INVENTORY CONTROL FOR JOINT MANUFACTURING AND REMANUFACTURING

by

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Inventory control for joint manufacturing and remanufacturing

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ABSTRACT

Product remanufacturing restores used products to an `as good as new' condition, after which they can be resold as new products. Inventory systems in which remanufacturing and production of new products is integrated, have typical characteristics such that they cannot be treated as traditional systems. This working paper presents a quantitative framework for a one-product, one-component inventory system with joint manufacturing and remanufacturing, to obtain insights into the structure of optimal control policies.

1 Introduction

The growing environmental burden of a 'throw-away-society' has made apparent the need for alternatives to landfilling and incineration of waste. Opportunities have been sought to reintegrate used products and materials into industrial production processes. Recycling of waste paper and scrap metal have been around for a long time. Collection and reuse of packages and recovery of electronic equipment are more recent examples. Efforts to efficiently reuse products and/or materials have introduced a wide range of novel and complex issues that affect the complete supply chain of recoverable products. Supply chain management in the light of product reuse is what we refer to as '*Reverse Logistics Management*'.

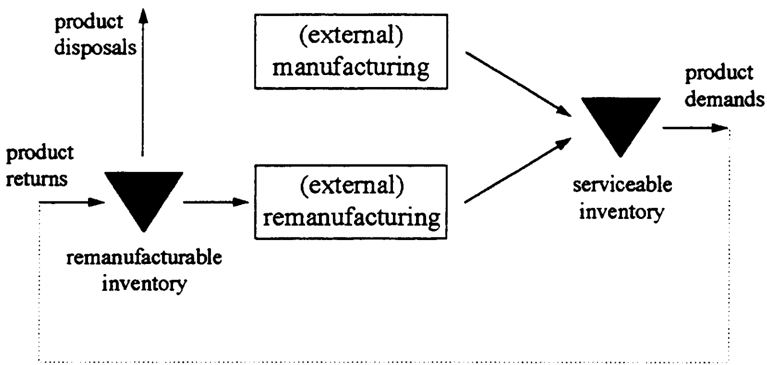
What makes efficient reuse even more complicated is the fact that the bulk of products that appear on the market are not designed for reuse. Product design is crucial since it determines to a large extent whether products and components can be easily disassembled, cleaned, tested and repaired if necessary. Nevertheless, product recovery has been successfully implemented for a wide variety of products. This indicates that even under imperfect conditions product recovery can be beneficial from an ecological point of view as well as from an economical point of view.

A specific type of product recovery is *product remanufacturing*. Product remanufacturing is the process that restores used products or product parts to an 'as good as new' condition, after which they can be resold on the market of new products. Examples of products that are actually being remanufactured in practice are automobile parts, commercial and military aircraft, diesel, gasoline and turbine engines, electronic equipment, machine tools, medical equipment, and railroad locomotives. The industrial operations involved with remanufacturing, like disassembly, testing, cleaning, repair, overhaul and refurbishing, are of a very stochastic nature due to the uncertainty in timing, quantity and quality of returned products (see [8, 28]). This results in a large uncertainty regarding the availability of inputs, and highly variable processing times.

Since remanufacturing restores a used product to an 'as good as new' condition it can serve as an alternative input resource in the fabrication of new products, but also *vice versa*. The former situation applies for instance to the electronics industry, where returned modules can be reused in new products. The latter may apply to the automobile industry (see [29]), where spare parts are made out of used parts and manufacturing of new parts is only used at times that the supply of remanufacturable parts is too low. A general framework for the situation in which demand can be supplied by both manufacturing and remanufacturing is given in Figure 1.

In this chapter we focus our attention on quantitative models for inventory control and production planning that apply to the situation depicted in Figure 1. Note that we are only concerned with simultaneously controlling the remanufacturable inventory, the serviceable inventory, the manufacturing

Figure 1: A hybrid system with manufacturing and remanufacturing operations, and stocking points for remanufacturables and serviceables.



source and the remanufacturing source, to satisfy *end item* demand. We do not address the issues related to shop floor scheduling and capacity planning for (re)manufacturing operations (see [3, 7, 8, 9, 10, 11, 15, 16, 27]). This means, for example, that we will not enter the discussion whether and how to use MRP (Material Requirements Planning) logic in a remanufacturing environment. For one view on this discussion we refer to [8]. For a general overview of quantitative models for reverse logistics we refer to [4].

We feel that any quantitative model that addresses our situation should at least contain the following characteristics:

I End item demand can be satisfied from two sources, i.e., the manufacturing source and the remanufacturing source, both having quite different characteristics (see II and III).

II In contrast with the manufacturing facility, the input of the remanufacturing facility is characterized by limited and random availability. This is due to the uncertainty in the *timing* of the returns, the *quantity* of the returns, and the *quality* of the returns.

III Due to the uncertainty in the *quality* of subassemblies and components, the operations with respect to disassembly, cleaning and repair are highly variable. Therefore, the remanufacturing facility is typically characterized by substantial and highly variable processing times.

The above indicates that we cannot just aggregate the manufacturing source together with the remanufacturing source, since both have quite different characteristics. The remanufacturing source may be seen as more unreliable than the manufacturing source, due to its limited availability of inputs and its variable holding costs. Since remanufacturing lead-times may be substantial, they need to be modeled explicitly. Assuming zero lead-times would be very inappropriate.

The complexities and the various process interactions that are prominent in a joint manufacturing/remanufacturing facility make efficient control a difficult task. Since little is known about the structure of the optimal policy, the main objective of this chapter is to obtain more insights in this issue.

The remainder of this chapter is organized as follows. In Section 2 we investigate if there are quantitative models in the literature that, although not originally developed for product remanufacturing, capture the above characteristics of a joint manufacturing/remanufacturing facility. Since the answer to this question appears to be negative, we present a quantitative framework that does capture the above characteristics (Section 3). The framework enables us to investigate the influence of system parameters, such as lead-times, the holding costs structure, and the return rate, on system performance under heuristic control policies. This may give us some clues about the structure of the optimal control policy and which information it should make use of (Section 4). We end this chapter with a summary and concluding remarks in Section 5.

2 Related models that do not apply to remanufacturing

A considerable number of articles in the production planning and inventory control literature have explicitly modeled both a return process and a demand process. Some of these articles deal with situations that appear to have a clear analogy with product remanufacturing. This literature comprises articles on repair and spare parts models, articles on traditional two-source models, and articles, originating from finance, on the so-called cash-balancing models. This tempts some people to believe that remanufacturing issues have already been studied extensively and therefore any recent literature that specifically deals with remanufacturing is redundant and obsolete. We show that this belief is false by reviewing the models in question, indicating why they do not capture the unique characteristics of a situation with joint manufacturing/remanufacturing. Recall that these characteristics are (i) two exogeneous supply sources, that is manufacturing and remanufacturing, (ii) time-dependent availability of the remanufacturing source, and (iii) non-zero stochastic lead-times for remanufacturing.

2.1 Repair and spare parts models

The literature dealing with spare parts management and repair is quite extensive. The models considered all apply to a closed system consisting of a fixed number of parts or products that are subject to failure. If a part or product fails it needs to be replaced by a spare part. If possible, the failed part is then repaired in order to satisfy future demand. Otherwise parts are procured from outside. The objective is to determine the number of spare parts in the system for which total system costs are minimized and/or for which prespecified service levels are guaranteed. An example is the well-known family of METRIC models introduced by [25].

Unfortunately, the above models do not apply to remanufacturing. A common assumption in these models is that demands for new products are generated by product failures only, i.e., product demands and product returns are *perfectly* correlated. This is not the case for remanufacturing, where product returns may or may not be caused by a product replacement. Even in case of a product replacement it may not be clear when the used product will be available for remanufacturing. The product need not be returned immediately after use, or the collection process may delay and randomize its time of arrival to the remanufacturing company. In general it may even be more appropriate to assume complete *independence* between the demand and return flows, than to assume correlation.

Another difference between models for spare parts management and models for remanufacturing lies in the objective: with spare parts management the objective is to determine the *fixed* number of spare parts in the system, such that the associated long-run average costs are minimized. With remanufacturing the objective is to develop a policy on when and how much to remanufacture, dispose, and produce, such that some cost function is minimized. Essential in remanufacturing is that the number of products in the system may vary over time.

Reviews on spare parts and repair management can be found in [23, 20, 2, 17].

2.2 Cash-balancing models

Another type of models that show some resemblance with remanufacturing stem from finance: the cash-balancing models. These models usually consider local cash of a bank with incoming money flows (customer deposits), and outgoing money flows (customer withdrawals). Local cash can be increased by ordering money from central cash, or decreased by transferring money to central cash. The objective is to determine the time and quantity of the cash transactions, such that the sum of fixed and variable transaction costs, backlogging costs, and interest costs related to the local cash are minimized. There exist continuous review and periodic review cash-balancing models.

The reason why we feel these models are not suitable for remanufacturing is that their analysis does not handle storage of returns (customer deposits) and lead-times for (re)manufacturing operations. Although storage of returns makes no sense from the perspective of cash-balancing, it is typical for remanufacturing systems. Additionally, the relation between the manufacturing and remanufacturing lead-times appears to be crucial. Therefore we believe that one should favor an approach that can handle a more detailed modeling of the remanufacturing source.

An extensive overview of cash-balancing models is given by [13].

2.3 Classical two-source models

Inventory models in which outside procurement orders may be placed at two different suppliers, so called two-source models (see e.g. [1, 18, 34]), show some analogy with hybrid remanufacturing systems in which product demand may be served by both manufacturing (supplier one) and remanufacturing (supplier two). Usually these models deal with the situation in which there is the regular supplier and an alternative supplier that offers smaller lead-times against a higher price.

The reason why these models are not suitable for remanufacturing lies in the fact that both suppliers are assumed to be continuously available. In remanufacturing though, the availability of the remanufacturing source varies over time, since it depends on the uncertain flow of product returns. As a consequence, the two types of models make different trade-offs in optimizing their associated costs. In classical two-source models the faster source is chosen if inventories are running dangerously low. In remanufacturing models the manufacturing source is chosen at times that a replenishment should occur but the remanufacturing source is not available. If disposal of excess remanufacturable products is allowed, one also has to make a trade-off between manufacturing and remanufacturing costs. However, instead of bringing the models closer together, the option of disposal introduces even more complexity in terms of control policy structure and system analysis.

A small subgroup of the multiple-source models considers random availability of suppliers ([12, 21]). However, these models contain simplifying assumptions such as deterministic demand, zero lead-times, and equal characteristics for all suppliers, so they do not match with our criteria.

3 An analytical framework for joint manufacturing and remanufacturing

In this section we follow the analytical framework of [29], who developed an *exact* procedure to study the system in Figure 26.1. This framework satisfies our three criteria and it allows to study various control policies under different conditions, such as stochastic lead-times, correlation between the return and demand flows, and Coxian-2 distributed return and demand flows. Formally, the characteristics of the system studied are as follows:

The *demand and return processes* are stochastic and may be modeled by any Markovian arrival process, although it is common practice to assume (compound) Poisson arrivals. In a straightforward manner we may use Coxian-2 arrival processes. These processes enable us to do a three moment fit of an arbitrary arrival process. If disposal of remanufacturable products is not allowed, we assume that the return intensity λ_R (the average number of returns per unit of time) is smaller than the demand intensity λ_D.

Testing process. It is assumed that every returned item is already tested and satisfies the quality requirements for remanufacturing.

The *remanufacturing process* has unlimited capacity. The remanufacturing lead-time L_r, which is the time that passes between the time at which a remanufacturing order is released to the remanufacturing facility and the time of actual delivery, is a random variable with mean μ_{L_r} and variance $\sigma^2_{L_r}$. The fixed remanufacturing costs are c_r^f per batch, and the variable remanufacturing costs are c_r^v per product. A remanufacturing release moves a batch of remanufacturables from the remanufacturable inventory to the remanufacturing work-in-process (WIP). After remanufacturing all products immediately enter the serviceable inventory.

The *manufacturing process* has unlimited capacity. The manufacturing costs consist of a fixed component c_m^f per order, and a variable component of c_m^v per product. The manufacturing lead-time L_m, which is the time that passes between the time at which a manufacturing order is placed and the time of actual delivery, is a random variable with mean μ_{L_m} and variance $\sigma^2_{L_m}$. Manufactured products enter the serviceable inventory.

The *inventory process* consists of two stocking points: one to keep remanufacturable inventory and one to keep serviceable inventory. The holding costs in remanufacturable inventory are c_r^h per product per time-unit, and the holding costs in serviceable inventory are c_s^h per product per time-unit. Although there is quite some controversy with respect to the valuation of these holding cost parameters, it is reasonable to assume that $c_r^h \leq c_s^h$. If disposal is not allowed, both stocking points have unlimited capacity.

If disposals are allowed, the *disposal process* depends on the actual control policy employed. Variable disposal costs are c_d^v per disposed product and fixed disposal costs are c_d^f.

Customer service: demands that cannot be fulfilled immediately are backordered against backorder costs c_b per product per unit of time.

Under some predefined control policy $\mathcal{P}$ with decision vector V, the long-run average system costs per unit of time, denoted $\overline{C}_{\mathcal{P}}(V)$, are the summation of the following components:

c_s^h	$\times$	average serviceable inventory per time unit
c_r^h	$\times$	average remanufacturable inventory per time unit
c_r^v	$\times$	average number of remanufactured products per time unit
c_r^f	$\times$	average number of remanufacturing batches per time unit
c_m^v	$\times$	average number of manufactured products per time unit
c_m^f	$\times$	average number of manufacturing batches per time unit
c_b	$\times$	average backordering position per time unit
c_d^v	$\times$	average number of disposed products per time unit
c_d^f	$\times$	average number of disposal batches per time unit

The objective is to choose the parameters of policy $\mathcal{P}$ such that total long run average costs per unit of time are minimized. However, it is still unclear what $\mathcal{P}$ should be, since the structure of the optimal policy is unknown. To start our investigations we therefore have to rely on heuristic control policies.

3.1 Definition of control policies

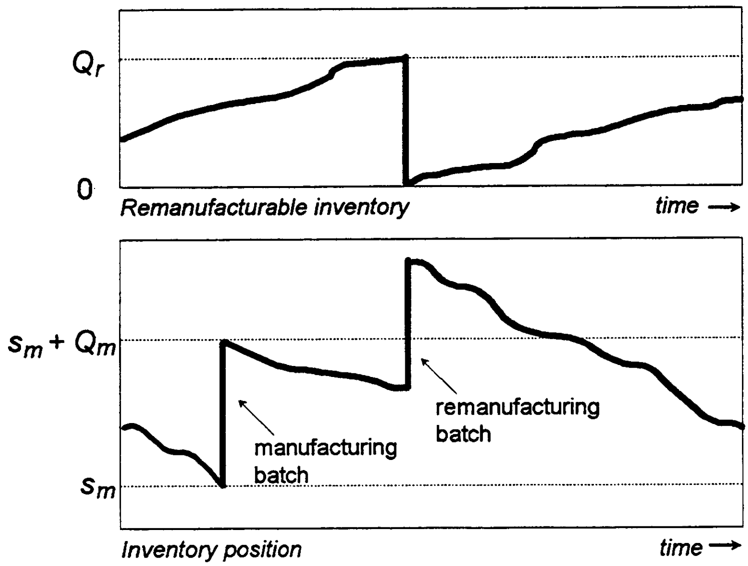
The two heuristic control policies that we have chosen to implement are all natural extensions of the classical (s, Q) policy. While manufacturing orders are controlled by an (s, Q) policy, remanufacturing batches are either being pushed through the remanufacturing facility, or they are being pulled when they are really needed. Although we suspect that the optimal policy will be a mixture of push and pull we look at two extremes. The next two paragraphs formally define our policies.

The (s_m, Q_m, Q_r) PUSH policy

Remanufacturing starts whenever a batch of returned products of size Q_r is available at the stocking point for remanufacturables. The remanufacturing order will arrive at the on-hand serviceable inventory after L_r time units. Manufacturing of Q_m products starts whenever the serviceable inventory position drops to the level s_m. At that time, the inventory position is increased to $s_m + Q_m$. The manufacturing order will arrive at the on-hand serviceable inventory after L_m time units (see Figure 26.2).

This policy is named *PUSH* policy, since remanufacturable inventory is *pushed* through the remanufacturing process as soon as possible.

Figure 2: A schematic representation of the PUSH policy.

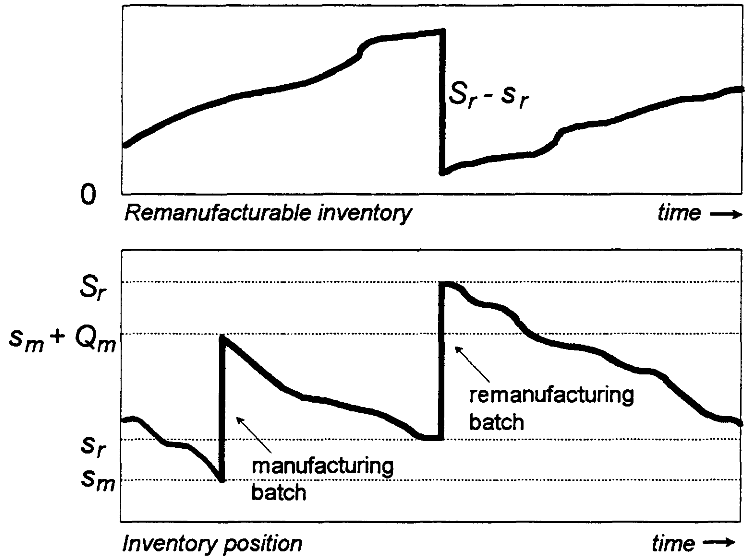


The (s_m, Q_m, s_r, S_r) PULL policy

Remanufacturing starts whenever the serviceable inventory position is at or below s_r, and sufficient remanufacturable inventory exists to increase the serviceable inventory position to S_r. The remanufacturing batch is exactly the amount that is necessary to increase the inventory position to S_r. If we denote this amount by Q then a remanufacturing order decreases the remanufacturable inventory with Q products, and increases the remanufacturing WIP and the inventory position with Q products. The remanufacturing order will arrive at the on-hand serviceable inventory after L_r time units. Manufacturing of Q_m products starts whenever the serviceable inventory position drops to the level s_m. At that time, the inventory position is increased to $s_m + Q_m$. The manufacturing order will arrive at the on-hand serviceable inventory after L_m time units (see Figure 26.3).

This policy is named *PULL* policy, since remanufacturable inventory is *pulled* into the remanufacturing process only when needed to fulfill customer demands for serviceables. Note that s_m should always be smaller than s_r, since otherwise the remanufacturing option would never be chosen, and remanufacturable inventory would accumulate infinitely.

Figure 3: A schematic representation of the PULL policy.



3.2 Mathematical analysis

Next we outline a procedure to calculate the long run average costs $\overline{C}_{\mathcal{P}}(V)$. The notation used in this outline is specified in Table 26.1.

Under the assumption that the control policy only depends on the inventory position and/or the remanufacturable inventory, the state transitions of the joint manufacturing/remanufacturing system at hand can be formulated as a continuous time Markov chain. This Markov chain, say $\mathcal{M}$, has a two-dimensional state variable $X(t) = \left(I_s(t), I_r^{OH}(t)\right)$ with discrete state space $\mathcal{S} = \{I_s(t)\} \times \{I_r^{OH}(t)\}$. By definition, $X(t) = (i_s, i_r^{OH}) \in \mathcal{S}$ whenever $I_s(t) = i_s$ *and* $I_r^{OH}(t) = i_r^{OH}$.

The limiting joint probability distribution $\pi(i_s, i_r^{OH})$, defined as

$$\pi(i_s, i_r^{OH}) = \lim_{t\to\infty} \Pr\{I_s(t) = i_s, I_r^{OH}(t) = i_r^{OH}\} \tag{1}$$

is obtained by solving the balance equations (inflow equals outflow) that are associated with $\mathcal{M}$. Although these balance equations are usually easy to write down, it is generally quite difficult to find a closed form expression for (1). Therefore, we have to rely on numerical procedures instead. More complicated is the calculation of the average on-hand serviceable inventory and the average

Table 1: Notation used for the analysis.

$I_s^{net}(t)$	=	The net serviceable inventory at time t, defined as the number of products in on-hand serviceable inventory minus the number of products in backorder at time t
$I_s(t)$	=	The serviceable inventory position at time t, defined as the net serviceable inventory plus the number of products in manufacturing work-in-process plus the number of products in remanufacturing work-in-process
$I_r^{OH}(t)$	=	The number of products in remanufacturable on-hand inventory at time t
$W_m(t)$	=	The number of products in manufacturing work-in-process at time t
$W_r(t)$	=	The number of products in remanufacturing work- in-process at time t
$D(t_0, t_1)$	=	The demands in time interval $(t_0, t_1]$
$Z(t-t_0, t-t_1)$	=	The number of products ordered to be (re)manufactured in the interval $(t-t_0, t-t_1]$ that enter serviceable inventory at or before time t *minus* the demands in the interval $(t-t_0, t-t_1]$
$\overline{B}$	=	The long-run average backordering position
$\overline{I}_s^{OH}$	=	The long-run average on-hand serviceable inventory
$\overline{I}_r^{OH}$	=	The long-run average on-hand remanufacturable inventory
$\overline{O}_r$	=	The long-run average number of remanufacturing orders
L^{min}	=	The minimum of all possible realisations of the manufacturing and remanufacturing lead-time
$L^{max}(<\infty)$	=	The maximum of all possible realisations of the manufacturing and remanufacturing lead-time

backorder position, i.e.,

$$\overline{I}_s^{OH} = \sum_{i_s^{net}>0} i_s^{net} \lim_{t\to\infty} \Pr\{I_s^{net}(t) = i_s^{net}\}, \tag{2}$$

and

$$\overline{B} = - \sum_{i_s^{net}<0} i_s^{net} \lim_{t\to\infty} \Pr\{I_s^{net}(t) = i_s^{net}\}. \tag{3}$$

These cannot be calculated directly using a Markov chain formulation, since the transitions of I_s^{net} are not Markovian. However, we can derive an expression for

$I_s^{net}(t)$ from which we can calculate its long run distribution. By definition, we have

$$I_s^{net}(t) = I_s(t) - W_m(t) - W_r(t). \tag{4}$$

Additionally, we have the following relation for the net inventory process:

$$\begin{aligned} I_s^{net}(t) = \; & I_s^{net}(t - L^{max}) \\ & + \text{ every ordered product that is delivered in } (t - L^{max}, t] \\ & - \text{ every demand that arrives in } (t - L^{max}, t] \end{aligned} \tag{5}$$

Here we mean by 'every ordered product' all products that were ordered for manufacturing *and* remanufacturing.

The number of ordered products that arrive in $(t - L^{max}, t]$ can be split into two groups: The products that are in (re)manufacturing WIP at time $t - L^{max}$ (these will all arrive before or at time t, since lead-times are never larger than L^{max}), and the number of products that were ordered during the interval $(t - L^{max}, t]$ *and also* arrived before time t. So, (5) becomes

$$\begin{aligned} I_s^{net}(t) = \; & I_s^{net}(t - L^{max}) \\ & + \text{ every product in (re)manufacturing WIP at time } t - L^{max} \\ & + \text{ every product that is both ordered } \textit{and} \text{ delivered} \\ & \quad \text{in } (t - L^{max}, t] \\ & - \text{ every demand that arrives in } (t - L^{max}, t] \\ = \; & I_s^{net}(t - L^{max}) + W_m(t - L^{max}) + W_r(t - L^{max}) \\ & + \text{ every product that is both ordered } \textit{and} \text{ delivered} \\ & \quad \text{in } (t - L^{max}, t] \\ & - \text{ every demand that arrives in } (t - L^{max}, t] \end{aligned} \tag{6}$$

Using (4) we simplify (6) as

$$\begin{aligned} I_s^{net}(t) = \; & I_s(t - L^{max}) \\ & + \text{ every product that is both ordered } \textit{and} \text{ delivered} \\ & \quad \text{in } (t - L^{max}, t] \\ & - \text{ every demand that arrives in } (t - L^{max}, t] \end{aligned}$$

Note that all products that are ordered after time $t - L^{min}$ arrive *after* time t. So finally we have

$$\begin{aligned} I_s^{net}(t) = \; & I_s(t - L^{max}) \\ & + \text{ every product ordered in } (t - L^{max}, t - L^{min}] \textit{ and } \text{delivered} \\ & \quad \text{before time } t \\ & - \text{ every demand that arrives in } (t - L^{max}, t - L^{min}] \\ & - \text{ every demand that arrives in } (t - L^{min}, t] \\ = \; & I_s(t - L^{max}) \; + \; Z(t - L^{max}, t - L^{min}) \; - \; D(t - L^{min}, t) \end{aligned} \tag{7}$$

We can use relation (7) to derive the long run distribution of $I_s^{net}(t)$ if we take into account the following stochastic (in)dependencies between $I_s(t - L_{max})$, $I_r^{OH}(t - L_{max})$, $Z(t - L_{max}, t - L_{min})$, and $D(t - L^{min}, t)$:

- $Z(t - L_{max}, t - L_{min})$ is correlated with $I_s(t - L_{max})$ and $I_r^{OH}(t - L_{max})$,
- $I_s(t - L_{max})$ and $I_r^{OH}(t - L_{max})$ are correlated,
- $D(t - L^{min}, t)$ is uncorrelated with $I_s(t - L_{max})$, $I_r^{OH}(t - L_{max})$, and $Z(t - L_{max}, t - L_{min})$.

Substituting $u = t - L^{max}$ and $\Delta L = L^{max} - L^{min}$, the long-run distribution of the net inventory is derived from (7) as

$$\begin{aligned} & \lim_{t\to\infty} \Pr\{I_s^{net}(t) = i_s^{net}\} \\ & \quad = \sum_{\Omega} \; \lim_{t\to\infty} \Pr\{D(t - L^{min}, t) = d\} \\ & \quad \qquad \times \lim_{u\to\infty} \Pr\{I_s(u) = i_s, I_r^{OH}(u) = i_r^{OH}, Z(u, u + \Delta L) = z\} \\ & \quad = \sum_{\Omega} \exp^{-\lambda_D L^{min}} \frac{(\lambda_D L^{min})^d}{d!} \times \pi(i_s, i_r^{OH}) \times h_{z|(i_s, i_r^{OH})}(\Delta L), \end{aligned} \tag{8}$$

where

$$\Omega = \{(i_s, i_r^{OH}, z, d) | i_s + z - d = i_s^{net}\}, \tag{9}$$

and

$$h_{z|(i_s, i_r^{OH})}(\Delta L) = \lim_{u\to\infty} \Pr\{Z(u, u + \Delta L) = z | I_s(u) = i_s, I_r^{OH}(u) = i_r^{OH}\}. \tag{10}$$

The conditional probability $h_{z|(i_s,i_r^{OH})}(\Delta L)$ is calculated as

$$h_{z|(i_s,i_r^{OH})}(\Delta L) = \sum_{(k,\ell)\in\mathcal{S}} q_{(k,\ell,z)|(i_s,i_r^{OH},0)}(\Delta L) \tag{11}$$

where $q_{(k,\ell,z)|(i_s,i_r^{OH},0)}(\Delta L)$ is the conditional probability that during the interval $(t - L^{max}, t - L^{min}]$ the initial system state changes from state

$$\{I_s(t - L^{max}) = i_s, I_r^{OH}(t - L^{max}) = i_r^{OH}, Z(t - L^{max}, t - L^{max}) = 0\}$$

into state

$$\{I_s(t - L^{min}) = k, I_r^{OH}(t - L^{min}) = \ell, Z(t - L^{max}, t - L^{min}) = z\}.$$

This conditional probability can be calculated with the transient analysis of an appropriate Markov chain, using a discretization technique. Stochastic lead-times complicate this technique, since the transition rates of the underlying Markov chain are not time-independent. However, for discretely distributed lead-times one can identify a series of time-intervals for which a time-independent Markov chain exists. Details of this approach can be found in [29].

Relations (8)–(11) now enable to calculate the average net inventory (2) and the average backorder position (3). All other cost function components are derived using (1) and the PASTA (Poisson Arrivals See Time Averages) property (see [35]).

Although the above analysis is *exact*, we have to evaluate the cost function *numerically*. The main problem here involves the truncation of infinite sums. While in some situations there may exist general rules for truncation, in other situations we have to commit ourselves to heuristic bounds and stopping criteria.

Note that the analysis does not depend on any specific assumptions regarding the demand and return processes, and the control policy involved, as long as they are Markovian. Main complicating factors are the dimensions of the Markov chain involved and the truncation of infinite state spaces. The reader should keep in mind however that this framework is meant as a means to compute expected costs for a general remanufacturing setting. It is not meant as an efficient numerical recipe. For the latter we refer to [19] and [31]. Details on how to model Coxian-2 arrival processes and how to incorporate correlation between the demand and the return processes can be found in [29].

Remark All the numerical examples presented in the next section were calculated using the analysis of this section and the following parameter settings: $\lambda_D = 1.0$, $\lambda_R = 0.7$, $L_m \equiv 2.0$, $L_r \equiv 2.0$, $c_s^h = 1.0$, $c_r^h = 0.5$, $c_b = 50$ unless specified otherwise. We initially choose $c_m^v = c_r^v = c_d^v = 0$, since these values are only relevant if product disposals are allowed (section 4.3). The influence of fixed costs are not considered in this paper, so we set $c_m^f = c_r^f = c_d^f = 0$.

4 On the structure of optimal policies

Now that we have a framework for a joint manufacturing/remanufacturing system we may ask what kind of control policies are reasonable, or even optimal, for such a system. Since this is not an easy question to answer, we start with studying this system under some simplifying assumptions. These assumptions are (i) stocking of remanufacturables is not allowed, (ii) disposal of remanufacturables is not allowed, and (iii) the remanufacturing lead-time is zero. This system applies to situations where products are returned because of delivery errors or special return agreements between supplier and customer. Except perhaps for repackaging, no remanufacturing operations need to take place and the products can enter the serviceable inventory immediately. In this light, disposal of returned products does not seem to make sense as long as handling costs are reasonably low. The above system is studied by [6]. We discuss the main outcomes below.

Since returned items immediately enter serviceable inventory the model considered comes down to a variant of a standard stochastic inventory model where demand may be both positive or negative. First, assume that demands and returns are generated by independent Poisson processes. Since disposal is not considered in this model the return rate is restricted to be smaller than the demand rate. Let $\gamma = \lambda_R/\lambda_D < 1$ denote the return ratio. Finally, assume the manufacturing lead-time to be constant.

Under these assumptions [6] show average–cost optimality of an (s, Q) control policy for manufacturing. Analogous to standard inventory control models with backordering it is easy to see that it suffices to consider control policies depending only on the inventory position $I_s(t)$. Note that in terms of the framework depicted in Figure 1 this is due to the fact that there is only one process to be controlled, namely manufacturing.

Let $G(l)$ denote the decision relevant expected variable costs, namely the expected holding and backorder costs at time $t + \mu_{L_m}$ when $I_s(t)$ equals l. Conditioning the cost function on $I_s(t)$ yields

$$G(l) = (c_s^h + c_b) \sum_{j=-\infty}^{l-1} D_j + c_b[(\lambda_D - \lambda_R)\mu_{L_m} - l],$$

where D_j is the probability that the net demand during a lead-time period is at most j.

It should be noted that in the above model any stationary policy based on the inventory position is an (s, Q) policy upto a transient start–up phase. (Take s as the largest value of $I_s(t)$ for which a manufacturing order is placed.) Therefore, the optimality proof boils down to showing that there exists an average cost optimal stationary policy in the above model. This can be achieved by applying results from general Markov decision theory. Using the results by [24], two conditions need to be verified: (i) the inventory levels for which it is optimal not to place an order under an α–discounted cost criterion are bounded below uniformly in α, and (ii) there exists a stationary policy inducing an irreducible, ergodic Markov chain yielding finite average costs in steady state. In the above model (i) can be shown by using convexity of $G(.)$ and the fact that $G(l) \to \infty$ for $l \to -\infty$ whereas the costs incurred during a first passage from level l to $l-1$ are bounded in α. (ii) is proven by showing that an arbitrary (s, Q) policy verifies this condition. Since the analysis provides some interesting insights we discuss this last step in somewhat more detail.

Due to [19] the process $I_s(t)$ in the above model under a given (s, Q) policy is known to be ergodic and to have the following stationary distribution

$$\lim_{t\to\infty} P\{I_s(t) = s+k\} = \begin{cases} \dfrac{1-\gamma^k}{Q}, & \text{for } 1 \le k \le Q \\ \dfrac{(\gamma^{-Q}-1)\,\gamma^k}{Q}, & \text{for } Q < k. \end{cases}$$

Note that in contrast with standard inventory models the state space is unbounded above. The long–run average costs for a given (s, Q) policy in this model, denoted by $C(s, Q)$, can now be written as

$$C(s,Q) = \frac{1}{Q}[c_m^f(\lambda_D - \lambda_R) + \sum_{l=1}^{Q}(1-\gamma^l)G(s+l) + (\gamma^{-Q}-1)\sum_{l=Q+1}^{\infty}\gamma^l\, G(s+l)].$$

The analysis of this expression is complicated due to the infinite sum whose coefficients depend on both control parameters s and Q. This difficulty can be overcome by introducing

$$H(k) := (1-\gamma)\sum_{l=0}^{\infty}\gamma^l G(k+l).$$

It is easy to verify that

$$C(s,Q) = [c_m^f(\lambda_D - \lambda_R) + \sum_{k=s+1}^{s+Q} H(k)] \,/\, Q\,.$$

This expression is remarkable since it can be interpreted as the average cost of a standard (s, Q) inventory model with expected backorder and holding cost function $H(.)$ (compare [36]). This shows that the above return flow inventory model can be transformed into an equivalent standard (s, Q) model. One consequence is that standard methods can be applied to compute optimal values of the control parameters.

The above approach can be extended in several directions. For example, considering compound Poisson demand and return processes leads to an (s, S) model in an analogous way. This case is discussed in detail in [5]. Furthermore, a remanufacturing lead–time that is shorter than the manufacturing lead–time can be incorporated, using a well–known approach from standard inventory models. Letting $\tilde{D}$ denote demand in time interval $(t, t + L_m]$ minus returns in time interval $(t, t + L_m - L_r]$ we have $I_s^{net}(t + L_m) = I_s(t) - \tilde{D}$ where the last two terms are independent. Hence, the decision relevant costs are $\tilde{G}(y) := E[G(I_s^{net}(t + L_m))|I_s(t) = y] = E[G(y - \tilde{D})]$ which are again convex in y. This puts the extended model in the form discussed above. See [5] for a more detailed discussion of this approach.

The above analysis shows that introducing return flows alone does not entail major changes in the mathematical structure of standard inventory models. In the next sections we see that adding a controllable remanufacturing process makes the model considerably more complex. The question arises how the structure of the optimal policy changes if we explicitly model the remanufacturing process by introducing remanufacturing lead-times (Section 4.1), if we introduce different holding costs for remanufacturables, work-in-process, and serviceables (Section 4.2), or if we allow disposal of returned products (Section 4.3).

4.1 The influence of lead-times on the structure of the optimal policy

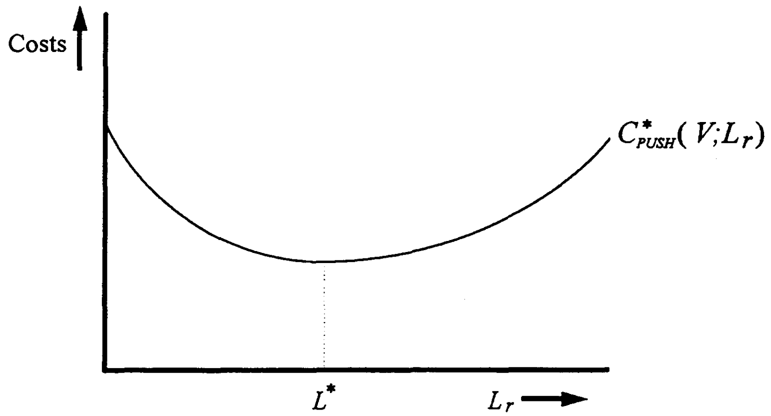
To address the influence of lead-times on the structure of the optimal policy, we consider a counter-intuitive result that is reported in [33]. The authors show that an increase in the remanufacturing lead-time may result in lower system costs under a PUSH policy.

Figure 4 shows a graphical representation of this effect for the PUSH policy. Initially, as the remanufacturing lead-time L_r increases from 0, optimized costs $C^*_{PUSH}(V; L_r)$, where $V = (s_m, Q_m, Q_r)$, monotonously decrease until L_r reaches a certain level L^* from which costs start to increase monotonously.

To understand this counter-intuitive effect it is important to note that at the time a remanufacturing batch is started, the inventory position is larger than the reorder point for manufacturing, s_m. Also note that in principle s_m is used as safety stock to protect against the demands during the *manufacturing* lead-time. Suppose for the sake of argument that a remanufacturing batch is started at the time the inventory position lies somewhere between s_m and $s_m + Q_m$. At least for moderate values of the return rate this is very probable. If the remanufacturing lead-time is very small compared to the manufacturing lead-time this batch will arrive in the serviceable inventory when the on-hand inventory is well above zero (see Figure 5), since the safety stock s_m is meant to protect against the longer manufacturing lead-time. In other words, each time a remanufacturing batch comes in there is too much safety stock, and therefore there are excessive holding costs (assuming that $c_r^h < c_s^h$).

On the other hand, if the remanufacturing lead-time is very large compared to the manufacturing lead-time this batch will arrive in the serviceable inventory when the net inventory is well below zero (see Figure 6), since the safety stock

Figure 4: The counter-intuitive effect of decreasing costs with increasing remanufacturing lead-time.



s_m is meant to protect against the much shorter manufacturing lead-time. In other words, each time a remanufacturing batch comes in, there may not have been enough safety stocks to protect against shortage, and therefore we may have excessive shortage costs.

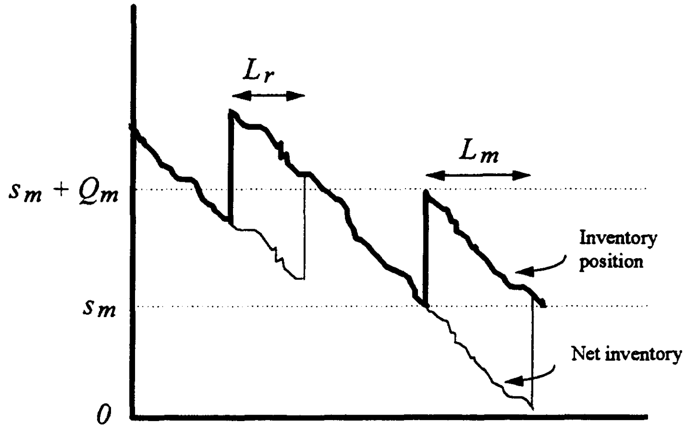
Of course, one could argue that the optimal value of s_m will be adjusted to meet the above issues, but then we will have similar problems with the manufacturing batches. Whatever the value of s_m, it seems that we can never time the manufacturing batches *and* the remanufacturing batches efficiently if L_r differs a lot from L^*. Setting the value of s_m always results in some kind of compromise, making the sum of total holding costs and backorder costs larger than seems necessary. Note that the lead-time 'imbalance' effect will only occur for moderate values of the return rate. If the return rate is very small, the small number of remanufacturing batches will only have a limited effect on system performance. Similarly, if the return rate is close to the demand rate, the influence of the manufacturing batches will be relatively small.

From the above heuristic argument we may conclude that we can improve the PUSH policy by changing the *timing* of the incoming remanufacturing batches.

An alternative policy

In order to improve the PUSH policy we will first consider the situation in which the remanufacturable holding costs are zero and we do not have holding costs for remanufacturables in WIP either. That is, one may keep any number of products in remanufacturable inventory or WIP against zero cost.

Figure 5: A remanufacturing order coming in too early.



Note that if L^* is the 'optimal' lead-time for remanufacturing, we may improve the PUSH policy by altering the remanufacturing lead-time, i.e., by altering the time-interval between the time that an order is put into inventory position and the time that the order arrives in the serviceable inventory. Note that for the PUSH policy this time-interval is always equal to the processing-time L_r.

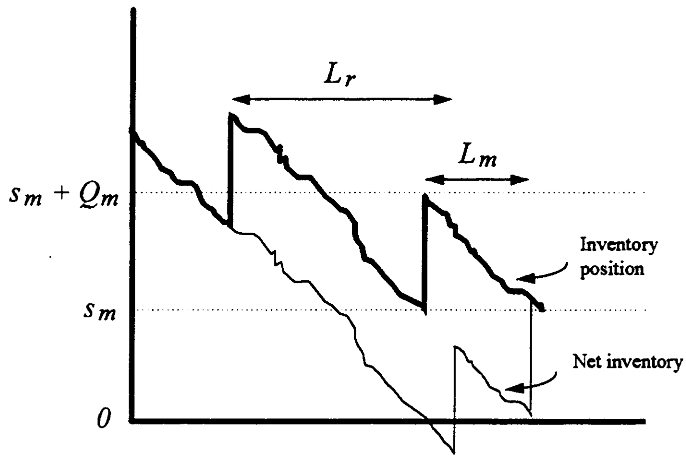
If L_r is smaller than L^* we would like to increase the remanufacturing lead-time. We can do this by increasing the time that an order spends in WIP (this means increasing the processing time) against WIP holding cost c_w^h. But this does not make sense if $c_r^h \leq c_w^h \leq c_s^h$, which is a reasonable assumption. Instead, we wait a fixed time $L^* - L_r$ before the order is released to the remanufacturing facility.

If L_r is larger than L^* we would like to decrease the remanufacturing lead-time. Assuming that we cannot decrease the remanufacturing processing time we do the following. As soon as a remanufacturing batch is available it is released to the remanufacturing facility, but we wait a fixed time $L_r - L^*$ before the order is put into the inventory position.

We now have the following control policy as an alternative to the PUSH policy:

Case A ($L_r < L^*$) The (s_m, Q_m, Q_r) PUSH policy is employed with the following alteration: As soon as Q_r remanufacturables become available they will enter the inventory position, but remanufacturing will only start after time $L^* - L_r$. In this way the remanufacturing processing time is still L_r, but the remanufacturing lead-time has changed into L^*. The system dynamics and costs under this

Figure 6: A remanufacturing order coming in too late.



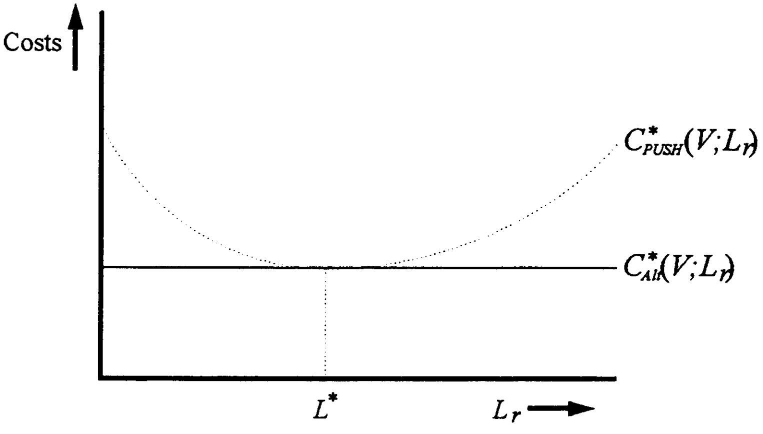
policy are exactly the same as those under a PUSH policy with remanufacturing lead-time L^*. Therefore, $C^*_{Alt}(V, L^*; L_r) \equiv C^*_{PUSH}(V; L^*) \leq C^*_{PUSH}(V; L_r)$. So, the alternative policy dominates the PUSH policy with respect to system costs as long as $L_r < L^*$.

Case B ($L_r \geq L^*$) The (s_m, Q_m, Q_r) PUSH policy is employed with the following alteration: As soon as Q_r remanufacturables become available, they will be remanufactured, but they will only enter the inventory position after time $L_r - L^*$. In this way the remanufacturing processing time is still L_r, but the remanufacturing lead-time has changed into L^*. The system dynamics and costs $C^*_{Alt}(V, L^*; L_r)$ that result from this alternative policy are exactly the same as those under the PUSH policy with remanufacturing lead-time L^*. Therefore, $C^*_{Alt}(V, L^*; L_r) \equiv C^*_{PUSH}(V; L^*) \leq C^*_{PUSH}(V; L_r)$, so the alternative policy dominates the PUSH policy with respect to system costs as long as $L_r \geq L^*$.

This control policy dominates the (s_m, Q_m, Q_r) PUSH policy with respect to costs for all values of L_r. Moreover, the associated costs do not depend on the remanufacturing lead-time (see Figure 7).

If we let go of the assumption of zero holding costs for remanufacturable inventory and we introduce holding costs for WIP, things are slightly more complicated. The cost function $C^*_{PUSH}(V; L_r)$ now contains the additional cost component $c^h_w \lambda_R L_r$, and $C^*_{Alt}(V, L; L_r)$ the cost component $c^h_w \lambda_R L_r + c^h_r 1_{\{L_r < L\}} \lambda_R (L - L_r)$ with L the chosen remanufacturing lead-time. Here, $1_{\{L_r < L\}}$ is an indicator function, which is assigned the value one if $L_r < L$

Figure 7: The alternative policy compared to the (s_m, Q_m, Q_r) PUSH policy in absence of holding costs for remanufacturables and WIP.



and the value zero otherwise. Consequently, we have

$$C^*_{Alt}(V, L; L_r) = C^*_{PUSH}(V; L) + c^h_w \lambda_R (L_r - L) + c^h_r 1_{\{L_r < L\}} \lambda_R (L - L_r) \quad (12)$$

Minimizing $C^*_{Alt}(V, L; L_r)$ for L we find that the local optima satisfy the equation

$$\frac{dC^*_{PUSH}(V; L)}{dL} = (c^h_w - c^h_r 1_{\{L_r < L\}})\lambda_R. \quad (13)$$

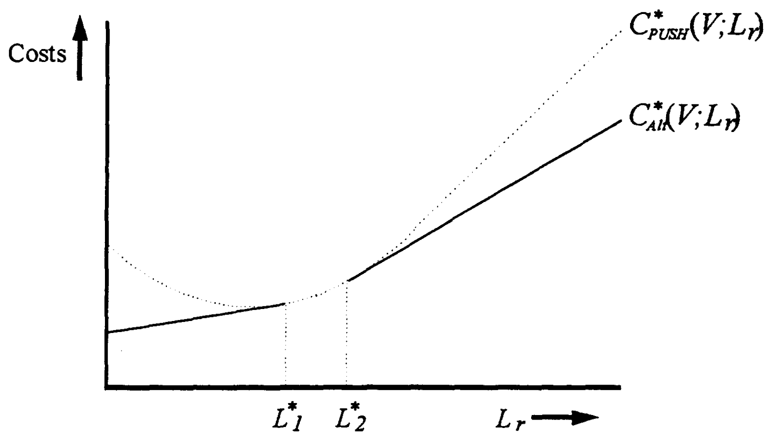
Using (13) it is not hard to see that the global minimum of (12) is reached for $L_1 > L_r$, where L_1 is that value of L for which the tangent with slope $(c^h_w - c^h_r)\lambda_R$ to $C^*_{PUSH}(V; L)$ as a function of L, intersects with $C^*_{PUSH}(V; L)$, or for $L_2 < L_r$, where L_2 is that value of L for which the tangent with slope $c^h_w \lambda_R$ to $C^*_{PUSH}(V; L)$ as a function of L, intersects with $C^*_{PUSH}(V; L)$.

We can now generalize the alternative policy for non-zero holding costs for remanufacturables and WIP:

- For $L_r < L_1$ we adopt the alternative policy as in Case A
- For $L_1 \leq L_r \leq L_2$ we adopt the (s_m, Q_m, Q_r) PUSH policy
- For $L_r > L_2$ we adopt the alternative policy as in Case B.

The optimal costs resulting from this policy are as in Figure 8.
Note that an increase in the remanufacturing lead-time results in an increase in costs for all L_r as long as $c^h_r < c^h_w$, i.e., the slope $(c^h_w - c^h_r)L_r$ is positive.

Figure 8: The alternative policy compared to the (s_m, Q_m, Q_r) PUSH policy in presence of holding costs for remanufacturables and WIP.



Implications

In this section we have established two things: *(i)* The effect of decreasing costs with increasing remanufacturing lead-time is due to the non-optimality of the (s_m, Q_m, Q_r) PUSH policy, possibly together with an unrealistic setting of c^h_w, and *(ii)* the PUSH policy is a non-optimal policy in general, because it is fully dominated by the alternative policy.

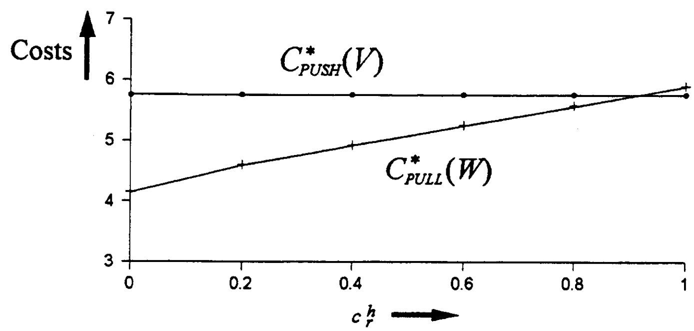
A derivative but more important result of the above is that *any* policy for which the ordering of a remanufacturing batch coincides with updating the inventory position, is a non-optimal policy for large remanufacturing lead-times. This type of policy will result in increasing backorder costs plus serviceable holding costs as the remanufacturing lead-time is increased. This is not the case for the alternative PUSH-policy, since its costs grow linearly in the remanufacturing lead-time. Hence, the alternative policy dominates all 'standard' policies for large remanufacturing lead-times.

4.2 The influence of holding costs on the structure of the optimal policy

In the previous section we found that holding costs may have an influence on the choice of the optimal policy parameters. But do they also influence the *structure* of the optimal policy? Up to now we have only considered push type policies, i.e., (batches of) incoming returns are remanufactured as soon as possible. However, in [29] it is shown that such a policy is not optimal if holding costs for remanufacturables are valued sufficiently lower than holding costs for serviceables. In that case a pull type policy may be considered instead.

A numerical example that compares the behaviour of the optimized costs under the PULL policy, $C^*_{PULL}(W)$, where $W = (s_m, Q_m, s_r, S_r)$, to that of the PUSH policy under different holding cost assumptions is given in Figure 9. As expected, the PULL policy performs better than the PUSH policy as long as holding costs for remanufacturables are valued sufficiently lower than holding costs for serviceable products.

Figure 9: The effect of the remanufacturable holding cost on system costs under the (s_m, Q_m, Q_r) PUSH policy and the (s_m, Q_m, s_r, S_r) PULL-policy.

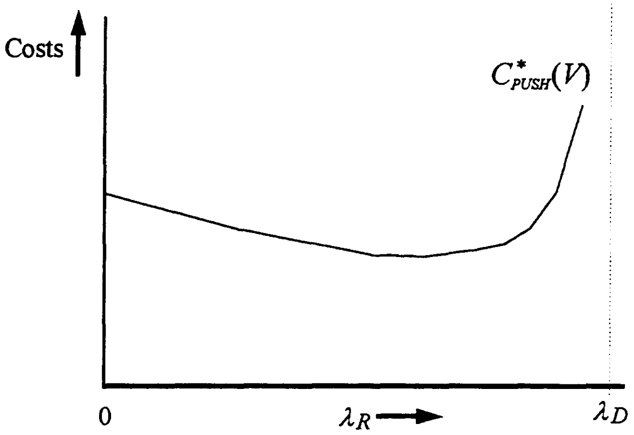


If we recall the conclusions from the previous section, we may expect that the PULL policy performs worse than the *alternative* PUSH policy if the remanufacturing lead-time differs considerably from the manufacturing lead-time. However, we can improve the PULL policy in a similar way as we did for the PUSH policy. In case of small remanufacturing lead-times we may delay remanufacturing for a fixed time interval; in case of large remanufacturing lead-times we can introduce a mixture of a push and pull policy: initially push the remanufacturables through a number of (re)manufacturing operations, store them in a work-in-process buffer and eventually pull them through the remaining operations when they are really needed. This is particularly useful with respect to disassembly and repair times, which are often very variable. Pushing the remanufacturables through these variable processes, and then pulling them further through the less variable assembly operations does not only improve system costs but also other performance measures, such as flow-time and lateness ([9]).

4.3 The influence of the return rate on the structure of the optimal policy

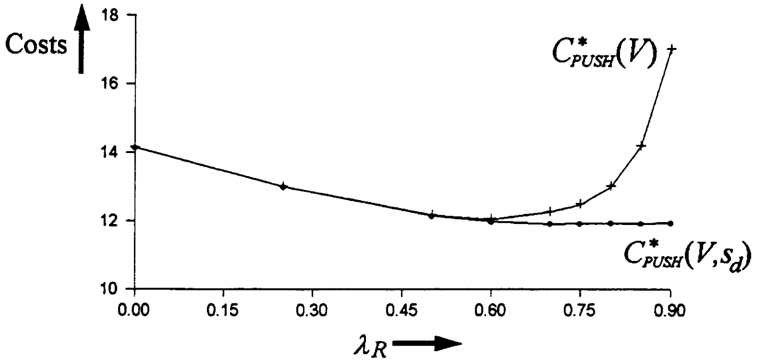
In the introduction of Section 4 we saw that one of the conditions for optimality of the classical (s, Q) policy is that disposal of incoming returns is not allowed. A typical picture of the effect of the return rate on costs behaviour under the (s_m, Q_m, Q_r) PUSH policy is given in Figure 10 (see e.g. [4, 32]).

Figure 10: The effect of the return rate on system costs under the (s_m, Q_m, Q_r) PUSH policy; $c_m^v = 10$, $c_r^v = 5.0$, $c_d^v = 0$.



Initially costs decrease as the return rate increases (assuming that the marginal cost of remanufacturing is lower than that of manufacturing), but after some point costs start to increase. As λ_R approaches λ_D the 'load' of the system grows bigger and bigger, leading to extremely high holding costs. A disposal policy can overcome this problem. For instance, one could extend the (s_m, Q_m, Q_r) PUSH policy with an extra parameter s_d, which specifies the inventory position at which returns should be disposed of upon arrival. Even this simple disposal policy may result in a dramatic improvement of system performance. This can be seen from the numerical example in Figure 11. Allthough in this example variable disposal costs are put to zero, one should realise that for any finite value of c_d^v average costs will tend to a constant if γ tends to λ. As a contrast, if disposals are not taken into consideration, costs will tend to infinity as γ tends to λ. Hence, even if disposal is costly, a disposal policy may reduce average costs considerably. It should be noted that $c_d^v = 0$ is not an extreme case, since returned items can have a positive scrap value (i.e. $c_d^v < 0$), and neither is $\gamma \approx \lambda$, since towards the end if a product's life cycle the number of product returns may exceed product demand.

Figure 11: The (s_m, Q_m, Q_r) PUSH policy compared to the (s_m, Q_m, Q_r, s_d) PUSH-disposal policy; $c_m^v = 10$, $c_r^v = 5.0$, $c_d^v = 0$.



Furthermore, in [32] it is shown that a disposal policy reduces the variability in the inventory processes by taking care of excess returns. Clearly, the above strongly suggests that the structure of the optimal policy should include the option of disposal. For a more detailed account of the effects of various disposal policies, we refer to [30] and [32].

4.4 Insights from periodic review models

In *periodic review* models the planning horizon is divided into discrete planning periods. At the beginning of each planning period n, decisions are taken according to the values of the following decision variables.

$Q_d^{(n)}$ = the quantity (batch-size) of remanufacturable products that is disposed of in planning period n,

$Q_p^{(n)}$ = the quantity (batch-size) of products that is procured outside or internally produced in planning period n,

$Q_r^{(n)}$ = the quantity (batch-size) of products that is remanufactured in planning period n.

All decision variables are assumed to be integer. The objective in periodic review models is to determine values for the decision variables, such that the total expected costs over the entire planning horizon are minimized.

Within this category, [14] considers a model with assumptions and characteristics as listed in Table 2. One of the objectives is to determine the structure of the optimal policy. It is argued that a nice structure can only be obtained for some special cases. These special cases relate to the assumptions made about the stocking policy of the returned products, and to the values of the manufacturing lead-time L_m and the remanufacturing lead-time L_r.

Table 2: Assumptions and characteristics of the remanufacturing model by [14].

demands/returns	All returns and demands per period are continuous time-independent random variables. The inter-arrival distributions are arbitrary distribution functions, which may be stochastically dependent.
testing	No testing facility.
remanufacturing	The remanufacturing lead-time L_r is non-stochastic and equal to μ_{L_r}; the remanufacturing facility has infinite capacity and variable remanufacturing costs.
production	The procurement lead-time L_m is non-stochastic and equal to μ_{L_m}; there are variable production costs.
inventories	Type I and Type II inventory buffers have infinite capacity; Type I and Type II inventory have (different) variable inventory holding costs.
disposal	Variable disposal costs.
service	Modeled in terms of backorder costs.
control policy	At the beginning of each period n decisions are taken on $Q_d^{(n)}$, $Q_p^{(n)}$, and $Q_r^{(n)}$, such that the total expected costs over the planning horizon are minimized.

For instance, for the case that returned items are not allowed to be stocked the following results hold:

- If $L_m = L_r$ the structure of the optimal policy can be formulated as a so-called (L, U) policy:

$$\begin{array}{llllr} Q_p^{(n)} = L^{(n)} - x, & Q_r^{(n)} = x_r, & Q_d^{(n)} = 0, & \text{for} & x < L^{(n)}, \\ Q_p^{(n)} = 0, & Q_r^{(n)} = x_r, & Q_d^{(n)} = 0, & & L^{(n)} \leq x \leq U^{(n)}, \\ Q_p^{(n)} = 0, & Q_r^{(n)} = U^{(n)} - x_s, & Q_d^{(n)} = x - U^{(n)}, & & x > U^{(n)}. \end{array}$$

  Here, x_r is the remanufacturable inventory, x_s is the inventory position of serviceables, and $x = x_s + x_r$. In words this policy states that every return is remanufactured in order to increase the inventory position to $U^{(n)}$. If there are not enough returns available to increase the inventory position up to $L^{(n)}$ an additional manufacturing order is placed to do so. Disposal of returns occurs such that the inventory position plus the number of returns never exceeds $U^{(n)}$.

- If $L_m = L_r + 1$ the structure of the optimal policy can be formulated as a so-called $(L, U, \hat{U})$ policy. IF $\hat{U} > U$ this policy is equal to the L, U policy. Otherwise, if the inventory is smaller than $\hat{U}$, which does not depend on period n, then procure up to $L^{(n)}$. If the inventory exceeds $\hat{U}$ then dispose down to $\hat{U}$, remanufacture the remaining remanufacturable products and procure $L^{(n)} - \hat{U}$ new products.
- If $L_m < L_r$ or $L_m > L_r + 1$ the structure of the optimal policy is not expected to be of a simple form.

Note that if returned products are not allowed to be stocked, all returned products will be remanufactured or disposed of at the end of each period. If returned products are allowed to be stocked there can be a more subtle control of the production, the remanufacturing and the disposal process. For the latter case Inderfurth derives the following results with respect to the structure of the optimal policy.

- If $L_m = L_r$ the structure of the optimal policy can be formulated as a so-called (L, M, U) policy:

$$\begin{array}{llllr}
Q_p^{(n)} = L^{(n)} - x, & Q_r^{(n)} = x_r, & Q_d^{(n)} = 0, & \text{for} & x < L^{(n)}, \\
Q_p^{(n)} = 0, & Q_r^{(n)} = x_r, & Q_d^{(n)} = 0, & & L^{(n)} \leq x < M^{(n)}, \\
Q_p^{(n)} = 0, & Q_r^{(n)} = M^{(n)} - x_s, & Q_d^{(n)} = 0, & & M^{(n)} \leq x \leq U^{(n)}, \\
Q_p^{(n)} = 0, & Q_r^{(n)} = M^{(n)} - x_s, & Q_d^{(n)} = x - U^{(n)}, & & x > U^{(n)}.
\end{array}$$

  Note that if $M^{(n)} = U^{(n)}$ this policy is equal to the abovementioned (L, U) policy. If $M^{(n)} < U^{(n)}$ and $M^{(n)} \leq x \leq U^{(n)}$, a fraction of the remanufacturables $(x - M^{(n)})$ is kept in stock, while the rest is remanufactured. If $x > U^{(n)}$ a fraction is also disposed of. This generalizes the results of [26] who studied this system for zero (re)manufacturing lead-times.

- If $L_m \neq L_r$ the structure of the optimal policy is much more difficult to obtain and becomes very complex, even if the manufacturing lead-time and remanufacturing lead-time differ only one period.

The above results show that under general conditions the optimal policy will be very complex and difficult to identify. Only if the (re)manufacturing lead-times are equal, a simple policy that does not depend on the timing of individual (re)manufacturing orders is optimal. This coincides with our findings in section 4.1, where it was shown that traditional policies can be improved considerably by redefining the inventory position in such a way that differences in (re)manufacturing lead-times are accounted for.

5 Summary and conclusions

In this paper we have addressed the issue of inventory control for joint manufacturing and remanufacturing. A short review of the existing literature on models that are related, but not specifically meant to apply to remanufacturing, showed that these models do not quite capture all of the characteristics that are typical for these types of systems. These characteristics are (i) two exogenous supply sources, that is manufacturing and remanufacturing, (ii) time-dependent availability of the remanufacturing source, and (iii) non-zero stochastic lead-times for remanufacturing. One framework that fits the above criteria was presented in Section 3. This framework enables the analysis of a broad variety of control policies under various system characteristics, like stochastic lead-times, Coxian-2 arrival processes, and correlation between the demand and return processes.

With respect to the structure of the optimal policy it was shown that the characterization of the (re)manufacturing lead-times plays an important role. Traditional policies can be improved considerably if additional lead-time information is used in the control policy and/or the definition of the inventory position. Other factors include the return and demand rate, which define the 'load' of the system. If this load is high it may be necessary to implement a disposal policy. The valuation of holding costs largely determines the choice between push and pull control systems. The analysis of periodic review models shows that, in general, the structure of the optimal policy will be very complex, even if the (re)manufacturing lead-times differ only slightly. The traditional (s, S) policy will only be optimal if stocking and disposal of remanufacturables is not allowed, and remanufacturing lead-times are sufficiently short.

More research is needed to identify the structure of the optimal policy for more general situations and to use the performance of the optimal policy as a benchmark for the performance of heuristic policies.

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