Product Warranties and Double Adverse Selection

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Abstract

There is extensive literature analysing the use of warranties and their application as a marketing tool. A frequently cited role for warranty policy is to facilitate price discrimination when consumer-types are unobservable. This issue is analysed by researchers including Kubo (1986), Matthews and Moore (1987), and Padmanabhan and Rao (1993). The use of warranties for price discrimination is analogous to the model for monopoly pricing of products of differing quality developed by Mussa and Rosen (1978). A second role for warranties is to signal product quality to consumers when quality is not observable. Nelson (1974) refers to products where buyers cannot evaluate the quality prior to buying as `experience' goods. When the quality of an `experience' good is variable, a long warranty can be used to signal better quality because sellers of premium quality have a cost advantage over sellers of lower quality in terms of offering warranty protection. This aspect of warranties is addressed by Spence (1977), Grossman (1981), Emons (1988), Lutz (1989) and Gal-Or (1989).

Frequently, a warranty does play one role or the other but there are markets in which there is a need for it to play both of these roles simultaneously. This need exists when buyers cannot observe product quality and sellers cannot identify buyers who have heterogeneous valuations for the product. Markets that meet these criteria include the used car market and the IBM-cloned personal computer market. The warranty literature has yet to consider the possibility of warranties acting as both a screen and signal simultaneously. Accordingly the objective of this study is to develop a model which examines optimal warranty policy under these conditions.

The model involves a seller who desires to sell a product and an optional extended warranty to heterogeneous consumers. Specifically, the seller chooses a base price and warranty for his product and the duration and pricing for optional extended coverage. The seller knows the quality of the product and the buyer does not. Using the terminology of Akerlof (1970), this situation of asymmetric information is described as a `lemons market' and it occurs when there is variability in the quality of a good and buyers cannot evaluate it before buying. Sellers are assumed to know the quality of the product and this provides them with an informational advantage over the buyer. The objective is to understand how sellers set warranty menus to maximise profit when the only potential signal of quality to buyers are the prices and warranty lengths chosen by sellers.

The main results are first, a seller of premium quality can use a warranty menu to signal premium quality and price discriminate simultaneously under most conditions. However, when the incremental benefit of premium quality to buyers is high, the warranty menu cannot play both roles simultaneously. The menu can signal premium quality to buyers, but it loses its ability to

screen. Second, when the incremental benefit associated with premium quality is above a certain level, a seller of premium quality will alter the menu in order to simultaneously maximise his profit and deter sellers of standard quality from mimicking. The optimal action for the seller of premium quality invariably involves offering longer warranties to both types of buyers and generally charging higher prices. However, when the incremental benefit of premium quality is relatively low, the menu chosen by a seller of premium quality is unaffected by the existence of sellers of standard quality. Finally, under conditions of two sided adverse selection, there are several equilibrium conditions that can occur in which sellers do not offer complete menus (a price/warranty bundle designed for each type of consumer). When the incremental benefit for premium quality is high, a seller of premium quality may be forced to offer a `collapsed' menu in which both types of buyers are offered the same bundle. In addition, there are regions in the feasible parameter space where a seller of standard quality may provide warranty coverage for high valuation buyers only (the base warranty is zero) or none at all (the base warranty is zero and extended warranties are not offered).

Key Words: extended warranty, signalling, screening, unobservable quality, adverse selection, price discrimination, menu of contracts

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1.0 Introduction

The following quote appeared in a brochure published by Compaq Computer Corp. in 1994 to assist rookies in buying a first personal computer:

"When comparing computer features, reliability is difficult to assess. But the length of the warranty is a clue to the dependability of its computers. Remember, it costs the company money to repair computers under warranty. A longer warranty period reflects the company's confidence that its products will last."

Even when consumers do not have the time or expertise to assess the quality of products, they can make useful inferences about a product's quality from the length of its warranty. Quite simply, when warranties act as signals, a longer warranty signals a better product.

On the other hand, a feature of buying products such as major appliances, power tools and stereo equipment is the persistent effort of a salesperson to sell some form of extended warranty protection[1]. Clearly, retailers are making money by convincing some consumers to buy extra warranty coverage. With heterogeneous consumers, a retailer can increase profit by offering different price/warranty combinations and having buyers choose the combination that best suits them (in the economics literature, this process is called "screening").

But why are retailers so aggressive in the marketing of extended warranties? The answer lies in their tremendous profitability. *Business Week* (January 14, 1991) reports that ½ of the operating profits for big consumer-electronic chains come from extended warranties and more recently, an article in the *New York Times* (July 23, 1995) indicates that retailers may earn as much as 75% of their gross income from the sales of extended warranty and service contracts.

These observations about the marketing of warranties underline two important facts. First, warranties (or commitments by the seller to repair defective products for a specific period of time) are an increasingly important element in the marketing mix for durable products. Not only are manufacturers such as Samsung (a major electronics manufacturer) investing substantial funds ($125 million) to enhance their warranty programmes (*Business Korea*, Vol. 12, No. 1, July 1994), but consumers too are buying extended warranties on a wider array of goods than ever before.

Second, warranties can do a lot more than provide extra value to purchasers of durable products by insuring them against failure. The two roles cited above, signalling and screening, underline the capacity warranties have to play different roles in different situations.

Given that warranties can either signal or screen, a question that comes to mind is “do

[1] Sears reports that its sales of extended warranties on durable goods exceeds $1 billion (*San Francisco Chronicle*, January 20, 1992).

warranties ever screen and signal simultaneously?". Researchers have examined how warranties signal product quality when buyers are uncertain about it and there is also extensive literature on the use of warranties to screen consumers when they have different unobservable `valuations' for warranty coverage. Interestingly, researchers have yet to consider a situation in which warranty policy both signals and screens simultaneously.

A first step in addressing this question is to ask whether the need for a warranty to play these roles simultaneously exists in observed markets? Second, assuming that there are markets where this need exists, can and how does warranty policy screen and signal simultaneously?

In the following section, we will discuss two well-known markets where there appears to be a need for warranties to signal and screen simultaneously. That is, there is both variability in the quality of products (and sellers know more about the quality of products than buyers) and buyers want some degree of warranty protection (and sellers cannot tell which buyers are most interested in buying warranty protection). The remainder of the paper is devoted to answering *can and how* warranty policy plays these roles simultaneously.

2.0 Background

Adverse selection in the economics literature refers to a class of problems where pre-contractual opportunism by parties possessing private information leads to inefficiency in the operation of a market[2]. The term is from the insurance industry where insurance companies face "adverse selection" in the sales of insurance policies. The problem is that the people most likely to purchase insurance policies are unfortunately those who are most likely to make claims (Rothschild and Stiglitz, 1976). When sellers know more about the quality of products than buyers, we clearly have an adverse selection problem for buyers. However, in the conditions described in section 1.0, sellers also lack information that is important for contracting: namely, information on the preferences of buyers. Accordingly, we describe this as a condition of *double adverse selection* because both parties (the buyer and the seller) posses private information that has the potential to create inefficiencies.

Two markets characterised by double adverse selection are the non-branded personal computer market and the used car market[3].

In the market for `IBM clone' computers, a number of manufacturers import components from the Far East and assemble personal computers in North America. When a consumer considers a purchase of a cloned computer, he/she cannot necessarily depend on a reliable brand name, *Consumer Reports* or advertising. Consumers would like a credible signal that might

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[2]See Mas-Colell, Whinston and Green (1995) for a review of adverse selection and its importance in the economic literature.

[3] In the United States, the used car market is estimated to be 100% larger than the new car market (Bennet, James, "Second-hand cars go first class", *Globe and Mail*, Tuesday, August 2, 1994, B7).

provide them with useful information. Because it is more expensive for a firm with low quality computers to offer the same length of warranty as a firm with better quality computers, a firm with better quality computers may signal its higher quality by offering a longer warranty. At the same time, all firms (regardless of the quality of computer they are selling) have an incentive to offer an extended warranty option to extract additional profit from those purchasers who are more risk averse. This is a situation where consumers cannot be identified a priori and consumers are not able to observe the quality of the product in question.

Another example of double adverse selection occurs in the used car market. Extended warranties are popular because used cars frequently need repairs. In addition, the *Consumer Reports 1997 Buying Guide* provides evidence of variability in automobile quality, across manufacturers, within manufacturers and even within make. Dealers usually have different qualities available because they purchase their stock through the wholesale market where quality is difficult to determine. Genesove (1993) points out the mechanics of the wholesale automobile auction provide little time for buyers to examine the cars and the reputation effects of sellers are not strong.

While dealers obtain a range of qualities from the auction, by the time the car is available for sale, dealers are invariably better informed than buyers about a car's condition (dealers have both experience and computer systems that are important in evaluating the condition of cars). The problem for a potential buyer is that the value of a car depends on its quality, an attribute of the product that she cannot observe. Conceivably, this problem could be quickly resolved if consumers could costlessly test cars to measure their quality (many dealers do allow potential buyers to take the cars to independent diagnostic centres for testing). The problem is that significant time and costs are associated with such testing and as a result, only a small minority of used car buyers actually pursue this option for getting better information about a car's quality.

To see how warranty policy might be affected by conditions of double adverse selection, we conducted a survey of used car dealers in a major metropolitan area. Following the motivation for the problem, information was collected by a "prospective used-car buyer" for cars in five categories that were ostensibly in the same condition. The survey involved discussions with 109 dealers and focussed on cars that were five years old (see Table 1)[4].

[4] In general, extended warranties for 5 year old used cars cover the power train and have a deductible of $50. The results of the survey are similar if the data is grouped by manufacturer or by specific model (however, the data points per cell are significantly reduced).

Table 1
Survey of Base and Extended Warranties in the Used Car Market

Car Class	base warranty		1 yr. extended warranty		2 yr. extended warranty		3 yr. extended warranty	
	Length (months)	Standard Deviation	Average Price	Standard Deviation	Average Price	Standard Deviation	Average Price	Standard Deviation
Domestic Compact	5	4.5	371	183	621	305	897	505
Imported Compact	6.8	6.9	376	170	643	215	905	370
Domestic Midsize	4.3	4.2	399	168	644	224	922	387
Imported Midsize	4.6	4.4	362	129	646	220	926	348
Minivans	4.3	2.9	294	114	449	122	614	143

By definition, used car dealers are "screening" because not all customers buy extended warranties (they are a useful but expensive option). Substantial variance in both the length of base warranties and the pricing of extended warranties was observed across all five categories. This variance could not be explained by differences in the base price, the features, the options or the appearance of the cars. In addition, differences of up to 50% in the pricing of extended warranties were observed for *identical* cars of the same year. This suggests that warranties and pricing are being used for more than just screening. It also suggests that the entire menu (the base price, the base warranty and the pricing of extended warranties) may have a role in helping buyers to learn about the hidden quality of a potential purchase. Of note, double adverse selection can be a problem in any second-hand market for used durable goods where different consumers desire different levels of warranty protection.

These examples show that conditions of double adverse selection do exist in markets where warranties are important. In addition, the information from the used car market suggests that under these conditions, warranties can and do have a role in both screening consumers and signalling quality. The objective of this paper is to provide insight about how this happens using a game theoretic approach. A model is developed to analyse the optimal contracts offered by sellers to buyers where the contract is comprised of a price for a product with a warranty of finite duration. The seller provides the product and will repair any breakdowns that occur during the warranty period. We now review the relevant literature on warranties.

3.0 Background Literature

There is a rich literature in both economics and marketing that relates to the marketing of warranties on durable goods. The objective of this section is to review two subsets of this literature: first, the literature that analyses the use of warranties for price discrimination and second, the

literature that considers the use of warranties to signal quality to potential buyers.

Warranties to Price Discriminate or Screen

In situations where a firm has market power, it can extract additional surplus from the consumer by including a repair warranty with the product. This is the basic bundling result as discussed by Tirole (1990). An extension to this idea is that a warranty can be used to screen consumers that have heterogeneous valuations for warranty protection; the length of warranty is a proxy for quantity in a typical second degree price discrimination model (consumers who have higher valuations for a product buy more of it but at a lower cost per unit). Since 1986, several papers have appeared which discuss this use of warranties.

Kubo (1986), for example, shows how a monopolist can increase its profits through the use of an optional quality guarantee when consumers are heterogeneous. Matthews and Moore (1987) extend this problem to a situation in which a monopolist has three decision variables: price, quality (which is fully observable), and warranty level, instead of two (price and warranty level).

Padmanabhan and Rao (1993) show how customer heterogeneity can arise from risk tolerances which vary across consumers. Given this heterogeneity, sellers can increase their profits by offering a menu of price/warranty bundles.

Warranties as Signals

Signalling is important when one agent to a contract is unfamiliar with the quality of the other agent (or his product) and that agent's quality cannot be observed prior to contracting. In the context of durable goods, the principal (the buyer) selects an agent (the seller) to perform a task (provide him/her with a durable good) but cannot observe the characteristics of the good before purchase. As noted by Bergen, Dutta and Walker (1992), this problem (which is also known as a problem of `Hidden Information') applies to many situations in marketing.

With many durable products, quality cannot be evaluated prior to purchase and it becomes evident only after prolonged use. Nelson (1974) refers to these products as "experience goods". Akerlof (1970) underlines how unobservable attributes can interfere with the operation of markets. In the used car market, he predicts that only poor quality cars (lemons) will be traded because buyers have poor information about individual cars that are offered for sale (and learn little by visually inspecting them).

Signalling can be used to mitigate the problem noted by Akerlof. When sellers of high quality have a cost advantage in providing warranty protection over sellers of low quality then warranty policy can facilitate the exchange of higher quality products. The use of warranties as signals is analysed by Spence (1977) who finds the amount of coverage offered by manufacturers is a perfect signal of quality in a competitive market. This result occurs in Spence's model due to profit maximising behaviour by firms and the requirement that consumers' beliefs about quality be

consistent with what firms actually offer[5].

As previously mentioned, no author has considered a situation in which warranty policy plays the dual role of screening and signalling at the same time. The rest of the paper is organised as follows. In section 4.0, we present the modelling framework that we use to analyse the problem of simultaneous signalling and screening. In section 5.0, present the findings of the study and discuss them in the context of the underlying assumptions. Finally, in section 6.0, we discuss the managerial implications of the study and provide a brief conclusion.

4.0 Overview of the model

As noted previously, the objective of this analysis is to identify the optimal contracts that a seller of durable goods (with warranty coverage) will offer to a buyer under conditions of double adverse selection. The market we consider is one in which a buyer purchases no more than 1 unit of product. We further assume that the seller has a degree of price setting ability and this follows from a situation in which buyers have positive search costs[6]. For example, when a buyer looks for a product (like a used car), she usually looks for a certain brand, with certain features (4 door for example, standard transmission, CD player, colour), in a certain price range. Once she finds a car that meets these criteria (at either the first or the second dealer or the *n*th dealer she visits), the dealer selling the car has a degree of price setting ability because the buyer will need to spend time going to more dealers to find an alternative. It should be noted that there is no guarantee (ahead of time) that the product found by the buyer will be of a certain quality. Several key assumptions underlie our analysis and we discuss these before proceeding with an exposition of the model.

Assumption 1. Sellers are risk neutral.

This assumption allows us to focus our analysis on the problem of screening and signalling without incorporating risk-sharing considerations. Moreover, sellers make numerous sales and this minimises their aversion to the risk associated with an individual transaction.

Assumption 2. Producer and purchaser moral hazard are insignificant.

Producer moral hazard can be a factor in the context of warranties; however, in the markets where

[5] In related work, Grossman (1981) discusses the informational content of warranties when statements about quality cannot be verified and Gal-Or (1989) examines the signalling role of warranties in the context of differentiated products.

[6] Diamond (1971) notes that a market which has subsequent search costs greater than zero results in monopoly-like pricing.

simultaneous screening and signalling take place, the seller has limited ability to affect the performance of the product after it has been sold. Downstream moral hazard is frequently limited with deductibles and maintenance programmes that induce careful behaviour on the part of purchasers[7]. This assumption is also made to focus our analysis on the problem of screening and signalling.

Assumption 3. Products can be either premium (P) or standard (S) quality and buyers cannot determine quality prior to buying.

As discussed in the context of the used car market, this assumption is the basis for the consumer's "lemon" problem. If a seller has a premium quality product, his objective is to communicate the premium quality to consumers as cheaply as possible[8].

Assumption 4. Consumers differ in their valuation for warranty coverage and sellers cannot observe these valuations.

Extended warranties are effectively "insurance" against uncertain repair costs. Consumers frequently place different values on these warranties and hence, we observe significant differences in the amount of warranty coverage purchased by consumers[9]. In our simplified market, we assume two different kinds of buyers: the first (type H) places higher value on warranty coverage than the second (type L).

Assumption 5. Over time, a premium quality product is associated with lower repair costs and higher consumer satisfaction.

A premium quality product is assumed to fail less than a standard quality product and accordingly, the expected cost for a seller of providing a repair contract (in this case, an extended warranty) is lower. Without loss of generality, we assume linear repair costs with an expected repair cost per unit time that is lower for premium quality products i.e. $c_p<c_s$. Given the difference in repair costs, the marginal value of warranty coverage to a buyer for a standard quality product is also higher (the warranty insures against greater potential expenditures). We also assume there are negative aspects to owning a standard quality product (such as the time spent getting repairs and poorer performance) beyond the expenditures on repairs. Thus, assuming

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[7] Several papers consider warranties in the context of producer and purchaser moral hazard including Cooper and Ross (1985) and Dybvig and Lutz (1993).

[8] From a modeling perspective, the need is to have two distinct levels of quality. We use the terms premium and standard here instead of the usual "high and low" because we wish to avoid confusion with the two types of buyers (high and low) that we have in our market.

[9] Heterogeneous preference for warranty protection can arise from differences in income [Shaked and Sutton (1982)] or differences in risk aversion [Padmanabhan and Rao, 1993].

equivalent warranty protection, a premium quality product is assumed to provide greater benefit to a buyer than a standard quality product.

Assumption 6. The value of a product and hence warranty coverage declines over time.

Virtually all durable goods depreciate over time due to wear and tear or obsolescence. Accordingly the value of a warranty which guarantees that broken products are repaired also declines over time.

Using these assumptions as a basis, we now outline the details of the model as it relates to the buyer's decision and the seller's decisions. We then discuss the informational assumptions and the extensive form of the model.

The Buyer's Decision

We assume that utility derived by a buyer from purchasing a product of known quality can be represented by a quasi-linear function in which there are three main components. The first component (B_Q) is the benefit that a buyer obtains from a product of quality Q (Q=P or S) without warranty protection. The second component is the benefit that the buyer obtains from the warranty offered with the car. The final component (*P*) is the total price paid for the product and the associated warranty coverage. The utility function for buyers (where x is the length of the warranty) is:

$$U(\theta, Q, x, P) = B_Q + \gamma_Q \theta_T V(x) - P \qquad \textbf{(1)}$$

The second term is the product of three items. Following from Assumption 5, the first item γ_Q is a parameter used to capture the assumption that the marginal value of warranty coverage for a premium quality product is less then the marginal value of warranty coverage for a standard quality product i.e. item $\gamma_P < \gamma_S$. The second item θ_T is a valuation parameter, which is different for the two types of buyers in our model. Consistent with Assumption 4, the valuation parameter for buyers who place a higher value on warranty coverage (type H) is larger $\theta_H > \theta_L$. We assume that a fraction `$\lambda$' of buyers are type H and `$1-\lambda$' are type L.

V(x) is a function that allows us to reflect Assumption 6 (the value of warranty coverage declines over time). Mathematically, this implies that *V(x)* has the following properties*:* $V'(x)>0$, and $V''(x)<0$. We further assume that a warranty of zero length has no value i.e. V(0)=0. To simplify the analysis, we utilise the following form for *V(x):*

$$V(x) = \frac{1-(1-x)^2}{2} \qquad \textbf{(2)}$$

This function exhibits the required properties for x ε [0,1] and satiation at $x=1$. Since most durable goods have a finite life, we model the warranty has having a limit beyond which it is of little value. Having chosen this functional form, we can now specify the constraint implied by Assumption 5.

When $B_P > B_{MIN} = \dfrac{B_S + \gamma_S \theta_L}{\gamma_P \theta_L}$, all buyers prefer premium quality to standard quality given equivalent prices and warranty coverage.

Two further comments are necessary to fully explain the buyer's decision process. First, a buyer will not purchase unless the offering (i.e. the price, warranty coverage and quality) is expected to yield a pre-determined level of reservation utility (this is referred to as the Individual Rationality Constraint). The level of reservation utility can be any number that reflects the outside options of buyer but to simplify the analysis, we assume that reservation utility is zero[10].

A second issue concerns the decision process of the buyer if he faces two options that both provide positive utility. We assume that the buyer chooses the option that yields the maximum utility and this is captured mathematically through an Incentive Compatibility Constraint.

Sellers

The expected profits of the seller are a function of the distribution of buyers in the market and the marginal cost of the product is assumed to be zero[11]. Thus, a seller will maximise the following function:

$$\pi_Q(x_H, P_H, x_L, P_L) = \lambda[P_H - c_Q x_H] + (1-\lambda)[P_L - c_Q x_L] \qquad \textbf{(3)}$$

The warranty/price bundles (x_H, P_H) and (x_L, P_L) are purchased by the high and low valuation buyers respectively and c_Q is the cost per unit time of providing warranty coverage for a seller of quality `Q' (Q=S or P). The warranty length x_L can be thought of as the base warranty and (x_H-x_L) is the length of the extended warranty that can be purchased for (P_H-P_L). The two options available to consumers (the product without an extended warranty, the product with an extended warranty) are referred to as two points (x_H,P_H) and (x_L,P_L) in warranty length/price space. Consistent with Assumption 5, the repair costs incurred by sellers are a linear function of the lengths of warranty coverage.

Two other conditions are worth mentioning in the context of the menus offered by sellers. In order for a seller to have an incentive to offer a positive warranty, the "most willing" buyer must be willing to pay more for the warranty than it costs the seller to provide it[12]. This reduces to 2 simple conditions:

$$\gamma_P \theta_H > c_P, \ \gamma_S \theta_H > c_S \qquad \textbf{(4)}$$

For the low valuation buyer to purchase warranty protection, these conditions must also be satisfied for θ_L.

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[10] This assumption implies that high and low valuation buyers have an incentive to purchase regardless of whether they believe the product to be standard or premium quality. Choice of a positive reservation utility allows for a case in which a low valuation consumer walks away from a seller that offers standard quality.

[11] Assuming zero marginal cost does not affect the results; production cost drops out of the first order conditions.

A second issue concerns the distribution of buyers in the marketplace. As we will see later, a key constraint faced by a seller in designing a menu for two types of buyers is that he cannot charge the high valuation buyer the maximum price he is willing to pay. Thus, when the percentage of low valuation buyers becomes sufficiently low, it can be optimal for a buyer to offer warranty coverage to high valuation buyers only (the low valuation buyer is offered the product without warranty coverage). We allow for this possibility in our analysis but focus on the interesting case when both low and high valuation buyers are offered positive levels of warranty protection[13].

Informational Assumptions

Sellers' cost structures and buyers' utility functions are assumed to be common knowledge. As previously mentioned, the buyer cannot tell a priori whether the seller is offering premium or standard quality and the seller cannot tell a priori whether the buyer has a low or high valuation for warranty coverage. In order for the buyer to figure out whether the seller is offering high or low quality, the buyer forms beliefs about the expected actions of a seller of premium quality versus a seller of standard quality. The Cho-Kreps criterion (1987) is used to find a set of beliefs (for buyers) that are *reasonable* and it is these beliefs that allow the derivation of a unique separating equilibrium.

Extensive Form of the Game

Although the game is modelled as a simultaneous single shot game in which the players choose optimal strategies, there is an implied order of play:

Stage 1. The seller chooses a menu of price/warranty bundles and will announce them to any buyer who has interest in the particular product that the seller is offering.

Stage 2. A buyer arrives at the seller's place of business and shows interest in the product that the seller is offering.

Stage 3. The buyer decides whether to purchase any of the bundles that the seller announces.

Following from our previous discussion, the seller's price setting ability is captured by having him choose prices and warranty lengths first. For the buyer, the existence of outside options is recognised through her Individual Rationality Constraint i.e. the more attractive (competitive) outside options are, the more likely it is that a buyer will leave the seller without making a purchase.

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[12] This condition is evaluated at x=0, the point at which the marginal value of warranty protection is highest.
[13] As shown in the technical appendix, a necessary condition for low valuation buyers to be offered positive

We now derive the equilibrium warranty menus in a market where conditions of double adverse selection are present. The approach we use is to first describe the outcomes that obtain in our model when product quality is fully observable. We then describe the outcomes that result when the second level of information asymmetry is added. This allows us to isolate and understand the impact that product quality uncertainty has on the actions of a seller that is using warranty policy to screen buyers.

5.0 Equilibrium Contracts in a Market under Conditions of Double Adverse Selection

First, we consider the action that a seller would take were product quality observable. Clearly the buyer's valuation of a given warranty/price bundle depends on the quality of the product. When quality is observable, the seller knows the value that the buyer will place on a product with a given warranty length.

With observable quality, we solve a constrained optimisation problem with the objective function shown in equation 3. This objective function is based on buyers selecting the appropriate bundle: (x_H,P_H) or (x_L,P_L) depending on their type (H or L). Since all buyers have the option of buying both bundles, it is critical that buyers voluntarily select the bundle that is designed for them. This leads to two Incentive Compatibility Constraints, one for each type of buyer.

For highs $$B_Q + \gamma_Q \theta_H V(x_H) - P_H \geq B_Q + \gamma_Q \theta_H V(x_L) - P_L \quad (5)$$

For lows $$B_Q + \gamma_Q \theta_L V(x_L) - P_L \geq B_Q + \gamma_Q \theta_L V(x_H) - P_H \quad (6)$$

As previously discussed, we also assume that both buyer types realise positive surplus by purchasing and this is reflected through Individual Rationality Constraints.

For highs $$B_Q + \gamma_Q \theta_H V(x_H) - P_H \geq 0 \quad (7)$$

For lows $$B_Q + \gamma_Q \theta_L V(x_L) - P_L \geq 0 \quad (8)$$

Finally, we constrain prices to be positive and warranty lengths to values between zero and one.

The key elements of the solution are that equations 5 and 8 bind with strict equality. This means that a high valuation buyer is indifferent between the bundle designed for her (x_H,P_H) and the bundle designed for a low valuation buyer (x_L,P_L) and a low valuation customer is indifferent between purchasing her bundle and not buying at all[14]. We summarise the nature of the solution to this problem in Proposition 1.

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warranty protection is $\lambda < (\gamma_Q\theta_L - c_Q)/(\gamma_Q\theta_H - c_Q)$.

[14] In spite of the high valuation buyer's indifference between the two bundles, she is assumed to choose the bundle designed for her. This is a typical assumption in self-selection problems justified because the high valuation buyer prefers her bundle strictly with an infinitesimal reduction in P_H.

Proposition 1

When product quality is observable, the profit maximising action for the seller is to offer a menu of price/warranty combination where each buyer-type self selects to a bundle designed for her.

Proposition 1 confirms the earlier results of Mussa and Rosen (1978) and Maskin and Riley (1984) that relate to second degree price discrimination (how can a firm price discriminate when consumers have unobservable heterogeneity). The basic challenge for the seller is that he wishes to serve two types of buyers, one of which is willing to pay more for warranty coverage than the other. On the one hand, he could choose to serve only the type who is willing to pay the most for warranty coverage (type H) but then he would serve only λ% of the market and leave a significant number of profitable buyers unserved. On the other hand, he could set a sufficiently low price on a single bundle such that all buyers happily purchase. The problem with this approach is first, that high valuation buyers would obtain a benefit that the seller could potentially charge for and second, the warranty coverage would be significantly less than the optimal coverage that could be sold to high valuation buyers. Essentially, the seller is looking for a way to sell high valuation buyers more warranty coverage at a higher price and still keep low valuation buyers in the market. The solution to this dilemma is to offer a number of warranty/price bundles to each buyer and construct the menu in such a way that high valuation buyers choose a more expensive bundle with more warranty coverage. If we restrict our attention to values of λ such that $\lambda < \dfrac{\gamma_Q\theta_L - c_Q}{\gamma_Q\theta_H - c_Q}$, the seller has an incentive to offer positive warranty lengths to both types and the optimal menu for the seller is:

$$(x_H, P_H) = \left(1 - \frac{c_Q}{\gamma_Q\theta_H}\ ,\ B_Q + \frac{\gamma_Q\theta_H}{2}\left[\frac{(1-\lambda)^2 c_Q^2}{\gamma_Q^2(\theta_L - \lambda\theta_H)^2} - \frac{c_Q^2}{\gamma_Q^2\theta_H^2}\right] + \frac{\gamma_Q\theta_L}{2}\left[1 - \frac{(1-\lambda)^2 c_Q^2}{\gamma_Q^2(\theta_L - \lambda\theta_H)^2}\right]\right) \quad \textbf{(9)}$$

$$(x_L, P_L) = \left(1 - \frac{(1-\lambda)c_Q}{\gamma_Q\theta_L - \lambda\gamma_Q\theta_H}, B_Q + \frac{\gamma_Q\theta_L}{2}\left[1 - \frac{(1-\lambda)^2 c_Q^2}{\gamma_Q^2(\theta_L - \lambda\theta_H)^2}\right]\right) \quad \textbf{(10)}$$

Mathematical comparisons lead us to Proposition 2 which underlines the principal elements of the seller's screening menu when product quality is observable.

Proposition 2

When quality is observable, the profit maximising menu for the seller has:

(a) A bundle designed for the high valuation buyer with warranty protection of efficient length and a price which leaves her with strictly positive utility.

(b) A bundle for the low valuation buyer with a warranty that is shorter than the efficient length and a price which leaves her indifferent between buying or not.

Proposition 2 underlines three key aspects of the optimal screening menu for a seller of known quality. First, offering two bundles to buyers certainly increases profit for the seller but it is not ideal (theoretical "profit" is left downstream with buyers). In a sense, the seller needs to provide high valuation buyers with a subsidy (or lower price) to buy the more expensive bundle. This situation obtains because regardless of how short the warranty is in the low valuation buyer's bundle, it will be attractive to a high valuation buyer since she is willing to pay more for warranty coverage than a low valuation buyer. Any bundle designed for the high valuation buyer must provide at least the benefit that the high valuation buyer would obtain from the low valuation buyer's bundle.

The second point is that this subsidy (or price reduction) to high valuation buyers is in some sense pre-determined (or fixed) by the characteristics of the low valuation buyer's bundle. Accordingly, the seller maximises his profit on sales to high valuation buyers by maximising the difference between the price he can charge for the high valuation buyer's bundle and his expected costs. This occurs at the warranty length where the marginal benefit of additional warranty coverage to the buyer is exactly equal to the marginal cost of providing warranty coverage: i.e. when $\gamma_Q\theta_H(1-x_H)=c_Q$. Thus, even though, the seller is unable to extract the maximum price from the high valuation customer, he nonetheless chooses a warranty length for her that is efficient.

Finally, a key drawback in meeting the Incentive Compatibility Constraint for the high valuation buyer is the cost of the "subsidy". The longer is the warranty coverage in the low valuation buyer's bundle, the greater is the subsidy (or price reduction) needed in the high valuation buyer's bundle. This explains why the optimal menu for the low valuation buyer involves a warranty that is strictly shorter than the efficient length. The seller gives up some profit on low valuation buyers in order to reduce the subsidy that he needs to pay high valuation buyers to ensure self-selection. Naturally, the degree of this reduction is a function of the distribution of types in the market. As the fraction of low valuation buyers in the market increases (i.e. λ decreases), the reduction in warranty coverage (x_L) from an efficient level gets smaller.

The Impact of Quality on the Screening Menu when Quality is Observable

When buyer types are not observable but the quality of the product is, the seller of premium quality offers strictly more expensive bundles than the seller of standard quality to each type of purchaser. This is illustrated graphically in Figure 1 where the menu offered by the seller of premium quality lies above the menu offered by the seller of standard quality. The high valuation buyer receives the surplus maximising bundle (i.e. this is reflected by the tangency of the isoprofit lines and indifference curves at P_H) regardless of whether she finds herself dealing with a seller of premium or standard quality. On the other hand, the low valuation buyer receives a bundle which lies on her reservation utility indifference curve but is *strictly shorter* than her surplus maximising bundle.

(Figure 1)

There are primarily two factors, which affect the relative location of the two menus in warranty length/price space. The first is the ratio of c_S to c_P. When the ratio is large, the isoprofit lines of the premium quality seller are much flatter than those of the standard quality seller. This will move the premium quality seller's menu to the right, as he will find it advantageous to offer his customers longer warranty protection. On the other hand when the difference between B_P and B_S is large (the fixed benefit difference between premium and standard quality), the prices a premium quality seller can charge are much higher and this tends to move the premium quality sellers upwards.

A seller of standard quality would love to obtain higher prices for his product/warranty combinations but on the other hand, he does not want to provide extra warranty coverage since his costs of doing so (c_P) are high. We can interpret this in the context of the Figure 1. When the menu of the premium quality seller lies significantly to the right (with long warranties), the standard quality seller is unlikely to want to mimic the premium quality menu. While he would get higher prices, he would also have to finance longer warranty coverage. In contrast, when the menu of the premium quality seller lies above the menu of the standard quality seller, the standard quality seller would love to offer the warranty/price combinations that are optimal for the premium quality seller. This is the essence of the double adverse selection problem. When quality is not observable, a wary buyer will not pay premium quality prices for a product/warranty combination unless she is sure that standard quality seller would be unwilling to offer her the same deal.

Optimal Screening Menus when Quality is Unobservable

A simple way of describing the problem created when product quality is unobservable is that we are looking for a market outcome in which sellers maximise profit and buyers actually obtain the quality that they think they are buying at the time of purchase. In other words, a situation in which a buyer thinks that she is purchasing premium quality but actually obtains standard quality is not an equilibrium[15]. Based on the discussion in the preceding paragraph, we need to address two situations. In the first, were quality observable, the premium quality seller's menu would lie in a region where the standard quality seller does not have an incentive to pretend to be a seller of premium quality i.e. the premium quality product is offered with long warranties. In the second, were quality observable, the premium quality seller's menu would lie in a region where a standard quality seller does have an incentive to pretend to be a seller of premium quality i.e. the premium quality seller's menu has high prices. The objective is first, to describe the boundary that delineates the two situations and second, to identify equilibrium action for the premium quality seller in each situation.

[15] Technically, it would not be an equilibrium for a buyer to expect standard quality before purchasing and actually receive premium quality. However, in our model, this situation is irrelevant since a premium quality

The Non-Mimic Constraint

The key problem when quality is unobservable rests with buyers who lack information about the quality of the products that they are buying. To help buyers make decisions, we assume that buyers form beliefs about product quality and these beliefs are based on prior information and actions (such as the warranty/price menus that are announced). This is the essence of signaling models in which uninformed players (i.e. the buyers) make inferences about informed players (i.e. sellers who know the quality they are selling) based on actions taken by informed players.

The technical approach to this problem involves Bayesian equilibria, in which the actions *and* beliefs (of players) are specified as reasonable outcomes in games of incomplete information. Following Mas-Collell, Whinston and Green (1995), the equilibrium concept to apply is that of the Perfect Bayesian Equilibrium (PBE). As noted by Fudenberg and Tirole (1992) and Mas-Collell, Whinston and Green, this concept imposes restrictions on the beliefs of players for actions that are part of the equilibrium. However, there are no restrictions on the beliefs of players for actions that are not part of the equilibrium. As a result, multiple separating equilibria as well as pooling equilibria (where both premium and standard quality sellers take identical actions) are possible in signalling problems under the PBE concept. However, in many cases, these equilibria are supported by off equilibrium beliefs that are inherently unreasonable.

To address this problem Cho and Kreps (1987) propose the Intuitive Criterion which places a restriction on the beliefs of buyers *off the equilibrium path*. Essentially, a buyer cannot attribute positive probability to a seller-type offering a given warranty/price menu if the warranty/price menu would make the seller-type worse off[16]. In addition, the Intuitive Criterion points to the equilibrium where the premium quality seller has no alternate action (i.e. a warranty/price menu), that yields a higher payoff given the restriction on buyer beliefs. The net effect of the Intuitive Criterion is to eliminate both pooling equilibrium and separating equilibrium with inefficient amounts of signalling. In fact, the only equilibrium not rejected by the Intuitive Criterion is a separating equilibrium with the least amount of inefficient signalling.

Going back to our problem, when the premium quality seller's optimal menu under observability is unattractive to the seller of standard quality, the least amount of inefficient signalling for the premium quality seller is zero (i.e. his actions are unaffected). This obtains since a buyer's beliefs about the quality will be inferred from the menu that is offered and a standard quality seller would lose money by doing the same thing.

In contrast, when the premium quality seller's optimal menu under observability is attractive to the seller of standard quality, the premium quality seller needs to signal premium quality through his actions. Here we use the Intuitive Criterion to identify a unique separating

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seller never has an incentive to pretend to be standard quality.

[16] The technical term for this restriction is that a player cannot attribute positive beliefs to a type taking an action for whom the said action is equilibrium dominated.

equilibrium. Following the Criterion, we need to find a warranty/price menu for the premium quality seller where no alternative will provide him with higher profit given "reasonable" beliefs by the buyers. This menu can be identified by introducing a constraint, the `no mimic' constraint, to the premium quality seller's optimisation problem. This constraint simply restricts the premium quality seller's choices to menus that are unattractive to a seller of standard quality. When the `no mimic' constraint binds, a seller of premium quality chooses a menu for which the seller of standard quality is indifferent between mimicking and not mimicking.

The no mimic constraint implies that if a standard quality seller chooses to offer the menu P_H^P, P_L^P, x_H^P and x_L^P (the menu chosen by the premium quality seller), he will make less profit than he makes by offering his optimal menu as a *known* seller of standard quality (π_S^{max})[17].

$$\pi_S^{max} > \lambda(P_H^P - x_H^P c_S) + (1-\lambda)(P_L^P - x_L^P c_S) \qquad \textbf{(11)}$$

The problem for a seller of premium quality is analogous to the problem considered when quality is observable subject to the additional constraint shown in equation 11. Thus, we maximise the objective function (equation 4) subject to incentive compatibility and individual rationality constraints (equation 5-8) and the no-mimic constraint (equation 11).

The solution in the technical appendix leads to the following proposition:

Proposition 3

When product quality is unobservable:

(a) and the fixed benefit B_P of a premium quality product is less than $\hat{B}$, the warranty/price menu offered by a seller of premium quality is identical to the one offered when product quality is observable.

(b) and the fixed benefit B_P of a premium quality product is greater than $\hat{B}$, the price/warranty menu offered by a seller of high quality is different from the one offered when product quality is observable.

Where: $\hat{B} = \pi_S^{MAX} + c_S - \frac{\gamma_P \theta_L}{2} - (2c_S - c_P)\left(\frac{(1-\lambda)^2 c_P}{2\gamma_P \tilde{\theta}} + \frac{\lambda c_P}{2\gamma_P \theta_H}\right)$, $\tilde{\theta} = \theta_L - \lambda\theta_H$ *and* π_S^{MAX} *is the profit realized by a seller of standard quality when quality is observable.*

The expression for π_S^{MAX} *is given by:*

$$\pi_S^{MAX} = B_S + \frac{\lambda\gamma_S\theta_H}{2}\left[\frac{(1-\lambda)^2 c_S^2}{\gamma_S^2\tilde{\theta}^2} - \frac{c_S^2}{\gamma_S^2\theta_H^2}\right] + \frac{\gamma_S\theta_L}{2}\left[1 - \frac{(1-\lambda)^2 c_S^2}{\gamma_S^2\tilde{\theta}^2}\right] - c_S + \frac{(1-\lambda)^2 c_S}{\gamma_S\tilde{\theta}} + \frac{c_S^2\lambda}{\gamma_S\theta_H}$$

Before providing the intuition for this result, we provide insight about the expression for $\hat{B}$, which appears very complex. $\hat{B}$ is a function of the relative costs of premium and standard quality sellers (c_P and c_S), the relative values of warranty coverage for buyers of premium and

[17] Since there are only 2 types of sellers in this model, a buyer believes that a seller is standard quality unless

standard quality respectively (γ_P and γ_S) and B_S the fixed benefit associated with standard quality. It describes the premium level B_P above which the menu of a premium quality seller *under observability* will be attractive to a seller of standard quality.

The intuition behind Proposition 3 is that when the fixed benefit for premium quality is low i.e. $B_P \in \{B_{MIN}, \hat{B}\}$, the cost advantage of the premium quality seller (i.e. the degree to which c_P is less than c_S) overshadows higher prices that buyers are willing to pay for premium quality (prices are a direct function of the premium B_P)[18]. As a result, the optimal menu chosen by the seller of premium quality is expensive for the standard quality seller to mimic (B_P is not sufficiently high in relation to B_S). In fact, when buyers have a high fixed benefit for standard quality (B_S) or when the difference between the marginal costs of providing coverage is high (c_S versus c_P), it is unlikely that a standard quality seller will mimic the optimal menu of the premium quality seller (under observability).

We should also discuss the role of the γ_P and γ_S (the marginal value of warranty protection parameters) in the equilibrium. First, the higher is γ_S (the marginal value of warranty coverage on standard quality products), the higher is $\hat{B}$ because a higher γ_S strictly increases π_S^{MAX}. The effect of an increase in γ_P (the marginal value of warranty coverage on premium quality products) on $\hat{B}$ is ambiguous because it has two effects. The first is to cause the premium quality seller to offer more coverage making the menu (under observability) less attractive to a seller of standard quality. The other is to increase the prices in the menu of the premium quality seller and this of course, makes the menu attractive to the seller of standard quality. Simulations suggest that for most values of γ_P, an increase in γ_P raises $\hat{B}$ and reduces the likelihood of the premium quality seller's menu being affected. However, when either the difference between c_P and c_S is small or γ_P is high, the effect can be negative.

When $B_P > \hat{B}$, the menu that would be chosen by a seller of premium quality (when quality is observable) is attractive to a seller of standard quality. In this situation, the solution to the constrained optimisation problem for the premium quality seller yields a positive Lagrangean multiplier on the no-mimic constraint (equation 11). This has three important economic interpretations. First, the solution to the problem involves the premium quality seller choosing a menu for which the seller of standard quality is precisely indifferent between mimicking and not mimicking. This menu is different from the menu that the premium quality seller would offer under conditions of observable quality. Second, relaxing the no-mimic constraint by one unit (for example by increasing B_S by one), the profit of the premium quality seller would increase by an amount equal to the value of the multiplier. Finally, the profit of the premium quality seller is strictly reduced due to the unobservability of quality (profit will increase when the constraint is relaxed and the profit under observability is obtained by solving the problem with the constraint fully relaxed).

We now consider Proposition 4 which considers the specific actions of the premium quality

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he has reason to believe otherwise.

[18] As noted earlier in the paper, B_{MIN} is the minimum value of B_P consistent with our basic assumptions.

seller.

Proposition 4

When product quality is unobservable and $B_P > \hat{B}$, the premium quality seller's menu involves more warranty protection and higher prices for all buyers (than the menu used when quality is observable). Additionally, extended warranties (i.e. $x_H - x_L$) are shorter when product quality is unobservable.

The intuition for Proposition 4 is that a premium quality seller will lengthen the warranties in his menu to make it `unaffordable' to a seller of standard quality. Admittedly, he charges more for the bundles in question, but the higher prices does not compensate for the added cost of providing the coverage (this difference is the cost of signalling for the premium quality seller). The reason that both x_L and x_H are longer is that the premium quality seller minimises his cost of signalling by making as small movements as possible from the optimal screening menu under observability. By spreading the signalling between the two bundles in the menu, he minimises this cost. In other words, the premium quality seller signals his quality to buyers through his menu while simultaneously sorting customers and getting high types to purchase more warranty coverage and pay more. However, there is a range in the parameter space where the premium quality seller loses his ability to screen because the signalling requirements are too great. This leads to Proposition 5.

Proposition 5

When product quality is unobservable and the fixed benefit for a premium quality product exceeds $B_{MAX} = \pi_S^{MAX} + c_S - \frac{\gamma_P \theta_L}{2}$, signaling considerations dominate screening considerations and a seller of premium quality offers the maximum warranty length to all buyers.

The collapse of the premium quality seller's menu occurs precisely because of the satiation property of buyers' utility functions. As previously discussed, we believe this to be a reasonable property for the utility function given that most durable products have a finite life. The intuition behind the result is that when $B_P > B_{MAX}$, both types of buyers are more than willing to pay $B_{MAX} + \frac{\gamma_P \theta_L}{2}$ for a premium quality product with a full warranty i.e. $x_L = x_H = 1$. The individual rationality constraints (equations 7 and 8) hold for neither buyer type and they both obtain positive surplus by buying. Nonetheless, a seller of premium quality is prevented from charging more for his menu because of the `no mimic' constraint. This contrasts with the menu that is chosen when product quality is unobservable and $B_P \in \{B_{MIN}, B_{MAX}\}$. When B_P falls in this range, the high valuation buyer always receives positive surplus and the low valuation buyer's individual rationality constraint is always binding (and hence she receives zero surplus). When $B_P > B_{MAX}$, the adverse

selection problem not only distorts the premium quality seller's menu, it also leaves the low valuation buyer strictly better off.

From the analysis it appears that there are three potential zones for B_P given B_S, λ, θ_L, θ_H, γ_P, γ_S c_P and c_S. The first is the zone $\{B_{MIN}, \hat{B}\}$ where the premium quality seller's action is unaffected by the unobservability of quality. The second is the zone $\{\hat{B}, B_{MAX}\}$ where the premium quality seller needs to lengthen his warranties to signal premium quality. The third zone is where $B_P > B_{MAX}$ and the premium quality seller offers one option with maximum warranty protection to everyone. If B_P falls into one of these zones then the findings above certainly hold. However, there is no guarantee that all three zones exist given B_S, λ, θ_L, θ_H, γ_P, γ_S c_P and c_S. If we examine the expression for $\hat{B}$, a low value of γ_P has no effect on the first two terms, drives the third term to zero and causes the final term to become large and negative. As a consequence, it is possible that $\hat{B}$ could be a value less than B_{MIN} for a given set of parameters B_S, λ, θ_L, θ_H, γ_P, γ_S c_P and c_S. . When this happens, a premium quality seller will be obliged to alter his menu to signal his quality *for any feasible value of* B_P. A similar argument can be used to show that for a sufficiently low value of γ_P, B_{MAX} is also less than B_{MIN}. In this situation, only one zone exists (for all possible B_P) which implies that whenever quality is unobservable, a premium quality seller will be obliged to offer a maximum warranty and one price to everybody.

Given B_P, B_S, λ, θ_L, θ_H, γ_P, γ_S c_P and c_S, it is possible to describe the equilibrium that will be observed in a market subject to double adverse selection using the information provided in Propositions 1-5. Consider the interesting case where the premium quality seller wishes to offer both buyer-types warranty protection i.e. $\lambda < \dfrac{\gamma_P \theta_L - c_P}{\gamma_P \theta_H - c_P}$ and γ_P is high enough such that three zones described in the previous paragraph do exist. Then the expected warranty offerings can be represented as 9 distinct regimes in cost-premium space (see Figure 2).

(Figure 2)

The legend in Figure 2 outlines the mathematical conditions that characterise each of the nine regimes: B_{MIN}, $\hat{B}$ and B_{MAX} are calculated as per the formulae in this paper.

In all market conditions, a seller of premium quality must ensure that his menu is unattractive to a seller of standard quality (i.e. the menu offered by a seller of high quality seller must satisfy a no-mimic constraint). The list of regimes in Figure 2 is complete for the assumptions that we have made. However, relaxing certain assumptions allows for even greater heterogeneity in the market outcomes. For example, in the absence of standard quality sellers, a premium quality seller might offer warranty protection to high-type customers only (leaving the base warranty at zero). However, double adverse selection can create a situation where a premium quality seller not only lengthens the coverage sold to high-type buyers but also introduces a non-zero base warranty

for low-types. It proves particularly interesting to relate some of the regimes in Figure 2 to observations made during the previously discussed used car market survey.

First, the incentive or incidence of warranty coverage is increased by the problem of double adverse selection. Unless a car is of the poorest quality, a seller frequently has incentives to offer warranty coverage as a way of sending a message to potential buyers. Consistent with this insight, 98% of all dealers surveyed discussed the extended warranty options during the selling discussion and 58% of the dealers actively promoted extended warranties (with point of purchase materials and banners). This speaks to both the strong profit incentive and the signalling role of warranty policy in the used car market.

Second, Figure 2 suggests that there are regions where a seller of standard quality offers cars with no base warranty but still makes extended warranties available. In fact, 32% of cars which had extended warranty options available, came with no base warranty. In addition, similar to standard quality sellers in Regime 1, 9% of the cars had no base or optional extended warranty coverage.

Propositions 4 and 5 imply that there are at most three possible sets of actions from the perspective of a seller of premium quality. In our survey, we did not observe a seller who offered a maximum warranty with his product (the result that one expects when $B_P>B_{MAX}$). This may be because the unobservable difference in qualities of used cars may not be strong enough to drive premium quality sellers to offer a maximum warranty. Nonetheless there are categories where quality differences are large and we do observe certain products like Swiss Army Knives and Craftsman tools being sold with lifetime warranties.

6.0 Numerical Example

We include a brief numerical example in this section to highlight three key effects of double adverse selection on a seller of premium quality[19]. It is useful to examine the effects in each of the three possible zones for B_P (we have chosen the parameter values carefully so that all three zones exist for $B_P>B_{MIN}$). The results of a numerical example *with* $\theta_H=15$, $\theta_L=10$, $c_P=3$, $c_S=7$, $\lambda=0.5$, $\gamma_P= 0.9$, $\gamma_S=1$, and $B_S=1$ are shown in Figures 3, 4, and 5. As a basis of comparison, we compare the outcomes to the outcomes that would be observed in a market where quality is observable.

(Figures 3, 4 and 5)

The first effect is to demonstrate the impact that double adverse selection has on warranty lengths. Figure 3 demonstrates that the fixed benefit associated with premium quality has no impact on warranty length when quality is observable. This obtains because the value of the warranty to

[19] In equilibrium, the actions of a seller of standard quality are unaffected by double adverse selection.

buyers obtains solely from their valuation of warranty protection and not the fixed benefit associated with using the product. However, once there is double adverse selection, we observe an interesting relationship between warranty length and the fixed benefit associated with premium quality. When the premium is less than $\hat{B}$, there is no relationship between warranty coverage and the fixed benefit B_P (similar to the situation when quality is observable). However, when the fixed benefit exceeds $\hat{B}$, we observe a positive relationship between the magnitude of the fixed benefit and warranty lengths (we show the base warranty length but the length of coverage purchased by the high-type buyer exhibits a similar relationship). This occurs because the premium quality seller uses his cost advantage to signal higher quality to buyers. Once B_P is greater than B_{MAX} , the premium quality seller signals by offering maximum warranty protection as a base warranty. As a result, in this range not only is the premium quality seller forced to offer longer warranties to everybody, he also loses his ability to offer a unique bundle for each type of buyer.

Figure 4 illustrates the relationship between pricing and the fixed benefit associated with premium quality. The simulation illustrates the positive relationship between the fixed benefit and pricing when quality is observable. This occurs because prices are a direct function of how much buyers are willing to pay for a product (premium quality sellers take this into account in setting their prices). Of course, when B_P is high, high prices are what make the premium quality seller's menu attractive to a seller of standard quality. Interestingly, when quality is not observable (and $B_P > \hat{B}$), the prices paid by buyers for premium quality are even higher than the prices they pay when quality is observable. However, the seller of standard quality is deterred from mimicking this menu by the higher costs that result from longer warranty coverage.

Finally in Figure 5, we see the effect that double adverse selection has on the profit realised by a seller of premium quality. When B_P is less than $\hat{B}$, the profit of a premium seller is unaffected. This is because a premium quality seller's action is unaffected when the fixed benefit for premium quality is low. In the intermediate zone for B_P i.e. when B_P lies in the interval ($\hat{B}$, B_{MAX}), the profit of the premium quality seller is reduced somewhat by double adverse selection. However, profits continue to rise because the cost of longer warranties is mitigated by the higher prices that can be charged. However, once B_P exceeds B_{MAX}, profits are drastically affected by double adverse selection. In this zone any benefit associated with a higher fixed benefit accrues directly to buyers since, as shown in Figure 4, the premium quality seller cannot raise his price at all.

7.0 Conclusion

The objective of this paper has been to analyse warranty policy that is used to support the sales of durable goods in markets characterised by double adverse selection. As noted in the

introduction, double adverse selection exists when a product or service is (to some degree) an experience good, sellers have an advantage over buyers in terms of identifying quality and buyers have unobservable preferences for the product.

The key insight of the paper is that warranties can be used to screen and signal simultaneously when sellers have price setting ability. The reason that warranties can play this dual role is that "warranty length" is a positive attribute (of the product) which can be metered and is cheaper for a seller of premium quality to provide.

The important findings of the paper are as follows. First, a seller of premium quality always needs to account for the existence of standard quality sellers in designing his warranty policy. When the fixed benefit for premium quality is relatively low (i.e. less than $\hat{B}$), the menu chosen by a seller of premium quality is the same regardless of whether quality is observable or not (in this region, a standard quality seller has no incentive to mimic the warranty policy of a seller of premium quality). In this situation, a premium quality seller offers the "high type" buyer an extended warranty that provides an "efficient length" of warranty coverage. On the other hand, the "low type" buyer buys the product with a standard base warranty and this provides her with less than efficient warranty coverage.

However, when B_P is greater than $\hat{B}$, a premium quality seller alters his warranty policy to account for the existence of standard quality sellers. The premium quality seller strategically makes his warranty policy "more expensive" for the standard quality seller to mimic. The optimal action for the premium quality seller entails both types of buyers purchasing more warranty coverage than they would were quality observable. Although optimal extended warranties are shorter when B_P is greater than $\hat{B}$, the net length of warranty coverage purchased by both buyer-types is longer. This "lengthening" of warranty coverage by premium quality sellers is costly but necessary in order for warranty policy to be an effective signal. Not surprisingly, a recent article highlights the efforts of sellers in second hand markets "to rein in fast talkers who promise coddled Cadillacs but deliver clunkers"[20]. The high cost of signalling with extended warranties provides an incentive for sellers of premium quality to find cheaper signalling alternatives (for example, certification).

A second important finding is that the ability of warranty policy to play a dual role is impaired when the fixed benefit for premium quality is overly high. Specifically, when the fixed benefit exceeds the cut-off point B_{MAX}, the signalling requirements on the premium quality seller are so severe that he cannot screen buyers. In this situation, the optimal action for the premium quality seller is to offer maximum warranty length to both types of buyers (the product is sold with a base warranty that lasts for the expected life of the product).

Finally, there are several equilibrium conditions that occur in which sellers do not offer a different warranty length for each type of consumer. As noted above, when the fixed benefit for premium quality is high, a seller of high quality may be forced to offer a 'collapsed' menu in which

[20] Bennet, James, (August 2, 1994) B7.

both types of buyers get the same deal. In addition, there are regions where a seller of standard quality may choose to offer no warranty coverage on his products or warranty coverage for high-type consumers only. A number of these situations were observed in our survey of the used car market in Toronto. It should be noted that a premium quality seller is constrained by the existence of standard quality sellers and not their equilibrium actions (for example, if the fixed benefit for premium quality is high, a standard quality seller could offer all of his products with no warranty coverage and nonetheless cause a premium quality seller to offer a collapsed menu).

It is important to mention that the relevance of this paper is broader than simply explaining the warranty policy offered with used durable goods. As indicated earlier, the reason that warranty policy can be used to screen and signal simultaneously is that it is a positive attribute which can be metered. There are many markets where a) the buying situation is characterised by double adverse selection and b) buyers desire an attribute which can be metered. Several notable examples include the service contracts offered by firms on industrial equipment, service contracts offered to home owners on heating or air conditioning systems, different coverage plans offered by health maintenance organisations (HMO's) in the United States and different redemption provisions on financial assets of unknown riskiness. In all of these cases, the seller of the product service knows more about the quality of the product (or service) than the buyer. In addition, the seller can offer variable amounts of positive attribute (degree/extent of service contracts, the size of the HMO's network and the number of services provided, or the penalty associated with early redemption of an asset) and these attributes are generally cheaper for a premium quality seller to provide. An interesting empirical extension to this paper would be to examine the question of simultaneous signalling and screening in a broad context across several markets with double adverse selection.

An assumption of the model that warrants discussion is that of setting the marginal (production) costs for both premium and standard quality products to zero. Clearly, sellers of used durable products do not obtain their inventory at zero cost; however, once a seller has a product to sell, the marginal cost of production (or procurement) does not affect the simultaneous screening and signalling problem examined in this paper (production cost does not affect the first order conditions for warranty length). However, if sellers make procurement decisions prior to announcing their warranty policy, differences in marginal cost may affect the selection of products purchased by a seller. A two-stage game would be needed to analyse this situation[21]. Nonetheless, this paper provides insights into markets in which screening and signalling occur simultaneously. These insights are as relevant to the second stage of a more complex game as they are to the solution of a simpler game in which product (seller) quality is exogenous.

[21] In the first stage, sellers would make a decision about which quality to sell. Assuming premium and standard quality sellers exist in equilibrium, the second stage would be analogous to the situation described in this paper.

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Figure 2

Potential Market Equilibria

Assume that $\tilde{\theta} = \theta_L - \lambda\theta_H$

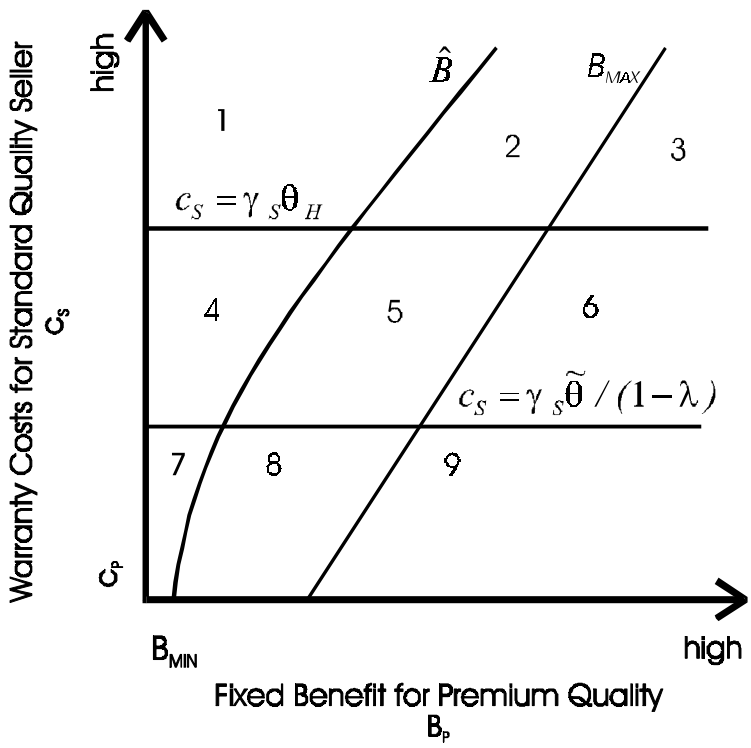

Table showing Conditions for Different Regimes that might be Observed		Actions of Premium Quality Seller		
		offers same bundles as if quality were observable	Offers complete menu but bundles are longer to meet signalling requirements	offers the same bundle to both types of buyers
Actions of Standard Quality Seller	offers no warranty coverage on his products	**1.** $c_S > \gamma_S\theta_H$ $B_P \in (B_{MIN}, \hat{B})$	**2.** $c_S > \gamma_S\theta_H$ $B_P \in (\hat{B}, B_{MAX})$	**3.** $c_S > \gamma_S\theta_H$ $B_P > B_{MAX}$
	Offers warranty coverage to high buyers only	**4.** $c_S < \gamma_S\theta_H$ $c_S > \gamma_S\tilde{\theta}/(1-\lambda)$ $B_P \in (B_{MIN}, \hat{B})$	**5.** $c_S < \gamma_S\theta_H$ $c_S > \gamma_S\tilde{\theta}/(1-\lambda)$ $B_P \in (\hat{B}, B_{MAX})$	**6.** $c_S < \gamma_S\theta_H$ $c_S > \gamma_S\tilde{\theta}/(1-\lambda)$ $B_P > B_{MAX}$
	offers warranty coverage to both high and low buyers	**7.** $c_S < \gamma_S\tilde{\theta}/(1-\lambda)$ $B_P \in (B_{MIN}, \hat{B})$	**8.** $c_S < \gamma_S\tilde{\theta}/(1-\lambda)$ $B_P \in (\hat{B}, B_{MAX})$	**9.** $c_S < \gamma_S\tilde{\theta}/(1-\lambda)$ $B_P > B_{MAX}$

Technical Appendix for Product Warranties and Double Adverse Selection

Proof of Proposition 1

When product quality is observable, the profit maximising action for the seller is to offer a menu of price/warranty combination where each buyer-type self selects to a bundle designed for her.

To prove this proposition, we find the optimal arguments for equation 3 subject to the constraints of equations 5, 6, 7, and 8 (all in the main text). With the assumptions, we have made equation 7 is redundant. The logic is as follows: if equation 8 is satisfied then $B_Q + \gamma_Q \theta_H V(x_L) - P_L > 0$ since $\theta_H > \theta_L$. Thus, equation 5 implies that $B_Q + \theta_H V(x_H) - P_H > 0$ so we can drop equation 7. We now write the problem as a Lagrangean function for equation 3 subject to equations 5, 6, and 8 (B_Q drops out of the third and fourth terms of this expression).

$$L = \lambda[P_H - c_Q x_H] + \lambda[P_L - c_Q x_L] + \mu_1(\gamma_Q \theta_H V(x_H) - P_H - \gamma_Q \theta_H V(x_L) + P_L)$$

$$+ \mu_2(\gamma_Q \theta_L V(x_L) - P_L - \gamma_Q \theta_L V(x_H) + P_H) + \mu_3(B_Q + \gamma_Q \theta_L V(x_L) - P_L) \qquad \textbf{(1)}$$

The Kuhn Tucker conditions for this problem in terms of x_H, x_L, P_H and P_L are:

$$\frac{\partial L}{\partial x_H} = \lambda(-c_Q) + \mu_1 \gamma_Q \theta_H V'(x_H) - \mu_2 \gamma_Q \theta_L V'(x_H) \leq 0$$

$$\textit{with complementary slackness condition } \frac{\partial L}{\partial x_H} x_H = 0 \qquad \textbf{(2)}$$

$$\frac{\partial L}{\partial x_L} = (1-\lambda)(-c_Q) - \mu_1 \gamma_Q \theta_H V'(x_L) + \mu_2 \gamma_Q \theta_L V'(x_L) + \mu_3 \gamma_Q \theta_L V'(x_L) \leq 0$$

$$\textit{with complementary slackness condition } \frac{\partial L}{\partial x_L} x_L = 0 \qquad \textbf{(3)}$$

$$\frac{\partial L}{\partial P_H} = \lambda - \mu_1 + \mu_2 \leq 0$$

$$\textit{with complementary slackness condition } \frac{\partial L}{\partial P_H} P_H = 0 \qquad \textbf{(4)}$$

$$\frac{\partial L}{\partial P_L} = 1 - \lambda + \mu_1 - \mu_2 - \mu_3 \leq 0$$

$$\textit{with complementary slackness condition } \frac{\partial L}{\partial P_L} P_L = 0 \qquad \textbf{(5)}$$

If positive warranty coverage is sold to the high type consumer, Equation 2 implies that $\mu_1 > 0$, because $\gamma_Q \theta_H V'(x_H) = \gamma_Q \theta_H (1 - x_H) > 0$ for the allowable range of x_H. Assuming non-zero prices, I add conditions 3 and 4 to obtain:

$$\lambda - \mu_1 + \mu_2 + 1 - \lambda + \mu_1 - \mu_2 - \mu_3 = 0 \quad \therefore \ \mu_3 = 1 \qquad \textbf{(6)}$$

The next step in this solution is to prove that equation 6 in the main text is not binding (i.e. $\mu_2 = 0$). First, add equations 5 and 6 in the main text:

$$\gamma_Q \, \theta_H V(x_H) - \gamma_Q \, \theta_H V(x_L) \geq \gamma_Q \, \theta_L V(x_H) - \gamma_Q \, \theta_L V(x_H) \qquad \textbf{(7)}$$

Evaluating the buyer utility function at x, we know that:

$$\gamma_Q \theta_T V(x) = \gamma_Q \, \theta_T \left[\frac{1-(1-x)^2}{2} \right] \qquad \textbf{(8)}$$

Substituting into equation 7 and simplifying, we obtain:

$$(\theta_H - \theta_L) \left[\frac{1-(1-x_H)^2}{2} - \frac{1-(1-x_L)^2}{2} \right] \geq 0 \qquad \textbf{(9)}$$

$$(\theta_H - \theta_L) \left[\frac{(1-x_L)^2 - (1-x_H)^2}{2} \right] \geq 0 \qquad \textbf{(10)}$$

Because $\theta_H - \theta_L > 0$, equation 10 implies that x_H is greater than or equal to x_L. We now rewrite constraint 5 from the main text:

$$P_H - P_L \geq \gamma_Q \, \theta_H V(x_H) - \gamma_Q \, \theta_H V(x_L) \qquad \textbf{(11)}$$

$$\therefore \ P_H - P_L \geq \gamma_Q \, \theta_H \int_{x_L}^{x_H} V'(x) \, dx \qquad \textbf{(12)}$$

But $V'(x) = 1 - x$ which implies that: $P_H - P_L \geq \gamma_Q \, \theta_H \int_{x_L}^{x_H} 1 - x \, dx$. Therefore,

$P_H - P_L > \gamma_Q \, \theta_L \int_{x_L}^{x_H} 1 - x \, dx$ strictly. This implies that

$$(P_H - P_L) > \gamma_Q \, \theta_L V(x_H) - \gamma_Q \, \theta_L V(x_L) \Rightarrow \gamma_Q \, \theta_L V(x_L) - P_L > \gamma_Q \, \theta_L V(x_H) - P_H \qquad \textbf{(13)}$$

Thus, $(x_L, P_L) \neq (x_H, P_H)$ and it is optimal to offer a different bundle to each type. Q.E.D.

Proof of Proposition 2

When quality is observable, the profit maximising menu for the seller has:

(c) A bundle designed for the high valuation buyer with warranty protection of efficient length and a price which leaves her with strictly positive utility.

(d) A bundle for the low valuation buyer with a warranty that is shorter than the efficient length and a price which leaves her indifferent between buying or not.

Equation 13 of this technical appendix implies that $\mu_2 = 0$. Therefore equation 4 implies that $\mu_1 = \lambda$. Substituting these values into equation 2 we obtain:

$$0 = \lambda(-c_Q) + \lambda \gamma_Q \, \theta_H V'(x_H), \because V'(x_H) = 1 - x_H \Rightarrow x_H = 1 - \frac{c_Q}{\gamma_Q \theta_H} \qquad \textbf{(14)}$$

This is the socially optimal length of warranty protection for the high-type buyer (this can be checked by setting the marginal benefit of warranty protection equal to the marginal cost of providing it). Substituting into equation 3 we obtain:

$$0=(1-\lambda)(-c)-\lambda\gamma_Q\theta_H V'(x_L)+\theta_L\gamma_Q V'(x_L), \because V'(x_L)=1-x_L \Rightarrow x_L=1-\frac{(1-\lambda)c_Q}{\gamma_Q\theta_L-\lambda\gamma_Q\theta_H}$$

(15)

It is easy to show that this is less than the socially optimal length for the low type buyer x_L* where: $x_L^*=1-\frac{c_Q}{\gamma_Q\theta_L}$. (Note: using equation 15, $x_L>0$ when $\lambda<(\gamma_Q\theta_L-c_Q)/(\gamma_Q\theta_H-c_Q)$. See footnote 10 in the main text). Because $\mu_3=1$, equation 8 in the main text binds. Thus, the low-type buyer is indifferent between buying and not buying. Using equation 8 in the main text:

$B_Q+\gamma_Q\theta_L V(x_L)-P_L=0 \Rightarrow B_Q+\gamma_Q\theta_H V(x_L)-P_L>0$ strictly but equation 5 implies that

$B_Q+\gamma_Q\theta_H V(x_H)-P_H \geq B_Q+\gamma_Q\theta_H V(x_L)-P_L \;\therefore\; B_Q+\gamma_Q\theta_H V(x_H)-P_H>0$ strictly. This means that the high type buyer is left with strictly positive surplus. Q.E.D.

Explicit Expression for Prices

Substituting the value for x_L in equation 15 into equation 8 in the main text, we obtain:

$$P_L=B_Q+\frac{\gamma_Q\theta_L}{2}\left[1-\frac{(1-\lambda)^2c_Q^2}{\gamma_Q^2(\theta_L-\lambda\theta_H)^2}\right] \qquad \textbf{(16)}$$

Because $\mu_1=\lambda$, equation 5 in the main text holds with strict equality i.e.

$B_Q+\gamma_Q\theta_H V(x_H)-P_H=B_Q+\gamma_Q\theta_H V(x_L)-P_L$. Substituting the values of x_H, x_L and P_L that obtain from equations 14, 15, and 16 respectively, we can use this equation to derive:

$$P_H=B_Q+\frac{\gamma_Q\theta_H}{2}\left[\frac{(1-\lambda)^2c_Q^2}{\gamma_Q^2(\theta_L-\lambda\theta_H)^2}-\frac{c_Q^2}{\gamma_Q^2\theta_H^2}\right]+\frac{\gamma_Q\theta_L}{2}\left[1-\frac{(1-\lambda)^2c_Q^2}{\gamma_Q^2(\theta_L-\lambda\theta_H)^2}\right] \qquad \textbf{(17)}$$

To obtain the explicit expression for seller profit, the values of x_H, x_L, P_L, and P_H are substituted into the objective function given by equation 3 in the main text.

Proof of Proposition 3

When product quality is unobservable:

(c) and the fixed benefit B_P of a premium quality product is less than $\hat{B}$, the warranty/price menu offered by a seller of premium quality is identical to the one offered when product quality is observable.

(d) and the fixed benefit B_P of a premium quality product is greater than $\hat{B}$, the price/warranty menu offered by a seller of high quality is different from the one offered when product quality is observable.

The problem for the premium quality seller when his quality is unobservable is analogous to the constrained optimisation problem of equations 3,5,6,7 and 8 with the added constraint of equation 11 (all in the main text). The Lagrangean function for this problem is:

$$L = \lambda[P_H - c_P x_H] + \lambda[P_L - c_P x_L] + \mu_1(\gamma_P \theta_H V(x_H) - P_H - \gamma_P \theta_H V(x_L) + P_L)$$
$$+ \mu_2(\gamma_P \theta_L V(x_L) - P_L - \gamma_P \theta_L V(x_H) + P_H) + \mu_3(B_Q + \gamma_P \theta_L V(x_L) - P_L) \quad \textbf{(18)}$$
$$+ \mu_4[\pi_S^{max} - \lambda(P_H - x_H c_S) - (1-\lambda)(P_L - x_L c_S)]$$

where: $\pi_S^{MAX} = B_S + \frac{\lambda\gamma_S\theta_H}{2}\left[\frac{(1-\lambda)^2 c_S^2}{\gamma_S^2\tilde{\theta}^2} - \frac{c_S^2}{\gamma_S^2\theta_H^2}\right] + \frac{\gamma_S\theta_L}{2}\left[1 - \frac{(1-\lambda)^2 c_S^2}{\gamma_S^2\tilde{\theta}^2}\right] - c_S + \frac{(1-\lambda)^2 c_S}{\gamma_S\tilde{\theta}} + \frac{c_S^2\lambda}{\gamma_S\theta_H}$

The Kuhn Tucker conditions for this problem in terms of x_H, x_L, P_H and P_L are:

$$\frac{\partial L}{\partial x_H} = \lambda(-c_P) + \mu_1\gamma_P\theta_H V'(x_H) - \mu_2\gamma_P\theta_L V'(x_H) + \mu_4\lambda c_S \le 0$$
$$\textit{with complementary slackness condition } \frac{\partial L}{\partial x_H} x_H = 0 \quad \textbf{(19)}$$

$$\frac{\partial L}{\partial x_L} = (1-\lambda)(-c_P) - \mu_1\gamma_P\theta_H V'(x_L) + \mu_2\gamma_P\theta_L V'(x_L) + \mu_3\gamma_P\theta_L V'(x_L) + \mu_4(1-\lambda)c_S \le 0$$
$$\textit{with complementary slackness condition } \frac{\partial L}{\partial x_L} x_L = 0 \quad \textbf{(20)}$$

$$\frac{\partial L}{\partial P_H} = \lambda - \mu_1 + \mu_2 - \mu_4\lambda \le 0$$
$$\textit{with complementary slackness condition } \frac{\partial L}{\partial P_H} P_H = 0 \quad \textbf{(21)}$$

$$\frac{\partial L}{\partial P_L} = 1 - \lambda + \mu_1 - \mu_2 - \mu_3 - \mu_4(1-\lambda) \le 0$$
$$\textit{with complementary slackness condition } \frac{\partial L}{\partial P_L} P_L = 0 \quad \textbf{(22)}$$

For any λ, c_P, γ_P, θ_H, and θ_L, there are unique warranty lengths (x_L and x_H) in the optimal menu under observability (equation 14 and 15). Only the prices (P_L and P_H) under observability (equations 16 and 17) depend on B_P. Using the no-mimic condition, we find a unique B_P ($\hat{B}$) given λ, c_P, γ_P, θ_H, and θ_L and π_S^{max} that solves

$\pi_S^{max} = \lambda(P_H - x_H c_S) + (1-\lambda)(P_L - x_L c_S)$ where x_L, x_H, P_L, and P_H are given by equations 14, 15, 16 and 17.

$$\hat{B} = \pi_S^{MAX} + c_S - \frac{\gamma_P\theta_L}{2} - (2c_S - c_P)\left(\frac{(1-\lambda)^2 c_P}{2\gamma_P\tilde{\theta}} + \frac{\lambda c_P}{2\gamma_P\theta_H}\right) \textit{ where } \tilde{\theta} = \theta_L - \lambda\theta_H \quad \textbf{(23)}$$

Whenever B_P exceeds $\hat{B}$, we know that the no mimic constraint is violated if the premium quality seller chooses x_L, x_H, P_L, and P_H as given by equations 14, 15, 16 and 17 because P_L, and P_H are strictly increasing functions of B_P. Conversely, when B_P is less than $\hat{B}$, the no mimic constraint will not be violated and μ_4=0. In this situation, the no-mimic constraint does not bind and the premium quality seller's menu is *unaffected* by the presence of standard quality.

When $B_P > \hat{B}$, $\mu_4 > 0$ and $\pi_S^{max} = \lambda(P_H^P - x_H^P c_S) + (1-\lambda)(P_L^P - x_L^P c_S)$. Note in this case, the superscript P indicates that the premium quality seller chooses a menu that is *different* from the

menu he chooses when quality is observable. Q.E.D.

Proof of Proposition 4

When product quality is unobservable and $B_P > \hat{B}$, the premium quality seller's menu involves more warranty protection and higher prices for all buyers (than the menu used when quality is observable). Additionally, extended warranties (i.e. $x_H - x_L$) are shorter when product quality is unobservable.

To simplify notation, we dispense with the superscript P introduced above. We now derive the optimal menu assuming that quality is unobservable. We add equation 21 and 22 to obtain:

$$1 - \mu_3 - \mu_4 = 0 \therefore \mu_4 = 1 - \mu_3 \qquad \textbf{(24)}$$

As with all Kuhn Tucker problems, the Lagrangean multipliers are restricted to non-negative values. Therefore, equation 24 implies that $\mu_4 \in [0,1]$ and we rewrite equation 21, $\{\lambda(1-\mu_4)+\mu_2\} - \mu_1 = 0$. The term in braces is clearly non-negative, therefore $\mu_1 \geq 0$. Unless $\mu_4=1$, $\mu_1>0$ strictly (we will show later that the maximum value for μ_4 is $c_P/c_S<1$).

Assuming $\mu_1>0$, we now prove that $\mu_2=0$. We add the incentive compatibility constraints (equation 5 and 6 in the main text) to obtain:

$B_P + \gamma_P \theta_H V(x_H) - P_H + B_P + \gamma_P \theta_L V(x_L) - P_L \geq B_P + \gamma_P \theta_H V(x_L) - P_L + B_P + \gamma_P \theta_L V(x_H) - P_H$

$\Rightarrow \gamma_Q(\theta_H - \theta_L)[V(x_H) - V(x_L)] > 0$. Since $\gamma_Q > 0$ *and* $\theta_H - \theta_L > 0 \therefore V(x_H) - V(x_L) \geq 0$. This implies that $x_H \geq x_L$. Since $\mu_1>0$, equation 5 in the main text holds with strict equality. Simplifying this equation and rearranging we obtain: $\gamma_Q \theta_H [V(x_H) - V(x_L)] = P_H - P_L$. Therefore, $\gamma_Q \theta_L [V(x_H) - V(x_L)] < P_H - P_L$ strictly $\Rightarrow B_Q + \gamma_Q \theta_L V(x_L) - P_L > B_Q + \gamma_Q \theta_L V(x_H) - P_H$ strictly and this proves that $\mu_2=0$.

Since $\mu_2=0$ and $\mu_4 = 1 - \mu_3$, equation 22 implies that $\mu_1=\lambda\mu_3$. We can now substitute these values into equation 19 to obtain an expression for μ_3 in terms of x_H.

$$\mu_3 = \frac{c_P - c_S}{\gamma_P \theta_H (1 - x_H) - c_S} \qquad \textbf{(25)}$$

We can now use equation 23 and the identity $\mu_1=\lambda\mu_3$, to write expressions for μ_1 and μ_4 in terms of x_H.

$$\mu_1 = \frac{\lambda(c_P - c_S)}{\gamma_P \theta_H (1 - x_H) - c_S}, \quad \mu_4 = \frac{\gamma_P \theta_H (1 - x_H) - c_P}{\gamma_P \theta_H (1 - x_H) - c_S} \qquad \textbf{(26)}$$

Because $\mu_1>0$, $\gamma_P \theta_H (1 - x_H) - c_S < 0$ because $\lambda(c_P - c_S) < 0$ by definition. Therefore,

$$\because \mu_4 > 0 \Rightarrow \gamma_P \theta_H (1 - x_H) - c_P < 0 \Rightarrow x_H > 1 - \frac{c_P}{\gamma_P \theta_H} \qquad \textbf{(27)}$$

Thus, x_H is strictly longer than the efficient length when $\mu_4>0$. Equations 20 can be used to derive an expression for x_L as a function of x_H:

$$x_L = 1 - \frac{(1-x_H)(1-\lambda)\theta_H}{\tilde{\theta}} \qquad \textbf{(28)}$$

Note that when the efficient length of x_H (equation 14) is substituted into equation 25, the value of x_L is equivalent to the value of x_L in the menu when quality is observable (equation 15). Since x_H is longer than the efficient length (equation 27), x_L is longer than the warranty length offered to the low type when quality is observable. Because μ_1 and μ_3 are greater than zero equations 5 and 8 in the main text hold strictly and we use these to express P_H and P_L as functions of x_H.

$$P_L = B_P + \frac{\gamma_P \theta_L}{2} - \frac{\gamma_P \theta_L \theta_H^2}{2\tilde{\theta}^2}(1-\lambda)^2(1-x_H)^2 \qquad \textbf{(29)}$$

$$P_H = B_p + \frac{\gamma_P \theta_L}{2} + \left[(\theta_H - \theta_L)\frac{\gamma_P \theta_H^2}{2\tilde{\theta}^2}(1-\lambda)^2 - \frac{\gamma_P \theta_H}{2}\right](1-x_H)^2 \qquad \textbf{(30)}$$

Both P_L and P_H are decreasing functions of x_H in the range (0,1). When the efficient length of x_H is substituted into equations 29 and 30, the expressions reduce to equations 16 and 17 (the prices when quality is observable). Since x_H is longer then the efficient length, P_L and P_H are greater than the prices observed when quality is observable.

Using equation 28, $x_H - x_L = \frac{(\theta_H - \theta_L)(1-x_L)}{(1-\lambda)\theta_H}$. If we take the derivative of this expression with respect to x_L, we obtain $\frac{\partial(x_H - x_L)}{\partial x_L} = \frac{\theta_H - \theta_L}{(\lambda-1)\theta_H} < 0$. Hence, since x_L is longer than the warranty length offered to the low type buyer when quality is observable, the extended warranty is shorter. Q.E.D.

Note: to solve for x_L, x_H, P_L, and P_H explicitly, substitute equations 28, 29, and 30 into the no-mimic constraint (equation 11 in the main text) to obtain a quadratic expression in x_H.

Proof of Proposition 5

When product quality is unobservable and the fixed benefit for a premium quality product exceeds $B_{MAX} = \pi_S^{MAX} + c_S - \frac{\gamma_P \theta_L}{2}$, *signaling considerations dominate screening considerations and a seller of premium quality offers the maximum warranty length to all buyers.*

Step 1 When x_H=1, show that x_L=1 and $P_H = P_L = B_p + \frac{\gamma_P \theta_L}{2}$. When μ_4>0, then equation 28 implies that x_L=1. Substituting x_H=1 into equations 29 and 30 produces the desired result.

Step 2 The maximal mimicking cost is placed on a standard quality seller by choosing a warranty length of 1.

Step 3 The value of B_P which necessitates maximal signalling can be determined by substituting equations 28, 29, and 30 into the no-mimic constraint (equation 11 in the main text), setting x_H=1 and solving for B_P. We call this expression $B_{MAX} = \pi_S^{MAX} + c_S - \frac{\gamma_P \theta_L}{2}$. Q.E.D.

Note: When B_P=B_{MAX}, the values of the Lagrangean multipliers are $\mu_1 = \frac{\lambda(c_S - c_P)}{c_S}$, $\mu_2 = (1-\lambda)\frac{c_P}{c_S}$, $\mu_3 = \frac{c_S - c_P}{c_S}$, $\mu_4 = \frac{c_P}{c_S}$. Thus, the maximum value of μ_4 where the constrained optimization problem yields an interior solution is c_P/c_S.

When B_P>B_{MAX}, the individual rationality constraint does not bind for either consumer type and the optimal price is found by solving $\pi_S^{MAX} = P - c_S$. This is simply the reduced form of equation 11 in the main text when (x_L, P_L)= (x_H, P_H).