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Abstract

This paper analyzes the impact of endogenous financing costs under capital market imperfections in a capacity investment setting. We focus on the strategic interaction between a single firm that decides on its technology choice (flexible vs dedicated), capacity level, and production quantities under budget and demand uncertainty, and a single creditor that provides funds to the firm to finance its operational investments. The creditor has perfect information about the firm and offers two loan commitment contracts to the firm, one for each technology. Capital market imperfections in the form of bankruptcy costs and underwriter fees impose financing frictions on the firm. We derive the creditor's contract decision and the firm's technology, capacity, production and external borrowing decisions in equilibrium. Our analysis contributes to the capacity investment literature by analyzing the effect of capital market imperfections (in the form of fixed bankruptcy cost and underwriter fee) on capacity investment and technology choice with endogenously determined financing costs. We demonstrate that the endogenous nature of financing costs in imperfect capital markets may modify or reverse conclusions concerning capacity investment and technology choice obtained under the perfect market assumption. For example, the value of flexible technology may increase with increasing demand correlation and decreasing demand variability. Even with identical cost structures, dedicated technology may be preferred to flexible technology.

KeyWords: *Capacity, Flexibility, Financing, Risk Management, Market Imperfection.*

1 Introduction and Literature Review

Capacity investment is subject to internal or external financing frictions, especially in capital-intensive industries. If the internal capital of the firm is not sufficient to finance the desired investment, then the firm may decide to raise external capital. External capital is more expensive because there exist capital market imperfections such as bankruptcy costs, underwriter fees, etc. (Froot et al. 1993) that create frictions in the borrowing process of the firm. However, as highlighted by Van Mieghem (2003, p. 275) "stochastic capacity models assume (often implicitly) [...] perfect capital markets, so that frictionless borrowing is possible [...]." The objective of this paper is to increase our understanding of how capital market imperfections affect technology choice and capacity investment. In imperfect capital markets, the investment decision and the cost of external capital are interdependent. A key feature of our paper is that we endogenize the cost of borrowing in a creditor-firm equilibrium.

To this end, we model a monopolist who produces and sells two products under budget and demand uncertainty. The firm chooses between flexible and dedicated technologies that incur variable investment costs, and determines the capacity level and the production quantities with the chosen technology. Differing from the majority of the stochastic capacity investment literature, we assume that the firm is budget-constrained and can relax its budget constraint by borrowing from a creditor. To capture capital market imperfections, we assume that the creditor incurs a fixed cost of bankruptcy if the firm defaults on the loan, and imposes an underwriting fee. The creditor offers technology-specific loan contracts to the firm, after which the firm makes its technology choice and subsequent decisions.

We derive the technology choice and external borrowing, capacity, and production level decisions of the firm and the creditor's optimal loan contract terms in equilibrium. Based on these results, we show how capital market imperfections affect the operational decisions and the performance of the firm in equilibrium. This comparison is made possible by the existence of a natural perfect-market benchmark in our model. In particular, we answer the following research questions:

1. How do capital market imperfections affect operational decisions and performance of the firm in equilibrium?
2. For a given technology, how do demand structure, budget level and capacity investment cost affect the equilibrium level of capacity investment and operational performance?

3. What are the main drivers of equilibrium technology choice?
4. Do these effects and drivers differ from those in perfect capital markets and if so, what explains these differences?

We note that the objective of this paper is not to solve for the optimal capital structure of the firm (equity financing, debt financing with debt contracts of different rates, maturities, covenants, etc); rather we focus on a prevalent type of financing contract and analyze the creditor-firm strategic interaction in that setting. Our results contribute to several streams of research, as detailed below.

The stochastic capacity investment literature analyzes the value of flexible technology in a variety of models. Van Mieghem (1998), Bish and Wang (2004) and Chod and Rudi (2005) are representative papers in this stream. We refer readers to Van Mieghem (2003) for an excellent review. As highlighted in this review paper, the operations management literature (often implicitly) assumes that capital markets are perfect, in which case operational and financial decisions decouple (Modigliani and Miller 1958). In practice, capital market imperfections such as agency costs, taxes, underwriter fees and bankruptcy costs exist (Harris and Raviv 1991) and impose deadweight costs of external financing, leading operational and financial decisions to interact with each other. There is a growing body of work in operations and finance that analyze these interactions. Our paper's overall contribution to this literature is to analyze the effect of capital market imperfections (in the form of fixed bankruptcy cost and underwriter fee) on capacity investment and technology choice with endogenously determined financing costs. We demonstrate that the endogenous nature of financing costs in imperfect capital markets may modify or reverse conclusions concerning capacity investment and technology choice obtained under the perfect market assumption.

In the Operations Management literature, among papers analyzing the joint financing and operating decisions of the firm (Lederer and Singhal 1994, Buzacott and Zhang 2004, Xu and Birge 2004, Babich and Sobel 2004, Caldentey and Haugh 2005, Babich et al. 2006 and Xu and Zhang 2006), the first three endogenize the financing cost. We compare our results to two of these papers in particular. Lederer and Singhal (1994) study the joint financing (optimal mix of debt and equity) and capacity investment problem in a multi-period setting and show how the technology choice of the firm is related to its financing decision. In a numerical example, they show that the value of flexible technology decreases in demand variability, and argue that this is because the default risk of the firm decreases, which allows the firm to secure lower financing cost in equilibrium. In our model, we demonstrate that this result is only valid at high demand correlations. At low demand correlations, the default risk of the firm is not affected by the change in demand variability because the diversification benefit of operating in two markets (which we call "financial pooling") is sufficiently large. It follows that at low demand correlations, the value of flexible technology increases in demand variability. This is because i) the value of flexible technology at a given financing cost increases in demand variability (due to capacity pooling), and ii) the equilibrium level of financing cost is robust to changes in demand variability (due to financial pooling).

Xu and Birge (2004) analyze the effect of taxes and bankruptcy costs on the firm's joint financing and operating decisions in a single-period single-product capacity investment setting. They demonstrate the value of integrated decision making and analyze the effect of demand variability and some other operational characteristics in imperfect capital markets. Our work is complementary to theirs. We provide a parallel result that is based on analytical proof for one of their numerical observations and extend the analysis to the two-product setting. In addition, we analyze the effect of capital market imperfections on the equilibrium technology choice of the firm.

Several finance papers also investigate the interaction of financing and operating decisions. Dotan and Ravid (1985) and Dammon and Senbet (1988) are examples of early research that demonstrates the effect of operational investments on the financing policy of the firm in a single-period setting. We refer the reader to Childs et al. (2005) for a recent review of papers in this stream. Froot et al. (1993) is closest to our paper in terms of its modelling framework. They model a budget-constrained monopolist that produces and sells a single product under budget and demand uncertainty. The firm can relax its budget constraint by borrowing from capital markets at an *exogenously given* external financing cost scheme. They demonstrate that financial risk management has positive *static* value: It adds value by generating sufficient internal funds to finance operational investments in imperfect markets for a *given* financing cost scheme. We extend their framework by further formalizing the operational investments (allowing technology choice in a two-product setting) and *endogenizing* the external financing costs, which creates a *strategic* effect. We report on our analysis of the financial risk management decision of the firm in §6. With endogenous financing costs, engaging in financial risk management has *strategic* value as it alters the equilibrium level of financing costs. Interestingly, we find that the strategic value may be negative: Engaging in financial risk management may increase the financing cost in equilibrium if it is effective before the loan is taken as it decreases the expected borrowing level of the firm.

More recently, a number of papers in the finance literature (Mauer and Triantis 1994, Mello et al. 1995, and Mello and Parsons 2000) analyze the effect of various forms of operational flexibility (e.g. shutting down the production plant) on the joint operational and financing decisions of firm in the contingent claims framework. The focus of these papers is on the financing policy of the firm with strong modeling assumptions concerning the firm's operations. As highlighted in MacKay (2003), without agency cost concerns, operational flexibility has a positive strategic value: Operational flexibility decreases the firm's default risk by generating higher returns due to its option value and this decreases the financing cost in equilibrium. We demonstrate that this argument may not hold in general with a more detailed formalization of the firm's operations. Anticipating the option value of operational flexibility (flexible technology in our case), the firm optimally adjusts other operational decisions (capacity investment and production quantity). As a result, the creditor may charge a higher financing cost in equilibrium, yielding a net negative strategic value. Even if operational flexibility is costless (in our model, this means flexible technology has the same cost structure as dedicated technology), the firm may be worse off with operational flexibility due to this negative strategic value.

Our work is also related to the recent stream of papers that analyze the interaction of operational and financial decisions from an integrated risk management point of view (Zhu and Kapuscinski 2004, Ding et al. 2005, Chod et al. 2006, Dong et al. 2006). These papers (implicitly) assume perfect capital markets so that borrowing frictions do not exist, and motivate financial risk management by the risk-aversion of the decision-maker (manager of the firm). In Boyabatlı and Toktay (2006), we analyze the interaction in imperfect capital markets, but take external financing costs to be *identical and exogenous* for each technology. This paper formalizes the capital market imperfections and *endogenizes* the external financing costs in a creditor-firm interaction. We provide new trade-offs and insights that arise from this strategic interaction.

The remainder of this paper is organized as follows: In §2, we describe the model and discuss the basis for our assumptions. §3 and §4 analyze the optimal strategy of the firm and the creditor, respectively. Our main results and contributions are provided in Section 5. We provide a perfect market benchmark in §5.1. §5.2 provides comparative static results with respect to several parameters in imperfect capital markets. §5.2.1 provides general structural properties of the comparative statics analysis. §5.2.2 and §5.2.3 focus on specific parameters (demand structure, internal budget level and unit investment cost) and analyze their impact in the single and two- product cases respectively. §5.3 investigates technology choice in imperfect capital markets. In §6, we present some extensions to our model and discuss the robustness of our results to some of our assumptions. We conclude in §7 by discussing the managerial insights, limitations of our research and future research directions.

2 Model Description and Assumptions

We consider a creditor-firm interaction where the creditor determines the borrowing terms before the firm takes any decisions. The firm is a budget-constrained monopolist that makes its technology choice under budget and demand uncertainty. After the budget uncertainty is resolved, it invests in capacity (potentially after borrowing from the creditor) under demand uncertainty, and produces and sells two products after the resolution of this uncertainty. The firm chooses the technology (dedicated versus flexible), and the borrowing, capacity investment, and production levels so as to maximize the expected equity value. We model the firm's decisions as a three-stage stochastic recourse problem. We focus on a stylized firm that lives for a single period and is liquidated at the end of the period. After operating profits are realized, the firm pays back its debt (if any); default occurs if it is unable to do so. The timeline is depicted in Figure 1. Before discussing the timeline in detail, we introduce our assumptions about the firm and the creditor.

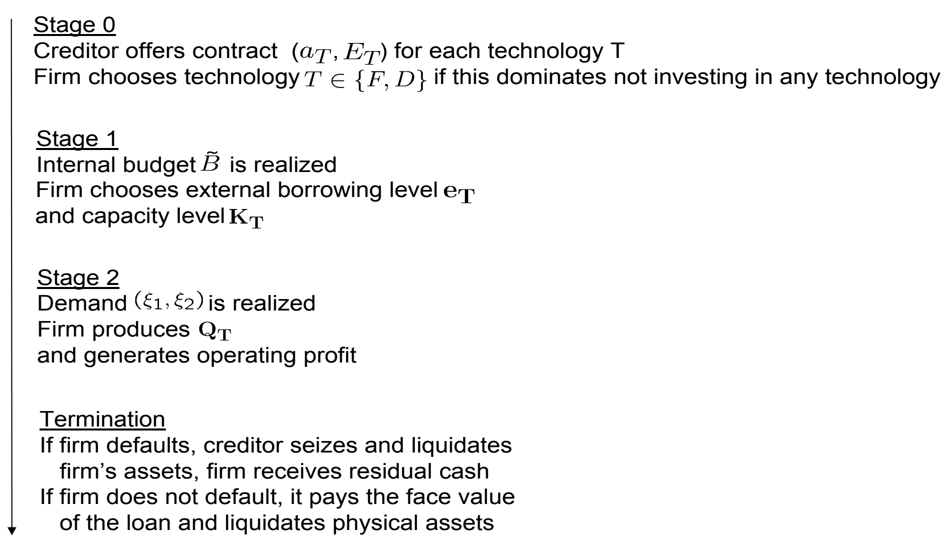

Figure 1: Timeline of events

The firm's objective is to maximize the expected shareholder wealth by maximizing the expected (stage 0) value of equity. Shareholders have limited liability. The risk-free rate r_f

is normalized to 0.

We assume that the creditor offers loan commitment contracts to the firm. Loan commitment is a promise to lend up to a pre-specified amount at pre-specified terms. We choose this type of contract for our analysis because in practice, most short-term industrial and commercial loans in the US are made under loan commitment contracts (Melnik and Plaut 1986). In the context of our model, when the creditor offers a contract at stage 0, this means that the firm owns the right to a loan contract that can be exercised in stage 1. We assume that the creditor has perfect information about the firm and determines the loan contract terms before the firm takes any decisions.

As discussed in Froot et al. (1993), outside capital is more expensive than internally generated funds. This is because there exist transaction costs of external financing that give rise to capital market imperfections. In our model, we focus on two of these imperfections, fixed bankruptcy cost and underwriter fees.

We assume that the creditor incurs a fixed bankruptcy cost BC if the firm defaults on its loan; this cost is incurred as an out-of-pocket fee. BC represents the direct cost of bankruptcy which includes the administrative and legal fees of the bankruptcy process (Altman 1980) and is often used in the literature to represent default-related capital market imperfections (e.g. Smith and Stulz 1985). Thus, the existence of bankruptcy cost introduces a market imperfection in our model. There also exist indirect costs of bankruptcy that are effective after default takes place (such as decreased product market-competitiveness, or costs associated with the disturbance of the relation with suppliers). Since we focus on a single-period firm, indirect costs of bankruptcy are not an issue in our model.

We assume that the creditor designs contracts to make an expected return of $U \geq 0$. U represents the reservation utility of the creditor and is an indicator of the competitiveness of the loan market. In the financial economics literature, the common assumption is to have perfectly competitive loan markets such that the creditor makes zero expected profit, i.e. $U = 0$ (Melnik and Plaut 1986). We allow strictly positive values of U which can be interpreted as underwriter fees. This introduces a second capital market imperfection in our model.

We assume that the creditor offers technology-specific loan commitment contracts. Contract T consists of terms a_T and E_T, where a_T is the unit financing cost and E_T is the credit limit. Most bank loans are secured by the company's assets (Weidner 1999) and modelled as such (Mello and Parsons 2000). Therefore, we assume that the firm has physical assets of value P (e.g. real estate) that are pledged to the creditor as collateral and that the loan commitment is fully secured by the firm's physical assets. We assume that at a given unit cost a_T, the creditor offers the maximum credit limit that can be secured with the collateral value P: $E_T(a_T) = \frac{P}{1+a_T}$. The value of the physical assets P is sufficient to finance the budget-unconstrained optimal capacity investment level of the firm.

The physical assets are illiquid; they can only be liquidated with a lead time. The implication of this assumption is that although the loan is fully collateralized, if the firm's final cash position is not sufficient to cover the face value of the debt, the firm cannot immediately liquidate the collateral assets to repay its debt. Under limited shareholder liability, this leads to default, in which case the creditor seizes these physical assets, liquidates them and uses their liquidation value to recover the loan.

Returning to the time line, in stage 0, the creditor offers its borrowing terms (a_T, E_T), $T \in \{D, F\}$. The firm then determines its technology choice $T \in \{D, F\}$ (if this dominates the doing nothing option in expectation) and accepts the corresponding loan commitment contract T. The flexible technology (F) has a single resource that is capable of producing two products and the dedicated technology (D) consists of two resources that can each produce a single product. At this point in time, the stage 1 budget B is uncertain. We assume B has a continuous distribution with positive support and bounded expectation $\overline{B}$. The technology choice decision is made in advance of the actual capacity investment decision that will be taken in stage 1. This is to reflect that technology investment typically incurs a lead time of acquisition (if outsourced) or development (if built in-house). This assumption also enables us to make direct comparisons between each technology because the firm chooses only one of the technologies at stage 0.

In stage 1, the budget uncertainty is resolved and the budget realization $\tilde{B}$ is observed. In this stage, the firm can borrow external capital up to the credit limit E_T from the unit financing cost of $a_T \geq r_f = 0$ for the chosen technology $T \in \{D, F\}$. The firm determines the amount of external borrowing and the capacity investment level under demand uncertainty. Technology T incurs unit investment cost c_T.

In stage 2, demand uncertainty is resolved. The firm then chooses the production quantities (equivalently, prices) to satisfy demand optimally. Price-dependent demand for each product is represented by the iso-elastic inverse-demand function $p_i(q_i; \xi_i) = \xi_i q_i^{1/b}$ for $i = 1, 2$. Here, $b \in (-\infty, -1)$ is the constant elasticity of demand, and p and q denote price and quantity, respectively. ξ_i represents the idiosyncratic risk component. $\boldsymbol{\xi}' = (\xi_1, \xi_2)$ is a bivariate random variable with continuous distribution that has bounded expectation $\overline{\boldsymbol{\xi}}' = (\overline{\xi}, \overline{\xi})$ with covariance matrix $\boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}_{ii} = \sigma^2$ and $\boldsymbol{\Sigma}_{ij} = \rho\sigma^2$ for $i \neq j$ and ρ

denotes the correlation coefficient. We make specific assumptions about the distribution of $\boldsymbol{\xi}$ throughout the text whenever necessary. For tractability, we assume that the marginal production costs of each product are 0. This is an assumption that is widely used in the literature (see Goyal and Netessine 2007 and references therein).

At termination, if the firm is able to repay its debt from its final cash position, it does so and, since the firm lives for a single-period, terminates by liquidating its physical assets. Otherwise, default occurs. In this case, because of the limited liability of the shareholders and the lead time involved with liquidating its physical assets, the firm goes to bankruptcy. The cash on hand and the ownership of the collateralized physical assets are transferred to the creditor. The firm receives the remaining cash after the creditor covers the face value of the loan from the seized assets of the firm.

With our assumption that the bankruptcy cost is incurred by the creditor as an out-of-pocket fee in the default states, this cost is effectively charged to the firm ex-ante in the equilibrium unit borrowing cost a_T. This assumption is made for analytical convenience and can be justified by the firm-specific nature of the loan commitment contracts (Melnik and Plaut 1986). We can show that the equilibrium level of unit financing cost decreases if the creditor can deduct the fixed cost of the bankruptcy from the seized assets. This demonstrates that the firm is penalized ex-ante (before borrowing) through a more severe loan contract for not being responsible for the fixed cost of bankruptcy after default. We discuss the robustness of our results with respect to a number of other assumptions in §6.

3 Analysis of the Firm's Problem

In this section, we describe the optimal solution for the firm's technology choice, and the levels of external borrowing, capacity investment and production. A realization of the random variable s is denoted by $\tilde{s}$ and its expectation is denoted by $\overline{s}$. Bold face letters represent vectors of the required size. Vectors are column vectors and $'$ denotes the transpose operator. $\mathbf{x}^a$ denotes the componentwise exponent a of the vector $\mathbf{x}$. $\mathbf{xy}$ denotes the componentwise product of vectors $\mathbf{x}$ and $\mathbf{y}$ with identical dimensions. We use the following vectors throughout the text: $\boldsymbol{\xi}' = (\xi_1, \xi_2)$ (demand), $\mathbf{K_D'} = (K_D^1, K_D^2)$ (dedicated capacity investment), $\mathbf{K_F} = K_F$ (flexible capacity investment)[1] and $\mathbf{Q_T'} = (q_T^1, q_T^2)$ (production vector with technology T). Pr denotes probability, $\mathbb{E}$ denotes the expectation operator, $(x)^+ \doteq \max(x, 0)$ and $\Omega^{01} \doteq \Omega^0 \bigcup \Omega^1$. Monotonic relations (increasing, decreasing) are

[1]Using vector notation for a scalar may seem counterintuitive; this notation was chosen because it significantly shortens the exposition to use the notation $\mathbf{K_T}$ and $\mathbf{Q_T}$, $T = F$ or D, to denote the capacity investment and production level decisions.

used in the weak sense otherwise stated. x^* denotes the optimal value of x. Tables 4 and 5 in the Technical Appendix summarize all the notation. All proofs are provided in Appendix A of the Technical Appendix. We solve the firm's problem by using backward induction starting from stage 2.

3.1 Stage 2: Production Decision

In stage 1, the firm will have observed the budget realization $\tilde{B}$, borrowed e_T and invested in capacity level $\mathbf{K_T}$, where the technology choice $T = D$ or F will have been made in stage 0. In this stage, the firm observes the demand realization $\tilde{\boldsymbol{\xi}}$ and determines the production quantities $\mathbf{Q_T}' = (q_T^1, q_T^2)$ within the existing capacity limit $\mathbf{K_T}$ to maximize the stage 2 equity value. Let $\Gamma_T^*\left(\mathbf{K_T}, \tilde{\boldsymbol{\xi}}\right)$ denote the optimal stage 2 operating profit. The production decision only affects the operating profit in stage 2. Thus, maximizing the stage 2 equity value is equivalent to maximizing the operating profit. Since we assume that production is costless, the optimal stage 2 operating profit is equal to the maximum sales revenue that can be obtained using the existing capacity $\mathbf{K_T}$:

$$\Gamma_T^*\left(\mathbf{K_T}, \tilde{\boldsymbol{\xi}}\right) = \max_{\mathbf{Q}\in\Theta_T} \mathbf{Q}'\mathbf{p}(\mathbf{Q}; \tilde{\boldsymbol{\xi}}) = \max_{\mathbf{Q}\in\Theta_T} \tilde{\boldsymbol{\xi}}'\mathbf{Q}^{1+\frac{1}{b}}, \tag{1}$$

where $\Theta_F \doteq \{\mathbf{Q} : \mathbf{Q} \geq \mathbf{0}, \mathbf{1}'\mathbf{Q} \leq K_F\}$ and $\Theta_D \doteq \{\mathbf{Q} : \mathbf{Q} \geq \mathbf{0}, \mathbf{Q} \leq \mathbf{K_D}\}$ are the feasibility sets for production quantity levels for each technology T.

Proposition 1 *The optimal production quantity vector in stage 2 with technology $T \in \{D, F\}$ for given $\mathbf{K_T}$ and $\tilde{\boldsymbol{\xi}}$ is given by*

$$\mathbf{Q_D^*} = \mathbf{K_D}, \quad \mathbf{Q_F^*} = \frac{K_F}{\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}} \ \tilde{\boldsymbol{\xi}}^{-b}.$$

Since the unit production cost is zero, the firm optimally utilizes the entire available capacity. With dedicated technology, the optimal production quantities are equal to the available capacity levels for each product. With flexible technology, the firm allocates the available capacity K_F in such a way that the marginal profits for each product are equal.

3.2 Stage 1: Capacity Choice and External Financing

In this stage, the firm observes the internal budget realization $\tilde{B} \in [0, \infty)$. For given $\tilde{B}$ and T, the firm determines the optimal capacity investment level $\mathbf{K_T^*}(\tilde{B})$ and the optimal external borrowing level $e_T^*(\tilde{B})$.

The optimal expected (stage 1) equity value of the firm, $\pi_T^*(\tilde{B})$, is

$$\begin{aligned}
\pi_T^*(\tilde{B}) \quad &= \quad \max_{\mathbf{K_T},\, e_T} \tilde{B} + e_T - c_T \mathbf{1'K_T} - (B + e_T - c_T \mathbf{1'K_T}) + \mathbb{E}\left[\Pi_T^*\left(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}\right)\right] \\
\text{s.t.} \quad & \quad e_T \geq c_T \mathbf{1'K_T} - \tilde{B} \\
& \quad e_T \leq E_T \qquad\qquad (2) \\
& \quad \mathbf{K_T} \geq \mathbf{0}, \quad e_T \geq 0.
\end{aligned}$$

The firm has available budget $\tilde{B}$ and borrows e_T from the creditor. Out of this sum $\tilde{B}+e_T$, the firm invests $c_T\mathbf{1'K_T}$ in capacity and places the remainder $(B + e_T - c_T\mathbf{1'K_T})$ into the cash account at the risk-free rate. The cash holdings and the operating profits from the capacity investment are included in the expected optimal value of the equity in stage 2, $\mathbb{E}\left[\Pi_T^*\left(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}\right)\right]$.

To derive Π_T^*, note that two outcomes are possible in stage 2: If the firm's final cash position (optimal operating profits and cash account holdings) is sufficient to cover the face value of the loan, i.e. $\Gamma_T^*(\mathbf{K_T}, \tilde{\boldsymbol{\xi}}) + (\tilde{B} + e_T - c_T\mathbf{1'K_T}) \geq e_T(1 + a_T)$, then the firm does not default; otherwise, it does. If the firm does not default, it repays the face value of its loan and liquidates the physical assets, generating P. If the firm defaults, the cash on hand and the ownership of the collateralized physical asset are transferred to the creditor. The firm receives the remaining cash after the face value of the loan is deducted from its seized assets. In either case, the optimal (stage 2) equity value can simply be written as

$$\Pi_T^*\left(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}\right) = \Gamma_T^*(\mathbf{K_T}, \tilde{\xi}) + (\tilde{B} + e_T - c_T\mathbf{1'K_T}) - e_T(1 + a) + P, \qquad (3)$$

where we invoke the assumptions that the bankruptcy cost BC is borne by the creditor as an out-of-pocket expenditure, the loan is fully-collateralized by the physical asset and the risk-free rate is 0. Obtaining this unique functional form preserves tractability and allows us to derive closed-form expressions for the firm's operational decisions.

Substituting this expression into the objective function, the latter can be rewritten as $\tilde{B} + P + \Gamma_T^*(\mathbf{K_T}, \tilde{\boldsymbol{\xi}}) - c_T\mathbf{1'K_T} - a_T e_T$. Here, the first two terms are equal to the equity value of the firm if the firm does nothing (does not borrow and does not invest). The last three terms are the net profit derived from borrowing and investing in capacity. The first constraint ensures that the amount of external borrowing is greater than the difference between the cost of the investment and the available budget. The second constraint states that the external borrowing level is less than the credit limit (E_T) of the firm.

Proposition 2 *The optimal capacity investment vector* $\mathbf{K_T^*}(\tilde{B})$ *and the optimal external borrowing level* $e_T^*(\tilde{B})$ *for technology* $T \in \{D, F\}$ *with a given budget level* $\tilde{B}$ *are*

$$\mathbf{K_T^*}(\tilde{B}) = \begin{cases} \mathbf{K_T^0} & \textit{if} \;\; \tilde{B} \in \Omega_T^0 \doteq \{\tilde{B} : \tilde{B} \geq c_T \mathbf{1'K_T^0}\} \\ \overline{\mathbf{K}}_\mathbf{T} & \textit{if} \;\; \tilde{B} \in \Omega_T^1 \doteq \{\tilde{B} : c_T \mathbf{1'K_T^1} \leq \tilde{B} < c_T \mathbf{1'K_T^0}\} \\ \mathbf{K_T^1} & \textit{if} \;\; \tilde{B} \in \Omega_T^2 \doteq \{\tilde{B} : c_T \mathbf{1'K_T^1} - E_T \leq \tilde{B} < c_T \mathbf{1'K_T^1}\} \\ \overline{\overline{\mathbf{K}}}_\mathbf{T} & \textit{if} \;\; \tilde{B} \in \Omega_T^3 \doteq \{\tilde{B} : 0 \leq \tilde{B} < c_T \mathbf{1'K_T^1} - E_T\} \end{cases} \tag{4}$$

$$e_T^*(\tilde{B}) = \left(c_T \mathbf{1'K_T^*}(\tilde{B}) - \tilde{B} \right)^+ . \tag{5}$$

The explicit expressions for the capacity vectors in the proposition are given in expression (26) of the proof. $\mathbf{K_T^0}$ is the optimal capacity investment in the absence of a budget constraint (the "budget-unconstrained optimal capacity"). If the budget realization is high enough to cover the corresponding cost $c_T \mathbf{1'K_T^0}$ ($\tilde{B} \in \Omega_T^0$), then $\mathbf{K_T^*}(\tilde{B}) = \mathbf{K_T^0}$ with no borrowing. Otherwise, for each budget level $\tilde{B} \in \Omega_T^{123}$, the firm determines to borrow or not by comparing the marginal revenue from investing in an additional unit of capacity over its available budget with the marginal cost of that investment including the external financing cost, $(1 + a_T)c_T$. For $\tilde{B} \in \Omega_T^1$, the budget is insufficient to cover $\mathbf{K_T^0}$, and the marginal revenue of capacity is lower than its marginal cost. Therefore, the firm optimally does not borrow, and only purchases the capacity level $\overline{\mathbf{K}}_\mathbf{T}$ that fully utilizes its budget $\tilde{B}$. For $\tilde{B} \in \Omega_T^{23}$, the marginal revenue of capacity is higher than its marginal cost $(1 + a)c_T$. Therefore, the firm optimally borrows from external markets to invest in capacity. $\mathbf{K_T^1}$ is the optimal capacity investment with borrowing, in the absence of a credit limit (the "credit-unconstrained optimal capacity"). If the budget realization and the credit limit can jointly cover its cost, $\mathbf{K_T^1}$ is the optimal capacity investment; otherwise, the firm purchases the capacity level $\overline{\overline{\mathbf{K}}}_\mathbf{T}$ that fully utilizes its budget and its credit limit.

The optimal external borrowing level $e_T^*(\tilde{B})$ is such that the firm borrows exactly what it needs to cover its capacity investment. Since production is costless, the firm does not incur any further costs beyond this stage. The firm only borrows for funding the capacity investment, which yields (5).

Using (4), the optimal expected (stage 1) equity value of the firm with a given budget level $\tilde{B}$, $\pi_T^*(\tilde{B})$, can be obtained in closed form, and is given in the proof of Proposition 2.

3.3 Stage 0: Technology Choice

In this stage, the firm decides on the technology choice $T \in \{D, F\}$. The optimal expected (stage 0) equity value Ψ^* is given by

$$\Psi^* = \max\left\{Y^*, \overline{B} + P\right\}. \tag{6}$$

Here, Y^* denotes the expected (stage 0) equity value of the more profitable technology where the expectation is taken over B and $\boldsymbol{\xi}$. In (6), the firm compares this equity value with $\overline{B} + P$, the expected (stage 0) equity value of not investing in any technology.

Since the credit limit E_T is uniquely determined by the unit financing cost a_T ($E_T = \frac{P}{1+a_T}$), we only use a_T to denote the loan terms. We define the vector $\mathbf{a} \doteq (a_D, a_F)$. The choice T^* between flexible versus dedicated technology is determined by a unit cost threshold that makes the firms indifferent between the two technologies. This characterization is valid for any continuous B and $\boldsymbol{\xi}$ distribution with positive support and bounded expectation.

Proposition 3 *For a given financing cost scheme* $\mathbf{a}$ *there exists a unique variable cost threshold* $\overline{c}_F(c_D, \mathbf{a})$ *such that when* $c_F < \overline{c}_F(c_D, \mathbf{a})$ *it is more profitable to invest in flexible technology* ($T^* = F$). *With symmetric financing costs* $a_F = a_D$,

$$\overline{c}_F(c_D, \mathbf{a}) = \overline{c}_F^S(c_D) \doteq c_D \left(\frac{\mathbb{E}^{-b}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]}{\mathbb{E}^{-b}[\xi_1] + \mathbb{E}^{-b}[\xi_2]} \right)^{-\frac{1}{b+1}} \geq c_D. \tag{7}$$

Investing in T^* *dominates not making any technology investment.*

The cost threshold developed in Proposition 3 reveals the more profitable technology. Proposition 3 also concludes that investing in this technology is more profitable than not investing at all. This completes the characterization of the firm's optimal decisions.

4 Analysis of the Creditor's Problem

Anticipating the firm's actions, the creditor determines credit terms to ensure an expected return $\widehat{\Lambda}_T(a_T, E_T)$ of U. We focus on the Pareto-Nash subgame perfect equilibrium that achieves a return of U for the creditor and the highest equity value for the firm. This is consistent with the financial economics literature where the creditor makes zero expected return in equilibrium ($U = 0$) and the creditor chooses the best contract for the firm (e.g. Melnik and Plaut 1986). In our model, the Pareto-optimality refinement guarantees the uniqueness of the equilibrium loan commitment contract, if the contract is offered. We use the $\dot{x}$ notation to denote equilibrium quantities for the creditor and the firm. For example, $\dot{\mathbf{K}}_{\mathbf{T}}(\tilde{B}) \doteq \mathbf{K}^*_{\mathbf{T}}(\tilde{B}; \dot{a}_T, \dot{E}_T)$, where $(\dot{a}_T, \dot{E}_T)$ is the equilibrium loan commitment contract.

Recall from §2 that at a given unit financing cost a_T, the creditor offers the credit limit that can be secured with the collateral value P: $E_T(a_T) = \frac{P}{1+a_T}$. Then we can write the expected return of the creditor with a loan commitment contract $(a_T, E_T(a_T))$ as a function of only a_T, as $\Lambda_T(a_T) \doteq \widehat{\Lambda}_T(a_T, \frac{P}{1+a_T})$.

Proposition 4 *If there exists a feasible $a_T \geq 0$ that satisfies $\Lambda_T(a_T) = U$ for the creditor, then in the Pareto-optimal equilibrium, the creditor offers a unique loan commitment contract for each technology T with parameters*

$$\begin{aligned} \dot{a}_T &= argmin_{a_T \geq 0}\ \Lambda_T(a_T) = U, \\ \dot{E}_T &= \frac{P}{1+\dot{a}_T}. \end{aligned}$$

If such an $a_T \geq 0$ that satisfies $\Lambda_T(a_T) = U$ does not exist, then in equilibrium the creditor does not offer a contract. In this case, we say $\dot{a}_T \to \infty$ and $\dot{E}_T \to 0$.

The minimal a_T (which also corresponds to maximal credit limit E_T) is Pareto-optimal for the firm because the optimal expected (stage 0) equity value of the firm increases as more external capital becomes available at a lower unit cost; the creditor is indifferent between all a_T's satisfying his reservation expected utility U. Since the firm's borrowing level depends on a_T, when the fixed bankruptcy cost or the reservation utility of the creditor (underwriter fee) is sufficiently large, there may not exist a feasible a_T that satisfies the creditor's requirement of U. In this case, the creditor does not offer a contract and the firm cannot raise external capital.

We now discuss properties of $\Lambda_T(a_T)$, the expected return of the creditor for a given a_T, to shed some light on the drivers of $\dot{a}_T$. We have

$$\Lambda_T(a_T) = a_T\,\mathbb{E}\left[e_T^*\right] - BC\,\mathbb{E}\left[Pr\left\{\Gamma_T^* < e_T^*(1+a_T)\right\}\right], \tag{8}$$

where the expectation is taken over the firm's internal budget distribution B. We call the first term in (8) EE_T, the expected earnings of the creditor (without default), and the second term ED_T, the expected default cost of the creditor. Since we focus on fully-secured loan commitment contracts, the creditor always retrieves the face value of the loan and generates expected earnings of $a_T\mathbb{E}[e_T^*]$ from the lending business. However, since the physical assets are illiquid, default can occur because the firm is not able to immediately pay back the debt with the liquid assets, in which case the creditor incurs the cost BC.

These two major drivers (the borrowing level of the firm, and the likelihood of bankruptcy given the borrowing level) work in opposite directions and create a trade-off for the creditor: The creditor wants the firm to borrow more to have a higher EE_T. On the other hand, a higher borrowing level leads to a higher default probability, and hence, a higher ED_T. A detailed characterization of (8) is given in Technical Appendix §B.1.

It is clear from (8) that the creditor's expected return for any loan contract depends on the firm's operational decisions. The borrowing level e_T^* of the firm depends on a_T because the optimal capacity investment level of the firm at each budget state $\tilde{B}$ and the regions in Proposition 2 are functions of a_T. The default probability also depends on a_T through the face value of the loan $e_T(1 + a_T)$ and the optimal production quantity that determines Γ_T^*.

It is easy to establish that the default probability of the firm decreases in the unit financing cost in our model. Melnik and Plaut (1986) derive several relations among the parameters of loan commitment contracts based on the assumption that the borrowing level is independent of the unit financing cost, and that the default probability increases in the unit financing cost. Our analysis demonstrates that these assumptions may not be valid with a more formal representation of operations.

5 Effect of Capital Market Imperfections on the Firm's Operational Decisions and Performance

The goal of this paper is to analyze the impact of endogenous financing costs under capital market imperfections in a capacity investment setting. As mentioned in the introduction, the capacity investment literature has implicitly assumed frictionless borrowing. We will show that many conclusions that would be obtained under this assumption are modified or reversed once capital market imperfections (transaction costs in the form of bankruptcy cost and/or underwriter fee) and endogenous financing costs are taken into account. To this end, in §5.1, we first identify the natural perfect market benchmark in our modeling framework (with zero transaction cost). §5.2 develops comparative statics analysis for a given technology choice under transaction costs and compares them to the no transaction cost case. §5.3 analyzes the impact of transaction costs on technology choice.

We use "transaction cost" and "capital market imperfection" interchangeably in our discussion. Since the market imperfections in our paper are bankruptcy cost and underwriter fee, "no transaction cost" refers to zero bankruptcy cost and underwriter fee; whereas "(positive) transaction cost" or "imperfect capital market" refers to a strictly positive value of at least one of the transaction costs. We distill the effect of the existence of bankruptcy cost by comparing the ($BC = 0,\ U > 0$) and ($BC > 0,\ U > 0$) cases.

5.1 The Perfect Capital Market Benchmark

Proposition 5 *If $BC = U = 0$, $\dot{a}_T = 0$ and $\dot{E}_T = P$.*

The perfect market assumption prohibits any transaction costs ($BC = U = 0$) and requires the fair valuation of the debt obligation in terms of its underlying default exposure. Since we focus on collateralized debt, in the absence of transaction costs, there is no risk for the creditor associated with default. Consequently, for either technology, the fair price of any secured debt obligation is the risk-free rate ($\dot{a}_T = 0$)[2] and the equilibrium credit limit is the value of the collateralized physical asset ($\dot{E}_T = P$). With positive transaction costs, $\dot{a}_T > 0$ and $\dot{E}_T < P$.

Proposition 6 *If the capital markets are perfect ($BC = U = 0$), the firm's equilibrium operational decisions are independent of the financing decision for a given technology:* $\dot{K}_F(\tilde{B}) = K_F^0 = \left(\frac{\mathbb{E}\left[\left(\xi_1^{-b}+\xi_2^{-b}\right)^{-\frac{1}{b}}\right]\left(1+\frac{1}{b}\right)}{c_F}\right)^{-b}$, $\dot{\mathbf{K}}_{\mathbf{D}}'(\tilde{B}) = \mathbf{K}_{\mathbf{D}}^{\mathbf{0}}{}' = \left(\left(\frac{\bar{\xi}_1\left(1+\frac{1}{b}\right)}{c_D}\right)^{-b}, \left(\frac{\bar{\xi}_2\left(1+\frac{1}{b}\right)}{c_D}\right)^{-b}\right)$ *and $\dot{e}_T(\tilde{B}) = [c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{0}} - \tilde{B}]^+$. Equilibrium technology choice $\dot{T}$ is determined by the variable cost threshold $\bar{c}_F^P(c_D) = \bar{c}_F^S(c_D)$ of Proposition 3 and the expected (stage 0) equity value in equilibrium is given by $\dot{\Psi} = \overline{B} + P + \frac{c_{\dot{T}}\mathbf{1}'\mathbf{K}_{\dot{\mathbf{T}}}^{\mathbf{0}}}{-(b+1)}$.*

Here, $\bar{c}_F^P(c_D)$ denotes the variable cost threshold in perfect capital markets and it equals the symmetric threshold $\bar{c}_F^S(c_D)$ of Proposition 3 because $\dot{a}_F = \dot{a}_D(= 0)$. The equilibrium investment level is the budget-unconstrained investment level $\mathbf{K}_{\mathbf{T}}^{\mathbf{0}}$ for any budget realization as in traditional stochastic capacity models: The firm simply chooses the optimal investment level without regard to the budget limit, and implements it by borrowing if necessary. This is consistent with the decoupling of operational and financial decisions in perfect markets (Modigliani and Miller 1958).

5.2 Comparative Statics Analysis

We now investigate the effect of demand structure, internal budget level and unit capacity investment cost on the equilibrium level of financing costs and the firm's operational decisions and performance for a given technology choice in the single and two- product cases, respectively. Since production vector $\mathbf{Q}_{\mathbf{T}}$ is a linear function of the capacity vector $\mathbf{K}_{\mathbf{T}}$, we only discuss the optimal capacity investment level as the firm's operational decision for a

[2] If the debt were not fully secured, then the unit cost of financing would be larger than the risk-free rate even in a perfect market (see Xu and Birge 2004 for a detailed treatment of this with unsecured spot lending).

given technology. We compare the results with the perfect market benchmark of Section 5.1 to identify the effect of capital market imperfections coupled with endogenous financing costs. Our goal is not to undertake a complete characterization of the equilibrium, but to show the existence of certain effects that arise from capital market imperfections. For this reason, we focus on specific demand distributions. For the single product case in §5.2.2, we assume that ξ is uniformly distributed between $[0, 2\bar{\xi}]$ as the uniform distribution allows us to derive all results analytically[3]. The explicit characterization of the equilibrium level of financing cost in the uniform case is presented in Technical Appendix §B.2. For the two-product case in §5.2.3, we assume that $\boldsymbol{\xi}$ has a symmetric bivariate normal distribution with standard deviation σ and correlation coefficient ρ.

5.2.1 General Structural Properties

This subsection develops some general properties of the comparative static analysis that are used in the discussion of the single- and two-product cases.

Proposition 7 *$\dot{a}_T$ and $\dot{E}_T$ increase and decrease, respectively, in BC and U. $\dot{\pi}_T$, $\dot{\mathbf{K}}_\mathbf{T}(\tilde{B})$ and $\mathbb{E}[\mathbf{1}'\dot{\mathbf{K}}_\mathbf{T}(B)]$ decrease in BC and U.*

Proposition 7 demonstrates the effect of capital market imperfections on the firm's operational decisions and performance for a given technology choice. It is intuitive that a higher transaction cost (higher BC or U) induces the creditor to ask for a higher level of unit financing cost (which results in a lower credit limit). Higher transaction costs in the financial markets are directly transferred to the firm's operations, resulting in a lower expected equity value and capacity investment level in equilibrium.

We now develop some properties concerning the effect of a change in an arbitrary parameter τ (other than unit financing cost a_T and credit limit E_T) from τ_0 to τ_1 on the operational decisions and performance of the firm under technology T. Since the credit limit is uniquely determined by the unit financing cost ($E_T = \frac{P}{1+a_T}$), we only use a_T to characterize the loan terms. Let $\dot{a}_T(\tau_i)$ denote the equilibrium level of financing cost when $\tau = \tau_i$ for $i = 0, 1$ and let $\Upsilon_T = \{\pi_T, \mathbb{E}[\mathbf{1}'\mathbf{K}_\mathbf{T}(B)]\}$ be generic notation that denotes the expected (stage 0) equity value and expected total capacity investment level of the firm using technology T. The effect of the change in τ from τ_0 to τ_1 on $\dot{\Upsilon}_T$ is $\dot{\Upsilon}_T(\tau_1) - \dot{\Upsilon}_T(\tau_0)$,

[3] We also carry out the same analysis with normally distributed ξ. Unfortunately, it is not possible to characterize the equilibrium level of financing cost in closed-form and we mostly resort to numerical results in that case. Our numerical study demonstrates that the same effects are observed with the normal distribution assumption.

or,

$$\Upsilon_T^*(\tau_1; \dot{a}_T(\tau_1)) - \Upsilon_T^*(\tau_0; \dot{a}_T(\tau_0)),$$

which can be rewritten as

$$\underbrace{[\Upsilon_T^*(\tau_1; \dot{a}_T(\tau_1)) - \Upsilon_T^*(\tau_1; \dot{a}_T(\tau_0))]}_{\text{Strategic Effect}} + \underbrace{[\Upsilon_T^*(\tau_1; \dot{a}_T(\tau_0)) - \Upsilon_T^*(\tau_0; \dot{a}_T(\tau_0))]}_{\text{Static Effect}}.$$

The total effect can be written as a summation of two separate effects. The *static effect* captures the effect of the change in τ at the initial equilibrium financing cost $\dot{a}_T(\tau_0)$. The *strategic effect* captures the effect of the change in τ on the creditor's expected returns and, hence, the effect of the change in the equilibrium level of financing cost.

In perfect capital markets, as follows from Proposition 5, we have $\dot{a}_T(\tau_1) = \dot{a}_T(\tau_0) = 0$ and the strategic effect does not exist:

Property 1 *If $BC = U = 0$, there only exists the static effect, with $\dot{a}_T = 0$.*

The *strategic effect* only exists in imperfect capital markets with endogenous financing costs. The sign of the strategic effect is determined by the ordering of the equilibrium level of financing costs.

Property 2 *The strategic effect is negative (positive) if $\dot{a}_T(\tau_1) > \dot{a}_T(\tau_0)$ ($\dot{a}_T(\tau_1) < \dot{a}_T(\tau_0)$).*

Results concerning capacity investment under the (implicit) perfect market assumption are driven by the *static effect in perfect markets*. Although the magnitude of the *static effect in imperfect capital markets* is different from the magnitude of the static effect in perfect capital markets because of the higher (non-zero) financing costs, we will see in our later analysis that the sign of both static effects coincide in our setting. Thus, if we were to assume exogenous financing costs, the insights concerning capacity investment under the (implicit) perfect market assumption would continue to hold. However, financing costs should be modeled endogenously in imperfect capital markets: The change in τ alters the creditor's expected earnings and default risk, which induces the creditor to change his financing terms. As we shall see, this *strategic effect* may work in the opposite direction from the static effect, and reverse conclusions obtained under the perfect market assumption.

We now focus on specific τ parameters (demand structure, internal budget level and unit capacity investment cost) and analyze their impact in the single and two- product cases, respectively. In the rest of the paper, we focus on deterministic internal cash flow $\tilde{B} = \overline{B}$ for expositional purposes. All the analytical results will continue to hold for random internal cash flows with positive support and bounded expectation with minor modifications in the proofs. Since the budget level is deterministic, $\Upsilon_T = (\pi_T, \mathbf{K_T}(\overline{B}))$. For notational convenience, we drop the argument $\overline{B}$ in the optimal capacity vector and use $\Upsilon_T = \{\pi_T, \mathbf{K_T}\}$. In our analysis, we focus on the cases in which the firm borrows in equilibrium initially, i.e. $\dot{a}_T(\tau_0)$ is finite.

5.2.2 The Single Product Case

In the single product setting, the firm uses a single resource and technology choice is not relevant so we eliminate the D and F subscripts. The capacity investment and production quantity decisions of the firm follow from our analysis in §3 by setting the range of one of the demand uncertainties ξ_i to 0; we also eliminate the i subscript. For convenience, we summarize the results of this subsection in Table 1.

An Increase in:	Perfect Market ($U = BC = 0$)	Imperfect Market ($BC > 0$, $U > 0$)	Imperfect Market ($BC = 0$, $U > 0$)
Demand variability	Does not impact capacity level and equity value	Decreases capacity level and equity value	Does not impact capacity level and equity value
Budget level	Does not impact capacity level, increases equity value	Decreases capacity level, may decrease equity value	Decreases capacity level, may decrease equity value
Unit capacity cost	Decreases capacity level and equity value	Decreases capacity level and equity value even more	Decreases capacity level and equity value even more

Table 1: Differences between perfect and imperfect markets in single-product investments with uniform $[0, 2\overline{\xi}]$ demand uncertainty. The differences between perfect and imperfect markets are driven by the strategic effect that only exists under capital market imperfections.

The effect of demand variability. As follows from Proposition 6, in perfect capital markets, the firm's equilibrium capacity decision and expected (stage 0) equity value $\dot{\Upsilon}$ depend on the expected value of demand but not its variability; the *static effect* of demand variability is 0 with no transaction cost. The following proposition demonstrates that independence from demand variability may not hold with positive transaction costs.

Proposition 8 *If $BC > 0$ and $\xi \sim U[0, 2\overline{\xi}]$, $\dot{a}$ increases and $\dot{\Upsilon} = (\dot{\pi}, \dot{K})$ decreases in σ.*

For an arbitrary financing cost a, increasing variability does not alter $\dot{\Upsilon}$; *the static effect in imperfect markets* is also zero. With the uniform distribution, higher variability at the same expected demand level corresponds to the mean-preserving spread of ξ – more probability mass is transferred to the tails, and in particular, the downside risk of the firm's operating cash flows increases. This leads to a higher expected default risk for the firm and lower expected returns for the creditor. To compensate for this reduction, the creditor charges a higher financing cost in equilibrium. Therefore, *the strategic effect* is negative as follows from Property 2. The total effect is therefore negative and an increase in demand variability decreases $\dot{\Upsilon}$. Xu and Birge (2004) demonstrate the negative strategic effect of demand variability with numerical observations in a price-taking newsvendor setting. Proposition 8 is a parallel result, and analytically proves this observation in our modelling context.

If there is no bankruptcy cost, then the increase in the downside risk of the operating cash flows does not have an effect on the creditor's expected returns. The equilibrium level of financing cost is not affected, and the *strategic effect* is zero. $\dot{\Upsilon}$ is not affected either.

The effect of the internal budget level. With an increase in the internal budget level, the *static effect in perfect capital markets* on the equilibrium capacity investment level is zero (because the firm is not budget-constrained), and the effect on the expected (stage 0) equity value is positive: The expected (stage 0) equity value is higher because the internal budget level is higher. The following proposition demonstrates that these results may not hold in imperfect capital markets.

Proposition 9 *If $U > 0$ and $\xi \sim U[0, 2\bar{\xi}]$, when $BC = 0$ ($BC > 0$), $\dot{a}$ increases (locally increases) and $\dot{K}(\overline{B})$ decreases (locally decreases) in $\overline{B}$.*

When the firm borrows in equilibrium, the *static effect* of the budget level is zero on the equilibrium capacity investment level (since $K^1 = \left(\frac{\bar{\xi}(1+\frac{1}{b})}{c(1+a)}\right)^{-b}$ is independent of $\overline{B}$), and is positive on the equilibrium expected (stage 0) equity value (since the budget level is higher). To analyze the strategic effect, we first analyze the creditor's expected return. From Technical Appendix §B.2, it follows that the uniform distribution assumption leads to a nice decomposition of the creditor's expected return for a given a under which the firm optimally borrows ($\overline{B} \in \Omega_2(a)$):

$$\Lambda(a) = \left(cK^0(1+a)^b - \overline{B}\right)\left(a - \frac{BC\;\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0(1+a)^b}\right). \tag{9}$$

The first term is the amount of lending. The second term is the unit marginal profit of lending. For each unit of the loan $cK^0(1+a)^b - \overline{B}$, the creditor earns a minus the expected default cost. At a given a, the unit marginal profit of the creditor is independent of the internal budget of the firm, but a higher budget level means a smaller loan as $\overline{B}$ is larger. With zero bankruptcy cost (and $U > 0$), the unit marginal profit is always positive. Therefore, to compensate for the reduction in the loan size, the creditor increases the unit financing cost in equilibrium, and the *strategic effect* is negative as follows from Property

2. With positive bankruptcy cost, this result can be proven only for local changes in $\overline{B}$. However, in all numerical experiments, we observed that the negative strategic effect is not confined to local changes in $\overline{B}$.

The total effect on the capacity investment level (both static and strategic) is negative, and the equilibrium capacity investment level decreases in $\overline{B}$. The total effect on the equity value is determined by the relative magnitudes of the positive static and negative strategic effects. Our numerical investigation reveals that the strategic effect may dominate the static effect and the equilibrium equity value may decrease in the internal budget level (see the example following the proof of Proposition 9 in Appendix).

To summarize, contrary to intuition, a higher internal budget may lead to reductions in the equilibrium capacity investment and performance of the firm due to the strategic effect. It could be argued that firms with higher internal cash flows can obtain lower financing costs, based on the premise that with more internal capital, the firm borrows less, and hence has lower default risk. Proposition 9 highlights the importance of the borrowing level, and demonstrates that a lower external borrowing need may induce the creditor to charge a higher unit financing cost in equilibrium.

The effect of unit capacity investment cost. The static effect of an increase in the capacity investment cost on $\dot{\Upsilon}$ is negative in perfect capital markets. In imperfect capital markets, an increase in c has an additional impact on $\dot{\Upsilon}$ due to an increase in $\dot{a}$:

Proposition 10 *If $U > 0$ and $\xi \sim U[0, 2\overline{\xi}]$, with $BC = 0$ ($BC > 0$), $\dot{a}$ increases (locally increases), and $\dot{\Upsilon} = \{\dot{\pi}, \dot{K}\}$ decreases (locally decreases) in c.*

With zero bankruptcy cost (and $U > 0$), a higher c reduces the expected amount of lending in (9) because the firm invests more conservatively in capacity. As a result, the creditor increases the equilibrium level of financing cost and the *strategic effect* is negative as follows from Property 2. With positive bankruptcy cost, a higher c also increases the unit marginal profit of lending in (9). It turns out that with a small increase in c, the former effect dominates the latter, and the creditor increases the financing cost to compensate for the reduction in expected returns. Therefore, the *strategic effect* is locally negative. In all numerical experiments, we observed that the negative strategic effect is not confined to local changes in c. The *static effect in imperfect capital markets* is also negative; hence the total effect is negative. Proposition 10 demonstrates that firms that are exposed to capital market imperfections may underestimate the effect of higher investment costs if they do not consider the resulting change in the equilibrium level of financing costs.

5.2.3 The Two-Product Case

In this subsection, we investigate the effect of demand uncertainty on the equilibrium capacity investment level and performance of the firm with each technology in the two-product case. We work with the symmetric bivariate normal distribution with covariance matrix $\mathbf{\Sigma}$, where $\mathbf{\Sigma}_{ii} = \sigma^2$ and $\mathbf{\Sigma}_{ij} = \rho\sigma^2$ for $i \neq j$. We use the same parameter set ($c_F = c_D = 2, P = 220, b = -2, \overline{\xi}_1 = \overline{\xi}_2 = 20, \overline{B} = 5, \sigma \in [2, 5.5], \rho \in [-0.995, 0.995], U = 20, BC \in \{25, 50, 75, 100\}$) for numerical examples throughout. Our main results are summarized in Table 2.

		Perfect Market ($BC = U = 0$)	Imperfect Market ($BC > 0$, $U > 0$)	Imperfect Market ($BC = 0$, $U > 0$)
Dedicated	Increase in σ	Does not impact capacity level and equity value	Decreases capacity level and equity value	Does not impact capacity level and equity value
	Increase in ρ	Does not impact capacity level and equity value	Decreases capacity level and equity value	Does not impact capacity level and equity value
Flexible	Increase in σ	Increases capacity level and equity value	May decrease capacity level and equity value	Increases capacity level and equity value even more
	Increase in ρ	Decreases capacity level and equity value	May increase capacity level and equity value	Decreases capacity level and equity value even more

Table 2: Differences between perfect and imperfect markets in two-product investments with bivariate normal demand uncertainty. Without bankruptcy cost, the direction of the effect of demand variability and correlation are identical to those in the perfect market benchmark. The existence of bankruptcy cost may reverse the conclusions obtained with the perfect market assumption.

Dedicated Technology. Similar to the single product case, in perfect capital markets, the firm's equilibrium capacity decision and the expected (stage 0) equity value with the dedicated technology depend only on the mean demand vector $\overline{\boldsymbol{\xi}}$ and not on the covariance matrix $\mathbf{\Sigma}$ in our model; the *static effect in perfect markets* is zero. In contrast, in imperfect capital markets, the elements of the covariance matrix (ρ and σ) may also matter:

Proposition 11 *If $BC > 0$ and $\boldsymbol{\xi}$ is symmetric bivariate normal, then $\dot{\Upsilon}_D = (\dot{\pi}_D, \dot{\mathbf{K}}_\mathbf{D})$ decreases in ρ and σ through an increase in $\dot{a}_D$.*

The *static effect in imperfect markets* is also zero; hence the changes in $\dot{\Upsilon}_D$ are driven by the negative *strategic effect*. As in the single product case, higher variability increases the downside risk and in turn, the default risk of the firm. With positive bankruptcy cost, this reduces the expected return of the creditor for an arbitrary financing cost a_D. The creditor increases the equilibrium level of financing cost to compensate for this reduction.

The result with respect to ρ follows from a financial risk-pooling argument. The firm's default probability for a given capacity level depends on the variability in operating revenues. Operating in two markets creates a *diversification benefit* for the firm: When the demands are negatively correlated, revenue variability and hence default risk are low. With high positive correlation, the firm generates similar revenues from both markets, and operates under higher default risk. As correlation increases, the diversification benefit decreases and the creditor increases $\dot{a}_D$ to compensate for the increase in expected default costs.

These results are demonstrated in Figure 2. In addition, in all our numerical experiments, we observe that the strategic effect of demand variability is insignificant at low correlations. This can also be observed in Figure 2: At low correlations, the diversification benefit is very high, therefore higher variability does not increase the default risk of the firm. Hence, the creditor does not alter the financing cost with an increase in variability (Panel A). In this case, only the static effect is active and the equilibrium capacity level (Panel B) and equity value (Panel C) are not affected by a change in σ. The financial risk-

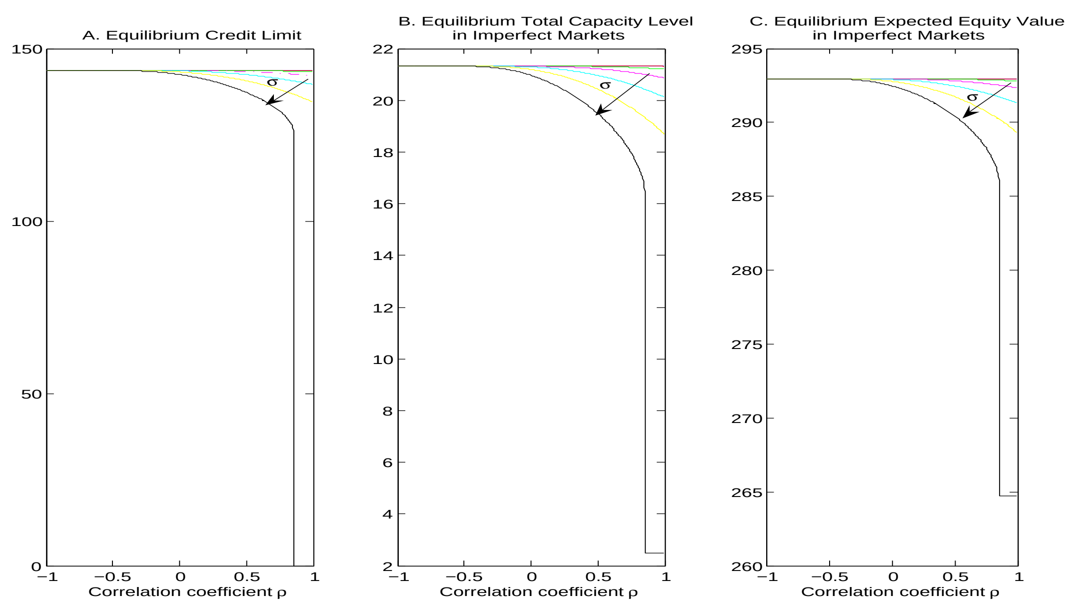

Figure 2: Effect of demand correlation ρ and demand variability σ on dedicated technology investment in imperfect markets with $BC = 50$ and $\sigma \in [2.5, 5.5]$ with 0.5 increments. The equilibrium level of credit limit $\dot{E}_D = \frac{P}{1+\dot{a}_D}$ is plotted rather than $\dot{a}_D$; this avoids scale compression when $\dot{a}_D \to \infty$. Observe that a higher ρ or σ lead to a lower credit limit $\dot{E}_D$ (a higher financing cost $\dot{a}_D$) in equilibrium (Panel A) and this decreases the $\mathbf{1}'\dot{\mathbf{K}}_\mathbf{D}$ (Panel B) and $\dot{\pi}_D$ (Panel C). Both quantities are robust to changes in demand variability at low correlations: The diversification benefit is high in this range and the equilibrium financing cost is robust to changes in demand variability.

pooling effect discussed in this section is different from the capacity-pooling effect of flexible technology that comes from the ability to switch capacity between products. The former effect only exists in imperfect capital markets with positive bankruptcy cost, whereas the latter is a demand effect and also exists in perfect capital markets.

Without any bankruptcy cost (but $U > 0$), the creditor is not concerned with the default risk of the firm; hence both the *strategic effect* and the total effect are zero.

Flexible Technology. In this section, we analyze the effect of increasing ρ and decreasing σ on $\dot{\Upsilon}_F$. In perfect capital markets, $\dot{\Upsilon}_F$ depends on the covariance matrix $\mathbf{\Sigma}$ of $\boldsymbol{\xi}$ through the term $M_F \doteq E\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$ (Proposition 6). This term captures the capacity-pooling feature of flexible technology (the ability to switch production between two products after demand uncertainty is resolved). Unfortunately, it is not possible to derive analytically the effect of ρ and σ on M_F for bivariate normal $\boldsymbol{\xi}$. In our numerical experiments, at a sufficiently low coefficient of variation $\frac{\sigma}{\bar{\xi}}$ to avoid negative demand realizations, we observe that M_F increases in σ and decreases in ρ. In the rest of the paper, to derive analytical results, we make the following assumption:

Assumption 1 $\frac{\partial}{\partial \sigma} M_F \geq 0$ *and* $\frac{\partial}{\partial \rho} M_F \leq 0$.

Assumption 1 is in line with the traditional argument on flexible technology investment: Its value increases in demand variability and decreases in demand correlation.

With Assumption 1, the *static effect in perfect capital markets* is negative: $\dot{\Upsilon}_F$ decreases with increasing ρ and decreasing σ. The *static effect in imperfect capital markets* is also determined by the change in M_F and is negative. To analyze the *strategic effect*, we focus on the creditor's expected return. Expanding (8) for flexible technology and deterministic budget $\overline{B}$, the expected return of the creditor is given by

$$\Lambda_F(a_F) = \left(c_F K_F^1 - \overline{B}\right) a_F - BC\ Pr\left\{\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}} < \frac{(c_F K_F^1 - \overline{B})(1 + a_F)}{K_F^{1\,1+\frac{1}{b}}}\right\}. \quad (10)$$

The *strategic effect* has three main drivers. First, with Assumption 1, an increase in ρ or a decrease in σ decreases the capacity investment level K_F^1. This decreases the first term in (10), the earnings (without default) of the creditor, at a given a_F. With Assumption 1, the right-hand side of the default probability expression decreases at a given a_F. The first effect incents the creditor to increase the unit financing cost, while the second incents him to decrease it. Third, a change in ρ or σ alters the distribution of $\boldsymbol{\xi}$, and in turn, the distribution of the random variable $H_F(\boldsymbol{\xi}) \doteq \left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}$, where $H_F(\tilde{\boldsymbol{\xi}})$ captures the capacity-pooling effect due to production switching and the financial-pooling effect due to the diversification benefit at a given capacity investment level K_F^1. The effect of an increase in ρ or a decrease in σ cannot be proven analytically except in special cases, but our numerical experiments indicate that they increase the default probability, which would incent the creditor to increase the unit financing cost.

To summarize, the net *strategic effect* with the flexible technology is a combination of three effects. Without bankruptcy cost ($BC = 0, U > 0$), only the reduction in the earnings (without default) of the creditor is effective and it can be shown that the *strategic effect*, and in turn the total effect, is negative. With positive bankruptcy cost ($BC > 0, U > 0$), the net strategic effect is indeterminate. Figure 3 demonstrates that the strategic effect can be sufficiently positive to dominate the negative static effect in imperfect markets: In contrast to the perfect market setting, $\dot{\Upsilon}_F$ may *increase* in ρ and *decrease* in σ. In Panels C and E of Figure 3, we observe that this happens at high correlations. This is because at these correlation levels, reducing σ increases the credit limit (decreases the financing cost; Panel A) and the strategic effect is sufficiently positive to dominate the negative static effect. A similar (but slight) effect is observed when increasing ρ at very high correlation levels (Panel F): With $\rho \approx 1$, the negative static effect of ρ is sufficiently low that the positive strategic effect can dominate it[4]. We also observe that the total effect of increasing σ is positive at low correlations (Panel C and E). This observation is in contrast with the numerical observation of Lederer and Singhal (1994) about the strategic effect of σ in a price-taking newsvendor setting. This is because the default risk is robust to changes in σ at these correlation levels: Low correlation implies high capacity pooling and diversification benefits, and the default risk is not affected by the changes in σ. The earnings (without default) of the creditor increase in σ (because the capacity investment level increases), and the strategic effect is negative. Therefore the total effect is also negative.

5.3 Technology Choice under Capital Market Imperfections

In §5.2, we performed comparative statics analysis for a given technology. In this section, we turn into equilibrium technology choice in imperfect capital markets. Our main results are summarized in Table 3.

We first start with technology choice with identical costs ($c_F = c_D$). As intuition suggests (and as also follows from Propositions 3 and 6), the firm then always prefers flexible technology in perfect capital markets due to the option value of production switching. In

[4]This effect also holds for the equilibrium capacity investment level. This can be observed from Figure 5 in Appendix A, after the proof of Proposition 11, in which we re-plot Figure 3 for $\rho \geq 0.95$.

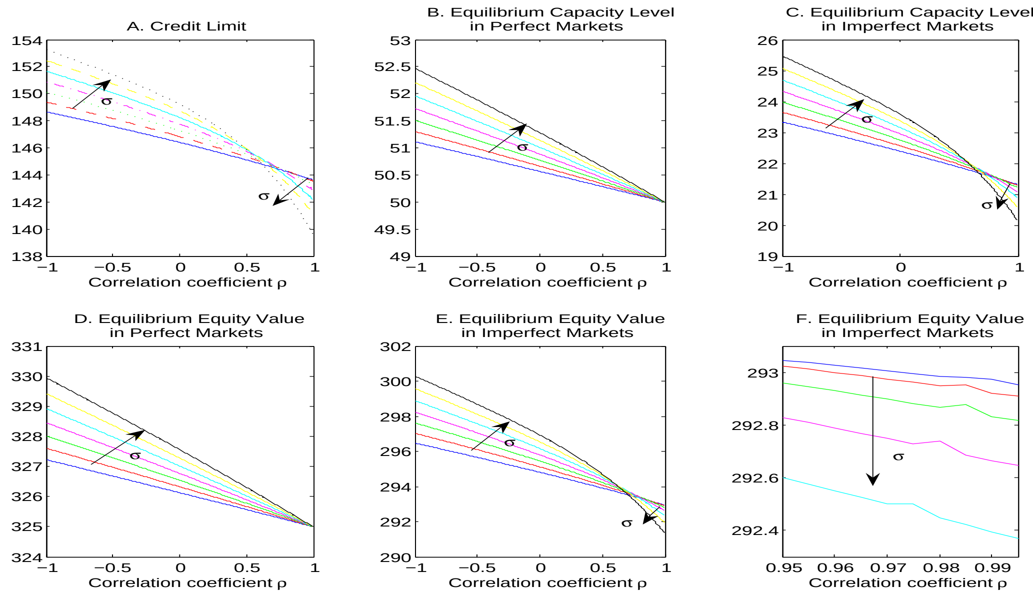

Figure 3: Effect of ρ and σ on flexible technology investment in imperfect markets with $BC = 50$ and $\sigma \in [3, 4.5]$ with 0.25 increments: A higher σ leads to a lower credit limit $\dot{E}_F$ (higher financing cost $\dot{a}_F$) at low correlation levels and a higher credit limit $\dot{E}_F$ at high correlation levels (Panel A). In perfect capital markets, $\dot{K}_F$ (Panel B) and $\dot{\pi}_F$ (Panel D) increase with an increase in σ and a decrease in ρ. In imperfect capital markets, $\dot{K}_F$ (Panel C) and $\dot{\pi}_F$ (Panel E) decrease with an increase in σ at high correlation levels. This is because the strategic effect is negative in this range. For the same reason, a decrease in ρ may lead to a higher $\dot{\pi}_F$ (Panel F, $\sigma \in [3, 3.25.., 4]$).

our numerical experiments, we observe that the firm may prefer dedicated technology over flexible technology in imperfect capital markets even with identical costs (at $BC = 100$, $\sigma \in \{4.5, 4.625\}$ and $\rho = 0.995$ in our numerical set). This result is driven by the higher financing cost of flexible technology in equilibrium, i.e. $\dot{a}_F > \dot{a}_D$. If the creditor offers identical financing costs for each technology, then the firm borrows more and invests more in capacity with flexible technology. Since the earnings (without default) are higher with flexible technology, this incents the creditor to charge a lower financing cost to flexible technology. On other hand, since the borrowing level is higher with flexible technology and capacity pooling is not of high value at this correlation level, the default risk of the firm is higher with flexible technology. This incents the creditor to charge a higher financing cost with flexible technology. At this numerical set, we observe that the second effect dominates the first effect and $\dot{a}_F > \dot{a}_D$. Similarly, in other numerical experiments, we observe the dominance of the dedicated technology at close to perfect positive correlations. This is

		Perfect Market ($BC = U = 0$)	Imperfect Market ($BC > 0$, $U > 0$)	Imperfect Market ($BC = 0$, $U > 0$)
Technology choice	$c_F = c_D$	Flexible is always preferred	Dedicated may be preferred to flexible	Flexible is always preferred
	$c_F > c_D$ $c_F \leq \bar{c}_F^P(c_D)$	Flexible is always preferred	Dedicated may be preferred to flexible	Flexible is always preferred

Table 3: Differences between perfect and imperfect markets in technology choice with bivariate normal demand uncertainty. The firm may prefer dedicated technology over flexible technology even with identical costs. This is because the financing cost is higher with flexible technology in equilibrium.

because the option value of production switching is very low at high correlations; and the negative strategic effect of the larger financing cost of the flexible technology can overcome this option value.

In the finance literature, operational flexibility is associated with positive strategic value: It is argued that since operational flexibility decreases the firm's default risk by generating higher returns due to its option value, the equilibrium financing cost decreases (MacKay 2003). We find that this argument may not hold in general with a stronger formalization of the firm's operations. Anticipating the option value of operational flexibility (flexible technology in our case), the firm optimally adjusts other operational decisions (capacity investment level). As a result, the creditor may charge higher financing cost in equilibrium, yielding a negative strategic value.

We now turn to technology choice with non-identical unit capacity investment costs. In general, the production switching capability of the flexible technology comes with higher unit capacity investment cost ($c_F > c_D$). As follows from Proposition 6, the firm prefers flexible technology if $c_F \leq \bar{c}_F^P(c_D)$ in perfect capital markets. If there is no bankruptcy cost ($BC = 0$), then the firm still prefers the flexible technology within the cost range $c_F \leq \bar{c}_F^P(c_D)$ in imperfect capital markets ($U > 0$):

Proposition 12 *If $BC = 0$ and $U > 0$, $\dot{T} = F$ for $c_F \leq \bar{c}_F^P(c_D)$.*

For any (c_F, c_D) in this range, the firm can always secure lower financing cost in equilibrium with flexible technology ($\dot{a}_F(c_F) < \dot{a}_D(c_D)$). This is because the creditor is only concerned with the earnings (without default) and the firm always borrows more with flexible technology if the financing costs are identical with each technology $a_F = a_D$. Therefore, the creditor always charges lower financing cost in equilibrium with flexible technology.

If there exists a positive bankruptcy cost ($BC > 0$), then the default risk concern of the creditor may induce alterations in the technology choice relative to the perfect market benchmark. Let $\bar{c}_F^P(c_D;\rho)$ be the unit cost threshold of Proposition 6 parametrized by demand correlation ρ and let $\boldsymbol{\xi}$ have positive support.

Proposition 13 *If* $BC > 0$, $\rho \approx 1$ *and at the unit capacity investment cost pair* $\left(c_D, \bar{c}_F^P(c_D;\rho)\right)$, $\dot{a}_D(c_D) < \dot{a}_F(\bar{c}_F^P(c_D;\rho))$ *and* $\dot{T} = D$.

In perfect markets, the firm is indifferent between the two technologies at the cost pair $\left(c_D, \bar{c}_F^P(c_D;\rho \approx 1)\right)$ as follows from Proposition 6. According to Proposition 13, with this cost pair, although the firm is indifferent between the two technologies in perfect capital markets, the firm strictly prefers dedicated technology in imperfect capital markets due to the higher financing cost the creditor offers for the flexible technology in equilibrium.

It follows from Proposition 3 that the firm would be indifferent between the two technologies at the cost pair $\left(c_D, \bar{c}_F^P(c_D;\rho \approx 1)\right)$ if both were exposed to identical financing costs. Therefore, the ordering of the financing cost in equilibrium determines the technology choice. To understand the intuition of the equilibrium ordering of the financing cost, suppose that the creditor offers identical financing costs for each technology. Then we show that the firm borrows the same amount from the creditor, and hence the earnings of the creditor (without default) are identical with each technology. However, the default risk with dedicated technology is lower: Production switching is not of high value (because of high correlation) and the firm optimally invests in a higher capacity level with the dedicated technology ($\mathbf{1}'\mathbf{K}_{\mathbf{D}}^*(a) > K_F^*(a)$) for a given a. The higher total capacity investment enables the firm to generate sufficient revenues to avoid default with the dedicated technology at some demand realizations in which the firm defaults with the flexible technology. Thus, in equilibrium, the financing cost with dedicated technology is lower. Consequently, the firm chooses the dedicated technology in equilibrium. This result demonstrates that ignoring the strategic effect may induce the firm to choose the sub-optimal technology.

6 Discussion of Assumptions and Extensions.

In this section, we briefly present highlights from analysis not included in the paper and that extends our model or relaxes some of our assumptions.

Engaging in financial risk management. If the firm's internal cash flow B is correlated with a financially tradable index, then the firm can engage in financial risk management in stage 0 to change the distribution of B. Financial risk management has value because it generates internal capital (Rao and Gutierrez 2007) and reduces the firm's dependence on external borrowing (Froot et al. 1993). If B is linearly dependent on a financial index

($B = \omega\alpha$ where ω is the number of assets, and α is the stochastic asset price), and the financial derivatives are fairly priced, we can show (building on the analysis for a random B presented in §3 and 4) that the firm optimally isolates the budget from the underlying risk exposure with financial risk management, and has a deterministic internal cash flow $\bar{B}$ at stage 1. Therefore, all the results in this paper continue to hold.

The value of engaging in financial risk management can be written as the summation of the static and the strategic effects. In a single-product setting with uniform demand, we show that the strategic effect of engaging in financial risk management is always negative: Engaging in financial risk management decreases the expected borrowing level of the firm, and the creditor increases the unit financing cost to compensate for this reduction. This is in contrast to Smith and Stulz (1985), who demonstrate that when the firm uses financial risk management to hedge its cash flows *after the loan is taken*, then engaging in financial risk management decreases the equilibrium level of financing cost by reducing the expected default risk. What we have shown is that if financial risk management is effective *before the loan is taken*, it increases the equilibrium level of financing cost by decreasing the expected earnings of the creditor.

Stage 0 internal cash flow and the fixed cost of technology. We assume that the firm starts with zero internal endowment at stage 0 and does not incur any fixed costs when the technology commitment is made. This assumption can be relaxed. Let F_T denote the fixed technology cost and B_0 the initial (stage 0) endowment of the firm. If $F_T \leq B_0$, all results of the paper hold by replacing $\overline{B}$ with $\overline{B} + B_0 - F_T$. This constraint is necessary because we only consider borrowing from the creditor at stage 1; hence the firm should have sufficient internal endowment to cover the fixed cost of technology at stage 0.

With this assumption, two new observations can be made. First, the equilibrium capacity investment level can be shown to be *strictly decreasing* in a linear fashion in the fixed cost for some budget realizations. This is in contrast to the traditional capacity investment models that implicitly assume perfect capital markets where only the invest/do not invest boundary decision depends on the fixed cost, but not the actual capacity investment level (Van Mieghem 2003, p.277). The difference is due to the fact that a higher fixed cost reduces the internal budget available for capacity investment. Second, the equilibrium equity value of the firm may *increase* in the fixed cost. This follows from Proposition 9 and our discussion in §5.2.2. In a single-product setting with uniform demand uncertainty, a higher fixed cost leads to a lower residual stage 1 internal budget level $\overline{B} + B_0 - F_T$, which may decrease the financing cost in equilibrium and increase the equilibrium level of equity value.

Positive production cost at stage 2. Let y denote the unit production cost for both products with either technology. To focus on secured loan commitment contracts, we assume that P is sufficiently large to cover the production and the capacity investment costs of the budget-unconstrained capacity investment level. With this assumption, the perfect market benchmark continues to hold by replacing c_T with $c_T + y$. In imperfect capital markets, with $y > 0$, the optimal production vector at stage 2 is limited by the cash availability of the firm in addition to the capacity constraints. With the $y > 0$ assumption, the literature often uses a clearing-pricing strategy for tractability that fully utilizes capacity (see for example, Chod and Rudi 2005). If we assume a clearing-pricing strategy, the firm optimally borrows so as to fully utilize its capacity in stage 2 and all the results of our paper continue to hold by replacing c_T with $c_T + y$.

7 Conclusion

This paper contributes to the stochastic capacity investment literature by modeling the impact of endogenous financing costs under capital market imperfections. In a parsimonious and tractable model of the creditor-firm interaction, we solve for the technology choice, external borrowing, capacity, and production level decisions of the firm and the creditor's loan contract terms in equilibrium. We write the total effect of a change in operational parameters on the equilibrium decisions of the firm as the sum of two effects that we call the *static effect* and the *strategic effect*. The *static effect* is the effect of a change in an operational parameter at a *given* financing cost. At the same time, a parameter change alters the equilibrium level of financing cost by impacting the creditor's expected returns through the firm's borrowing level and/or its default risk. The *strategic effect* captures the effect of the change in the equilibrium level of financing cost.

The strategic effect only exists with market imperfections and endogenous financing cost. The static effect in imperfect capital markets has the same sign (but a different magnitude) as the static (and only) effect in perfect capital markets. Thus, an operational parameter change acts in the same direction in perfect capital markets and in imperfect capital markets with exogenously modelled financing cost. Our most revealing finding is that the strategic effect may act in the opposite direction, generating a fundamentally different total effect. This finding highlights that in creditor-firm interactions where the financing cost depends on the firm's operational characteristics, ignoring the endogeneity of financing costs may lead to not only suboptimal, but in some cases diametrically opposed operational choices in response to a change in operational parameters.

This paper documents a rich set of implications of the endogeneity of financing costs, and in particular, of the strategic effect, for capacity investment and technology choice. Below we primarily highlight those where conclusions that would be obtained under the (often implicit) perfect market assumption are modified or reversed, and discuss their drivers.

1. An increase in capital market imperfection costs decreases the expected equity value of the firm because it leads to higher financing costs in equilibrium. The equilibrium capacity investment level should be reduced with higher imperfection costs.
2. With transaction costs, an increase in the internal budget of the firm may *decrease* the equilibrium expected equity value, as demonstrated in a single product setting with uniform demand: A higher internal budget increases the equilibrium financing cost because it decreases the borrowing level of the firm at a given financing cost. Due to the higher borrowing cost, the equilibrium capacity level should be decreased. With no transaction costs, the expected equity value increases in the budget level and the capacity level is not affected.
3. Consider a two-product setting with dedicated technology and symmetric bivariate normal demand. With no transaction costs, the expected equity value and capacity level are independent of ρ and σ. With transaction costs, an increase in demand correlation (ρ) or variability (σ) decreases the equilibrium expected equity value: A higher ρ reduces the diversification benefits of investing in two markets ("financial pooling"), which leads to a higher default risk and hence higher financing costs in equilibrium. A higher σ leads to higher downside risk, higher default risk and hence higher financing costs in equilibrium. The equilibrium capacity investment level should be decreased with an increase in ρ or σ.
4. Consider a two-product setting with flexible technology and symmetric bivariate normal demand. With the perfect market assumption, the expected equity value and capacity level always increase with an increase in σ and a decrease in ρ. This is because of the increasing value of capacity-pooling. With bankruptcy cost, an increase in σ increases (decreases) the expected equity value at low (high) correlation levels. At low correlation levels, diversification benefits are high and the default risk is not affected by a change in σ. The firm borrows more if σ increases, so the equilibrium financing cost decreases. The net effect is that the equilibrium expected equity value increases in σ because the capacity-pooling value at a given financing cost increases, and the equilibrium level of financing cost decreases. At high correlation levels, the diversification benefit is lower and a higher σ increases the default risk, and hence the

equilibrium level of financing cost. The increase in the equilibrium level of financing cost dominates the increase in the capacity-pooling value and the expected equity value decreases in σ. The equilibrium capacity level should be increased (reduced) with an increase in σ at low (high) correlation levels. Similar arguments are applicable to the effect of ρ. The expected equity value may increase in ρ at strongly positive correlations. This is because a higher ρ decreases the financing cost in equilibrium, and this dominates the declining value of capacity pooling.

5. In perfect markets, the firm always prefers flexible technology if its cost is the same as dedicated technology due to the option value of production switching. In contrast, the firm may prefer dedicated technology over flexible technology in imperfect capital markets even with identical investment costs. This result is driven by the potentially higher financing cost of flexible technology in equilibrium: If the operational decisions are made under identical financing costs, the firm borrows more with flexible technology and may have a higher default risk. This induces the creditor to charge a higher financing cost in equilibrium.

This paper brings constructs and assumptions motivated by the finance literature into a classical operations management problem and develops new insights. In turn, by formalizing operational decisions of the firm in more detail than the finance literature, we provide novel insights on issues discussed in this literature. For example, in contrast to arguments summarized in McKay (2003), we show that operational flexibility (flexible technology in our case) may increase financing costs in equilibrium. This is because the firm optimally adjusts other operational decisions (capacity investment level) with operational flexibility. We extend the modeling framework of Froot et al. (1993) and enrich the argument that engaging in financial risk management has positive strategic value (Smith and Stulz 1985) by showing that it can have negative strategic value when financial risk management is effective before the loan is taken: This decreases the expected borrowing level of the firm and raises its financing costs.

In §6, we discussed the implication of relaxing some of our assumptions. Other interesting research directions remain. For example, we focus on a particular type of financing contract and two types of capital market imperfections. The firm can also issue equity or raise external capital by spot borrowing (not forward borrowing as in the case of the loan commitment contract). The firm can also be exposed to different imperfection costs. We assume that the creditor has perfect information about the firm. In reality, the creditor may not have perfect information about the risk-profile of the operational investments, or perfectly monitor the firm after the loan is taken, or have the same valuation of the collateralized assets as the firm. These would create agency costs and impose additional financing frictions. While the strategic effect would always exist with any type of financing friction, the operational implications are expected to be model-specific: Our analysis shows that the type of market imperfection matters. There are significant differences between the zero bankruptcy cost ($BC = 0, U > 0$) and positive bankruptcy cost ($BC > 0, U > 0$) cases.

We assume a stylized firm that is liquidated at the end of a single period. Shubik et al. (2003) model the dynamic capital structure choice of the firm without bankruptcy. Extending our analysis to a multi-period setting is certainly a non-trivial task and requires further research. Besides the usual operational dynamics of multi-period models (capacity and inventory carryover), there also exist additional dynamics coming from capital carryover: The firm may decide to hold some of the earlier loans as internal capital for future expenditures. In addition, the indirect costs of bankruptcy should be incorporated because the financing cost in equilibrium depends on the earlier loan performance of the firm.

This paper focuses on a monopolistic firm. In a competitive setting, joint capacity investment and financing decisions has not received much attention. Goyal and Netessine (2007) analyze the value of flexible technology in a two-product setting under product market competition in perfect capital markets. They delineate the conditions under which operational flexibility is beneficial. Maksimovic (1990) demonstrates the value of financial flexibility (being able to raise external capital) under product market competition in a single-product setting. It would be interesting to look jointly at the operational and financial flexibilities of a firm in a competitive setting.

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TECHNICAL APPENDIX TO:
STOCHASTIC CAPACITY INVESTMENT AND TECHNOLOGY CHOICE IN IMPERFECT CAPITAL MARKETS

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In this technical appendix, we provide the proofs of the technical statements in our paper. Appendix A includes the main proofs. Appendix B characterizes the expected (stage 0) return of the creditor and the equilibrium level of financing cost with uniform demand in the single-product setting. Appendix C contains the proofs for the supporting lemmas that we use in Appendix A. Table 4 summarizes the decision variables by stage. Table 5 summarizes all the other notation. Superscript * is appended to denote the optimal value; $\dot{}$ is appended to denote the equilibrium level.

Decision Maker	Stage	Name	Meaning
Creditor	Stage 0	a_T	Unit financing cost with technology T
		E_T	Credit limit with technology T ($E_T = \frac{P}{1+a_T}$)
Firm	Stage 0	$T \in \{D, F\}$	Technology choice, dedicated or flexible
	Stage 1	e_T	Borrowing level with technology T
		$\mathbf{K_T}$	Capacity investment level with technology T
	Stage 2	$\mathbf{Q_T}$	Production quantity with technology T

Table 4: Decision variables by stage

A Appendix A: Main Proofs

Proof of Proposition 1: Let $f(\mathbf{Q}) \doteq \tilde{\boldsymbol{\xi}}'\mathbf{Q}^{1+\frac{1}{b}}$ and $\mathbf{Q}^*_\mathbf{T}$ denote the optimal production vector that solves (1) for technology $T \in \{F, D\}$. It is easy to establish that $f(\mathbf{Q})$ is strictly concave in $\mathbf{Q}' = (q_1, q_2)$. Since the constraints are linear, KKT conditions are necessary and sufficient for optimality and $\mathbf{Q}^*_\mathbf{T}$ is unique. Since $\frac{\partial f}{\partial q_i} = (1 + 1/b)\tilde{\xi}_1 q_i^{1/b} > 0$, and with $b \in (\infty, -1)$, $\lim_{q_i \to 0^+} \frac{\partial f}{\partial q_i} \to \infty$, the non-negativity constraints will be non-binding and the

Name	Meaning
BC	fixed bankruptcy cost
U	underwriter fee (reservation utility of the creditor)
$r_f(=0)$	risk-free rate
P	value of the physical assets of the firm
b	constant elasticity of demand
$\boldsymbol{\xi} = (\xi_1, \xi_2)$	multiplicative demand intercept in product markets
$\boldsymbol{\Sigma}$	covariance matrix of $\boldsymbol{\xi}$
ρ	correlation coefficient in $\boldsymbol{\Sigma}$
σ	standard deviation of ξ_i for $i = 1, 2$
Pr	probability
$\mathbb{E}$	expectation operator
$\bar{c}_F^P(c_D)$	unit variable cost threshold in perfect markets
$\bar{c}_F(c_D, \mathbf{a})$	unit variable cost threshold in imperfect markets
$\bar{c}_F^S(c_D)$	unit variable cost threshold for $a_F = a_D$
Γ_T	stage 2 operating profits of the firm with technology T
Π_T	stage 2 equity value of the firm with technology T
π_T	expected (stage 1) equity value of the firm with technology T
Y^*	expected (stage 0) equity value of the firm with better technology for $T \in \{D, F\}$
Ψ^*	optimal expected (stage 0) equity value of the firm
$\Lambda_T(a_T)$	expected (stage 0) return of the creditor with loan contract offered for technology T

Table 5: Summary of Notation

capacity constraint will be binding at optimality. With the dedicated technology, this yields $\mathbf{Q_D^*} = \mathbf{K_D}$ and

$$\Gamma_D^*\left(\mathbf{K_D}, \tilde{\boldsymbol{\xi}}\right) = f(\mathbf{Q_D^*}) = \tilde{\boldsymbol{\xi}}' \mathbf{K_D}^{1+\frac{1}{b}}.$$

With the flexible technology, according to the KKT conditions, $\mathbf{Q_F^*}$ solves $\left.\frac{\partial f}{\partial q_1}\right|_{q_F^1} = \left.\frac{\partial f}{\partial q_2}\right|_{K_F - q_F^1}$. After some algebra, we obtain $\mathbf{Q_F^*} = \frac{K_F}{\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}} \; \tilde{\boldsymbol{\xi}}^{-b}$ and

$$\Gamma_F^*\left(K_F, \tilde{\boldsymbol{\xi}}\right) = f(\mathbf{Q_F^*}) = \frac{K_F^{1+\frac{1}{b}}}{\left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right)^{1+\frac{1}{b}}} \left[\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right] = \left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right)^{-\frac{1}{b}} K_F^{1+\frac{1}{b}}.$$

■

Proof of Proposition 2: Let $\mathbf{K_T^*}(\tilde{B})$ be the optimal capacity investment vector in (2). Since $a_T > 0$, the firm optimally does not borrow if it does not invest in capacity ($e_T^*(\tilde{B}) = 0$ if $\mathbf{K_T^*}(\tilde{B}) = \mathbf{0}$) and only borrows exactly enough to cover the capacity investment when this investment level is positive ($e_T^*(\tilde{B}) = \left(c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^*(\tilde{B}) - \tilde{B}\right)^+$ if $\mathbf{K}_\mathbf{T}^*(\tilde{B}) > \mathbf{0}$). Let $\mathbf{N_F} \doteq \left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right)^{-\frac{1}{b}}$ and $\mathbf{N_D} \doteq \tilde{\boldsymbol{\xi}}$. Substituting Γ_T^* from Proposition 1 in (2), we obtain the equivalent formulation

$$\begin{aligned} \pi_T^*(\tilde{B}) = \max_{\mathbf{K_T}} \quad & \tilde{B} - c_T \mathbf{1}'\mathbf{K_T} - a_T \left(c_T \mathbf{1}'\mathbf{K_T} - \tilde{B}\right)^+ + \mathbb{E}\left[\mathbf{N_T}\right]'\mathbf{K_T}^{1+\frac{1}{b}} + P \\ \text{s.t.} \quad & c_T \mathbf{1}'\mathbf{K_T} - \tilde{B} \leq E_T \\ & \mathbf{K_T} \geq \mathbf{0}. \end{aligned} \tag{11}$$

Let $g(\mathbf{K_T})$ denote the objective function in (11). The function $g(\mathbf{K_T})$ has a kink and is not differentiable at $\mathbf{1}'\mathbf{K_T} = \frac{\tilde{B}}{c_T}$. We rewrite (11) as a combination of two sub-problems $i = 0, 1$ with

$$\pi_T(\tilde{B}) = \max_i \Xi_T^i(\tilde{B}) \tag{12}$$

such that

$$\begin{aligned} \Xi_T^i(\tilde{B}) = \max_{\mathbf{K_T}} \quad & \tilde{B} - c_T \mathbf{1}'\mathbf{K_T} - a_T^i \left(c_T \mathbf{1}'\mathbf{K_T} - \tilde{B}\right) + \mathbb{E}\left[\mathbf{N_T}\right]'\mathbf{K_T}^{1+\frac{1}{b}} + P \\ \text{s.t.} \quad & Z_L^i \leq c_T \mathbf{1}'\mathbf{K_T} - \tilde{B} \leq Z_U^i \\ & \mathbf{K_T} \geq \mathbf{0}, \end{aligned} \tag{13}$$

where $a_T^0 = 0, a_T^1 = a_T$ and $Z_L^0 = -\infty,\ Z_L^1 = 0,\ Z_U^0 = 0,\ Z_U^1 = E_T$. Subproblem 0 (1) is the restriction of the problem to the no borrowing (borrowing) regions. Let $g^i(\mathbf{K_T})$ denote the objective function and $\mathbf{K}_\mathbf{T}^{\mathbf{p_i}}(\tilde{B})$ be the optimal solution of sub-problem i. We have

$$g(\mathbf{K_T}) \quad = \quad \begin{cases} g^0(\mathbf{K_T}) & \text{if} \quad c_T \mathbf{1}'\mathbf{K_T} \leq \tilde{B} \\ g^1(\mathbf{K_T}) & \text{if} \quad c_T \mathbf{1}'\mathbf{K_T} > \tilde{B}. \end{cases}$$

The remainder of the proof has the following structure:

1. We show that $g^i(\mathbf{K_T})$ is strictly concave and solve each sub-problem i for $\mathbf{K}_\mathbf{T}^{\mathbf{p_i}}(\tilde{B})$.

2. We show that $g(\mathbf{K_T})$ is strictly concave. It follows that

$$\mathbf{K}_\mathbf{T}^*(\tilde{B}) = \mathbf{K}_\mathbf{T}^{\mathbf{p_i}}(\tilde{B}) \ \text{ where } \ i = arg \max_i \Theta_T^i(\tilde{B})$$

   We derive $\pi_T^*(\tilde{B})$ by using $\mathbf{K}_\mathbf{T}^{\mathbf{p_i}}(\tilde{B})$.

1. Solution for $\mathbf{K}_\mathbf{T}^{\mathbf{p_i}}(\tilde{B})$

1.a. Flexible Technology:

Let $M_F \doteq \mathbb{E}[\mathbf{N_F}] = \mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$. The first and second order conditions in (13) are

$$\begin{aligned} \frac{\partial g^i}{\partial K_F} &= -c_F - a_F^i c_F + (1+1/b)\ M_F\ K_F^{1/b}, \\ \frac{\partial^2 g^i}{\partial K_F^2} &= \frac{1}{b}(1+1/b)\ M_F\ K_F^{(1/b-1)}. \end{aligned}$$

Since $b < -1$, we have $\lim_{K_F \to 0^+} \frac{\partial^2 K_F^{(1/b-1)}}{\partial K_F^2} \to \infty$ and $\frac{\partial^2 K_F^{(1/b-1)}}{\partial K_F^2} > 0 \quad \forall K_F \geq 0$. With $b < -1$, it follows that $\frac{\partial^2 g^i}{\partial K_F^2} < 0$ for $K_F \geq 0$ and the function $g^i(K_F)$ is strictly concave for $i = 0, 1$. Since the constraints in (13) are linear, first-order KKT conditions are necessary and sufficient for optimality for each sub-problem i and $K_F^{p_i}(\tilde{B})$ is unique.

From KKT conditions if i has a non-empty feasible region then the optimal solution is either the solution of $\frac{\partial g^i}{\partial K_F} = 0$, $K_F^{p_i}(\tilde{B}) = \left(\frac{M_F\left(1+\frac{1}{b}\right)}{c_F(1+a_F^i)}\right)^{-b}$, or is a boundary solution. Since $\lim_{K_F \to 0^+} \frac{\partial g^i}{\partial K_F} \to \infty$, $K_F = 0$ is never optimal. If $\frac{Z_L^i + \tilde{B}}{c_F} > 0$ and $\frac{\partial g^i}{\partial K_F} < 0$ at this point, then $K_F^{p_i}(\tilde{B}) = \frac{Z_L^i + \tilde{B}}{c_F}$, i.e., the optimal solution occurs at the lower bound of the financing constraint. If $\frac{\partial g^i}{\partial K_F} > 0$ at $K_F = \frac{Z_U^i + \tilde{B}}{c_F} > 0$, then $K_F^{p_i}(\tilde{B}) = \frac{Z_U^i + \tilde{B}}{c_F}$, i.e., the optimal solution occurs at the upper bound of the financing constraint. To summarize, $K_F^{p_i}(\tilde{B})$ for $i = 0, 1$ is characterized by

$$\begin{aligned} K_F^{p_0}(\tilde{B}) &= \begin{cases} K_F^0 \doteq \left(\frac{M_F\left(1+\frac{1}{b}\right)}{c_F}\right)^{-b} & \text{if } \ c_F K_F^0 - \tilde{B} \leq 0 \\ \overline{K}_F \doteq \left(\frac{\tilde{B}}{c_F}\right) & \text{if } \ c_F K_F^0 - \tilde{B} > 0, \end{cases} \\ K_F^{p_1}(\tilde{B}) &= \begin{cases} \overline{K}_F \doteq \left(\frac{\tilde{B}}{c_F}\right) & \text{if } \ c_F K_F^1 - \tilde{B} \leq 0 \\ K_F^1 \doteq \left(\frac{M_F\left(1+\frac{1}{b}\right)}{c_F(1+a_F)}\right)^{-b} & \text{if } \ 0 < c_F K_F^1 - \tilde{B} \leq E_F \\ \overline{\overline{K}}_F \doteq \left(\frac{E+\tilde{B}}{c_F}\right) & \text{if } \ c_F K_F^1 - \tilde{B} > E_F. \end{cases} \end{aligned} \tag{14}$$

Here, K_F^0 is the budget-unconstrained optimal capacity investment and K_F^1 is the credit-unconstrained optimal capacity investment.

1.b. Dedicated Technology:

We obtain

$$\begin{aligned} \frac{\partial^2 g^i}{\partial (K_D^j)^2} &= \frac{1}{b}(1+1/b)\ \overline{\xi}_j\ (K_D^j)^{(1/b-1)} < 0, \\ \frac{\partial^2 g^i}{\partial (K_D^1)^2}\frac{\partial^2 g^i}{\partial (K_D^2)^2} - \left[\frac{\partial^2 g^i}{\partial K_D^1 K_D^2}\right]^2 &= \prod_j \frac{1}{b}(1+1/b)\ \overline{\xi}_j\ (K_D^j)^{(1/b-1)} - 0 > 0 \end{aligned}$$

for $i = 0, 1$ and $j = 1, 2$. Therefore, the Hessian matrix $D^2 g^i(\mathbf{K_D})$ is negative definite for $\mathbf{K_D} \geq \mathbf{0}$ and $g^i(\mathbf{K_D})$ is strictly concave. Since the constraints in (13) are linear, first-order KKT conditions are necessary and sufficient for optimality in each sub-problem i and $\mathbf{K_D^{p_i}}(\tilde{B})$ is unique.

If $\mathbf{K_D^{p_i}}(\tilde{B})$ is an optimal solution to (13), then there exist $\boldsymbol{\lambda}^{i'} = (\lambda_1^i, \lambda_2^i)$ and $\boldsymbol{\mu}^{i'} = (\mu_1^i, \mu_2^i)$ that satisfy

$$\begin{aligned}
c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) - \tilde{B} \;&\leq\; Z_U^i, & (15)\\
c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) - \tilde{B} \;&\geq\; Z_L^i, & (16)\\
\mathbf{K_D^{p_i}}(\tilde{B}) \;&\geq\; \mathbf{0}, & (17)\\
-(1+a_D^i)c_D + (1+1/b)\;\overline{\boldsymbol{\xi}}\;\mathbf{K_D^{p_i}}(\tilde{B})^{1/b} - c_D(\lambda_1^i - \lambda_2^i) + \boldsymbol{\mu}^i \;&=\; \mathbf{0}, & (18)\\
\lambda_1^i[Z_U^i - c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) + \tilde{B}] \;&=\; 0, & (19)\\
\lambda_2^i[-Z_L^i + c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) - \tilde{B}] \;&=\; 0, & (20)\\
\boldsymbol{\mu}^i\mathbf{K_D^{p_i}}(\tilde{B}) \;&=\; \mathbf{0} & (21)
\end{aligned}$$

with $\boldsymbol{\lambda}^i \geq \mathbf{0}$ and $\boldsymbol{\mu}^i \geq \mathbf{0}$ for $i = 0, 1$. Observe that $\lim_{K_D^j \to 0^+} \frac{\partial g^i}{\partial K_D^j} \to \infty$ for $j = 1, 2$, so it is never optimal to invest in only one of the resources. This implies $\boldsymbol{\mu}^i = \mathbf{0}$ for (21) to be satisfied.

Case 1: $c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) - \tilde{B} < Z_U^i$ and $c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) - \tilde{B} > Z_L^i$

In this case $\boldsymbol{\lambda}^i = \mathbf{0}$, and (18) yields

$$\mathbf{K_D^{p_i}}(\tilde{B}) = \mathbf{K_D^i} \doteq \left(\frac{(1+\frac{1}{b})}{c_D(1+a_D^i)}\right)^{-b} \overline{\boldsymbol{\xi}}^{-b}.$$

For (15), (16) and (17) to be satisfied, and the solution $\mathbf{K_D^{p_i}}(\tilde{B}) = \mathbf{K_D^i}$ to be valid, we need $Z_L^i < c_D\mathbf{1}'\mathbf{K_D^i} - \tilde{B} < Z_U^i$. Here, $\mathbf{K_D^0}$ is the budget-unconstrained optimal capacity investment and $\mathbf{K_D^1}$ is the credit-unconstrained optimal capacity investment.

Case 2: $c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) - \tilde{B} = Z_U^i$

In this case (16) holds as a strict inequality, so $\lambda_2^i = 0$ for (20) to be satisfied. Rewriting the equality as $K_D^2 = \frac{Z_U^i + \tilde{B} - c_D K_D^1}{c_D}$, and combining this with (18) yields

$$\mathbf{K_D^{p_i}}(\tilde{B})' = \left(\left(\frac{Z_U^i + B}{c_D}\right)\left(\frac{\overline{\xi}_1^{-b}}{\overline{\xi}_1^{-b} + \overline{\xi}_2^{-b}}\right), \left(\frac{Z_U^i + B}{c_D}\right)\left(\frac{\overline{\xi}_2^{-b}}{\overline{\xi}_1^{-b} + \overline{\xi}_2^{-b}}\right)\right). \qquad (22)$$

The condition $\lambda_1^i \geq 0$ should be satisfied at optimality. After some algebra, this condition implies that (22) is optimal if $\tilde{B} \leq c_D\mathbf{1}'\mathbf{K_D^i} - Z_U^i$.

Case 3: $c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) - \tilde{B} = Z_L^i$

This case is only relevant for $i = 1$ since $Z_L^0 = -\infty$. In this case, (15) holds as a strict inequality, so $\lambda_1^1 = 0$ for (19) to be satisfied. Rewriting the equality as $K_D^2 = \frac{Z_L^1 + \tilde{B} - c_D K_D^1}{c_D}$, and combining with (18) yields

$$\mathbf{K_D^{p_1}}(\tilde{B})' = \left(\left(\frac{Z_L^1 + B}{c_D}\right)\left(\frac{\overline{\xi}_1^{-b}}{\overline{\xi}_1^{-b} + \overline{\xi}_2^{-b}}\right), \left(\frac{Z_L^1 + B}{c_D}\right)\left(\frac{\overline{\xi}_2^{-b}}{\overline{\xi}_1^{-b} + \overline{\xi}_2^{-b}}\right)\right). \tag{23}$$

The condition $\lambda_2^1 \geq 0$ should be satisfied at optimality. After some algebra, this condition implies that (23) is optimal if $\tilde{B} \geq c_D \mathbf{1}'\mathbf{K_D^1} - Z_L^1$.

Combining cases 1, 2 and 3, $\mathbf{K_D^{p_i}}(\tilde{B})$ for $i = 0, 1$ is characterized by

$$\mathbf{K_D^{p_0}}(\tilde{B}) = \begin{cases} \mathbf{K_D^0} = \left(\frac{(1+\frac{1}{b})}{c_D}\right)^{-b} \boldsymbol{\xi}^{-b} & \text{if } \ c_D\mathbf{1}'\mathbf{K_D^0} - \tilde{B} \leq 0 \\ \overline{\mathbf{K}}_{\mathbf{D}}' = \left(\left(\frac{B}{c_D}\right)\left(\frac{\overline{\xi}_1^{-b}}{\overline{\xi}_1^{-b}+\overline{\xi}_2^{-b}}\right), \left(\frac{B}{c_D}\right)\left(\frac{\overline{\xi}_2^{-b}}{\overline{\xi}_1^{-b}+\overline{\xi}_2^{-b}}\right)\right) & \text{if } \ c_D\mathbf{1}'\mathbf{K_D^0} - \tilde{B} > 0, \end{cases} \tag{24}$$

$$\mathbf{K_D^{p_1}}(\tilde{B}) = \begin{cases} \overline{\mathbf{K}}_{\mathbf{D}}' = \left(\left(\frac{B}{c_D}\right)\left(\frac{\overline{\xi}_1^{-b}}{\overline{\xi}_1^{-b}+\overline{\xi}_2^{-b}}\right), \left(\frac{B}{c_D}\right)\left(\frac{\overline{\xi}_2^{-b}}{\overline{\xi}_1^{-b}+\overline{\xi}_2^{-b}}\right)\right) & \text{if } \ c_D\mathbf{1}'\mathbf{K_D^1} - \tilde{B} \leq 0 \\ \mathbf{K_D^1} = \left(\frac{(1+\frac{1}{b})}{c_D(1+a_D)}\right)^{-b} \boldsymbol{\xi}^{-b} & \text{if } \ 0 < c_D\mathbf{1}'\mathbf{K_D^1} - \tilde{B} \leq E_D \\ \overline{\overline{\mathbf{K}}}_{\mathbf{D}}' = \left(\left(\frac{E+B}{c_D}\right)\left(\frac{\overline{\xi}_1^{-b}}{\overline{\xi}_1^{-b}+\overline{\xi}_2^{-b}}\right), \left(\frac{E+B}{c_D}\right)\left(\frac{\overline{\xi}_2^{-b}}{\overline{\xi}_1^{-b}+\overline{\xi}_2^{-b}}\right)\right) & \text{if } \ c_D\mathbf{1}'\mathbf{K_D^1} - \tilde{B} > E_D. \end{cases}$$

2. Solution for $\mathbf{K_T^*}(\tilde{B})$ and $\pi_T^*(\tilde{B})$:

To show that $g(\mathbf{K_T})$ is strictly concave, we need to show that $\forall\ \mathbf{K_T^I}, \mathbf{K_T^{II}} \geq 0$ and $\lambda \in (0,1)$,

$$g(\lambda\mathbf{K_T^I} + (1-\lambda)\mathbf{K_T^{II}}) - \lambda g(\mathbf{K_T^I}) - (1-\lambda)g(\mathbf{K_T^{II}}) > 0. \tag{25}$$

Since $g^i(\mathbf{K_T})$ is strictly concave, we only need to focus on $\mathbf{K_T^I}, \mathbf{K_T^{II}}$ such that $c_T\mathbf{1}'\mathbf{K_T^I} + F_T \leq \tilde{B}$ and $c_T\mathbf{1}'\mathbf{K_T^{II}} + F_T > \tilde{B}$. We have two cases to consider. First, if $c_T\mathbf{1}'\left(\lambda\mathbf{K_T^I} + (\mathbf{1}-\lambda)\mathbf{K_T^{II}}\right) + F_T \leq \tilde{B}$ then after some algebra, the left-hand side of (25) becomes

$$\mathbb{E}\left[\mathbf{N_T}\right]'\left(\lambda\mathbf{K_T^I} + (\mathbf{1}-\lambda)\mathbf{K_T^{II}}\right)^{1+\frac{1}{b}} - \lambda\mathbb{E}\left[\mathbf{N_T}\right]'\mathbf{K_T^I}^{1+\frac{1}{b}} - (1-\lambda)\mathbb{E}\left[\mathbf{N_T}\right]'\mathbf{K_T^{II}}^{1+\frac{1}{b}} + (1-\lambda)a_T(c_T\mathbf{1}'\mathbf{K_T^{II}} - \tilde{B}).$$

Since $x^{1+\frac{1}{b}}$ is strictly concave for $x \geq 0$ and $c_T\mathbf{1}'\mathbf{K_T^{II}} - \tilde{B}$ is positive by definition, the above equation is strictly greater than 0. Second, if $c_T\mathbf{1}'\left(\lambda\mathbf{K_T^I} + (\mathbf{1}-\lambda)\mathbf{K_T^{II}}\right) > \tilde{B}$ then after some algebra, the left-hand side of (25) becomes

$$\mathbb{E}\left[\mathbf{N_T}\right]'\left(\lambda\mathbf{K_T^I} + (\mathbf{1}-\lambda)\mathbf{K_T^{II}}\right)^{1+\frac{1}{b}} - \lambda\mathbb{E}\left[\mathbf{N_T}\right]'\mathbf{K_T^I}^{1+\frac{1}{b}} - (1-\lambda)\mathbb{E}\left[\mathbf{N_T}\right]'\mathbf{K_T^{II}}^{1+\frac{1}{b}} - \lambda a_T(c_T\mathbf{1}'\mathbf{K_T^I} - \tilde{B}).$$

Since $x^{1+\frac{1}{b}}$ is strictly concave for $x \geq 0$ and $c_T\mathbf{1}'\mathbf{K_T^I} - \tilde{B}$ is negative by definition, the equation above is strictly greater than 0. Since (25) is satisfied for both cases, $g(\mathbf{K_T})$ is strictly concave. It follows that

$$\mathbf{K_T^*}(\tilde{B}) = \mathbf{K_T^{p_i}}(\tilde{B}) \ \text{ where } \ i = arg\max_i \Theta_T^i(\tilde{B})$$

is the unique maximizer of g. Combining (14) and (24), the unique optimal solution to problem (11) and the corresponding optimal amount of borrowing are given by

$$\mathbf{K_T^*}(\tilde{B}) = \begin{cases} \mathbf{K_T^0} & \text{if } c_T \mathbf{1}'\mathbf{K_T^0} \le \tilde{B} \\ \overline{\mathbf{K}}_\mathbf{T} & \text{if } c_T \mathbf{1}'\mathbf{K_T^1} \le \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^0} \\ \mathbf{K_T^1} & \text{if } c_T \mathbf{1}'\mathbf{K_T^1} - E_T \le \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^1} \\ \overline{\overline{\mathbf{K}}}_\mathbf{T} & \text{if } \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^1} - E_T, \end{cases} \tag{26}$$

$$e_T^*(\tilde{B}) = \left(c_T \mathbf{1}'\mathbf{K_T^*}(\tilde{B}) - \tilde{B}\right)^+$$

where

$$\mathbf{K_D^0}' = \left(\left(\frac{\bar{\xi}_1\left(1+\frac{1}{b}\right)}{c_D}\right)^{-b}, \left(\frac{\bar{\xi}_2\left(1+\frac{1}{b}\right)}{c_D}\right)^{-b}\right)$$

$$\overline{\mathbf{K}}_\mathbf{D}' = \left(\left(\frac{B}{c_D}\right)\left(\frac{\bar{\xi}_1^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right), \left(\frac{B}{c_D}\right)\left(\frac{\bar{\xi}_2^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right)\right)$$

$$\mathbf{K_D^1}' = \left(\left(\frac{\bar{\xi}_1\left(1+\frac{1}{b}\right)}{c_D(1+a_D)}\right)^{-b}, \left(\frac{\bar{\xi}_2\left(1+\frac{1}{b}\right)}{c_D(1+a_D)}\right)^{-b}\right)$$

$$\overline{\overline{\mathbf{K}}}_\mathbf{D}' = \left(\left(\frac{E_D+B}{c_D}\right)\left(\frac{\bar{\xi}_1^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right), \left(\frac{E_D+B}{c_D}\right)\left(\frac{\bar{\xi}_2^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right)\right)$$

$$K_F^0 = \left(\frac{M_F\left(1+\frac{1}{b}\right)}{c_F}\right)^{-b}$$

$$\overline{K}_F = \left(\frac{B}{c_F}\right)$$

$$K_F^1 = \left(\frac{M_F\left(1+\frac{1}{b}\right)}{c_F(1+a_F)}\right)^{-b}$$

$$\overline{\overline{K}}_F = \left(\frac{E_F+B}{c_F}\right).$$

We substitute (26) in (11) and it follows that the optimal expected (stage 1) equity value of the firm with a given budget level $\tilde{B}$ is given by

$$\pi_T^*(\tilde{B}) = \begin{cases} \tilde{B} + \frac{c_T \mathbf{1}'\mathbf{K_T^0}}{-(b+1)} + P & \text{if } c_T \mathbf{1}'\mathbf{K_T^0} \le \tilde{B} \\ M_T\left(\frac{\tilde{B}}{c_T}\right)^{1+\frac{1}{b}} + P & \text{if } c_T \mathbf{1}'\mathbf{K_T^1} \le \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^0} \\ \tilde{B}(1+a_T) + \frac{c_T \mathbf{1}'\mathbf{K_T^1}(1+a_T)}{-(b+1)} + P & \text{if } c_T \mathbf{1}'\mathbf{K_T^1} - E_T \le \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^1} \\ -E_T(1+a_T) + M_T\left(\frac{E_T+\tilde{B}}{c_T}\right)^{1+\frac{1}{b}} + P & \text{if } \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^1} - E_T. \end{cases} \tag{27}$$

where $M_F = \mathbb{E}\left[\left(\xi_1^{-b}+\xi_2^{-b}\right)^{-\frac{1}{b}}\right]$ and $M_D = \left(\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}\right)^{-\frac{1}{b}}$. ∎

Proof of Proposition 3: We first prove the existence of $\overline{c}_F(c_D, \mathbf{a})$. The expected (stage 0) equity value $\mathbb{E}\left[\pi_T^*\left(c_T\right)\right]$ is a continuous function of c_T. For a finite $c_D > 0$, $\mathbb{E}\left[\pi_D^*\left(c_D\right)\right]$ is finite. It is easy to prove that

$$\begin{aligned} \lim_{c_F \to \infty} \mathbb{E}\left[\pi_F^*\left(c_F\right)\right] &= \overline{B} + P, \\ \lim_{c_F \to 0} \mathbb{E}\left[\pi_F^*\left(c_F\right)\right] &\to \infty. \end{aligned}$$

Since the equity value is continuous in c_F, there exists a c_F such that the equity values with both technologies coincide. This concludes the proof for the existence of $\overline{c}_F(c_D, \mathbf{a})$.

To prove the uniqueness of $\overline{c}_F(c_D, \mathbf{a})$, we first provide the following lemmas and relegate the proof to Appendix C:

Lemma 1 *For any argument κ_T of π_T, the expectation and the derivative operators can be interchanged, i.e.* $\frac{\partial}{\partial \kappa_T}\mathbb{E}[\pi_T^*] = \mathbb{E}\left[\frac{\partial}{\partial \kappa_T}\pi_T^*\right]$.

Lemma 2 *The optimal expected (stage 0) equity value of the firm with technology T strictly decreases in the unit capacity investment cost* ($\frac{\partial}{\partial c_T}\mathbb{E}[\pi_T^*(c_T)] < 0$).

Lemma 2 implies the uniqueness of $\overline{c}_F(c_D, \mathbf{a})$.

With symmetric financing costs, we establish the functional form of $\overline{c}_F^S(c_D)$ with the following Lemma and relegate the proof to Appendix C:

Lemma 3 *When the financing costs of the two technologies are symmetric, at $c_F = \overline{c}_F^S(c_D)$ optimal expected (stage 1) equity values at a given budget level $\tilde{B}$ and the optimal expected (stage 0) equity values are the same for both technologies, i.e.* $\pi_F^*(\tilde{B}; c_F)\Big|_{c_F = \overline{c}_F^S(c_D)} = \pi_D^*(\tilde{B}; c_D)$ *for* $\tilde{B} \geq 0$ *and* $\mathbb{E}[\pi_F^*(\overline{c}_F^S(c_D))] = \mathbb{E}[\pi_D^*(c_D)]$.

It follows from Lemma 3 that $\overline{c}_F^S(c_D)$ is the unique threshold in the symmetric case ($\overline{c}_F(c_D, \mathbf{a}) = \overline{c}_F^S(c_D)$). We now prove the relation $\overline{c}_F^S(c_D) \geq c_D$. It is sufficient to show

$$\mathbb{E}^{-b}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right] \geq \mathbb{E}^{-b}[\xi_1] + \mathbb{E}^{-b}[\xi_2].$$

From Hardy et al. (1988, p.133,146) if $\gamma \in (0, 1)$ and X and Y are non-negative random variables then the following is true:

$$E^{1/\gamma}\left[(X+Y)^\gamma\right] \geq \mathbb{E}^{1/\gamma}[X^\gamma] + \mathbb{E}^{1/\gamma}[Y^\gamma] \tag{28}$$

where the equality only holds when X and Y are effectively proportional, i.e. $X = \lambda Y$. In the expression for $\overline{c}_F^S(c_D)$ we have $\gamma = -\frac{1}{b} \in (0, 1)$ and $\boldsymbol{\xi} > \mathbf{0}$ therefore we can use this inequality. Replacing X with ξ_1^{-b} and Y with ξ_2^{-b} gives the desired result. Notice that $\overline{c}_F^S(c_D) = c_D$ only if $\xi_1 = k\xi_2$ for $k > 0$. This is only possible if either $\boldsymbol{\xi}$ is deterministic or it is perfectly positively correlated and has a proportional bivariate distribution.

The dominance of investing in T^* over not making any technology investment follows from the optimality of $\mathbf{K}^*_\mathbf{T}(\tilde{B})$ at stage 1. ■

Proof of Proposition 4: To establish this result, we provide the following lemma and relegate the proof to Appendix C:

Lemma 4 *The optimal expected (stage 0) value of the equity with technology T*

i) strictly decreases in unit financing cost ($\frac{\partial}{\partial a_T}\mathbb{E}[\pi_T^*(a_T)] < 0$),

iii) increases in credit limit ($\frac{\partial}{\partial E_T}\mathbb{E}[\pi_T^*(E_T)] \geq 0$), and the equality only holds for E_T such that $c_T\mathbf{1'K^1_T} - E_T < 0$.

Since $E_T = \frac{P}{1+a_T}$, it follows From Lemma 4 that the firm prefers the smallest a_T that satisfies $\Lambda_T(a_T) = U$.

If there exists a feasible $a_T^{'} < \infty$ such that $\Lambda_T(a_T^{'}) \geq U$, since $\Lambda_T(0) < 0$ and $\Lambda_T(a_T)$ is a continuous function of a_T (which can be easily verified), it follows from the Mean-value theorem that such $\dot{a}_T \leq a_T^{'}$ always exists and is unique. If there does not exist a feasible a_T that satisfies $\Lambda_T(a_T) = U$ then in equilibrium the creditor does not offer any contract, and $\dot{a}_T \to \infty$ and $\dot{E}_T \to 0$. ■

Proof of Proposition 5: When $U = BC = 0$, it follows from (8) and Proposition 4 that $\dot{a}_T = argmin_{a_T \geq 0}\, a_T\mathbb{E}\left[e_T^*\right] = 0$. This implies that $\dot{a}_T = 0$ (minimum feasible unit financing cost) and from Proposition 4, we obtain $\dot{E}_T = P$. ■

Proof of Proposition 6: If the capital markets are perfect, Proposition 5 states that $\dot{a}_T = 0$ and $\dot{E}_T = P$. Since $P > c_T\mathbf{1'K^0_T}$ by assumption, with this financing cost scheme we obtain from Proposition 2 that $\Omega_T^1 = \Omega_T^3 = \emptyset$, $\mathbf{K^1_T} = \mathbf{K^0_T}$, and for $\tilde{B} \in \Omega_T^2$, the optimal expected (stage 1) equity value is $\tilde{B} + \frac{c_T\mathbf{1'K^0_T}}{-(b+1)} + P$. Therefore, we have from Proposition 2 that the firm invests in the budget-unconstrained capacity investment level for any budget realization $\dot{\mathbf{K}}_\mathbf{T}(\tilde{B}) = \mathbf{K^0_T}$, and borrows to finance this capacity level $\dot{e}_T(\tilde{B}) = [c_T\mathbf{1'K^0_T} + -B]^+$. It follows that equilibrium expected (stage 1) equity value at each budget state $\tilde{B} > 0$ is $\tilde{B} + \frac{c_T\mathbf{1'K^0_T}}{-(b+1)} + P$, and the equilibrium expected (stage 0) equity value is $\overline{B} + \frac{c_T\mathbf{1'K^0_T}}{-(b+1)} + P$. Proposition 3 implies that the equilibrium technology choice $\dot{T}$ is determined by the variable cost threshold $\overline{c}_F^P(c_D) = \overline{c}_F^S(c_D)$ (because $\dot{a}_D = \dot{a}_F = 0$) and $\dot{T}$ is more profitable than not investing in technology option. ■

Proof of Proposition 7: Since $\dot{E}_T = \frac{P}{1+\dot{a}_T}$, we only focus on $\dot{a}_T$. We will only provide the proof for the results related to the bankruptcy cost BC. The results related to the underwriter fee U follow in a similar fashion. To establish the result with respect to $\dot{a}_T$, we provide the following lemma and relegate the proof to Appendix C:

Lemma 5 *Let τ be an arbitrary parameter and $\dot{a}_T(\tau_0) < \infty$[5], $\dot{a}_T(\tau_1)$ denote the equilibrium financing cost with parameter level τ_0 and τ_1 respectively with $\tau_1 > \tau_0$. The equilibrium financing cost with technology T strictly increases in τ ($\dot{a}_T(\tau_1) > \dot{a}_T(\tau_0)$) if the expected stage 0 return $\Lambda_T(a_T)$ of the creditor strictly decreases in τ for any feasible a_T, and $\Lambda_T(0;\tau_0) < 0$ for $a_T = 0$.*

Let $\dot{a}_T(BC_0)$ denote the equilibrium financing cost with $BC_0 > 0$. From (8), we obtain $\frac{\partial}{\partial BC}\Lambda_T(a_T) = -\mathbb{E}\left[Pr\left\{\Gamma_T^* < e_T^*(1+a_T)\right\}\right] < 0$ for any feasible a_T, i.e. $\overline{B} \in \Omega_T^2(a_T)$, and we have $\Lambda_T(0; BC_0) < 0$. Substituting $\tau = BC$, it follows from Lemma 5 that the equilibrium level of financing cost decreases in BC.

The result with respect to the expected (stage 0) equity value follows from Lemma 4 in the proof of Proposition 4 and the identity $\dot{E}_T = \frac{P}{1+\dot{a}_T}$. The results related to the equilibrium capacity investment level at each budget state and the equilibrium level of expected total capacity investment can be established in a similar fashion and are omitted. ■

Proof of Property 2: The result with $\Upsilon_T^* = \pi_T^*$ follows from Lemma 4 in the proof of Proposition 4 and the identity $\dot{E}_T = \frac{P}{1+\dot{a}_T}$. The result with respect to $\Upsilon_T^* = \mathbb{E}[\mathbf{1'K_T^*}(B)]$ can be established in a similar fashion and is omitted. ■

Proof of Proposition 8: We prove this result for general uniform distributions with mean $\overline{\xi}$ and support $[\overline{\xi} - d, \overline{\xi} + d]$ where $0 \le d \le \overline{\xi}$. We use the mean-preserving spread of the uniform distribution to characterize an increase in the demand variability. For uniform distributions, this can be achieved by symmetrically increasing the support by keeping the mean constant, i.e. $[\overline{\xi} - d - \epsilon, \overline{\xi} + d + \epsilon]$ for $\epsilon \in (0, \overline{\xi} - d)$. Higher ϵ leads to a higher variance of ξ_1[6]. Let $\epsilon_0 \in (0, \overline{\xi} - d)$ ($\epsilon_1 \in (0, \overline{\xi} - d)$) be the initial (final) level of the range parameter that corresponds to initial (final) demand variability σ_0 (σ_1) and $\dot{a}(\epsilon_0) < \infty$ ($\dot{a}(\epsilon_1)$) denote the corresponding equilibrium level of financing cost with $\epsilon_1 > \epsilon_0 > 0$ (or equivalently

[5] $\dot{a}_T(\tau_0)$ is finite because of our assumption in §5.2.1 that the creditor offers a contract initially before the change in parameter level τ. In the same line of reasoning of Lemma 5 we can show that if $\dot{a}_T(\tau_0) \to \infty$, i.e. the creditor does not offer a contract at τ_0, then the contract is also not offered for τ_1, i.e. $\dot{a}_T^1 \to \infty$.

[6] Variance is the correct indicator of risk for uniform distributions in the Rothschild-Stiglitz sense (Eeckhoudt and Gollier 1995, p.82)

$\sigma_1 > \sigma_0 > 0$). We have

$$\Lambda(a) = \left(cK^0(1+a)^b - \overline{B}\right) a - BC\left[\frac{\overline{\xi}(1+1/b)\left(1-\frac{\overline{B}}{cK^1}\right) - \left(\overline{\xi} - d - \epsilon\right)}{2(d+\epsilon)}\right]$$

for a given a satisfying $\overline{B} \in \Omega^2(a)$. We obtain

$$\frac{\partial \Lambda(a)}{\partial \epsilon} = \frac{BC}{2(d+\epsilon)^2}\left[\overline{\xi}(1+1/b)\left(1-\frac{\overline{B}}{cK^1}\right) - \overline{\xi}\right] < 0$$

for any feasible $\overline{B} \in \Omega^2(a)$. Since the expected (stage 0) return of the creditor strictly decreases in ϵ for any feasible a and $\Lambda(0, \epsilon_0) < 0$, substituting $\tau = \epsilon$ in Lemma 5 in the proof of Proposition 7, it follows that the equilibrium level of financing cost increases in ϵ, and hence in demand variability, i.e. $\dot{a}(\sigma_1) > \dot{a}(\sigma_0)$. From §5.2.1, we have

$$\Upsilon^*(\sigma_1; \dot{a}(\sigma_1)) - \Upsilon^*(\sigma_0; \dot{a}(\sigma_0)),$$

which can be rewritten as

$$\left[\Upsilon^*(\sigma_1; \dot{a}(\sigma_1)) - \Upsilon^*(\sigma_1; \dot{a}(\sigma_0))\right] + \left[\Upsilon^*(\sigma_1; \dot{a}(\sigma_0)) - \Upsilon^*(\sigma_0; \dot{a}(\sigma_0))\right].$$

The rest of the proof follows from Property 2 (the first term in brackets is negative) and the independence of Υ^* from σ for a given financing cost a (the second term in brackets is zero). ∎

Proof of Proposition 9: Let $\dot{a}(\overline{B}_0) < \infty$, $\dot{a}(\overline{B}_1)$ denote the equilibrium financing cost with budget level $\overline{B}_0$ and $\overline{B}_1$ respectively with $\overline{B}_1 > \overline{B}_0 > 0$. Since the creditor offers a contract with $\overline{B}_0$, we have $\overline{B}_0 < cK^0$. From (9), we obtain

$$\frac{\partial \Lambda(a)}{\partial \overline{B}} = -\left(a - \frac{BC\,\overline{\xi}(1+1/b)}{2\overline{\xi}cK^0(1+a)^b}\right) \tag{29}$$

for any a satisfying $\overline{B} \in \Omega^2(a)$.

If $U > 0, BC = 0$ then $\frac{\partial}{\partial \overline{B}}\Lambda(a) = -a < 0$ in (29). Since the expected (stage 0) return of the creditor strictly decreases in $\overline{B}$ for any feasible a and $\Lambda(0, \overline{B}_0) < 0$, substituting $\tau = \overline{B}$ in Lemma 5 in the proof of Proposition 7, it follows that the equilibrium level of financing cost increases in $\overline{B}$. Since $\dot{K}(\overline{B}_0) = \left(\frac{\overline{\xi}(1+\frac{1}{b})}{c(1+\dot{a}(\overline{B}_0))}\right)^{-b}$, and $\dot{a}(\overline{B}_1) > \dot{a}(\overline{B}_0)$, $\dot{K}(\overline{B})$ decreases in $\overline{B}$.

If $U > 0, BC > 0$, $\frac{\partial}{\partial \overline{B}}\Lambda(a)$ is not necessarily negative for any a satisfying $\overline{B} \in \Omega^2(a)$. It follows from Proposition 4 that $\dot{a}(\overline{B}_0)$ satisfies

$$\Lambda(\dot{a}(\overline{B}_0)) = (cK^0(1+\dot{a}(\overline{B}_0))^b - \overline{B}_0)\left(\dot{a}^0 - \frac{BC\,\overline{\xi}(1+1/b)}{2\overline{\xi}cK^0(1+\dot{a}(\overline{B}_0))^b}\right) = U$$

in equilibrium. It follows that $\frac{\partial}{\partial \overline{B}}\Lambda(a)\Big|_{\dot{a}(\overline{B}_0)} < 0$ in (29). With a small change in $\overline{B}$ to $\overline{B}_1$, we can guarantee that $\Lambda(a;\overline{B}_1) < U$ for $\forall\ a < \dot{a}(\overline{B}_0)$ because i) $\Lambda(a;\overline{B}_0) < U$ for $\forall\ a < \dot{a}(\overline{B}_0)$ by the definition of equilibrium, and ii) $\left|\frac{\partial}{\partial \overline{B}}\Lambda(a)\right|$ and $\left|\frac{\partial}{\partial a}\Lambda(a)\right|$ are bounded. Therefore the equilibrium financing cost increases locally in $\overline{B}$. The result with respect to $\dot{K}(\overline{B})$ also holds locally. ■

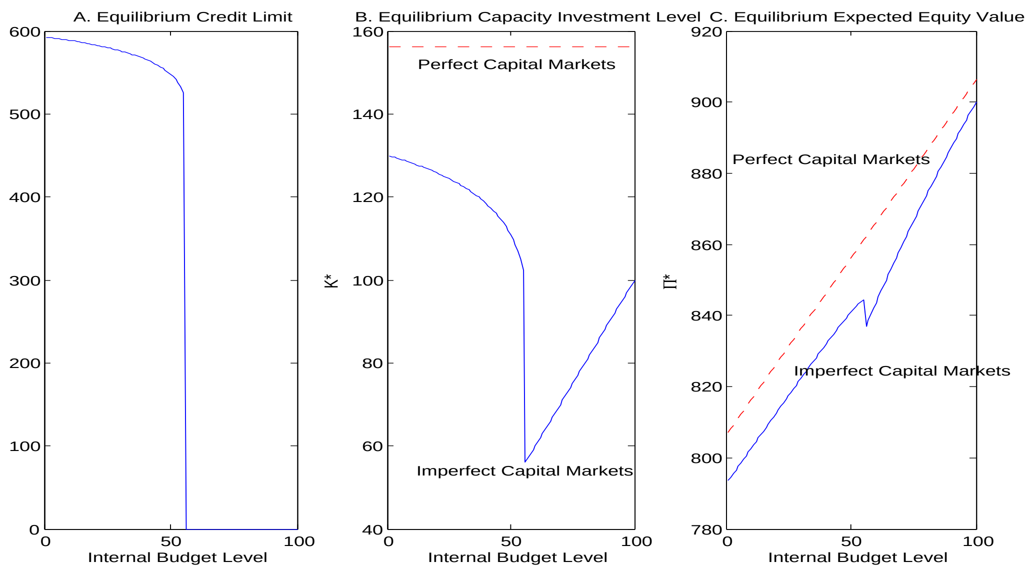

Figure 4: Increasing internal budget level decreases $\dot{E}$ (and in turn increases $\dot{a}$) (Panel A). The creditor does not offer a loan contract after sufficient increase in $\overline{B}$. The equilibrium capacity level decreases in $\overline{B}$ when the firm borrows and increases in $\overline{B}$ when the firm does not borrow (Panel B). For small levels of $\overline{B}$, the positive effect of an increase in $\overline{B}$ dominates the negative effect of an increase in $\dot{a}$ and the equity value increases in $\overline{B}$. With a sharp decline in $\dot{E}$, the equilibrium expected (stage 0) equity value decreases (Panel C).

For the effect on the expected (stage 0) equity value in equilibrium, we have two drivers, $\overline{B}$ and $\dot{a}$ that work in opposite directions. Figure 4 demonstrates that either effect may dominate; therefore expected (stage 0) equity value of the firm in equilibrium may increase or decrease in the budget level $\overline{B}$. The numerical example is generated by using the parameter levels $c = 1, b = -2, P = 650, \overline{\xi} = 25, BC = 10, U = 10$. We observe the reduction in the equilibrium level of expected (stage 0) equity value when an increase in $\overline{B}$ induces the creditor not to offer a loan contract, i.e. $\dot{E} \to 0$: A higher internal cash flow decreases the expected returns of the creditor, and no feasible a can yield the expected return U. The unavailability of the loan contract leads to a sharp decrease in the equity value.

Proof of Proposition 10: Let $\dot{a}(c_0) < \infty$, $\dot{a}(c_1)$ denote the equilibrium financing cost with unit capacity investment cost c_0 and c_1 respectively with $c_1 > c_0 > 0$. From (9), we obtain

$$\frac{\partial \Lambda(a)}{\partial c} = (b+1)\left[K^0(1+a)^b\left(a - \frac{BC\,\overline{\xi}(1+1/b)}{2\overline{\xi}cK^0(1+a)^b}\right) + \left(cK^0(1+a)^b - \overline{B}\right)\frac{BC\,\overline{\xi}(1+1/b)}{2\overline{\xi}c^2K^0(1+a)^b}\right] \quad (30)$$

for any a satisfying $\overline{B} \in \Omega^2(a)$.

If $U > 0, BC = 0$ then $\frac{\partial}{\partial c}\Lambda(a) < 0$ in (30) for any feasible a. Since the expected (stage 0) return of the creditor strictly decreases in c for any feasible a and $\Lambda(0; c_0) < 0$, substituting $\tau = c$ in Lemma 5 in the proof of Proposition 7, it follows that the equilibrium level of financing cost increases in c. The effect on the equilibrium level of equity value $\dot{\pi}$ follows from Lemma 2 in the proof of Proposition 3 and Property 2. The result with respect to $\dot{K}$ can be proven in a similar fashion and is omitted.

If $U > 0, BC > 0$, $\frac{\partial}{\partial c}\Lambda(a)$ is not necessarily negative for any a satisfying $\overline{B} \in \Omega^2(a)$. It follows from Proposition 4 that $\dot{a}(c_0)$ satisfies

$$\Lambda(\dot{a}(c_0)) = (c_0K^0(1+\dot{a}(c_0))^b - \overline{B})\left(\dot{a}(c_0) - \frac{BC\,\overline{\xi}(1+1/b)}{2\overline{\xi}c_0K^0(1+\dot{a}(c_0))^b}\right) = U$$

in equilibrium. It follows that $\frac{\partial}{\partial c}\Lambda(a)\big|_{\dot{a}(c_0)} < 0$ in (30). With a small change in c to c_1, we can guarantee that $\Lambda(a; c_1) < U$ for $\forall\ \ a < \dot{a}(c_0)$ because i) $\Lambda(a; c_0) < U$ for $\forall\ \ a < \dot{a}(c_0)$ by the definition of equilibrium, and ii) $\left|\frac{\partial}{\partial c}\Lambda(a)\right|$ and $\left|\frac{\partial}{\partial a}\Lambda(a)\right|$ are bounded. Therefore the equilibrium financing cost increases locally in c. The results with respect to $\dot{\Upsilon}$ also holds locally. ■

Proof of Proposition 11: Let $\dot{a}_D(\sigma_0) < \infty$, $\dot{a}_D(\sigma_1)$ ($\dot{a}_D(\rho_0) < \infty$, $\dot{a}_D(\rho_1)$) denote the equilibrium financing cost with demand variability (correlation) σ_0 and σ_1 (ρ_0 and ρ_1) respectively with $\sigma_1 > \sigma_0$ ($\rho_1 > \rho_0$). From Proposition 14, we obtain

$$\Lambda_D(a_D) = (c_D\mathbf{1}'\mathbf{K_D^1} - \overline{B})a_D - BC\ Pr\left(\xi_1 + \xi_2 < 2\overline{\xi}(1+\frac{1}{b})\left[1 - \frac{\overline{B}}{c_D\mathbf{1}'\mathbf{K_D^1}}\right]\right) \quad (31)$$

for any $\overline{B} \in \Omega_D^2(a_D)$ where $\mathbf{K_D^1}' = \left(\left(\frac{\overline{\xi}(1+\frac{1}{b})}{c_D(1+a_D)}\right)^{-b}, \left(\frac{\overline{\xi}(1+\frac{1}{b})}{c_D(1+a_D)}\right)^{-b}\right)$. Since (ξ_1, ξ_2) is bivariate normal with $N(\boldsymbol{\xi}, \boldsymbol{\Sigma})$, $\xi_1 + \xi_2$ is also Normally distributed with mean $2\overline{\xi}$ and standard deviation $\overline{\sigma} = \sigma\sqrt{2(1+\rho)}$. Let C_D denote the right-hand side of the default probability in (31). Since $b < -1$ and $\overline{B} < c_D\mathbf{1}'\mathbf{K_D^1}$ (because $\overline{B} \in \Omega_D^2(a_D)$), we obtain $C_D < 2\overline{\xi}$. We have $Pr\left(\xi_1 + \xi_2 < C_D\right) = \Phi\left(\frac{C_D - 2\overline{\xi}}{\overline{\sigma}}\right)$ where $\Phi(.)$ is the cumulative distribution function of the standard normal random variable. For an arbitrary a_D that satisfies $\overline{B} \in \Omega_D^2(a_D)$, since $\frac{\partial}{\partial \rho}\overline{\sigma} = \frac{\sigma}{\overline{\sigma}} > 0$, $\frac{\partial}{\partial \sigma}\overline{\sigma} = \sqrt{2(1+\rho)} > 0$, and $C_D < 2\overline{\xi}$ it follows that

$$\begin{aligned} \frac{\partial \mathbb{E}\left[\Lambda_D(a_D)\right]}{\rho} &= -BC\phi\left(\frac{C_D - 2\overline{\xi}}{\overline{\sigma}}\right)\left(\frac{2\overline{\xi} - C_D}{\overline{\sigma}^2}\right)\frac{\partial \overline{\sigma}}{\partial \rho} < 0, \\ \frac{\partial \mathbb{E}\left[\Lambda_D(a_D)\right]}{\sigma} &= -BC\phi\left(\frac{C_D - 2\overline{\xi}}{\overline{\sigma}}\right)\left(\frac{2\overline{\xi} - C_D}{\overline{\sigma}^2}\right)\frac{\partial \overline{\sigma}}{\partial \sigma} < 0 \end{aligned}$$

where $\phi(.)$ is the density function of the standard normal random variable. Since the expected (stage 0) return of the creditor strictly decreases in σ and ρ for any feasible a_D, $\Lambda(0, \sigma_0) < 0$ and $\Lambda(0, \rho_0) < 0$, substituting $\tau = \sigma$ and $\tau = \rho$ respectively in Lemma 5 in the proof of Proposition 7, it follows that the equilibrium level of financing cost increases in σ and ρ, i.e. $\dot{a}_D(\sigma_1) > \dot{a}_D(\sigma_0)$ and $\dot{a}_D(\rho_1) > \dot{a}_D(\rho_0)$. The result related to $\dot{\Upsilon}_D = \dot{\pi}_D$ follows from Property 2 and the independence of π_D^* from $\mathbf{\Sigma}$ for a given financing cost a_D. The result related to $\dot{\Upsilon}_D = \dot{\mathbf{K}}_\mathbf{D}$ can be established in a similar fashion and is omitted. ■

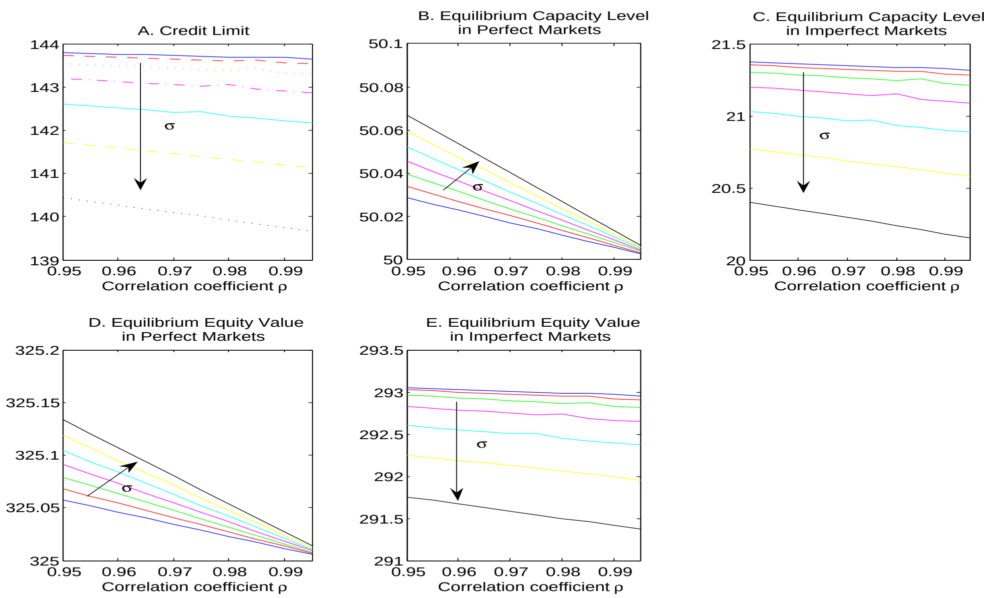

Figure 5: Re-plot of Figure 3 for $\rho \geq 0.95$ (Effect of ρ and σ on flexible technology investment in imperfect markets with $BC = 50$ and $\sigma \in [3, 4.5]$ with 0.25 increments): A higher ρ may lead to a higher credit limit $\dot{E}_F$ (lower financing cost $\dot{a}_F$) at high correlations (Panel A). In perfect capital markets, $\dot{K}_F$ (Panel B) and $\dot{\pi}_F$ (Panel D) decrease with an increase in ρ. In imperfect capital markets, $\dot{K}_F$ (Panel C) and $\dot{\pi}_F$ (Panel E) may increase with an increase in ρ at high correlation levels. This is because the strategic effect is positive in this range.

Proof of Proposition 12: With $BC = 0$, the creditor's expected (stage 0) return with technology T is given by $\Lambda_T(a_T) = (c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1} - \overline{B})a_T$ for any a_T such that $\overline{B} \in \Omega_T^2(a_T)$.

We first start with the unit investment cost pair $(c_D, \overline{c}_F^P(c_D))$. It is easy to verify that $c_F K_F^0|_{\overline{c}_F^P(c_D)=\overline{c}_F^S(c_D)} = c_D \mathbf{1'K_D^0}$. Therefore with identical financing costs, $a_D = a_F = a$, we have $\Lambda_F(a) = \Lambda_D(a)$ and the creditor offers identical financing cost for each technology in equilibrium $(\dot{a}_D(c_D) = \dot{a}_F(\overline{c}_F^P(c_D)))$. It follows from Proposition 3 that the firm is indifferent between both technologies in equilibrium at this cost pair.

For unit investment cost pair (c_D, c_F) such that $c_F < \overline{c}_F^P(c_D) = \overline{c}_F^S(c_D)$ it follows from Proposition 3 that if the firm is exposed to the identical financing costs with each technology at this cost pair, the firm strictly prefers flexible technology over dedicated technology. We now prove that at this cost pair the creditor always offers lower financing cost with flexible technology in equilibrium $(\dot{a}_D(c_D) \geq \dot{a}_F(c_F)$ for $c_F < \overline{c}_F^P(c_D))$, and we will conclude from Property 2 that the firm chooses flexible technology in equilibrium.

Let $\dot{a}_F(c_F^0)$, $\dot{a}_F(c_F^1)$ denote the equilibrium financing cost with unit capacity investment cost c_F^0 and c_F^1 respectively with $c_F^1 > c_F^0 > 0$. We obtain $\frac{\partial}{\partial c_F}\Lambda(a_F) = (b+1)K_F^1 a_F < 0$ for any a_F satisfying $\overline{B} \in \Omega_F^2(a_F)$. Since the expected (stage 0) return of the creditor strictly decreases in c_F for any feasible a_F and $\Lambda_F(0; c_F^0) < 0$, substituting $\tau = c_F$ in Lemma 5 in the proof of Proposition 7, it follows that the equilibrium level of financing cost strictly increases in c_F if the creditor offers a contract in equilibrium with c_F^0 (if $\dot{a}_F(c_F^0) < \infty$). Within the same line of arguments, it can also be shown that if $\dot{a}_F(c_F^0) \to \infty$ then $\dot{a}_F(c_F^1) \to \infty$. This implies that $\dot{a}_F(c_F) \leq \dot{a}_F(\overline{c}_F^P(c_D))$ for $c_F < \overline{c}_F^P(c_D)$. From the first part of the proof, we already established that $\dot{a}_D(c_D) = \dot{a}_F(\overline{c}_F^P(c_D))$. This concludes the proof. ■

Proof of Proposition 13: Since $\overline{c}_F^P(c_D;\rho) = \overline{c}_F^S(c_D;\rho)$ from Proposition 6, it follows from Proposition 3 that if both technologies are exposed to identical financing costs then the firm is indifferent between both technologies at the cost pair $\left(c_D, \overline{c}_F^P(c_D;\rho)\right)$. From Proposition 14 in Appendix B, we have

$$\Lambda_T(a_T) = (c_T \mathbf{1'K_T^1} - \overline{B})a_T - BC\ Pr\left(H_T(\boldsymbol{\xi}) < M_T(1+\frac{1}{b})\left(1 - \frac{\tilde{B}}{c_T\mathbf{1'K_T^1}}\right)\right) \tag{32}$$

for $\overline{B} \in \Omega_T^2(a_T)$ where $M_F = \mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$, $M_D = 2^{-1/b}\overline{\xi}$, $H_F(\boldsymbol{\xi}) = \left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}$, and $H_D(\boldsymbol{\xi}) = \frac{\xi_1+\xi_2}{2^{1+1/b}}$. It is easy to verify that $c_F K_F^0|_{\overline{c}_F^P(c_D)} = c_D \mathbf{1'K_D^0}$. Therefore with identical cost scheme, earnings (without default) of the creditor is the same with each technology. The ordering of the equilibrium financing cost between each technology is determined by the ordering of the default risk calculated at the identical financing costs with each technology. We first show that for $\rho \approx 1$, the default risk of the firm is higher with flexible technology if both technologies are exposed to identical financing costs $a_F = a_D < \infty$.

Figure 6 demonstrates the default region with each technology with identical financing cost scheme for $c_F = \bar{c}_F^P(c_D;\rho)$. The overall default probability is determined by superimposing

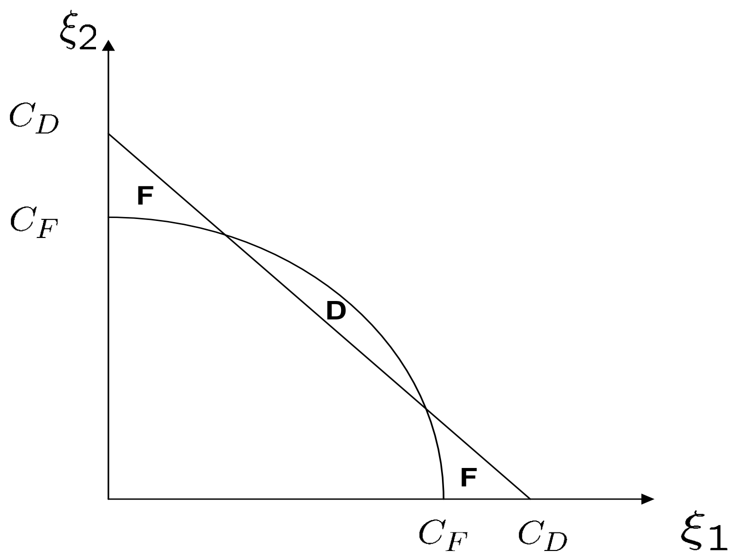

Figure 6: Default regions in (ξ_1, ξ_2) space with each technology: C_D and C_F are the right-hand side terms in the default probability in (32). The area below the straight line (curve) is the default region with the dedicated (flexible) technology. F (D) represents the $\boldsymbol{\xi}$ realizations that the firm does not default with the flexible (dedicated) technology and defaults with the other technology.

the $\boldsymbol{\xi}$ distribution and taking the expectation over the regions. Let C_T denote the right-hand side of the default probability in (32). At the technology cost pair $\left(c_D, c_F^S(c_D;\rho)\right)$, since $\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}} < \left((\xi_1+\xi_2)^{-b}\right)^{-\frac{1}{b}}$ for any $\boldsymbol{\xi}$ realization, it follows that $M_F < 2\bar{\xi}$ and we have $C_D = C_F \frac{2\bar{\xi}}{M_F} > 1$. Therefore the region denoted with F always exists around the points $(0, C_D)$ and $(C_D, 0)$. The point $(\frac{C_D}{2}, \frac{C_D}{2})$ that is on the default line of the dedicated technology is in the default region of the flexible technology if $\left(\frac{C_D}{2}^{-b} + \frac{C_D}{2}^{-b}\right)^{-\frac{1}{b}} \leq C_F$. At the technology cost pair $\left(c_D, c_F^P(c_D;\rho)\right)$ this condition is equivalent to $2^{-\frac{1}{b}}\bar{\xi} \leq M_F$. It follows from (28) and the related discussion in the proof of Proposition 3, this condition is satisfied with equality only if $\rho = 1$, otherwise the inequality is always satisfied for $\rho \neq 1$. Therefore, D region only exists if $\rho \neq 1$. With close to perfect positive correlation, all the $\boldsymbol{\xi}$ realizations are located around $\xi_1 = \xi_2$ line (which also passes through the point $(\frac{C_D}{2}, \frac{C_D}{2})$). It follows that after taking the expectation over the default regions, we have higher default risk with the flexible technology because of the existence of region D.

We now prove that since the dedicated technology has lower default risk with identical financing cost scheme, the equilibrium financing cost with dedicated technology is lower at the cost pair $\left(c_D, c_F^P(c_D;\rho)\right)$ for $\rho \approx 1$. Let $\dot{a}_D(c_D)$ and $\dot{a}_F(c_F^P(c_D;\rho))$ be the equilibrium financing cost with dedicated technology and flexible technology respectively for $\rho \approx 1$. Suppose that $\dot{a}_F(c_F^P(c_D;\rho)) < \dot{a}_D(c_D)$. For $\dot{a}_F(c_F^P(c_D;\rho)) < \infty$, it follows from Proposition 4 that, we have $\Lambda_F(\dot{a}_F(c_F^P(c_D;\rho))) = U$. Since the default probability with flexible technology is higher, it follows that $\Lambda_D(\dot{a}_F(c_F^P(c_D;\rho))) > U$ for $a_D = \dot{a}_F(c_F^P(c_D;\rho))$. This implies from Proposition 4 that there exists $\dot{a}_D^{'}(c_D) < a_D = \dot{a}_F(c_F^P(c_D;\rho)) < \dot{a}_D(c_D)$. This contradicts with $\dot{a}_D(c_D)$ being the Pareto-optimal equilibrium.

Since the firm is indifferent between two technologies at the cost pair $\left(c_D, \overline{c}_F^P(c_D;\rho)\right)$ for $\rho \approx 1$, if both technologies are exposed to identical financing costs, and since $\dot{a}_D(c_D) < \dot{a}_F(c_F^P(c_D;\rho))$, it follows from Property 2 that the firm prefers dedicated technology in equilibrium at this cost pair. ■

B Appendix B. Characterization of Equilibrium Financing Cost

B.1 Characterization of $\Lambda_T(a_T)$

Proposition 14 *Let $R_B(.)$ denote the probability distribution function of the internal budget B. The expected (stage 0) return of the creditor $\Lambda_T(a_T)$ for technology $T \in \{D, F\}$ is given by*

$$\int_{\Omega_T^2(a_T)} \left[(c_T \mathbf{1'K_T^1} - \tilde{B}) a_T - BC\ Pr\left(H_T(\boldsymbol{\xi}) < M_T(1+\frac{1}{b}) \left(1 - \frac{\tilde{B}}{c_T \mathbf{1'K_T^1}}\right)\right)\right] dR_B(\tilde{B}) \quad (33)$$

where $M_F = \mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$, $M_D = 2^{-1/b}\overline{\xi}$, $H_F(\boldsymbol{\xi}) = \left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}$, and $H_D(\boldsymbol{\xi}) = \frac{\xi_1+\xi_2}{2^{1+1/b}}$.

Proof of Proposition 14: For $a_T > 0$, it follows from Proposition 2 that the firm borrows to finance the credit-unconstrained optimal capacity level $c_T \mathbf{1'K_T^1} - \tilde{B}$ for $\tilde{B} \in \Omega_T^2(a_T)$ and borrows up to the credit limit $E_T = \frac{P}{1+a_T}$ for $\tilde{B} \in \Omega_T^3(a_T)$. Let $V_T^3(a_T) \doteq c_T \mathbf{1'K_T^1} - \frac{P}{1+a_T}$ denote the upper bound of the region $\Omega_T^3(a_T)$. We now establish $V_T^3(a_T) < 0$ and $\Omega_T^3(a_T) = \emptyset$. We obtain

$$\frac{\partial V_T^3}{\partial a_T} = \frac{1}{(1+a_T)^2} \left[\frac{b c_T \mathbf{1'K_T^0}}{(1+a_T)^{-(b+1)}} + P \right].$$

It follows that $\frac{\partial}{\partial a_T} V_T^3 < (>) 0$ for $a_T < (>) \left(\left(\frac{-bc_T \mathbf{1'K_T^0}}{P} \right)^{\frac{-1}{b+1}} - 1 \right)^+$. Therefore V_T^3 is first decreasing (if $(.)^+ \neq 0$) then increasing in a_T. We have $V_T^3(0) = c_T \mathbf{1'K_T^0} - P < 0$ by assumption (since P is sufficient to finance the budget-unconstrained optimal capacity investment level) and $\lim_{a_T \to \infty} V_T^3(a_T) = 0$. It follows that $V_T^3(a_T) < 0$ for any $a_T > 0$ and $\Omega_T^3(a_T) = \emptyset$. Equation (33) follows from

$$\Lambda_T(a_T) = \int_{\Omega_T^2(a_T)} \left[(c_T \mathbf{1'K_T^1} - \bar{B}) a_T - BC\ Pr \left(\mathbf{N_T'1'K_T^1}^{1+\frac{1}{b}} < (c_T \mathbf{1'K_T^1} - \bar{B})(1 + a_T) \right) \right] dR_B(\tilde{B})$$

after some algebra. ■

B.2 Characterization of $\dot{a}$ for $\xi \sim U[0, 2\overline{\xi}]$ in the single-product setting

In the single product setting, from Proposition 14 the expected return of the creditor for a given a satisfying $\overline{B} \in \Omega_2(a)$ can be written as

$$\Lambda(a) = (cK^0(1+a)^b - \overline{B})a - BC\ \Phi \left(\frac{\overline{\xi}(1+1/b)}{cK^0(1+a)^b} \left(cK^0(1+a)^b - \overline{B} \right) \right), \tag{34}$$

where $\Phi(.)$ is the cdf of ξ. With uniform distribution assumption, the default probability in (34) is a linear function of the amount of borrowing and we obtain

$$\Lambda(a) = \left(cK^0(1+a)^b - \overline{B} \right) \left(a - \frac{BC\ \overline{\xi}(1+1/b)}{2\overline{\xi} cK^0(1+a)^b} \right).$$

The first term is the amount of lending. The second term is the unit marginal profit of lending. For each unit of the loan, the creditor earns a and incurs an expected default cost.

Proposition 15 *If $\xi \sim U[0, 2\overline{\xi}]$ and $U \geq 0$, there exists a unique bankruptcy cost threshold $\widehat{BC} = \left(\frac{\overline{\xi}(1+1/b)^2}{c} \right)^{-b-1} \frac{2\overline{\xi}}{-b}$ such that for $BC > \widehat{BC}$, the creditor does not offer any contract in equilibrium ($\dot{a} \to \infty$). For $BC \leq \widehat{BC}$, if U and $\overline{B}$ are sufficiently small, the creditor offers $\dot{a} < \left(\frac{cK^0}{\overline{B}} \right)^{-1/b} - 1$; otherwise the creditor does not offer a contract ($\dot{a} \to \infty$) in equilibrium.*

Proof of Proposition 15: With $\xi \sim U[0, 2\overline{\xi}]$, the expected returns of the creditor can be written as

$$\Lambda(a) = \left(cK^0(1+a)^b - \overline{B} \right) \left(a - \frac{BC\ \overline{\xi}(1+1/b)}{2\overline{\xi} cK^0(1+a)^b} \right). \tag{35}$$

for any a that satisfies $\overline{B} \in \Omega^2(a)$. We first focus on the second term, the expected unit marginal profit of lending. We define $x \doteq 1 + a$ and after some algebra to the second term in (35), we obtain $G(x) = -\frac{BC\ \overline{\xi}(1+1/b)}{2\overline{\xi} cK^0} x^{-b} + x - 1$. We have $\frac{\partial}{\partial x} G(x) > 0$ for $x < \bar{x} =$

$\left(\frac{2\bar{\xi}cK^0}{-bBC\,\bar{\xi}(1+1/b)}\right)^{\frac{-1}{b+1}}$ and $\frac{\partial}{\partial x}G(x) < 0$ for $x > \bar{x}$. Note that if $G(\bar{x}) < 0$ then the marginal profit is always negative and the creditor does not lend in equilibrium. This is the case if $BC > \widehat{BC} = \left(\frac{\bar{\xi}(1+1/b)^2}{c}\right)^{-b-1}\frac{2\bar{\xi}}{-b}$. We now show that for $BC \le \widehat{BC}$ and for sufficiently small U and $\overline{B}$, the creditor always offers a contract and the firm borrows in equilibrium.
We obtain $\lim_{x\to\infty} G(x) = -\infty$ and $\frac{\partial}{\partial x}G(x)\big|_{x=1} > 0$ for $BC \le \widehat{BC}$; therefore there exist two positive roots x_1, x_2 such that $1 < x_1 < \bar{x} < x_2$. Since we focus on Pareto-optimal equilibrium, we are interested in the smallest root x_1. Since the feasible set of the unit financing cost a is $[0, \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1]$, we are interested in the roots of $G(x) = 0$ in the range of $1 \le x \le \bar{\bar{x}}$ where $\bar{\bar{x}} = \left(\frac{cK^0}{\overline{B}}\right)^{-1/b}$. We now check if $a = x_1 - 1$ is feasible. For x_1 to be infeasible, i.e. $\bar{\bar{x}} < x_1$, the conditions

$$\begin{aligned} G(\bar{\bar{x}}) &= -\frac{BC\,\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0}\left(\frac{cK^0}{\overline{B}}\right) + \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1 < 0, \\ \frac{\partial G(x)}{\partial x}\bigg|_{x=\bar{\bar{x}}} &= \frac{b\,BC\,\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0}\left(\frac{cK^0}{\overline{B}}\right)^{\frac{b+1}{b}} + 1 > 0 \end{aligned}$$

should be satisfied. The second condition is equivalent to $\overline{B} > \widehat{B} = c\left(\frac{-b\,BC}{2\bar{\xi}}\right)^{\frac{b}{b+1}}$. Two cases can arise:

Case $i, \overline{B} \le \widehat{B}$: We have $\frac{\partial}{\partial x}G(x)\big|_{x=\bar{\bar{x}}} < 0$, therefore x_1 is feasible. Since $x_1 < \bar{\bar{x}}$, if U is sufficiently small then there exists an equilibrium financing cost that satisfies $\dot{a} > x_1 - 1$ and $\dot{a} < \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$. If U is large enough, it is not feasible to generate U because both terms in (35) are bounded. In that case the creditor does not offer a contract ($\dot{a} \to \infty$).

Case $ii, \overline{B} > \widehat{B}$: We have $\frac{\partial}{\partial x}G(x)\big|_{x=\bar{\bar{x}}} > 0$. Using $G(\bar{\bar{x}})$, we define

$$H(\overline{B}) \doteq -\frac{BC\,\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0}\left(\frac{cK^0}{\overline{B}}\right) + \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1.$$

It is easy to establish that $\frac{\partial}{\partial \overline{B}}H(\overline{B}) > 0$ for $\overline{B} < \widehat{B}$ and $\frac{\partial}{\partial \overline{B}}H(\overline{B}) < 0$ for $\overline{B} > \widehat{B}$.
For $BC = \widehat{BC}$, we obtain $H(\widehat{B}) = 0$ and it follows that for $BC = \widehat{BC}$, $H(\overline{B}) < 0$ for $\overline{B} > \widehat{B}$ and hence $G(\bar{\bar{x}}) < 0$ is satisfied for such $\overline{B}$. Therefore, x_1 is infeasible. In this case, the creditor does not offer a contract ($\dot{a} \to \infty$).
For $BC < \widehat{BC}$, we have $H(\widehat{B}) = 0$, therefore for some $\overline{B} > \widehat{B}$, we have $G(\bar{\bar{x}}) > 0$, and x_1 is feasible. For such $\overline{B}$, the creditor offers $\dot{a} \in \left(x_1 - 1, \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1\right)$ for sufficiently small $U > 0$. For significantly large U, the creditor does not offer any contract ($\dot{a} \to \infty$). Since $\frac{\partial}{\partial \overline{B}}H(\bar{B}) < 0$ for $\overline{B} > \widehat{B}$, after sufficient increase in $\overline{B}$, we may have $G(\bar{\bar{x}}) < 0$, and x_1 becomes infeasible. In this case, the creditor does not offer a contract ($\dot{a} \to \infty$) for any $U > 0$. ■

C Appendix C. Proofs of Supporting Lemmas

Proof of Lemma 1: The expectation and differentiation operators can be interchanged if the function under expectation is integrable and satisfies the Lipschitz condition of order one (Glasserman 1994, p.245). The function $\pi_T^*(\tilde{B})$ satisfies the Lipschitz condition of order one if

$$\frac{|\pi_T^*(\tilde{B}') - \pi_T^*(\tilde{B}'')|}{|\tilde{B}' - \tilde{B}''|} \leq Y_{\pi_T^*} \ \ \forall\ (\tilde{B}', \tilde{B}'') > 0 \text{ for some } Y_{\pi_T^*} \text{ with } \mathbb{E}[Y_{\pi_T^*}] < \infty. \tag{36}$$

Clearly, condition (36) is satisfied if $\left|\frac{\partial \pi_T}{\partial \tilde{B}}\right|$ is bounded. From (27) in the proof of Proposition 2, we obtain

$$\frac{\partial \pi_T^*(\tilde{B})}{\partial \tilde{B}} = \begin{cases} 1 & \text{if } \ \tilde{B} \in \Omega_T^0 \\ \frac{M_T}{c_T}(1+1/b)\left(\frac{\tilde{B}}{c_T}\right)^{\frac{1}{b}} & \text{if } \ \tilde{B} \in \Omega_T^1 \\ 1 + a_T & \text{if } \ \tilde{B} \in \Omega_T^2 \\ \frac{M_T}{c_T}(1+1/b)\left(\frac{E_T+\tilde{B}}{c_T}\right)^{\frac{1}{b}} & \text{if } \ \tilde{B} \in \Omega_T^3 \end{cases}$$

at the points where $\pi_T(\tilde{B})$ is differentiable. It is easy to verify that $\lim_{\tilde{B}\to\tilde{B}_k^+} \frac{\partial}{\partial \tilde{B}}\pi_T^*(\tilde{B}) = \lim_{\tilde{B}\to\tilde{B}_k^-} \frac{\partial}{\partial \tilde{B}}\pi_T^*(\tilde{B})$ for $\tilde{B}_k \in \Omega_T^{0123}$, and $\pi_T^*(\tilde{B})$ is differentiable everywhere in its domain. If $\tilde{B} \in \Omega_T^1$ we have

$$\frac{\partial \pi_T^*}{\partial \tilde{B}} \leq \frac{M_T}{c_T}(1+\frac{1}{b})\left(\mathbf{1'K_T^1}\right)^{\frac{1}{b}} \leq (1+a_T),$$

and for $\tilde{B} \in \Omega_T^3$ we have

$$\frac{\partial \pi_T^*}{\partial \tilde{B}} \leq \frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{E_T}{c_T}\right)^{\frac{1}{b}} \leq Y_T$$

where $1 + a_T < Y_T < \infty$. It follows that $\left|\frac{\partial \pi_T^*}{\partial \tilde{B}}\right| \leq Y_T < \infty$ for $\tilde{B} \geq 0$. Since $\pi_T^*(\tilde{B})$ is integrable, the interchange of the derivative and expectation is justified. ■

Proof of Lemma 2: Let $R_B(.)$ denote the probability distribution function of the internal budget B. From Lemma 1 in the proof of Proposition 3, we can interchange the derivative and the expectation operators and using the Leibniz' rule we obtain

$$\begin{aligned} \frac{\partial \mathbb{E}[\pi_T^*(B)]}{\partial c_T} &= \int_{\Omega_T^0} -\mathbf{1'K_T^0} \ dR_B(\tilde{B}) \\ &+ \int_{\Omega_T^1} -(1+\frac{1}{b})\frac{M_T}{c_T}\left(\frac{\tilde{B}}{c_T}\right)^{1+\frac{1}{b}} dR_B(\tilde{B}) \\ &+ \int_{\Omega_T^2} -\mathbf{1'K_T^1}(1+a_T) \ dR_B(\tilde{B}) \\ &+ \int_{\Omega_T^3} -(1+\frac{1}{b})\frac{M_T}{c_T}\left(\frac{E_T+\tilde{B}}{c_T}\right)^{1+\frac{1}{b}} dR_{B(H)}(\tilde{B}). \end{aligned}$$

It follows that $\frac{\partial}{\partial c_T}\mathbb{E}[\pi_T^*(B)] < 0$. ■

Proof of Lemma 3: It is easy to verify that we have $c_F K_F^j\Big|_{\bar{c}_F^S(c_D)} = c_D \mathbf{1'K_D^j}$ for $j = 0, 1$. Since $a_F = a_D$ and $E_F = E_D$, the regions in (27) also overlap, i.e. $\Omega_F^i \equiv \Omega_D^i$ for $i = 0, .., 3$. Therefore, we have $\pi_F^*(\tilde{B}) = \pi_D^*(\tilde{B})$ in (27). This concludes the proof. ■

Proof of Lemma 4: Let $R_B(.)$ denote the probability distribution function of the internal budget B. From Lemma 1 in the proof of Proposition 3, we can interchange the derivative and the expectation operators and using the Leibniz' rule we obtain

$$\frac{\partial \mathbb{E}[\pi_T^*(B)]}{\partial a_T} = \int_{\Omega_T^2} \left(\tilde{B} - c_T \mathbf{1'K_T^1}\right) \ dR_B(\tilde{B}) - \int_{\Omega_T^3} E_T \ dR_B(\tilde{B}).$$

It follows that $\frac{\partial}{\partial a_T}\mathbb{E}[\pi_T^*(B)] < 0$.

Similarly, we obtain

$$\frac{\partial \mathbb{E}[\pi_T^*(B)]}{\partial E_T} = \int_{\Omega_T^3} \left(-(1 + a_T) + \frac{M_T\left(1 + \frac{1}{b}\right)}{c_T} \left(\frac{E_T + \tilde{B}}{c_T}\right)^{\frac{1}{b}} \right) \ dR_B(\tilde{B}).$$

For $\tilde{B} \in \Omega_T^3$, since $\tilde{B} < c_T \mathbf{1'K_T^1} - E_T$,

$$\frac{M_T}{c_T}(1 + \frac{1}{b}) \left(\frac{E_T + \tilde{B}}{c_T}\right)^{\frac{1}{b}} > \frac{M_T}{c_T}(1 + \frac{1}{b}) \ \left(\mathbf{1'K_T^1}\right)^{\frac{1}{b}} = 1 + a_T.$$

It follows that $\frac{\partial}{\partial E_T}\mathbb{E}[\pi_T^*(B)] \geq 0$ with equality holding if $\Omega_T^3 = \emptyset$, i.e. $c_T \mathbf{1'K_T^1} - E_T < 0$. ■

Proof of Lemma 5: It follows from Proposition 4 that $\dot{a}_T(\tau_0) = argmin_{a_T \geq 0} \Lambda_T(a_T; \tau^0) = U$. Let us assume that the unit financing cost decreases in τ, i.e. $\dot{a}_T(\tau_1) < \dot{a}_T(\tau_0)$. We will show by contradiction that this is not possible.

Since $\dot{a}_T(\tau_0) < \infty$, we also have $\dot{a}_T(\tau_1) = argmin_{a_T \geq 0}\Lambda_T(a_T; \tau^1) = U$. If $\frac{\partial}{\partial \tau}\Lambda_T(a_T) < 0$ for any feasible a_T then we have $U = \Lambda_T(\dot{a}_T(\tau_1); \tau^1) < \Lambda_T(\dot{a}_T(\tau_1); \tau^0)$. It follows from *i)* $\Lambda_T(0; \tau^0) < 0$, *ii)* the continuity of $\Lambda_T(a_T; \tau^0)$ in a_T, and *iii)* the Mean-value theorem that there exist $\dot{a}'_T(\tau_0) < \dot{a}_T(\tau_0)$ such that $\Lambda_T(\dot{a}'_T(\tau_0); \tau^0) = U$. This is a contradiction with $\dot{a}_T(\tau_0)$ being the Pareto-optimal equilibrium for τ^0. Therefore, if there exists $\dot{a}_T(\tau_1)$ that satisfies $\Lambda_T(\dot{a}_T(\tau_1); \tau^1) = U$, we have $\dot{a}_T(\tau_1) > \dot{a}_T(\tau_0)$. If such $\dot{a}_T(\tau_1)$ does not exist, then we have $\dot{a}_T(\tau_1) \to \infty > \dot{a}_T(\tau_0)$. ■

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