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Abstract

The diffusion of new technology among competing firms is of long-standing interest in industrial organization. There is an extensive theoretical literature on technology adoption in which firms can instantaneously deploy a new technology in the market at a cost that is exogenously falling over time. While such models explain diffusion (firms adopt asynchronously), Fudenberg and Tirole (1985) show that the incentives to preemptively adopt in subgame perfect equilibria can cause rents to be equalized across firms. In contrast, we study technology development where costly and time consuming effort is required to deploy a new technology. With diminishing returns to instantaneous effort, delaying deployment reduces the firm's cost, as in adoption models. However, the incentive to preempt is lower: with its development already partially complete, a preempted firm delays deployment less than with adoption. We provide reasonable conditions under which the subgame perfect equilibrium outcome corresponds that in the pre-commitment equilibrium first proposed by Reinganum (1981a, 1981b), yielding both diffusion and first mover advantages for the case of technology development.

Key words: preemption, rent equalization, sustainable competitive advantage, pre-commitment equilibria, lead times

1. Introduction

The incorporation of new technology into products and services is an important driver of economic growth and a potentially significant source of advantage for competing firms. For example, advances in information technology have created new internet-based retail channels, allowed firms to offshore an increasing number of productive activities, and led to the development of advanced product features in telecommunications, automobiles and consumer electronics. The impact of new technology depends on how quickly it diffuses; that is, on how quickly firms choose to deploy the new technology in their products and services and in their production processes. The time required to deploy new technology can be substantial: the launch of a new version of an automobile, microchip or airplane takes years from the start of the design process until products start rolling off the production line. Some firms choose to lead the introduction of new technology in their industry, such as Mercedes in automobiles, Sony in consumer electronics, and Samsung in memory chips. One can ask, however, whether such early deployment actually leads to a first mover advantage where the technology leader enjoys superior financial performance (Robinson et al, 1994; Lieberman and Montgomery, 1988).

Although bringing new technology to product markets can be a very time consuming activity, the formal literature on diffusion (see Hoppe (2002) for a survey) has focused on adoption decisions where deployment is instantaneous once a firm has decided to adopt. The modern game theoretic treatment of this topic starts with Reinganum (1981a), although it is the analytic approach of Fudenberg and Tirole (1985) that is followed by subsequent authors such as Katz and Shapiro (1987) and Riordan (1992). New technology diffuses over time in these models because the cost of adoption is assumed to fall exogenously over time (either due to advances in fundamental research or due to the increasing efficiency of technology suppliers). A firm's adoption time then trades off cost reductions from waiting with the forgone profits from delayed access to the technology.[1]

[1] Although Katz and Shapiro (1987) frame their paper as being about development, in their model "once chosen, development is assumed to take place instantaneously" (p. 405) and hence it is usually seen as part of the literature on adoption (Hoppe, 2002). For us, a defining feature of development is that it is a time consuming activity.

In contrast to the adoption literature, we consider time consuming and costly development as the driver of diffusion. In our theory, the cost of deploying the new technology at a given time is endogenously determined by the ease with which firms can compress their development processes. The completion of development and the deployment of the new technology in the market is observable, but following Hopenhayn and Squintani (2006), internal development efforts are not. We consider the same core issues as the adoption literature, namely whether or not competing firms deploy the technology at different times and when they do, whether there are first mover advantages. We get fundamentally different answers. In contrast to adoption, the timing of development is not driven by preemption incentives that serve to dissipate the rents of first movers. The reason is that a preempted firm has partially completed its development and hence it delays its deployment less than in the case of adoption and this lowers the returns to preempting.

We follow the literature on adoption timing in how we model the effect of the new technology on firm profits and specifically use the continuous time formalization introduced by Reinganum (1981a) and subsequently studied by Fudenberg and Tirole (1985). There are two competing firms that are initially symmetric and which receive a flow of profits that are initially the same. When a firm chooses to adopt the new technology, or in our case completes its development, its profit flows increase while those of its competitor (weakly) decrease. The main restriction on the profit flows is that the profits of the first firm to deploy the new technology increase more than the profits of the second mover.[2] When firms adopt or develop the technology at different times, we say that the first firm to deploy the technology in the market is the first mover.

1.1. The Adoption of New Technology

In their classic paper on adoption, Fudenberg and Tirole (1985) show that while there may be a first mover in equilibrium, competitive forces dissipate rents so that the payoffs of the first and second mover are the same. Their paper is notable both for the striking result of rent

[2] This profit ordering is a weak assumption given the assumption of initially symmetric firms. For the case of a new market, it simply requires that the profits of a monopolist are greater than those of a duopolist.

equalization and for the introduction of the subgame perfection equilibrium (SPE) concept to the adoption timing literature. Subgame perfection is a widely accepted refinement that rules out equilibria based on incredible threats. In the case of adoption games, subgame perfection requires that firms are expected to respond optimally to prior adoption by a competitor.

Fudenberg and Tirole (1985) show that there is a unique Pareto optimal SPE and that it can take one of two possible forms. The first is a preemption equilibrium where one firm is a first mover. The adoption time of the second firm is fixed by the tradeoff between the declining cost of adoption and the forgone profit flows from deploying the new technology later. The first mover has superior profit flows for an interval of time, but also higher costs due to the earlier adoption. In equilibrium, the increase in profit flows must be exactly offset by the costs. Otherwise, the second mover could profitably deviate and adopt just before the equilibrium adoption time of the first mover, which would cause the firm that was supposed to be the first mover in equilibrium to optimally respond by adopting at the time of the second mover.

A second possibility arises when moving first leaves both firms worse off than if adoption were sufficiently delayed. Then, there exists a joint adoption equilibrium where the firms adopt late–at the point where joint payoffs are maximized.[3] With the firms adopting at the same time, rents are again equalized.

Reinganum (1981a) solves the same model as Fudenberg and Tirole (1985) for pre-commitment equilibria where firms simultaneously choose an adoption time at the start of the game and are unable to deviate later on. In particular, a firm does not change its adoption time if the other firm deviates from its equilibrium adoption time. Such pre-commitment equilibria have nice properties. There are unique equilibrium adoption times and one firm is always a first mover. Moreover, there is a first mover advantage in that the first firm to adopt has a higher payoff than the second adopter. Unlike subgame perfect equilibria, which have larger strategy spaces, it is straightforward to generalize the analysis to an arbitrary number of firms (Reinganum, 1981b; Quirmbach, 1986). However, subsequent authors have followed Fudenberg and Tirole

[3] The joint adoption equilibrium only arises when there is a sufficiently large negative externality from the first adopter on the other firm. In the case of a new market, where there is no such externality, the preemptive outcome is the unique SPE.

(1985) in viewing SPE as the appropriate solution concept for adoption games.[4]

1.2. Costly and Time Consuming Development

While we model the effect of new technology on profit flows as in the received literature on diffusion, we do not assume an exogenously given cost of the new technology that is falling over time. Instead, we make three key assumptions. The first is that exploiting the new technology requires costly and time consuming internal development efforts.[5] For example, exploiting internet technology to offer online technical support for a product requires a firm to develop the website. We model development as requiring that firms complete a certain number of steps. For example, the development of an offshore call center to exploit advances in telecommunications and information technology would require the design of the call center, the leasing of the facility, the design and implementation of the IT and telecommunication system, and the recruiting and training of the staff.[6]

The second key assumption is that there are diminishing returns to effort at any point in time. Thus, ten engineers working for a month on a crash project to develop a new product are less productive than two engineers working for five months. One reason is the increasing fraction of time taken up with coordination as the size of a team grows. An interest in diminishing returns dates back to the early literature on research and development (Mansfield, 1971). Scherer (1967) is one of the earliest authors to discuss "diminishing returns to time compression" so that "as time is compressed, total development cost increases at an increasing rate." The specific model of resource development we employ is due to Lucas (1971), who solves the monopoly problem.[7] We focus on the case of quadratic cost of progress, which is

[4] In an empirical study of technology adoption, Schmidt-Dengler (2006) characterizes the extent to which adoption decisions reflect preemption incentives. He finds a small but statistically significant effect of preemption on the adoption of MRI machines by US hospitals.

[5] There are two ways to interpret development in the model. It could literally be the time and costs required to develop the new technology. Alternatively, it could be the time and cost required to develop the products and services that exploit the new technology.

[6] Some of what is required to exploit a new technology might be available from the market such as the facility for a call center which could be rented rather than built. Nonetheless, market solutions usually require time consuming and costly effort including searching for suppliers and negotiating terms.

[7] Pacheco de Almeida and Zemsky (2006) extend the model of Lucas (1971) to characterize sequential development by competing firms. The current paper analyzes the more complicated setting of simultaneous development, which is the setting that closely parallels the adoption literature.

particularly tractable given the complexity of our problem.

We can solve for the firm's cost minimizing effort profile, which depends on the target completion date, the number of steps to complete and the cost of capital. The optimal effort profile balances the benefit of spreading effort over time due to the diminishing returns to effort and the benefit of delaying effort due to the time value of money. Given these optimal effort profiles, there is a decreasing, convex relationship between development time and development cost. Such a time-cost tradeoff is also a longstanding feature of the literature on technology development (Gilbert and Newbery, 1982; Mansfield, 1971). Moreover, the specific time-cost tradeoff that we derive satisfies the assumptions in the adoption literature (c.f. Fudenberg and Tirole, 1985).

With equivalent profit flows and an identical time-cost tradeoff, our model of development simplified to a single monopolist is equivalent to a monopoly model of adoption, and hence it has the same optimal deployment time. This equivalence carries over to the pre-commitment equilibrium outcomes, which are the same for adoption and development. However, as in the case of adoption, it seems that SPE is more compelling in that firms should respond optimally to observable actions of their rivals. This brings us to our third key assumption: what is observable in our model?

We follow Fudenberg and Tirole (1985) and assume that firms can instantaneously observe when a competitor deploys the new technology in the market. Unlike adoption models, there is the additional issue of the observability of a firm's internal development efforts. We follow the recent work of Hopenhayn and Squintani (2006), and assume that a firm's internal state variable, which in our case is effort towards technology development, is unobservable by competitors. Accurately observing the status of a competitor's ongoing internal development projects is infeasible in many settings.

Two of the defining features of our model of technology development – namely, development time is significant and internal processes are secret – are consistent with the survey results of Cohen et al. (2000) on the appropriability of the returns to innovation. They report that "patents tend to be the least emphasized by firms in the majority of manufacturing industries, and secrecy and lead time tend to be emphasized most heavily" (p. 1). These results also justify the study of profit flows that are not winner take all, as in the adoption literature that we build on.

1.3. Overview of Results

Although the pre-commitment equilibrium outcomes are the same for adoption and development, we show that the subgame perfect equilibrium outcomes are radically different. First, joint development equilibria (where both firms complete their development at the same time) are not possible. Equilibria where both firms adopt late at the same time are supported by the threat that if one firm adopts earlier, the other firm best responds by adopting immediately. With time consuming development, such an immediate response is not possible. A firm may speed up its development activities in response to an earlier than expected development by its competitor, but this turns out to be insufficient to support joint development.

A striking result is that the pre-commitment outcome can be supported in a subgame perfect development equilibrium and, moreover, that this happens under fairly weak conditions. For example, the pre-commitment outcome is necessarily supported when profit flows are given by Cournot competition with linear demand and constant marginal costs that are reduced arbitrarily by the new technology. We show that there may be other subgame perfect development equilibria, with these involving some amount of preemption. However, full preemption where rents are equalized is not possible, except in the new market case. Moreover, whenever the pre-commitment outcome is supported as an SPE, it is the unique Pareto optimal equilibrium.

We thus offer a formal justification for the pre-commitment equilibria in Reinganum (1981a) where preemption incentives are not considered. Our justification does not depend on information lags or on costs to changing strategies. Rather, our results arise because the incentives to preempt are less for time consuming development than for adoption. A firm that is preempted slows down its development (because the payoff to being second are lower than the payoff to being first). However, the effect is less than under adoption because the preempted firm has already completed some of its development at the faster pace of a first mover.

We compare the diffusion of new technology when it is more of an adoption decision (e.g. hospitals buying new medical equipment) and when it is more of a development decision (e.g. automobile manufacturers launching a new model with a hybrid engine). We find that the first mover's competitive advantage is more sustainable (i.e. lasts longer) for adoption, but it yields a higher payoff for development.[8] The preemptive behavior that occurs with adoption leads to earlier deployment by the first mover, but does not alter the adoption date for the second mover, which results in both a more sustainable advantage and a lower payoff for the first mover.

We extend the model to allow firms to differ in their development capabilities. We find that such asymmetries widen the region in which the pre-commitment outcome can be supported as a SPE, which reinforces our argument that preemption is less of an issue with development than with adoption.

The paper proceeds as follows. Section 2 defines the model and Section 3 characterizes the cost minimizing development profiles. Section 4 reviews the key results on technology adoption in the context of our model, while Section 5 contains the main results on the existence of development equilibria. Section 6 compares adoption and development. Section 7 extends the model to incorporate differing development capabilities while Section 8 concludes.

2. The Model

We analyze a continuous time game played by two firms indexed by $i = 1, 2$ that compete in an output market. Time is indexed by $t \geq 0$ and there is an infinite horizon. Each firm can engage in the costly and time consuming development of a new technology. Initially, firms are identical but they may vary in the speed at which they choose to develop the new technology. We now detail our assumptions about product market competition and technology development, before turning to the equilibrium concepts we employ.

[8] We follow Pacheco de Almeida and Zemsky (2006) and say that a firm has a competitive advantage at a point in time when it has deployed the new technology and the competitor has not. The sustainability of a competitive advantage is then the interval of time over which the asymmetry persists. When defined in this way, sustainable competitive advantage is distinct from the overall financial performance of the firm, which is net of technology development costs.

2.1. Product Market Competition

Firms receive a flow of profits from the output market and these flows are affected by technology development. When no firm has completed development of the new technology, firms have instantaneous profits of $\pi_{00} \geq 0$. When only one firm has completed development, its profit flow increases to $\pi_{10} > \pi_{00}$, while the profit flows of its disadvantaged competitor (weakly) decrease to $\pi_{01} \in [0, \pi_{00}]$. Finally, profit flows when both firms have developed the new technology are given by $\pi_{11} \in [\pi_{00}, \pi_{10})$. As is standard in the diffusion literature (e.g., Reinganum, 1981a; Fudenberg and Tirole, 1985), we assume that the increase in profit flows is greater for the first firm to deploy the technology: $\pi_{10} - \pi_{00} > \pi_{11} - \pi_{01}$.

The case of a new market, where developing the technology is required to have positive profit flows, is a special case of this model with $\pi_{00} = \pi_{01} = 0$. In this case, the restriction that the first mover has a greater increase in profits is $\pi_{10} > \pi_{11}$, which is just the property that the profits of a monopolist are greater than the profits of a duopolist.

2.2. Technology Development

Technology development is a deterministic process requiring the completion of a continuum of $K > 0$ steps. Firm i chooses an effort profile that specifies the level of effort to be exerted over time on technology development as given by the function $z_i(t) \geq 0$. There are diminishing returns to effort so that development progresses at a rate $\sqrt{z_i(t)}$. With a slight abuse of notation, firm i's progress up to time t is then $K_i(t) = \int_0^t \sqrt{z_i(s)}ds$. For a given completion time $T_i \geq 0$, the effort profile $z_i(t)$ must satisfy the feasibility condition $K_i(T_i) = K$, so that all steps are finished by the completion date.

The flow of costs associated with technology development are proportional to the effort exerted by firm i, $c_i(t) = z_i(t)/\omega$. Instantaneous costs are then a quadratic function of the rate of progress. Without loss of generality, we take $\omega = 1$.[9] The discounted cost of technology

[9] The parameter ω represents the (common) productivity of firms in terms of new technology development. What matters for the analysis is the number of steps relative to firm productivity K/ω and hence taking $\omega = 1$ is without loss of generality. In Section 7, we generalize the analysis to allow firms to have different development capabilities and hence different ω_i.

development for a given completion time T_i is then

$$\int_0^{T_i} z_i(t)e^{-rt}dt,$$

where $r > 0$ is the common cost of capital (discount rate) for the two firms. This model of technology development was first proposed by Lucas (1971), who solves the monopoly case.[10]

We assume that $\pi_{11} - \pi_{01} > (rK)^2$, which assures that firms have an incentive to develop the technology even if they expect to be second to market.[11]

Firms observe when their competitor completes technology development, but internal development efforts are not observable.

2.3. Solution Concepts

We assume that firms choose their effort levels simultaneously at any point in time and we solve for pure strategy equilibria. We consider both pre-commitment and subgame perfect equilibria, although our primary interest is with the subgame perfect equilibria. In a **pre-commitment development equilibrium** each firm's (pure) strategy consists of a profile $z_i(t)$ that together constitute a Nash equilibrium at the initial date. This implies that firms are committed to their strategies and do not change them in response to the arrival of information. In particular, they do not adjust their development profiles in response to the competitor's deployment of the technology in the market prior to the time implied by the equilibrium effort schedules.

In a **subgame perfect development equilibrium,** firm strategies must constitute a Nash equilibrium in any well defined subgame. There are subgames after one of the two firms completes development, in addition to the overall game starting at time $t = 0$. There are no intermediate subgames given the assumption that a firm's progress is not observable, as in

[10] Other papers building on Lucas (1971) are Grossman and Shapiro (1986) who extend the analysis, Toxvaerd (2006) who incorporates an agency problem, and Pacheco de Almeida and Zemsky (2006) who consider sequential development problems.

[11] Therefore, the model does not apply to winner take all contexts such as patent races, for which $\pi_{11} = \pi_{01}$.

Hopenhayn and Squintani (2006).[12]

We will sometimes invoke the equilibrium refinement of Pareto optimality, which limits attention to those equilibria for which it is not possible to find another equilibrium in which one firm is better off and the other is no worse off. We use this to refine the set of subgame perfect equilibria.

2.4. Payoffs

We say that there is a first mover if the firms develop at different times, that is, if $T_1 \neq T_2$. Let subscript L denote the identity of the first mover and F the identity of the second mover, and for convenience when $T_1 = T_2$ let $L = 1$ and $F = 2$. Thus, T_L is the development time of the first mover and T_F is the development time of the second mover.

Firm payoffs for any pair of effort profiles are the net present value of their cash flows given the implied development times. For the leader, the payoff is

$$\Pi_L(T_L, T_F) = \int_0^{T_L} \pi_{00} e^{-rt} dt + \int_{T_L}^{T_F} \pi_{10} e^{-rt} dt + \int_{T_F}^{\infty} \pi_{11} e^{-rt} dt - \int_0^{T_L} z^*(t; T_L, K) e^{-rt} dt,$$

and for the follower

$$\Pi_F(T_L, T_F) = \int_0^{T_L} \pi_{00} e^{-rt} dt + \int_{T_L}^{T_F} \pi_{01} e^{-rt} dt + \int_{T_F}^{\infty} \pi_{11} e^{-rt} dt - \int_0^{T_F} z^*(t; T_F, K) e^{-rt} dt,$$

where $z^*(t; T_i, K)$ is the cost minimizing effort profile for a firm that plans to complete development at T_i, which is derived in Lemma 3.1.

We say that there is a first mover advantage if there is a first mover and the first mover has a strictly higher payoff than the second mover: $T_L < T_F$ and $\Pi_L(T_L, T_F) > \Pi_F(T_L, T_F)$.

[12] With the aim of being more formal, let us define

$$1_i(t) = \begin{cases} 1 \text{ if } K_i(t) \geq K \\ 0 \text{ else} \end{cases},$$

that is, $1_i(t)$ denotes whether or not firm $i \in \{1, 2\}$ has developed the technology by date t. The time-t history for any firm consists of how many steps are left for completion as well as whether or not the competitor has already developed the new technology by date t. More precisely, the date-t history for firm $i \in \{1, 2\}$, denoted by h_i^t, is defined as follows: $h_i^t = \{z_i(s), 1_{3-i}(s)\}_{0 \leq s < t}$. Letting H_i^t denote the set of all histories for firm i up to time t, a strategy for firm i is an infinite sequence of mappings from H_i^t into an effort level.

3. Cost Minimizing Technology Development

We begin the analysis by characterizing a firm's cost minimizing effort profile for a given development time. This allows us to express the cost of development as a function of the development time, which is a key input into the equilibrium analysis. As we want to consider SPE, where firms may want to reoptimize their effort profiles in mid course, we characterize the cost minimizing effort profile for any number of remaining steps $k \leq K$, not just the initial number of steps K. We have the following result, originally shown by Lucas (1971).

Lemma 3.1. *Suppose that a firm seeks to develop the new technology over an interval of time of length T when the number of remaining steps is $k \in (0, K]$. The cost minimizing effort profile is*

$$z^*(t; T, k) = \left(\frac{rke^{rt}}{e^{rT} - 1} \right)^2,$$

and the progress at any time $t \leq T$ is given by

$$K^*(t; T, k) = \frac{k(e^{rt} - 1)}{e^{rT} - 1}.$$

The minimized cost of technology development is

$$C(T, k) = \frac{rk^2}{e^{rT} - 1}.$$

When $k = K$, we drop the parameter k from these functions (e.g. we write $C(T)$ for $C(T, K)$ or $K^*(t, T)$ for $K^*(t; T, K)$). Figure 3.1 illustrates the progress of the cost minimizing development path as given by $K^*(t, T)$ for the completion times $T = T_L^*$ and $T = T_F^*$, which are defined in the next section. The convexity of the curves arises because $z^*(t; T, k)$ is increasing over time: firms concentrate effort towards the end of the process due to discounting, although the effect is limited by the diminishing returns to effort at a point in time.

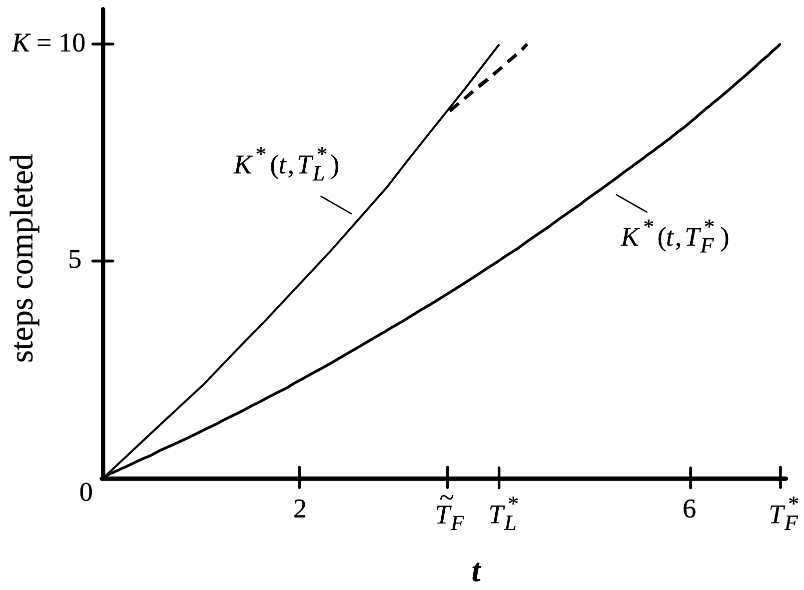

Figure 3.1: Cost minimizing development paths when $K = 10$ and $r = .1$ for the development times T_L^* and T_F^*. These are equilibrium development times when $r = .1$, $\pi_{10} = 9$, $\pi_{11} = 4$, $\pi_{00} = \pi_{01} = 0$. The dotted line shows how the firm that was supposed to be the leader alters its development at the follower's optimal preemption time.

The cost function $C(T, k)$ is an increasing, convex function of the number of steps k and a decreasing, convex function of the development time T. This sort of a time-cost tradeoff goes back to the early literature on development.[13] Note that the expression for development costs in the payoff functions, which is $\int_0^{T_i} z^*(t; T_i, K)e^{-rt}dt$, can be replaced with $C(T_i) = \dfrac{rK^2}{e^{rT_i} - 1}$.

It is useful to consider the optimal development time of a monopolist that increases its flow profits from π to $\pi + \Delta$ when it develops the new technology and development requires K steps. Such a firm's payoff is given by $\Pi(T) = \int_0^T \pi e^{-rt}dt + \int_T^\infty (\pi + \Delta)e^{-rt}dt - C(T)$. There is a unique solution to the first order condition $\Pi'(T) = 0$, given by

$$T^* = \frac{1}{r} \ln(1 - \frac{rK}{\sqrt{\Delta}})^{-1}.$$

If $(rK)^2 < \Delta$, then the firm maximizes its payoff at $T = T^*$, while if $(rK)^2 \geq \Delta$, then its payoff is maximized at $T = \infty$ (i.e., non development). Note that there is no incentive to

[13] Gilbert and Newbery (1982) write "In its simplest representation, T, is a deterministic function of the time paths of expenditures. The present value of an optimal expenditure path defines a cost function $C(T)$, that is a decreasing function of the invention date. The cost function is the same for all firms engaged in research and development," which could equally well describe our model. See also Mansfield (1971).

delay deployment after development is complete as the firm would be better off to slow down development so as to complete just at the deployment date.

4. Adoption of a New technology

Our model of development is very close to models of adoption timing. Not only do we have (by construction) an equivalent structure for the profit flows, but the specific $C(T)$ time-cost tradeoff that we derive satisfies the general properties assumed in the adoption literature (c.f. Fudenberg and Tirole, 1985). While we derive our cost function from an underlying development process, the adoption literature assumes an exogenously given tradeoff.[14] As one of our objectives is to compare and contrast outcomes under development and adoption, we now state the classic results for adoption games for our cost function $C(T)$. For proofs, the reader is referred to the original papers.

We begin with the pre-commitment equilibrium studied by Reinganum (1981a). It is useful to define the following development times

$$T_L^* \equiv \frac{1}{r} \ln(1 - \frac{rK}{\sqrt{\pi_{10} - \pi_{00}}})^{-1}$$

and

$$T_F^* \equiv \frac{1}{r} \ln(1 - \frac{rK}{\sqrt{\pi_{11} - \pi_{01}}})^{-1}.$$

T_L^* is the optimal deployment time for a single firm that expects its profit flow to increase by $\Delta = \pi_{10} - \pi_{00}$ when it deploys the new technology and when adoption costs are given by $C(T) = \dfrac{rK^2}{e^{rT} - 1}$. T_F^* is the optimal time for a single firm that expects its profit flows to increase by $\Delta = \pi_{11} - \pi_{01}$.

Proposition 4.1. *In a pre-commitment adoption equilibrium, there is a first mover and adoption times are* $(T_L, T_F) = (T_L^*, T_F^*)$. *There is a first mover advantage:* $\Pi_L(T_L^*, T_F^*) >$

[14] While the adoption literature usually proceeds by imposing general properties on the function $C(T)$, Katz and Shapiro (1987) propose the functional form $C(T) = Ke^{(r-\lambda)T}$ with $\lambda > r$. This differs from the one we derive. For example, their function has a finite cost of instantaneous development unlike in our case.

$\Pi_F(T_L^*, T_F^*)$.

In a pre-commitment adoption equilibrium, there are no strategic interactions once the order of adoption is fixed. The equilibrium adoption times mirror those of the single firm analysis (with profit flows suitably defined). An attractive feature of this outcome is that there is diffusion (in that firms adopt asynchronously) even though there are symmetric firms and complete certainty. This is due entirely to the curvature of the cost function and the falling benefits of deploying the technology (i.e., $\pi_{11} - \pi_{01} < \pi_{10} - \pi_{00}$). It is straightforward to generalize the pre-commitment analysis to $n > 2$ firms (Reinganum, 1981b; Quirmbach, 1986). Finally, the model has a clear prediction of first mover advantage.[15]

As Fudenberg and Tirole (1985) point out, pre-commitment is critical for Reinganum's results. The equilibrium is supported by the leader's threat to adopt at time T_L^* regardless of the adoption time of the other firm. Without some form of pre-commitment, such a threat is not credible. Once one firm has adopted, the optimal adoption time for the other firm is T_F^*. Combining this observation with the first mover advantage, one has that (T_L^*, T_F^*) cannot constitute a subgame perfect adoption equilibrium outcome. Figure 4.1 shows the profits of a firm when the competitor plans to adopt at T_L^* but best responds to preemptive adoption prior to this time by adopting at T_F^*. The best response by the supposed follower is to adopt just prior to T_L^*. The firm that was supposed to adopt first now adopts at T_F^* and the other firm obtains the superior payoff of the first mover.

As in an auction, the price of being the first mover (in terms of the extra cost of adoption) gets bid up through earlier and earlier adoption in a SPE until the payoffs are equalized. Define the resulting preemption time $\hat{T}_L$ by the smaller of the two solutions to $\Pi_L(\hat{T}_L, T_F^*) = \Pi_F(\hat{T}_L, T_F^*)$, which yields

$$\hat{T}_L = \frac{1}{r}\ln(1 - \frac{rK\sqrt{\pi_{11} - \pi_{01}}}{\pi_{10} - \pi_{01}})^{-1}.$$

As is typical in a duopoly investment game, it is possible that equilibrium investment levels can have a Prisoner's Dilemma structure, where the firms end up over investing because of

[15] To see the existence of a first mover advantage, note that $\Pi_L(T_L^*, T_F^*) > \Pi_L(T_F^*, T_F^*) = \Pi_F(T_F^*, T_F^*) \geq \Pi_F(T_L^*, T_F^*)$.

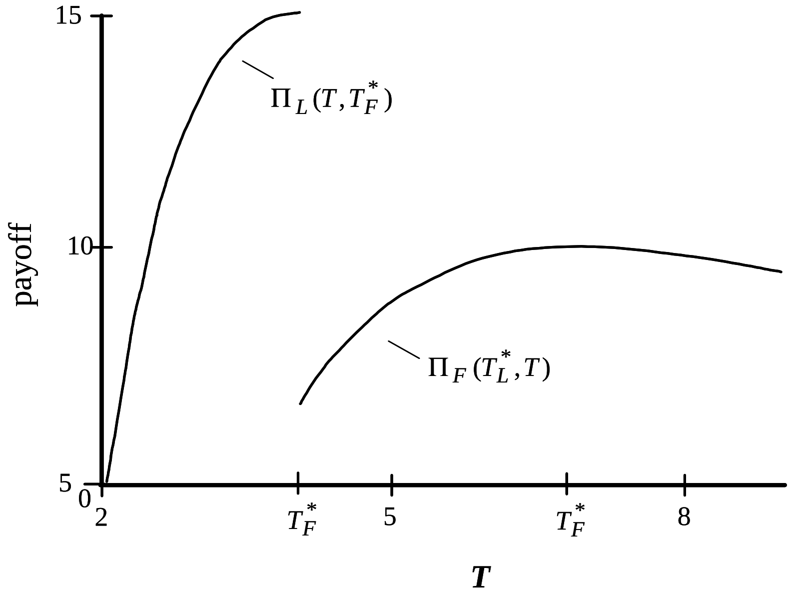

Figure 4.1: The profits of the follower if the leader is planning on adopting at time T_L^* but best responds to pre-emptive adoption by the follower by adopting at T_F^* for the parameters as in Figure 3.1

the negative externality on each other. Specifically, it is possible that firms would be better off if they both waited to adopt until the technology cost had fallen substantially. Such considerations turn out to matter for subgame perfect equilibria. Define the profit maximizing joint adoption time by $\hat{T}_J \equiv \arg\max_T \Pi_L(T, T)$, which is just

$$\hat{T}_J = \frac{1}{r} \ln(1 - \frac{rK}{\sqrt{\pi_{11} - \pi_{00}}})^{-1}.$$

Fudenberg and Tirole (1985) establish the following.

Proposition 4.2. *There is a unique Pareto optimal subgame perfect adoption equilibrium. If $\Pi_L(T_L^*, T_F^*) > \Pi_L(\hat{T}_J, \hat{T}_J)$ then the equilibrium times are $(T_L, T_F) = (\hat{T}_L, T_F^*)$ and otherwise they are $(T_L, T_F) = (\hat{T}_J, \hat{T}_J)$. In either case, there is rent equalization.*

When the maximal payoff from being a first mover exceeds the maximal payoff from waiting and adopting together (i.e., $\Pi_L(T_L^*, T_F^*) > \Pi_L(\hat{T}_J, \hat{T}_J)$), then the unique SPE is the preemption equilibrium $(\hat{T}_L, T_F^*)$. Otherwise, there exist equilibria where the firms wait and adopt jointly at later times.[16] These equilibria are supported by the threat that if a firm adopts prior to the joint adoption time the other firm's best response is to adopt immediately (since $\hat{T}_J > T_F^*$ holds in this case).[17]

While there may or may not be a first mover and hence diffusion in a subgame perfect adoption equilibrium, there is no first mover advantage.

5. Development of a New Technology

This section characterizes both pre-commitment and subgame perfect development equilibria. We are interested in how these equilibria compare to those under adoption, particularly in terms of whether there is a first mover and whether moving first leads to a higher payoff.

5.1. Pre-Commitment Equilibria and their Breakdown

The analysis of pre-commitment development equilibria is straightforward. Given that firms do not reoptimize their effort profiles during the course of the game, there is no loss in assuming that firm strategies involve the choice of a development time T_i given the cost function $C(T_i)$. Hence, the pre-commitment development game is equivalent to the pre-commitment adoption game and we have the following result.

Proposition 5.1. *The pre-commitment development equilibrium has the same development times as the pre-commitment adoption equilibrium,* $(T_L, T_F) = (T_L^*, T_F^*)$.

As with the adoption game, it seems appropriate to consider subgame perfect equilibria that allow firms to reoptimize their strategies in response to the unexpected development of a competitor. The question is whether the pre-commitment outcome breaks down for the case of development, as it does for adoption.

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[16] The joint adoption equilibrium does not arise in discrete time models even in the limit as the period length goes to zero. Papers such as Riordan (1992) avoid the joint development equilibrium by using discrete time models.

[17] The restriction to Pareto optimal equilibria matters when $\Pi_L(T_L^*, T_F^*) \leq \Pi_L(\hat{T}_J, \hat{T}_J)$ as then there are multiple SPE. Typically there are a continuum of joint adoption equilibrium (since it is optimal to adopt earlier if the other firm is expected to adopt then as well) and $(\hat{T}_L, T_F^*)$ continues to be a possible equilibrium outcome.

Consider subgames in which one of the firms has completed development at some time T. Suppose the other firm has $k \in (0, K]$ steps remaining. This is then a single firm development problem where there are k steps to complete, the shift in flow profits is $\Delta = \pi_{11} - \pi_{01}$, and the problem starts at time $t = T$ rather than at $t = 0$. The firm's best response is then

$$\tilde{T}(T, k) \equiv T + \frac{1}{r} \ln(1 - \frac{rk}{\sqrt{\pi_{11} - \pi_{01}}})^{-1}.$$

Figure 3.1 illustrates a reoptimized development path for a firm that is initially developing to be the first mover at time T_L^* but is unexpectedly preempted at a time $\tilde{T}_F < T_L^*$. The dotted line shows the optimal development path after the preemption. When the firm is no longer expecting to be the first mover, it slows down its development, which causes it to complete development after time T_L^*. However, unlike with adoption, it develops before time T_F^* since some of its development has already been completed at the faster pace of a first mover.

Now consider strategies for the full game that lead to the outcome (T_L^*, T_F^*). Firm effort profiles on the equilibrium path must be given by $z^*(t, T_L^*)$ for the leader and $z^*(t, T_F^*)$ for the follower, as this assures cost minimization for the equilibrium development times. We can assume that the follower continues to develop according to $z^*(t, T_F^*)$ even if the leader fails to develop by time T_L^*.[18] All other off equilibrium path strategies are determined by the requirement of subgame perfection as incorporated into the best response function $\tilde{T}(T, k)$.

Observe that the leader does not have a profitable deviation. Since there is no impact on the follower's development time, T_L^* is optimal because $T_L^* = \arg\max_T \Pi_L(T, T_F^*)$ by construction. What about the follower? Because $\Pi_F(T_L^*, T_F^*) > \Pi_F(T_L^*, T_F)$ for $T_F \geq T_L^*$, any profitable deviation would involve preemption at some time $T_F < T_L^*$. Figure 5.1 illustrates the follower's payoff as a function of its development time (for the same parameters used in the previous figures). Unlike the case of adoption illustrated in Figure 4.1, there is no discontinuity at

[18] The follower's effort choice when the leader fails to develop by time T_L^* is off the equilibrium path and not part of a well defined subgame (since the leader's progress is not common knowledge). Hence, the follower's strategy is not restricted by subgame perfection. However, as long as the follower believes that the leader is about to complete develop, then continuing to develop according to $z^*(t, T_F^*)$ is optimal. Moreover, having the follower speed up because it now expects to be first to market with some probability, would not affect the analysis. See subsection 5.4 for further analysis.

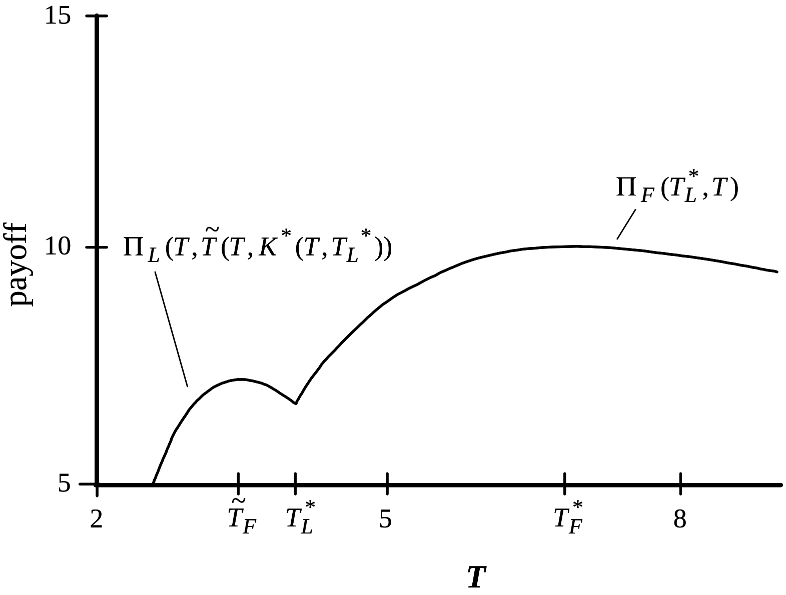

Figure 5.1: The profits of the follower if the leader is planning on developing at time T_L^* but reoptimizes its effort profile in response to preemptive development by the follower for the parameters as in figures 3.1 and 4.1.

$T = T_L^*$. The reason is that the leader has all but completed its development for preemption times just before T_L^* and hence its development time is still close to T_L^*. The earlier a firm preempts the less progress the other firm has made and the more its completion date is delayed, but the greater are the preempting firm's development costs. Define the optimal preemptive deviation given that the leader plans to develop at T_L^* as

$$\tilde{T}_F \equiv \arg\max_T \Pi_L(T, \tilde{T}(T, K - K^*(T, T_L^*)).$$

The question is whether such a deviation is profitable (i.e., $\Pi_L(\tilde{T}_F, \tilde{T}(\tilde{T}_F, K - K^*(\tilde{T}_F, T_L^*)) > \Pi_F(T_L^*, T_F^*))$, in which case (T_L^*, T_F^*) fails to be a subgame perfect development equilibrium. For the parameters used to generate Figure 5.1, the deviation is not profitable. The general conditions required to support (T_L^*, T_F^*) are as follows.

Proposition 5.2. *(i) For a new market (i.e. $\pi_{00} = \pi_{01} = 0$), (T_L^*, T_F^*) constitutes a subgame* *perfect development equilibrium outcome if and only if*

$$\frac{\pi_{10}}{\pi_{11}} \leq 9.$$

(ii) In general, a sufficient condition for (T_L^*, T_F^*) to be a subgame perfect development equilibrium outcome is

$$\frac{\pi_{10} - \pi_{00}}{\pi_{11} - \pi_{01}} \leq 6.85.$$

The pre-commitment outcome is a subgame perfect development outcome when the incentives to preempt are not too large. This occurs, for example, when the flow profits from being first to market π_{10} are not too large and when the flow profits from being second π_{11} are not too small. The bounds in Proposition 5.2 are quite weak, as becomes clear when we apply the result to standard models of competition.

5.2. Application to Cournot and Bertrand Competition

Consider how Proposition 5.2 applies to Cournot competition. Specifically, suppose that the profit flows (π_{00}, π_{10}, π_{11}, π_{01}) result from Cournot competition in which there is linear demand $p = a - b(q_1 + q_2)$ and constant marginal costs that are reduced from c to $c' \geq 0$ when a firm develops the new technology. We assume that $c' < a$ as otherwise all profit flows are zero. There are three cases depending on the value of c.

Suppose $c \geq a$. This is the new market case ($\pi_{00} = \pi_{01} = 0$) and Cournot profits are $\pi_{10} = (a - c')^2/(4b)$ when one firm has completed development and $\pi_{11} = (a - c')^2/(9b)$ when both have completed development. We have that $\pi_{10}/\pi_{11} = 9/4 < 9$. By Proposition 5.2 (i), we have that (T_L^*, T_F^*) is a subgame perfect development equilibrium for any value of c', a and b (such that $c \geq a$).

Now suppose that $(a + c')/2 \leq c < a$, in which case the new technology is a drastic innovation in that the output and profit of a disadvantaged firm is zero ($\pi_{01} = 0$). We have that $\pi_{10} = (a-c')^2/(4b)$, $\pi_{00} = (a-c)^2/(9b)$ and $\pi_{11} = (a-c')^2/(9b)$. Hence, $(\pi_{10}-\pi_{00})/(\pi_{11}-$

$\pi_{01}) < \pi_{10}/\pi_{11} = 9/4 < 6.85$. By Proposition 5.2 (ii), we have that (T_L^*, T_F^*) is a subgame perfect development equilibrium for all parameter values consistent with this case.

Finally, suppose that $c < (a + c')/2$ in which case the new technology is not a drastic innovation and both firms always produce positive amounts. In this case we have that

$$\begin{aligned}\frac{\pi_{10} - \pi_{00}}{\pi_{11} - \pi_{01}} &= \frac{(a + c - 2c')^2/(9b) - (a - c)^2/(9b)}{(a - c')^2/(9b) - (a + c' - 2c)^2/(9b)} \\ &= \frac{a - c'}{a - c} < 2,\end{aligned}$$

where the inequality follows from setting c equal to the upper bound of $(a + c')/2$. Again, we have that (T_L^*, T_F^*) is a subgame perfect development equilibrium. Thus, we have shown the following.

Corollary 5.3. *If flow profits are given by Cournot competition with linear demand and constant marginal costs that are reduced by the new technology, then (T_L^*, T_F^*) is a subgame perfect development equilibrium for all parameter values.*

A similar analysis for Bertrand competition shows that (T_L^*, T_F^*) is a subgame perfect development equilibrium outcome as long as products are slightly differentiated. Again, suppose that firms have constant marginal costs of c without the new technology and $c' < c$ with it. Suppose that there exists a representative consumer whose preferences over the products offered by the two firms are:

$$U(q_1, q_2) = \alpha(q_1 + q_2) - \frac{\beta}{2}(q_1^2 + q_2^2 + 2\theta q_1 q_2) - p_1 q_1 - p_2 q_2,$$

where $\alpha > c'$, $\beta > 0$ and where p_i and q_i respectively denote the price charged and the quantity sold by firm $i \in \{1, 2\}$. Noting that the inverse of parameter $\theta \in (0, 1)$ measures the degree of product differentiation, we have the following sufficient, but not necessary, condition for (T_L^*, T_F^*) to be a subgame perfect development equilibrium outcome in this setting:

Corollary 5.4. *If flow profits are given by Bertrand competition with differentiated products and constant marginal costs that are reduced by the new technology, then (T_L^*, T_F^*) is a subgame* *perfect development equilibrium outcome if* $\theta \leq 0.918$.

5.3. Discussion

Our model of development offers a justification for the pre-commitment equilibrium analysis in Reinganum (1981a, 1981b). Interestingly, many of the elements of our model are present in her verbal descriptions except that what we call development she refers to as "adjustment". For example, Reinganum (1981b) states that $C(T)$ "represents the purchase price of the new technology plus the adjustment costs required to bring the new technology online by date" T and later she refers to the cost savings from "spreading out adjustment". It appears that Reinganum was influenced by the prior literature on time consuming development and that it is Fudenberg and Tirole (1985) who focus the literature on pure adoption problems where deployment is instantaneous. Writing about her diffusion models in the *Handbook of Industrial Organization* after the criticism by Fudenberg and Tirole, Reinganum (1989) is more explicit: "The justification for this assumption [of pre-commitment] is that adoption of a process innovation is a time-consuming activity, with installation and adjustment costs a function of the planned adjustment path. Thus, the choice of an 'adoption date' really represents a time at which adoption will be completed (assuming it begins immediately)."

While our model formalizes the notion of adjustment (i.e., development) costs that are incurred over time, our results go beyond Reinganum's verbal arguments. In the end, the argument in Reinganum (1989) comes down to invoking a large cost to changing effort profiles: "it may be very costly to alter the planned path of adjustment once it has been selected... We assume that such alterations of plans are prohibitively costly". We show that the pre-commitment outcome can be supported even when effort profiles can be changed costlessly. The assumption that adjustment takes place over time and is unobservable is sufficient to obtain the result.[19]

Intuitively, the leader's past development efforts act as a form of partial commitment to an early deployment date. The incentives to preempt are then dampened because the leader

[19] Of course, any costs to changing effort profiles would only reinforce the result and enlarge the parameter space in which (T_L^*, T_F^*) is supported as a subgame perfect development equilibrium.

only responds to preemption to the extent to which it has not already sunk its development efforts.

5.4. Other Possible Subgame Perfect Development Equilibria

What about other possible subgame perfect development equilibrium (SPDE) outcomes, in particular the joint development and preemption outcomes that exist in adoption models? We consider each of these types of equilibria in turn.

Lemma 5.5. *There cannot be a subgame perfect development equilibrium where both firms develop the technology at the same time.*

Late joint development does not occur in subgame perfect equilibria of our model. In these equilibria, firms have an incentive to deviate and develop the technology shortly before the joint date if they can get the first mover profit flows. For the case of adoption, equilibrium joint development times are supported by the credible threat that earlier adoption triggers immediate adoption by the other firm (and hence results in second mover profit flows). In our setting, a deviation does cause the competitor to speed up its development and to deploy earlier than it would have, but this lagged response turns out to be insufficient to support a joint development outcome.[20]

We now further restrict the form of any SPDE. Given Lemma 5.5, there is a firm that expects to develop first and another that expects to develop second in any subgame perfect development equilibrium, i.e. $T_L < T_F$. It must be that the leader develops at some time $T_L < T_F^*$, as otherwise the follower would increase its payoff by developing earlier than the leader. On the other hand, note that the follower's choice of development time has no impact on the leader's development time as long as $T_F \geq T_L$. Since $T_F^* = \arg\max_{T \geq T_L} \Pi_F(T_L, T)$ for $T_L \leq T_F^*$, we have that the follower must develop at time T_F^* in any subgame perfect development equilibrium. Finally, with the follower developing at T_F^* in equilibrium and also

[20] Our result highlights the fragility of a joint deployment outcome. This outcome also fails to be a subgame perfect adoption equilibrium in a discrete time model even in the limit as the period length goes to zero. In both cases, it is because the response to a deviation needs to be instantaneous.

developing at this time for any $T_L < T_F^*$, it cannot be that $T_L > T_L^*$, since the leader does better developing at time T_L^*. In summary, we have that $T_L \leq T_L^*$ and $T_F = T_F^*$ in any (candidate) SPDE.

The above arguments do not rule out preemptive behavior where the leader develops at some time $T_L < T_L^*$. In such equilibria, the leader would delay its development time if the follower's development time were to be fixed at T_F^*. However, this need not be the case, as discussed below. We can rule out full preemption, where $T_L = \widehat{T}_L$ and rents are equalized except for the new market case. With full preemption, the leader would get $\Pi_L(\widehat{T}_L, T_F^*) = \Pi_F(\widehat{T}_L, T_F^*)$. However a deviation by the leader to T_F^* is necessarily more profitable. Let $T' > \widehat{T}_L$ denote the follower's development time when the leader deviates and develops at time T_F^*. For any such T', we have that the leader's deviation is profitable since $\Pi_F(T', T_F^*) > \Pi_F(\widehat{T}_L, T_F^*)$ for $\pi_{00} > 0$.[21] For $T_L < \widehat{T}_L$, a deviation to T_F^* is always profitable. We are left with the following.

Proposition 5.6. *In any subgame perfect development equilibrium the leader develops at some time $T_L \in [\hat{T}_L, T_L^*]$ and the follower develops at time $T_F = T_F^*$. For $\pi_{00} > 0$, $T_L > \hat{T}_L$.*

In general, we are not able to rule out SPDE with some preemption and indeed such equilibria exist. These equilibria involve some rent dissipation relative to the pre-commitment outcome (T_L^*, T_F^*). They are supported by an expectation that the follower will speed up its development if the leader misses its equilibrium development time. Because the state of internal development is private information, the requirement of subgame perfection does not restrict off equilibrium path behavior in this case.[22]

One could go beyond the SPE requirement and ask whether speeding up by the follower is consistent with common knowledge of rationality. The rationality of speeding up requires that the follower now expects to be the first to market (as otherwise the best response is still T_F^*).

[21] For $T' \in (\widehat{T}_L, T_F^*]$ we have that $\Pi_F(T', T_F^*) > \Pi_F(\widehat{T}_L, T_F^*)$ and the deviation is profitable. For $T' > T_F^*$, we have that $\Pi_L(T_F^*, T') > \Pi_L(T_F^*, T_F^*) = \Pi_F(T_F^*, T_F^*) \geq \Pi_F(\widehat{T}_L, T_F^*)$ and the deviation is profitable.

[22] To be precise, let $T_L < T_L^*$ denote the leader's equilibrium development time. The follower's strategy would then be to develop according to $z^*(t, T_F^*)$ for $t \in [0, T_L]$ and to develop according to $z^*(t, T_F, K - K^*(T_L, T_F^*))$ for any $t \in [T_L, T_F]$ for which the leader has not developed for some $T_F \in (T_L, T_F^*)$; after the leader develops the follower's strategy is given by subgame perfection.

This suggests that one could further refine the set of equilibria by restricting off equilibrium path beliefs in some way. We do not pursue such refinements. First, there is no universally accepted refinement of off equilibrium path beliefs, especially in a setting such as ours where player types (the state of internal development) are endogenously determined. Second, we have already restricted the set of equilibria sufficiently that they can be Pareto ranked.

Proposition 5.7. *The unique Pareto optimal subgame perfect development equilibrium involves the timing* $(T_L, T_F) = (T_L^*, T_F^*)$ *whenever this is a SPE outcome.*

Preemption makes each firm worse off. Hence the refinement of Pareto optimality is sufficient to assure uniqueness of the pre-commitment outcome (T_L^*, T_F^*) whenever it exists as a SPE. Thus, we have identified conditions on the payoffs (Proposition 5.2) and a simple equilibrium refinement (Pareto optimality) such that the original outcome in Reinganum (1981a) is the unique subgame perfect equilibrium outcome.

6. Sustainability and Performance

Whether or not diffusion of new technology is driven by preemption matters. Here we focus on the implications for firm performance and the sustainability of the leader's competitive advantage. We follow Pacheco de Almeida and Zemsky (2006) and say that a firm has a competitive advantage at a point in time when it has deployed the new technology and the competitor has not. The sustainability of a competitive advantage is then the interval of time over which the asymmetry persists. When defined in this way, sustainable competitive advantage is distinct from the overall financial performance of the firm, which is net of technology development costs and is captured by the payoff functions Π_L and Π_F in our model.

Formally, we compare the preemption outcome $(\hat{T}_L, T_F^*)$ associated with technology adoption with the outcome (T_L^*, T_F^*) associated with technology development. The sustainability

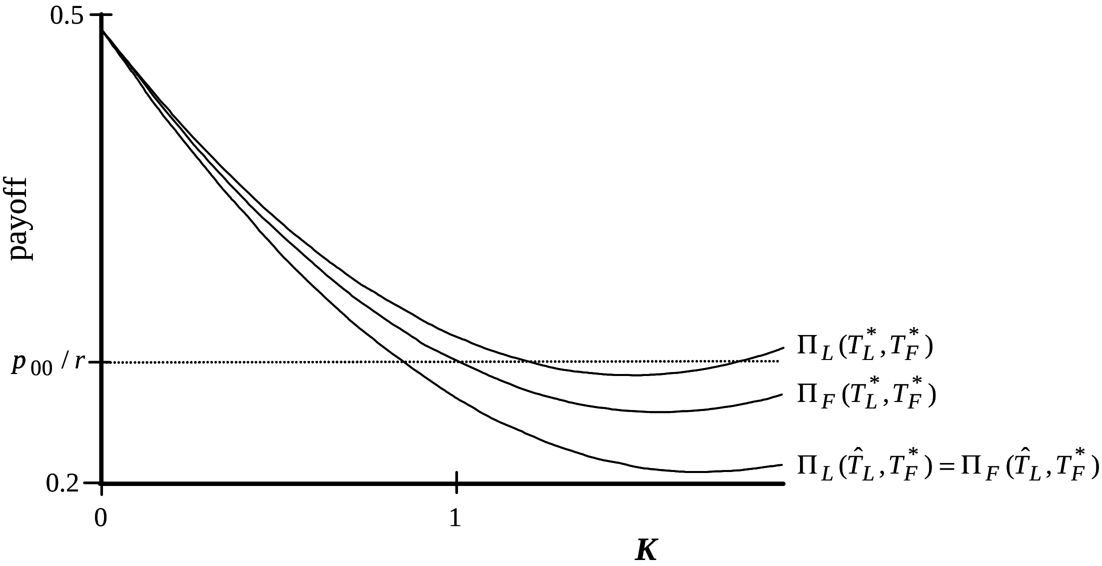

Figure 6.1: Payoffs of leader and follower in subgame perfect development and adoption equilibria as a function of K when flow profits are given by Cournot competition with $c = 1/2$, $c' = 1/3$, $a = b = 1$ and $r = 0.1$.

of the first mover's competitive advantage for each outcome is denoted

$$\hat{S} \equiv T_F^* - \hat{T}_L,$$

$$S^* \equiv T_F^* - T_L^*.$$

Proposition 6.1. *Although $S^* < \hat{S}$, so that the leader's competitive advantage is less sustainable under development than under preemption, the leader's payoff is higher. The payoff ordering is $\Pi_L(T_L^*, T_F^*) > \Pi_F(T_L^*, T_F^*) \geq \Pi_F(\hat{T}_L, T_F^*) = \Pi_L(\hat{T}_L, T_F^*)$ and the follower's payoff is higher under development as well.*

Preemption not only serves to equalizes rents, the greater competitive pressures also serve to depress industry payoffs relative to what occurs under development. Figure 6.1 shows how payoffs vary with the parameter K.[23] The payoff π_{00}/r in Figure 6.1 is what the firms would get if the new technology were not available.

[23] For the case of development, we have interpreted the parameter K as the number of required steps. For the case of adoption, K is a shift parameter in the cost function. A general way to interpret K is as the complexity of the new technology.

In order to provide linkages to potential empirical work, we now consider comparative statics with respect to the parameters K and r. In addition to results on payoffs, we also consider results on timing and sustainability because these may be much easier to observe.

Proposition 6.2. *(i) All of the equilibrium times T_L^*, T_F^*, $\hat{T}_L$ and $\hat{T}_J$ are increasing in K and r. (ii) Sustainability of the leader's competitive advantage is increasing in K and r; the increase is faster for adoption than for development:*

$$\begin{aligned} 0 &< \frac{\partial S^*}{\partial r} < \frac{\partial \hat{S}}{\partial r}, \\ 0 &< \frac{\partial S^*}{\partial K} < \frac{\partial \hat{S}}{\partial K}. \end{aligned}$$

Proposition 6.3. *Restrict attention to the development times (T_L^*, T_F^*). (i) The payoff of the leader and follower are both falling in r and are both non-monotonic in K, first decreasing and then increasing (except for the new market case where the follower's payoffs are everywhere decreasing). (ii) The extent of the leader's first mover advantage, $\Pi_L(T_L^*, T_F^*) - \Pi_F(T_L^*, T_F^*)$, is increasing in K and does not vary with r.*

The differing effects of the parameters on the sustainability of competitive advantage and the overall performance reinforces the point that these are distinct concepts within the model.

7. Differing Capabilities

We now relax the assumption of symmetric firms. In the adoption literature, both Katz and Shapiro (1987) and Riordan (1992) consider asymmetric firms. Specifically, these authors focus on asymmetries in the profit flows across firms. In the case of development, there is another natural avenue for introducing asymmetries, namely differences in firm development capabilities. For example, in empirical work on time-cost tradeoffs, Mansfield (1988) finds that Japanese firms tend to have cost and time advantages over US firms in the commercialization of new technologies. We now generalize our equilibrium analysis to consider asymmetric capabilities. Our primary interest is in whether the conditions under which the pre-commitment outcome can be supported as a SPE get easier or harder to satisfy. A secondary question arises as well, which is whether or not it is possible for the less capable firm to be the first mover.

7.1. Extending the model

Firms vary in the effective number of steps K_i that they must complete in order to develop the new technology. There is a strong firm, denoted by $i = S$, that has greater development capabilities than a weaker rival, denoted by $i = W$. Specifically, $K_S < K_W$.[24]

The payoff functions are as in the base model except that we limit the analysis to the new market case where $\pi_{00} = \pi_{01} = 0$. We define the ratio $\rho \equiv \pi_{10}/\pi_{11} > 1$. The cost of technology development for firm $i \in \{S, W\}$ is still given by Lemma 3.1:

$$C(T_i, K_i) = \frac{rK_i^2}{e^{rT_i} - 1}.$$

We need to generalize the pre-commitment outcome (T_L^*, T_F^*) to reflect the asymmetries. There are now two possible outcomes depending on which firm moves first. Define the strong first mover outcome as (T_{SL}^*, T_{WF}^*) where S is the first mover and where

$$\begin{aligned} T_{SL}^* &= \frac{1}{r}\ln(1 - \frac{rK_S}{\sqrt{\pi_{10} - \pi_{00}}})^{-1}, \\ T_{WF}^* &= \frac{1}{r}\ln(1 - \frac{rK_W}{\sqrt{\pi_{11} - \pi_{01}}})^{-1}. \end{aligned}$$

Similarly, define the weak first mover outcome as (T_{WL}^*, T_{SF}^*) where W is the first mover and where

$$\begin{aligned} T_{WL}^* &= \frac{1}{r}\ln(1 - \frac{rK_W}{\sqrt{\pi_{10} - \pi_{00}}})^{-1}, \\ T_{SF}^* &= \frac{1}{r}\ln(1 - \frac{rK_S}{\sqrt{\pi_{11} - \pi_{01}}})^{-1}. \end{aligned}$$

[24] By "effective number of steps" we have the following in mind. Recall that firm productivity in the base model is paramaterized by ω which we normalize to $\omega = 1$ since what matters is K/ω. We have in mind that firms have differing productivities $\omega_S > \omega_W$. We can then define the effective number of steps for each firm as $K_W = K/\omega_S$ and $K_W = K/\omega_W$.

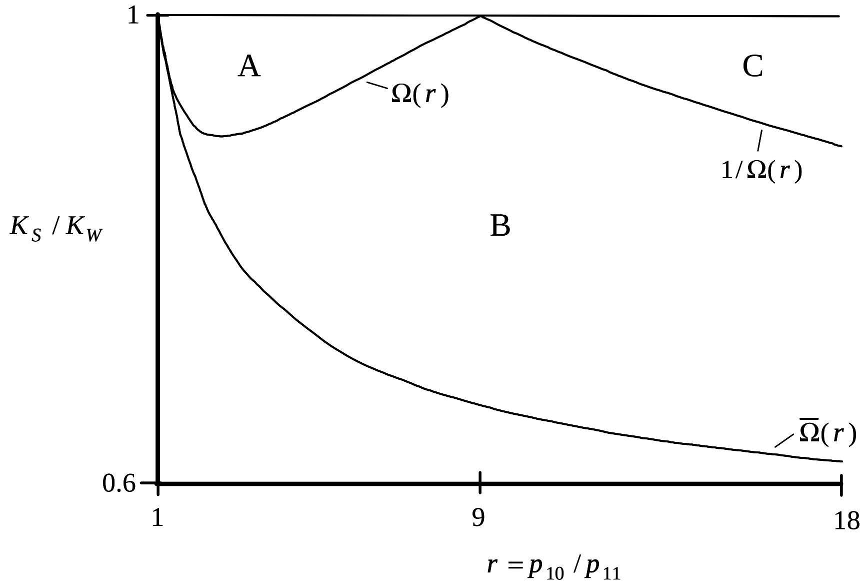

Figure 7.1: The existence regions of different equilibria when firms differ in their development capabilities.

7.2. Equilibrium existence

When $K_W = K_S$, Proposition 5.2 (i) states that $(T_L^*, T_F^*) = (T_{SL}^*, T_{WF}^*) = (T_{WL}^*, T_{SF}^*)$ is a subgame perfect development equilibrium for $\rho = \pi_{10}/\pi_{11} \leq 9$. We are interested in how the support for these equilibria varies as the asymmetries in the firms' development capabilities increase. It turns out that existence depends only on the flow profit ratio ρ and the relative capabilities K_S/K_W. Define

$$(\rho) \equiv \frac{\sqrt{1 + (\rho - 1)\sqrt{\rho}} + 1}{2\sqrt{\rho}},$$

and note that $\quad (9) = 1$ and that $K_S/K_W < 1$.

Proposition 7.1. *(i) The strong first mover outcome is a subgame perfect development equilibrium if and only $K_S/K_W \leq 1/\quad(\rho)$. (ii) The weak first mover outcome is a subgame perfect development equilibrium if and only $K_S/K_W \geq \quad(\rho)$.*

Figure 7.1 presents the results graphically. Asymmetric capabilities make it easier to support the strong first mover equilibrium because preemption by the weak firm gets more and more costly and has less and less effect on the strong firm's development time. Thus, the strong first mover outcome can always be supported as a subgame perfect equilibrium for $\rho \leq 9$ (since it can be supported when there is no asymmetry). For larger values of ρ, the equilibrium can now be supported as long as the capability asymmetry is sufficiently large. Specifically, it can be supported for all parameter values except those in Region C of Figure 7.1. Conversely, asymmetric capabilities make it harder to support the weak first mover outcome since deviation by the strong firm are less costly and have a greater impact on the weak firm's development time. Such outcomes are only supported when $\rho < 9$ and the asymmetries are sufficiently small. In particular, the weak first mover outcome can only be supported in Region A. Since $\ (\rho) > 0.89$, large capability asymmetries are not consistent with this outcome being an equilibrium.

One drawback of the (T_L^*, T_F^*) outcome in the base model is that there is multiplicity: either firm can be the first mover. The determination of which firm gets the rewards of being the first mover lies outside of the model. Once the model is extended to allow for asymmetries, multiplicity is less of an issue. It is only in Region A that there is both the strong and the weak first mover equilibria.

In the base model, (T_L^*, T_F^*) always exists as a pre-commitment equilibrium outcome. Define

$$\overline{\ }(\rho) = \frac{1}{2} + \frac{1}{2\sqrt{\rho}},$$

and note that $\overline{\ }(\rho) < \ (\rho)$ and $\overline{\ }'(\rho) < 0$.

Proposition 7.2. *(i) The strong first mover outcome is a pre-commitment equilibrium for all parameter values. (ii) The weak first mover outcome is a pre-commitment equilibrium if and only if* $\overline{\ }(\rho) \leq K_S/K_W$.

The requirements of the pre-commitment equilibrium are easier to satisfy for both the strong and weak first mover outcome. Thus, the pre-commitment concept overstates the extent to which a weak firm can be a first mover. Figure 7.1 graphs $\overline{\ }(\rho)$. Note that in Region B, the weak first mover outcome is a pre-commitment equilibrium but not a subgame perfect equilibrium, while in Region C, the strong first mover outcome is a pre-commitment development equilibrium but not a subgame perfect development equilibrium.

8. Conclusion

Pre-commitment equilibria are often justified by invoking information lags such that firm actions are only observed with a delay. We develop an alternative justification in which firm actions are implemented over time and hence only partially adjust in response to the arrival of new information. We show that such partial adjustment can reduce the incentives to deviate sufficiently to sustain the pre-commitment outcome as a subgame perfect equilibrium.

Although we have emphasized the assumption of time consuming development, there are several other important assumptions in our theory. We now highlight these and speculate on what would happen if they were relaxed. First, it is important that internal development is unobservable as otherwise firms might compete from the beginning to establish a sufficient lead to emerge as the first mover. Second, we assume a particular curvature for the diminishing returns to effort, specifically the cost of effort is quadratic. We hypothesize that as the extent of diminishing returns goes to zero and the cost of effort becomes linear, then development becomes instantaneous and we would expect preemption incentives to again drive behavior. This suggests that our results require that the time-cost trade-off be sufficiently great. The extent of diminishing returns to effort and the resulting time-cost tradeoff is an empirical question that merits more study. Finally, there is no uncertainty in our development process. We note that adding uncertainty is likely to prove difficult.[25]

We would like to highlight a couple of avenues where future work might prove to be interesting. The first is to relax the assumption that internal development efforts are completely unobservable prior to completion. One way to do this is to consider milestones, such as a working prototype, which are observable.[26] The adoption literature has derived interesting

[25] Lucas (1971) and Grossman and Shapiro (1986) extend the development model we use to include uncertainty about the number of required steps. Their analysis of the single firm problem is already quite complex.

[26] At the other extreme, Harris and Vickers (1985) examine patent races with continuous observation of development efforts, but find no equilibrium when firms move simultaneously (even if firms are allowed to employ mixed strategies). Such a result is likely to persist in settings that are not winner takes all.

results for the case of asymmetric flow profits, which can be interpreted as arising from one firm being an incumbent and the other being an entrant. It would be interesting to extend our analysis to consider the effect of such asymmetries in development problems.

More broadly, IO models often make the simplifying assumption that the actions that firms take are not time consuming. In many settings, such assumptions are hard to justify. Yet they can be critical for the nature of the strategic interactions among firms. In this paper, we show that the timing of new technology deployment can depend critically on whether or not deployment is time consuming. Pacheco de Almeida and Zemsky (2003) show that the classic tradeoff in models of strategic investment under uncertainty between commitment and flexibility depend on whether or not investment is time consuming. What other problems could benefit from explicit modeling of the fact that actions are time consuming?

References

[1] Cohen, W. M., Nelson, R.R., and Walsh, J.P. (2000), "Protecting their Intellectual Assets: Appropriability Conditions and Why U.S. Manufacturing Firms Patent (or Not)", *NBER Working Paper* 7552.

[2] Fudenberg, D. and Tirole J. (1985), "Preemption and Rent Equalization in the Adoption of New Technology", *Review of Economic Studies*, **52**, 383–401.

[3] Gilbert, R.J. and Newbery, D. (1982), "Preemptive Patenting and the Persistence of Monopoly", *American Economic Review*, **72** (3), 514–526.

[4] Grossman, G. and Shapiro, C. (1986), "Optimal Dynamic R&D Programs", *Rand Journal of Economics*, **17** (4), 581–593.

[5] Harris, C. and Vickers, J. (1985), "Perfect Equilibrium in a Model of a Race", *Review of Economic Studies*, **52** (2), 193–209.

[6] Hopenhayn, H. and Squintani, F. (2006), "Preemption Games with Private Information", mimeo, University College of London.

[7] Hoppe, H. (2002), "The Timing of New Technology Adoption: Theoretical Models and Empirical Evidence", *The Manchester School,* **70**, 56–76.

[8] Katz, M. and Shapiro, C. (1987), "R&D Rivalry with Licensing or Imitation", *American Economic Review*, **77** (3), 402–420.

[9] Lieberman, M.B. and Montgomery, D.B. (1988), "First Mover Advantages", *Strategic Management Journal*, **9**, 41-58.

[10] Lucas, R. (1971), "Optimal Management of a Research and Development Project", *Management Science,* **17** (11), 679–697.

[11] Mansfield, E. 1971. *Industrial Research and Technological Innovation: An Econometric Analysis.* Norton: New York.

[12] —— (1988), "The Speed and Cost of Industrial Innovation in Japan and the United States: External vs. Internal Technology", *Management Science,* **34**, 1157–1168.

[13] Pacheco de Almeida, G. and Zemsky, P. (2003), "The Effect of Time-to-Build on Strategic Investment Under Uncertainty", *Rand Journal of Economics*, **34** (1), 166–182.

[14] —— and —— (2006), "The Timing of Resource Development and Sustainable Competitive Advantage", INSEAD working paper 2006/13/ST.

[15] Quirmbach, H.C. (1986), "The Diffusion of New technology and the Market for an Innovation", *Rand Journal of Economics*, **17**, 33–47.

[16] Reinganum, J. (1981 *a*), "On the Diffusion of New Technology: A Game Theoretic Approach", *Review of Economic Studies,* **48**, 395–406.

[17] Reinganum, J. (1981 *b*), "Market Structure and the Diffusion of New Technology", *Bell Journal of Economics,* **153** (2), 618–624.

[18] Reinganum, J. (1989), "The Timing of Innovation: Research, Development and Diffusion", in R. Schmalensee and R. D. Willig (eds) *Handbook of Industrial Organization*, Vol 1, Amsterdam, North-Holland, 850–908.

[19] Riordan, M. H. (1992), "Regulation and Preemptive Technology Adoption", *Rand Journal of Economics*, **23**, 334-349.

[20] Robinson, W. T., Kalyanaram, G. and Urban, G. L., (1994), "First-Mover Advantages from Pioneering New Markets: A Survey of Empirical Evidence", *Review of Industrial Organization*, **9**, 1–23.

[21] Scherer, F. (1967), "Research and Development resource Allocation under Rivalry", *Quarterly Journal of Economics*, **3**, 359–394.

[22] Schmidt-Dengler, P. (2006), "The Timing of New Technology Adoption: The Case of MRI", mimeo, London School of Economics.

[23] Toxvaerd, F. (forthcoming), "A Theory of Optimal Deadlines", *Journal of Economic Dynamics & Control.*

9. Appendix

Proof of Lemma 3.1 Recall that $K(t) = \int_0^t \sqrt{z(s)}ds$ denotes the number of steps completed by a firm at date t when following effort profile $\{z(s)\}_{s\in[0,t]}$. Therefore, if the firm targets at developing the new technology by date T, then it solves the following program:

$$\min_{z(t)\geq 0} \int_0^T z(t)e^{-rt}dt$$
$$\text{s.t. } \frac{dK}{dt} = \sqrt{z(t)},\ K(0) = 0,\ K(T) = k.$$

Denoting the costate variable by $\lambda(t)$, we can write the Hamiltonian function as follows:

$$H(z) = -ze^{-rt} + \lambda(t)\sqrt{z}.$$

The necessary (and sufficient) conditions for an interior maximum require that the following conditions hold:

$$\begin{aligned} \frac{\lambda(t)}{2\sqrt{z}} &= e^{-rt} && (9.1)\\ \frac{dK}{dt} &= \sqrt{z} && (9.2)\\ \frac{d\lambda}{dt} &= 0 && (9.3)\\ K(0) &= 0,\ K(T) = k && (9.4) \end{aligned}$$

Using (9.3), we have that $\lambda(t) = \lambda$ for some constant λ, so $z(t) = \left(\frac{\lambda e^{rt}}{2}\right)^2$ by (9.1). This, together with (9.2), implies that $K(T) - K(0) = \frac{\lambda(e^{rT}-1)}{2r}$. Hence, (9.4) implies that $\lambda = \frac{2rk}{e^{rT}-1}$, and thus we have that the optimal effort profile given T and k takes the following form:

$$z^*(t; T, k) = \left(\frac{rke^{rt}}{e^{rT}-1}\right)^2.$$

Therefore, the number of steps completed at date t by a firm that behaves optimally and wishes to complete k steps at time T is

$$K^*(t; T, k) = \int_0^t \sqrt{z^*(s; T, k)}ds = \frac{k(e^{rt}-1)}{e^{rT}-1}.$$

The minimum cost for a firm of developing at time T a technology that requires the completion of k steps is

$$C(T, k) = \frac{rk^2}{e^{rT}-1}.$$

Proof of Proposition 5.2 Recall that the most profitable preemptive development by the follower should take place at time $\tilde{T}_F$. To show that $\tilde{T}_F$ exists and is uniquely defined, we prove that $\Pi_L(T, \tilde{T}(T, \frac{K(e^{rT_L^*}-e^{rT})}{e^{rT_L^*}-1}))$ is strictly quasi concave in T. Note that

$$\Pi_L(T, \tilde{T}(T, \tfrac{K(e^{rT_L^*}-e^{rT})}{e^{rT_L^*}-1})) = \int_0^T \pi_{00}e^{-rt}dt + \int_T^{\tilde{T}} \pi_{10}e^{-rt}dt + \int_{\tilde{T}}^{\infty} \pi_{11}e^{-rt}dt - \frac{rK^2}{e^{rT}-1}, \quad (9.5)$$

where we have suppressed the arguments of $\tilde{T}(T, \frac{K(e^{rT_L^*}-e^{rT})}{e^{rT_L^*}-1})$ for simplicity. Differentiating

(9.5) with respect to T yields:

$$\frac{d\Pi_L(T,\tilde{T}(T,\frac{K(e^{rT_L^*}-e^{rT})}{e^{rT_L^*}-1}))}{dT} = \frac{(rK)^2e^{rT}}{(e^{rT}-1)^2}-(\pi_{10}-\pi_{00})e^{-rT}+ (\frac{d\tilde{T}(T,\frac{K(e^{rT_L^*}-e^{rT})}{e^{rT_L^*}-1})}{dT})e^{-r\tilde{T}(T,\frac{K(e^{rT_L^*}-e^{rT})}{e^{rT_L^*}-1})}(\pi_{10}-\pi_{11}).$$

Note that

$$\begin{aligned}\frac{d\tilde{T}(T,\frac{K(e^{rT_L^*}-e^{rT})}{e^{rT_L^*}-1})}{dT} &= 1-\frac{rKe^{rT}}{\sqrt{\pi_{11}-\pi_{01}}(e^{rT_L^*}-1)-rK(e^{rT_L^*}-e^{rT})} \\ &= \frac{rKe^{rT_L^*}-\sqrt{\pi_{11}-\pi_{01}}(e^{rT_L^*}-1)}{rK(e^{rT_L^*}-e^{rT})-\sqrt{\pi_{11}-\pi_{01}}(e^{rT_L^*}-1)}.\end{aligned}$$

Therefore, letting $d\Pi_L(T,\tilde{T}(T,\frac{K(e^{rT_L^*}-e^{rT})}{e^{rT_L^*}-1}))/dT=0$ and rearranging yields the following:

$$(\pi_{10}-\pi_{00})e^{-rT}-\frac{(rK)^2e^{rT}}{(e^{rT}-1)^2}+(\pi_{10}-\pi_{11})\left(\frac{rKe^{rT_L^*}-\sqrt{\pi_{11}-\pi_{01}}(e^{rT_L^*}-1)}{rK(e^{rT_L^*}-e^{rT})-\sqrt{\pi_{11}-\pi_{01}}(e^{rT_L^*}-1)}\right)\times \left(\frac{rK(e^{rT_L^*}-e^{rT})-\sqrt{\pi_{11}-\pi_{01}}(e^{rT_L^*}-1)}{\sqrt{\pi_{11}-\pi_{01}}(e^{rT_L^*}-1)e^{rT}}\right)=0,$$

or, equivalently,

$$\left(\frac{rKe^{rT}}{e^{rT}-1}\right)^2+(\pi_{11}-\pi_{00})+\frac{rK(\pi_{10}-\pi_{11})}{\sqrt{\pi_{11}-\pi_{01}}(1-e^{-rT_L^*})}=0.$$

Hence,

$$\tilde{T}_F=-\frac{1}{r}\ln(1-\frac{rK}{(\pi_{11}-\pi_{00}+\frac{rK(\pi_{10}-\pi_{11})}{(1-e^{-rT_L^*})\sqrt{\pi_{11}-\pi_{01}}})^{\frac{1}{2}}}), \tag{9.6}$$

which shows that the most profitable deviation by the follower is unique.[27]

To conclude the proof, it remains to examine the conditions under which $\Pi_F(T_L^*,T_F^*)\geq \Pi_L(\tilde{T}_F,\tilde{T}(\tilde{T}_F,\frac{K(e^{rT_L^*}-e^{r\tilde{T}_F})}{e^{rT_L^*}-1}))$. Very tedious algebra shows that

$$\Pi_F(T_L^*,T_F^*)-\Pi_L(\tilde{T}_F,\tilde{T}(\tilde{T}_F,\frac{K(e^{rT_L^*}-e^{r\tilde{T}_F})}{e^{rT_L^*}-1}))\geq 0$$

if and only if the following holds:

$$2\sqrt{(\pi_{10}-\pi_{00})(\pi_{11}-\pi_{00})(\pi_{11}-\pi_{01})+(\pi_{10}-\pi_{11})(\pi_{10}-\pi_{00})\sqrt{(\pi_{10}-\pi_{00})(\pi_{11}-\pi_{01})}}- (\pi_{11}+\pi_{10}-2\pi_{01})\sqrt{\pi_{10}-\pi_{00}}+(\pi_{00}-\pi_{01})\sqrt{\pi_{11}-\pi_{01}}\geq 0 \tag{9.7}$$

[27] It is simple to show that $d^2\Pi_L(\tilde{T}_F,\tilde{T}(\tilde{T}_F,\frac{K(e^{rT_L^*}-e^{r\tilde{T}_F})}{e^{rT_L^*}-1}))/dT^2<0$ so that $\Pi_L(T,\tilde{T}(T,\frac{K(e^{rT_L^*}-e^{rT})}{e^{rT_L^*}-1}))$ is indeed single-peaked, as we claimed.

In the case of new markets (i.e., $\pi_{00} = \pi_{01} = 0$), it is straightforward to see that this condition holds if and only if $\dfrac{\pi_{10}}{\pi_{11}} \leq 9$,[28] so let us focus now on the cases in which $\pi_{00} > \pi_{01} \geq 0$, recalling that we have already imposed the additional following parameter restrictions: $\pi_{10} > \pi_{11}$, $\pi_{11} \geq \pi_{00}$ and $\pi_{10} - \pi_{00} > \pi_{11} - \pi_{01}$. Since (9.7) is expressed in terms of differences, we can normalize so that $\pi_{01} = 0$. Then dividing by $(\pi_{10})^{3/2} > 0$ yields an equivalent condition:

$$2\sqrt{(1-\hat{\pi}_{00})(\hat{\pi}_{11}-\hat{\pi}_{00})\hat{\pi}_{11} + (1-\hat{\pi}_{11})(1-\hat{\pi}_{00})\sqrt{(1-\hat{\pi}_{00})\hat{\pi}_{11}}} - (\hat{\pi}_{11}+1)\sqrt{1-\hat{\pi}_{00}} + \hat{\pi}_{00}\sqrt{\hat{\pi}_{11}} \geq 0, \tag{9.8}$$

where $\hat{\pi}_{00} = \dfrac{\pi_{00}-\pi_{01}}{\pi_{10}-\pi_{01}}$ and $\hat{\pi}_{11} = \dfrac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{01}}$. We now have the following parameter restrictions: $1 > \hat{\pi}_{11} \geq \hat{\pi}_{00} > 0$ and $1 - \hat{\pi}_{00} > \hat{\pi}_{11}$. Subject to these constraints, extensive numerical analysis shows that (9.8) is automatically satisfied if $\hat{\pi}_{11} > .1274$.[29] Equivalently, it is satisfied if $7.85(\pi_{11} - \pi_{01}) \geq (\pi_{10} - \pi_{00}) + (\pi_{00} - \pi_{01})$ holds. Because $\pi_{11} - \pi_{01} \geq \pi_{00} - \pi_{01}$, a sufficient condition for (9.7) to be satisfied is $6.85(\pi_{11} - \pi_{01}) \geq \pi_{10} - \pi_{00}$.

Proof of Corollary 5.4 We show the result by partitioning the possible values of α into four ranges. First of all, if $c' < \alpha \leq c$, then we have the new market case. In such a situation, it is standard to show that the demand system assumed and price competition between the two firms imply that the profits of the first firm to develop the new technology are $\pi_{10} = \dfrac{(\alpha - c')^2}{4\beta} > \pi_{01} = 0$, while the profits when both firms have the technology are $\pi_{11} = \dfrac{(1-\theta)(\alpha-c')^2}{(2-\theta)^2(1+\theta)\beta} > \pi_{00} = 0$. As a result, we have that $\dfrac{\pi_{10}}{\pi_{11}} = \dfrac{(2-\theta)^2(1+\theta)}{4(1-\theta)}$, which is not larger than 9 if and only if $\theta \leq .939$ holds.

Second, if $c < \alpha \leq c + \dfrac{\theta(c-c')}{2-\theta}$, then both firms are profitable whenever they share the same technology. However, the development of the new technology results in a drastic innovation and leaves the competitor with zero profit. More specifically, it is easy to show that $\pi_{10} = \dfrac{(\alpha-c')^2}{4\beta} > \pi_{01} = 0$ and $\pi_{11} = \dfrac{[(\alpha-c')(1-\theta)(\theta+2)]^2}{(4-\theta^2)^2(1-\theta^2)\beta} > \pi_{00} = \dfrac{[(\alpha-c)(1-\theta)(\theta+2)]^2}{(4-\theta^2)^2(1-\theta^2)\beta}$. Hence,

$$\frac{\pi_{10}-\pi_{00}}{\pi_{11}} = \frac{(2-\theta)^2(1+\theta)}{4(1-\theta)} - \left(\frac{\alpha-c}{\alpha-c'}\right)^2.$$

Because $\dfrac{(2-\theta)^2(1+\theta)}{4(1-\theta)} \leq 6.85$ if $\theta \leq .918$, it follows that $\dfrac{\pi_{10}-\pi_{00}}{\pi_{11}} < 6.85$ if $\theta \leq .918$ holds.

Third, if $c + \dfrac{\theta(c-c')}{2-\theta} < \alpha < c + \dfrac{\theta(c-c')}{(1-\theta)(2+\theta)}$, it can be shown that each firm attains $\pi_{00} = \dfrac{[(\alpha-c)(1-\theta)(\theta+2)]^2}{(4-\theta^2)^2(1-\theta^2)\beta}$ as long as no firm has developed the new technology. Because we have a non-drastic innovation in this case, the first firm in developing the technology prices the rival

[28] To prove it, let $\rho \equiv \pi_{10}/\pi_{11}$ and note that we simply have to show that $\sqrt{4\rho^2[1+(\rho^2-1)\rho]} \geq \rho(\rho^2+1)$ if and only if $\sqrt{\rho} \leq 3$. This can be readily seen from the set of solutions to the following equation, recalling that ρ exceeds 1: $\rho^4 - 4\rho^3 + 2\rho^2 + 4\rho - 3 = 0$.

[29] Although this can be easily checked graphically, we followed two different numerical procedures to verify this claim. On the one hand, we randomly generated over a million numbers for $\hat{\pi}_{00}$ and $\hat{\pi}_{11}$ (between 0 and .5 and 0 and 1, respectively). Among those pairs satisfying the constraints, including $\hat{\pi}_{11} > .1274$, we checked whether or not (9.8) held. All of them did. On the other hand, we generated over a million points using a grid and again found no counter example to (9.8).

out of the market, which can be easily shown to imply that $\pi_{10} = \dfrac{(\alpha - c)(c - \theta c' - (1-\theta)\alpha)}{\beta\theta^2} > \pi_{01} = 0$. In addition, each firm makes a profit of $\pi_{11} = \dfrac{[(\alpha - c')(1-\theta)(\theta+2)]^2}{(4-\theta^2)^2(1-\theta^2)\beta}$ when both have developed the technology. Therefore, we have that:

$$
\begin{aligned}
\frac{\pi_{10} - \pi_{00}}{\pi_{11}} &= \frac{(4-\theta^2)^2(1-\theta^2)(\alpha - c)(c - \theta c' - (1-\theta)\alpha) - [(\alpha - c)\theta(1-\theta)(\theta+2)]^2}{[(\alpha - c')\theta(1-\theta)(\theta+2)]^2} \\
&= \frac{(2-\theta)^2(1+\theta)(\alpha - c)(\theta(\alpha - c') - (\alpha - c))}{(\alpha - c')^2\theta^2(1-\theta)} - \left(\frac{\alpha - c}{\alpha - c'}\right)^2 \\
&= \frac{(2-\theta)^2(1+\theta)}{\theta(1-\theta)}\left(\frac{\alpha - c}{\alpha - c'}\right) - \frac{2(2-\theta^2)}{\theta^2(1-\theta)}\left(\frac{\alpha - c}{\alpha - c'}\right)^2.
\end{aligned}
$$

The right hand side is a strictly concave function of $(\alpha - c)/(\alpha - c')$. Because the constraints on this case imply that $\dfrac{\alpha - c}{\alpha - c'} \in (\dfrac{\theta}{2}, \dfrac{\theta}{2-\theta^2})$ and the unconstrained global maximum is attained at $\dfrac{(2-\theta)^2(1+\theta)\theta}{4(2-\theta^2)} < \dfrac{\theta}{2}$, $\dfrac{\pi_{10} - \pi_{00}}{\pi_{11}}$ can be at most $\dfrac{(2-\theta)^2(1+\theta) - (2-\theta^2)}{2(1-\theta)}$ for feasible values of α, β, θ, c and c'. This implies that $\dfrac{\pi_{10} - \pi_{00}}{\pi_{11}} \leq 6.85$ for any $\theta \leq .92$, since $\dfrac{(2-\theta)^2(1+\theta) - (2-\theta^2)}{2(1-\theta)}$ is monotone increasing in θ.

Fourth and last, when $\alpha \geq c + \dfrac{\theta(c - c')}{(1-\theta)(2+\theta)}$, both firms are always profitable, and it is straightforward to prove that

$$
\pi_{10} = \frac{[(\alpha - c')(1-\theta)(\theta+2) - \theta(c' - c)]^2}{(4-\theta^2)^2(1-\theta^2)\beta} > \pi_{01} = \frac{[(\alpha - c)(1-\theta)(\theta+2) + \theta(c' - c)]^2}{(4-\theta^2)^2(1-\theta^2)\beta}
$$

and

$$
\pi_{11} = \frac{[(\alpha - c')(1-\theta)(\theta+2)]^2}{(4-\theta^2)^2(1-\theta^2)\beta} > \pi_{00} = \frac{[(\alpha - c)(1-\theta)(\theta+2)]^2}{(4-\theta^2)^2(1-\theta^2)\beta}.
$$

Hence,

$$
\begin{aligned}
\frac{\pi_{10} - \pi_{00}}{\pi_{11} - \pi_{01}} &= \frac{[(\alpha - c')(1-\theta)(\theta+2) - \theta(c' - c)]^2 - [(\alpha - c)(1-\theta)(\theta+2)]^2}{[(\alpha - c')(1-\theta)(\theta+2)]^2 - [(\alpha - c)(1-\theta)(\theta+2) + \theta(c' - c)]^2} \\
&= \frac{[(1-\theta)(\theta+2) + \theta]^2 + 2(1-\theta)(\theta+2)[(1-\theta)(\theta+2) + \theta]\left(\frac{\alpha - c}{c - c'}\right)}{[(1-\theta)^2(\theta+2)^2 - \theta^2] + 2(1-\theta)(\theta+2)[(1-\theta)(\theta+2) + \theta]\left(\frac{\alpha - c}{c - c'}\right)} \\
&= 1 + \frac{2\theta[(1-\theta)(\theta+2) + \theta]}{[(1-\theta)^2(\theta+2)^2 - \theta^2] + 2(1-\theta)(\theta+2)[(1-\theta)(\theta+2) + \theta]\left(\frac{\alpha - c}{c - c'}\right)} \\
&= 1 + \frac{2\theta}{(1-\theta)(\theta+2) - \theta + 2(1-\theta)(\theta+2)\left(\frac{\alpha - c}{c - c'}\right)}.
\end{aligned}
$$

Because the right hand side is monotone decreasing in $\dfrac{\alpha - c}{c - c'}$ and $\dfrac{\alpha - c}{c - c'} \geq \dfrac{\theta}{(1-\theta)(2+\theta)}$, $\dfrac{\pi_{10} - \pi_{00}}{\pi_{11} - \pi_{01}}$ can be at most $1 + \dfrac{2\theta}{(1-\theta)(\theta+2) + \theta} = 1 + \dfrac{2\theta}{2-\theta^2}$, which is smaller than 6.85 for any $\theta \in (0,1)$, since $\dfrac{2\theta}{2-\theta^2}$ increases with θ.

Proof of Lemma 5.5 Suppose to the contrary that there exists some $\overline{T}$ such that both firms develop simultaneously at $\overline{T}$ in equilibrium.[30] Keep the effort profile of one of the firms fixed (as long as development by the rival does not take place), and observe that sticking to the equilibrium effort profile that results in a development time of $\overline{T}$ yields a payoff of $\Pi_L(\overline{T},\overline{T}) = \Pi_F(\overline{T},\overline{T})$ for any of the firms. Suppose first that $\overline{T} \le T_L^*$ and consider a unilateral deviation by any of the firms such that it delays development until T_F^* so as to attain $\Pi_F(\overline{T}, T_F^*)$. To see that doing so would increase the firm's payoff, note that the payoff to following is maximized at $T_F^* > T_L^* \ge \overline{T}$ for any development date by the leader, so $\Pi_F(\overline{T}, T_F^*) > \Pi_F(\overline{T},\overline{T})$ indeed holds. Hence, we cannot have $\overline{T} \le T_L^*$.

Suppose now that $\overline{T} \in (T_L^*, T_F^*)$, and consider a deviation by one of the firms such that it preempts its competitor at $T_L^* < \overline{T}$. Such a move would imply that the competitor should develop by date $\tilde{T}(T_L^*, \frac{K(e^{r\overline{T}}-e^{rT_L^*})}{e^{r\overline{T}}-1}) > \overline{T}$, that is, the preempted firm slows down once the competitor develops the new technology at T_L^*.[31] Therefore, the preempting firm attains a payoff equal to $\Pi_L(T_L^*, \tilde{T}(T_L^*, \frac{K(e^{r\overline{T}}-e^{rT_L^*})}{e^{r\overline{T}}-1}))$, which is greater than $\Pi_L(T_L^*, \overline{T})$ because $\tilde{T}(T_L^*, \frac{K(e^{r\overline{T}}-e^{rT_L^*})}{e^{r\overline{T}}-1}) > \overline{T}$. Note that $\overline{T} > T_L^*$, so $\Pi_L(T_L^*, \overline{T}) > \Pi_L(\overline{T},\overline{T})$, and hence we have shown that

$$\Pi_L(T_L^*, \tilde{T}(T_L^*, \frac{K(e^{r\overline{T}} - e^{rT_L^*})}{e^{r\overline{T}}-1})) > \Pi_L(T_L^*, \overline{T}) > \Pi_L(\overline{T},\overline{T}),$$

which implies that any firm could improve its payoff by unilaterally deviating and preempting its rival at $T_L^* < \overline{T}$, a contradiction.

Therefore, we must have that $\overline{T} \ge T_F^*$. We proceed to show that any of the firms could unilaterally preempt its competitor at T_L^* and improve its payoff despite speeding up development by the competitor. This will constitute a contradiction and hence will conclude the proof. It clearly suffices to show that $\widehat{\Pi}(\overline{T}) \equiv \Pi_L(T_L^*, \tilde{T}(T_L^*, \frac{K(e^{r\overline{T}}-e^{rT_L^*})}{e^{r\overline{T}}-1})) - \Pi_L(\overline{T},\overline{T}) > 0$ for any $\overline{T} \ge T_F^*$. Straightforward manipulations yield that

$$\begin{aligned}\widehat{\Pi}(\overline{T}) \;=\; & \left(\frac{\pi_{11}-\pi_{00}}{r}\right)(e^{-rT_L^*} - e^{-r\overline{T}}) + \left(\frac{\pi_{10}-\pi_{11}}{r}\right)\left(\frac{rK}{\sqrt{\pi_{11}-\pi_{01}}}\right)\left(\frac{e^{r\overline{T}} - e^{rT_L^*}}{e^{r\overline{T}}-1}\right)e^{-rT_L^*} \\ & + \frac{rK^2}{e^{r\overline{T}}-1} - K(\sqrt{\pi_{10}-\pi_{00}} - rK).\end{aligned}$$

Differentiating and performing some algebra after using $T_L^* = -\dfrac{1}{r}\ln(1 - \dfrac{rK}{\sqrt{\pi_{10}-\pi_{00}}})$ yields that

$$\frac{d\widehat{\Pi}(\overline{T})}{d\overline{T}} = e^{-r\overline{T}}\left[\pi_{11} - \pi_{00} + \left(\frac{rK}{1-e^{-r\overline{T}}}\right)^2 \left(\frac{\pi_{10}-\pi_{11}}{\sqrt{\pi_{10}-\pi_{00}}\sqrt{\pi_{11}-\pi_{01}}} - 1\right)\right].$$

[30] Note by Lemma 3.1 that this would require both firms to exert effort at time t as dictated by $z^*(t;\overline{T},K) = \left(\dfrac{rKe^{rt}}{e^{r\overline{T}}-1}\right)^2$.

[31] This holds because the preempted firm was completing steps at a faster pace than had it chosen to move optimally in the second place (i.e., at T_F^*) from the beginning of the game.

Hence, $\dfrac{d\widehat{\Pi}(\overline{T})}{d\overline{T}} > 0$ if the following inequality holds

$$\frac{\pi_{10}-\pi_{11}}{\sqrt{\pi_{10}-\pi_{00}}\sqrt{\pi_{11}-\pi_{01}}} \geq 1. \tag{9.9}$$

This, together with the fact that $\widehat{\Pi}(T_F^*) > 0$, which can be shown to hold doing some algebra because $\sqrt{\pi_{10}-\pi_{00}} > \sqrt{\pi_{11}-\pi_{01}}$, implies that $\widehat{\Pi}(\overline{T}) > 0$ for any $\overline{T} \geq T_F^*$. When inequality (9.9) is not satisfied, $\widehat{\Pi}(\overline{T})$ can be easily seen to be strictly quasi convex with a global minimum attained at

$$\overline{T}^* = -\frac{1}{r}\ln(1 - \frac{rK}{\sqrt{\pi_{11}-\pi_{00}}}\sqrt{\frac{\sqrt{\pi_{10}-\pi_{00}}\sqrt{\pi_{11}-\pi_{01}} - (\pi_{10}-\pi_{11})}{\sqrt{\pi_{10}-\pi_{00}}\sqrt{\pi_{11}-\pi_{01}}}}).$$

It is easy to show that in such a case $\overline{T}^* < T_F^*$,[32] so strict quasi convexity implies that $\widehat{\Pi}(\overline{T})$ is monotonically increasing for $\overline{T} \geq T_F^*$, and hence $\widehat{\Pi}(\overline{T}) > 0$ follows from $\widehat{\Pi}(T_F^*) > 0$.

Proof of Proposition 6.2 We first perform comparative statics for K. Part (i) follows by inspection, so we just prove part (ii). Because

$$\frac{\partial S^*}{\partial K} = \frac{\partial T_F^*}{\partial K} - \frac{\partial T_L^*}{\partial K} = \frac{1}{\sqrt{\pi_{11}-\pi_{01}} - rK} - \frac{1}{\sqrt{\pi_{10}-\pi_{00}} - rK} > 0$$

and

$$\frac{\partial \widehat{T}_L}{\partial K} = \frac{\sqrt{\pi_{11}-\pi_{01}}}{\pi_{10}-\pi_{01} - rK\sqrt{\pi_{11}-\pi_{01}}} < \frac{1}{\sqrt{\pi_{10}-\pi_{00}} - rK} = \frac{\partial T_L^*}{\partial K},$$

we have that $\dfrac{\partial \hat{S}}{\partial K} > \dfrac{\partial S^*}{\partial K} > 0$.

We conclude the proof by performing comparative statics for r. Thus, take derivatives so as to get

$$\frac{\partial T_F^*}{\partial r} = \frac{\dfrac{rK}{\sqrt{\pi_{11}-\pi_{01}} - rK} + \ln\left(1 - \dfrac{rK}{\sqrt{\pi_{11}-\pi_{01}}}\right)}{r^2},$$

$$\frac{\partial T_L^*}{\partial r} = \frac{\dfrac{rK}{\sqrt{\pi_{10}-\pi_{00}} - rK} + \ln\left(1 - \dfrac{rK}{\sqrt{\pi_{10}-\pi_{00}}}\right)}{r^2},$$

$$\frac{\partial \hat{T}_J}{\partial r} = \frac{\dfrac{rK}{\sqrt{\pi_{11}-\pi_{00}} - rK} + \ln\left(1 - \dfrac{rK}{\sqrt{\pi_{11}-\pi_{00}}}\right)}{r^2}$$

[32] Some simple algebra yields that $\overline{T}^* < T_F^*$ if and only if

$$\frac{\pi_{10}-\pi_{11}}{\sqrt{\pi_{10}-\pi_{00}}} - \frac{\pi_{00}-\pi_{01}}{\sqrt{\pi_{11}-\pi_{01}}} > 0.$$

The left hand side is increasing in π_{10}, so using the lower bound on π_{10} (namely, $\pi_{10} > \pi_{11} + \pi_{00} - \pi_{01}$) yields the desired result.

and

$$\frac{\partial \widehat{T}_L}{\partial r} = \frac{\dfrac{rK\sqrt{\pi_{11}-\pi_{01}}}{\pi_{10}-\pi_{01}-rK\sqrt{\pi_{11}-\pi_{01}}} + \ln\left(1 - \dfrac{rK\sqrt{\pi_{11}-\pi_{01}}}{\pi_{10}-\pi_{01}}\right)}{r^2}.$$

The four derivatives can be easily shown to be increasing in K, so the fact that each of the expressions becomes zero for $K = 0$ implies that the derivatives are positive for any feasible K, which proves part (i). Although considerably more tedious, a similar argument applies to the differences $T_F^* - T_L^*$ and $T_L^* - \widehat{T}_L$ using respectively that $\sqrt{\dfrac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{00}}} < 1$ and $\dfrac{\pi_{10}-\pi_{01}}{\sqrt{\pi_{10}-\pi_{00}}\sqrt{\pi_{11}-\pi_{01}}} > 1$. This shows that $\dfrac{\partial \hat{S}}{\partial r} > \dfrac{\partial S^*}{\partial r} > 0$, and hence completes the proof.

Proof of Proposition 6.3 We first deal with (i). On the one hand, note that

$$\frac{\partial \Pi_L(T_L^*, T_F^*)}{\partial r} = \frac{\partial \Pi_F(T_L^*, T_F^*)}{\partial r} = \frac{(rK)^2 - \pi_{11}}{r^2} < 0,$$

which shows that the leader and the follower's equilibrium payoff both fall in r. On the other, an application of the envelope theorem yields

$$\frac{\partial \Pi_F(T_L^*, T_F^*)}{\partial K} = \frac{\pi_{00}-\pi_{01}}{\sqrt{\pi_{10}-\pi_{00}}} - 2(\sqrt{\pi_{11}-\pi_{01}} - rK).$$

Because $\dfrac{\partial^2 \Pi_F(T_L^*, T_F^*)}{\partial K^2} = 2r > 0$, there exists a $K' = \dfrac{\sqrt{\pi_{11}-\pi_{01}}}{r} - \dfrac{\pi_{00}-\pi_{01}}{2r\sqrt{\pi_{10}-\pi_{00}}} \leq \dfrac{\sqrt{\pi_{11}-\pi_{01}}}{r}$ (where the inequality is weak if and only if $\pi_{00} = \pi_{01} = 0$) such that $\dfrac{\partial \Pi_F(T_L^*, T_F^*)}{\partial K} < 0$ for $0 < K < \max(0, K')$ and $\dfrac{\partial \Pi_F(T_L^*, T_F^*)}{\partial K} > 0$ for $\max(0, K') < K < \dfrac{\sqrt{\pi_{11}-\pi_{01}}}{r}$. In turn,

$$\frac{\partial \Pi_L(T_L^*, T_F^*)}{\partial K} = \frac{\pi_{10}-\pi_{11}}{\sqrt{\pi_{11}-\pi_{01}}} - 2(\sqrt{\pi_{10}-\pi_{00}} - rK),$$

so the strict convexity of $\Pi_L(T_L^*, T_F^*)$ implies that there exists a $K'' = \dfrac{\sqrt{\pi_{10}-\pi_{00}}}{r} - \dfrac{\pi_{10}-\pi_{11}}{2r\sqrt{\pi_{11}-\pi_{01}}} < K'$ such that $\dfrac{\partial \Pi_L(T_L^*, T_F^*)}{\partial K} < 0$ for $0 < K < \max(0, K'')$ and $\dfrac{\partial \Pi_L(T_L^*, T_F^*)}{\partial K} > 0$ for $\max(0, K'') < K < \dfrac{\sqrt{\pi_{11}-\pi_{01}}}{r}$.[33] This proves part (i).

As for part (ii), note that

$$\begin{aligned}\frac{\partial(\Pi_L(T_L^*, T_F^*) - \Pi_F(T_L^*, T_F^*))}{\partial K} &= \frac{\pi_{10}+\pi_{11}-2\pi_{01}}{\sqrt{\pi_{11}-\pi_{01}}} + \frac{\pi_{00}+\pi_{01}-2\pi_{10}}{\sqrt{\pi_{10}-\pi_{00}}} \\ &= \frac{[\pi_{10}-\pi_{01}-\sqrt{(\pi_{11}-\pi_{01})(\pi_{10}-\pi_{00})}][\sqrt{\pi_{10}-\pi_{00}}-\sqrt{\pi_{11}-\pi_{01}}]}{\sqrt{(\pi_{11}-\pi_{01})(\pi_{10}-\pi_{00})}}.\end{aligned}$$

The last expression is positive because $\pi_{10} > \pi_{00} \geq \pi_{01}$, $\pi_{10} > \pi_{11} > \pi_{01}$ and $\pi_{10} - \pi_{00} > \pi_{11} - \pi_{01}$, which completes the proof since we have already shown that $\dfrac{\partial \Pi_L(T_L^*, T_F^*)}{\partial r} = \dfrac{\partial \Pi_F(T_L^*, T_F^*)}{\partial r}$.

[33] It can be shown that $K'' < K'$. The proof is analogous to that of $\dfrac{\partial(\Pi_L(T_L^*, T_F^*) - \Pi_F(T_L^*, T_F^*))}{\partial K} > 0$.

Proof of Proposition 7.1 For the purpose of this proof with asymmetric players, it is necessary to extend our notation to allow for asymmetries. Thus, for $i, j \in \{S, W\}$ with $i \neq j$ let

$$\Pi_{iL}(T_{iL}, T_{jF}) = \int_{T_{iL}}^{T_{jF}} \pi_{10} e^{-rt} dt + \int_{T_{jF}}^{\infty} \pi_{11} e^{-rt} dt - \frac{rK_i^2}{e^{rT_{iL}} - 1}$$

and

$$\Pi_{iF}(T_{jL}, T_{iF}) = \int_{T_{iF}}^{\infty} \pi_{11} e^{-rt} dt - \frac{rK_i^2}{e^{rT_{iF}} - 1}.$$

Also, let

$$\tilde{T}_{iF} \equiv \arg\max_{T} \Pi_{iL}(T, \tilde{T}(T, \frac{K_j(e^{rT^*_{jL}} - e^{rT})}{e^{rT^*_{jL}} - 1})) \text{ for } i, j \in \{S, W\},\ i \neq j.$$

It is standard to generalize (9.6) so as to get:

$$\tilde{T}_{iF} = -\frac{1}{r} \ln(1 - \frac{rK_i}{(\pi_{11} + \frac{rK_j(\pi_{10} - \pi_{11})}{(1 - e^{-rT^*_{jL}})\sqrt{\pi_{11}}})^{\frac{1}{2}}}).$$

That is, $\tilde{T}_{iF}$ is date at which the firm labeled $i \in \{S, W\}$ could most profitably deviate if its rival indexed by $j \in \{S, W\}$ $(i \neq j)$ were targeting technology development in the first place at time T^*_{jL}.

We now turn to conditions for existence of equilibria. As for a SPDE in which the weak firm leads, it exists if the following two conditions hold at the same time: the strong firm has no incentives to preempt the weak firm at $\tilde{T}_{SF}$, and the weak firm has no incentives to delay discovery up to T^*_{WF} given that the strong firm is developing at T^*_{SF} (and that the order of development is fixed even if the weak firm deviates). The first condition implies that the following should hold:

$$\Pi_{SF}(T^*_{WL}, T^*_{SF}) - \Pi_{SL}(\tilde{T}_{SF}, \tilde{T}(\tilde{T}_{SF}, \frac{K_W(e^{rT^*_{WL}} - e^{r\tilde{T}_{SF}})}{e^{rT^*_{WL}} - 1})) \geq 0. \tag{9.10}$$

Because

$$e^{-r\tilde{T}_{SF}} = 1 - \frac{rK_S}{(\pi_{11} + \frac{\sqrt{\pi_{10}}(\pi_{10} - \pi_{11})}{\sqrt{\pi_{11}}})^{\frac{1}{2}}}$$

and

$$e^{-r\tilde{T}(\tilde{T}_{SF}, \frac{K_W(e^{rT^*_{WL}} - e^{r\tilde{T}_{SF}})}{e^{rT^*_{WL}} - 1})} = e^{-r\tilde{T}_{SF}}(1 - \frac{rK_W(\pi_{11} + \frac{\sqrt{\pi_{10}}(\pi_{10} - \pi_{11})}{\sqrt{\pi_{11}}})^{\frac{1}{2}} - rK_S\sqrt{\pi_{10}}}{\sqrt{\pi_{11}}((\pi_{11} + \frac{\sqrt{\pi_{10}}(\pi_{10} - \pi_{11})}{\sqrt{\pi_{11}}})^{\frac{1}{2}} - rK_S)}),$$

the left hand side of (9.10) can be written as follows after some tedious manipulations:

$$\begin{aligned}
&\frac{\pi_{11}}{r}e^{-rT^*_{SF}} - \frac{rK_S^2}{e^{rT^*_{SF}}-1} - \frac{\pi_{10}(e^{-r\tilde{T}_{SF}} - e^{-r\tilde{T}})}{r} - \frac{\pi_{11}e^{-r\tilde{T}}}{r} + \frac{rK_S^2}{e^{r\tilde{T}_{SF}}-1} \\
= \;& \frac{\pi_{11}}{r}(1 - \frac{rK_S}{\sqrt{\pi_{11}}}) - \frac{rK_S^2(\sqrt{\pi_{11}} - rK_S)}{rK_S} - \frac{\pi_{10}}{r}(\frac{rK_W}{\sqrt{\pi_{11}}} - \frac{rK_S\sqrt{\pi_{10}}}{\sqrt{\pi_{11}}(\pi_{11} + \frac{\sqrt{\pi_{10}}(\pi_{10}-\pi_{11})}{\sqrt{\pi_{11}}})^{\frac{1}{2}}}) \\
& - \frac{\pi_{11}}{r}(\frac{rK_S(\sqrt{\pi_{10}} - \sqrt{\pi_{11}}) - (rK_W - \sqrt{\pi_{11}})(\pi_{11} + \frac{\sqrt{\pi_{10}}(\pi_{10}-\pi_{11})}{\sqrt{\pi_{11}}})^{\frac{1}{2}}}{\sqrt{\pi_{11}}(\pi_{11} + \frac{\sqrt{\pi_{10}}(\pi_{10}-\pi_{11})}{\sqrt{\pi_{11}}})^{\frac{1}{2}}}) \\
& + \frac{rK_S^2((\pi_{11} + \frac{\sqrt{\pi_{10}}(\pi_{10}-\pi_{11})}{\sqrt{\pi_{11}}})^{\frac{1}{2}} - rK_S)}{rK_S} \\
= \;& \sqrt{\pi_{11}}K_W - 2\sqrt{\pi_{11}}K_S - \frac{\pi_{10}K_W}{\sqrt{\pi_{11}}} + 2K_S(\pi_{11} + \frac{\sqrt{\pi_{10}}(\pi_{10}-\pi_{11})}{\sqrt{\pi_{11}}})^{\frac{1}{2}}.
\end{aligned}$$

Hence, (9.10) holds if and only if the following holds:

$$\frac{K_S}{K_W} \geq \frac{1}{2}\frac{(\rho-1)}{\sqrt{1+(\rho-1)\sqrt{\rho}}-1} = \frac{\sqrt{1+(\rho-1)\sqrt{\rho}}+1}{2\sqrt{\rho}} \equiv \quad (\rho),$$

where $\rho \equiv \pi_{10}/\pi_{11}$. Furthermore, we have that $\Pi_{WL}(T^*_{WL}, T^*_{SF}) - \Pi_{WF}(T^*_{SF}, T^*_{WF}) \geq 0$ should also be satisfied. As shown in the proof of Proposition 7.2 (see expression (9.11)), this implies that $\frac{K_S}{K_W} \geq \frac{2}{\sqrt{\rho}+1}$ should also hold. Because $\frac{2}{\sqrt{\rho}+1} < \frac{\sqrt{\rho}+1}{2\sqrt{\rho}} < \quad (\rho)$ for any $\rho > 1$, we have that a SPDE in which the weak firm leads exists if and only if $K_S \geq K_W \quad (\rho)$.

In turn, a SPDE in which the strong firm leads exists if and only if

$$\Pi_{WF}(T^*_{SL}, T^*_{WF}) - \Pi_{WL}(\tilde{T}_{WF}, \tilde{T}(\tilde{T}_{WF}, \frac{K_S(e^{rT^*_{SL}} - e^{r\tilde{T}_{WF}})}{e^{rT^*_{SL}}-1})) \geq 0,$$

since it is clear that the strong firm has no incentives to deviate and develop later. Note that this expression is similar to (9.10) except for the indices of the firms, which are interchanged. Therefore, a necessary and sufficient condition for existence is the following:

$$\frac{K_S}{K_W} \leq \quad (\rho).$$

Proof of Proposition 7.2 It is clear that a pre-commitment development equilibrium in which the strong firm is the first mover exists for any feasible value of the parameters. However, a pre-commitment development equilibrium in which the weak firm moves first need not exist for certain parameter configurations. In order for this to be an equilibrium, no firm should have incentives to develop the technology at some other time. On the one hand, existence of such an equilibrium requires that the strong firm has no incentives to deviate from completing development in the second place at T^*_{SF}, given that its competitor is going to develop the technology by T^*_{WL}. Some algebra shows that $\Pi_{SL}(T^*_{SL}, T^*_{WF}) \leq \Pi_{SF}(T^*_{WL}, T^*_{SF})$ if and only if

$$\frac{K_S}{K_W} \geq \overline{\ }(\rho) \equiv \frac{1}{2} + \frac{1}{2\sqrt{\rho}}.$$

On the other hand, the weak firm should have no incentives to deviate from discovering first at T^*_{WL}, given that its rival is going to discover by date T^*_{SF}.[34] Notwithstanding, it is straightforward to show that $\Pi_{WL}(T^*_{WL}, T^*_{SF}) - \Pi_{WF}(T^*_{SF}, T^*_{WF}) \geq 0$ holds if and only if the following is satisfied:

$$\frac{K_S}{K_W} \geq \frac{2}{\sqrt{\rho}+1}. \tag{9.11}$$

Because $\dfrac{2}{\sqrt{\rho}+1} < \dfrac{1}{2} + \dfrac{1}{2\sqrt{\rho}} = \dfrac{\sqrt{\rho}+1}{2\sqrt{\rho}}$ for $\rho > 1$, it follows that a necessary and sufficient condition for the existence of an pre-commitment development equilibrium in which the weak firm moves first is that ρ be such that $\dfrac{K_S}{K_W} \geq \overline{\ }(\rho)$ is satisfied.

[34] It can be easily checked that conditions on the parameters such that $T^*_{SF} \leq T^*_{WL}$ holds are too strong.

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