INSEAD

The Business School for the World®

Faculty & Research Working Paper

Explaining Households' Investment Behavior

Dmitry S. MAKAROV
Astrid V. SCHORNICK
2010/44/FIN

Explaining Households' Investment Behavior

Dmitry S. Makarov*

Astrid V. Schornick **

March 31, 2010

This paper has benefitted from conversations with Suleyman Basak, Francisco Gomes, Anna Pavlova and Raman Uppal. All errors are ours.

* Assistant Professor of Finance at New Economic School, Nakhimovsky pr. 47, Moscow 117418, Russia. Ph: (+7-495) 129-3911 Email: dmakarov@nes.ru

** Assistant Professor of Finance at INSEAD, Boulevard de Constance, 77305 Fontainebleau, France. Ph: +33 (0) 1 60 72 49 92 Email: astrdid.schornick@insead.edu

Abstract

Building on a framework of heterogeneous uncertainty across the population, this paper provides a unified theoretical explanation for several salient features of household investment behavior: First, a fraction of households will choose not to participate in the stock market, with poorer households less likely to participate. Second, among the part of the population that does choose to invest, wealthier households choose to invest a larger share of their wealth into risky assets than less wealthy households. The model suggests that as aggregate wealth rises, we should see higher levels of stock market participation. We build on a CARA-normal framework with uncertainty-averse investors and link the level of uncertainty to investor wealth, given the empirical evidence that wealthier investors tend to acquire more costly information. The model is able to reconcile previous conflicting findings: our model shows that the intuition of models with exogenous participation restrictions—limited stock market participation leads to a higher equity premium—can indeed be retained when the participation decision is endogenized.

1 Introduction

The availability of detailed data on household decision making has revealed that households seem to follow particular patterns in their consumption and investment decisions, across different countries and different forms of data collection. However, these patterns are not easily matched by traditional representative-agent models of a utility maximizing investor. According to the 2001 Survey of Consumer Finances, 89% of households whose wealth is above the median participate in the stock market, while the participation rate is only 15% among those whose wealth is below the median. Within the group that does hold stocks in their portfolio, household wealth predicts the degree of exposure to risky assets: wealthier households invest not just larger sums into stock markets, but indeed a larger proportion of their wealth.

The robustness of these findings suggests that they are the result of optimal—albeit constrained—decision making, which should be reflected in stock markets and the compensation they provide for carrying risk. It is then perhaps not surprising that asset pricing models with a single representative agent have had limited success in explaining aggregate market behavior like returns and the risk premium. And by construction, they could provide no insight into these observed differences between individuals' investment or trading decisions.

While the theoretical literature has addressed the issues of limited participation as well as the impact of wealth on investment decisions respectively, in this paper we provide a unified theory that can explain these salient features of household investment decisions jointly. We show that costly access to information, and investors' aversion to the uncertainty that remains, leads less wealthy households to opt out of participation in the stock market. At the same time, among the group that chooses to invest, wealthier households invest a larger fraction of their wealth in risky assets. As access to information becomes less costly, the model predicts that stock market participation will increase—a trend that has been observed in the past decades. In addition to matching these empirical features of portfolio choice, the model is also able to reconcile conflicting findings of earlier papers on the resulting market risk premia. When a fraction of the population chooses not to participate in the stock market, equity risk premia rise. This is in contrast to the findings of, for example, Cao, Wang and Zhang (2005), who find in their model that endogenizing the participation decision leads to lower risk premia when

investors opt out of the stock market. Earlier papers such as Basak and Cuoco (1998) suggested a model of exogenously restricted participation to explain the high market risk premium seen in the data. Taking advantage of the tractability provided by our model, we show that this discrepancy in results can be reconciled by using a more realistic utility function for investors when endogenizing the participation decision.

The empirical literature has identified two particularly robust features. Consistent with the 2001 survey mentioned, Haliassos and Bertaut (1995) and Mankiw and Zeldes (1991) for example show that a large fraction of households do not participate at all in the stock market, with poorer households more likely not to hold any stocks. However, when looking only at the the fraction of the population that does invest in the stock market, Vissing-Jorgenssen (2002), Bertaut and Starr-McCluer (2000), as well as Perraudin and Sorensen (2000) find that the wealth share invested in risky assets tends to increase with wealth.

While there are models that explain each of these patterns individually, ours is the first, to our knowledge, to provide a unified explanation. Gomes and Michaelides (2006) as well as Cao, Wang and Zhang (2005) obtain *endogenous* limited participation, but do not explain the positive correlation between wealth and the relative amounts invested into stocks. Peress (2004) and Wachter and Yogo (2007) on the other hand provide an explanation for the increasing wealth share, but do not have limited participation—their models suggest that all agents should optimally hold at least some amount of risky assets.

The theoretical literature in asset pricing has diverged into two main directions in the effort to explain empirical regularities. One that focuses on modeling the features of the fundamental economy and the underlying risk more comprehensively. Other efforts have instead focused on sources of heterogeneity across investors as a possible explanation for observed asset market behavior. This paper fits squarely into the latter category. Fundamental risks in the economy follow a standard structure, the focus is on how heterogeneity in the wealth distribution can explain empirical findings. Our model combines a relatively standard notion of model uncertainty with heterogeneous CARA investors, who differ in the level of uncertainty they face regarding the distribution of asset payoffs.

Several studies have documented that expenditures on information about the stock market increase with a person's income, as early as Lewellen, Lease, and Schlarbaum (1977) and more

recently Donkers and Van Soest (1999). To reflect this relation in our model, we assume that wealthier households have less uncertainty about the risky assets. As a result of higher expenditures on information, investors become more familiar with financial markets, leaving less uncertainty. We impose this exogenously and do not formulate the underlying decision process about optimal information acquisition itself. While a tractable analysis of information acquisition is possible in models with Bayesian investors who learn within a single-prior setting – as in Verrecchia (1982) – a rigorous analysis of this issue in multi-prior settings is much more challenging and would impede any tractability. Investors are assumed to have constant absolute risk aversion, but we allow the parameter to be investor-specific. An investor's absolute risk tolerance increases proportionally with his wealth. This assumption, as already suggested in Merton (1987), addresses the common common criticism of CARA-normal settings, namely that CARA investors choose to invest a fixed amount into risky assets, irrespective of their level of wealth. Our assumption retains tractability while taking into consideration the empirical evidence that individuals' absolute risk aversion decreases with wealth.[1]

These two deviations from standard models—linking risk aversion as well as uncertainty to wealth—endogenously generate limited participation as well as rising *fractions* of wealth being invested as investors of the participating group become wealthier. The equilibrium returns resulting from matching these observed patterns of household stock holding, are able to reconcile previous conflicting conclusions.

Investors averse to uncertainty in the sense formalized by Gilboa and Schmeidler (1998) display a kink in their indifference curves—among the distributions they consider as possible to be driving fundamental payoffs, uncertainty averse investors will base their decisions on the 'worst-case' distribution: the one that would be the least beneficial to them as investors. Accordingly, investors who face a higher degree of uncertainty may choose not to participate at all, if equilibrium returns are not high enough to compensate them for the additional perceived uncertainty. Earlier papers like Verrecchia (1982) have shown that when information about asset payoffs is costly, less wealthy individuals will purchase less information, a link that we take as given in this paper. In a related paper, Makarov and Schornick (2010), we show that the positive relationship between wealth and amount of information acquired follows through in

[1]See e.g. Wolf and Pohlman (1983), Saha, Shumway, and Talpaz (1994), and Guiso and Paiella (2001).

equilibrium for the setup used here, with the adjustment made to CARA utility. In our setting, poorer households have higher levels of uncertainty, creating the link between wealth an non-participation. At the same time, among the group of households who do choose to participate, households with higher wealth optimally choose to invest a higher share of wealth into risky assets. This feature is the joint implication of the wealth-adjustment to CARA utility and uncertainty aversion. Simply setting an investor's absolute risk-tolerance parameter linear to his wealth would, by itself, result in decisions akin to CRRA utility: risky investment making up a constant fraction of an investor's wealth. Also taking uncertainty about the possible distribution of payoffs into consideration delivers the result that matches observed investment behavior: wealthier households tend to invest a larger share of their wealth into assets with risky payoffs.

While this does suggest that uncertainty aversion may be an explanation for the observed features of individual portfolio choice, we do not claim that this is the only plausible source of the findings. Other papers, for example At-Sahalia, Parker and Yogo (2004) link the equity premium to the consumption of luxury goods—in contrast to basic goods. Rather, we want to show that utility functions that capture multiple empirical regularities about investor behavior are better able to explain features of the stock market, in particular the equilibrium implications of limited participation for asset prices and returns. Cao et al. (2005) likewise obtain limited stock-market participation in their setting with uncertainty aversion. However, in their model, the market risk premium *decreases* when market participation declines: endogenizing the decision of investors to participate in the market makes the equity premium puzzle even worse compared to the full-participation case. This is a surprising result as it stands in contrast to the conclusion of models with *exogenous* limited participation, e.g. Basak and Cuoco (1998). They who argue that limited participation in the stock market can help resolve the puzzle. The intuition behind their result is that as aggregate risk is shared among a smaller group when a subset of the population is restricted from investing in the risky asset market, the active investors demand a higher return for carrying this risk. In our analysis we generate an endogenous level of participation from investors' optimal decisions and find support for Basak and Cuoco's result. The underlying reason for the inverse result of Cao et al. (2005) is the lack of wealth effect in investors' risk aversion, not the presence of uncertainty aversion—the source

of endogenous limited participation. Considering a utility function that better matches other features of households' investment decisions, lower market participation leads to an increase in the equity premium.

These results raise the question of which types of questions about investment can be reliably answered in a setting based on CARA utility investors. The undeniable benefit of tractability in CARA-normal settings unfortunately generates some counterintuitive results, due to the very particular ways in which CARA investors share risk across the population. Many other common utility specifications are also unable to precisely capture observed investment decisions of individuals or households. But absolute risk aversion has a significant *qualitative* impact on other pertinent economic quantities like asset prices. For example, Bernardo and Judd (2000) solve numerically a variation of the Grossman and Stiglitz (1980) model in which investors have CRRA preferences, and find that the original model' predictions about the general equilibrium effects, e.g. regarding price informativeness, are not robust once CARA is changed by CRRA. The authors go as far as claiming that "exponential utility ... is an unreasonable assumption making dynamic, general equilibrium extensions of this model unrealistic." Our results seem to support the argument that general-equilibrium conclusions of CARA-models need to be carefully examined for robustness, especially when being considered in conjunction with other features that capture heterogeneity across the population of (potential) investors.

The remainder of the paper is organized as follows. Section 2 described the economic setting. Section 3 solves for an equilibrium. Section 4 demonstrates how the model's predictions can be used to explain several empirical regularities concerning stockholdings and equity premium. Section 5 concludes. The Appendix contains all proofs.

2 Economic Setting

We consider a one-period economy populated by a continuum of investors indexed by i. There are two traded assets—a risk-free bond whose rate is normalized to zero, and a risky stock. The stock's payoff at the terminal date, $\tilde{u}$, is normally distributed with mean μ and variance σ^2. At the initial date, each investor chooses their investment portfolio by maximizing expected utility over terminal wealth, subject to the budget constraint set by their personal endowment as well

as their individual access to information. While investors know the exact value of σ, they are uncertain about the true value of expected payoff μ. Averse to this uncertainty—in addition to the fundamental risk posed by σ, investors form their investment decisions by considering the distributions of $\tilde{u}$ they deem possible, and basing their decision on their perceived 'worst-case scenario'. This approach to modeling uncertainty aversion, as distinct from risk aversion, was formalized by Gilboa and Schmeidler (1989) and has subsequently been used by various other papers.[2] Investor i believes that true mean is contained in the set $[\mu - \phi_i, \mu + \phi_i]$, where ϕ_i defines investor i's level of uncertainty. Investors differ in the degree of uncertainty they face, and across the population, ϕ_i is uniformly distributed on the interval $[\bar{\phi} - \delta, \bar{\phi} + \delta]$, where δ is a measure of uncertainty dispersion across investors.

There are a number of conceivable reasons that levels of uncertainty may vary across investors, most of which are based on a notion of (costly) access to and processing of information.[3] Our interpretation relates the uncertainty dispersion to the incentives of information acquisition, various aspects of which have been studied in the literature. Very recently, Veldkamp and Van Nieuwerburgh (2010) have linked costly information processing to optimal information acquisition decisions and under-diversification in a setup with multiple assets. To keep the model of investors' portfolio choice tractable while capturing the cross-sectional characteristics of the wealth distribution, we do not provide a formal analysis of the information acquisition in this setting. We exogenously assume a positive relation between an investor's wealth and the optimal amount of information they acquire about the financial market, consistent with the literature. Buying more information leads to a greater reduction in the level of uncertainty, implying that wealthier investors have a narrower interval of uncertainty, $[\mu - \phi_i, \mu + \phi_i]$, around the true mean μ.

We assume the following relationship between the initial wealth and the level of uncertainty:

$$\phi_i = 1/x_i. \tag{1}$$

where x_i is investor $i \in [0, 1]$'s wealth—his endowment with units of risky stock.

[2] For example Dow and Werlang (1992), Gilboa and Schmeidler (1993), Epstein and Wang (1994) as well as Hansen and Sargent (2001), who link the formulation of Gilboa and Schmeidler (1998) to robust control theory.

[3] Cao, Wang and Zhang (2005) also mention alternative explanations for the presence of limited participation, e.g. transaction or liquidity costs, and other forms of market imperfections.

This relationship expresses the idea that if investor i has little wealth, they will spend less on reducing uncertainty about the stock market, and the investor's level of ϕ_i remains higher. The particular functional form chosen is without much loss of generality, but aims to satisfy two features consistent with the data.

First, the relationship in (1) reflects decreasing returns to information acquisition — even very wealthy investors are not able to entirely eliminate uncertainty with respect to the financial market and learn μ precisely: ϕ_i converges to zero slowly. Importantly, this relationship also ensures that the level of uncertainty cannot be negative—one cannot be more than 100% about the properties of an asset's payoff distribution.Second, this relationship between uncertainty and wealth ensures that imposing a realistic cross-sectional distribution of wealth can indeed lead to ϕ_i being uniformly distributed. The functional form in (1) is consistent with wealth having a power distribution $\propto w^{-1}$, which is broadly consistent with the empirical evidence.[4] Characteristic for this distribution is that fewer people are associated to increasing levels of wealth. This property is sometimes referred to as "80-20 rule"—meaning that 20% of the population owns 80% of the wealth.

The average supply $\bar{x}$ of the risky stock in the population can be expressed in terms of the distribution of investors' uncertainty ϕ_i:

$$\bar{x} = \int_{\bar{\phi}-\delta}^{\bar{\phi}+\delta} \frac{1}{\phi_i} \frac{1}{2\delta} d\phi_i = \frac{1}{2\delta} \ln \frac{\bar{\phi}+\delta}{\bar{\phi}-\delta}. \tag{2}$$

The literature on the equilibrium effects of information acquisition have often relied on the use of CARA utility to retain tractability. This, however, comes at the cost of lacking 'wealth effects', which empirically seem to be a stable feature of individuals' portfolio choice: the extent to which one invests in risky assets rises as one becomes wealthier. In this paper we try to reconcile the two aspects by introducing the wealth effect directly in our CARA-normal setting. We link an investor's level of absolute risk tolerance r_i to their initial endowment:[5]

$$r_i = x_i/a. \tag{3}$$

[4]This inversely proportional distribution is a special case of a more general Pareto distribution often used to describe the distribution of wealth in various countries. See, e.g. Persky (1992).

[5]This ad-hoc adjustment to the utility function was used early on by Merton (1987), and is also discussed in more detail in the context of optimal information acquisition and investment in the authors' related work Makarov and Schornick (2010).

In the absence of any other factors, here uncertainty aversion or information acquisition, this investor would hold the same portfolio as a CRRA investor would in this setting: investing a constant *fraction* of his wealth into the risky asset rather than a constant *amount*, as is typical for a CARA investor.

3 Equilibrium

3.1 Portfolio Choice

In the model's single trading period, we denote by P the stock price and by D_i investor i's demand for the stock. The optimal portfolio for a CARA investor is standard and is thus given here without derivation.[6]

$$D_i = \begin{cases} \frac{r_i}{\sigma^2}(\mu - \phi_i - P) & \text{if} \quad \mu - P > \phi_i, \\ 0 & \text{if} \quad -\phi_i \leq \mu - P \leq \phi_i, \\ \frac{r_i}{\sigma^2}(\mu + \phi_i - P) & \text{if} \quad \mu - P < -\phi_i. \end{cases} \tag{4}$$

The fraction of wealth w_{0i} invested into the stock, θ_i, is given by

$$\theta_i = \frac{PD_i}{w_{0i}}.$$

Using $w_{0i} = Px_i$, θ_i can also be expressed as

$$\theta_i = \begin{cases} \frac{1}{a\sigma^2}(\mu - \phi_i - P) & \text{if} \quad \mu - P > \phi_i, \\ 0 & \text{if} \quad -\phi_i \leq \mu - P \leq \phi_i, \\ \frac{1}{a\sigma^2}(\mu + \phi_i - P) & \text{if} \quad \mu - P < -\phi_i. \end{cases} \tag{5}$$

3.2 Equilibrium with Full Participation

An equilibrium with full participation endogenously arises when the equilibrium stock price P is low enough to attract even the investors with the highest uncertainty (the highest ϕ_i). All investors will have non-zero demand for the stock and participate in the market. From (1), (3), and (4) the demand of investor i in this case is

$$D_i = \frac{1}{a\phi_i\sigma^2}(\mu - \phi_i - P).$$

The next proposition characterizes the equilibrium price.

[6] For more details in a similar setting, see also the portfolio choice in Cao et al. (2005).

Proposition 3.1 *The equilibrium price with full participation, P, is given by*

$$\mu - P = a\sigma^2 + \frac{1}{\bar{x}} \tag{6}$$

It is of interest to compare (6) with the corresponding equation derived by Cao, Wang and Zhang (2005). Using the notation from this paper, the equilibrium price in their setup is

$$\mu - P = a\sigma^2\bar{x} + \bar{\phi}. \tag{7}$$

The first term on the right-hand side of (6) and (7), respectively, represents the risk premium, which is proportional to the relative risk aversion a and the stock's variance. However, in (7), unlike our expression (6), the risk premium depends on the average supply of the risky stock. Due to the lack of a wealth effect inherent in a CARA setup, the aggregate demand is independent of the stock's supply. So as average endowment $\bar{x}$ increases, the risky stock has to become more attractive for the market to clear. This is achieved through increasing the equity premium. The setting of this paper, taking into account that wealthier investors are willing (or able) to carry more risk than poorer ones, the increase in asset supply is matched by the corresponding increase in absolute risk tolerance—investors are happy to hold a larger position in the stock at the same price.

The second term in (7), the total premium of Cao, Wang and Zhang (2005), represents the premium for uncertainty.[7] It depends on the *average* uncertainty in the economy $\bar{\phi}$, while the uncertainty dispersion δ does not affect the equilibrium price. In our model, which takes into account the interaction of wealth and information, the premium for uncertainty is inversely proportional to the average supply of the risky asset—a proxy for the average initial wealth. The intuition is that when the wealth of the average investor increases, she buys more information about the stock, which reduces the average uncertainty in the economy. As a result, the uncertainty premium component of the price decreases.

Notice that (2) links average wealth to the uncertainty dispersion δ. A higher $\bar{x}$ implies a higher uncertainty dispersion, showing that when taking into account wealth effects, the dispersion matters, not just the average uncertainty in the economy: Higher uncertainty dispersion leads to a lower ambiguity premium. Looking at the effect of average uncertainty, $\bar{\phi}$,

[7] As is common in the literature, we use the terms 'ambiguity' and 'uncertainty' interchangably, but of course distinct from 'risk'.

on the ambiguity premium, the result is in line with what we expect from the previous insights of the ambiguity aversion literature: the ambiguity premium increases as average uncertainty increases.

In order for all potential investors to participate in the stock market, the model parameters must satisfy certain conditions. If the investor with the highest level of uncertainty holds a long position in stock, the other investors—facing less uncertainty—will also participate in the stock market. Hence, full participation requires

$$\mu - (\bar{\phi} + \delta) - P > 0. \tag{8}$$

Denote the wealth of the poorest investor by x^{min}. From (1) it follows that $x^{min} = 1/(\bar{\phi} + \delta)$. Therefore full participation implies

$$\frac{1}{x^{min}} - \frac{1}{\bar{x}} < a\sigma^2, \tag{9}$$

meaning that it is most likely to occur when the dispersion of the initial wealth is low, and accordingly information will be relatively evenly distributed across the population of investors.

3.3 Equilibrium with Limited Participation

When wealth dispersion is high, some investors may choose not to hold the risky stock, as it does not provide a sufficient premium for these investor to take on the risk it has for them. This is a notable distinction from a classic Bayesian setting, where investors also have imperfect knowledge of a parameter. In such a setting, investors with less information (a more diffuse prior) will elect to invest less, but still participate: non-participation will occur only in cases of measure zero. Because of the kink in the indifference curve that uncertainty aversion—essentially first-order risk aversion—induces, this is not the case in the present setup, non-participation can occur.

Denote by ϕ^* the threshold level of uncertainty at which investors are at the margin between participating and opting out of the market for risky assets. Investors with a higher level of uncertainty, $\phi_i > \phi^*$, do not participate in the stock market, while the rest do. The threshold level ϕ^* can be determined by (4), looking at the marginal investor choosing to invest:

$$\mu - \phi^* - P = 0. \tag{10}$$

The next proposition characterizes the equilibrium P and ϕ^* for the limited participation equilibrium.

Proposition 3.2 *In the equilibrium with limited market participation, the threshold value of uncertainty ϕ^* is implicitly given by*

$$\ln \frac{\bar{\phi}+\delta}{\bar{\phi}-\delta} = \frac{\phi^*}{a\sigma^2} \ln \frac{\phi^*}{\bar{\phi}-\delta} - \frac{1}{a\sigma^2}(\phi^* - \bar{\phi} + \delta). \tag{11}$$

The equilibrium price is given by

$$P = \mu - \phi^*. \tag{12}$$

We now turn to analyzing the model's predictions and relating them to the empirical evidence.

4 Wealth Share, Market Participation and Risk Premium

Given the fact that even in developed countries a large fraction of households do not participate in the stock market, we focus mainly on which observed investment patterns can be explained by this limited participation equilibrium. Below we look at three features: 1) stock market participation and its relation to wealth, 2) wealth share invested into the risky asset, 3) equity premium.

Stock Market Participation

Investors with high levels of uncertainty are more likely not to participate in the stock market. Given the link between initial wealth and uncertainty, it is the poor households that stay away from the stock market in our model, which is consistent with the empirical evidence .

It is not unusual to observe non-participation even among the households whose wealth exceeds \$100,000, as shown by Mankiw and Zeldes (1991). From our model it follows that if an investor with some wealth A participates in the stock market, then an investor with wealth B, such that $B > A$, will also participate. While this seems to be at odds with the findings of Mankiw and Zeldes, this model could easily be extended to account for this fact. We assumed that investors' uncertainty intervals are symmetric around the true μ. This means that the

worst-case expected return of a rich investor, whose interval is narrow, is always higher than the worst-case expected return of a poor investor, whose interval is wide. In this sense, a rich investor in our model *always* seems more optimistic than a poor one.[8] As a result, we cannot have a situation with a poor investor investing and a rich investor not investing.

However, in the style of Bayesian updating, different investors may have different prior means around which their ambiguity intervals are centered. Under this assumption it is possible to have a situation of a wealthy investor, while having less ambiguity, being more pessimistic in the "worst-case scenario" sense than a poor one. Given this, a wealthier household may optimally have zero holdings in the stock market, while a poorer one participates in the stock market. Such an extension would not change our main qualitative results. Indeed, we will still have that the minimal value (left boundary) for a narrower interval is higher *on average* than that for a wider interval.[9] Wealthier households will still on average be more optimistic about the risky stock's payoff – consistent with our assumptions.

To investigate the predictions of our model regarding stock market participation, we look at the proportion of participating investors among the whole population, denoted by π, where

$$\pi = \frac{\phi^* - (\bar{\phi} - \delta)}{2\delta}. \tag{13}$$

Empirically, economic growth has led households' wealth to increase over time. Looking at this effect within an essentially static model makes use of the comparative static results, analyzing the effect of higher initial endowments on the stock market participation. Suppose the initial stock endowment of all investors increases by a factor of k, so that the new endowment of investor i equals $k * x_i$. In terms of the ensuing uncertainty, this affects both the average level of uncertainty, which becomes $\bar{\phi}/k$, and the uncertainty dispersion, which becomes δ/k.

Proposition 4.1 *Equilibrium market participation π rises as aggregate wealth increases: have*

[8] This is the case for long positions being held in the market on average, i.e. with positive net supply assets.

[9] To see the intuition behind this, consider the following simple example. Suppose that after purchasing the information the resulting uncertainty interval has a width Y and is equally likely to have any position around the true μ. As is easy to see, the left boundary of the interval is uniformly distributed between $\mu - Y$ and μ. Hence, the average worst case scenario is given by $\mu - Y/2$. The wider the interval, i.e. the higher Y, the lower is the average worst-case value.

that

$$\frac{d\pi}{dk} > 0.$$

Proposition 4.1 shows that the proportional increase of the investors' initial endowments leads to higher stock market participation. Indeed, from (9) it follows that a proportional increase of both x^{min} and $\bar{x}$ decreases the left-hand side of this inequality, thus moving the economy towards the full-participation scenario. A recent comprehensive study of the household stockholding in Europe by Guiso, Haliassos and Japelli (2003) documents that stock market participation has increased over time. This paper provides a model consistent with this finding in Proposition 4.1.

An alternative but related explanation for limited participation, also described by Guiso, Haliassos and Japellli, involves entry costs. The increase in participation is explained by the fact that these costs have been decreasing over time as a result of the increasing competition among financial institutions. Another cost-based explanation concerns transaction costs. For example, it used to be expensive to have a well-diversified portfolio of stocks since transaction costs were incurred on each individual stock. Today there are many mutual funds that allow any investor to own a certain index at a small cost.

However, cost-based explanations cannot account for some features of the data. For example, many households with high wealth, for whom entry costs are a very small fraction of the assets, do not participate in the stock market. Models with uncertainty-averse investors, such as the one analyzed in this paper, are able to explain such findings. However, for our explanation to work it is essential that investors are "sufficiently" heterogeneous in terms of their level of ambiguity. Otherwise, the full participation case is likely to occur.

Welch (2000) reviews several papers on the estimation of the equity premium and concludes: "Unfortunately, there is neither a uniformly accepted precise definition nor agreement on how the equity premium should be computed and applied." Given that even academics and professionals studying the stock market cannot agree on how to estimate the equity premium, it is natural to expect a great deal of heterogeneity across households, including wealthy ones, regarding the precision of their estimates of μ.

Wealth Share

Our model implies that wealthier investors will choose to invest a larger fraction of their wealth into risky asset markets. As mentioned earlier, making the parameter of investors' absolute risk tolerance depend on wealth only goes part of the way here: this adjustment by itself would imply investors devoting a *constant* fraction of their wealth to the risky asset. Only when also considering uncertainty aversion, do investors' portfolio shares in the risky asset *increase* in wealth. As investors become wealthier, their level of uncertainty is reduced as they purchase more information.

One of the findings of Guiso, Haliassos and Japelli (2003) is that initial wealth has a positive but small effect on the asset share invested in the stock market – for those investors who do participate. They interpret this evidence as supporting the relevance of participation costs. The reasoning is that, while wealth is important for deciding whether to participate in the stock market or not, once investors have incurred these costs there is not much difference in their stockholdings.

Our model provides an alternative explanation for this finding. The relationship between the initial wealth and the level of uncertainty is decreasing and convex. This essentially implies that returns to scale in information acquisition are decreasing. So at very high levels of wealth, a small increase in endowment has only a small effect on the level of uncertainty, and hence on portfolio choice. Effects are larger for those with lower levels of wealth—among the group even participating in the stock market, already the wealthier section of the population.

Equity Premium

Various studies investigate the relationship between limited participation and equity premium. Some papers assume, without modelling the underlying mechanisms, that some investors do not participate in the stock market, for example Basak and Cuoco (1998), Mankiw and Zeldes (1991), and Brav, Constandinides and Geczy (2002). These studies suggest that limited participation increases the equilibrium equity premium compared to the full-participation case and hence can help resolve the equity premium puzzle as described by Mehra and Prescott (1985).

Surprisingly, when endogenizing the decision whether to participate or not, Cao, Wang and

Zhang (2005) show that the opposite is true. They find that increasing the uncertainty in the economy decreases both the participation and the equity premium. This would imply that limited participation in fact makes the equity premium puzzle even worse.

The insights from our model can reconcile these findings. Cao et al. primarily want to look at the endogeneity of the participation decision, which they generate through ambiguity aversion. But the underlying utility specifications they use also differ from the papers looking at exogenous participation restriction: Basak and Cuoco use CRRA, whereas Cao et al. use CARA. Our model allows us to combine the nice features of the two settings: we have both endogenous participation and the more desirable wealth effect as present in CRRA settings.

The equilibrium in our model depends on several parameters, and changing each parameter is likely to affect both the participation rate and the risk premium. Cao et al. choose to vary the uncertainty dispersion δ. However, in our model δ is not an exogenous parameter but rather is determined by the value of the initial endowment, via the information acquisition. For this reason, when looking at comparative statics, we alter the investors' endowments.

As before, we consider a proportional increase $k > 1$ of the investors' endowments. As we have already shown, this leads to an increase in stock market participation due to more information being purchased. In the next proposition we look at the effect on the equity premium.

Proposition 4.2 *Suppose the initial stock endowment of all investors is multiplied by a factor of $k > 1$. In equilibrium, the equity premium will fall, as*

$$\frac{d(\mu - P)}{dk} < 0.$$

Proposition 4.2 can be interpreted to reveal the same conclusion as Basak and Cuoco – lower stock market participation leads to a higher equity premium. This result is also able to link the empirical evidence on two distinct relationships, that have to date been studied separately. The equity premium has been steadily declining over several decades, as demonstrated in e.g. Blanchard, Shiller and Siegel (1993), Fama and French (2002), as well as Jagannathan, McGrattan and Scherbina (2000). At the same time, stock market participation has been increasing over recent decades, shown by Bertaut and Starr-McCluer (2000) and Mankiw and Zeldes (1991).

5 Conclusion

We incorporate the so-called wealth effect into the CARA portfolio choice of ambiguity averse investors by linking an investor's absolute risk aversion to her wealth. The resulting model explains several salient features of households' stockholding. Namely, we show that the wealth share invested into risky assets increases with wealth. In addition, the model predicts that wealthier households are more likely to participate in the stock market than poorer ones. Finally, the model provides an explanation for the fact that market participation increases over time, while the equity premium decreases. Combining the features of wealth-dependent absolute risk aversion and uncertainty aversion, we can reconcile the conflicting conclusions regarding the equity premium when limited market participation is endogenous vs. exogenous. By adjusting the portfolio choice properties of CARA utility to reflect the impact of wealth on portfolio choice, the intuition that limited stock market participation leads to a higher risk premium can indeed also be generated in a setting of endogenous market participation and ambiguity aversion.

A Appendix

Proof of Proposition 3.1.

The market clearing condition is

$$\frac{1}{2\delta}\ln\frac{\bar{\phi}+\delta}{\bar{\phi}-\delta}=\int_{\bar{\phi}-\delta}^{\bar{\phi}+\delta}\frac{1}{a\phi_i\sigma^2}(\mu-\phi_i-P)\frac{1}{2\delta}d\phi_i \tag{14}$$

Computing the integral on the right-hand side yields:

$$\int_{\bar{\phi}-\delta}^{\bar{\phi}+\delta}\frac{1}{a\phi_i\sigma^2}(\mu-\phi_i-P)\frac{1}{2\delta}d\phi_i=\frac{1}{2\delta}\frac{\mu-P}{a\sigma^2}\ln\frac{\bar{\phi}+\delta}{\bar{\phi}-\delta}-\frac{1}{a\sigma^2}$$

Plugging this into (14) and dividing both sides by $\frac{1}{2\delta}\ln\left[(\bar{\phi}+\delta)/(\bar{\phi}-\delta)\right]$ gives

$$1=\frac{\mu-P}{a\sigma^2}-\frac{2\delta}{a\sigma^2\ln\left[(\bar{\phi}+\delta)/(\bar{\phi}-\delta)\right]}.$$

Finally, multiplying both sides by $a\sigma^2$, rearranging, and using the expression for $\bar{x}$ yields:

$$\mu-P=a\sigma^2+\frac{1}{\bar{x}}.$$

Q.E.D.

Proof of Proposition 3.2.

The Proof is the same as that of Proposition 3.1, when instead of $\mu-P$ we need to use ϕ^*, and the upper limit of integration of individuals demands should now be ϕ^* instead of $\bar{\phi}+\delta$.

Q.E.D.

Proof of Proposition 4.1.

From (13), it follows that

$$\frac{d\pi}{dk}=\frac{d\left[\phi^*/(2\delta)\right]}{dk}=\frac{2\delta\frac{d\phi^*}{dk}-\phi^*\frac{2\delta}{dk}}{4\delta^2} \tag{15}$$

Here we used the fact that the numerator and the denominator in $(\bar{\phi}-\delta)/(2\delta)$ are both proportional to k and so the ratio is not affected when k varies.

Denote by $F(\phi^*,k)$ the right-hand side of (11). We also need to replace $\bar{\phi}$ and δ by $\bar{\phi}/k$ and δ/k, respectively, to reflect how the model's parameters change when the investors' endowments are multiplied by k. We have

$$F(\phi^*,k)=\frac{\phi^*}{a\sigma^2}\ln\frac{k\phi^*}{\bar{\phi}-\delta}-\frac{1}{a\sigma^2}\left(\phi^*-\frac{\bar{\phi}-\delta}{k}\right).$$

To differentiate the implicit function $\phi^*(k)$, we need to compute F_{ϕ^*} and F_k. We have

$$\frac{dF}{d\phi^*} = \frac{1}{a\sigma^2}\ln\frac{k\phi^*}{\bar{\phi}-\delta} + \frac{\phi^*}{a\sigma^2}\frac{1}{\phi^*} - \frac{1}{a\sigma^2} = \frac{1}{a\sigma^2}\ln\frac{k\phi^*}{\bar{\phi}-\delta},$$

and

$$\frac{dF}{dk} = \frac{\phi^*}{a\sigma^2}\frac{1}{k} - \frac{1}{a\sigma^2}\frac{\bar{\phi}-\delta}{k^2}.$$

We now have that

$$\frac{d\phi^*}{dk} = -\frac{dF/dK}{dF/d\phi^*} = \frac{\frac{\bar{\phi}-\delta}{k^2} - \frac{\phi^*}{k}}{\ln\frac{k\phi^*}{\bar{\phi}-\delta}} \tag{16}$$

Plugging this into (15) and ignoring the denominator as we are only interested in the sign of $d\pi/dk$, we get

$$2\delta\frac{\frac{\bar{\phi}-\delta}{k^2} - \frac{\phi^*}{k}}{\ln\frac{k\phi^*}{\bar{\phi}-\delta}} + \phi^*\frac{2\delta}{k^2}$$

We now multiply the RHS by

$$\frac{k^2}{2\delta a\sigma^2}\ln\frac{k\phi^*}{\bar{\phi}-\delta}$$

which, being positive, does not change the sign of $d\pi/dk$. This yields

$$\frac{\phi^*}{a\sigma^2}\ln\frac{k\phi^*}{\bar{\phi}-\delta} - \frac{k}{a\sigma^2}\left(\phi^* - \frac{\bar{\phi}-\delta}{k}\right).$$

Evaluating the last expression at $k = 1$, we see that it equals the right-hand side in (11) and so is positive since the left-hand side is positive. Hence, an infinitesimal increase (decrease) in endowments increases (decreases) the market participation. But since this is true for any ϕ^* it means that $\phi^*(k)$ increases for all k-s until the full participation is achieved.

Q.E.D.

Proof of Proposition 4.2.

The result immediately follows from (16) because $\phi^* > \bar{\phi} - \delta$.

Q.E.D.

References

At-Sahalia, Y., J. A. Parker and M. Yogo, 2004, "Luxury Goods and the Equity Premium," *Journal of Finance*, 59(6), 2959–3004.

Basak, S., and D. Cuoco, 1998, "An Equilibrium Model with Restricted Stock Market Participation," *Review of Financial Studies*, 11, 309–341.

Bernardo, A.E., and K.L. Judd, 2000, "Asset market equilibrium with general tastes, returns, and informational asymmetries," *Journal of Financial Markets*, 3, 17–43.

Bertaut, C.C., and M. Starr-McCluer, 2000, "Household Portfolios in the United States," FEDS Working Paper No. 2000-26.

Blanchard, O.J., R. Shiller, and J.J. Siegel, 1993, "Movements in the equity premium," *Brookings Papers on Economic Activity, Macroeconomics*, 2, 75–118.

Brav, A., G. Constandinides, and C. Geczy, 2002, "Asset Pricing with Heterogenuous Consumers and Limited Participation: Empirical Evidence," *Journal of Political Economy*, 110, 793–824.

Cao H.H., T. Wang, and H.H. Zhang, 2005, "Model Uncertainty, Limited Market Participation, and Asset Prices," *Review of Financial Studies*, 18, 1219–1251.

Dow, J., and S. Ribeiro da Costa Werlang, 1992, "Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio," *Econometrica*, 60(1), 197–204.

Donkers, B., and A. van Soest, 1999, "Subjective Measures of Household Preferences and Financial Decisions," *Journal of Economic Psychology*, 20(6), 613–642.

Dow, J., and S. Werlang, 1992, "Uncertainty aversion, risk aversion and the optimal choice of portfolio," *Econometrica*, 60, 197–204.

Epstein, L.G., and T. Wang, 1994, "Intertemporal Asset Pricing under Knightian Uncertainty," *Econometrica*, 62(2), 283–322.

Fama, E.F., and K.R. French, 2002, "The equity premium," *Journal of Finance*, 57(2), 637–659.

Gilboa, I., and D. Schmeidler, 1989, "Maxmin Expected Utility Theory with Non-Unique Prior," *Journal of Mathematical Economics*, 18, 141–153.

Gilboa, I., and D. Schmeidler, 1993, "Updating Ambiguous Beliefs," *Journal of Economic Theory*, 59(1), 33–49.

Gomes, F., and A. Michaelides, 2006, "Asset Pricing with Limited Risk Sharing and Heterogeneous Agents," *Working Paper*.

Grossman, S.J., and J.E. Stiglitz, 1980, "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, 70, 393–408.

Guiso, L., M. Haliassos, and T. Japelli (2003), 2003, "Household Stockholding in Europe: Where Do We Stand and Where Do We Go?" *Economic Policy*, April, 117–164.

Guiso, L., and M. Paiella, 2005, "The Role of Risk Aversion in Predicting Individual Behavior," *Working Paper*.

Jagannathan, R., E.R. McGrattan, and A. Scherbina, 2000, "The declining U.S. equity premium," *Quarterly Review*, Federal Reserve Bank of Minneapolis, 24, 3–19.

Haliassos, M., and C.C. Bertaut, 1995, "Why Do So Few Hold Stocks?," *The Economic Journal*, 105, 1110–1129.

Hansen and Sargent (2001) " Robust Control and Model Uncertainty " Author(s): Lars Peter Hansen and Thomas J. Sargent " Source: The American Economic Review, Vol. 91, No. 2, Papers and Proceedings of the Hundred Thirteenth Annual Meeting of the American Economic Association (May, 2001), pp. 60-66

Hansen, L.P., and T. J. Sargent, 2001, "Robust Control and Model Uncertainty," *American Economic Review*, Papers and Proceedings of the Hundred Thirteenth Annual Meeting of the American Economic Association, 91(2), 60–66.

Hellwig, M.F., 1980, "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory*, 22, 477–498.

Lewellen, W.G., R.C. Lease, and G.G. Schlarbaum, 1977, "Patterns of Investment Strategy and Behavior Among Individual Investors," *Journal of Business*, 50(3), 296–333.

Makarov, D., and A. Schornick, 2010, "A Note on Wealth Effects Under CARA Utility," working paper.

Merton, R.C., 1987, "A Simple Model of Capital Market Equilibrium with Incomplete Information," *Journal of Finance*, 42(3), 483–510.

Mankiw, N. G., and S. P. Zeldes, 1991, The Consumption of Stockholders and Non-Stockholders, *Journal of Financial Economics*, 29, 97–112.

Mehra, R., and Prescott, E. C., 1985, "The equity premium: A puzzle," *Journal of Monetary Economics*, 15, 145–61.

Peress, J., 2004, "Wealth, Information Acquisition and Portfolio Choice," *Review of Financial Studies*, 17(3), 879–914.

Perraudin, W.R.M., and B.E. Sorensen, 2000, "The demand for risky assets: Sample selection and household portfolios," *Journal of Econometrics*, 97(1), 117–144.

Persky, J., 1992, "Retrospectives: Pareto's Law," *Journal of Economic Perspectives*, 6(2), 181–192.

Saha, A., C.R. Shumway, and H. Talpaz, 1994, "Joint Estimation of Risk Preference Structure and Technology Using Expo-Power Utility ," *American Journal of Agricultural Economics*, 76(2), 173–184.

Van Nieuwerbergh, S., and L. Veldkamp, 2010, "Information Acquisition and Under-Diversification", *Review of Economic Studies*, forthcoming.

Verrecchia, R., 1982, "Information Acquisition in a Noisy Rational Expectations Economy," *Econometrica*, 50(6), 1415–1430.

Vissing-Jorgensen, A., 2002, "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economics*, 110, 825–853.

Vissing-Jorgensen, A., 2003, "Perspectives on Behavioral Finance: Does "Irrationality" Disappear with Wealth? Evidence from Expectations and Actions," *NBER Macroeconomics Annual.*

Wachter, J., and M. Yogo, 2007, "Why Do Household Portfolio Shares Rise in Wealth?," *Working Paper.*

Welch, I., 2000, "Views of Financial Economists on the Equity Premium and on Professional Controversies," *Journal of Business*, 73(4), 501–537.

Wolf, C., and L. Pohlman, 1983, "The Recovery of Risk Preferences From Actual Choices," *Econometrica*, 51(3), 843–850.

Europe Campus
Boulevard de Constance
77305 Fontainebleau Cedex, France
Tel: +33 (0)1 60 72 40 00
Fax: +33 (0)1 60 74 55 00/01

Asia Campus
1 Ayer Rajah Avenue, Singapore 138676
Tel: +65 67 99 53 88
Fax: +65 67 99 53 99

www.insead.edu

Printed by INSEAD

INSEAD

The Business School
for the World®