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Managing Satisfaction in Relationships over Time

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by

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Abstract

Consider a firm that has the flexibility to actively manage and customize the service offered to its customers in a repeat business context. What is the long-term value of such flexibility, and how should firms manage the service relationship over time? We propose a dynamic model of the firm--client relationship that relies on behavioral theories and empirical evidence to model the endogenous evolution of service expectations and customer satisfaction, as well as their impact on repurchase decisions. In general, we find that firms can extract higher long-term value by managing service experiences and expectations over time. Varying service in the long run is not optimal, however. We characterize the firm's optimal dynamic service policy and show that it converges over time to a steady-state service level. Loss aversion expands the range of constant optimal service policies, suggesting that behavioral asymmetries limit the value of responsive service. Our results provide insights for service suppliers seeking to leverage customer-level data and service flexibility in order to prioritize clients and improve long-term performance.

Keywords: Service Management; Quality; Satisfaction; Managing Relationships; Managing Expectations

1. **Introduction**

Largely facilitated by recent advances in information technology, the shift from managing transactions to managing customer relationships has become an important part of corporate strategy in both business-to-customer (B2C) markets. Companies such as Harrah's Entertainment and Yves Rocher use customer-level data to develop service delivery strategies that are finely tuned to customer needs in order to increase customer equity and long-term profitability. Traditionally product-focused firms, such as IBM and Roche, are now focused increasingly on building service relationshipsby offering customized support and maintenance contracts to their business clients. In such repeated interaction settings, clients' assessment of the value of the relationship and subsequent repatronagedecisions depend largely on prior experiences with

the firm and subsequent expectations.

How should firms manage service delivery and customer experience over time in order to capture the most value from a customer relationship? In particular, when is it desirable for the firm to use responsive service strategies as opposed to maintaining a constant service level? Which customers are more valuable in this context, and how should the firm prioritize service delivery? This paper aims to provide guidance for firms that seek to use responsive service strategies and knowledge of individual customers in order to improve long-term profitability by actively managing service and satisfaction in relationships over time.

We propose a behavioral dynamic model of a single firm–client relationship in which the firm's objective is to maximize expected long-run discounted profit from the client relationship—that is, customer lifetime value (CLV). In each period, the provider decides what service level to offer the customer. Improving service is costly, but it increases customer satisfaction and retention rate. We rely on behavioral decision theories (adaptive expectations, prospect theory) and empirical evidence from the marketing literature to model realistic effects of service experiences on satisfaction as well as their impact on repurchase decisions.

Although practitioners recognize the importance of understanding and responding to customer behavior in repeated service interactions, "surprisingly little time ... has been spent examining service encounters from the customer's point of view" (Chase and Dasu 2001). "Every year companies have thousands, even millions of interactions with human beings, also known as customers. Their perceptions of an interaction are influenced by ... the sequence of painful and pleasurable experiences. ... Yet the application of behavioral science to service operations seems spotty at best" — a recent McKinsey study reveals (DeVine and Gilson 2010).

Our research addresses this practical need by drawing on the behavioral literature to provide prescriptive models for managing service encounters over time. From a modeling perspective, our work fills a gap at the interface of the service operations and marketing literatures by addressing the need (i) to "incorporate findings from psychology and marketing into OM models of service

management" (Bitran et al. 2008, p. 80) and (ii) to develop dynamic models of customer lifetime value in marketing (Rust and Chung 2006, Table 1).

Our contribution to the literature is three fold: (i) we propose the first dynamic model of managing service relationships that endogenizes retention and incorporates realistic customer behavior; (ii) we characterize the structure of the firm's optimal policy in the long run and also in a transient regime; and (iii) we explain how customer behavior affects the firm's policy and profits and how it should prioritize business.

We define responsive service broadly as the level of extra effort that the firm expends on retaining the customer; this can be any costly driver of customer satisfaction (excluding price) that the firm can customize and control in a responsive manner. Examples include sales-force effort or response time (e.g for insurance or service contracts), personalized offers and gifts (e.g. Yves Rocher or DBS bank), impressions (or make-goods) delivery for advertising contracts or, for online information providers, content relevance. For lifestyle memberships, service consists of customized information and access to special deals and events. At Harrah's, responsive service refers to a range of complimentary benefits, known as 'comps' (e.g., free room, shows, chips, faster lines), which are fine-tuned to customer needs to improve retention and create customer goodwill.

We rely on behavioral theories and evidence from the marketing literature to model the effect of service on satisfaction and repatronage behavior. Repurchase probability is modeled as a general increasing function of satisfaction with the service provider (there are no assumptions on its shape). Customer satisfaction depends on prior service experiences with the firm and follows an adaptive expectations process that is consistent with behavioral theories on belief formation (Hogarth and Einhorn 1992) and empirical findings (Bolton 1998). Our basic model focuses on an exponential smoothing process for satisfaction updating, and it extends to capture more complex, non linear, and asymmetric effects, such as loss aversion. Prospect theory (Tversky and Kahneman 1991) predicts that customers are loss averse—that is, more sensitive to perceived downgrades than upgrades in service—as widely evidenced in the satisfaction literature (Boulding et al. 1993; Bolton 1998).

In this context, we show that firms can increase CLV by appropriately adjusting service and managing customer expectations at a tactical level. However, we find that it is not optimal in the long run to oscillate service level. The optimal dynamic service policy converges to an ideal long-run service level from which it is suboptimal to deviate. This steady state is lower than the optimal static service policy; it is also lower the more the firm is focused on the short term and the more consumers anchor on past experiences. Loyalty has an inverse U-shaped effect on optimal service levels: the firm's incentive to spend on inherently sticky customers diminishes after a point.

It is interesting that behavioral asymmetries drive the structure of our results and also limit the benefits of responsive service. Loss aversion leads to a range of optimal constant policies that becomes wider as the adaptation process becomes more asymmetric. In contrast, if consumers are "gain seeking" (i.e., if service levels above expectations are more salient) then the service policy oscillates.

For firms that do not manage service experiences over time, there is a unimodal (inverse U-shaped) relationship between satisfaction and profitability. By contrast, we find that service flexibility always allows firms to extract more value from customers who are more satisfied, without necessarily offering them better service. The optimal policy should maintain a consistent satisfaction ranking across customers over time.

We establish that firms in a transient regime should either increase or decrease satisfaction monotonically, depending on initial consumer expectations. Yet the sequence of service levels need not be monotonic unless retention is a convex function of satisfaction (e.g., in more competitive markets; Jones and Sasser 1995). In this case we find, as in Ho et al. (2006), increasing marginal long-run returns to satisfaction. In particular, it is more effective to increase satisfaction for consumers who are already more satisfied, and the more satisfied customers receive better service. The opposite need not hold even if the relationship between satisfaction and retention is concave. These results suggest that, in order to prioritize customers, it is important for firms to understand the shape of the retention function.

The robust structure of a policy based on satisfaction suggests that firms should focus on satisfaction, rather than service, as a metric for managing relationships over time. In sum, our results show how service suppliers can leverage their customer-level data and service flexibility to increase retention and improve long-term performance by gradually managing service expectations.

2. Related Literature and Customer Behavior Model

This section reviews related work and then develops our customer behavior model, which is based on psychology and marketing literature.

2.1. Related Literature

Our work emerges from the interface of a relatively large literature on satisfaction and service-relationship management in marketing, reviewed by Zeithaml (2000) and Rust and Chung (2006), respectively, and the growing literature in behavioral operations; see Ferrer and Rocha e Oliveira (2008) and Loch and Wu (2007) for reviews.

Several marketing frameworks capture the trade-off between cost and customer retention, including the customer lifetime value (CLV) framework (Rust et al. 2004; Venkatesan and Kumar 2004), the return on quality (ROQ) framework (Rust et al. 1995), and the customer equity framework (Blattberg and Deighton 1996).

Few papers consider optimal investment decisions in customer satisfaction. Ho et al. (2006) model customer purchases as a Poisson process whose rate depends on customer satisfaction, and the probability p that a customer is satisfied in a given period is controlled by the firm. Focusing on static policies, these authors find that customer value is increasing and convex in satisfaction. However, their model neither captures the evolution of customer experiences over time nor its endogenous effect on retention. Liu et al. (2007) consider the problem of inter-temporal allocation of a limited capacity (sales-force effort) to a customer who forms adaptive expectations based on prior service experiences. These expectations affect short-term profit but not retention. The focus on allocating an exhaustible resource over a limited time period renders their model and insights inherently different from ours.

There is a growing operations literature that examines endogenous models of demand in repeated interaction settings. In this context, customers form adaptive expectations based on the firm's past policies, including pricing (Popescu and Wu 2007; Nasiry and Popescu 2009), capacity (Liu and van Ryzin 2009), and quality (Caulkins et al. 2006). Closest to our work is that of Gaur and Park (2007), who derive steady-state results in an oligopoly where loss-averse customers form adaptive expectations about retailers' fill rates. Their asymmetric adaptation model is similar to our model in Section 7.1. These authors find that loss aversion leads to lower industry profits (consistent with our findings) and higher service levels (an effect of strategic interaction).

In the service operations literature, Gans (2003) characterizes the firm's optimal stationary service policy in an oligopoly when customers respond in a Bayesian fashion to any change in service. Our model is different in that it leverages the firm's service flexibility by analyzing dynamic service policies in the absence of strategic interactions. In our model, competition is captured indirectly, through the retention function. In a duopoly setting, Hall and Porteus (2000) consider a finite-horizon dynamic model where demand is a function of "service failures", which are determined by the firm's investment in capacity. Their customers are purely reactive in their switching behavior, with no memory of experiences prior to the current period. In contrast, our work endogenizes retention by explicitly modeling customers' evolution of satisfaction.

Two combined features distinguish our work from this literature. First, we draw on behavioral theories—such as adaptive expectations and loss aversion—to endogenize retention based on past service experiences in a dynamic service-satisfaction framework. Second, we go beyond steady-state analysis to investigate the dynamics of customized service decisions over time, which are critical for managing individual customer relationships.

2.2. The Customer Behavior Model

The effect of variations in service on customer lifetime value is more pronounced in B2B contexts as well as "continuously provided services", in which case revenue streams are closely tied with retention. In such settings, customers dynamically adjust their perception of service quality based

on recent service encounters, and these perceptions ultimately determine repurchase behavior. This section models the relationships among service, satisfaction, and retention based on evidence from behavioral sciences and from a vast marketing literature on the antecedents and consequences of satisfaction (see Anderson and Sullivan 1993).

2.2.1. The link between service and satisfaction. In the marketing literature, "customer satisfaction" refers to a dynamic construct, formed after each service encounter (Bolton and Drew 1991a), as a function of the customer's immediate *perception* of service and her service *expectation* (see Tse and Wilton 1988 and the references therein).[1] We assume that customer satisfaction reflects previous experiences with the firm, which are used as a basis to form endogenous expectations about what will happen in the next service encounter. Satisfaction is then updated based on perceived service levels.

The most common model of satisfaction updating is an adaptive expectation model, where satisfaction s_{t+1} is a weighted average of past satisfaction s_t and current service x_t. For simplicity we assume that both service x and satisfaction s are measured in percentage terms: $x, s \in [0,1]$.[2] The following exponential smoothing model captures the essence of theoretical models on belief formation (Hogarth and Einhorn 1992) and satisfaction (discussed below); it is also the simplest model that renders the main insights from our framework:

$$s_{t+1} = \lambda s_t + (1-\lambda)x_t. \tag{1}$$

The adaptation parameter $\lambda \in [0,1)$ is the weight that the customer puts on prior experiences and beliefs in order to update her satisfaction level. Customers with lower λ focus more on more recent experiences; in particular, the satisfaction of the customers for whom $\lambda = 0$ is solely determined by the most recent service experience. Exponential smoothing models have been used extensively to

[1] Boulding et al. (1993) distinguish two types of expectations: *will expectations*, closely tied with the customer's previous experiences with the firm, refer to customer's expectations about what *will* happen in the next service encounter; *should expectations* reflect normative beliefs about what service levels *should* be, based on exogenous variables such as firm's reputation, industry standards, and expert opinions. Our focus is on the endogenous effects of the firm's service decisions and hence on *will* expectations.

[2] This allows us to capture a variety of service decisions, including the binary (service, no service) models typically used in the literature; in this case, x represents the fraction of time that service is offered over a given time period.

model adaptive expectations in an operational context (see, e.g., Caulkins et al. 2006; Gaur and Park 2007; Liu et al. 2007; Popescu and Wu 2007).

Cronin and Taylor (1992) propose a general model of satisfaction updating based on expectations anchoring and disconfirmation. In their model, a customer's next level of satisfaction s_{t+1} is a function of previous satisfaction s_t and *disconfirmation*, which is the gap between that customer's perception of service, $g(x_t)$, and the status quo: $s_{t+1} = H(s_t, x_t) = G(s_t, g(x_t) - s_t)$. A similar general model is proposed by Anderson and Sullivan (1993). In Section 8.1 we extend our basic model (1) to allow for such non linearities in anchoring and disconfirmation. Without loss of generality, the perception of service, g, is assumed to be the identity function; our results apply to any g that is both increasing and concave, reflecting diminishing marginal perception.

Our model (1), expressed as $s_{t+1} = s_t + (1 - \lambda)(x_t - s_t)$, is consistent with the anchoring and disconfirmation framework of satisfaction, where $1 - \lambda$ captures the effect of disconfirmation on satisfaction. Section 7.1 extends our model to capture asymmetries in disconfirmation motivated by prospect theory, notably loss aversion. That negative disconfirmation is more salient than its positive counterpart is consistent with behavioral theories (Tversky and Kahneman 1991) and empirical marketing evidence (Bolton 1998).

2.2.2. The link between satisfaction and retention. Longitudinal studies have established the positive effect of customer satisfaction on repatronage behavior (Bolton 1998; Bolton and Lemon 1999; Bolton et al. 2006). These authors suggest that customers form beliefs (based on prior satisfaction) about the value of a relationship, which leads to behavioral intentions of renewal and eventually to repurchase decisions. The relationship between satisfaction s and retention is captured by an increasing function, $F(s)$.

Most empirical studies in the marketing literature either find or assume diminishing marginal effects of satisfaction on retention (i.e., F concave). The most commonly estimated retention function is a logit choice model—for examples, see Zahorik and Rust (1993) for retail banking, and Rust et al. (2004) for airlines. Berger and Nasr (1998) use an exponential, whereas Bolton et al. (2006) estimate a double exponential in the context of information technology service contracts.

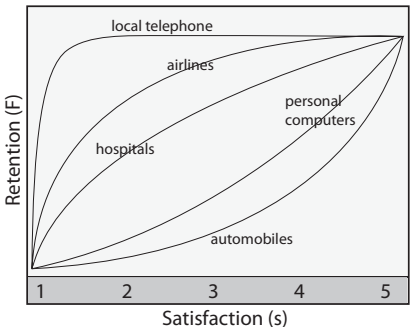

Figure 1 **"How the competitive environment affects the satisfaction–loyalty relationship" (Jones and Sasser 1995, p. 91).**

Jones and Sasser (1995) argue that the marginal effect of satisfaction on loyalty need not be diminishing and depends on the degree of market competition and customers' switching costs. Specifically, they find that F becomes more convex when the degree of competition in the market increases, as illustrated in Figure 1 (replicated from their paper). They cite evidence from Xerox, which found that increasing satisfaction ratings from 4 to 5 (on a 5-point scale) for its office products customers increased the retention rate by a factor of 6, significantly more than for lower-rating customers. Increasing marginal effects of satisfaction on loyalty (F convex) have also been observed in the automotive industry (Mittal and Kamakura 2001).

We make no structural assumptions on the retention rate F beyond that it is increasing; that is, more satisfied customers are more likely to renew, as evidenced in much of the marketing literature (see Zeithaml 2000). In section 7.2 we relax this assumption by investigating an alternative model in which value (and hence retention) is driven by disconfirmation, the gap between experience and expectation: $F(x - s)$. Such a model suggests that more satisfied customers are more difficult to retain because they become more demanding, consistent with the treadmill effect (Brickman and Campbell 1971) and with models of habituation (Baucells and Sarin 2010).

3. The Firm's Problem

Marketing research suggests that "retaining customers is a far more profitable strategy than gaining market share or reducing costs" (Zeithaml 2000), and goes so far as to recommend "separate marketing plans — or even building two marketing organizations – for acquisition and retention

efforts" (Blattberg and Deighton 1996). Our model focuses on maximizing net present value from the retention of existing customers (so-called defensive marketing), and does not consider acquisition and word of mouth-effects.

For simplicity of exposition we assume that, for each period during which the customer is active, the revenue (or margin) is fixed and equal to P. So the short-term profit from offering service level x is $\pi(x) = P - c(x)$, where c is an increasing convex cost of service. This model is consistent with service contracts in B2B and subscription services in B2C (e.g., insurance, information service providers, lifestyle memberships, credit cards, telecom, cable/TV).

In broader contexts, satisfaction may affect customer spending (Seiders et al. 2005). In the case of Harrah's, for example, satisfaction may not influence a customer's overall spending on gambling, but it can affect the share of his gambling budget spent with the company (and not with competitors). As discussed in Section 8.2, all our results extend without loss of generality to allow for the effect of satisfaction on customer spending $P = P(s)$— or, more general short-term profit models $\pi(x, s)$ — as long as the marginal cost of service is increasing. The assumption of convex costs, which is ubiquitous in the context of service operations, can be motivated by explicitly deriving costs in various service delivery systems, such as queueing systems or inventory problems (cf. Ho et al. 2006).

3.1. Dynamic Model

In our stylized model, the firm is fully informed about customer behavior (owing to extensive customer-level data), and has the capability to adjust service based on this information. The firm's objective is to maximize the long-term value of each customer by dynamically adjusting the customized service level x_t, in response to the endogenous consumer behavior process described in section 2.2. The main trade-off is between the short-term cost of providing high service and the long-term benefit of increasing retention and future revenue streams. Under the exponential smoothing model (1) for satisfaction updating, this problem can be formulated as a discounted dynamic program with the following Bellman equation:

$$J(s) = \max_{x \in [0,1]} \pi(x) + \beta F(\lambda s + (1-\lambda)x) J(\lambda s + (1-\lambda)x). \tag{2}$$

This is equivalent to an infinite horizon stochastic shortest path problem, with discount factor $\beta \in [0,1]$, representing how much the firm discounts future profits. Alternatively, β is a proxy for the frequency of service encounters (the further away the next purchase is, the more heavily it is discounted). Our model and results extend to capture non linear satisfaction formation mechanisms as well as the effect of satisfaction on purchase frequency and customer spending (or short-term profitability), as shown in Section 8.

LEMMA 1. *The value function $J(\cdot)$ is increasing, and $P \leq J(s) \leq \frac{P}{1-\beta}$ for all $s \in [0,1]$.*

Lemma 1 follows as a direct consequence of the satisfaction–retention relationship F being increasing (Bolton 1998). This result shows that a firm with the flexibility to vary service level optimally over time is always able to extract more value from customers who are ex ante more satisfied, thereby realizing customers' full profit potential. This is a benefit of service flexibility, as the profitability relationship need not be monotone for firms that keep service and satisfaction constant over time (Blattberg and Deighton 1996), as discussed in the next section.

3.2. Static Benchmark Model

If a firm does not have the flexibility to change service level x over time, then the long-term profit from maintaining a constant service level x (consistent with customer expectations) is given by the well-known CLV formula (Rust et al. 2004):

$$\Pi(x) = \sum_{t=0}^{\infty} \beta^t F^t(x)\pi(x) = \frac{\pi(x)}{1-\beta F(x)} = \pi(x)L(x). \tag{3}$$

We use $L(x) = 1/(1-\beta F(x))$ to denote the *expected lifetime* of the customer, given a constant service level x; $L(x)$ is increasing in x. If we assume unit demand in each period, then $L(x)$ can be interpreted as the expected total demand from the customer over her lifetime. The *expected customer lifetime value* given a constant satisfaction level x, $\Pi(x)$, is the product of the short-term profit $\pi(x)$ of each active customer and her expected lifetime $L(x)$. Because π is decreasing and L is increasing, Π is typically not monotone.

The marketing literature (see Rust et al. 1995) provides ample evidence of the diminishing marginal returns to increasing satisfaction (i.e., that Π is concave), so Π has a unique maximizer $\bar{s}$.

To guarantee uniqueness of optimal solutions throughout the paper, we make the following milder assumption.

ASSUMPTION 1. *$c'(s)/L'(s)$ is strictly increasing in s.*

Assumption 1 is satisfied for common parametric models used in the literature, such as logistic $F(x) = 1/(1+\exp(-\alpha x))$ or exponential $F(x) = 1-\exp(-\alpha x)$ retention and for power cost $c(x) = x^{1+\theta}$ with $\theta \geq \alpha$. Other assumptions that guarantee uniqueness of optimal solutions throughout the paper are strict concavity of Π or alternatively, strict quasi-concavity of Π and $\lambda\pi + (1-\lambda)\Pi$.

Assumption 1 ensures that Π has a unique maximizer $\bar{s}$, which is the optimal static service level. In order to characterize this level, define the marginal cost per marginal customer lifetime as

$$C(x) = \frac{(c(x)L(x))'}{L'(x)} = c(x) + c'(x)\frac{L(x)}{L'(x)} = c(x) + c'(x)\frac{1-\beta F(x)}{\beta F'(x)}. \tag{4}$$

LEMMA 2. *There exists a unique optimal static service level $\bar{s} \in [0,1]$ that solves: (a) $P = C(x)$ as given by (4) when $P \in [C(0), C(1)]$, (b) $\bar{s} = 1$ for $P \geq C(1)$, and (c) $\bar{s} = 0$ for $P \leq C(0)$.*

Rewriting $P = C(x)$ as $(c(x)L(x))' = (PL(x))'$ reveals that the optimal static service level balances the long-run marginal cost and marginal revenue of offering service, which is consistent with the standard microeconomics insight. An implication of Lemma 2 is that Π is quasi-concave and, in particular, non monotone for $P \in [C(0), C(1)]$, consistent with marketing literature (Rust et al. 1995; Blattberg and Deighton 1996).

4. Long-Run Policy

Consider a firm that offers a constant service level $s_t \equiv \bar{s}$ (as determined by Lemma 2) in each period, so that customer expectations are anchored to $\bar{s}$. Is it possible for the firm to extract more profit in the long run from this customer by varying service? If so, how?

Figure 2 suggests that the firm can improve profits (by at least 10% relative to the static optimal policy $\bar{s} = 0.7$) simply by reducing the service quality in the first two periods and offering higher service in the long run. Observe that this policy makes both the firm and the consumers better off over the long run, though at the expense of a short-lived reduction in service. Alternatively, the

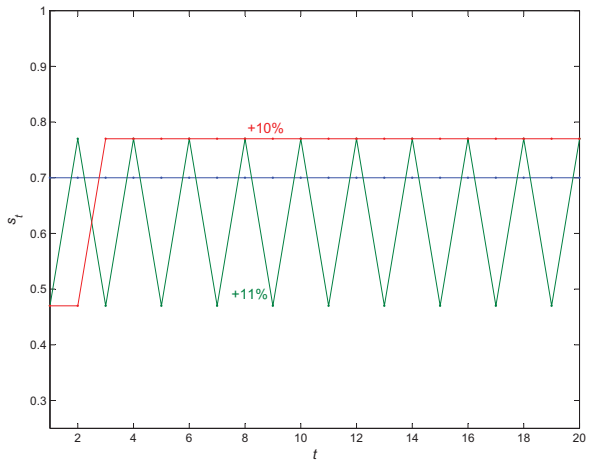

Figure 2 **The value of flexibility in service.** $F(s) = s^{0.9}$, $c(x) = x^2$, $\lambda = 0.8$, $\beta = 0.94$, $P = 1$.

firm can obtain even higher profits (an 11% improvement) by adopting an oscillating, high–low service policy; this is illustrated by the zigzag path in Figure 2.

The example indicates that firms can indeed do better by varying service, even if only temporarily. It remains for us to establish whether these insights are robust and what the optimal service policy actually looks like. In particular, the following questions should be addressed. (i) Is it optimal to oscillate between high and low service in the long run, or is there an ideal long-run service level for which the firm should aim? (ii) Is it always possible for the firm to improve profits by varying service in the short run (as compared with implementing a constant service policy), and (iii) when can the firm do so, while also providing better service to the customer in the long run? We provide formal answers to these questions in what follows.

4.1. Steady State

This section characterizes an ideal long-run service level from which it is suboptimal to deviate. By definition, s^{**} is a steady state if the firm has no incentive to move away from it—that is if it is a fixed point of the optimal service policy: $x^*(s^{**}) = s^{**}$, where $x^*(\cdot)$ is the policy that optimizes (2). In other words, if customers' satisfaction is in steady state, s^{**}, then it is optimal for the firm to offer $x = s^{**}$ in each period.

Now consider the memory-adjusted marginal cost of service per marginal lifetime value:

$$C(s;\lambda) = c(s) + \left(\frac{\lambda}{1-\lambda} + L(s)\right)\frac{c'(s)}{L'(s)} = c(s) + c'(s)\frac{(1-\beta F(s))(1-\lambda\beta F(s))}{\beta(1-\lambda)F'(s)}, \tag{5}$$

where $C(s;0) = C(s)$ as defined in (4). Because $c(s)$ and $L(s)$ are increasing, Assumption 1 guarantees that $C(s;\lambda)$ is strictly increasing in s therefore, $\underline{C} = C(0,\lambda) < C(1,\lambda) = \overline{C}$.

The next result provides necessary conditions for a steady state to exist. Existence and global stability are subsequently confirmed in Proposition 3.

Proposition 1. *If problem* (2) *admits a steady state* s^{**}*, then this is unique and: (a)* s^{**} *is the unique solution of* $P = C(s;\lambda)$ *if* $\underline{C} < P < \overline{C}$*; (b)* $s^{**} = 1$ *if* $P \geq \overline{C}$*; (c)* $s^{**} = 0$ *if* $P \leq \underline{C}$*. Furthermore,* s^{**} *is decreasing in* λ *and increasing in* β *and* P*.*

Proposition 1 relates the firm's long-run service policy to the firm's strategic outlook as well as to marketing (price) and operational (cost) factors. A firm with a short-term outlook puts less weight β on future cash flows (e.g., $\beta = 0$ for a fully myopic firm), and provides lower long-run service and satisfaction, because it focuses on (short-term) cost savings. All else equal, steeper costs or lower prices reduce the firm's incentive to invest in service and customer satisfaction.

Our results quantify the relationship between service level and contract price or customer spending, and they provide bounds on revenue indicating what it is worth for the firm to spend on retention; see Figure 3. Some firms, such as the Ritz-Carlton, strive for full service and 100% satisfaction across the board (Jones and Sasser 1995) whereas other businesses, such as Facebook and Google's Nexus phone, offer minimal customer service (Cachon and Terwiesch 2010). Our results qualify the profitability of such practices by providing bounds on revenue for extreme policies to be optimal in the long run. Specifically, below a relatively low price ($P \leq \underline{C} \geq c(0)$), the firm should not increase its service expenditures (i.e., $s^{**} = 0$). But for sufficiently high margins ($P \geq \overline{C} \geq c(1)$), the firm should aim to maximize retention by offering full service over the long run.

Proposition 1 explains how the firm's long-run policy is affected by customer behavior— in particular by adaptation (or memory, λ) and loyalty (F), as discussed next. The effect of behavioral factors on profitability J is addressed in Section 6.2.

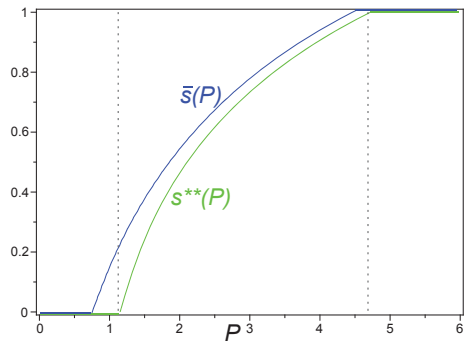

Figure 3 **Steady state s^{**} and optimal static service $\bar{s}$ as functions of price P, where** $F(s) = 1/(1 + \exp(-3s))$, $c(x) = x^2 + x$, $\lambda = 0.5$, $\beta = 0.94$; $\underline{C} = 1.15$, $\overline{C} = 4.7$.

The firm provides higher long-run satisfaction to customers who are more adaptive (or forgetful), i.e. those who anchor on more recent experiences, captured by a lower λ. In particular, if satisfaction is determined solely by the most recent service encounter ($\lambda = 0$), then the steady state coincides with the optimal static service level: $s^{**}(\lambda = 0) = \bar{s} = \arg\max_{s\in[0,1]} \Pi(s)$. In general, the steady-state service level $s^{**}(\lambda)$ is always lower than the static optimal policy $\bar{s}$; see Figure 3. This follows because $C(s;\lambda)$ is increasing in λ, since $\bar{s}$ solves $P = C(s) = C(s;0)$ and s^{**} solves $P = C(s;\lambda)$.

Remark 1. $s^{**} \leq \bar{s}$.

In order to understand the effect of loyalty on the steady state, consider a parametric logit model $F(s;\alpha) = 1/(1 + \exp(-\alpha s))$, the most common retention model estimated in the literature. The retention rate $F(s;\alpha)$ increases with α, which acts as a proxy for customer loyalty. Figure 4 shows that the steady state is non monotone in α: it increases up to a certain point but then decreases. This suggests that there is an "ideal" loyalty level beyond which the firm will treat the customer as a captured audience and hence reduce costly investment in retention.

In this model, α also determines the degree of concavity of F, which is linked to the level of competition in the market environment (as argued in Jones and Sasser 1995). This suggests that, from the customers' point of view, there is an optimal level of market competition that results in the highest steady-state service level. These insights are partially consistent with Hall and Porteus (2000), who find that loyalty decreases long-run service levels in an oligopoly. Our model captures competitive effects implicitly, through customer choice, but does not account for strategic

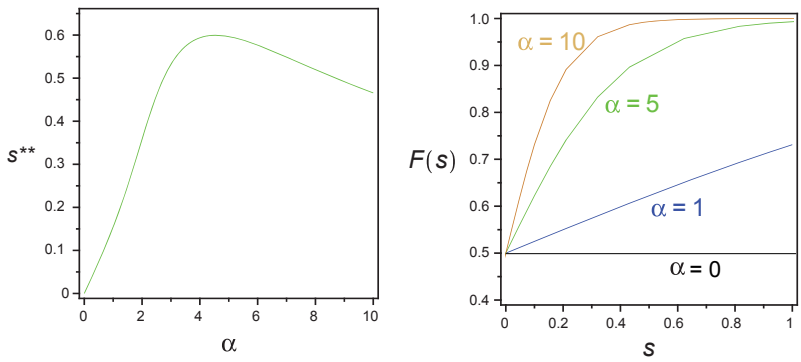

Figure 4 **The effect of loyalty α on steady state service s^{**} and retention F;** $F(s;\alpha) = 1/(1+\exp(-\alpha s))$, $c(x) = x^2$, $\lambda = 0.5$, $\beta = 0.94$.

interaction.

Corollary 1. *The steady state s^{**} of problem* (2) *maximizes over $s \in [0,1]$ the function:*

$$W(s) = \lambda\pi(s) + (1-\lambda)\Pi(s). \tag{6}$$

This result shows that an alternative to Assumption 1, which ensures uniqueness of the steady state, is that W be strictly quasi-concave (in particular that Π be strictly concave). Corollary 1 suggests that in steady state the firm balances short term-profit (π) and long-term customer value (Π), as weighed by customer memory λ, by solving $\lambda\pi'(s) + (1-\lambda)\Pi'(s) = 0$. From a technical standpoint, W as defined in (6), is an important construct for problem (2) because it is a Lyapounov function.[3]

4.2. Limited Flexibility

In Section 4.1 we characterized a unique service level s^{**} from which it is suboptimal to deviate. We now show that, unless $s_0 = s^{**}$ (i) the firm can always do better by varying service in the short run (first two periods); and, moreover, (ii) a policy can always be devised where both firm and customer are better off in the long run, at the expense of a short disturbance in service. These results validate the robustness of the example presented in Figure 2.

[3] Specifically, W describes a "hill" that solutions to (2) are always climbing, with s^{**} at the top, which ensures global stability (see the proof of Proposition 3). In general, Lyapounov functions are hard to come by; "there is no way other than unsystematic ingenuity to find them" (Stokey et al. 1989, p. 140).

The long-term expected customer profit associated with a given service path $\wp = \{x_t, t \geq 1\}$, as a function of the prior satisfaction level s_0, is denoted $J^{\wp}(s_0)$. In particular, for the path $x_t \equiv x = s_0$ we have $J^{\wp}(x) = \Pi(x)$.

PROPOSITION 2. *For any $s_0 \neq s^{**}$, there exists a service path $\wp = \{x_1, x_2, x_t = s_0, t \geq 3\}$ that is more profitable than the constant service path $\{x_t = s_0, t \geq 0\}$, i.e. $J^{\wp}(s_0) > \Pi(s_0)$.*

This result shows that, if expectations are not in steady state, then the firm can obtain higher profits by varying service temporarily (over the first two periods). Moreover, the result extends for strategic transitions to a prespecified service level $\mathrm{x} = x_t$, $t \geq 2$. For all but finitely many values $\mathrm{x} \neq s_0$, it is more profitable to vary service temporarily (so as to manage expectations in the transition) than to shift directly from s_0 to the new service level x. Smooth service downgrading (or upgrading) is practiced in various industries, such as airlines (Cruz and Papadopoulos 2003).

Finally, we argue that both the firm and the customer can be better off over the long run if the firm has the flexibility to temporarily vary service.

COROLLARY 2. *For any initial customer expectation $s_0 \neq s^{**}$, there exists a service level $\mathrm{x} > s_0$ and a service path $\wp(\mathrm{x}) = \{x_0, x_1, x_t = \mathrm{x}, t \geq 2\}$ that is more profitable than the constant service path $\{x_t = s_0, t \geq 0\}$; that is $J^{\wp(\mathrm{x})}(s_0) > \Pi(s_0)$.*

Consider, for example, a firm that follows the optimal static service policy determined in Section 3.2 and offers a constant service level $\bar{s}$. In this case, it is reasonable to assume that a customer's expectation of service is $s_0 = \bar{s}$. Proposition 2 shows that exploiting the behavioral effects of changes in service on customer satisfaction and retention enables firms to improve customer profitability by temporarily varying service. Moreover, Corollary 2 indicates that long-run win–win solutions can be designed whereby the firm can obtain higher net present value than the best static policy ($\Pi(\bar{s})$) and simultaneously offer better service ($\mathrm{x} > \bar{s} = s_0$) to customers over the long run (albeit at the cost of a temporary disturbance in service).

5. Transient Satisfaction Policy

In Section 4 we showed that the firm can benefit from varying service, either temporarily or in the long run, when expectations are not in steady state. Naturally the question then arises as to what is the best way to leverage service flexibility.

In this section we characterize the transient structure of the firm's optimal policy. Our results suggest that it is conceptually (and technically) more effective to focus on satisfaction, rather than service, as a decision variable because doing so leads to unified, robust insights. Because the firm is fully informed about the satisfaction updating process, deciding on a service level is equivalent in our setting to choosing the next level of customer's satisfaction.

Define $\bar{\pi}(r,s) = \pi(\frac{r-\lambda s}{1-\lambda})$; then, in terms of the variable $s_{t+1} = \lambda s_t + (1-\lambda)x_t$, problem (2) becomes

$$J(s_t) = \max_{s_{t+1} \in \mathbf{s}(s_t)} \bar{\pi}(s_{t+1}, s_t) + \beta F(s_{t+1}) J(s_{t+1}). \tag{7}$$

Here $\mathbf{s}(s_t) = [\lambda s_t, \lambda s_t + (1-\lambda)]$ is the feasible set of next-period satisfaction s_{t+1} associated with the constraint $x_t \in [0,1]$. The optimal satisfaction policy $s^*(\cdot)$ solves (7) and satisfies $s^*(s) = \lambda r + (1-\lambda)x^*(s)$ for the optimal service policy $x^*(\cdot)$, that solves (2). Technically, the advantage of this model over its equivalent formulation (2) is that it avoids the complex interaction between state and decision variables in the expected profit-to-go. Thus, supermodularity of the argument of the Bellman equation (7) is ensured by concavity of short-term profits π, with no assumption on the renewal rate F.

The following result establishes existence and global stability of the unique steady state determined by Proposition 1, and it characterizes the structure of the optimal transient policy.

Proposition 3. *The optimal satisfaction policy $s^*(\cdot)$ is increasing. Moreover, the optimal satisfaction path $\{s_t^*\}$ converges monotonically to the unique steady state s^{**} characterized by Proposition 1. The optimal service path $\{x_t^*\}$ also converges to the steady state s^{**}.*

Proposition 3 shows that there does exist an ideal satisfaction level that the firm will deliver in the long run, regardless of customers' prior expectations. If initial expectations are not in steady state,

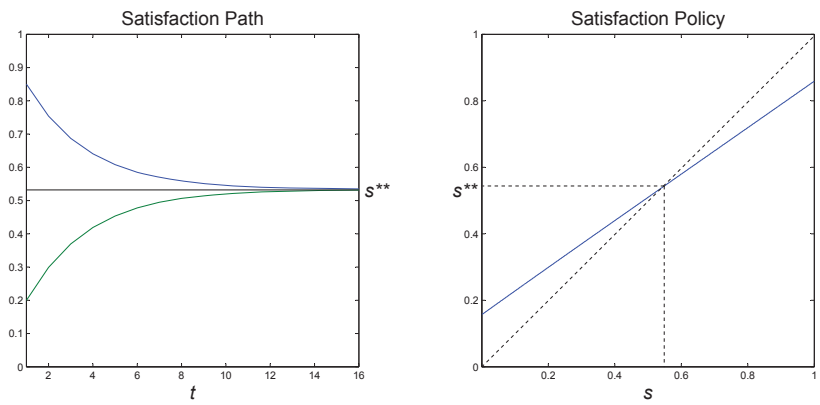

Figure 5 **(a) Optimal dynamic satisfaction paths $s_{t+1} = s^*(s_t)$ as a function of prior satisfaction s_0; (b) optimal satisfaction policy $s^*(\cdot)$ and steady state s^{**};** $F(s) = 1/(1+\exp(-3s))$, $c(x) = x^2$, $\lambda = 0.5$, $\beta = 0.94$.

$s_0 \neq s^{**}$, then the firm benefits from adjusting service level and the optimal way of doing so induces a monotone satisfaction path that converges to s^{**}. As illustrated in Figure 5, if customers have low initial expectations, the firm will gradually increase satisfaction to s^{**} by systematically offering service above expectations; and the opposite holds if customers have high initial expectations. Relative to a constant policy, which maintains service at current expectations, the optimal policy increases expected lifetime for less satisfied customers by improving their service experience and also capitalizes on short-term profits for more satisfied customers. The steady state balances the marginal short-term cost of increasing satisfaction against the corresponding marginal expected return over the long run.

Proposition 3 indicates that the service path will converge to the same steady state but makes no statement regarding monotonicity of the service policy. Indeed, it is possible that the optimal service policy is not increasing or even monotone. So even though it is optimal for the firm to deliver higher satisfaction to ex ante more satisfied customers, they may not actually receive higher service, as discussed in Section 6.1. Given the robust nature of insights concerning the satisfaction (vs. service) policy, we conclude that firms should consider monitoring satisfaction rather than service. The cost of doing so may not be an issue, given that "many companies routinely measure customer satisfaction rather than service quality" (Rust et al. 1995).

6. Customer Value and Prioritization

Section 5 characterized the optimal satisfaction policy over the long run within a transient regime. Firms can use this information to infer which customers are more valuable and benefit more from an optimal dynamic service strategy. In particular, should customers who are more satisfied receive higher service? Should the firm prioritize increasing satisfaction for more or less satisfied customers? Which types of customers (e.g., more loyal, more adaptive) are more profitable for the firm? These issues are addressed in this section.

6.1. Marginal Return on Satisfaction and Optimal Service Policy

Here we investigate the structure of the optimal service policy and the nature of marginal returns to satisfaction as captured by the shape of the value function J. We argue that both depend on the nature of customer loyalty—that is, the shape of the retention function F (see Section 2.2.2).

PROPOSITION 4. *If $F(s)$ is convex, then the optimal service policy $x^*(s)$ is increasing and the value function $J(s)$ is convex. The opposite need not hold when $F(s)$ is concave.*

Proposition 4 implies that more satisfied customers receive better service if satisfaction has an increasing marginal impact on retention; this appears to be the case in more competitive industries (Jones and Sasser 1995). The result is not robust when F is non convex or in particular, concave. When F is concave, the optimal policy can be decreasing; that is, more satisfied customers actually receive lower service although their overall satisfaction remains higher (see Figure 6). Concavity of F, however, does not generally guarantee this result. Indeed, Figure 7 illustrates cases when F is concave but $x^*(\cdot)$ is increasing.

Increasing marginal returns to satisfaction over the long run may seem to contradict conventional wisdom (Rust et al. 1995). Yet, Ho et al. (2006) find that, for static policies, customer value can be convex if costs are not too steep; they attribute this effect to a disaggregate view of customer behavior in their model. Our result is not driven by cost but rather by the shape of the retention function, which does not figure in their paper.

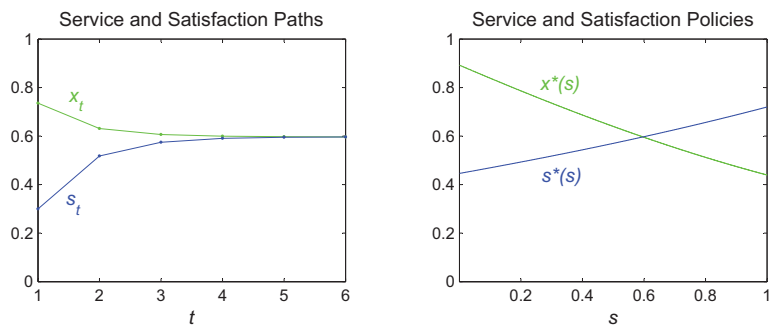

Figure 6 **The optimal service policy and path can be decreasing, while the satisfaction policy and path are increasing for F concave;** $F(s) = 1/(1+\exp(-5s))$, $c(x) = x^2$, $\lambda = 0.5$, $\beta = 0.94$, $s_0 = 0.3$.

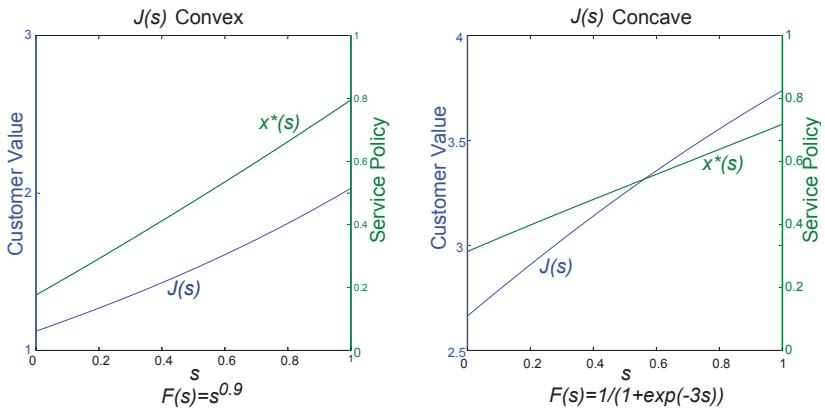

Figure 7 **Customer value J can be convex or concave and the optimal service policy x^* can be increasing for F concave;** $c(x) = x^2$, $\lambda = 0.5$, $\beta = 0.94$.

The controversial suggestion that firms may want to focus on (increasing satisfaction for) customers who already are relatively more satisfied was advanced by Jones and Sasser (1995), who suggested that such priorities should be determined by whether the retention rate is convex or concave. Proposition 4 partially confirms their intuition for the case when F is convex. However, in contrast to these authors, we find that firms may experience increasing marginal returns to satisfaction even when the latter has a diminishing marginal effect on retention. An example is illustrated in Figure 7, where J can be either convex or concave for F concave.

Intuition suggests that the shape of the value function J depends on the degree of concavity of F. Characterizing this relationship analytically is difficult because of the multiplicative interaction effects in the value-to-go (2) or (7), which make preservation of structural properties (e.g., con-

cavity) problematic. A sufficient condition for $J(s)$ to be concave (resp. convex) and the optimal service policy $x^*(s)$ to be decreasing (resp. increasing) is the concavity (convexity) of $(FJ)(s)$ (see the proof of Proposition 4). This property is generally not preserved, however, since FJ may not be concave even when both F and J are (increasing and) concave. Numerical results for parametric logit, exponential, and power retention rates F suggest that, for sufficiently concave F, customer value J (but not necessarily FJ) becomes concave and the optimal service policy may remain increasing.

6.2. The Effect of Behavioral Parameters on the Value Function

A key premise of our work is that behavioral factors, such as customer loyalty and memory, affect demand and profitability. Understanding these effects facilitates identifying the customer types that are more valuable to the firm. Our results in Section 4.1 show that customers who adapt faster (or are more forgetful) receive better service in the long run. But are these customers also more valuable to the firm? What about customers who are more loyal? Do they remain more profitable when they do not receive better service? (See Figure 4.)

Proposition 5. *(a) The marginal effect of memory on the value function $\frac{d}{d\lambda}J(s;\lambda)$ is positive for $s > s^{**}(\lambda)$, negative for $s < s^{**}(\lambda)$, and zero at $s = s^{**}(\lambda)$. (b) If $F(s;\alpha)$ is increasing in the loyalty parameter α, then $J(s;\alpha)$ is also increasing in α.*

Part (a) states that the value function is locally monotone in the adaptation parameter λ but that the direction of monotonicity depends on customer satisfaction. All else equal, the firm can extract more value from satisfied customers when prior experiences are more salient (i.e., when customers adjust their expectations more slowly). Empirical evidence suggests that "customers who have many months' experience with the organization weigh prior cumulative satisfaction more heavily and new information (relatively) less heavily" (Bolton 1998). In this case, Proposition 5 suggests that, ceteris paribus, the focus should be on customers who have a longer history with the firm if they are satisfied. Among the unsatisfied customers, the firm should focus on the recently acquired ones because they are less anchored on past experiences and adapt faster. Figure 8(a)

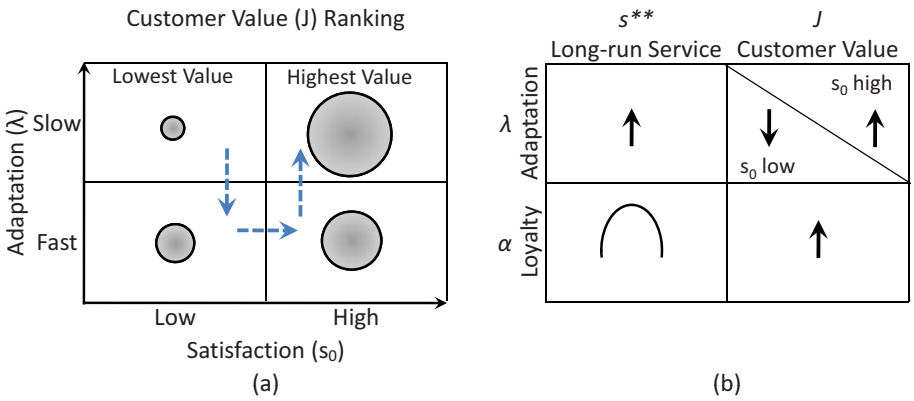

Figure 8 **(a) Effect of customer adaptation speed on value depends on prior satisfaction s_0; (b) Effect of adaptation and loyalty on service and customer value.**

summarizes these insights, suggesting that 'old' customers are the most valuable assets for the firm if they are happy, but otherwise they may be the least valuable.

Part (b) of the proposition confirms the intuitive result that latent loyalty (higher α) is always valuable to the firm. Recall, however, that more loyal customers may not receive better long-run service, as illustrated in Figure 4 for $F(x;\alpha) = 1/(1+\exp(-\alpha x))$. The effect of behavioral parameters (adaptation and loyalty) on customer value and long-run service is summarized in Figure 8(b).

7. Asymmetries in Satisfaction and Renewal Processes

In this section we investigate, along lines that are consistent with prospect theory and marketing evidence, the effect of behavioral asymmetries on adaptation and decision processes. We find that these asymmetries, especially loss aversion, have important effects on the firm's policy: (i) they make constant service policies more prevalent, leading to a range of steady states; and (ii) optimal service policies oscillate if the asymmetries are reversed.

These insights are preserved in an alternative retention model that is driven by disconfirmation, in which customers react more to changes in satisfaction, rather than to absolute levels.

7.1. Loss Aversion and Satisfaction

Prospect theory postulates that decision makers code new information as gains or losses relative to a status quo, a principle that applies to both decision and experience utility (Tversky and

Kahneman 1991; Kahneman et al. 1997). "Experienced utility" is the decision maker's hedonic value at the moment of experience which is captured by the concept of satisfaction in our model. For both types of utility, a negative change from the status quo has a larger effect on value than a positive change of the same magnitude. Generally known as loss aversion, this phenomenon has received vast empirical support in the satisfaction literature (Boulding et al. 1993; Bolton 1998).

Consider the following asymmetric (kinked) satisfaction updating process:

$$s_{t+1}^K = \begin{cases} s_{t+1}^G = s_t + (1-\lambda_G)(x_t - s_t) & \text{if} \quad x_t \geq s_t, \\ s_{t+1}^L = s_t + (1-\lambda_L)(x_t - s_t) & \text{if} \quad x_t < s_t, \end{cases} \tag{8}$$

where $1-\lambda_L > 1-\lambda_G$ expresses loss aversion. Such a kinked learning model is used by Gaur and Park (2007) for consumers who form expectations about product availability (fill rate) and by Nasiry and Popescu (2009) in a dynamic pricing context.

The Bellman equation for loss-averse adaptation can be written as follows:

$$J^K(s_t) = \max_{x_t \in [0,1]} \pi(x_t) + \beta F(s_{t+1}^K) J^K(s_{t+1}^K), \tag{9}$$

where s_{t+1}^K follows the transition dynamics (8). Let J_L and J_G denote the value functions of the smooth problems (2) corresponding to λ_L and λ_G, respectively. Loss aversion implies that, by Proposition 1, the corresponding steady states satisfy $s_L^{**} = s^{**}(\lambda_L) > s^{**}(\lambda_G) = s_G^{**}$. The kinked adaptation process can be expressed as the minimum of two smooth exponential smoothing mechanisms, $s_{t+1}^K = \min\{s_{t+1}^G, s_{t+1}^L\}$, implying $J^K(s) \leq \min\{J_L(s), J_G(s)\}$ (see the Appendix). This suggests that loss aversion—that is, the asymmetric effect of disappointing experiences relative to pleasurable ones—has a negative effect on profitability. This is consistent with the findings in Gaur and Park (2007).

Proposition 6. *Assume that customers are loss averse (i.e., $\lambda_L < \lambda_G$). Then problem* (9) *admits a range of steady states $[s_L^{**}, s_G^{**}]$: starting from any $s_0 \in [s_L^{**}, s_G^{**}]$, a constant service and satisfaction path $s_t = s_0$ is optimal. For $s_0 > s_L^{**}$, the satisfaction path s_t monotonically decreases to s_L^{**} and $J^K(s_0) = J_L(s_0)$. For $s_0 < s_G^{**}$, the satisfaction path monotonically increases to s_G^{**} and $J^K(s_0) = J_G(s_0)$. The service path converges to the same steady state as the corresponding satisfaction path.*

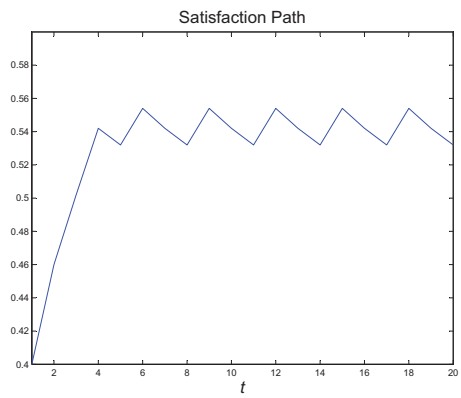

Figure 9 **The optimal satisfaction path oscillates for gain-seeking customers;** $F(s) = 1/(1+\exp(-3s))$, $c(x) = x^2$, $\lambda_G = 0.4$, $\lambda_L = 0.6$, $\beta = 0.94$, $P = 1$.

Adding loss aversion to the model does not affect the structure of the transient policy, but it does widen the set of steady states and so makes constant service policies more prevalent. Intuitively, the firm has less leverage to improve perception when customers anchor more on negative experiences. Technically, this is due to the kink in satisfaction updating.

For completeness, we briefly consider here the case where customers are "gain seeking" (i.e., $\lambda_G < \lambda_L$). This means that service experiences above expectations (positive disconfirmation) are more salient than those below expectations (negative disconfirmation). Bolton et al. (2000) find evidence that members of loyalty rewards programs tend to discount or overlook negative service experiences. In this case, we find that no interior steady state exists. Under a high–low policy, the firm benefits in the long-run by manipulating customer satisfaction and their expectations. The benefits to the firm in this case are attributable to the positive net effect of increasing service and then decreasing it back.

Proposition 7. *If $\lambda_L > \lambda_G$ then problem* (9) *admits no interior steady state. If, moreover, $C(0;\lambda_G) < P < C(1;\lambda_L)$, then any optimal service path oscillates.*

7.2. Renewal Is Driven by Disconfirmation

The results so far have been based on a model where more satisfied customers are more likely to renew; this assumption has the widest empirical support in the marketing literature. One could

argue, however, that more satisfied customers also have higher expectations and thus are more likely to react negatively to lower service quality than those who expect less. This section considers such a model where retention F depends only on disconfirmation, $x-s$, or the perceived gain or loss relative to expectations. This model is a good proxy for cases when customers react more to *changes* in service.

Evidence that disconfirmation is a better predictor of retention, than is customer satisfaction, may be found, for example, in Bolton and Drew (1991b). The key difference from our previous models is that here customers with higher expectations are *less* likely to renew. Intuitively, they become harder to please as they grow accustomed to better service owing to the satisfaction *treadmill* effect. Indeed, our model is consistent with the treadmill effect (Brickman and Campbell 1971), and habituation models (Baucells and Sarin 2010).

Loss aversion in customer's decision process suggests that a perceived downgrade in service (negative disconfirmation) has a stronger impact on retention than a perceived improvement (positive disconfirmation) of the same magnitude. Formally, we model the kinked retention rate as

$$F_K(x-s) = \begin{cases} F(\delta_L(x-s)) & \text{if } x < s, \\ F(\delta_G(x-s)) & \text{if } x \geq s, \end{cases} \tag{10}$$

where $\delta_L > \delta_G$ captures loss aversion. The corresponding Bellman equation is

$$J_K(s) = \max_{x\in[0,1]} \pi(x) + \beta F_K(x-s) J_K(\lambda s + (1-\lambda)x). \tag{11}$$

Lemma 3. *$J_K(s)$ is decreasing in s. Moreover, if F is convex then J_K is also convex.*

This result is in sharp contrast with Lemma 1. If retention is driven by the gap between experience and expectation, then customers who are more satisfied require higher maintenance, and this makes them less profitable in the long run. However, the rest of our insights are preserved under this conceptually and structurally different model.

Proposition 8. *If customers are loss averse in their renewal decisions (i.e., $\delta_L \geq \delta_G$) then problem* (11) *admits a range of steady states $[s^{**}(\delta_G), s^{**}(\delta_L)]$, where $s^{**}(\delta)$ solves*

$$(1-\lambda\beta F(0))\pi'(s) + \beta\delta F'(0)\pi(s) = 0. \tag{12}$$

If $s_0 > s^{**}(\delta_L)$ then the optimal satisfaction path decreases to $s^{**}(\delta_L)$, and if $s_0 < s^{**}(\delta_G)$, then the optimal satisfaction path increases to $s^{**}(\delta_G)$. If $\delta_L < \delta_G$, then no steady state exists.

8. Extensions: Alternative Satisfaction and Revenue Models

We now show that the main insights in this paper are robust when allowing for non linearity in the satisfaction updating process as well as for alternative revenue models that account for the effect of satisfaction on spending, purchase frequency, and social social planning objectives.

8.1. Non Linear Models of Satisfaction

Our results can be extended to account for general nonlinear effects in the satisfaction updating process, $s_{t+1} = G(s_t, x_t - s_t) = H(x_t, s_t)$, as long as this satisfies the following assumption.

Assumption 2. *(a) $H(x,s)$ is increasing in x and s, and $H(s,s) = s$; (b) $H_{11}(x,s) \leq 0$; (c) $H_{12}(x,s) \geq 0$; (d) $H_{22}(s,s) \geq 0$.*

For convenience, partial derivatives are here denoted by corresponding subscripts. Part (a) states that all else equal, both service level and ex ante satisfaction have a positive effect on ex post satisfaction; this is consistent with empirical evidence (Oliver and DeSarbo 1988). In particular, it suggests that the absolute effect of satisfaction dominates the effect of disconfirmation $x - s$; the opposite effect was captured in Section 7.2. Satisfaction does not change if service remains constant, $H(s,s) = s$, while controlling for exogenous effects on satisfaction. Diminishing marginal sensitivity to service, (b), is a natural assumption. Part (c) and part (d) are technical conditions stating that more satisfied customers are more sensitive to a change in service and to a change in satisfaction, respectively. To the best of our knowledge, these hypotheses have not yet been tested in the literature.

Assumption 2 is satisfied, for example, for an exponential smoothing model in which the weight attached to the current experience depends on the current service level: $H(x,s) = \lambda(x)s + (1 - \lambda(x))x$, provided that $\lambda(x)$ is increasing and has a bound on its curvature, $\lambda'(x) \geq |\lambda''(x)|/2$ (as for, e.g., $\lambda(x) = x, e^{x-1}$, and $1 - e^{-x}$). In this model, lower service is more salient in memory— that

is, the lesser the current experience, the more it weighs on satisfaction. Section 7.1 described a model in which experiences below expectations are more salient than those above expectations.

The Bellman equation corresponding to this problem is:

$$J(s) = \max_{x \in [0,1]} \pi(x) + \beta F(H(x,s)) J(H(x,s)). \tag{13}$$

Proposition 3 extends under this general adaptation process by replacing λ with $\lambda(s) = H_2(s,s)$, the derivative of $H(x,s)$ with respect to s, evaluated at $x = s$. In particular, for the exponential smoothing model (1), we recover precisely $\lambda(s) = \lambda$.

Proposition 9. *Under Assumption 2, the results in Proposition 1, Proposition 3 and Corollary 1 apply to model* (13) *once we redefine* $C(s;\lambda(s)) = c(s) + \left(\frac{\lambda(s)}{1-\lambda(s)} + L(s)\right)\frac{c'(s)}{L'(s)}$. *In particular, the corresponding steady state solves:* $\lambda(s)\pi'(s) + (1-\lambda(s))\Pi'(s) = 0$.

We find it interesting that the steady state depends on the service–satisfaction relationship only via $\lambda(s)$, the marginal effect of expectations on satisfaction, when service is aligned with expectations.

8.2. Alternative Profit Models

Our structural results are essentially driven by the concavity of the short-term profits π (supermodularity of $\bar{\pi}$), with no assumption on the retention function except monotonicity. This allows one to extend the application of our insights to more complex revenue models, which capture the effect of satisfaction on customer spending and purchase frequency, and also to nonprofit and public-service contexts.

Satisfaction affects purchase frequency. Our structural results extend to capture the effect of satisfaction on purchase frequency. Technically, this is equivalent to replacing β with $\beta^{\nu(s_{t+1})}$ in problem (7). Practical evidence suggests that more satisfied customers increase their frequency of purchase (Sharma et al. 1999); in other words, $\nu(\cdot)$ is decreasing. This transformation has no effect on supermodularity of (7)'s argument, implying that our monotonicity and convergence results hold without any additional assumptions on $\nu(\cdot)$.

Satisfaction affects spending. Our results also extend for a general model $\pi(x,s)$ where the firm's instant profit is a function of the offered service and satisfaction level, provided that $\pi(x,s)$ is supermodular and concave in service x. This holds, for example, if more satisfied customers are more sensitive to changes in service, or if the marginal cost of serving them is lower. In particular, this condition holds if customer spending $P(s)$ is increasing in satisfaction but unaffected by current service x; whereas convex costs depend only on current service $c(x)$, so $\pi(x,s) = P(s) - c(x)$.

Public services. In particular, consider a model in which the firm's objective has a social component: $P(\cdot) = (1-w)P + wS(\cdot)$. Here $w \in [0,1]$ is the weight that the firm puts on consumer surplus, which is an increasing concave function of service $S(x)$ or, alternatively, of satisfaction $S(s)$. As $w \to 1$, this becomes a public-services problem for a social planner who cares only about customer surplus and not profit. All our results extend in this context. Moreover, customer satisfaction increases as the firm puts more weight on consumer surplus (i.e., $s^{**}(w)$ and $s^{*}(w)$ are increasing in w). In particular, a profit-focused monopolist ($w = 0$) will satisfy customers less than a public planner with the same cost structure. But if social concerns are of major importance to the firm and if the marginal social benefits of full satisfaction exceed marginal costs (specifically, if $w \geq c'(1)/S'(1)$), then it is easy to see that offering full service is optimal, $x^* \equiv 1$.

9. Conclusions

We relied on behavioral theories to develop a dynamic programming model of using responsive service strategies to manage satisfaction over time in a customer relationship. In this context, we showed that firms can extract more value in the long run by gradually managing service experiences and expectations over time towards an "ideal" steady-state service level from which it is suboptimal to deviate. Interestingly, behavioral asymmetries (such as loss aversion) drive the structure of optimal long-run policies by ensuring convergence and increasing the prevalence of constant service policies. Indeed, the more customers are averse to service downgrades, the wider is the range of optimal constant service policies offered by the firm and the lower are its profits.

Our results provide insight into how firms with responsive service capabilities can use customer-level information—such as prior satisfaction, loyalty, and adaptation—to prioritize customers in

terms of value and customized service. The relationships between various behavioral factors and the firm's policy and profits are summarized in Table 1.

More loyal customers are more valuable, but they may not receive better service if they are inherently too sticky. Our model predicts a unimodal relationship between loyalty and service. From the customers' perspective, this means that there exists an ideal, intermediate level of loyalty (driven, e.g., by switching costs or the level of market competition) that fetches the best service. The firm offers less service in the long run to less adaptive customers, such as those with a longer history; these are more profitable than customers who focus on recent experiences, but only if they are sufficiently satisfied.

We find that the firm can always extract more value from more satisfied customers if they are also more likely to renew. This is a direct benefit of service flexibility that would not be realized if firms were constrained to maintain the same service level, or if renewal is mainly driven by disconfirmation. The optimal policy preserves satisfaction ranking: ex ante more satisfied customers remain more satisfied. However, they receive better service only if there are increasing marginal returns to satisfaction. Contrary to existing predictions (Jones and Sasser 1995), this may be the case even if satisfaction has diminishing marginal effects on retention.

Table 1 Effects of Consumer Behavior on Policy and Value

Behavioral Factor	**Long-run Service (steady state s^{**})**	**Customer Value** (J)
Loyalty (α)	Inverse U-shape	Increasing
Adaptation (λ)	Increasing	Increasing for high s_0; decreasing for low s_0
Loss aversion ($\rho = \lambda_G/\lambda_L > 1$)	Expands range (oscillates if $\rho < 1$)	Decreasing
Initial satisfaction/ Expectation (s_0)	No effect	Depends on what drives renewal: increasing, if satisfaction; decreasing, if disconfirmation.

This paper is a first step toward capturing behavioral effects of service dynamics in a business relationship. We have therefore strived for parsimony in developing the simplest stylized model

capable of transmitting the main insights from this framework. Ample opportunities exist for future research to extend this model and address its limitations from an operational, marketing, or economic perspective—for example, by incorporating strategic interaction, customer heterogeneity and fairness perceptions, acquisition and network effects, and/or richer operational service structures. For expository purposes, we cast our model and results in the context of managing service relationships. However, our setup can be expanded to broader contexts, such as employee retention, effort and quality management.

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Appendix: Proofs

Proof of Lemma 1. Monotonicity of the value function holds because $F(\lambda s + (1-\lambda)x)$ is increasing in x. Monotonicity is preserved by induction for the corresponding finite-horizon model and then at the limit for our infinite-horizon formulation. The firm can at least extract P from the customer by offering $\{x_t \equiv 0\}$, so $P \leq J(s)$. An upper bound on customer value is obtained if the customer never defects and the firm offers no added service, so $J(s) \leq \sum_{t=0}^{\infty} \beta^t P = \frac{P}{1-\beta}$.

Proof of Lemma 2. Assumption 1 guarantees that Π is unimodal, so its maximizer is unique. We write the first-order condition with respect to x to obtain, after rearrangement, $P = C(x)$. By Assumption 1, $C(x)$ is increasing and so $C(0) < C(1)$. The rest of the result follows from the unimodality of Π.

Proof of Proposition 1. *(a)* An interior steady state is given by the following Euler equation:

$$\frac{\partial}{\partial s_{t+1}}\left\{\pi\Big(\frac{s_{t+1}-\lambda s_t}{1-\lambda}\Big)+\beta F(s_{t+1})\left(\pi\Big(\frac{s_{t+2}-\lambda s_{t+1}}{1-\lambda}\Big)+\beta F(s_{t+2})\Pi(s_{t+2})\right)\right\}_{s_t=s_{t+1}=s_{t+2}=s}=0. \tag{14}$$

Defining $W(s)=\lambda\pi(s)+(1-\lambda)\Pi(s)$, we can rewrite (14) as

$$W'(s)=(1-\lambda)\beta F'(s)(P-C(s;\lambda))=0, \tag{15}$$

which is equivalent to $P=C(s;\lambda)$. Under Assumption 1, $C(s;\lambda)$ is increasing and so the solution to $P=C(s;\lambda)$ is unique if it exists. Strict quasi-concavity of W (or strict concavity of Π) is also sufficient for this.

It remains to show that (i) if $P>\underline{C}$ then $s^{**}=0$ cannot be a steady state; and (ii) if $P<\overline{C}$ then $s^{**}=1$ cannot be steady state. For this we show that, starting from $s_0=0$ (resp. $s_0=1$), the corresponding constant satisfaction paths with profits $\Pi(0)$ (resp. $\Pi(1)$), are suboptimal.

(i) Consider the service path $\wp_0(\delta)=\{x_1=\delta,\, x_t=0,\, t\geq 2\}$, and let $J^{\wp_0(\delta)}(0)$ denote the corresponding discounted profit starting at $s_0=0$. We then have

$$\begin{aligned}\lim_{\delta\to 0^+}\frac{J^{\wp_0(\delta)}(0)-\Pi(0)}{\delta} &\geq \lim_{\delta\to 0^+}\frac{1}{\delta}\Big(J^{\wp_0(\delta)}(0)-\pi(0)+\beta(F((1-\lambda)\delta)-F(0))\pi(0)\Big)\\ &=\pi'(0)+\beta(1-\lambda)F'(0)\pi(0)\\ &=[1-\beta F(0)]^2W'(0)>0,\end{aligned} \tag{16}$$

where the last inequality holds because $P>\underline{C}$ implies $W'(0)>0$.

(ii) Similarly, consider the service path $\wp_1(\delta)=\{x_1=1-\delta,\, x_t=1,\, t\geq 2\}$ and let $J^{\wp_1(\delta)}(1)$ denote the corresponding profit starting at $s_0=1$. Then

$$J^{\wp_1(\delta)}(1)=\pi(1-\delta)+\beta\pi(1)\sum_{k=0}^{\infty}\beta^k F(1-\delta\lambda^k(1-\lambda)). \tag{17}$$

Therefore,

$$\begin{aligned}\lim_{\delta\to 0^+}\frac{J^{\wp_1(\delta)}(1)-\Pi(1)}{\delta} &= \lim_{\delta\to 0^+}\frac{1}{\delta}\Big(\pi(1-\delta)-\pi(1)+\beta\pi(1)\sum_{k=0}^{\infty}\beta^k(F(1-\delta\lambda^k(1-\lambda))-F(1))\Big)\\ &=-\pi'(1)-\beta(1-\lambda)\pi(1)F'(1)\sum_{k=0}^{\infty}\beta^k\lambda^k\end{aligned}$$

$$
\begin{aligned}
&= -\pi'(1) - \frac{\beta(1-\lambda)}{1-\lambda\beta} F'(1)\pi(1) \\
&> -\pi'(1) - \frac{\beta(1-\lambda)}{(1-\lambda\beta)(1-\beta)} F'(1)\pi(1) \\
&= -\frac{\beta(1-\lambda)}{(1-\lambda\beta)(1-\beta)} F'(1)(P-\overline{P}) > 0. \qquad (18)
\end{aligned}
$$

The last inequality holds because $P < \overline{P}$ by definition. Hence, starting from $s_0 = 1$, a profitable deviation from the constant satisfaction path $\{s_t \equiv 1\}$ exists and so $s^{**} = 1$ cannot be a steady state.

Moreover, $C(s;\lambda)$ is decreasing in β because

$$
\frac{\partial C(s;\lambda)}{\partial \beta} = -\frac{(1-\lambda\beta^2[F(r)]^2)c'(r)}{\beta(1-\lambda)F'(r)} \leq 0.
$$

Finally, since $C(s;\lambda)$ is increasing in s and decreasing in β and since s^{**} solves $P = C(s;\lambda)$, it follows that s^{**} is increasing in β.

Proof of Corollary 1. This follows directly from the proof of Proposition 1.

Proof of Proposition 2. For any $s_0 \in (0,1)$, consider the variational path $\wp(\varepsilon) = \{x_1 = s_0 + \varepsilon, x_2 = s_0 - \lambda\varepsilon, x_t = s_0, t \geq 3\}$. We show that, for any initial expectation $s_0 \neq s^{**}$, there exists $|\varepsilon| \leq \min\{s_0, 1-s_0\}$ such that at least one of the feasible paths $\wp(\varepsilon)$ or $\wp(-\varepsilon)$ is more profitable than the constant path $s_t \equiv s_0$; that is, $\Pi(s_0) < \max\{J^{\wp(\varepsilon)}(s_0), J^{\wp(-\varepsilon)}(s_0)\}$.

It is easy to verify that under $\wp(\varepsilon)$ we have $s_2 = s_0$ for any feasible ε (i.e., $|\varepsilon| \leq \min\{s_0, 1-s_0\}$), which implies that

$$
J^{\wp(\varepsilon)}(s_0) = \pi(s_0+\varepsilon) + \beta F(s_0 + (1-\lambda)\varepsilon)\Big(\pi(s_0-\lambda\varepsilon) + \beta F(s_0)\Pi(s_0)\Big). \qquad (19)
$$

For $\varepsilon > 0$ we use $\Pi(s_0) = \pi(s_0) + \beta F(s_0)\Big(\pi(s_0) + \beta F(s_0)\Pi(s_0)\Big)$ to obtain

$$
\begin{aligned}
\lim_{\varepsilon \to 0^+} \frac{J^{\wp(\varepsilon)}(s_0) - \Pi(s_0)}{\varepsilon} &= \beta(1-\lambda)F'(s_0)\Big(\pi(s_0) + \beta F(s_0)\Pi(s_0)\Big) + (1-\lambda\beta F(s_0))\pi'(s_0) \qquad (20) \\
&= \beta(1-\lambda)F'(s_0)\Pi(s_0) + (1-\lambda\beta F(s_0))\pi'(s_0) \qquad (21) \\
&= (1-\beta F(s_0))W'(s_0). \qquad (22)
\end{aligned}
$$

Similarly, using (19) for $\wp(-\varepsilon)$, we obtain

$$\lim_{\varepsilon \to 0^+} \frac{J^{\wp(-\varepsilon)}(s_0) - \Pi(s_0)}{\varepsilon} = -(1 - \beta F(s_0))W'(s_0). \tag{23}$$

The result follows because, by Assumption 1, $W'(s_0) \neq 0$ for $s_0 \neq s^{**}$.

Proof of Corollary 2. By Proposition 2, for any $s_0 \neq s^{**}$ there exist x_0 and x_1 such that $J(s_0; x_0, x_1, s_0) > \Pi(s_0)$, where $J(s_0; x_0, x_1, x)$ is the value of the path $\{x_0, x_1, x_t = x, t \geq 2\}$ starting at s_0. The result then follows because $J(s_0; x_0, x_1, x)$ is continuous in x.

Proof of Proposition 3. By the concavity of $\pi(\cdot)$, the term on the right-hand side of the Bellman equation (7) is supermodular in (s_{t+1}, s_t). Furthermore, the feasible sets $\mathbf{s}(s_t)$ are ascending in s_t; that is, for any $s_t \leq s'_t$, $r \in \mathbf{s}(s_t)$, and $r' \in \mathbf{r}(s'_t)$ we have $\min(r, r') \in \mathbf{s}(s_t)$ and $\max(r, r') \in \mathbf{s}(s'_t)$. Therefore, by Topkis's theorem (Topkis 1998, Thm. 2.8.2), the policy function $s^*(\cdot)$ is increasing on $[0, 1]$. It follows that $s^*(\cdot)$ must have a fixed point, $s^{**} = s^*(s^{**})$, which is a steady state of problem (7).

Moreover, monotonicity of $s^*(\cdot)$ implies that the state path $\{s^*_t\}$ is monotone (by induction), and because the feasible set $\mathbf{s}(\cdot)$ is compact, $\{s^*_t\}$ must converge to a steady state. Hence a steady state exists, and by Proposition 1 it is unique. Finally, in steady state $s^{**} = \lambda s^{**} + (1 - \lambda)x^{**}$, so $s^{**} = x^{**}$. In particular, because the satisfaction paths converge monotonically to s^{**} (which maximizes the unimodal function W), it follows that W is a Lyapounov for our problem.

Proof of Proposition 4. The result holds because convexity is preserved by maximization and limits. Indeed, if $F(s)$ is convex, it follows by induction that the corresponding finite horizon value function is also convex. By value iteration, we can take limits to obtain that the infinite horizon value function $J(s)$ is convex, so $V(s) = F(s)J(s)$ is convex.

We further show that V convex (resp. concave) is sufficient for the service policy to be increasing (decreasing) and the value function J to be convex (concave), confirming the statement at the end of Section 6.1. Now, V convex (concave) is equivalent to the argument on the right-hand side of the Bellman equation, $Q(x, s) = \pi(x) + \beta V(\lambda s + (1 - \lambda)x)$, being supermodular (submodular); Monotonicity of the optimal policy then follows by Topkis's theorem. Moreover, V convex implies

that $Q(x,s)$ is convex in s, so J is convex. On the other hand, V concave implies that $Q(x,s)$ is jointly concave, so J is concave.

Proof of Proposition 5. For a given s_0, consider the optimal service path $s_{t+1} = s^*(s_t)$ for all t. By the envelope theorem, for all t we have

$$\frac{\partial}{\partial\lambda} J(s_t;\lambda) = \frac{s_{t+1}-s_t}{(1-\lambda)^2}\pi'\Big(\frac{s_{t+1}-\lambda s_t}{1-\lambda}\Big) + \beta F(s_{t+1})\frac{\partial}{\partial\lambda} J(s_{t+1};\lambda). \tag{24}$$

In particular, at steady state $s_0 = s^{**}(\lambda) = s_t$ we have $(1-\beta F(s))\frac{\partial}{\partial\lambda}J(s;\lambda)|_{s=s^{**}} = 0$, which implies the stated result. For $s_0 \geq s^{**}(\lambda)$, by Proposition 3 the optimal satisfaction path is decreasing, $s_{t+1} \leq s_t$ for all t, so the first term in the right-hand side of (24) is positive. Thus, for any $t > 0$, we have

$$\begin{aligned}
\frac{\partial}{\partial\lambda} J(s_0;\lambda) &\geq \beta F(s_1)\frac{\partial}{\partial\lambda} J(s_1;\lambda) && (25)\\
&\geq \beta^2 F(s_1)F(s_2)\frac{\partial}{\partial\lambda} J(s_2;\lambda) \geq \cdots && (26)\\
&\geq \lim_{t\to\infty}\beta^t\left(\prod_{i=1}^{t} F(s_i)\right)\frac{\partial}{\partial\lambda} J(s_t;\lambda) = 0. && (27)
\end{aligned}$$

The last derivative is bounded as $n \to \infty$ because $s_t \to s^{**}(\lambda)$ (Proposition 3). The case $s_0 \leq s^{**}(\lambda)$ is proved similarly.

Proof of Proposition 6. We transform problem (9) to obtain

$$J^K(s_t) = \max_{s^K_{t+1}\in\mathbf{s}(s_t)} \pi\Big(\frac{s^K_{t+1}-\lambda s_t}{1-\lambda}\Big) + \beta F(s^K_{t+1})J^K(s^K_{t+1}). \tag{28}$$

For any $\theta \in [0,1]$, let (P_θ) denote problem (28) where $s^\theta_{t+1} = \theta s^G_{t+1} + (1-\theta)s^L_{t+1}$. This is equivalent to a smooth model (7) with $\lambda_\theta = \theta\lambda_G + (1-\theta)\lambda_L$. Let J_θ and s^{**}_θ denote (respectively) the value function and the steady state of this problem. In particular, $s^{**}_{\theta=0} = s^{**}_L$ and $s^{**}_{\theta=1} = s^{**}_G$.

Lemma 4. *(a) $J^K(s) \leq J_\theta(s)$ for all s. (b) If s^{**}_θ is a steady state for (P_θ), then it is also a steady state for problem* (28).

Proof of Lemma 4. (a) The claim follows from $s^\theta_{t+1} = \theta s^G_{t+1} + (1-\theta)s^L_{t+1} \geq \min\{s^G_{t+1}, s^L_{t+1}\} = s^K_{t+1}$, by induction on the finite-horizon versions of the corresponding problems, and then using value

iteration. (b) Starting from s_θ^{**}, a constant satisfaction path is optimal for (P_θ). This path is feasible for our kinked problem (28) and achieves the same value $J^K(s_\theta^{**}) = J_\theta(s_\theta^{**})$. Therefore, using part (a) of Lemma 4, the constant path s_θ^{**} must be optimal for problem (28) and so s_θ^{**} is also a steady state for this problem. □

By Proposition (3), if we start from $s_0 > s_{\theta=0}^{**} = s_L^{**}$ then the optimal satisfaction path in (P_L) decreases to s_L^{**}. This path is feasible for problem (28) and gives the same value in both problems. Because $J_L(s) \geq J^K(s)$, for all s, the optimal path for (P_L) is also optimal for problem (28). The case $s_0 < s_{\theta=1}^{**} = s_G^{**}$ is analogous.

By Proposition 1, s_θ^{**} solves $P = C(s;\lambda_\theta)$ where $\lambda_\theta = \theta\lambda_G + (1-\theta)\lambda_L$. For $\theta = 0$ (resp. $\theta = 1$), the solution to $P = C(s;\lambda_\theta)$ is s_L^{**} (resp. s_G^{**}). Continuity of $C(s;\lambda)$ ensures that for all $s \in [s_G^{**}, s_L^{**}]$ there exists a $\theta \in [0,1]$ such that $P = C(s;\lambda_\theta)$; in other words, s is a steady state for (P_θ). By Lemma 4, it is also a steady state for problem (28) and hence for problem (9).

Proof of Proposition 7. Suppose that the interior steady state exists and is equal to x. It follows that, starting from $s_0 = \mathrm{x}$, any deviation from the constant service path $\{x_t \equiv \mathrm{x}, \forall t\}$ is not profitable. Consider the following two deviations:

$$\wp_+(e) = \{x_1 = \mathrm{x} + e,\, x_2 = \mathrm{x} - d,\, x_t = \mathrm{x},\, t \geq 3\}, \tag{29}$$

$$\wp_-(e) = \{x_1 = \mathrm{x} - e,\, x_2 = \mathrm{x} + f,\, x_t = \mathrm{x},\, t \geq 3\}; \tag{30}$$

here $d = \frac{\lambda_L(1-\lambda_G)}{1-\lambda_L}e$ and $f = \frac{\lambda_G(1-\lambda_L)}{1-\lambda_G}e$. It follows that for both paths we have $s_2 = \mathrm{x}$. The profit associated with each path is equal to

$$J^{\wp_+(e)}(\mathrm{x}) = \pi(\mathrm{x}+e) + \beta F(\lambda_G \mathrm{x} + (1-\lambda_G)(\mathrm{x}+e))\Big(\pi(\mathrm{x}-d) + \beta F(\mathrm{x})\Pi(\mathrm{x})\Big), \tag{31}$$

$$J^{\wp_-(e)}(\mathrm{x}) = \pi(\mathrm{x}-e) + \beta F(\lambda_L \mathrm{x} + (1-\lambda_L)(\mathrm{x}-e))\Big(\pi(\mathrm{x}+f) + \beta F(\mathrm{x})\Pi(\mathrm{x})\Big). \tag{32}$$

Suboptimality of any deviation from the constant path $\{\mathrm{x}\}$ implies that the derivative of $J^{\wp_+(e)}(\mathrm{x})$ and $J^{\wp_-(e)}(\mathrm{x})$ with respect to e, evaluated at $e = 0$, should be negative. Therefore,

$$\frac{d}{de}J^{\wp_+(e)}(\mathrm{x}) + \frac{d}{de}J^{\wp_-(e)}(\mathrm{x})\Big|_{e=0} = \beta(\lambda_L - \lambda_G)\left(F'(\mathrm{x})\Pi(\mathrm{x}) - \frac{1-\lambda_L\lambda_G}{(1-\lambda_G)(1-\lambda_L)}F(\mathrm{x})\pi'(\mathrm{x})\right) \leq 0 \tag{33}$$

which contradicts $\lambda_L > \lambda_G$; thus an interior steady state does not exist. Ruling out the boundary steady states follows the same line of argument as in Proposition 1.

Proof of Lemma 3. The monotonicity result for the finite-horizon version of the problem follows by induction because $F_K(x-s)$ is decreasing in s and monotonicity is preserved by maximization. Since monotonicity is preserved by limits, the infinite-horizon result follows by value iteration. To show convexity of the value function we use that F is increasing and convex, and J is decreasing, so the function on the right-hand side of the Bellman equation is convex. The result follows because convexity is preserved by maximization and limits.

Proof of Proposition 8. First we characterize the steady state of the smooth problem (11) when $\delta_L = \delta_G = \delta$. The Euler equation states that the following expression, evaluated at $s_t = s_{t+1} = s_{t+2} = s^{**}(\delta)$ equals zero:

$$\frac{\partial}{\partial s_{t+1}}\left\{\pi\Big(\frac{s_{t+1}-\lambda s_t}{1-\lambda}\Big)+\beta F\Big(\frac{\delta(s_{t+1}-s_t)}{1-\lambda}\Big)\left(\pi\Big(\frac{s_{t+2}-\lambda s_{t+1}}{1-\lambda}\Big)+\beta F\Big(\frac{\delta(s_{t+2}-s_{t+1})}{1-\lambda}\Big)\Pi(s_{t+2})\right)\right\}; \tag{34}$$

Equivalently,

$$\frac{1}{1-\lambda}\pi'(s)+\delta\beta F'(0)\left(\pi(s)+\beta F(0)\Pi(s)\right)+\beta F(0)\left(-\frac{\lambda}{1-\lambda}\pi'(s)-\frac{\delta\beta}{1-\lambda}F'(0)\Pi(s)\right)\Big|_{s=s^{**}}=0. \tag{35}$$

After simplification, this is equation is precisely (12).

The proof for the loss-averse case is analogous to that of Proposition 6. In particular, by defining $F_\theta(x-s) = F((\theta\delta_G + (1-\theta)\delta_L)(x-s)) \geq F_K(x-s)$ with $\theta \in [0,1]$, we can prove a result similar to Lemma 4. In particular, the resulting smooth problems for $\theta \in [0,1]$ provide value function upper bounds $J_\theta \geq J_K$. Their steady states (characterized previously) span the range $[s^{**}(\delta_G), s^{**}(\delta_L)]$ and are also steady states for our kinked problem. The proof of the gain-seeking case ($\delta_L < \delta_G,$) is similar to that of Proposition 7 and follows by considering the following variational paths:

$$\wp_+(e) = \{x_1 = \mathrm{x}+e,\, x_2 = \mathrm{x}+e(1-\lambda),\, x_t = \mathrm{x},\, t \geq 3\} \tag{36}$$

$$\wp_-(e) = \{x_1 = \mathrm{x}-e,\, x_2 = \mathrm{x}-e(1-\lambda),\, x_t = \mathrm{x},\, t \geq 3\}. \tag{37}$$

Proof of Proposition 9. Let h denote the inverse function of $H(x,s)$; that is, $h(H(x,s),s)=x$. problem (13) can be rewritten with respect to the variable s_{t+1} as

$$J(s_t)=\max_{s_{t+1}\in\mathbf{s}(s_t)}\pi(h(s_{t+1},s_t))+\beta F(s_{t+1})J(s_{t+1}), \tag{38}$$

where $\mathbf{s}(\cdot)$ represents the corresponding feasible set. Much as in the proof of Proposition 3, we show that the argument inside problem (38) is supermodular and that the feasible sets are ascending, which (by Topkis's theorem) implies monotonicity of the policy function $s^*(\cdot)$. Because $H(x_t,s_t)$ is increasing in s_t, the feasible sets $\mathbf{s}(s_t)$ are ascending. The cross partial derivative of the argument of problem (38) is

$$\pi''(h(s_{t+1},s_t))h_1(s_{t+1},s_t)h_2(s_{t+1},s_t)+\pi'(h(s_{t+1},s_t))h_{12}(s_{t+1},s_t), \tag{39}$$

where derivatives are denoted by corresponding subscripts. Differentiating

$$H(h(s_{t+1},s_t),s_t)=s_{t+1} \tag{40}$$

with respect to s_{t+1} and to s_t, we obtain (respectively)

$$h_1(s_{t+1},s_t)=\frac{1}{H_1(x_t,s_t)}>0 \quad \text{and} \quad h_2(s_{t+1},s_t)=-\frac{H_2(x_t,s_t)}{H_1(x_t,s_t)}\leq 0 \tag{41}$$

by Assumption 2. Finally, we take the cross partial derivative of (40) with respect to (s_{t+1},s_t) and substitute $h_1(s_{t+1},s_t)$ and $h_2(s_{t+1},s_t)$ from (41); this yields

$$h_{12}(s_{t+1},s_t)=\frac{H_2(x_t,s_t)H_{11}(x_t,s_t)-H_{12}(x_t,s_t)H_1(x_t,s_t)}{[H_2(x_t,s_t)]^3}\leq 0. \tag{42}$$

Because π is decreasing and concave, it follows from (41) and (42) that (39) is positive, confirming the desired supermodularity result. The rest of the proof is similar to that of Proposition 1.

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