Project Management Under Risk: Using the Real Options Approach to Evaluate Flexibility in R&D

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Abstract

Managerial flexibility has value in the context of uncertain R&D projects, as management can repeatedly gather information about uncertain project and market characteristics and, based on this information, change its course of action. This value is now well-accepted and referred to as "real option value". We introduce, in addition to the familiar real option of abandonment, the option of corrective action that management can take during the project. The intuition from options pricing theory is that higher uncertainty in project payoffs increases the real option value of managerial decision flexibility. However, R&D managers face uncertainty not only in payoffs, but also from many other sources. We identify five example types of R&D uncertainty, in market payoffs, project budgets, product performance, market requirements, and project schedules. How do they influence the value from managerial flexibility? We find that if uncertainty is resolved or costs/revenues occur *after* all decisions have been made, more variability may "smear out" contingencies and thus reduce the value of flexibility. In addition, variability may reduce the probability of flexibility ever being exercised, which also reduces its value. This result runs counter to established option pricing theory intuition and contributes to a better risk management in R&D projects. Our model builds intuition for R&D managers as to when it is and when it is not worthwhile to delay commitments, for example, by postponing a design freeze, thus maintaining flexibility in R&D projects.

Keywords: Real options, R&D projects, project evaluation, decision trees, stochastic dynamic programming, managerial flexibility, project management.

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1 Introduction and Literature Overview

Most investment decisions (and R&D projects in particular) are characterized by irreversibility and uncertainty about their future rewards: once money is spent, it can not be recovered if the payoffs hoped for do not materialize. However, a firm usually has some leeway in the *timing* of the investment: it has the right but not the obligation to buy an asset (e.g., access to a profitable market in the case of an R&D project) at some future time of its choosing, and thus, it is holding an option, analogous to a financial call option (Dixit and Pindyck 1994). As new information arrives and uncertainty about the investment's rewards is gradually resolved, management often has the flexibility to alter the initial operating strategy adopted for the investment. As with options on financial securities, this flexibility to adapt in response to new information enhances the investment opportunity's value by improving its upside potential while limiting downside losses relative to the initial expectations (Trigeorgis 1997). Using the analogy with options on financial assets, such investment flexibility is often called a "real option." A real option may significantly enhance the value of an investment (e.g., Kogut and Kulatilaka 1994).

This flexible decision structure of options is valid in an R&D context: after an initial investment, management can gather more information about project progress and market characteristics and, based on this information, change its course of action (e.g., Dixit and Pindyck 1994, Lint and Pennings 1997). The real option value of this managerial flexibility enhances the R&D project value; a pure net present value analysis understates the value. Five basic sources of flexibility have been identified (e.g., Trigeorgis 1997): A *defer option* refers to the possibility of waiting until more information has become available. An *abandonment option* offers the possibility to make the investment in stages, deciding at each stage, based on the newest information, whether to proceed further or whether to stop (this is applied by venture capitalists). An *expansion or contraction option* represents the possibility to adjust the scale of the investment (e.g., a production facility) depending

on whether market conditions turn out favorably or not. Finally, a *switching option* allows changing the mode of operation of an asset, depending on factor prices (e.g., switching the energy source of a power plant, or switching raw material suppliers).

One key insight generated by the real options approach to investment is that *higher uncertainty in the payoffs of the investment increases the value of managerial flexibility, or the value of the real option* (Dixit and Pindyck 1994, p. 11; this was also shown by Roberts and Weitzman (1981) in a sequential decision model without referring to real options at all). The intuition is clear – with higher payoff uncertainty, flexibility has a higher potential of enhancing the upside while limiting the downside. An important managerial implication of this insight is that the more uncertain the project payoff is, the more efforts should be made to delay commitments and maintain the flexibility to change the course of action. This intuition is appealing. Yet, there is hardly any evidence of real options pricing of R&D projects in practice (see Smith and McCardle 1998; this is confirmed in our conversations with R&D managers) despite reports that Merck uses the method (Sender 1994). Moreover, there is recent evidence that more uncertainty may *reduce* the option value if an alternative "safe" project is available (Kandel and Pearson 1998).

We view this evidence as a gap between the financial payoff variability, as addressed by the real options pricing literature, and operational uncertainty faced at the level of R&D management. For example, R&D project managers encounter uncertainty about budgets, schedules, product performance, or market requirements, in addition to financial payoffs. The relationship between such operational uncertainty and the value of managerial flexibility (option value of the project) is not clear. For example, should the manager respond to increased uncertainty about product performance in the same way as to uncertainty about project payoffs, by delaying commitments?

The first contribution of this article lies in connecting these operational sources of uncertainty to the real option value of managerial flexibility. In a simple model, we demonstrate

that operational uncertainty (in particular, uncertainty in product performance, market requirements and schedule adherence) may *reduce* the real option value. We interpret this counter-intuitive result in terms of when the underlying uncertainty is resolved: if operational uncertainty is resolved *before* decisions are made and costs or revenues are incurred, flexibility can be applied in order to protect the project against a downside. In this case, more uncertainty enhances the option value of managerial flexibility. However, if operational uncertainty is resolved *after* decisions are made, or if it reduces the probability that flexibility is useful, more variability reduces the ability to respond and, thus, diminishes the option value of flexibility.

As a second contribution, we extend the usual taxonomy of types of real options (delay, abandon, contract, expand, switch) by "improvement." Mid-course actions during R&D projects to improve the performance of the product (or to correct its targeting to market needs) are commonly used. The availability of such improvement actions represent an additional source of option value.

The literature on real options is quite extensive – readers are referred to textbooks such as Dixit and Pindyck (1994) or Trigeorgis (1997) for overviews. Most applications of real options have been in the area of commodities (such as oil exploration) because financial markets are well developed in this environment and allow to replicate risks by traded assets. Recently, research has been carried out on the application of real options pricing to R&D projects (e.g., Brennan and Schwartz 1985, Faulkner 1996, McDonald and Siegel 1985, Mitchell and Hamilton 1988, Teisberg 1994).

2 Five Types of Operational Uncertainty

Figure 1 shows a simple conceptual picture of the drivers of project value: an R&D project is characterized by its lead time, its cost over time, and the resulting product performance.

The market is characterized by its payoff from the project (caused by market size and attractiveness) and by its performance requirements, indicating how the payoff increases with product performance. Project and market characteristics together determine the project value. Formally, we can express this as follows: A project's value V is a function of five "value drivers" which will be further defined in Section 3:

$$V = f(\text{performance, cost, time, market requirement, market payoff}). \tag{1}$$

Real options theory has shown that *uncertainty* in the market payoff enhances the project value V if management has the flexibility to respond to contingencies. It creates option value in the presence of uncertainty because it can eliminate the payoff downside while retaining the benefits of the upside. This is known to R&D managers (although rarely formally valued): when the market potential of a project is unknown, managers strive to *delay* decisions in order to be able to react to new market information, and they know that this flexibility has value (e.g., delaying the specification freeze or the commitment to an engineering change, Bhattacharya *et al.* 1999 or Terwiesch *et al.* 1999).

The question we examine in this article is whether this insight holds as well for uncertainty in the other value drivers in Equation (1). Each of the five drivers is typically characterized by uncertainty, which is graphically represented in Figure 1. Uncertainty corresponds to *stochastic variability* of parameter distributions, and in the remainder of this article, we use uncertainty and (stochastic) variability interchangeably.

1. *Market Payoff Variability.* The market payoff (e.g., price and sales forecast) depends on uncontrollable factors such as competitor moves, demographic changes, substitute products, etc. It has, therefore, a significant random (unforeseeable) component.
2. *Budget Variability.* This refers to the fact that the running development costs of the project are not entirely foreseeable. Budget overruns are common, and less frequently, under-budget completion also occurs.

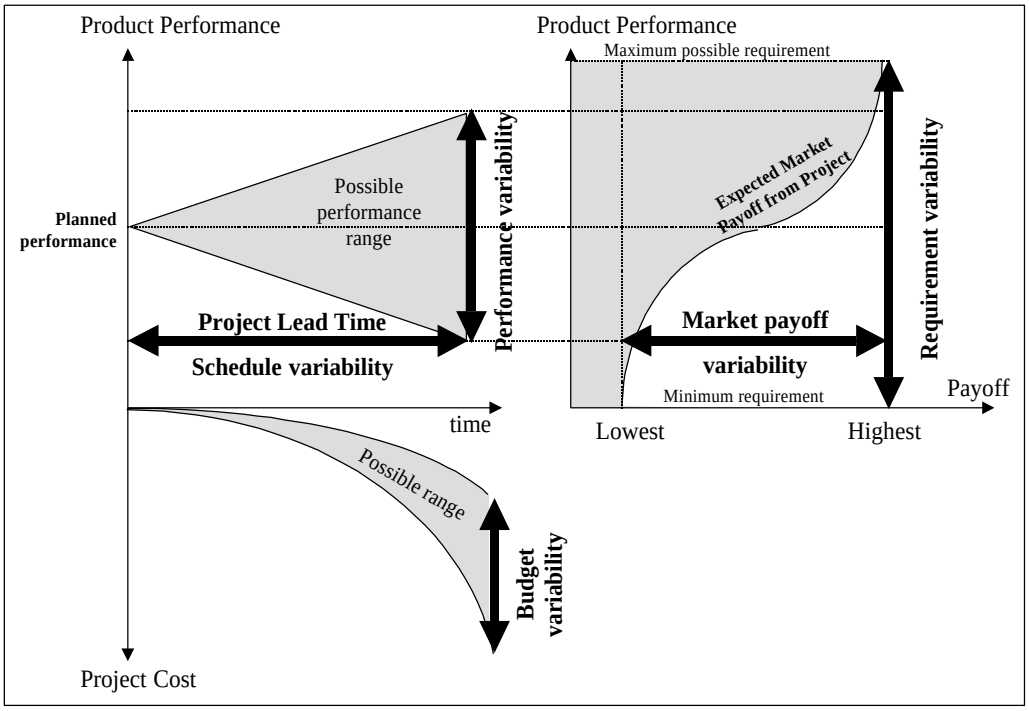

Figure 1: Five Types of Operational Variability

3. *Performance Variability.* This corresponds to uncertainty in the performance of the product being developed. Initially targeted performance can often not be fully achieved, as tradeoffs must be resolved among multiple technical criteria, which together determine performance in the customer's eye. The greater the technical novelty of a product, the higher is this uncertainty (Roussel *et al.* 1991).
4. *Market Requirement Variability.* This corresponds to uncertainty about the performance level required by the market. Performance targets for a product are often only imperfectly known (especially for conceptually new products, see Chandy and Tellis 1998 or O'Connor 1998).
5. *Schedule Variability.* Project may finish unpredictably ahead of or behind schedule. In the latter case, reduced market payoffs (in terms of market share or prices) may result, as empirical work shows (Datar *et al.* 1997).

The influence of variability in these operational drivers, in addition to variability in market payoffs, on the value of managerial flexibility has not been examined. It is important to understand the impact of operational drivers because often, different functional managers in an organization are responsible for the different drivers. For example, a project manager may control project cost and time, and product performance, a marketing manager may be in charge of understanding and influencing performance requirements, and a finance manager may be responsible for the budget approval. It is important for them to understand in which cases managerial flexibility creates value. Only then is it worth postponing commitments to maintain flexibility. After setting up our basic model in Section 3, we show in subsections 4.1 and 4.2 that increased variability in market payoffs, as well as in budgets, may indeed enhance the option value of managerial flexibility, consistent with option pricing theory. The other types of operational variability, however, may have the effect of *reducing* the value of flexibility, as we show in subsections 4.3 to 4.5.

3 The Basic Model

3.1 Contingent Claims Analysis

The real option value of managerial flexibility can be evaluated using contingent claims analysis, developed for pricing options in financial markets. This approach, however, requires a complete market of risky assets capable of *exactly*[1] replicating the project's risk by the stochastic component of some traded asset (Dixit and Pindyck 1994, p. 121). Such replicability often does not apply in R&D projects, whose risks are typically idiosyncratic and uncorrelated with the financial markets. Merton (1998) proposes an approximation, where a dynamically traded asset portfolio is used to *track* the project value as closely as

[1]Here, "exactly" means for every sample path of the realization of the uncertainty.

possible. The approximating portfolio can then be used to derive an option value from the financial markets. This is complex and beyond the scope of this article, as the key parameter continuously tracked during the project is product performance (see below), which is a non-financial parameter.

We, therefore, revert to an equivalent approach to option evaluation, dynamic programming (Dixit and Pindyck 1994, p. 7, Smith and Nau 1995), which does not require asset replication. Thus, we develop in this section a dynamic programming model of an R&D investment.[2]

The drawback of the dynamic programming approach is that it does not address the question of the correct risk-adjusted discount rate. Dynamic programming requires an exogenously specified discount rate that reflects the decision-maker's risk attitude. However, the risk of an R&D project is typically due to factors unique to this project and thus *unsystematic* or *diversifiable*. Therefore, a rational investor can diversify the project risk away by holding a portfolio of securities without requiring a risk premium. A reasonable assumption for a large firm is, therefore, a risk-neutral attitude toward the project with discounting at the risk-free rate (Trigeorgis 1997, p. 43). All our results are insensitive to the discount rate assumed.

3.2 A Dynamic Programming Model of an R&D Project

Consider an R&D project proceeding in T discrete stages (corresponding to regular design reviews) toward market introduction. The market success is determined by the product performance, which is modeled by a one-dimensional parameter i, such as processor speed in a computer, or in the case of a multi-attribute product, the customer utility derived

[2]Smith and McCardle 1998 propose an "integrated" approach for oil exploration projects, where they use option pricing for risks that can be replicated in the market and dynamic programming for risks that cannot be priced.

from a conjoint analysis (see, e.g., Aaker and Day 1990). The project is subject to uncertainty stemming from the market and from technical development risk. Performance uncertainty manifests itself in the *variability* of a probability distribution. A distribution is said to exhibit higher variability than a second distribution if both have the same mean and the former has a higher variance. This definition corresponds to Rothschild and Stiglitz's (1970) definition of higher risk. Focusing on variability distinguishes changes in distribution means from changes in risk.

Performance variability causes the product performance i to "drift" between review periods of the project. The state of the system is characterized by (i, t), the level of product performance i at project review t. From the viewpoint of an R&D manager, (i, t) signifies "expected final product performance given information at review t". R&D teams commonly perform design reviews, where expected performance is estimated based on tests, simulations or prototypes (e.g., Thomke 1998). For example, a prototype test may reveal whether a chip is stable at a certain clock speed and provide an estimate of achievable speed at market introduction. Such a test may also trigger corrective action, which is discussed below. Typically, reviews are not directed toward an estimate of the value of maintaining further flexibility, depending on the newest information about market requirements or project state. This is modeled below.

The performance drift follows a binomial distribution in each period, independent of the previous history of project progress. From period t to the next period, the performance may unexpectedly improve with probability p, or it may deteriorate with probability $(1-p)$ due to unexpected adverse events. We generalize the binomial distribution by allowing the performance improvement and deterioration, respectively, to be "spread" over the next N

performance states with transition probabilities.

$$p_{ij} = \begin{cases} \frac{p}{N} & \text{if } j \in \{i + \frac{1}{2}, \ldots, i + \frac{N}{2}\} \\ \frac{1-p}{N} & \text{if } j \in \{i - \frac{1}{2}, \ldots, i - \frac{N}{2}\} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The mean of this distribution is $\frac{N+1}{4}(2p-1)+i$, and the variance is $\frac{N+1}{8}[\frac{N}{3}+(N+1)(\frac{1}{3}-\frac{(2p-1)^2}{2})]$. With two parameters, this discrete distribution can approximate the first two moments of a range of continuous distributions. Moreover, this approximation leads to a recombining lattice tree model, which reduces the size of the state space and, thus, computational complexity. If $p = 0.5$ (a particularly relevant case for the analysis below), N characterizes the variability of the product performance. The state space of product performance over two periods is illustrated in Figure 2, in which the left section corresponds to the transition probabilities (2).

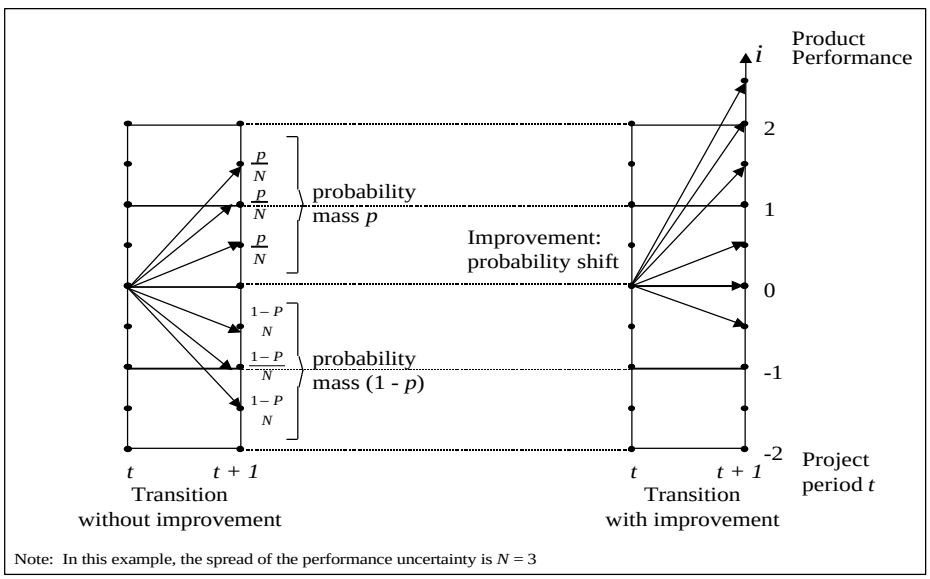

Figure 2: Transition Probabilities of Product Performance

If $p = 0.5$, the expected performance state for the product launch is $Ei = 0$ at time zero, which means that the project plan is initially unbiased. If $p > 0.5$, the project plan is "pessimistic", and the true expected performance at launch is $Ei > 0$. If $p < 0.5$, the project plan is "optimistic", and the true expected performance is $Ei < 0$.

At each period t, management can take any one of three possible actions: abandon, continue, or improve. The first two options are standard in real option theory. Abandonment terminates the project immediately, cutting any further costs and foregoing any further revenues. Continuation proceeds to the next stage $(t+1)$ at a continuation cost of $c(t)$. The continuation cost usually increases over time for R&D projects: $c(t) \leq c(t+1)$, but this is not required for our results. Over the period, the performance state evolves according to the transition probabilities shown above.

In addition to these two possibilities, management can also choose to take corrective action and inject additional resources in order to improve expected product performance by one level. For example, the gate layout of a processor chip is changed to eliminate cross-line interference at high frequencies. Such improvement imposes an improvement cost of $\alpha(t)$ in addition to the continuation costs. The improvement cost typically also increases over time since engineering changes become more difficult as more of the product design is completed (again, this is not required for our results): $\alpha(t) \leq \alpha(t+1)$. The improvement results in a "mean shift" of the transition probabilities (right section of Figure 2).[3]

$$p_{ij} = \begin{cases} \frac{p}{N} & \text{if } j \in \{i+1+\frac{1}{2}, \ldots, i+1+\frac{N}{2}\} \\ \frac{1-p}{N} & \text{if } j \in \{i+1-\frac{1}{2}, \ldots, i+1-\frac{N}{2}\} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

At the start of the project, an initial investment of I must be made (e.g., to put the project infrastructure in place). Costs and revenues are discounted at the risk-free rate r. Project continuation and improvement costs have to be paid at the beginning of each period.

When the project is launched at time T with a performance level i, it will generate an *expected* market payoff Π_i in the form of an S-curve, that is, Π_i is convex-concave in i

[3]This implies that the improvement can be carried out purely with additional resources (such as engineers, or experimental lab. capacity), without an additional time delay. Time delays, or schedule risk, will be treated separately in subsection 4.5.

(left section of Figure 3). The S-curve is a general market payoff model, which includes linear, convex or concave payoff functions as special cases. It is intuitive to assume that a performance improvement makes little difference when performance is very low (improvement does not save a bad product) or very high (improvement makes little difference for an already great product). Performance matters most for intermediate performance levels (Kalyanaram and Krishnan 1997, Bhattacharya *et al.* 1999).

In particular, the S-curve may be the result of a competitive performance threshold that is not known in advance. In such a case, the market requires a certain level of performance D, dictated by competitive dynamics. If the project meets or exceeds this performance level, the market will yield a premium profit margin M. But if the project misses the target, it must compete on cost, and produces only a smaller margin m (right section of Figure 3). The required market performance is not known to the firm in advance and is resolved only after the product launch. The firm has an efficient forecast in the form of a probability distribution F of D (center in Figure 3).

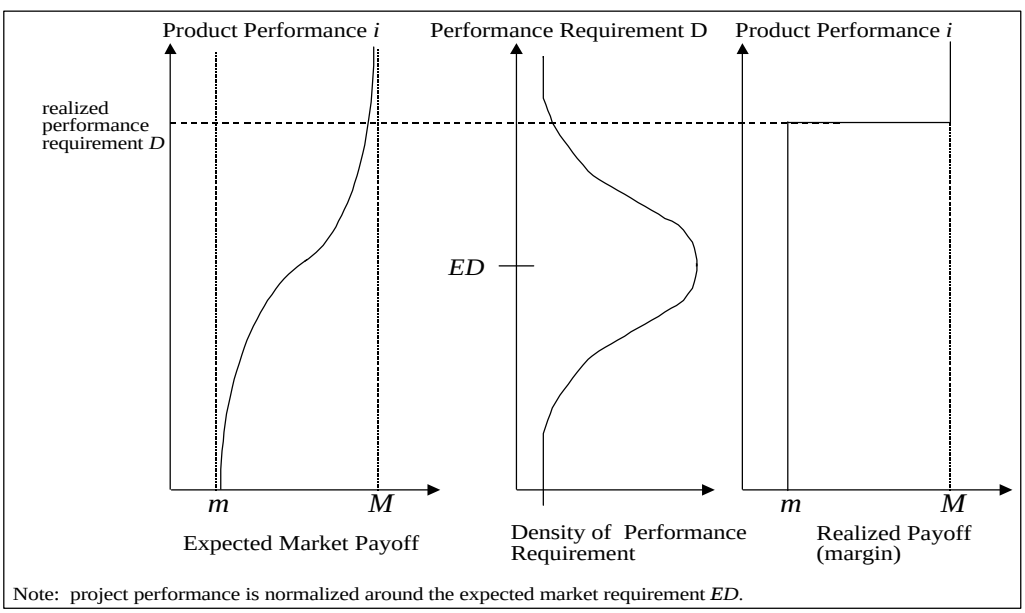

Figure 3: Market Uncertainty and Project Payoffs

Thus, if the project launches a product of performance level i, the *expected* payoff can be

written as $\Pi_i = m + F(i)(M - m)$, where $F(i)$ represents the probability that performance i exceeds the market requirement D. It can easily be shown that for any distribution F with a density that has a single maximum, Π_i is strictly convex-concave increasing in i.[4] For simplicity of exposition, we assume that the mode of F (that is the point where Π turns from convex to concave) is at the expectation ED.[5]

The sequential decision problem resulting from the above described setup can be formulated as a stochastic dynamic program with the following value function, which can be solved with the standard backward recursion:

$$V_i(T) = max \begin{cases} \text{abandon:} & 0; \\ \text{continue:} & -c(T) + \frac{\sum_{j=1}^{N}[p\Pi_{i+j/2} + (1-p)\Pi_{i-j/2}]}{N(1+r)}; \\ \text{improve:} & -c(T) - \alpha(T) + \frac{\sum_{j=1}^{N}[p\Pi_{i+1+j/2} + (1-p)\Pi_{i+1-j/2}]}{N(1+r)} \end{cases} \tag{4}$$

$$V_i(t) = max \begin{cases} \text{abandon:} & 0; \\ \text{continue:} & -c(t) + \frac{\sum_{j=1}^{N}[pV_{i+j/2}(t+1) + (1-p)V_{i-j/2}(t+1)]}{N(1+r)}; \\ \text{improve:} & -c(t) - \alpha(t) + \frac{\sum_{j=1}^{N}[pV_{i+1+j/2}(t+1) + (1-p)V_{i+1-j/2}(t+1)]}{N(1+r)} \end{cases} \tag{5}$$

We can characterize the optimal decision rule, or policy, for this dynamic program. Proposition 1 describes the optimal policy for an increasing and convex-concave Π_i (this includes as special cases Π_i convex or concave).

Proposition 1 *If the payoff function Π_i is convex-concave increasing, the optimal policy in period t is characterized by control limits $L_u(t) \geq L_m(t)$ and $L_d(t)$ (all may be outside the range $[-Nt/2, (N/2+1)t]$) such that it is optimal to:*
choose abandonment if $L_d(t) \geq i$. Otherwise: choose continuation if $i > L_u(t)$, choose improvement if $L_u(t) \geq i > L_m(t)$, and choose continuation if $L_m(t) \geq i$.

[4]For example, Kalyanaram and Krishnan (1997) use an equivalent setup, expressed in terms of fraction of customers questioned who like the design, where F is assumed to be a normal distribution.

[5]This is true, for example, for the normal distribution. Our results are slightly simplified by this assumption, but do not depend on it.

Moreover, the optimal value function $V(t)$ is also convex-concave increasing in i, and $L_u(t)$ lies in its concave region and $L_m(t)$ in its convex region.

Proof

For easier readability of the text, all proofs are shown in the Appendix.

Figure 4 demonstrates the structure of the optimal policy. In the center, where the convex-concave payoff-function is at its steepest, improvement is worthwhile. However, in the flatter regions of the payoff function, the higher payoff does not justify the improvement cost α_t. The lower control L_d cuts off the project whenever the expected payoff (over the $2N$ reachable states) is too low to justify the continuation cost c_t. If Π_i is concave, $L_m = -\infty$, and if Π_i is convex, $L_u = \infty$.

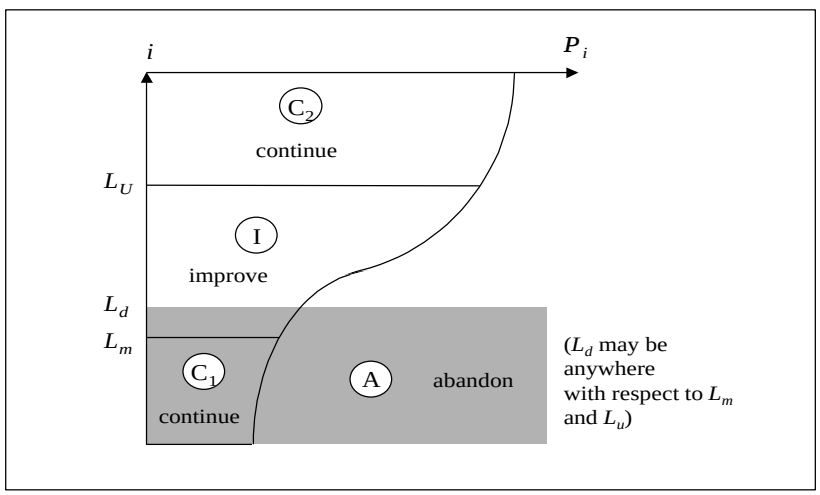

Figure 4: Control Limits of Optimal Policy

Figure 5 demonstrates the policy and the value function on an example. To the right is the market payoff function Π_i. The lattice tree, corresponding to the increasing number of possible states over time, contains the values of the optimal value function. Below each state in the tree, the optimal value function is shown, along with the corresponding decision. In this example, the uncertainty N has been set at 1 for easier exhibition. $V_0(t = 0)$ corresponds to the optimal value of the project before the investment costs of

$I = 50$ are deducted. Below the tree, the optimal project value after deducting I is shown. $V_0(t = 0)$ includes a *compound real option*, namely, the value of the managerial flexibility to choose improvement *or* abandonment in any period.

Along with the optimal project value, two benchmark values are shown: first, the project value resulting from having the possibility to abandon, but not to improve, in each period. It comes as no surprise that it is lower than the optimal value, as it includes an abandonment option only. The second benchmark is the "traditional" net present value (NPV), which corresponds to setting all decisions equal to "C" (continue) and deciding at the beginning if the project should be carried out or not, depending on whether $V_0(0)$ exceeds I. If the project plan is unbiased, this is equivalent to discounting the expectation of the payoffs minus the appropriately discounted continuation costs. At the bottom, finally, the compound option value itself, or the value of the managerial flexibility, is shown (the difference between the optimal project value and the NPV). The example shows that the value of flexibility can be substantial.

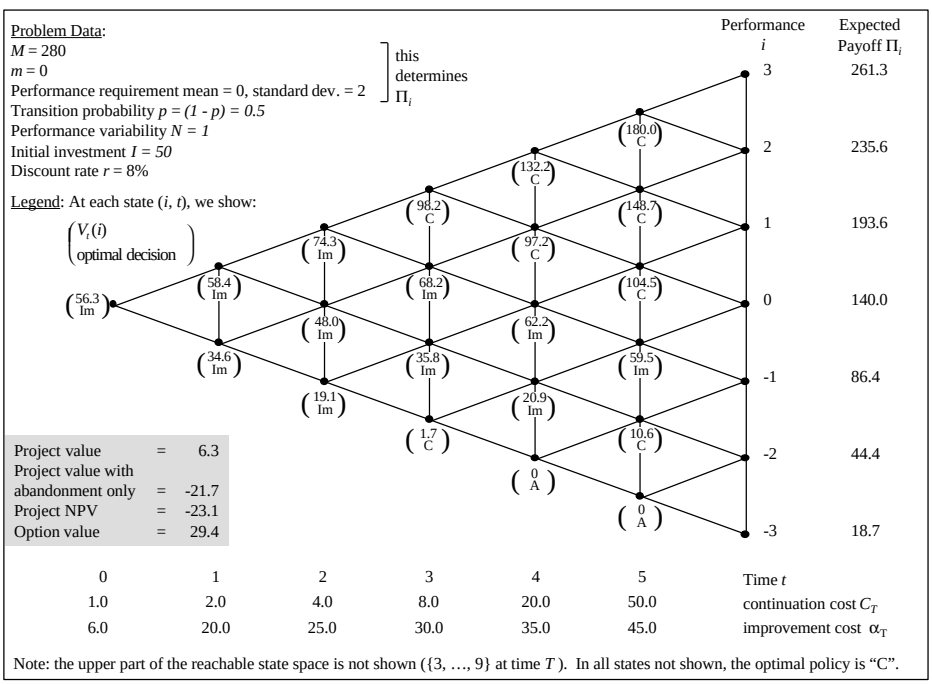

Figure 5: Example of Optimal Policy and Value Function

Improvement represents a source of managerial flexibility different from the standard "expansion" or "contraction", which are strategic options, such as additonal target markets or capacity enlargements. Mid-course improvement in the way of a delayed design freeze, engineering changes, or a change in the project team, poses challenges not for project strategy, but for project execution.

4 Uncertainty and the Value of Flexibility

The value function of the dynamic program specifies function (1). Market payoff is represented by Π, cost by c_t, schedule by T, performance by i, and the market requirement by D. Our question is how the variances (increasing uncertainty) of these drivers influence the value of managerial flexibility. The variances correspond to the price spread $(M - m)$, the variance of c_t, schedule variance ν, transition spread N and requirement variance σ^2, respectively.

4.1 Market Payoff Variability

In the context of our model, payoff variability corresponds to the difference $(M - m)$, holding the average constant. Suppose we have two financial payoff functions Π_i and $\overline{\Pi}_i$, both are convex-concave increasing, and $\overline{\Pi}_i$ exhibits greater variability in the Rothschild-Stiglitz (1970) sense: $\overline{\Pi}_0 = \Pi_0$, and $\overline{\Pi}_i - \Pi_i = \Pi_{-i} - \overline{\Pi}_{-i} \geq 0$ for all i.

Proposition 2 *Assume the project plan is unbiased, that is, $p = 0.5$. Then the option value corresponding to the payoff function with larger variability, $\overline{\Pi}_i$, is larger: $\overline{V}_0(0) \geq V_0(0)$ while the NPV remains unchanged.*

The reader may note that this proposition is valid in the case that Π_i is convex, convex-concave, or concave (that is, the result is independent of where the expected performance

requirement ED is relative to the expected product performance $Ei = 0$). The condition that the project plan be unbiased serves to distinguish variance from mean effects. Consider the case of an optimistic project plan with $p < 0.5$. Then product performance will drift downward over time as the project progresses, and the payoff will be biased toward lower values. If the payoff function has higher variability, the project is likely to end up in the lower half of the performance range where the expected payoff *decreases* with the higher variability. In other words, the mean project value is likely to suffer. The options of improvement and abandonment may or may not suffice to offset this suffering of the mean payoff. If the project plan is pessimistic ($p > 0.5$), product performance is biased toward the upper end of the performance range, and even the straight NPV already benefits from a payoff variability increase. Thus, the unbiased case that we analyze in Proposition 2 is the limit case where the NPV is not affected by the increase in variability.[6]

Corollary. *If the market payoff difference* $(M - m)$ *increases while the mean* $(M + m)/2$ *remains constant, the option value of managerial flexibility increases.*

This result confirms the real option theory intuition in our model: the value of managerial flexibility is enhanced by an increase in market payoff variability.

4.2 Budget Variability

Budget varability is already included in the model to the extent that improvement, the occurrence of which is stochastic according to the optimal policy, carries a cost α_t. The question is how the value of flexibility is impacted if the continuation cost becomes stochas-

[6]The requirement that the two payoff functions cross at $i = 0$ and their differences are symmetric is required to ensure that the NPV for both is the same, which makes the change in $V_0(0)$ equal to the change in option value. Even if the two payoff functions have non-symmetric differences, the ideas described here remain valid, although exposition becomes more complicated because the change in NPV has to be factored into the analysis.

tic, independent of whether improvement is chosen or not. Variability is represented by the variance θ_t^2 of the continuation cost c_t.

We need to consider two cases. First, if c_t is independent of c_{t-1}, the optimal policy continues to hold, and both $V_0(0)$ and the NPV are unchanged. Thus, the option value of abandonment and improvement is unaffected, although the *variance* of the project payoff increases. The reason for this is that past variations of the project costs carry no information about the future. The value of flexibility is neither enhanced nor reduced.

This changes if project costs are *correlated* over time, that is, if a budget overrun in c_t makes a future budget overrun more likely. In this case, the realization of c_t carries information about the future, based on which flexibility can be used to improve the expected payoff. Formally, we can use the realization of c_t to update our estimate of the future value function. Suppose that c_t becomes known at the beginning of period t and encapsulates all information from previous costs. We can then expand the state space to (t, i, c_t), and the value function becomes $V(t, i, c_{t-1}) = c_t + E[V(t+1) \mid c_t]$. Qualitatively, a higher variance θ_t of the continuation cost implies a higher variance of the (updated) $V(t+1)$, which, by Proposition 2, means that the option value of flexibility increases. We conclude that the real option intuition continues to hold for the case of budget variability.

4.3 Performance Variability

The product performance i of the product varies stochastically because of the state transitions from one period to the next. Performance variability increases with parameter N and thus the variance in the transition probabilities (a larger N makes a greater number of states reachable in a transition). We now show that performance variability may *reduce* financial payoff variability and thus the real option value.

Proposition 3 *Assume the project plan is unbiased, that is, $p = 0.5$, and the expected*

performance requirement $ED = Ei = 0$. *Then the option value* $V_0(0)$ *decreases when the performance variability* N *increases.*

The negative impact on the option value stems from the higher uncertainty "smearing out," or averaging out, the achievable performance over a wider range. This smearing out reduces the available payoff variability. The intuition is represented in Figure 6. From any current performance state during the project, higher performance uncertainty increases the reachable performance range. Thus, the expected payoff function flattens out, which reduces the downside protection the decision-maker can achieve by intelligently choosing improvement or abandonment of the project. Therefore, the value of managerial flexibility suffers.

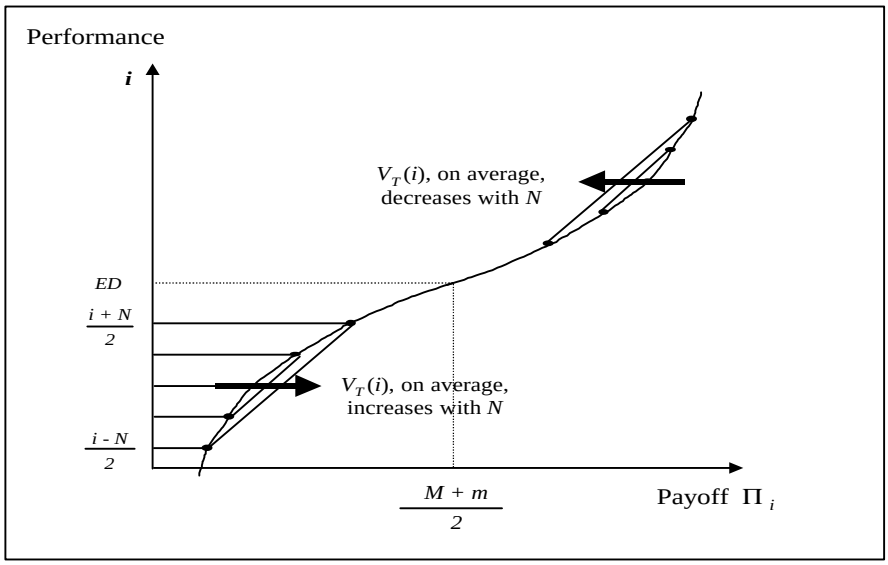

Figure 6: The Effect of Larger Performance Variability

This effect does not appear if the payoff function Π_i is *linear* in the stochastic variable, e.g., the option value follows directly the stochastically varying project performance. The convex-concavity in our model stems from the fact that the performance requirement in the market itself is stochastic (unforeseeable). Convex-concavity is the essential driver of our result that performance variability "washes out" payoff variability. Performance uncertainty is only revealed *in the future*, after possible decisions have been made. This

uncertainty may cause "mean reversion" that reduces the payoff variance for which flexibility is valuable. Mean reversion does not occur for a linear payoff function, in which case, performance variability does not impact flexibility value.

As before, this result has been "isolated" from other effects by assuming an unbiased project plan. If the project plan is optimistic, i.e., $p < 0.5$, an increase in the performance variability parameter N shifts the expected project performance downward, making improvement and abandonment even more important, thus boosting the option value for an increasing N.[7] This can be illustrated using the example that was introduced in Figure 5. If N is increased from 1 to 2, the option value decreases from 29.4 to 17.5. If, however, the upward transition probability is reduced to $p = 0.1$, the option value increases to 43 for $N = 1$ and even higher to 53 for $N = 2$.

4.4 Market Requirements Variability

In the context of our model, market requirement variability is represented by the variance σ^2 of the market performance requirement, while holding the mean market requirement ED constant. Proposition 4 shows another negative effect of operational variability on the option value.

Proposition 4 *Assume the project plan is unbiased, that is, $p = 0.5$. Then the option value $V_0(0)$ decreases if σ, the market requirement variability, increases. Furthermore, if $V_0(0) \geq 0$ for any σ, then there is a $\overline{\sigma}$ such that for all $\sigma \geq \overline{\sigma}$ the optimal policy is to "continue" in all states (i, t), in which case $V_0(0) = NPV$.*

[7]Similarly, if the expected market requirement $ED < 0$, the expected performance is larger than the expected requirement, in which case more of the distribution of i lies in the concave region of Π. In this case, $V_0(0)$ and the NPV both decrease, so the option value may increase or decrease. The converse holds if $ED > 0$. As in Proposition 2, the assumption in the proposition distinguishes the variability effect on the option value from the mean effect on the NPV.

The reason for the real option value to be diminished by market requirement variability is summarized in Figure 7. When market requirements are more spread out without a corresponding increase in payoff variability, part of the probability mass "escapes" beyond the performance range in reach of the development project. As a result, the information about payoff variability offered by the current performance state i is reduced, which destroys the value of flexibility responding to this information. When variability becomes so great that no expected payoff difference exists over the reachable performance range, there remains no option benefit. The project decision becomes equivalent to the static NPV criterion since the performance states carry no information about payoffs.

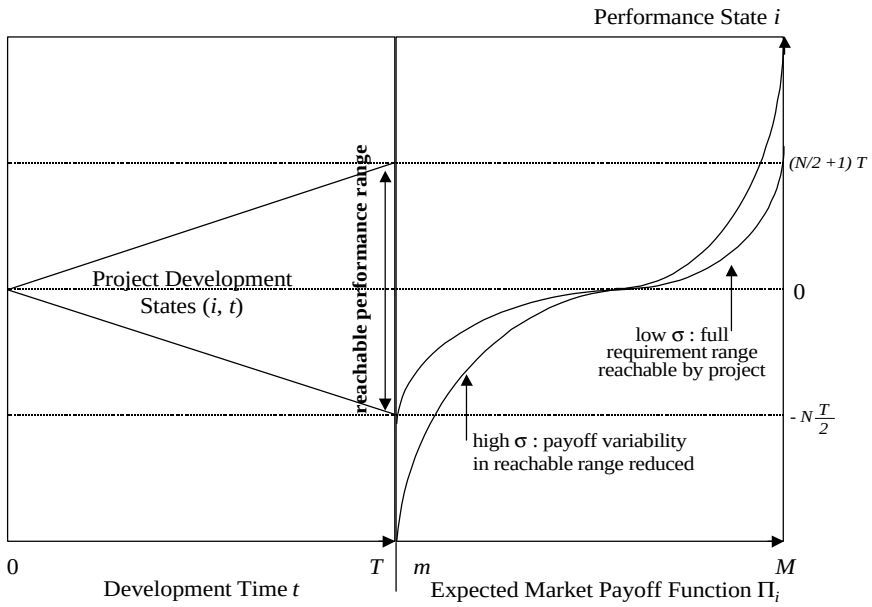

Figure 7: Increased Market Requirements Variability

The decrease in option value is demonstrated in Figure 8 on the same example as in Figure 5, with market requirement variability increased from $\sigma = 2$ to $\sigma = 3$. The NPV value of the project has remained unchanged, but the value of both the abandonment option and the improvement option has been reduced. This becomes apparent when comparing the optimal policies of Figures 4 and 7. The number of states in which it is worthwhile to choose improvement has shrunk because the payoff function is flatter.

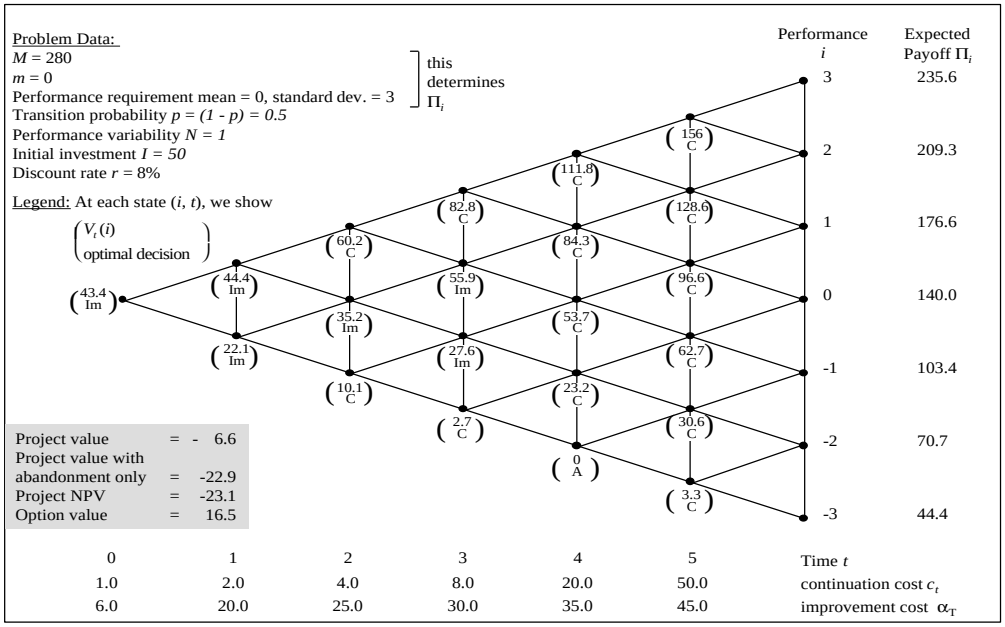

Figure 8: Example With Increased Requirements Variability

As in subsection 4.3, option value is lost because uncertainty is resolved *after* all decisions are made. Thus, a variability increase causes mean-reversion in the variability against which flexibility can be exercised. Yet, the reason for the lost option value in Proposition 4 is very different from that in Proposition 3. The effect in Proposition 4 has nothing to do with payoff nonlinearity (or with convex-concavity, for that matter). Indeed, the effect would persist with a linear function Π_i, which would be "rotated" around $i = 0$ such that its extreme values in the reachable performance cone would be pushed closer together. The key phenomenon is that the end point of the payoff distribution is pushed beyond the reachable performance range, and therefore, the *reachable* payoff variability is reduced.

A similar effect of probability mass escaping beyond a reachable "capacity" limit is very important in a different context as well. Consider an investment in a flexible production facility with a capacity limit. More variability can be detrimental if probability mass of demand, and thus part of the upside of the option, escapes beyond the capacity limit (Jordan and Graves 1995, Cohen and Huchzermeier 1999).

4.5 Schedule Variability

Suppose that the expected market payoff Π_i is sensitive to the time-to-market: a product launch delay by δ beyond the planned launch time T reduces $\Pi_i(\delta)$. This is consistent with empirical results that a time-to-market delay may destroy development project payoffs (Datar *et al.* 1997). In order to focus on schedule variability and to make its effects very clear, we simplify our basic model by collapsing the product performance states, i.e., by considering a situation where the target performance is well-known and reachable.[8] We consider a two-stage project, in which stage 1 may be delayed, as is shown in Figure 9.

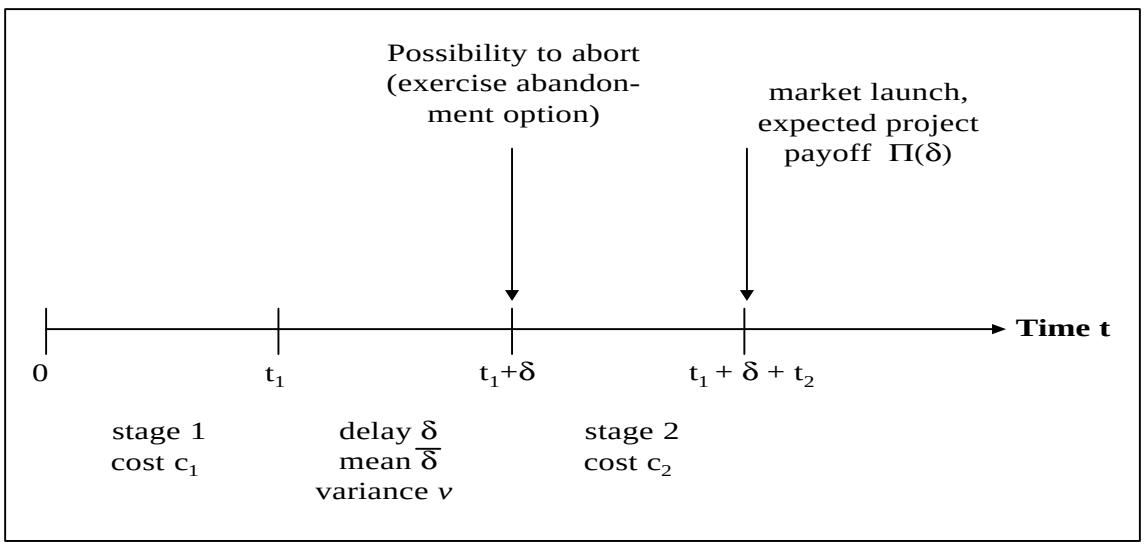

Figure 9: A Development Project With Schedule Variability

The first stage may be interpreted as technical development, with the risk of a delay δ, and the second stage as the marketing and launch campaign. The expected project payoff is a strictly decreasing function of the delay δ. Management may, after the delay has been observed, decide to abort the project before the launch costs are incurred (exercising an

[8]The impact of delays on revenues can be incorporated into the basic model from Section 3 by expanding the state space from (i, t) to (i, t, Δ), where Δ is the accumulated delay up to time t. This would complicate the model without adding clarity to the argument. Similarly, we formally leave discounting out of the model. Discounting alone would correspond to Π decreasing convexly with δ, which is incorporated in our analysis as a special case.

abandonment option).

The decision rule at the beginning of stage 2 (at which point δ has been revealed) is clear: continue if $\Pi(\delta) - c_2 > 0$, and abort otherwise. We can invert $\Pi(\delta)$ and thus write the prior probability of continuation as $P\{\delta \leq \Pi^{-1}(c_2)\}$, where Π^{-1} stands for the inverse. This allows us to write the optimal project value and the NPV of the project (which assumes continuation regardless of the delay). The value of the abandonment option is the difference:

$$\begin{aligned} V_0(0) &= -c_1 + P\{\delta \leq \Pi^{-1}(c_2)\}E[\Pi(\delta) - c_2 \mid \delta \leq \Pi^{-1}(c_2)]; \\ NPV &= -c_1 - c_2 + E[\Pi(\delta)]; \\ \text{Option value } &= P\{\delta > \Pi^{-1}(c_2)\}(c_2 - E[\Pi(\delta) \mid \delta > \Pi^{-1}(c_2)]). \end{aligned} \tag{6}$$

As this expression shows, the option value lies in the avoidance of the loss-making case where the reduced payoffs are too small to cover the launch costs. The option value depends on where the critical cutoff delay $\Pi^{-1}(c_2)$ lies with respect to the distribution of δ. This critical delay indicates how "bad" things must become before the project is aborted. If the critical delay is very large, revenues are very unlikely to be reduced so much as to make the project unprofitable. Thus, the option is unlikely to be exercised and not worth much. If the critical delay is small, the option is likely to be "in the money," and thus worth more.

How does increased schedule uncertainty, represented by the delay variance v^2, influence this option value? The answer depends on two effects: first, the probability of exercising the option is determined by the distribution of the delay δ, and second, the shape of the payoff over time $\Pi(\delta)$ influences the effect of averaging. Proposition 5 summarizes the result.

Proposition 5 *If the critical delay $\Pi^{-1}(c_2)$ is small (large) relative to the expected delay $\overline{\delta}$, an increasing schedule variability v^2 may decrease (increase) the option value of flexibility.*

If the payoff function $\Pi(\delta)$ is convex (concave), an increasing schedule variability v^2 may decrease (increase) the option value of flexibility.

Two simple examples best illustrate the essence of the argument. First, suppose there is a critical introduction date δ_{crit}, for example, the announced introduction date by a competitor or a regulatory deadline, beyond which revenues suffer discontinuously. $\Pi(\delta)$ is unaffected at $H > c_2$ as long as $\delta < \delta_{crit}$, but it drops to $L < c_2$ if $\delta \geq \delta_{crit}$. Suppose also that the delay is normally distributed with parameters $(\overline{\delta}, v)$. Then the option value (6) becomes $[1 - \Phi((\delta_{crit} - \overline{\delta})/v)](c_2 - L)$. The derivative of this option value with respect to the standard deviation v is $\frac{c_2 - L}{v^2}\phi((\delta_{crit} - \overline{\delta})/v)(\delta_{crit} - \overline{\delta})$. This is positive for $\delta_{crit} > \overline{\delta}$ and negative for $\delta_{crit} < \overline{\delta}$. This means that when the expected delay is large, the project will only be carried through in the left tail of the distribution. This left tail increases with v, so the probability of the option being exercised shrinks with v. As the low payoff L is constant in δ, the probability of exercise determines the value of the option.

The second example shows the effect of averaging over convex functions. A convex $\Pi(\delta)$ corresponds to a situation where a small delay does a lot of damage, but then further delays matter less and less. Suppose the payoff $\Pi(\delta) = 50e^{-0.1\delta}$, and the continuation cost $c_2 = 20$. Then the cutoff delay at which the abandonment is exercised is $\Pi^{-1}(c_2) = 9.16$. Now, suppose that the delay is normally distributed with parameters $(30, v)$. We find that with $v = 10$, the option value is 16.1, with $v = 20$ it is 13.9, and with $v = 30$ it shrinks to 12.7. A larger variance of δ spreads the possible delays and thus increases the expected NPV payoff (over the region where the option is exercised). In this example, a change in variability cannot be separated from a change in the NPV.

Similarly to performance and requirement variability, schedule uncertainty smears out payoff variability against which flexibility has value. However, in the first example, this is not due to uncertainty being revealed after decisions are made. More variability may reduce the probability that flexibility will ever be exercised, which diminishes its value.

5 Conclusion

In this article, we have developed a simple real option model of an R&D project, in which not only the market payoff is subject to uncertainty, but also operational variables of budget, product performance, market performance requirement, and schedule. In each of T stages of the project, management has the flexibility of improving or abandoning the project when additional information becomes available. "Improvement" represents an extra source of option value, in addition to continuation, abandonment, expansion, contraction, or switching. Improvement is the capability of an operational mid-course correction during the execution of the project.

Standard real options intuition states that more variability increases the value of managerial flexibility, as more "downside" can be avoided. Our results imply that this intuition is not always correct. The structure of uncertainty resolution determines whether or not variability makes flexibility valuable. If uncertainty is resolved and then a decision can be made *before* costs or revenues accrue, the intuition holds: more variability creates more downside to be avoided, making flexibility worth more. This applies to payoff and budget variability in our model. However, if uncertainty is resolved or costs/revenues occur *after* all decisions are made, more variability may smear out contingencies and thus reduce the value of flexibility. In our model, this is the case for performance and market requirements variability. In addition, operational variability may reduce the probability of flexibility ever being exercised, which also reduces its expected value. This is demonstrated on the example of schedule variability in our model.

The results of our model have clear managerial implications, indicating when it is most important to delay commitments. For example, project management flexibility offers a significant option value for an incremental innovation project in a mature market with low performance and requirement variability. Thus, a good project plan with a little flexibility

may be the right approach. However, in a radically new project with high performance and requirement uncertainty, the option value of flexible project management may be reduced. Thus, a lot of flexibility is required to effectively react to contingencies. Recent findings in the empirical product development literature suggest a trial-and-error approach for such projects (e.g., O'Connor 1998, Chandy and Tellis 1998). But such high flexibility is expensive. Our model implies that management should perform regular, formal reviews to obtain information on *all* sources of uncertainty, in order to target flexibility to where it is needed. For example, if performance uncertainty is most critical, testing and prototyping capacity should be provided to resolve technical problems as quickly as possible when they arise. If, in contrast, market requirement uncertainty is most critical, management should be prepared to quickly respond to customer feedback by changing features or product aethetics.

More generally, our results suggest that managers should be willing to pay for flexibility after new information becomes available and before major costs or revenues occur, if the probability of that flexibility being exercised is significant. It is worth maintaining flexibility until additional information becomes available about product performance (e.g., through testing or simulation) or customer requirements (e.g., through prototypes or lead users). There is an option value of additional information. If information (from *any* source) is not updated, the default decision may be "just continue and see what happens". This may represent a "continuation trap" if additional information could be obtained leading to abortion.[9]

The model proposed in this paper makes a conceptual step toward understanding the effects of operational variability on the value of managerial flexibility in R&D projects. There remains a great need for empirical evaluation of managerial flexibility in real R&D environments. In addition, many issues remain to be explored, e.g., dynamic R&D in-

[9]We are grateful to an anonymous referee for suggesting this point.

vestment policies for several R&D projects in parallel, or reduction of market requirement uncertainty over time. Such considerations may lead to additional types of variability with surprising effects. As R&D project costs and risks increase, evaluation of flexibility will become even more important.

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6 Appendix

6.1 Proof of Proposition 1

We proceed by induction. By assumption, Π_i is convex-concave increasing. We show first that the control policy is optimal as claimed. Then we show that the resulting value function $V_i(t)$ is convex-concave increasing as well.

Lemma 1. If $V_i(t+1)$ is convex-concave increasing, $V_i(t)$ has the described optimal policy.

Proof. The values corresponding to continuation and improvement in (5) both increase in i because $V_i(t+1)$ does. Thus, if we find an $L_d(t)$ for which abandoning is the best action, abandoning is also best for all $i \leq L_d(t)$. This determines region A in Figure 4. The reader should note that region A is independent of regions C_1, C_2 and I.

Improvement is preferred over continuation in state i iff

$$\alpha(t) < R_i \equiv \frac{\sum_{j=1}^{N}[pV_{i+1+j/2}(t+1)+(1-p)V_{i+1-j/2}(t+1)]}{N(1+r)} - \frac{\sum_{j=1}^{N}[pV_{i+j/2}(t+1)+(1-p)V_{i-j/2}(t+1)]}{N(1+r)}. \tag{7}$$

By convex-concavity of $V_i(t+1)$, the right-hand side of (7) first increases, then decreases in i. Thus, if there is an $L_m(t)$ with $R_{L_m(t)} < R_{L_m(t)+1}$ such that $\alpha(t) \geq R_{L_m(t)}$ but $\alpha(t) \leq R_{L_m(t)+1}$, continuation is preferred for all $i \leq L_m(t)$. For state $L_m(t)+1$, improvement is preferred. This describes region C_1 in Figure 4.

If there is an $L_u(t)$ with $R_{L_u(t)} < R_{L_u(t)-1}$ (i.e., $V_i(t)$ is locally concave), such that $\alpha(t) \geq R_{L_u(t)}$ but $\alpha(t) \leq R_{L_u(t)-1}$, then continuation is preferred, for all $i \geq L_u(t)$. For state $L_u(t)-1$, improvement is preferred. This determines region C_2 in Figure 4. Finally, by convex-concavity of $V_i(t+1)$, there can be no additional switch of condition (7) in between, which settles region I in Figure 4. □

Lemma 2. $V_i(t)$ is convex-concave increasing in i.

Proof. Within the regions of Figure 4, $V_i(t)$ is convex-concave increasing since it is a linear combination of summands from $V_i(t+1)$. The borders between the regions remain to be checked.

At $i = L_u(t)+1$, continuation is optimal, and at $L_u(t)$ improvement is optimal.

$$\begin{aligned} V_{i+1}(t) - V_i(t) &= \alpha(t); \\ V_i(t) - V_{i-1}(t) &= \frac{\sum_{j=1}^{N}[pV_{i+1+j/2}(t+1)+(1-p)V_{i+1-j/2}(t+1)]}{N(1+r)} \\ &\quad - \frac{\sum_{j=1}^{N}[pV_{i+j/2}(t+1)+(1-p)V_{i-j/2}(t+1)]}{N(1+r)} \\ &\geq \alpha(t) \text{ by Equation (7).} \end{aligned}$$

Thus, $V_i(t)$ is concave at $L_u(t)$. A symmetric argument at $i = L_m(t)$ implies that $V_i(t)$ is convex at $L_m(t)$.

Finally, for $L_d(t)$ we must consider two cases. First, if $L_d(t)$ is in the convex region of

$V_i(t)$, we can write the increments of $V_i(t)$ as follows:

$$\begin{aligned}
V_{L_d(t)-1}(t) = V_{L_d(t)}(t) = \;& 0 \text{ by definition of } L_d(t); \text{ thus:} \\
V_{L_d(t)+1}(t) - V_{L_d(t)}(t) = \;& \frac{p\sum_{L_d(t)+3/2}^{L_d(t)+(N+2)/2} V_j(t+1) + (1-p)\sum_{L_d(t)+1/2}^{L_d(t)-(N-2)/2} V_j(t+1)}{N(1+r)} \\
& - \frac{p\sum_{i+1/2}^{i+N/2} V_j(t+1) + (1-p)\sum_{i-1/2}^{i-N/2} V_j(t+1)}{N(1+r)} - 0 \\
\leq \;& V_{L_d(t)+2}(t) - V_{L_d(t)+1}(t) \text{ because } V_{L_d(t)}(t) \text{ is bounded below at zero} \\
& \text{and by local convexity of } V_i(t+1).
\end{aligned}$$

Thus, $V_i(t)$ is convex at $L_d(t)$. Second, if $L_d(t)$ is in the concave region of $V_i(t)$, the symmetric argument can be used to establish that $V_i(t)$ is concave at $L_d(t)$. □

6.2 Proof of Proposition 2

Lemma 3. Suppose there are two value functions $V_i(t+1)$ and $\overline{V}_i(t+1)$, both convex-concave increasing in i, with the following characteristics:

$$\text{larger increments: } \overline{V}_i(t+1) - \overline{V}_{i-1}(t+1) \geq V_i(t+1) - V_{i-1}(t+1) \;\forall i; \tag{8}$$

$$\text{equal value: } \sum_{i_{min}(t+1)}^{i_{max}(t+1)} \overline{V}_i(t+1) \geq \sum_{i_{min}(t+1)}^{i_{max}(t+1)} V_i(t+1), \tag{9}$$

where i_{min} is the lowest performance possible at launch if continuation is chosen in all project states, and i_{max} is the highest. Then $V_i(t)$ and $\overline{V}_i(t)$ are convex-concave increasing and fulfill conditions (8) and (9).

Proof. Convex-concavity of both value functions follows directly from Proposition 1. Each value function is, by its definition (5), a linear combination within the regions of improvement (I) and continuation (C) separately. This implies condition (8) for the regions I and C separately. In addition, by the definition (7) of L_m and L_u, $\overline{L}_m \leq L_m$, and $\overline{L}_u \geq L_u$, that is, the range of action I is larger for $\overline{V}_i(t)$ because $\overline{V}_i(t+1)$ is steeper by (8).

At the transition $\overline{L}_m$, where $\overline{V}_i(t)$ switches to I while $V_i(t)$ still stays with C, condition (8) also holds because the expected improvement payoff more than makes up for additional improvement costs α_t. Similarly, at the upper transition L_u, $V_i(t)$ switches back to C while $\overline{V}_i(t)$ still stays with I because $\overline{V}_i(t+1)$ is still steep enough to justify the improvement cost, while $V_i(t+1)$ is not. Thus condition (8) holds here as well. Finally, at the abandonment control L_d, we can argue that $\overline{V}_i(t)$ is prevented from "dipping below zero," which limits its disadvantage where it is below $V_i(t)$. (The algebraic details of these comparisons are omitted). This establishes condition (8) for $\overline{V}_i(t)$ and $V_i(t)$.

To see that condition (9) holds, recall that if continuation was chosen everywhere, (9) would hold for time t because both period t value functions are symmetric linear combinations of the period $t+1$ value functions. But as the improvement range of $\overline{V}_i(t)$ is enlarged, condition (7) ensures that the improvement enhances $\overline{V}_i(t)$ at least for some states, and similarly, abandonment limits $\overline{V}_i(t)$ from below. Therefore, (9) holds for time t. □

The proposition can now be proved by induction: $\overline{\Pi}_i$ and Π_i are convex-concave increasing and fulfill conditions (8) and (9) by assumption, crossing over at $i = 0$. Then Lemma 3 establishes an induction backwards from T to time 0, where $\overline{V}_0(0) \geq V_0(0)$.

At the same time, $\overline{\Pi}_i$ and Π_i have the same project NPV (continuation at every state) because the compounded probability distribution of the payoffs (as $p = 0.5$), as well as the differences $\overline{\Pi}_i - \Pi_i$ (by assumption), are symmetric around zero. Therefore, the option value of flexibility, corresponding to the difference $V_0(0) - NPV$, is larger for $\overline{\Pi}_i$. □

The corollary follows directly: Consider two payment distributions (m, M) and $(\overline{m}, \overline{M})$ such that $(\overline{M} - \overline{m}) > (M - m)$, but the averages are equal. Then $\overline{\Pi}_i > \Pi_i$ for all $i > ED = 0$ and *vice versa*. Moreover, $\overline{\Pi}_i - \overline{\Pi}_{i-1} \geq \Pi_i - \Pi_{i-1}$ for all i, by the definition of the payoff function. Thus, the two payoff functions fulfill the conditions for the proposition.

6.3 Proof of Proposition 3

We include N as an explicit parameter in the value function $V_{T,N}(i)$. We prove that for every N, there exists an i_N^* such that $V_{T,N+1}(i) \geq V_{T,N}(i)$ for all $i < i_N^*$ and $V_{T,N+1}(i) \leq V_{T,N}(i)$ for all $i \geq i_N^*$. That is, the value function increases with the performance uncertainty N below an inflection point, and decreases with N above the inflection point. As a result, the value function $V_{T,N}(i)$ is "squeezed" more closely and has thus smaller increments. Therefore, by Proposition 2, the option value $V_0(0)$ decreases in N, reflecting the reduced potential for risk-hedging. Figure 5 summarizes the intuition of this argument.

First, consider the expected payoff in period T from continuation. From (4) and $p = 0.5$,

$$V_{T,N}(i)(\text{cont.}) = -c(T) + \frac{\sum_{j=1}^{N} \Pi_{i+j/2} + \Pi_{i-j/2}}{N(1+r)}.$$

Convex-concavity of Π_i implies that at $i = 0$, the first summand in the numerator decreases with N, and the second summand in the numerator increases with N. As $i > 0$, the convex combination in the numerator shifts more toward the concave part of Π_i and thus toward decreasing in N, and *vice versa*. Therefore, we can define $i_{cont.}(T)$ analogously to

Proposition 2 such that $V_{T,N}(i)(\text{cont.})$ increases in N for all $i \leq i_{cont.}(T)$ and $V_{T,N}(i)(\text{cont.})$ decreases in N for all $i > i_{cont.}(T)$. Moreover, by symmetry of Π_i, $i_{cont.}(T) = 0$.

Thus, there exists an $i_{impr.}(T)$ such that $V_{T,N}(i)(\text{impr.})$ (defined in the same way as $V_{T,N}(i)(\text{cont.})$ above) increases in N for all $i \leq i_{impr.}(T)$ and $V_{T,N}(i)(\text{impr.})$ decreases in N for all $i > i_{impr.}(T)$. Moreover, $i_{impr.}(T) = i_{cont.}(T) - 1 = -1$, which can be seen from the fact that the two expected payoffs only shift by one performance level.

By convex-concavity of Π_i and (7), $L_m(T, N)$ increases in N, and $L_u(T, N)$ decreases in N. Therefore, when considering $N_1 < N_2$, the two corresponding value functions fit the structure in Figure 7 with $V_{T,N_1}(i)$ corresponding to the higher variability value function $\overline{V}_T(i)$ in Lemma 3. Proposition 2 implies that the option value $V_0(0)$ decreases in N. □

6.4 Proof of Proposition 4

Consider two performance requirement distributions with equal mean ED but $\overline{\sigma} > \sigma$. Denote with the upper bar all policies and results corresponding to the distribution with $\overline{\sigma}$. The payoff function $\overline{\Pi}_i$ has the same mean but *lower* variability: $(\overline{\Pi}_i - \overline{\Pi}_{i-1}) < (\Pi_i - \Pi_{i-1})$ such that $(\overline{\Pi}_i - \Pi_i) < 0$ for $i > ED$ and vice versa for $i < ED$. Therefore, Proposition 2 applies with Π_i and $\overline{\Pi}_i$ exchanged. This proves statements 1 and 2 of the proposition.

Finally, suppose that $\overline{\Pi}_{ED}/(1+r)^T > \sum_{t=1}^{T-1} c(t)$ – the project exceeds its expected variable cost. There is a σ^* such that $(\overline{\Pi}_i - \overline{\Pi}_{i-1})/(1+r) < \alpha(t)$ for all $\overline{\sigma} > \sigma^*$ and all (i, t) and $\overline{\Pi}_{-NT/2}/(1+r)^T > \sum_{t=0}^{T-1} c(t)/(1+r)^t$. Then continuation is optimal in all states (i, t): payoff increments are too small to justify improvement, and even the worst case scenario permits continuation. □