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Preferences: Response Error,
Heterogeneity, Adaptive
Questionnaires, and Experimental
Evidence from Mortgagers

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2010/52/DS

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by

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July 2010

The authors would like to thank Martine Baldassi, Jacob Abernethy, Amy Krosh and Steven Atlas for their assistance in this project. The authors are listed in reverse alphabetical order.

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Estimation of Risk and Time Preferences: Response Error, Heterogeneity, Adaptive Questionnaires, and Experimental Evidence from Mortgagers

Abstract

We develop a methodology for the measurement of the parameters of cumulative prospect theory and time discounting models based on tools from the preference measurement literature. These parameters are typically elicited by presenting decision makers with a series of choices between hypothetical alternatives, gambles or delayed payments. We present a method for adaptively designing the sets of hypothetical choices presented to decision makers, and a method for estimating the preference function parameters which capture interdependence across decision makers as well as response error. We apply our questionnaire design and estimation methods to a study of the characteristics of homeowners who owe more on their mortgage than the current value of the underlying real estate asset. Our estimates indicate that such homeowners have larger discount rates and present bias than others, but do not differ in their risk preferences.

Keywords: *Prospect Theory, Time Discounting, Bayesian Statistics, Adaptive Experimental Design, Revealed Preference.*

1. Introduction

The development of models of behavior, such as prospect theory (PT) (Kahneman and Tversky 1979, Tversky and Kahneman 1992) or time discounting models (Frederick et al. 2002 and references therein) has enabled a vast amount of research that relies on such models to explain behaviors of people in different experimental as well as field setups (e.g., Barberis et al. 2001, Harrison et al. 2002, Tanaka et al. 2010, Camerer et al. 2003 and references therein). These papers typically elicit the parameters of a model, such as PT, by asking subjects to make decisions about alternatives, such as gambles in the case of PT, and then use these parameters to reach empirical conclusions about people's behavior. For example, the recent work of Tanaka et al. (2010) examines how Vietnamese villagers' risk preferences relate to various socio-economic variables. The authors measure risk and time preferences by eliciting the switching points of subjects in monotonic series of lottery pairs. The empirical conclusions such experimental or field studies are able to reach may depend, of course, on the quality of the parameter estimates.

In this paper we present a novel methodology for eliciting parameters of cumulative prospect theory (CPT) (Tversky and Kahneman 1992) as well as of a quasi-hyperbolic time discounting (QTD) model (Frederick et al. 2002, Benhabib et al. 2010). Although the methodology is developed for estimating parameters of specific models, namely CPT and QTD, it can be also adapted to other models. We then test this methodology in a study of the characteristics of "underwater" homeowners, namely homeowners who owe more on their mortgage than the current value of the underlying real estate asset.

The main contribution of this paper is methodological. Our parametric methodology for eliciting and estimating parameters of CPT and QTD relies on concepts from the vast marketing literature of preference measurement and conjoint analysis (e.g., Green and Rao 1972, Srinivasan and Shocker 1973, Lenk et al. 1996, Allenby and Rossi 1999, Rossi and Allenby 2003). Therefore our work bridges the preference measurement literature in marketing with the preference assessment literature in decision theory. Our methodology addresses certain limitations of many existing parametric methods for eliciting risk and time preferences: the modeling of response error, the capture of similarities/heterogeneity across individuals, and the efficiency of the questions used for gathering data to estimate the parameters. We introduce and test an estimation method that addresses the first two limitations, and a questionnaire design method that addresses the third. The proposed estimation and questionnaire design methods may be used together or independently (i.e., the estimation method may be ap-

plied to questionnaires that were not designed using the proposed questionnaire design method, and vice versa).

We apply the proposed methodology to a study that examines risk and time preferences of underwater homeowners – as defined above. The proposed methodology produces parameter values consistent with those reported in the literature in terms of ranges and averages. We find that underwater respondents have on average significantly larger present bias (smaller present bias parameters) and discount rates. On the other hand, over- and underwater respondents are found to have similar CPT parameters. We also compare the estimated parameter values with those elicited for the same study with a standard non-adaptive methodology used in the past. This comparison highlights the criticality of measurement tools for empirical behavioral research. In particular, while the approach proposed in this paper allows us to uncover statistically significant relationships between the time preferences of a homeowner and the status of their mortgage, such relationships do not appear to be significant when the parameters are measured using the standard non-adaptive method.

1.1 Related Methodologies

A number of methodologies for measuring parameters of preference models such as CPT or QTD have been developed in the decision analysis area (e.g. Abdellaoui et al. 2008, Delqu   1997, Laskey and Fischer 1987, Wakker and Deneffe 1996 among others). These methods may differ on a number of features, including the type of responses they elicit (e.g., payoff or probability), whether choices or indifference judgments are elicited, whether the questions are chained or not, whether parametric forms are assumed or not, and so on. However, to the best of our knowledge, none of these methods and devices explicitly address/exploit individuals' heterogeneity. One exception is the recent work of Bruhin et al. (2010) who apply latent-class analysis to the estimation of probability distortion.¹ In contrast, our approach captures heterogeneity both when designing questionnaires and when estimating the parameters for all risk and time preference parameters. Moreover, traditional methods often use adaptive questions, but in a somewhat restricted manner: typically, choice questions are dynamically iterated according to a bisection search process so as to zero in on a point or

¹ Latent-class analysis has a long history in marketing (see Kamakura and Russell 1989 for one of the earlier applications). Comparisons of latent-class with hierarchical Bayes approaches have suggested that both fit the data equally well overall (see for example Andrews, Ainslie and Currim 2002 and Andrews, Ansari and Currim 2002).

small interval where preference switches. The questions in our designs are adaptively developed with no restrictions from previous questions except that each question adds the most information in a certain statistical sense at each point given the subject's responses to the previous ones.

Our estimation and questionnaire design approach is based on two key developments in the large literature on preference measurement methods: hierarchical Bayes estimation (Allenby and Rossi 1999, Rossi and Allenby 2003, Rossi et al. 2005), experimental design (Ford et al., 1989, Kuhfeld et al., 1995, McFadden 1974, and references therein) and in particular the recently developed adaptive designs (Sawtooth Software 1996, Toubia et al. 2003, Toubia et al. 2004, Abernethy et al. 2008). The first is a statistical approach to modeling heterogeneity of people and estimating parameters of models for different subjects simultaneously. The latter develops designs of various information gathering methods and studies their statistical properties. Although both have been widely and successfully used for preference measurement in marketing both by researchers and practitioners, to the best of our knowledge they have not been used for the purpose of eliciting parameters of either risk or time preferences.

The estimation and questionnaire design frameworks that we develop apply similarly to the estimation of risk and time preferences. We focus on particular risk and time preference models here but the framework can also apply to others. We present our estimation method in Section 2 and our questionnaire design method in Section 3. We apply and test these methods in Section 4 in an online study of the decision-making characteristics of underwater homeowners. We conclude in Section 5.

1.2 Background and Notation

For simplicity we discuss in Sections 2 and 3 the methodology using a general notation that applies to any parametrically specified model of risk and/or time preferences. This reflects some overlap in the notation for CPT and QTD that we clarify, as necessary.

1.2.1 Risk Preferences and Cumulative Prospect Theory

PT (Kahneman and Tversky 1979) and its extension CPT (Tversky and Kahneman 1992) are widely regarded as the most prominent descriptive models of choice under risk. CPT has three main features: a value function defined on gains and losses, which accounts for the fact that people are sensitive to changes in wealth rather than total wealth; loss aversion, which reflects that people are more sensitive to losses than to gains of the same magnitude; and probability weighting, which captures the fact that people tend to weigh probabilities in a

non-linear fashion, particular in the presence of uncertainty. CPT allows probability weighting to differ for gains and losses (Tversky and Kahneman 1992), but for simplicity's sake we assume here, as in other work (e.g., Tanaka et al. 2010), that weighting is the same for gains and losses.

The gambles we use are defined by $\{x,p,y,q\}$ such that the outcome of the gamble is x with probability p , and y with probability q ($= 1 - p$). We work within the framework of CPT reviewed above and assume that the probability weighting function is as proposed by Prelec (1998). Therefore, the value derived by a decision maker from a given gamble is defined by three parameters $\{\alpha,\sigma,\lambda\}$ which capture respectively the distortion of probabilities, the curvature of the value function, and loss aversion. Formally, this value $U(x,p,y,q,\alpha,\sigma,\lambda)$ is given by (without loss of generality, we assume $|x| > |y|$; otherwise $\{x,p\}$ and $\{y,q\}$ may be swapped):

$$U(x,p,y,q,\alpha,\sigma,\lambda) = \begin{cases} v(y) + \pi(p)(v(x) - v(y)) & \text{if } x > y > 0 \text{ or } x < y < 0 \\ \pi(p)v(x) + \pi(q)v(y) & \text{if } x < 0 < y \end{cases}$$

where
$$v(x) = \begin{cases} x^\sigma & \text{for } x > 0 \\ -\lambda(-x)^\sigma & \text{for } x < 0 \end{cases}$$

and
$$\pi(p) = \exp[-(-\ln p)^\alpha]$$

We elicit the CPT parameters by asking decision makers to make a series of choices between pairs of gambles. We index decision makers by i ($i=1,\dots,I$) and denote by w_i the vector of parameters for decision maker i , namely $w_i = [\alpha_i; \sigma_i; \lambda_i]$. We index questions by j ($j=1,\dots,J$), such that question j for respondent i consists in choosing between a gamble characterized by $\{x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1\}$ and a gamble characterized by $\{x_{ij}^2, p_{ij}^2, y_{ij}^2, q_{ij}^2\}$. Without loss of generality, we assume that the first gamble $\{x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1\}$ is always chosen over the second gamble $\{x_{ij}^2, p_{ij}^2, y_{ij}^2, q_{ij}^2\}$ (otherwise we just re-label the gambles).

1.2.2 Time Preferences and Quasi-Hyperbolic Discounting

In experimental studies of time preferences, subjects are typically faced with choices between a smaller-sooner reward and a larger-later reward. The choice alternatives take the form (x, t) , meaning a reward x to be received at time t , or t periods (e.g., days) from now. The basic model to represent preferences for payoffs occurring in time is Discounted Utility $U(x,t) = v(x)d(t)$. By and large, the literature on delayed reward preferences is concerned with the shape and nature of the discount function d . Classic forms are exponential discounting (the economically normative model, with constant discount rate) and hyperbolic discount-

ing, which implies a discount rate decreasing with time, as observed in behavioral studies. Some models allow discounting to be a function of payoff x , in addition to delay. Here we use a “quasi-hyperbolic” discount function (Angeletos et al. 2001, Benhabib et al. 2010) and a linear value function of payoff. Quasi-hyperbolic discounting refers to the fact that the valuation of rewards declines more sharply for near future rewards than those in the more distant future. Specifically, the QTD model we estimate is of the form (Benhabib et al. 2010):

$$U(x, t) = xd(t)$$

$$\text{where } d(t) = \begin{cases} 1 & \text{for } t = 0 \\ \beta \exp(-rt) & \text{for } t > 0 \end{cases}$$

For $\beta < 1$, the discount function presents a discontinuous drop at $t = 0$, which reflects the empirical observation that the present $t = 0$ is overweighed relative to any future $t > 0$. This is also called “present bias” (O'Donoghue and Rabin 1999). Notice the use of notation U both for QTD and CPT. Our approach may be extended to other functional forms, but we focus on quasi-hyperbolic discounting here for simplicity and due to the popularity of this model.

We elicit the vector of QTD parameters $[\beta_i, r_i]$ of decision maker i through a series of choices between pairs of delayed payments – where the delay of an immediate payment is zero. Question j for respondent i consists of choosing between $\{x_{ij}^1, t_{ij}^1\}$ and $\{x_{ij}^2, t_{ij}^2\}$. Again, we label as 1 the alternative chosen by the respondent.

2. Hierarchical Bayes Estimation of Cumulative Prospect Theory and Quasi-Hyperbolic Discounting Parameters

We begin with the parameter estimation methodology, which may be used in conjunction with any questionnaire design method that produces a series of independent pairwise choices between gambles (for CPT) or delayed payments (for QTD). Although we work with pairwise choice data, the methodology can be applied for other types of questions (e.g., certainty equivalents, amounts of payments, etc).

We assume that we have responses to J choice questions from I respondents as noted above. We show how the estimation of the respondents’ value function parameters may be performed using a hierarchical Bayes framework (Allenby and Rossi 1999, Rossi and Allenby 2003, Rossi et al. 2005). This framework allows us to (i) capture response error, (ii) capture similarities across decision makers, and (iii) model how these similarities relate to similarities in covariates that describe the decision makers (e.g., gender, age, etc). For ease of exposition, we build the method up by introducing each of these three features in sequence,

and then discuss how to estimate the parameters by sampling from the posterior distribution assuming commonly-used distributions for the priors.

2.1. Setup of the Estimation Method

2.1.1 Response Error

We assume that faced with a choice between two options (gambles or delayed payments) a decision maker will not systematically choose the one with the higher value. There is some noise in the respondents' choices, as has long been recognized in the literature (e.g. Luce 1958). There are many ways to model response error (e.g., Hey and Orme 1994). We make a particular choice here but other models could also be used. We introduce response error by modeling the probability that decision maker i chooses option 1 over option 2 in question j using a logistic specification common in choice modeling and in particular in preference measurement (Louviere et al. 2000), namely:

$$P_{ij} = \frac{\exp(\delta U(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, w_i))}{\exp(\delta U(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, w_i)) + \exp(\delta U(x_{ij}^2, p_{ij}^2, y_{ij}^2, q_{ij}^2, w_i))}$$

for CPT, and

$$P_{ij} = \frac{\exp(\delta U(x_{ij}^1, t_{ij}^1))}{\exp(\delta U(x_{ij}^1, t_{ij}^1)) + \exp(\delta U(x_{ij}^2, t_{ij}^2))}$$

for QTD. Parameter δ captures the amount of response error (it is equivalent to a logit scale parameter). Higher values of δ imply less response error (the choice probabilities converge to 0 or 1).

2.1.2 Similarities across Decision Makers

Instead of treating each decision maker independently, as in traditional decision analysis elicitation methods, we model similarities between them by formulating a typical (Allenby and Rossi 1999, Rossi and Allenby 2003, Rossi et al. 2005) normally distributed Bayesian prior distribution on w_i :

$$w_i = [\alpha_i; \sigma_i; \lambda_i] \sim N(w_0, D) \text{ for CPT; and } w_i = [\beta_i; r_i] \sim N(w_0, D) \text{ for QTD.}$$

This prior distribution effectively shrinks w_i towards a common vector w_0 . The amount of shrinkage is governed by the covariance matrix of the prior distribution, D . Note that matrix D and vector w_0 are not the same for CPT and QTD: D is a 3x3 matrix for CPT and 2x2 for QTD, and w_0 is a 3-dimensional vector for CPT and 2-dimensional for QTD.

It is important to note that the fact that the prior distribution on w_i is normal does not imply that the final estimates will follow a normal distribution. Indeed, Bayesian statistics combine the prior distribution with the likelihood implied by the logistic probabilities defined above, and the shape of the posterior distribution does not necessarily coincide with the shape of the prior distribution.

The values of the parameters of the prior distribution on w_i , w_0 and D , are usually not known a priori. For example, the decision makers could indeed be completely independent, in which case the “correct” value for D may be a very large matrix. The decision makers could also be in fact very similar, in which case the “correct” value for D would be much smaller. Hierarchical Bayes allows capturing this uncertainty on w_0 and D by treating them as random variables themselves. The posterior distributions of these random variables are driven by the data and by a prior that is specified by the researcher. In other words, we formulate a prior distribution on the parameters of the prior distribution themselves, hence the hierarchical nature of the model. The prior on w_i is referred to as the first-stage prior, and the priors on w_0 and D as the second-stage priors. The second-stage priors are usually selected to be as uninformative as possible, in order to let the value of w_0 and D be determined primarily by the data (see below).

2.1.3 Capturing the Impact of Covariates on the Parameters

As a more general setup, it is possible that the similarities across decision makers be driven by similarities in covariates that influence the preference model parameters. For example, Tanaka et al. (2010) explore the relation between the CPT parameters and various demographic variables for Vietnamese villagers. Our model allows us to capture the effect of such covariates on the parameters through the prior distribution on w_i . In particular, in situations in which a given set of covariates are thought to influence w_i , the prior distribution on w_i may be replaced with:

$$w_i = [\alpha_i; \sigma_i; \lambda_i] \sim N(\Theta.z_i, D) \text{ for CPT and } w_i = [\beta_i; r_i] \sim N(\Theta.z_i, D) \text{ for QTD}$$

where z_i is a set of covariates for respondent i (including an intercept), and Θ is a matrix capturing the relationship between these covariates and the mean of the first-stage prior (this matrix is estimated) – note again the abuse of notation by using Θ for both CPT and QTD. For example, if the covariates are age (in years) and gender (binary equal to 1 for male and 0 for

female) then for CPT the 3x3 dimensional matrix Θ is $\Theta = \begin{bmatrix} \theta_{1,1} & \theta_{1,2} & \theta_{1,3} \\ \theta_{2,1} & \theta_{2,2} & \theta_{2,3} \\ \theta_{3,1} & \theta_{3,2} & \theta_{3,3} \end{bmatrix}$ where $\theta_{1,1}$, $\theta_{2,1}$,

$\theta_{3,1}$ are the intercept parameters for α , σ and λ respectively, $\theta_{1,2}$, $\theta_{2,2}$, $\theta_{3,2}$ capture the effect of age on the parameters α , σ and λ respectively, and $\theta_{1,3}$, $\theta_{2,3}$, $\theta_{3,3}$ capture the effect of gender on the parameters α , σ and λ respectively. In this case the second-stage prior is defined on Θ and D instead of w_0 and D . This specification allows us to capture the effects of covariates on the value function parameters directly, instead of estimating the parameters first and then regressing them on covariates (e.g., Tanaka et al. 2010).

Note that the case without covariates is a special case of this formulation, in which the covariates are limited to an intercept (the vector w_0 corresponds to the first column of Θ). Therefore we use the formulation with covariates as the default formulation, as it nests the formulation without covariates.

2.2 Hierarchical Bayes Estimation

We now combine the likelihood function (capturing the link between value and response probabilities), the first-stage prior (capturing similarities across decision makers), and a second-stage prior (prior distribution on the parameters of the first-stage prior) in the following hierarchical Bayes model for CPT:

$$\text{Likelihood: } \frac{\exp(\delta U(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, \alpha_i, \sigma_i, \lambda_i))}{\exp(\delta U(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, \alpha_i, \sigma_i, \lambda_i)) + \exp(\delta U(x_{ij}^2, p_{ij}^2, y_{ij}^2, q_{ij}^2, \alpha_i, \sigma_i, \lambda_i))}$$

$$\text{First-stage prior: } w_i = [\alpha_i; \sigma_i; \lambda_i] \sim N(\Theta \cdot z_i, D)$$

$$\delta: \text{diffuse (improper) on } \mathfrak{R}^+$$

$$\text{Second-stage prior: } \Theta: \text{diffuse (improper) on } \mathfrak{R}^{+3}$$

$$D \sim \text{Inverse Wishart}(\eta_0, \eta_0 \cdot \Delta_0)$$

and a similar one for QTD, (replacing U , w_0 , and w_i accordingly). With the exception of the specification of the likelihood function which is specific to risk and time preferences, the specifications of all our prior distributions are standard in the hierarchical Bayes literature (see for example Rossi and Allenby 2003 or Rossi et al. 2005). We select an inverse Wishart distribution on D because it is conjugated with the likelihood function implied by $w_i \sim N(w_0, D)$, i.e., the posterior distribution on D is inverse Wishart as well. We use typical parameter values for the inverse Wishart prior on D that make this prior uninformative (Allenby and Rossi 1999, Rossi and Allenby 2003): $\eta_0 = p+3$ and $\Delta_0 = I$, where p is the number

of parameters in the model ($p=3$ for CPT and $p=2$ for QDT). The use of diffuse improper priors on δ and Θ (i.e., the priors on these parameters are completely “flat”) ensures that the posterior distribution of these parameters is driven only by the data.

Hierarchical Bayes estimation consists of sampling from the posterior distribution of the parameters. The posterior distribution is simply given by Bayes’ rule: $P(\{w_i\}, \Theta, D, \delta | data) \propto P(data | \{w_i\}, \delta).P(\{w_i\} | \Theta, D).P(\Theta).P(D).P(\delta)$ where $P(data | \{w_i\}, \delta)$ is given by the likelihood function, $P(\{w_i\} | \Theta, D)$ is given by the first-stage prior, and $P(\Theta), P(D), P(\delta)$ are the priors on Θ, D , and δ respectively. Drawing from this posterior distribution is achieved by using a Markov Chain Monte Carlo (MCMC) algorithm. Details are provided in the Appendix. MCMC provides a set of values drawn from the posterior distribution, which may be used to produce point estimates of the parameters, or to make other types of inference. Point estimates are typically obtained by averaging the draws from the MCMC, which approximates the mean of the posterior distribution.

3. Adaptive Selection of Choice Alternatives

In the previous section we proposed a method for estimating the model parameters given choices between gambles or delayed payments made by a panel of decision makers. We now propose a new methodology for selecting the pairs of gambles or delayed payments presented to each decision maker. We focus on choices between gambles for CPT and delayed payments for QTD as often done in the literature (e.g., Fehr and Goette 2007, Tom et al. 2007, Tanaka et al. 2010). However, the key questionnaire design criterion discussed below and the approach used for this methodology can be applied for other types of questions (e.g., certainty equivalents, amounts of payments, etc).

Our approach is based on principles from the experimental design literature (Ford et al. 1989, Kuhfeld et al. 1995, McFadden 1974, Steinberg and Hunter 1984, and references therein). Although these principles have been used to develop methodologies for preference elicitation before, for example for conjoint analysis in Marketing, to the best of our knowledge they have not been used for the purpose of eliciting risk or time preference parameters.

Experimental designs have been traditionally constructed in a non-adaptive fashion (i.e., before the collection of the experimental data). However, *adaptive* experimental designs have received a lot of attention with the advent of online data collection. Indeed, when data is collected online, computations may be performed during the experiment. In a preference measurement context this implies that each new question may be constructed based on the re-

spondent's answers to the previous questions (Abernethy et al. 2008, Sawtooth Software 1996, Toubia et al. 2003, 2004, 2007). Our method, designed for online data collection, is adaptive as well.

Before describing the method in details, we present and explain the main criterion that constitutes the foundations of the method.

3.1. Questionnaire Design Criterion

Let us consider a decision maker with true (CPT or QTD) parameter w_i , who has responded to q binary choice questions. Our challenge is to construct the $(q+1)^{\text{th}}$ question for that decision maker, which in the case of CPT will consist of a pair of gambles denoted by $(\{x_{i(q+1)}^A, p_{i(q+1)}^A, y_{i(q+1)}^A, q_{i(q+1)}^A\}, \{x_{i(q+1)}^B, p_{i(q+1)}^B, y_{i(q+1)}^B, q_{i(q+1)}^B\})$, and in the case of QTD will consist of a pair of delayed payments denoted by $(\{x_{ij}^A, t_{ij}^A\}, \{x_{ij}^B, t_{ij}^B\})$ (A and B will be labeled 1 and 2 after the decision maker's choice, i.e., the preferred gamble will be relabeled as gamble 1).

A central idea in the rich literature on experimental design is to design experiments (in our case questionnaires) such that the asymptotic covariance matrix of the maximum likelihood estimate (MLE) of the relevant parameters is as "small" as possible, according to some defined measure. Intuitively, this ensures that the parameters are elicited with as little uncertainty as possible. It has been shown (see for example McFadden 1974 and Newey and McFadden 1994) that under mild conditions, the asymptotic covariance matrix of the MLE is equal to the inverse of the Hessian (i.e., second derivative matrix) of the log-likelihood function (taken at the maximum likelihood estimate). Therefore reducing the asymptotic covariance matrix of the MLE is achieved by maximizing some norm of the Hessian of the likelihood function. Different norms can and have been used, such as the absolute value of the determinant, the absolute value of the largest eigenvalue, the trace norm, etc.

Given our Bayesian framework, a reasonable design criterion using the same insight is that each new question should maximize the Hessian of the posterior distribution at its mode. The mode of the posterior distribution is the maximum a posteriori estimate (DeGroot 1970). Intuitively, maximizing the Hessian of the posterior distribution at its mode is likely to decrease the variance of the posterior distribution, therefore decreasing our uncertainty on the decision maker's value function parameters.²

²One could also consider the Hessian at the mean or other summary statistics of the posterior, which, however, would be computationally challenging as estimating the posterior mean requires sampling.

In summary, the design criterion behind our method is to construct questions that maximize the Hessian of the posterior distribution at its mode. Applying this criterion decreases the variance of the posterior distribution on the decision maker’s vector of parameters. We now show how we implement this criterion.³

3.2. Details of the Method

One of the key constraints in adaptive experimental design in our context (and for preference measurement in general) is that computations between questions have to be completed quickly enough to avoid delays that would be noticeable by the decision maker. Implementing the design criterion outlined above requires performing the following computations between the q^{th} and $(q+1)^{\text{th}}$ question for each decision maker and for each value of q : (i) identify the mode of the posterior distribution (the posterior distribution changes after each new question), (ii) identify the question that maximizes a norm of the expected value of the Hessian of the posterior at its mode. These computations are performed as follows:

(i) Identify the mode of the posterior distribution: Allowing the parameters w_0 , D and δ to be updated between each question or even between each respondent would result in excessive delays. Therefore, our method relies on a prior distribution formed before starting to collect the data: $w_i \sim N(\hat{w}_0, \hat{D})$ and on a prior point estimate of δ , $\hat{\delta}$. The parameters \hat{w}_0 , \hat{D} and $\hat{\delta}$ are set before the start of the data collection and are not updated until after the completion of the data collection (at which point the full estimation method described in the previous section may be applied to the entire dataset). These values could be obtained from a pre-test with a relatively small number of respondents (Toubia et al. 2007), from prior beliefs (Huber and Zwerina 1996, Sándor and Wedel 2001) or from previous studies (e.g., Wu and Gonzalez 1996 for CPT parameters). Note that the prior $N(\hat{w}_0, \hat{D})$ does not have to be informative. Given the assumed prior distribution, the posterior likelihood on decision maker i ’s parameters after q questions is:

Using the mode only provides a conservative estimate of the benefits of using the proposed approach, and it is our hope that future work will explore variations. Note also that unlike the MLE case, we are not aware of any formal link between the Hessian of the mode of the posterior and the asymptotic covariance matrix of the mode (or other summary statistics) of the posterior. We limit ourselves to an empirical investigation of this criterion.

³ Both the estimation and questionnaire design code are available from the authors, as well as the code used to implement the adaptive questionnaire design methods online.

$$P(w_i | data, \hat{w}_0, \hat{D}, \hat{\delta}) \propto P(data | w_i, \hat{\delta}) \cdot P(w_i | \hat{w}_0, \hat{D})$$

$$\propto \prod_{j=1}^q \frac{\exp(\hat{\delta} \cdot U(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, w_i))}{\exp(\hat{\delta} \cdot U(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, w_i)) + \exp(\hat{\delta} \cdot U(x_{ij}^2, p_{ij}^2, y_{ij}^2, q_{ij}^2, w_i))} \cdot \exp\left(-\frac{1}{2}(w_i - \hat{w}_0)^T \hat{D}^{-1}(w_i - \hat{w}_0)\right)$$

for CPT, and similarly for QTD after replacing U . The mode of this posterior may be computed very quickly by maximizing the log of this expression using Newton's method. Let \hat{w}_{iq} be the mode of the posterior based on q questions.

(ii) *Identify the question that maximizes the Hessian*: We refer to the set of possible questions as “candidate” questions. These questions consist of all possible pairs of gambles or delayed payments from a candidate set – see below the choices we made in our specific field study. Candidate pairs of alternatives are evaluated based on their effect on the Hessian of the posterior at its mode, and the $(q+1)^{\text{th}}$ question is chosen to maximally increase a norm of this Hessian. Following the literature on experimental design, we use the absolute value of the determinant as the norm of the Hessian. In the case of CPT the Hessian of the posterior likelihood on decision maker i 's parameters after q questions, computed at \hat{w}_{iq} , is equal

$$\text{to } H_{iq} = \sum_{j=1}^q h(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, x_{ij}^2, p_{ij}^2, y_{ij}^2, q_{ij}^2, \hat{w}_{iq}, \hat{\delta}) - \frac{\hat{D}^1}{2}, \quad \text{where}$$

$h(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, x_{ij}^2, p_{ij}^2, y_{ij}^2, q_{ij}^2, \hat{w}_{iq}, \hat{\delta})$ is the Hessian corresponding to one question. We identify the pair $(\{x_{i(q+1)}^A, p_{i(q+1)}^A, y_{i(q+1)}^A, q_{i(q+1)}^A\}, \{x_{i(q+1)}^B, p_{i(q+1)}^B, y_{i(q+1)}^B, q_{i(q+1)}^B\})$ that maximizes:⁴

$$p_A \cdot |\det(H_{iq} + h(x_{i(q+1)}^A, p_{i(q+1)}^A, y_{i(q+1)}^A, q_{i(q+1)}^A, x_{i(q+1)}^B, p_{i(q+1)}^B, y_{i(q+1)}^B, q_{i(q+1)}^B, \hat{w}_{iq}, \hat{\delta}))|$$

$$+ p_B \cdot |\det(H_{iq} + h(x_{i(q+1)}^B, p_{i(q+1)}^B, y_{i(q+1)}^B, q_{i(q+1)}^B, x_{i(q+1)}^A, p_{i(q+1)}^A, y_{i(q+1)}^A, q_{i(q+1)}^A, \hat{w}_{iq}, \hat{\delta}))|$$

where

$$p_A = \frac{\exp(\hat{\delta} \cdot U(x_{i(q+1)}^A, p_{i(q+1)}^A, y_{i(q+1)}^A, q_{i(q+1)}^A, \hat{w}_{iq}))}{\exp(\hat{\delta} \cdot U(x_{i(q+1)}^A, p_{i(q+1)}^A, y_{i(q+1)}^A, q_{i(q+1)}^A, \hat{w}_{iq})) + \exp(\hat{\delta} \cdot U(x_{i(q+1)}^B, p_{i(q+1)}^B, y_{i(q+1)}^B, q_{i(q+1)}^B, \hat{w}_{iq}))}$$
 is the

probability that the decision maker chooses gamble A (computed based on \hat{w}_{iq}), and

$H_{iq} + h(x_{i(q+1)}^A, p_{i(q+1)}^A, y_{i(q+1)}^A, q_{i(q+1)}^A, x_{i(q+1)}^B, p_{i(q+1)}^B, y_{i(q+1)}^B, q_{i(q+1)}^B, \hat{w}_{iq}, \hat{\delta})$ is the value of the Hessian

⁴ Our approach is consistent with Bayesian Experimental Design (Chaloner and Verdinelli, 1995). Our utility function is the Hessian of the posterior at the mode of the posterior. Our optimization is approximate to the extent that we approximate $H_{i(q+1)}(\hat{w}_{i(q+1)})$ with $H_{i(q+1)}(\hat{w}_{iq})$, and we compute p_A and p_B based on the point estimate \hat{w}_{iq} instead of the entire posterior after q questions.

sian after $(q+1)$ questions if gamble A is chosen. The case of QTD is similar, with the appropriate changes in notations and definitions.

In summary, questions are constructed adaptively by performing the following computations between the q^{th} and the $(q+1)^{\text{th}}$ question, for each respondent i and for all values of q :

- Update the value of the mode of the posterior distribution, $\hat{w}_{i,q}$.
- Out of all candidate questions that have not been put to that respondent, select the one that will maximize the expected value of the determinant of the Hessian of the posterior distribution evaluated at its mode.

3.3. Practical Considerations

When applying the proposed approach in practice, three considerations are (i) the choice of the very first question, (ii) the set of candidate gambles (for CPT) and delayed payments (for QTD) from the which the questions will be constructed, and (iii) the values of the prior parameters \hat{w}_0 , \hat{D} and $\hat{\delta}$. For the very first question we do not have any information about the respondents hence we cannot adapt it. In our application we set this first question arbitrarily to a choice between gambles $\{100,0.1,-10,0.9\}$ and $\{1,0.9,-10,0.1\}$ for the CPT questionnaire and $(120,0)$ and $(300,90)$ for the QTD questionnaire. Regarding the second issue, for the CPT case the candidate set of gambles was a fractional factorial set of gambles, defined on a range of outcomes similar to the one used by Tanaka et al. (2010). In particular we considered gambles $\{x,p,y,q\}$ where $x \in \{1,30,40,100,1000\}$, $p \in \{0.1,0.3,0.5,0.7,0.9\}$, $y \in \{-20,-15,-10,-5,5,10,30\}$, $q=1-p$, with x and y in US dollars. For the QTD case we used a fractional factorial set of alternatives (x,t) where $x \in \{5,10,15,20,25,30,40,50,60,80,100,120,150,160,200,240,250,300\}$ and $t \in \{0,3,7,14,30,60,90\}$, with x in US dollars and t in days, based on the questions in Benhabib et al (2010). Clearly other values can be used depending on the subjects and other environmental conditions. Regarding the value of the prior parameters, we used $\hat{w}_0 = [0.6;0.8;2.2]$, $\hat{D} = 100I$ and $\hat{\delta} = 1$ for CPT and $\hat{w}_0 = [0.8;0.008]$, $\hat{D} = 100I$ and $\hat{\delta} = 0.5$ for QTD, where I is the identity matrix. These values make the prior uninformative (large D), and therefore focus more heavily on the information provided by previous questions. Recall that these parameters are estimated using hierarchical Bayes once the data have been collected.

4. Field Experiment

We used the proposed questionnaire design and estimation methods with an online panel of homeowners for whom we also gathered information regarding their mortgages. The panel consisted of 233 respondents, all homeowners in the US, drawn from a larger sample of past survey participants who had been screened for homeownership. They consisted of 152 women and 81 men. The average age was 39.6 with 92 homeowners being between 25 and 34, 82 between 35 and 44, and 59 over 44. The data was the result of an email request for participation from 500 homeowners, with an oversampling of underwater homeowners. Out of the 233 respondents, 119 were underwater homeowners while the remaining 114 were homeowners who were not underwater. Our interest in underwater homeowners is based on recent work linking time preferences to increased credit card debt (Meier and Sprenger 2010) and problems keeping up with that debt (Meier and Sprenger 2009).

We compare the estimated parameter values both with typical values in the literature as well as with parameter values estimated using a frequently used methodology that we discuss in Section 4.2.

The study consisted of five sets of questions:

1. As set of demographic and mortgage related questions including a self-report (0 or 1) of whether the respondent perceived herself as being underwater.
2. A standard non-adaptive questionnaire to elicit the loss aversion parameter λ for CPT as well as a standard non-adaptive questionnaire to elicit an exponential discount parameter r , discussed in Section 4.2. Only 178 of the 233 respondents fully completed this step.
3. A filler task.
4. Sixteen questions of choosing among pairs of gambles designed according to the proposed adaptive method for the elicitation of CPT parameters were completed. A screenshot of one such question is shown in the Appendix.
5. Finally, twenty questions of choosing among pairs of delayed payments designed according to the proposed adaptive method for the elicitation of QTD parameters were completed. A screenshot of one such question is shown in the Appendix.

For steps 2, 4, and 5, subjects saw a welcome page with instructions about each step, and were asked to answer a few simple questions to ensure they understood the subsequent choice questions. For example, for the CPT questionnaire in step 4 they were asked “What is the maximum amount one could win in a gamble where there is 30% chance of winning \$20 and

70% chance of winning \$5". Respondents who would not answer correctly these comprehension questions returned to the instructions page.

Steps 2, 4, and 5 were all designed to be incentive compatible. The respondents were informed throughout that one in every 100 participants would be selected and make some money based on the preferences that he or she indicated during the survey. For the winners, one of the questions answered would be selected randomly and the option chosen delivered.

4.1 Parameter Estimates

Table 1 reports the mean values of all five parameters ($\alpha, \lambda, \sigma, r, \beta$) based on the adaptive questions and estimated using hierarchical Bayes. For each of the five parameters we report mean values for all 233 respondents, and for the self-reported over- and underwater homeowners separately. More detailed information about the parameters can be seen in the histograms shown in Figure 1.

The estimated CPT parameters are generally consistent with those obtained by others (e.g. Tversky and Kahneman 1992, Wu and Gonzalez 1996), providing face validity to the proposed methodology. Our estimates are like those from the study by Tanaka et al. (2010), one of the few studies that simultaneously assess preferences over risk and across time. Our aggregate estimate of λ for money is comparable to the value of 2.25 reported by Tversky and Kahneman (1992) and well within the range between 1.42 and 4.8 reported in other studies (see for example Abdellaoui et al. 2007 for details). Our time preference estimates are also in the range obtained from incentive compatible field settings (Ashraf et al. 2006, Meier and Sprenger 2009, Meier and Sprenger 2010) which estimated QTD preferences.

Table 2 shows the correlations between the CPT and QTD parameters. We notice that there are two correlations that are relatively large. The first is between loss aversion λ and the measure of diminishing sensitivity σ : the negative sign of the correlation indicates that if the value function for gains and losses is more linear, then it is also less kinked at 0. The second is between the two time parameters, β and r . In both cases, the correlation is not high enough to suggest that these two parameters measure the same construct. Similar correlations between discount rates and present bias have been reported elsewhere (Meier and Sprenger 2009).

	Probability parameter α	Loss aversion parameter λ	Value function parameter σ	Discount rate parameter r	Present bias parameter β
All respondents N=233	0.74	2.26	0.64	0.0032	0.88
Underwater N=119	0.73	2.25	0.64	0.0038	0.84
Not Underwater N=114	0.76	2.27	0.65	0.0026	0.91

Table 1: Mean values of the estimated parameters α , λ , σ of CPT and r , β of QTD. The first row is across all 233 respondents, the second is across only the 119 respondents who self-reported as being underwater, and the third is across the remaining 114 respondents. For the last two rows bold indicates that the means are significantly different between the over- and underwater populations, both at $p < 0.01$.

	Probability parameter α	Loss aversion parameter λ	Value function parameter σ	Discount rate parameter r	Present bias parameter β
Probability parameter α	1.0000				
Loss aversion parameter λ	-0.2023	1.0000			
Value Function parameter σ	0.0793	-0.5974	1.0000		
Discount rate parameter r	0.0354	-0.1314	0.1472	1.0000	
Present bias parameter β	0.0126	0.1283	-0.1336	-0.5447	1.0000

Table 2: Correlations between the CPT and QTD parameters. Bold indicates significance at the $p < .05$ level.

The low correlations between the CPT and QTD parameters are interesting for another reason. Economists have expressed concern that the observation of non-standard time preferences is due, in part at least, to unobserved risk preferences (Andersen et al. 2008). Along with recent data (Tanaka et al. 2010) gathered in the field, the low correlations re-

ported in Table 2 suggest that the relationships between CPT and QTD parameters are limited. At the same time, however, the relationships within the CPT and QTD parameters are substantial, suggesting that simultaneous estimation is desirable to avoid biasing these estimates.

Since we are interested in differences in behavior (e.g. credit behavior) as a function of preference parameters, we also examine in Table 1 the parameter estimates for the different types of homeowners separately. We observe that over- and underwater populations do not differ significantly regarding any of the CPT parameters but they do so regarding the QTD parameters. Underwater respondents have significantly larger discount rates as well as present bias (smaller present bias parameter). This is consistent with the idea that present bias overweighs the immediate value of home possession, and that high discount rates lead to underestimating the difficulty of meeting payments in the longer run. To further explore this difference we study how the five parameters $(\alpha, \lambda, \sigma, r, \beta)$ relate to whether the respondents self-reported as being underwater or not. Table 3 presents the results of a logistic regression with $(\alpha, \lambda, \sigma, r, \beta)$ as the independent variables and a dummy variable equal to 1 for subjects who reported being underwater and 0 for the others as the dependent variable. In agreement with Table 1, QTD parameters r and β are significant and indicate that underwater respondents have a larger discount rate parameter r but smaller present bias parameter β . These results are consistent with results in other areas of credit (Ashraf et al. 2006, Meier and Sprenger 2009, Meier and Sprenger 2010), providing further face validity to the proposed methodology. However, the impact of these credit decisions seems to be independent of the prospect theory parameters.

Probability parameter α	-. 419992
Loss aversion parameter λ	-.045288
Value function parameter σ	-.5414976
Discount rate parameter r	145.5938
Present bias parameter β	-2.559471*
Intercept	2.60497

Table 3: Logistic regression of underwater status (1 if underwater, 0 otherwise) on the CPT and QTD parameters $(\alpha, \lambda, \sigma, r, \beta)$. Bold indicates significant at $p < .05$ and * at $p < .1$

Finally, to illustrate the methodology, we estimated the parameters $(\alpha, \lambda, \sigma, r, \beta)$ based on the adaptive questions and using hierarchical Bayes, this time also using two covariates: age (in years) and gender (binary variable equal to 1 for male) since these have been occasionally found to correlate with preferences, although with no conclusive findings in the literature. That is, the effect of these covariates on the parameters $(\alpha, \lambda, \sigma, r, \beta)$ and the parameters themselves were estimated simultaneously using the estimation method as described in Section 2.1.3. We report the results for our sample of respondents in Table 4. For our population, significant gender differences are found for time discounting, present bias, and loss aversion.

<i>on...</i>	Effect of...	
	Age	Gender
<i>a</i>	0.00181	0.0797
<i>λ</i>	-0.01453	-1.5711
<i>σ</i>	-0.00016	0.0507
<i>r</i>	0.00001	0.0016
<i>β</i>	0.00213	-0.1112

Table 4: Relations between covariates (*Age*, *Gender*) and estimated parameters $(\alpha, \lambda, \sigma, r, \beta)$. Bold indicates significant at $p < .05$.

4.2 Qualitative Comparison using a Standard Methodology

In order to explore the importance of the methodologies used for empirical research, we also compare the estimated parameter values to estimates based on a methodology used in past studies. As we cannot know the true parameter values, the purpose of this comparison is not to support one or the other methodology based on the accuracy of the estimation, but only to explore whether or not one can reach qualitatively similar empirical findings from estimates using different methodologies. Different findings indicate the importance of methodology for empirical research, as expected.

In psychology and marketing the “standard” methodology used is termed a titration method (Weber et al. 2007, Zauberman 2003) and in economics it is called a multiple price list (Harrison et al. 2002, Meier and Sprenger 2009, Ashraf et al. 2006). As implemented, the methodology allows the estimation of only parameter λ for CPT and an exponential discount factor r . This methodology was administered in step 2 of the study outlined above, but only a subset of 178 respondents fully completed this step. This methodology is as follows.

For the CPT case respondents were shown two sets of 9 gambles and were asked to indicate whether they would accept them. All gambles were based on a fair coin toss. In the first set there was 50% probability of gaining \$6 (for tails) for all 9 gambles while the loss was ranging from \$0.50 to \$7 across the 9 gambles – see a screenshot of the question in the Appendix. In the second set of gambles there was 50% probability of gaining \$20 (for tails) while the loss was ranging from \$2 to \$24 across the 9 gambles. Effectively the respondents were evaluating 18 gambles and asked to indicate the point at which they would switch from accepting to not accepting a gamble as the amount of losses increased in each of the two sets. Based on the “switching points” from the two sets we calculated two values of λ and took their average.

We estimated time preferences similarly. In this case respondents were shown two sets of 10 choices between two delayed payment options. In each set one of the options was fixed while the other was changing from least to most desirable. The first set of choices is shown in the Appendix. Effectively the respondents were evaluating 20 choices and asked to indicate the point at which they would switch from accepting the fixed choice to accepting the other one for each of the two sets. Based on the two “switching points” from the two sets we calculated r using the midpoint between the switching points. The mean values of parameters λ and r for the same subset of 178 respondents and for both the proposed methodology and the standard one are reported in Table 5.

To qualitatively compare the adaptive and non-adaptive methods, we reproduced the analysis of mortgage consequences using the standard methodology. In this case we find no significant difference between the underwater homeowners and the ones who are not underwater for neither the values of λ nor the values of r (the t-statistic of the difference of the means across the two types of homeowners for r is 1.22 for the standard method in Table 5, while it is 2.21 for the proposed method in Table 5). Moreover, the correlation between λ and r across the 178 common respondents is -0.17 and significant at $p < .05$ for the proposed method in Table 5, while it is 0.14 and not significant at $p < .05$ for the standard method.

Like in Table 3, we also show in Table 6 how the estimated parameters λ and r relate to whether the respondents self-reported as being underwater or not. As the simple method only elicits λ and r , we report the results of the logistic regression as in Table 3 but use only these two parameters both for our methodology and the simple benchmark. In agreement with Table 5, none of parameters λ and r are significant when their values are estimated using the

benchmark method. On the other hand, in agreement with Table 3, parameter r is significant when estimated using the proposed adaptive methodology.

		Loss aversion parameter λ	Discount rate parameter r
All respondents N = 178	Proposed Method	2.31	0.0030
	Standard Method	3.09	0.0013
Under-Water Only N = 83	Proposed Method	2.32	0.0034
	Standard Method	2.98	0.0014
Not Under-Water Only N=95	Proposed Method	2.29	0.0026
	Standard Method	3.20	0.0012

Table 5: Mean values of the estimated parameters λ of CPT and r of QTD using a simple benchmark as well as the proposed method (as in Table 1) for the subset of 178 respondents common for the two methods. The first row is across all these 178 respondents, the second is across only the 83 respondents who self-reported as being underwater, and the third is across the remaining 95 respondents. Bold indicates that the means are significantly different between the over- and underwater populations at $p < 0.05$.

	Standard Method	Proposed Method
Loss aversion parameter λ	-.0271649 (-0.60)	.0845182 (0.58)
Discount rate parameter r	177.6565 (1.29)	137.8952 (2.19)
Intercept	-.2805077 (-1.09)	-.7371219 (-1.68)

Table 6: Logistic regression of underwater status (1 if underwater, 0 otherwise) on the CPT and QTD parameters λ and r estimated using either the proposed adaptive method or the non-adaptive benchmark for the 178 common respondents. Logistic regression coefficients and t-statistics in parentheses are shown; bold indicates significant at $p < .05$.

This is clearly an initial assessment of the potential of the proposed adaptive method, yet it suggests promise. The adaptive estimates relate to credit behavior in a way that is suggested in theory and supported by recent empirical results, unlike the results based on the standard non-adaptive method that do not indicate significant relations.

5. Discussion

There is a growing interest in relating behavioral decision theories to real world empirical phenomena in financial decision making (e.g., credit, retirement savings, investment, insurance) or health decision making (e.g. nutrition, exercising, substance abuse, medical testing). Success in establishing such links will be highly dependent on our ability to develop efficient and accurate methods to measure the behavioral characteristics of economic agents.

We proposed a novel methodology for estimating parameters of decision models such as CPT and QTD. The method augments traditional approaches to preference elicitation in decision analysis. An essential difference is that traditional decision analytic methods aim to measure indifference (or switching) points, as efficiently as possible, and calculate parameters from there. By contrast, the proposed method is *not* concerned with eliciting indifference points; rather it is focused on dynamically gathering the best possible data in a certain statistical sense, and using it to estimate parameters. This can substantially increase the efficiency of model calibration. For example, our survey required only 16 binary choice questions to estimate an individual's CPT parameters, less than half of the 35 questions used in Tanaka et al.'s (2010) switch list.

Our survey of over 200 mortgagers based on this new method produced preference model estimates in line with values obtained in previous studies. Moreover, it allowed us to reach statistically significant conclusions that were not accessible using a standard method, e.g. in the case of differential time preferences of underwater and non-underwater mortgagers.

Traditionally, the appeal of adaptive methods has been statistical efficiency and minimization of potentially expensive respondents' time. However there are reasons to suspect that adaptive methods are attractive for behavioral reasons. First, adaptive methods may minimize the cognitive resources required to assess preferences by minimizing the number of questions that are posed to the respondent. The possibility that resource depletion occurs with an increased number of questions seems likely (Vohs et al. 2008), and recent evidence suggests that some context effects, such as selection of a default option, increase with depletion (Levav et al., in press). Thus it seems possible that by limiting the number of questions, and focusing them on parts of the preference function that are most (statistically) informative, that adaptive methods might produce estimates that are less contaminated by context effects as well as less influenced by the random error that may accompany fatigue. A second possible advantage of adaptive methods is that they focus more quickly on the part of the preference

function that seems most relevant to portraying the decision-maker's preferences. While the procedure is seen from the perspective of the algorithm as decreasing our uncertainty on the decision maker's value function parameters, it can be seen from the decision-maker's perspective as asking the most informative questions about what he or she may desire. By asking more relevant questions, the technique may also limit the possibility of range effects due to extreme values irrelevant to the decision-maker.

Finally, as mentioned above, while we have focused on the estimation of parameters for specific models, namely CPT and QTD, the proposed method can be applied to any preference model. The present results also suggest that it would be important to further explore the potential of the proposed methodology in a broader range of behavioral decision making studies, for example in financial, health, or other domains.

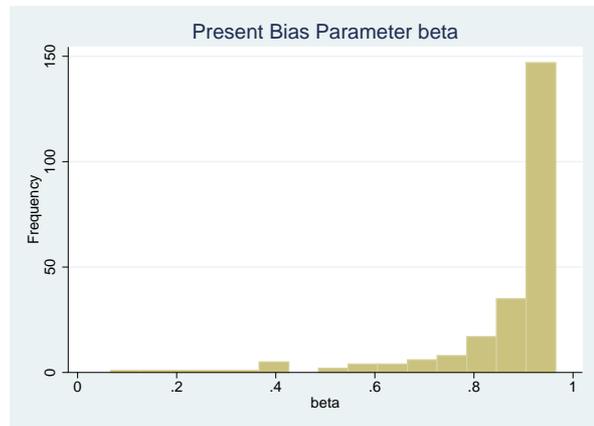
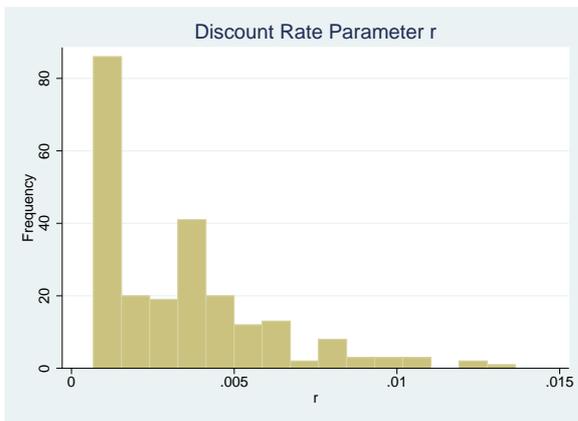
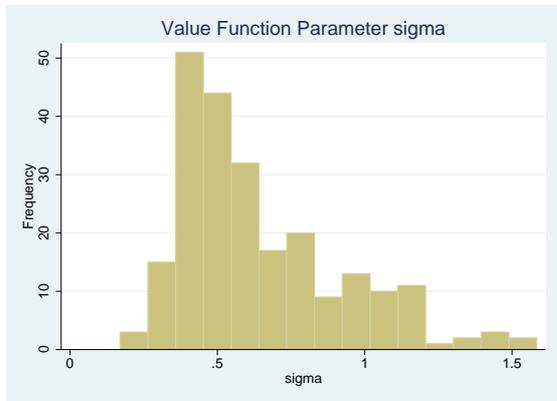
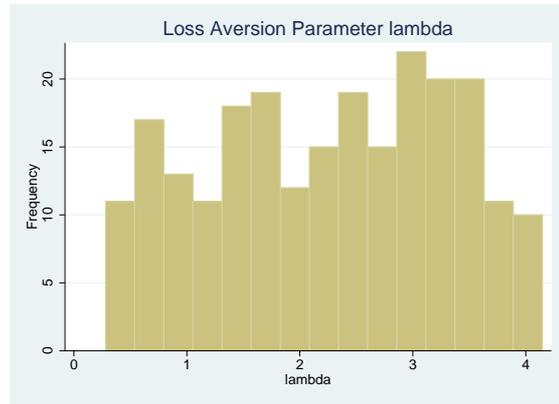
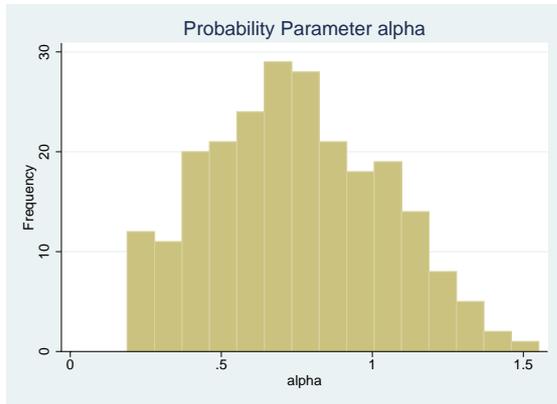


Figure 1: Histograms of parameters α , λ , σ of CPT and r , β of QTD for the population of the 233 homeowners in the study.

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Appendix A

MCMC algorithm for estimating the CPT and QTD parameters

MCMC draws successively each parameter from its posterior distribution conditioning on the data and the other parameters. Each parameter is drawn once in each iteration. The resulting Markov Chain has the posterior distribution as its equilibrium distribution. We use 10,000 iterations as “burn-in” (i.e., these iterations allow the Markov chain to converge to its equilibrium distribution and are not saved), followed by 40,000 iterations in which one iteration is saved every 10 iterations. The outcome is 4,000 draws from the posterior distribution. We now describe how each parameter is updated at each iteration.

- Update of D : this matrix is drawn directly from its conditional posterior distribution:
 $P(D | rest, data) \sim IW(\eta_0 + I, \eta_0 \Delta_0 + \sum_{i=1}^I (w_i - w_0)(w_i - w_0)^T)$ where IW is the inverse Wishart distribution.
- Update of $\{w_i\}$: we use a Metropolis Hastings algorithm, with a normal random walk proposal density (with a jump size adapted to keep the acceptance ratio around 30%). Constraints on the parameters are enforced with rejection sampling (Allenby, Arora and Ginter 1995). For each w_i , the acceptance ratio is obtained from:

$$\prod_{j=1}^J \frac{\exp(\delta U(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, w_i))}{\exp(\delta U(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, w_i)) + \exp(\delta U(x_{ij}^2, p_{ij}^2, y_{ij}^2, q_{ij}^2, w_i))} \cdot \exp\left(-\frac{1}{2}(w_i - w_0)^T D^{-1}(w_i - w_0)\right)$$

where we replace U appropriately for the QTD case.

- Update of Θ : this matrix is drawn directly from its conditional posterior distribution:

$$P(VEC(\Theta) | rest, data) \sim N(V(Z^T \otimes D^{-1})VEC(W), V)$$

where $V = ((Z^T Z) \otimes D^{-1})^{-1}$, Z is the matrix of covariates z_i 's (one row per decision maker), W is the matrix of w_i 's (one row per decision maker), $VEC(X)$ is the column vector obtained by stacking the columns of a matrix X , \otimes is the Kronecker product.

- Update of δ : we use a Metropolis Hastings algorithm, with a normal random walk proposal density (with variance 0.001). The acceptance ratio is obtained from:

$$\prod_{i=1}^I \prod_{j=1}^J \frac{\exp(\delta U(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, w_i))}{\exp(\delta U(x_{ij}^1, p_{ij}^1, y_{ij}^1, q_{ij}^1, w_i)) + \exp(\delta U(x_{ij}^2, p_{ij}^2, y_{ij}^2, q_{ij}^2, w_i))} \quad \text{where we replace again } U$$

appropriately for the QTD case.

Appendix B

Screenshots of Online Adaptive Questionnaire

RiskSense Survey

Please consider the two gambles below. Which of these two gambles would you rather play?

Option A	Option B
50% Chance to Win \$100	90% Chance to Win \$40
50% Chance to Lose \$15	10% Chance to Win \$10
<input type="button" value="Choose A"/>	<input type="button" value="Choose B"/>

[More Information on Prizes](#)

Questions Completed

3/16

RiskSense Survey

Please consider the two options below. Which of these two options do you find more attractive?

Option A	Option B
Receive \$250 today	Receive \$300 in 1 week
<input type="button" value="Choose A"/>	<input type="button" value="Choose B"/>

[More Information on Prizes](#)

Questions Completed

5/20

Appendix C

Screenshots of Benchmark Questions

*Consider a one time gamble based on the toss of a fair coin (the probability of heads equals the probability of tails, both being .5). If the coin turns up heads then you lose \$0.50, and if the coin turns up tails, you win \$6. Would you play this gamble? Consider the following similar gambles and record your choices.

<u>Heads</u>	<u>Tails</u>	YES	NO
Lose \$0.50	Gain \$6.00	<input type="radio"/>	<input type="radio"/>
Lose \$1.00	Gain \$6.00	<input type="radio"/>	<input type="radio"/>
Lose \$1.50	Gain \$6.00	<input type="radio"/>	<input type="radio"/>
Lose \$2.00	Gain \$6.00	<input type="radio"/>	<input type="radio"/>
Lose \$3.00	Gain \$6.00	<input type="radio"/>	<input type="radio"/>
Lose \$4.00	Gain \$6.00	<input type="radio"/>	<input type="radio"/>
Lose \$5.00	Gain \$6.00	<input type="radio"/>	<input type="radio"/>
Lose \$6.00	Gain \$6.00	<input type="radio"/>	<input type="radio"/>
Lose \$7.00	Gain \$6.00	<input type="radio"/>	<input type="radio"/>

Loss aversion benchmark questionnaire.

Imagine you just won an Amazon gift card, worth \$150, which will be paid to you in **twelve months**. However, the lottery commission is giving you the option of receiving a card worth a different amount instead, paid to you **six months** from now.

Important: 1 in every 100 participants will be selected and make some money based on the preferences that he or she indicated during the survey. You should therefore respond to all questions truthfully and carefully. You will find out whether you are one of our lucky winners right at the end of the survey. If you are among the winners - 1 in every 100 respondents is a winner - we will randomly select one of the questions that you answered and we will offer you the option that you chose.

Please choose which option you prefer in **each** pair:

Payment in Twelve Months	Payment in Six Months
<input type="radio"/> Receive \$150 in twelve months	<input type="radio"/> Receive \$155 in six months
<input type="radio"/> Receive \$150 in twelve months	<input type="radio"/> Receive \$150 in six months
<input type="radio"/> Receive \$150 in twelve months	<input type="radio"/> Receive \$145 in six months
<input type="radio"/> Receive \$150 in twelve months	<input type="radio"/> Receive \$132 in six months
<input type="radio"/> Receive \$150 in twelve months	<input type="radio"/> Receive \$115 in six months
<input type="radio"/> Receive \$150 in twelve months	<input type="radio"/> Receive \$98 in six months
<input type="radio"/> Receive \$150 in twelve months	<input type="radio"/> Receive \$82 in six months
<input type="radio"/> Receive \$150 in twelve months	<input type="radio"/> Receive \$68 in six months
<input type="radio"/> Receive \$150 in twelve months	<input type="radio"/> Receive \$57 in six months
<input type="radio"/> Receive \$150 in twelve months	<input type="radio"/> Receive \$48 in six months
<input type="button" value="Submit"/>	

Time discounting benchmark questionnaire.

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