*}\right)\right] = 0 \tag{13}$$

Hence, the critical fractile of the yield distribution satisfying the first-order condition and the value of x that eliminates shortage cost define the optimal policy:

$$\begin{cases} F\left(\frac{D}{N^*}\right) = \dfrac{p\tilde{y}\left(\frac{D}{N^*}\right) - (k+h)}{r-h} \\ x^* = (D - yN^*)^+ \end{cases}$$

Because of the cost function's convexity, the solution is unique. When the optimal policy is applied, the expected value of cost function C_3 is:

$$E_Y C_3(N^*,x^*,Y) = pD\int_0^{Y_3}\left(1-\frac{y}{Y_3}\right)dF(y) + \frac{kD}{Y_3}$$

$$+rD\left(\int_{Y_3}^{1}\frac{dF(y)}{y} + \int_0^{Y_3}\frac{dF(y)}{Y_3}\right) + hD\int_{Y_3}^{1}\left(\frac{1}{Y_3}-\frac{1}{y}\right)dF(y)$$

After some manipulation, it simplifies to

$$E_Y C_3(N^*,x^*,Y) = D\left\{pF(Y_3) + (r-h)\int_{Y_3}^{1}\frac{dF(y)}{y}\right\} \tag{14}$$

where Y_3 is the unique solution to $F(Y_3) = \dfrac{p\tilde{y}(Y_3) - (k+h)}{r-h}$. (Notice the similarity between the expressions for $F(Y_{N1})$ and $F(Y_3)$.)

7. The Value of Information: Actual Yield Is Known

If the first-scenario offered an upper bound for the remanufacturing cost, this is a good candidate for a lower bound, because it eliminates most of the yield uncertainty. Figure 6 depicts this strategy. It takes in consideration that the manager makes the disassembly and purchase decision fully informed about the actual yield. Still, he has to identify the machines containing parts that can be successfully repaired:

1. Learn yield y.
2. Order x parts from the outside supplier.
3. Disassemble N machines, while identifying the Ny repairable parts in the lot.
4. Repair Ny parts, pre-identified.
5. Receive x new parts.
6. Deliver D = x + Ny parts.

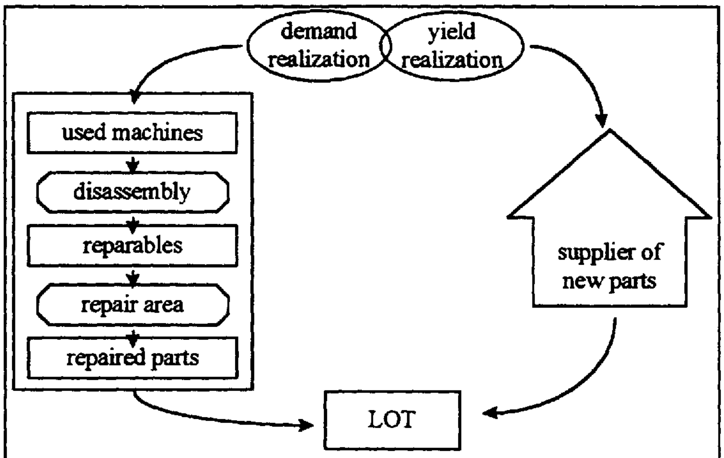

Figure 6: Actual Yield Is Known

The cost function has the same general form as in the previous scenarios. However, the manager is able to avoid holding or shortage cost altogether:

$$C_4(N,x,y) = px + kN + rNy \tag{15}$$

7.1 Optimal Policy when Yield Is Previously Known

Problem P_4 identifies the optimal policy when the yield is known but the machines are identified at the disassembly stage.

P4 $\quad \min_{N,x} C_4(N,x,y)$

This simple policy solves the problem:

$$\begin{cases} y < \dfrac{k}{p-r} \\ y > \dfrac{k}{p-r} \end{cases} \Rightarrow \begin{matrix} N^* = 0 & x^* = D \\ N^* = D/y & x^* = 0 \end{matrix}$$

In this case, the expected value of the cost function with respect to the yield Y is

$$E_Y \min_{N,x} C_4(N^*,x^*,Y) = D\left\{(p-r)F\left(\frac{k}{p-r}\right) + k\int_{\frac{k}{p-r}}^{1} \frac{dF(y)}{y} dF(y) + r\right\} \tag{16}$$

8. Comparison between scenarios

I have just produced the optimal policies for each of the four scenarios and determined the expected value of the remanufacturing cost, granted that the optimal policies are observed. The first scenario provided the upper bound of the remanufacturing costs in a single-part single-period job shop operation. The last scenario provided the lower bound. The second and third scenarios represent different types of relaxation of the base case. Let's compare them:

Table 2 - Experimental design

Variable Name	Symbol	Values
Remanufacturing costs (disassemble, repair)	(k, r)	(1.2;1.4) or (2.4;1.2)
Purchase cost of new part	p	5 or 8
Holding cost of disassembled part	h	1
Shortage cost	s	10 or 15
Yield distribution	F(y)	$\beta(6;2)$, $\beta(18;6)$ or $\beta(168;56)$

8.1 Disassembly builds knowledge vs. Lead-time of outside supplier is short

If under the strategy *disassembly builds knowledge* the parameters induce an optimal policy such that $x = D$, no part comes from disassembly. In this case, this strategy is dominated because the cost is at its upper-bound. Suppose that this is not the case: the optimal policy in *disassembly builds knowledge* chooses $x = 0$. Using the same positive value of N in both scenarios, the point-wise comparison, after simplification, becomes:

$$C_2 - C_3 = (s-p)(D-Ny)^+ - \left(\frac{1}{y}-1\right)\left[r \min\{D, Ny\} + h(Ny-D)^+\right]$$

This expression says that:

1. Higher values of shortage cost favor the third strategy, which depends on quick supplier.
2. Higher values of purchase cost favor the second strategy. Because in the third strategy the yield is learned during repair, high values of repair or holding costs favor the second strategy, based on learning yield during disassembly.
1. There is a realization of yield, $y = \dfrac{(s-p)D - rN}{(s-p-r)N}$, for which either strategy incurs the same cost. Hence, if the expected yield is below such value, it is more advantageous to have a supplier willing to deliver with a short lead-time. Otherwise, it is better to have the capability of learning yield during disassembly.

9. Numerical Illustrations

Now, the performance of each remanufacturing strategy is evaluated under a wide range of numerical parameters, to illustrate the cost advantage of each attribute (knowledge acquired during operation, fast delivery, full information). Table 2 discloses the values used in the experiment design. All possible scenarios were examined with a full-factorial experiment. All of them satisfy the reasonable conditions already described. They are:

$$s > p > \frac{k+r}{\bar{y}};\ h_r > h \text{ and } \frac{p}{r-h} > \int_0^1 \frac{dF(y)}{y}$$

These conditions limit the problem to situations where remanufacturing is inherently competitive. The yield distribution is assumed to follow the Beta distribution. That provides more flexibility in the choice of

Table 3 - Scenario comparison. Beta distribution.

Operating Strategy	Distribution	Minimum Gain	Average Gain	Maximum Gain
Base case (the hard way)	β (6; 2)	–	–	–
	β (18; 6)	–	–	–
	β (168; 56)	–	–	–
Disassembly builds knowledge (the value of learning while working)	β (6; 2)	–	7%	12%
	β (18; 6)	–	9%	12%
	β (168; 56)	7%	11%	14%
Lead-time of outside supplier is short (the value of speed)	β (6; 2)	3%	7%	16%
	β (18; 6)	2%	5%	9%
	β (168; 56)	–	1%	3%
Yield is previously known (the value of information)	β (6; 2)	13%	22%	30%
	β (18; 6)	12%	19%	24%
	β (168; 56)	10%	13%	16%

parameters. In all cases, the mean yield is 0.75 (typical of a remanufacturing operation), and the variance covers a wide range of possibilities. In this case, the β (6; 2) has the same mean and variance as the uniform distribution with parameters U (0.5; 1.0). Likewise, the β (18; 6) has the same mean and variance as the U (0.6; 0.9) and the β (168; 56) as the U (0.7; 0.8).

Table 3 indicates the indifference values of the three attributes (knowledge acquired during operation, fast delivery, full information) compared to the base case, with the three distributions used in the experiment. (Note that the expected yield is the same in the three cases.) It is interesting to notice that variance reduction has a different effect in each case. Figure 7 shows this phenomenon.. If the variance is sufficiently high, as when the yield follows β(6; 2), the advantages of having a fast supplier or an operator that can identify the good parts early is almost the same. However, implementing a comprehensive information system in the field may provide savings of up-to 30% of the operating costs.

On the other hand, as the variance decreases, the fast supplier provides fewer advantages. In such case, one should prefer to have a trained operator, capable of diagnosing the used machine as early as possible. If the variance is sufficiently low, as when the yield follows β(168; 56), having a trained operator is almost as interesting as having full knowledge about the quality of the inputs. In such case, the fast supplier brings almost no benefit at all.

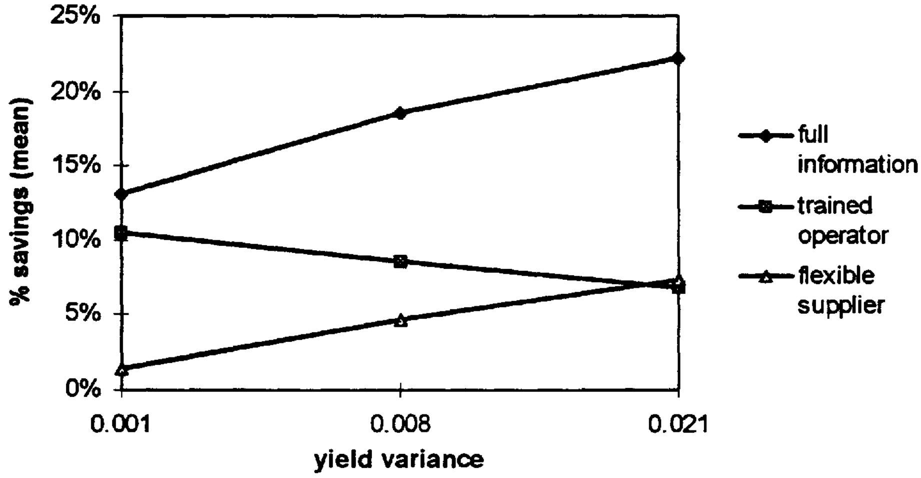

Figure 7: Impact of variance changes in the different scenarios when the mean yield equals 0.75.

The full information displays a clear advantage over the other attributes, at any variance. It confirms that remanufacturing industries can build a significant cost advantage by developing an information system to keep track of the quality of individual machines before they return to the part's recovery plant. What strikes in these tables is that *disassembly builds knowledge* offers considerable gains. Given that this strategy is generally easier to implement than a comprehensive IS, it emerges as an attractive alternative.

10. Summary

I have described four scenarios corresponding to different remanufacturing strategies:

1. A base case, where the manager knows just the probability distribution of the yield in the part's repair process;
2. A scenario where disassembly can be used to identify the repairable parts from the disassembled lot;
3. A scenario where, after the yield has been identified, new parts can be ordered by means of a fast supplier;
4. A scenario where yield knowledge is known before the disassembly and procurement decisions are made.

The optimal policy for each of these scenarios was identified and the expression for the expected value of the cost function was developed, granted that the optimal policy is observed. Then, I compared these remanufacturing strategies to identify the parameter that gave the greatest advantage to each of them. Finally, a numerical example was developed, covering a variety of reasonable cost structures, to illustrate the gains that certain remanufacturing strategies have over the base case. I tried to cover a broad variety of cost structures, but the experiment design is not exhaustive. For instance, one could examine the situations with even wider uncertainties or lower yield expectations than those represented here. I forgo this opportunity because it does not fit well with the remanufacturing examples that I have observed in practice. Some of our findings include:

- The relative gain of having a quick supplier diminishes significantly when the yield uncertainty is low.

- The ability to learn yield early, even if the use of this information is limited, can enhance the operating costs significantly. It seems that it can bring greater advantage than having a quick supplier.
- Having full information about the process yield brings the greatest advantage (confirming the intuition).

It is difficult to predict the impact of remanufacturing in the workforce, and how it affects employment. This question was not the objective of this research. Sophisticated machinery, or increased automation can improve manufacturing operations. The uncertainty in the remanufacturing shop does not allow for significant automation with the technology available today. It requires a great deal of labor in the disassembly operation, usually recruited among the unskilled. However, it is a surprising result that the economics of remanufacturing can be greatly improved with the help of skilled workers. Managers of remanufacturing projects should look carefully before embracing a full knowledge approach. The marginal gain compared to that of alternative strategies may not justify the costs of implementing a comprehensive information system.

11. References

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