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2010/05/TOM/ISIC

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February 1, 2010

The authors acknowledge helpful comments on any earlier draft from Thomas Weber and by participants at the Rutgers 28th Annual Eastern Conference, Skytop, PA, May 13-15, 2009.

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1. Introduction

The energy business is in transition from a cost structure based on fuel and capital to a business that also accounts for the carbon footprint of energy production, transmission and end use. In the European Union (EU), this is already the case, with a liquid and functioning carbon emissions trading market, and with full subscription to the Kyoto Protocol, including use of additionality offsets from projects certified under the Clean Development Mechanism (CDM) and the Joint Implementation (JI) process. Under the cap and trade system implemented in the European Union, and under any of the similar plans envisaged in various bills under discussion in the US Congress, the value of an energy investment will be the joint product of its cash flows (evaluated in the normal fashion) and the implied value/cost of carbon emissions or reductions associated with the investment. Valuation and risk management for energy projects will therefore involve hedging for both energy, inputs and outputs, as well as for carbon outputs. The objective of this paper is to provide an overview of the practicality of extending the normal portfolio problem in electricity supply to encompass the new markets for carbon, and hedging instruments defined on these.

Under a cap and trade system like that of the EU, the price of carbon has important implications for the valuation of generation assets, pricing of power purchasing agreements, demand-side projects and investments in new generation technologies. In addition to these long-term decisions, there are short-run issues of the impact ofcarbon prices on efficient dispatch and operational decisions. This paper addresses this shorter term problem in the context of the multi-period energy portfolio optimization problem, as described in Kleindorfer and Li (2005). In this application, we imagine a distribution company or competitive supply organization which has contractual or regulatory load obligations. The company can meet and hedge its contingent obligations over the course of the next quarter or year through a variety of means based on short-term and long-term contracts and market-based derivatives executed directly with other agents in the marketplace or via an exchange. The company can also build or contract for physical capacity to meet or hedge its contractual obligations. The problem of interest is the optimization of portfolios of such real and contractual assets, subject to a multi-period Value-at-Risk (VaR) constraint.

Adding carbon emissions (and associated risk hedge instruments) to the standard portfolio problem is relatively straightforward, but there are interesting wrinkles arising from the fact that carbon emission permits are required at a point in time following actual emissions (e.g., in the EU, these are due in April of the calendar year in which the emission liability was incurred), while energy prices and related financial instruments are evaluated in on-going spot markets throughout the year. There are also political and regulatory risks and the possibility of hedging carbon risks using instruments defined in correlated markets, e.g., crude oil and other market-traded commodities whose price is correlated with carbon credit prices. The resulting problem has a structure not unlike that of hedging spark spread (e.g., Li and Kleindorfer, 2010), but with some differences affecting both optimal dispatch and hedging strategies. The main objective of the paper is to extend our earlier work on optimal portfolio analysis to encompass the cost of carbon emissions in a cap and trade environment.

The paper proceeds as follows. In the next section, we describe briefly the institutional background related to cap and trade systems, following the EU model for specificity. In section 3, we indicate how carbon emissions liabilities can be included in the portfolio optimization problem. The solution to this problem gives rise to predicted carbon liabilities for the focal company, say for the next 12 months. We then develop in section 4 a framework for optimizing carbon trading (the acquisition of the required carbon certificates to cover carbon emission liabilities of the company), given the emissions liabilities resulting from the portfolio optimization and the stochastic process for carbon price evolution. In the concluding section, we raise a number of questions associated with regulation and with energy company strategies, including the development of risk management competencies and the impact of longer-term technology mix choices that are likely to be increasingly important in a carbon economy.

2. Background

Driven by concerns with the impact of industrial activity on climate change, there has been considerable activity in the past decade directed towards measuring and limiting man-made emissions of the primary greenhouse gases (GHG), CO_2, Methane and N_2O. An index of CO_2 equivalents is now used to reflect total GHG emissions with the primary focus for electric power on CO_2 itself, and that will be our focus here. The Kyoto Protocol of 1997, which came into full force in 2005,[4] requires countries in Annex I to the Protocol (the developed countries) to establish measures to limit their annual GHG emissions to levels that are at least 5% below 1990 levels in the first enforcement period 2008-2012, with a meeting of the parties in Copenhagen in 2012 to determine what are likely to be further reductions in this target.

Implementing the Protocol is left to the individual countries and a mix of regulations, standards and market mechanisms has been implemented in countries that

[4] The reason for the delay between 1997 and 2005 was that the Kyoto Protocol had to be ratified by at least 55 Parties to the Convention, including developed countries representing 55% of total baseline emissions in 1990. Russia’s joining the Protocol in February, 2005, achieved the required threshold. We note that the largest emitter of GHG, the USA (accounting for some 25% of total global emissions) has not yet ratified the Protocol but is expected to do so under the Obama Administration.

have endorsed the Protocol.[5] With respect to electric power, following the success of SO_2 and NO_x markets in the USA, attention has been focused on cap and trade systems, in which a target cap on emissions from covered sectors (the most energy-intensive sectors in the economy) is imposed on an annual basis. The implication of this for an electric power company or a cement manufacturer with facilities operating in a Protocol country is that the company must measure its CO_2 emissions throughout the year, subject to audit, and for each calendar year the company must then provide to its Kyoto Regulator (the agency in the country certified to collect and verify emissions data) emissions credits at some defined date after the calendar year has been completed. For example, in the European Union (EU), which is treated as a single "country" for the purposes of the Protocol, Member States/countries in the EU negotiated targets for which they would be responsible (not every country accepted the same target relative to 1990, based on energy technologies used and other political and economic factors). For each country in the EU, emissions permits for all emissions liabilities of covered companies for the previous calendar year must be provided to the country regulator (and from them to the EU Regulator) no later than April 15th of the following year. Focusing just on electric power, one of the main covered sectors, the implication of this is that electric power companies operating in the EU must measure their emissions permits in facilities located in each EU country, report these to the country Kyoto Regulators, and provide by April 15th of the year following their calendar year emissions sufficient carbon emissions credits to cover their emission liabilities.

What is the source of these carbon emissions credits for a given company covered by the cap and trade system? To start with, the government in each country may give out free allocations to the company at the beginning of the year, perhaps based on a benchmark such as 80% of emissions levels for some previous year's demand served and for the mix of technologies used in the focal year by the company. Secondly, some of the total cap target may be auctioned off at the beginning of the year. Thirdly, the company may purchase (or sell if it has excess) credits on the open market. In the EU, carbon trades are mediated by the European Trading System (ETS).[6] Since liabilities are on a calendar year basis, the main instrument traded is the end of December futures contract for delivery of credits. There is also an active options and swap market associated with the ETS. Finally, the company can buy emissions credits from brokers, or directly invest in projects that are certified under so-called Joint Implementation (JI) and the Clean Development Mechanism (CDM) of the Kyoto Protocol. JI and CDM projects themselves might rely for part of project financing from the carbon offsets they sell for the projects through brokers in the carbon market. We will not go into the details of these markets, as our focus here is on trading and acquisition policies for an electric power company covered by such a cap and trade mechanism. All that need be understood for the purposes of this paper is that a covered company has an obligation to cover its CO_2 emissions liabilities for each calendar year at some fixed date in the following year and that there are active and liquid markets where trading of the CO_2 certificates occurs on a routine basis. Against this background, we now consider the optimal sourcing portfolio and

[5] For an excellent introduction to the Protocol and its implications for carbon trading, see Mansanet-Bataller and Pardo (2008).

[6] For details on the ETS, as well as other carbon markets, andhistorical data on carbon trades and prices, see New Carbon Finance, a research and information provider for the global carbon market: http://www.newcarbonfinance.com/.

generation technology choices faced by an electric power company that has obligations of the sort described above under a cap and trade system.

3. Modeling Carbon Emissions in the Context of Portfolio Planning

This section extends the model developed in Kleindorfer and Li (2005) to cover carbon contracts. We imagine an integrated utility, called the "Company", which may own or lease generation, that has a trading division which can sign contracts for Power Purchase Agreements (PPAs), as well as puts, calls and forwards based on an underlying wholesale spot market. We abstract here from transmission constraints or markets.[7] We also imagine that the Company has some retail operations that are regulated.[8] We will take the simplest possible approach to the regulated sector, assuming a fixed, exogenously determined regulated price per kWh. More complicated regulatory scenarios are easily incorporated into the framework developed.

We are interested in formulating an optimal portfolio problem for sourcing power to meet the demands of the Company, sourcing carbon emissions credits sold on carbon markets and simultaneously engaging in trading. The portfolio will be characterized by different levels of time-indexed instruments (puts, calls, forwards, etc.) that might be called upon either to fulfill retail demand, to purchase or sell carbon emission credits or simply as part of profit-oriented trading/hedging activities by the Company's trading division. We refer to all potential assets for the portfolio, including owned/leased generation and PPAs, as "instruments".

We can think of each instrument as having a capacity (measured in MW or for emissions credits in terms of tons of CO_2), that can be bought or sold in a specific period (consisting of specified hours during a given week or month, typically the 5x16 hours of "peak" or the 7x8 hours of "off-peak"). Each instrument entails a reservation price, possibly zero, per MW (or ton) to reserve[9], and an execution price, per MWh or ton, if used. In this framework, instruments such as own generation and certain PPAs that have been pre-committed have a fixed execution price (e.g., the marginal running cost of own generation), but may be thought of as available at a reservation price of zero. Purchased forwards, which are fixed obligations to deliver power, may be viewed as call options having a zero execution price that therefore will be executed by the Company on the day. Forwards sold by the Company have the same characteristic, i.e., they may be viewed as call option contracts with a zero execution price (that therefore will be executed on the day) and a negative execution volume $Q_i < 0$ whose absolute value is the size of the forward.

[7] To the extent that these are based on principles of Locational Marginal Prices (LMP), as in the PJM market, transmission constraints and options could also be included as part of the portfolio optimization described below.

[8] The model developed below applies equally to trading and wholesale power brokers, whether or not they have retail sector commitments at regulated prices.

[9] Note that, in practice, reservation prices are also quoted in MWh's with the period of use/applicability of the instrument being clear (e.g., each peak hour during the month of October). This is clearly equivalent to our formulation.

To set up the model, we will assume that there are T planning periods (weeks or months) in the horizon of interest for the portfolio (e.g., quarterly or annual). The periods $t = 1, \ldots, T$ are further divided into hours denoted by the set H_t. We will consider additional subsets of H_t below, such that certain instruments may be valid only for certain subsets of time (e.g., peak hours, peak days, …). We need the following additional notation:

D_τ= subscribed demand for the Company in hour τ
Q_i = the amount (in MW) of instrument i that is purchased/sold
r_i = reservation price per MWh[10]
c_i = execution price per MWh of a call option (including own generation)
s_i = reservation price per MWh
p_i = execution price per MWh of a put option
$H_{jt} \subseteq H_t$, where H_{jt} is the index of those hours of type "j", $j = 1, \ldots, J$ in period t, representing, for example, peak hours, off-peak hours, weekend hours, etc.
I_{cj} = the set of indices for all call instruments (used in the summation below) that can be executed during hours of type j
I_{pj} = the set of indices for all put instruments (used in the summation below) that can be executed during hours of type j
n = the number of all instruments (so $n = \sum_{j=1}^{J} \left| I_{cj} \cup I_{pj} \right|$)
m_i = Lower bound or minimum amount allowed for instrument Q_i
M_i = Upper bound or maximum amount allowed for instrument Q_i

The typical contract traded for carbon credits is a futures contract, which is traded on an hourly market just like energy, and expires in December of each year. Such a contract covers any and all carbon emission liabilities during the year of its validity date and, under banking, for a specific number of years thereafter as well (i.e., excess credits can be banked for use the following year). For the energy instruments, options and forward contracts will be of much greater complexity. There is no need for our present purposes to get into the details. The aggregate cash flows for each period from all instruments will be our basic focus here.

Following Kleindorfer and Li (2005), under some mild regularity conditions the efficient frontier (trading off expected profits against a Value-at-Risk constraint) is determined by maximizing E - kV = Expected{Profit} – k Variance(Profit) for varying k. The indicated maximization is over the instruments Q_i available for the portfolio, where some instruments, within each month, may apply to different periods of time (e.g., just the peak periods or just the off-peak periods, or to some other selection of periods). The portfolio problem of interest can then be written as follows:

$$Maximize \left[E\{\Pi\} - kVAR\{\Pi\} \right] \tag{1}$$

[10] Thus, to reserve Q_i (MW) of callable capacity during a specific period of length L, the price paid is $r_i Q_i L$, where the maximum reserved/callable capacity during any hour of the period is Q_i (MW). The reader can think of the reservation price in \$/MWh as the allocated cost to each of the hours of the period in question, though instruments will be typically traded for a groups of hours in a month, e.g., 100 MW of capacity callable for any peak hour during the month.

where the maximization is over standard power market instruments and carbon market instruments (the details will become apparent below). We separate the non-carbon and carbon cash flows as follows:

$$\Pi = \Pi^N + \Pi^C = \sum_t \Pi_t^N + \sum_t \Pi_t^C \tag{2}$$

We discuss the structure of the non-carbon and carbon cash flows separately.

To make clear the impact of carbon liabilities, let us first note that in the absence of carbon emission liabilities, following Kleindorfer and Li (2005), we can write the cash flows from all power instruments in period *t* as follows:

$$\begin{aligned}\Pi_t = & \sum_{\tau \in H_t} (P_{c\tau} - P_{s\tau}) D_\tau \\ & + \sum_{j=1}^{J} \sum_{\tau \in H_{jt}} \left[\sum_{i \in I_{cj}} \left[(P_{s\tau} - c_{i\tau})^+ - r_i \right] Q_i + \sum_{i \in I_{pj}} \left[(p_{i\tau} - P_{s\tau})^+ - s_i \right] Q_i \right]\end{aligned} \tag{3}$$

where $x^+ = max[x, 0]$, H_t are the hours within month $t, P_{c\tau}$ is the exogenously determined regulated price per MWh and $P_{s\tau}$ is the spot price at time τ. We have indexed the execution prices c_i and p_i for the options by time periods, in case these execution prices are time sensitive (e.g., different prices for peak or off-peak for the same instrument). Once an instrument is purchased, it is available for all hours covered by the instrument (e.g., a peak-period call option of Q_i MW is available for any peak hour in the month). The first term in (3) represents the profits from retail sales; the second the profits from call options (including own generation and tolling agreements); and the third the profits from put options. The "+" sign indicates that these options contracts are only executed when they are in the money relative to the spot price.

Let us now consider the impact of carbon emissions in more detail in modifying (3). First, these will affect generation costs as the company will need to buy emission credits to cover its emission liabilities. These liabilities will also affect the cost of competing supply contracts, as well as the correlation between these contracts and own generation. Secondly, as envisaged in the cash flow equation above, the company may trade emission derivatives in its hedging activities. We assume in what follows that the only direct carbon liabilities for the Company accrue for generation (either owned or leased). Indirect liabilities could result from PPAs that pass through the carbon liability, and these could be treated in a similar fashion to our approach for generation.

In formula (3), in the case of dispatchable generation, the full marginal cost $C_{i\tau}$, including emissions liabilities, can be written as follows:

$$C_{i\tau} = c_{i\tau} + U_{s\tau} f_i = F_{s\tau} h_i + v_i + U_{s\tau} f_i \tag{4}$$

where $c_{i\tau}$ is the non-carbon operating cost, consisting of the spot fuel price $F_{s\tau}$ times the unit heat rate h_i of generation unit *i* plus the variable cost (per MWh) v_i, and where the carbon cost is $U_{s\tau} f_i$, the "spot emissions price" $U_{s\tau}$ times the carbon emission conversion factor f_i for unit *i* .

A reasonable starting point for the dispatch decision (and similarly for bids into any pool-based system) is that it should be solely based on unit profitability for each hour. At the time of dispatch, everything is visible at the time of decision except the cost of emissions. Emission prices evolve over the course of the year and the final liability is only completely clear at the end of the year. But the operations manager has to make the dispatch/bid decision based on some assumption on the emission cost. One approach is to assume a constant emission price. This price could be the forecast for the emissions price at the maturity date for emissions liabilities (e.g., the end of the calendar year in which they are incurred). At the end of the year, one may find that the generator was over dispatched or under dispatched depending on whether the final emission price was under estimated or over estimated. Another method is to consider the market emission futures price (e.g., for futures contracts with December expiration date) as a benchmark and calculate the total costs based on this price at decision time for dispatch. Although this is still not the realized price, in a competitive market for emissions permits, it would contain "the best current information" about the future cost of emission permits (and it would naturally already discount this price back to the decision time). We will assume in what follows that this "futures contract" method is what is used to trigger dispatch.

Taking the current price $U_{s\tau}$ of the end of year futures contract in hour τ as the trigger for dispatch, we will separate out the cash flows for carbon credits and for other generation costs. We do this as follows. We define the dispatch decision for unit/instrument *i* in (3) as $\varphi_{i\tau}$ = 0 (not dispatched) or 1 dispatched. We then use the rule just announced above to trigger dispatch, namely:

$$\varphi_{i\tau} = \begin{cases} 0 & \text{if } P_{s\tau} < C_{i\tau} \\ 1 & \text{if } P_{s\tau} \ge C_{i\tau} \end{cases} \tag{5}$$

where $C_{i\tau}$ is given in (4) and represents the "full" price of the generation call option or forward *i* in hour τ. Given this dispatch rule, we now reformulate (3) to account for carbon liabilities, obtaining the desired expression for Π_t^N in (2) as:

$$\begin{aligned} \Pi_t^N = & \sum_{\tau \in H_t} (P_{c\tau} - P_{s\tau}) D_\tau \\ & + \sum_{j=1}^{J} \sum_{\tau \in H_{jt}} \left[\sum_{i \in I_{cj}} [\varphi_{i\tau}(P_{s\tau} - c_{i\tau}) - r_i] Q_i + \sum_{i \in I_{pj}} \left[(p_{i\tau} - P_{s\tau})^+ - s_i \right] Q_i \right] \end{aligned} \tag{6}$$

The reader will note that the only difference between (3) and (6) is the presence of an explicit "dispatch" decision for call options (or dispatch choices) represented by the choice variable $\varphi_{i\tau}$ defined in (5). For regular call options and forwards, not entailing any explicit carbon liability, this amounts to the standard call option that will be exercised if and only if $P_{s\tau} \geq c_{i\tau}$. However, for instruments that do entail a carbon liability (e.g., owned fossil fuel generation), the dispatch decision is made on the basis of the full cost of the dispatch, as measured by $C_{i\tau}$ given in (4). Note that the cash flows for a unit dispatched are identical between (3) and (6) and include only the operational costs of units. We will account for the carbon costs separately and optimize these below.

Returning to the issue of the (implicit) carbon cash flows[11], given the above assumptions on valuation of these liabilities, and with an eye on (2), we see that

$$\Pi_t^C = \sum_{\tau \in H_{jt}} \sum_{i \in I_{cj}} - \varphi_{i\tau} f_i Q_i U_{s\tau} \tag{7}$$

Using (6) in place of (3), and adding the carbon emissions liabilities (7), one obtains a transformed portfolio optimization problem, including the effects of carbon emissions liabilities on execution choices. With these small changes, the approach formulated in Kleindorfer and Li (2005) for determining the efficient frontier defined by the solution to (1) goes through without change. We note that, with the substitutions (6) and (7), the profit equation is linear in the instrument variables Q_i. In particular, (1) gives rise to a quadratic programming problem, which can be readily solved to determine the efficient frontier between expected portfolio profits and the multi-period VaR of a portfolio, now including the carbon emission liabilities. In practice, a simulation model is used to obtain the requisite correlations for this quadratic programming problem, as explained and illustrated in Kleindorfer and Li (2005). The needed inputs for this simulation are the list of feasible instruments for each period, the stochastic models of spot price $P_{s\tau}$ and carbon credit price $U_{s\tau}$, including the correlation structure between spot and carbon credit price.

In principle, the above framework is all that is needed to solve for the efficient frontier in (1). However, lest we pass over the details too quickly, let us note some issues that are not dealt with in the above formulation regarding alternative types of generation. First, nuclear generators and other non-carbon instruments are relatively simple and entail no changes from the standard portfolio optimization problem. Also, the dispatch of peaker units can also be understood to track (4)-(5) fairly well. However, a coal unit typically is not an hourly spark spread option (with dispatch represented by (4)). It needs to be treated in a special way (e.g. the minimum run time

[11] We call these "implicit cash flows" since the Company may decide to buy or sell carbon certificates at a different time than when the carbon liabilities are actually incurred. Thus, there is a mild abuse of the term "cash flows" here. What (7) actually represents are the valuation of liabilities for dispatch decisions, with valuation at the then prevailing futures price for carbon certificates. The actual cost and cash flows for covering these liabilities will be the subject of a separate optimization, described in section (4) below. Note that if the Company starts the year with some allocation *B* of certificates from the government or from banking from previous years, the beginning of the year value of the asset B, namely $U_{s0}B$, would be added to Π^C. This beginning allocation would, of course, affect the expected value of Π in (1), but would have no other effect on the VaR optimization and the solution for the efficient frontier. It would be like any other sunk cost or windfall gain. Such an allocation would have other effects on needed purchases of carbon liabilities, as we will examine in more detail in Section 4 below.

and minimum off time are among the operational constraints). A simulation algorithm could optimize the dispatch decision based on the economic outcome or mimic the real operation rule one actually follows. The definition for the cash flows Y_i corresponding to a coal generation unit would still be given by

$$Y_i = \sum_{\tau \in H_j} \varphi_{i\tau}(P_{s\tau} - c_{i\tau})Q_i, i \in I_{cj} \tag{8}$$

as shown in (6), where Q_i is the capacity of coal unit *i*, but the definition of $\varphi_{i\tau}$ would no longer be the simple hourly dispatch rule (4), but would entail more complex choice rules.[12] As the reader can readily see, the foundation for the above approach is the actual dispatch rule used by the operations manager, which can then be simulated to give rise to the requisite statistics needed to formulate the multi-period quadratic programming problem of interest representing the VaR-constrained portfolio problem.

The above formulation provides the optimal choices of power instruments and the associated cash flows for these. As captured in (7), it also provides the implicit liabilities associated with carbon emissions. A further issue is the actual timing of accumulation of the necessary emissions permits to cover emission liabilities. One might imagine that these would be purchased from the market in an on-going fashion as liabilities are incurred (so that (7) would represent the actual cash flows associated with carbon liabilities). However, it is likely that some or all of these will be purchased or allocated from sources other than the futures market, so that this is itself an important decision. One approach would be to allocate allowances received for each specific time period, week or month, and periodically adjust the accumulated emission credits accordingly, e.g. buy more credits if actual use is beyond the allowance and sell otherwise. We will discuss this matter in detail in the next section.

A further issue that has confused some operating managers is generation-specific allowances. These arise when different types of generation may receive different emission allowances from the government. As a company, it is not necessary to distribute these credits to each generation unit; only the total amount allocated to the company is of interest. Of course, the value of this total allowance may change as the emission market changes. Let the total (annual) allowance be *B*, then its value as a component of the portfolio is $U_{s\tau}B$ at time τ. However, as with any sunk cost or revenue source, when making a dispatch decision, the Company should act as if it has to purchase the emission credit it needs from the market regardless of how many credits the company actually has (this is the essence of the cash flow model (2), (6), (7)). This is analogous to serving a load where the Company buys power from the spot market, but then delivers regardless of how much the generation division can produce.

[12] A simple heuristic would be to dispatch whenever spot price was projected to be above a critical dispatch level for some specified time. The question of how to best represent dispatch rules for coal plants in the context of the above portfolio optimization is an interesting one for future research. Our sense is that a reasonable approximate rule for such dispatch is sufficient for the purposes of the optimal portfolio problem studied here. On the other hand, the well established unit commitment and dispatch decisions for coal plants clearly needs to be updated to account for carbon prices based on a full cost model like (3).

The remaining question is how to buy/sell the credits from/into the carbon credit market; when and how much? Intuitively, we can think of this as purchasing (or banking from previous months) for each month exactly the amount of credits needed and selling these to each instrument in (6) to offset the emission liability of that instrument. If the time horizon is one year, for example, one might operate on a "pay as you go system", buying emission credits as emission liabilities are accrued, making adjustments for example on a weekly or monthly basis. This involves a buy if the Company does not have enough credits or a sell if they have excess credits. An alternative way is not to adjust the credit amount at all and buy or sell credits at the end of the year. In the next section, we pose and solve this problem for a given set of emissions liabilities, namely the projected set of emissions liabilities that result from solving the VaR-constrained portfolio optimization problem (1).

4. On Allocation, Trading and Banking of Carbon Emission Credits

Simply buying and selling of credits to cover each period's emission liabilities is overly simplistic. In particular, the Company will almost certainly have banked allocations from previous years and may be allocated (depending on its technology mix) "free credits" at the beginning of the year by the regulator. How should these banked and allocated credits be used over the year, and how should the Company react to price movements in the carbon credit market over the course of the year? We start from the solution derived in Section 3, which (at the particular portfolio selected from those on the efficient frontier) implies a set carbon emission liabilities from owned or leased generation for period $t = 1, \ldots, T$ as follows:

$$L_t = \sum_{\tau \in H_t} \sum_{i \in G_t} \varphi_{i\tau} f_i Q_i , \quad t = 1,\ldots,T \tag{9}$$

where G_t = index for instruments giving rise to carbon liabilities (e.g. owned generators) in period t, and where $\varphi_{i,\tau} = 0$ or 1 for the operation status "on" or "off" as determined by (4)-(5) (or some other more complex rule for dispatch).

At the end of the year, the Company may have excess credits to sell in the market or it may face a fine proportional to its shortfall in credits. In each time period, the Company may purchase or sell the credit futures, and the company also may receive credits from the government at the beginning of the year. In order to discuss different strategies, we define M_t as "Market Trading" of credits at time period t, positive for purchasing and negative for selling, and B as the credits available at the beginning of the year, either banked from a previous year or allocated to the Company by the government in the present year. Then the total cash flows for the carbon credits is

$$\Pi(C) = U_T \left(\sum_t M_t + B - \sum_t L_t \right)^+ - \Psi \left(\sum_t L_t - M_t - B \right)^+ - \sum_t U_t M_t \tag{10}$$

where U_T is the end of year price for the credits (think of the first term as last-minute purchases to cover liabilities) and $\Psi > 0$ is the non-compliance fine for failing to have sufficient credits to cover total annual emissions. The final term in (10) represents the cash flows for sales or purchases during the year. Now consider a few different trading and acquisition strategies for obtaining these credits.

Strategy A (Proportional Allocation): Allocate the existing credits and make purchases or sales in each period such that the credit available is exactly the amount of carbon emission liability incurred during the period in question. For this strategy, one needs to decide B_t for each time-period t such that the given total allocation $B = \sum B_t$, and with remaining credits purchased or sold to ensure a monthly netting to zero, so that:

$$M_t = L_t - B_t, \quad t = 1,...,T \tag{11}$$

A simple rule would be $B_t = B/12$, assuming monthly time periods. Alternatively, available credits B could be allocated to each month proportional to the expected liabilities incurred in each month.

Strategy B (Dynamic Programming for Assumed Emission Liabilities): A fully optimal approach to dealing with procurement and allocation of emission permits is complex because of the interactions of the carbon-emission credit/acquisition problem with the portfolio procurement problem. To get some insights on the structure of the optimal solution for carbon emissions trading and permit procurement problem, we solve a version of the problem in which we assume that a stochastic set of emissions liabilities are given exogenously and these must be covered by either banked or allocated emissions permits or purchases during the year. The natural set of liabilities to assume are those corresponding to the particular portfolio, chosen from the efficient frontier, determined by the optimization in section 3. We will see that for the formulation noted below, all that is needed to solve for the optimal timing of purchases is the expected value of total emissions in each month, corresponding to the portfolio chosen, and the correlation of total monthly liabilities with the spot price for carbon. These statistics are readily available from the portfolio optimization itself.

Besides the stochastic process of carbon emissions over time, we need also to have a model for carbon futures price. The optimization procedure below would work in general for any auto-regressive model of such prices. To be specific, we will illustrate this for the case where the carbon futures price is a mean-reverting process of Ornstein-Uhlenbeck form, which is a reasonable representation of price movements for many commodities (see Geman (2005)). Thus, suppose:

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$$dU = \alpha(\mu - U)dt + \sigma dw \tag{12}$$

For this process, the mean and variance at time T will be

$$E(U(T)) = U(0)e^{-\alpha T} + \mu(1 - e^{-\alpha T})$$
$$Var(U(T)) = (1 - e^{-2\alpha T})\frac{\sigma^2}{2\alpha} \tag{13}$$

Note that the variance is increasing with respect to time T. In the long term, it converges to $\sigma^2 / 2\alpha$. The speed of convergence depends on the mean-reverting speed α. In particular, note that for any given parameters, it is not difficult to use Monte-Carlo simulation to find the related portfolio variances for the problem (1)-(2), (6)-(7), with (12) describing the evolution of prices for carbon credits.

We assume (as is the case in the EU) that the non-compliance fine Ψ in (10) is large relative to U_{sD} so that it will be optimal to ensure that all liabilities are met at the end of the year, if not sooner, with last-minute purchases M_T as needed. We formulate this in terms of quadratic penalties from specific targets B_t^* for ending credit balances in each of the periods $t = 1, \ldots, T$, with a marginal penalty parameter for the final period that is equal to Ψ at an ending credit balance of 0. The general thought here is that the intent is not to engage in speculative activity relative to the accumulation of carbon credits, but rather to cover emissions liabilities without excess exposures. Specifying targets for ending credit balances thus serves the dual purpose of planning for the accumulation of credits while controlling the risks of excessive long or short positions in credits.

The problem of interest then takes the form of a quadratic stabilization problem:

$$Max \quad E\left\{-\sum_{t=1}^{T}\delta^t\Lambda_{t+1}(B_{t+1}-K_t-B^*_{t+1})^2-\sum_{t=1}^{T}\delta^t U_t M_t\right\} \tag{14}$$

Subject to:

$$B_{t+1} = B_t + M_t ; B_1 = B \tag{15}$$

$$K_t = K_{t-1} + L_t ; K_0 = 0 \tag{16}$$

$$U_{t+1} = U_t + \alpha(\mu - U_t) + \sigma Z_t ; U_0 \text{ given} \tag{17}$$

In the above, δ is the period (i.e. monthly) discount rate, B_t represents the total allowances "on the book" at the beginning of period t and K_t represents the total (i.e., cumulative) emissions liabilities at the end of period t (after observing L_t). At the start of the problem, period 1, $B_1 = B$ equals the total allowances provided by the government plus any previous banked allowances of the company. Equation (16) represents the accumulation of emissions liabilities, so that the end of period accumulated credits net of the period target for each period t are given by $B_{t+1} - K_t - B^*_{t+1}$. As noted above, we assume for the purposes of the present problem that the distribution of L_t is known to the planner/trader solving the problem

(14)-(17). We will see below that only the mean of L_t and the correlation of L_t with Z_t are required for the solution.

The key parameters in the above problem are Λ_t and B_t^*. While these are arbitrary, it seems reasonable that the planner would assume Λ_t to be increasing over time and B_t^* to be decreasing. For example, the stabilization trajectory might be induced by the following assumptions:

$$0 < \Lambda_2 \le \Lambda_3 \le \ldots \le \Lambda_{T+1}; \qquad B_{t+1}^* = B_{T+1}^* + \frac{(T-t)B}{12}, t = 2,\ldots,T \tag{18}$$

With the parameter Λ_{T+1} in (14) set so that the marginal opportunity cost of decreasing ending credits when the credit balance $B_{T+1} - K_{T+1} = 0$ is the fine Ψ, i.e. $-2\Lambda B^* = -\Psi$, or

$$\Lambda = \frac{\Psi}{2B^*} > 0 \tag{19}$$

Equation (17) is the discrete form of a mean-reverting process for U_t, the end-of-month spot price for the year-end futures contract, where $\{Z_t | t = 1,2,\ldots,T\}$ is a family of independent standard normal random variables, μ is the long-run mean of the process, $\alpha > 0$ is the speed of adjustment, and σ is the volatility.

The timing of decisions in the problem is this. B_t, K_t, U_t are all observed prior to making the choice of M_t at the end of each of the T periods. Applying dynamic programming (DP) to this problem, we have the following recursion relationship for the optimal return function:

$$\begin{aligned} &V_t(B_t, K_t, U_t) = \\ &\quad Max\, E\left\{-\Lambda_{t+1}(B_{t+1} - K_t - B_{t+1}^*)^2 - U_t M_t + \delta V_{t+1}(B_{t+1}, K_{t+1}, U_{t+1})\right\} \Big| M_t \in \Re \Big\} \\ &\quad = Max\left[E\left\{-\Lambda_{t+1}(B_t + M_t - K_t - B_{t+1}^*)^2\right\} \right. \\ &\quad\quad \left. + E\left\{-U_t M_t + \delta V_{t+1}(B_t + M_t, K_t + L_{t+1}, U_t + \alpha(\mu - U_t) + \sigma Z_t)\right\} \Big| M_t \in \Re \right] \end{aligned} \tag{20}$$

where the ending value function at period $T+1$ is given by $V_{T+1} = 0$. Since $V_{T+1} = 0$ is clearly concave in B_T, and since all the dynamic equations involved are linear, it is straightforward to show by induction that the optimal return function is concave in B_t (and therefore in M_t) and continuous for all t. Indeed, the solution to this linear-quadratic problem is well known and is summarized below. Let us first illustrate the nature of the tradeoffs involved by considering the solution to the problem at period T. From (20), we have:

$$V_T(B_T, K_T, U_T) = Max\ E\left\{-U_T M_T - \Lambda_{T+1}(B_T + M_T - K_T - B^*_{T+1})^2\right\} \quad (21)$$

which is easily solved for the optimal M_t to yield the final year-end sale/purchase of

$$M_T^* = \frac{-U_T}{2\Lambda_{T+1}} - \left(B_T - L_T - B^*_{T+1}\right) \qquad M_T^* = -\frac{U_T}{2\Lambda_{T+1}} - (B_T - K_T - B^*_{T+1}) \quad (22)$$

so that, substituting this in (20), the optimal solution value at T is:

$$V_T(B_T, K_T, U_T) = U_T(B_T - K_T - B^*_{T+1}) + \frac{(U_T)^2}{4\Lambda_{T+1}} \quad (23)$$

The general solution to (20) for $t = 1, 2, \ldots, T-1$ is the following:

$$M_t^* = \frac{\delta\left[U_t + \alpha(\mu - U_t)\right] - U_t}{2\Lambda_{t+1}} - \left(B_t - K_t - B^*_{t+1}\right); \quad t = 1, \ldots, T-1 \quad (24)$$

$$V_t(B_t, K_t, U_t) = U_t(B_t - K_t - B^*_{t+1}) + R_t(U_t); \quad t = 1, \ldots, T-1 \quad (25)$$

where $R_t(U_t)$ is computed recursively as:

$$R_T(U_T) = \frac{(U_T)^2}{4\Lambda_{T+1}}$$

$$R_t(U_t) = \frac{\delta[U_t + \alpha(\mu - U_t)] - U_t}{4\Lambda_{t+1}} \qquad (t = 1, 2, \ldots, T\text{-}1)$$
$$+ \delta[U_t + \alpha(\mu - U_t)]\left[B^*_{t+1} - B^*_{t+2} - E\{L_{t+1}\}\right]$$
$$- \delta\sigma E\{Z_{t+1} L_{t+1}\} + \delta E\{R_{t+1}(U_{t+1}) | U_t\}$$

(26)

The proof of (23)-(26) follows an induction argument (e.g., Kleindorfer & Kleindorfer, 1968). For $t = T\text{-}1$, one substitutes V_T from (23) into (20) and readily verifies that (24)-(26) is the result at $T\text{-}1$. Assuming that (24)-(26) holds for $t = T\text{-}1, \ldots, t+1$, one then verifies by substitution of the assumed form of $V_{t+1}(B_{t+1}, K_{t+1}, U_{t+1})$ into (20) that this induction hypothesis verifies the solution (24)-(26) also at t, completing the proof.

To understand the optimal solution (22) and (24) intuitively, assume $\delta = 1$ (as the monthly discount factor, it will obviously be close to 1). Then we see in (24) that, if $U_t = \mu$, the optimal rule calls for adjusting M_t roughly so as to hit the monthly ending target of credits B^*_{t+1}, net of accumulated sales/purchases and emissions liabilities. However, if $U_t < \mu$, then one can expect an upward trend in the price of the futures contract in the period ahead, and this leads in the optimal solution to an M^*_t that overshoots the end-of-month target, with the amount of optimal overshoot depending on the stabilization cost parameter Λ_{t+1}. An opposite (undershoot) effect occurs when $U_t > \mu$. Thus, the optimal rule tracks the desired stabilization path $\left\{B^*_t \,\middle|\, t = 1,\ldots,T\right\}$, with some departures in monthly sales and purchases of emission credits depending on the realized price of the end-of-year futures contract. While the above process is clearly grounded on the mean-reverting process (12), other processes would yield similar conditions in terms of tradeoffs between current price opportunities in the market and the need to accumulate sufficient emission credits to meet expected year-end obligations.

Finally, we note also that this optimization process for the timing of sales and purchases of carbon credits was based on the demands for carbon certificates generated through the instrument optimization problem in Section 3. However, based on the stabilization framework proposed in (14)-(17), the structure of the optimal solution for the timing of carbon credit purchases depends only on the mean and the correlation of the emission liabilities L_t with carbon prices, which can be easily generated as an output of the efficient frontier problem (1), (6)-(7). Thus, our heuristic of separating optimal VaR-constrained portfolio selection from the timing of carbon credit sales and purchases is consistent with the optimization framework proposed.

5. Implications, Conclusions and Future Research

The above introduction to carbon trading raises a number of questions associated with regulation and with energy company strategies, including new risk management competencies and the impact of longer-term technology mix choices that are likely to be increasingly important in a carbon economy. Let us just note a few of these here as a prelude to future research on this topic.

We can begin by noting that the above portfolio approach (1)-(2) & (6)-(7), as in the original Kleindorfer and Li (2005) paper, is an open-loop approach and needs to be extended to include dynamic consistency considerations and updating (see Geman and Ohana, 2008). The additional heuristic embedded in the two-stage approach analyzed in this paper is to separate out the optimization and risk management of the cash flows associated with instrument choice and dispatch from the timing of decisions associated with the management of carbon credits. Of course, in the initial portfolio problem, the cost and interdependent risks of carbon liabilities is accounted for, but a fully optimal joint optimization of both instrument choice and the timing of carbon credit purchases and sales is not envisaged above. Nonetheless, it seems reasonable to us to consider at an initial stage the optimization of the instrument portfolio (properly accounting for carbon costs and risks at their market value) as the continuing dominant problem for planning, dealing with timing or the associated purchases of carbon credits as a separate, follow-on problem. This judgment is based in part on the relative magnitudes of the cash flows likely to be associated with these two decision domains (with the main act remaining instrument choice and dispatch). Nonetheless, further research may find better ways of integrating these two choices than our initial proposal here.

As a second immediate point, even this heuristic approach points to some fundamental challenges for strategy and technology planning in electric power. These include integrating "carbon competencies" with strategy and risk management, and long-term planning of technology choices (both in terms of owned generation as well as leased and long-term PPA). It should be clear from the above description that the price of carbon emissions should and will have a fundamental impact on the appropriate technology mix. We have posed the above problem in the context of a fixed set of available technologies, but stepping back from this, with an eye on the carbon intensity of such technologies and the projected evolution of carbon credit prices gives rise to an interesting problem of diverse technology planning. In contrast to the original work on optimal technology mix of Crew and Kleindorfer (1976), the problem here is to select an optimal set of operating technologies from a group of technologies with specified capital and operating costs (and perhaps reliability features as well). However, in the carbon context, the operating cost now includes additional uncertainties embodied in (3) associated with the price of carbon (imputed back on a per MWh basis to the underlying technology). This decision is obviously a key implication of the move to cap and trade systems, and involves not just market-related risk but also political and regulatory risks that could influence the price of carbon credits and the allocation of these credits to power companies. These have yet to be well quantified.

Valuation and accounting matters related to banking of allocated or accumulated permits will be a central matter as well. Valuation and risk management for PPA's and lease contracts will also price the carbon risk in contracts, likely giving rise to performance-based risk sharing of carbon emissions as part of the overall negotiated price structure for these traditional power contracting arrangements.

Finally, regulatory issues associated with transitioning from green certificates, RPS, REFIT tariffs, white certificates, etc to a cap and trade system and the new carbon economy will require clear thinking and research to avoid double counting and unintended consequences.

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