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Start-ups and Established Firms  
in New Markets

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# The Timing of Capacity Investment by Start-ups and Established Firms in New Markets\*

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## Abstract

We analyze the competitive capacity investment timing decisions of both established firms and start-ups entering new markets which are characterized by a high degree of demand uncertainty. Firms may invest in capacity early (when the market is highly uncertain) or late (when market uncertainty has been resolved), possibly at different costs. In our model, established firms choose investment timing and capacity level to maximize expected profits. Start-ups are prone to bankruptcy if profit turns out to be too low, and hence choose investment timing and capacity level to maximize the probability of survival. Surprisingly, we find that in monopoly situations, a start-up is more likely to prefer early investment than an established firm, despite the presence of demand uncertainty. In duopoly situations with one start-up and one established firm competing in the same market, we characterize the equilibria of a strategic capacity investment timing game in which firms choose when to build capacity. We find that when demand uncertainty is high and costs do not decline too severely over time, the unique equilibrium of this game is for the start-up to take a leadership role and invest first in capacity while the established firm follows; by contrast, when two established firms compete in an otherwise identical game, high demand uncertainty leads to both firms investing late. Thus, the threat of bankruptcy leads to an increase in sequential investment outcomes in which the start-up leads, a result that we demonstrate persists even if the start-up is concerned with both profit and bankruptcy risk or profit above the bankruptcy threshold. We conclude that the threat of firm failure significantly impacts the dynamics of competition involving start-ups.

## 1 Introduction

Firms entering new markets face numerous operational challenges. Among the most crucial are issues related to capacity investment. Particularly when the size of a market is uncertain, two

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common yet difficult decisions are how much capacity to invest in, and when to do it. When choosing how much capacity to build or reserve with a supplier, the trade-off is clear: too much capacity results in underutilized facilities (if output is reduced to match market demand) or depressed prices (if output remains high despite low demand), while too little capacity results in reduced sales and suboptimal profit and growth.

Timing the capacity investment decision presents even subtler considerations. Uncertainty surrounding market size typically reduces over time, meaning a firm that invests in capacity early is subject to a higher degree of demand uncertainty than a firm that postpones the investment decision. On the other hand, in competitive situations, a firm investing earlier than its rivals becomes the first-mover in the market, which may yield a strategic advantage. Indeed, the cost of capacity itself may change over time, either increasing (e.g., if contract capacity becomes scarce as the market matures) or decreasing (e.g., if learning enables lower-cost processes). These factors combine to make the decision of *when* to invest in capacity just as difficult and perilous, if not more so, as the decision of *how much* capacity to build or purchase.

The timing of capacity investment when entering new markets is precisely the issue that we consider. We first examine stylized monopoly models in which the sole entrant to a new market must build or source capacity in anticipation of future demand. Eventual market size is uncertain, and the firm is allowed to invest in one of two periods: if the firm invests *early*, then it makes the capacity decision before knowing market size, whereas if it invests *late*, all demand uncertainty is eliminated and capacity is built or sourced after learning market size. The cost of capacity is allowed to vary between periods. Thus, a monopolist firm must trade off the value of information (which is gained if the investment decision is delayed) with potential cost advantages from early investment.

Because new markets are often pursued by nascent firms, we focus on how the timing of capacity investment differs between *start-ups* and *established firms*. We consider the primary difference between these two types of firms to be the threat of bankruptcy or firm failure. Large established firms diversifying into new markets are unlikely to face imminent peril should demand in that market turn out to be low; start-ups, on the other hand, are typically smaller firms wholly invested in a single market, and thus, to a far greater extent than their established counterparts, face potentially disastrous consequences should the market fail to materialize as expected. The presence of this risk,

combined with the high degree of demand uncertainty that typically accompanies the development of a new market, implies that start-ups should have a utility function which takes into account the risk and consequences of failure. Hence, in our model, the objective of a start-up is to time the capacity investment decision to maximize the *probability of survival*. Established firms, by contrast, do not face an imminent risk of failure, and hence make capacity decisions to maximize *expected profit*.

In the monopoly setting, we examine how start-ups differ in their capacity timing decisions from established firms, characterizing how market uncertainty, capacity costs, and the threat of failure influence both capacity levels and investment timing. We find that established firms are likely to prefer late investment even if early investment is cheaper, because the flexibility to respond to market conditions engendered by late investment allows the firm to capture higher profits, particularly in high demand states. By contrast, start-ups prefer to invest in capacity whenever capacity is least expensive—that is, if capacity costs increase over time, start-ups prefer early investment—because lower capacity costs minimize the threshold market size that results in firm survival and hence maximize the probability of survival.

We then proceed to analyze duopoly models in which two firms simultaneously consider entry into a new market. In addition to all of the trade-offs inherent in the monopoly model, the competitive interaction introduces a strategic aspect to the capacity investment timing decision: a firm investing earlier than a competitor may gain a leadership position in a sequential game. We find that when a start-up competes with an established firm, if market uncertainty is high (as in a new market) and costs do not decline severely over time, then the unique equilibrium is for the start-up to invest early, while the established firm invests late. By contrast, when two established firms compete, the only equilibrium when demand uncertainty is high is simultaneous: both firms invest late. We thus conclude that the threat of failure experienced by a start-up tends to push capacity investment earlier—in both monopolistic and competitive situations—and leads to asymmetric investment timing equilibria in which start-up firms, remarkably, act as first-movers in new markets, despite the apparent advantages of established firms in terms of resources and technology.

In this regard, our findings relate to several streams of research, for example the literature on disruptive innovation. The seminal works on this topic are Bower and Christensen (1995), Chris-

tensen and Bower (1996), and Christensen (1997); Schmidt and Druehl (2008) provide a recent review. A *disruptive innovation* is an improvement in a product or service that fundamentally changes its cost, performance, or target market in new or unexpected ways.<sup>1</sup> Such innovations are typically enabled by scientific, technological, or process advancements; for example, the rise of inexpensive, physically compact desktop computers enabled the emergence of the personal computing market over the minicomputer and mainframe markets, and the development of cheap, tiny digital flash storage technologies helped contribute to the dominance of digital photography over film photography. A recurring question in this literature is: why do large, established firms typically fail to embrace disruptive innovations early, while smaller start-up firms often take a leadership role in bringing the innovations to market? Our model supports one possible answer to this question, namely, that it is the natural equilibrium of an endogenous timing game between a start-up and an established firm.

The remainder of this paper is organized as follows. §2 provides a brief review of the literature. §3 analyzes the monopoly model, while §§4–5 analyze the duopoly model. §6 presents several extensions to the basic model, and §7 concludes the paper.

## 2 Related Literature

There are three primary streams of research related to our work: the operations literature on capacity investment under uncertainty; the economics literature on competitive capacity investment; and the strategic management literature on new market entry and disruptive innovation. The latter topic was discussed in §1; here, we briefly review the remaining two broad areas, with further references to relevant works included throughout the remainder of the paper.

Our model is one of capacity investment with stochastic demand. As such, it is related to the extensive operations literature on this topic—see the comprehensive review by Van Mieghem (2003). A number of papers consider the value of delaying capacity investment in order to obtain more accurate demand information—see, e.g., the literature on postponement, such as Van Mieghem

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<sup>1</sup>We abstract from the details of innovation and focus on the outcome of innovation resulting in highly uncertain new markets; thus, while we use the term “disruptive” as it invokes an image of significant market upheaval and uncertainty, innovation in our context could in fact be any of the four types of technological change described by Lange et al. (2009)—sustaining, disruptive, architectural, and competence destroying discontinuities—so long as the result is uncertainty in the size of the resultant market.

and Dada (1999), Anand and Girotra (2007), and Anupindi and Jiang (2008), though these works differ from ours in that they do not consider the possibility of firm bankruptcy and the implications it may have on the timing incentives of a start-up firm. Some works of particular relevance in this stream include Archibald et al. (2002), Babich et al. (2007), Babich (2008), Swinney and Netessine (2009), and Boyabatli and Toktay (2007), all of which consider the impact of bankruptcy risk on capacity or inventory decisions. Tanrisever et al. (2008) consider the related issue of simultaneous investment into capacity and process improvement in the presence of bankruptcy. While these papers address various consequences of bankruptcy on operational decisions (including process development, capacity levels, financial subsidies to suppliers, and contracting and sourcing strategies), no paper in the literature, to our knowledge, considers the impact of bankruptcy or firm failure on capacity investment *timing*. Indeed, there is a relative lack of research in the operations literature on the topic of capacity investment timing for entry into new markets.

We analyze duopoly models consisting of two firms strategically investing in capacity before either begins to sell in the market. Similar models, frequently referred to as “endogenous leadership games” in the economics literature, have been studied by Gal-Or (1985), Saloner (1987), Hamilton and Slutsky (1990), Maggi (1996), and Bhaskaran and Ramachandran (2007). Maggi (1996) considers an endogenous leadership game with demand uncertainty, much like ours, although two key differences are that the differing objectives of start-ups (and hence the impact of bankruptcy) are not considered, and further capacity investment may occur in multiple periods (whereas in our model, capacity investment occurs in at most one period, due to, e.g., high fixed costs). Also related along these lines is the long stream of research on capacity investment for entry deterrence, pioneered by Spence (1977).

Lastly, there is an extensive literature on entry timing for reasons not related to strategic capacity investment. Some examples include social influence (Joshi et al. 2009), quality or cost improvements (Lilien and Yoon 1990), product technology (Bayus and Agarwal 2007), and product design (Klastorin and Tsai 2004). Our paper differs from these by focusing solely on the impact of bankruptcy risk on capacity investment timing under demand uncertainty, and exploring how such risk impacts timing in duopolistic settings.

### 3 Monopolistic Firms

In this section, we introduce and analyze two different monopoly models of capacity investment timing in a new market with uncertain demand: §3.1 discusses an established, profit-maximizing firm, while §3.2 considers a start-up prone to bankruptcy. The established firm model is a relatively standard formulation, and serves as a vehicle to introduce the dynamics of our setting and also as a baseline for comparison with the bankruptcy-prone start-up. We defer all discussion of competition until §4.

#### 3.1 A Monopolistic Established Firm

An established firm (denoted by the subscript  $e$ ) sells a single product.<sup>2</sup> The quantity of the product released to the market is  $Q_e$ . The market price is given by the linear demand curve  $p(Q_e) = A - Q_e$ . Prior to determining the production quantity, the firm must invest in production capacity  $K_e$  which determines its maximum output. This capacity may be internal to the firm (e.g., if the firm in question is a manufacturer) or external (e.g., if the firm outsources production to a contract manufacturer). There is no constraint on the total amount of capacity that can be built or reserved in either case.

Capacity investment may occur at one of two times: either *early* or *late*. Early investment is sufficiently far in advance of the selling season that the total market size is uncertain. The uncertainty in market size is reflected in the demand intercept,  $A$ , which is modeled as a continuous random variable with positive support, distribution function  $F$ , mean  $\mu$ , and variance  $\sigma^2$ .

Late investment, on the other hand, is sufficiently close to the start of the selling season that all uncertainty in  $A$  is eliminated—hence, capacity investment is made after observing the realized value of  $A$ . Demand uncertainty may be reduced or eliminated via a variety of mechanisms. For example, uncertainty may be resolved exogenously if demand depends highly on overall market or economic conditions at the time of product release, or if demand is a function of overriding consumer trends in the category. The firm may take actions to resolve demand uncertainty, such as performing extensive market research, employing consumer focus groups, or working with retailers to improve

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<sup>2</sup>We implicitly assume that the established firm—diversifying into the new market—has already evaluated the impact (if any) that market entry will have on sales of its existing products, and determined that entry is profitable; Druehl and Schmidt (2008) analyze this related problem of how new market entry can encroach on sales of existing (substitutable) products.

forecasts. Lastly, the firm may even produce some (economically insignificant) number of units (e.g., using outsourced capacity) to sell in test markets, postponing full capacity investment until a later date.

Regardless of when the firm chooses the capacity, the production *quantity* ( $Q_e$ ) is determined after  $A$  has been observed and  $K_e$  has been fixed (i.e., just before the selling season), and hence output is subject to the constraint  $Q_e \leq K_e$ . We assume that capacity investment, whenever it is made, is irreversible. Furthermore, capacity investment can occur in at most one period.<sup>3</sup> The total capacity cost is linear in the amount of capacity reserved, and the marginal cost of capacity may vary over time. The unit cost in the early period is denoted  $c_1$  and the unit cost in the late period is denoted  $c_2$ . We make no *ex-ante* assumption on the ordering of  $c_1$  and  $c_2$ . Costs that decrease over time (i.e.,  $c_1 > c_2$ ) may be reflective of exogenous technological or process cost improvements, innovation, or raw materials cost decreases; similarly, costs that increase over time ( $c_1 < c_2$ ) could occur if contract manufacturers offer a discount for early investment, if capacity in the later period is scarce, or if second period capacity must be installed more quickly, incurring expedited construction or configuration costs. The reasons behind inter-temporal cost variation are outside the scope of this paper; rather, we will present results that hold conditional on a particular cost trend.

The marginal production cost is zero, and for analytical tractability, we assume that the firm adheres to a production clearance strategy: that is, the firm always produces up to its capacity and releases the maximum quantity to the market,  $Q_e = K_e$ . (The issue of holdback, i.e., producing a quantity less than the total capacity, is discussed in §6.3.) The established firm, being a large, diversified company, faces minimal risk of bankruptcy as a result of entry into this new market—hence, facing uncertainty in market size ( $A$ ), the established firm seeks to maximize *expected profit*, which is denoted  $\mathbb{E}(\pi_e(K_e))$ , where the absence of the expectation operator,  $\pi_e(K_e)$ , is used to denote profit for a particular realization of  $A$ . Throughout the analysis, optimal values (capacities, profits, etc.) are denoted by the superscript  $*$ .

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<sup>3</sup>In reality, firms may be able to invest in capacity in multiple periods. Allowing such an option clearly does not impact the evaluation of deferred (late) investment, though it may increase the value of early investment. If fixed costs of capacity installation or expansion are high, then the value of an option to invest in both periods is relatively low—in the extreme case, if fixed costs are high enough, then firms will only invest in capacity in one period. This is the case that we consider.

Given this formulation, the firm's optimal expected profit from early capacity investment is

$$\mathbb{E}(\pi_e^*) = \max_{K_e \geq 0} \mathbb{E}((A - K_e - c_1) K_e), \quad (1)$$

while the firm's optimal expected profit from late capacity investment is

$$\mathbb{E}(\pi_e^*) = \mathbb{E}\left(\max_{K_e \geq 0} ((A - K_e - c_2) K_e)\right). \quad (2)$$

Thus, when the firm is deciding whether or not to invest in capacity in the early period, it must compare (1) with (2). The following theorem provides the details of the optimal capacity timing and investment level.

**Theorem 1** *A monopolist established firm prefers early investment if and only if*

$$\sigma^2 < (\mu - c_1)^2 - (\mu - c_2)^2, \quad (3)$$

*yielding optimal capacity  $K_e^* = (\mu - c_1)/2$  and expected profit  $\mathbb{E}(\pi_e^*) = (\mu - c_1)^2/4$ . Otherwise, the firm prefers late investment, yielding optimal capacity  $K_e^* = (A - c_2)/2$  and expected profit  $\mathbb{E}(\pi_e^*) = (\mu - c_2)^2/4 + \sigma^2/4$ .*

**Proof.** All proofs appear in the appendix. ■

As Theorem 1 demonstrates, an established monopolist prefers early investment if and only if equation (3) holds, i.e., if demand uncertainty is low and early investment is cheaper than late investment ( $c_1 < c_2$ ). Note that if capacity costs decrease over time ( $c_1 > c_2$ ), the firm prefers late investment for any feasible variance (i.e., for any  $\sigma^2 \geq 0$ ). If capacity costs increase over time ( $c_1 < c_2$ ), the firm may prefer early or late investment, depending on the level of demand uncertainty.

### 3.2 A Monopolistic Start-up

A common feature of new markets, particularly those enabled by ground-breaking or unforeseen technological innovation, is that they are characterized, *ex ante*, by a large amount of demand uncertainty. Thus, far more so than their established counterparts, smaller start-up firms are

exposed to a serious risk: the risk of bankruptcy or firm failure, should demand turn out to be low. Consequently, while it is quite natural to assume that established firms make decisions to maximize expected profits, it is less clear that start-ups should or do behave in the same way: as Radner and Shepp (1996) and Dutta and Radner (1999) demonstrate, a firm prone to bankruptcy that purely maximizes expected profit over an infinite horizon will fail with probability one. The objective of a start-up should, then, take into account the acute risk of failure associated with entry into a new market. This implies that firms particularly prone to bankruptcy—for our purposes, start-ups entering new markets—in fact have a utility function that depends both on *operating profit* and the *risk of failure*, e.g.,

$$\text{Total Utility} = \text{Operating Profit} - \text{Cost of Bankruptcy} \times \text{Probability of Bankruptcy}, \quad (4)$$

where the “cost of bankruptcy” represents either real costs (e.g., default penalties on loans), or a virtual penalty term embodying the expected consequences of bankruptcy.<sup>4</sup> This type of utility function can be found, for example, in the seminal paper by Greenwald and Stiglitz (1990) and in Brander and Lewis (1988) and Walls and Dyer (1996). If the probability of default due to the outcome of this particular market is very low, then the firm may safely ignore the last term and simply maximize expected operating profits; this would be the case with large, established firms considering diversifying entry into a new market that represents a small potential fraction of their total business. Our model in the preceding section addressed precisely this scenario.

Alternatively, if the cost of bankruptcy is large compared to the assets of the firm and would result in financial ruin, or if the probability of bankruptcy is high (either of which is likely to be the case for a start-up), the second term dominates the expression; the maximization problem may then be thought of as approximately equal to minimizing the probability of bankruptcy or, equivalently, maximizing the probability of survival. As a result, in what follows, we assume that the presence of failure risk implies that start-ups have a different objective than established firms: instead of maximizing expected profits, they maximize their chance of survival. Essentially, while any firm has a true profit function that accounts for both operating profits and the chance of bankruptcy as

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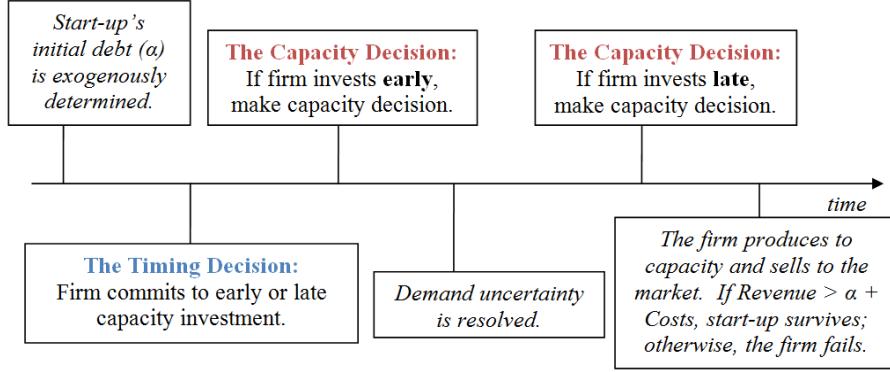
<sup>4</sup>From the accounting and financial points of view, the meaning of the word “bankruptcy” is often complex and does not necessarily imply that the company fails; the actual event of bankruptcy can have varying degrees of consequence to a firm, ranging from reorganization (Chapter 11 bankruptcy) to total liquidation (Chapter 7 bankruptcy). When using this term we simply imply that the company becomes insolvent and ceases to exist due to the negative cash-flow.

depicted in (4), we examine extreme cases: established firms are entirely concerned with operating profits while start-ups are entirely concerned with the probability of bankruptcy. As Chod and Lyandres (2008) discuss, the owners of private firms (e.g., start-ups in our model) are typically less diversified than the owners of public firms (established firms in our model), and hence are more sensitive to the risk inherent in a single venture and the corresponding chance of failure. Thus, it's reasonable that start-ups and established firms have different objectives—see Chod and Lyandres (2008) and references therein for a detailed discussion of this matter. This dichotomization of the objective function, while stylized, allows us to obtain sharp results; we extend our analysis numerically to the case of other, more complicated, objective functions in §6.4.

Consequently, the details of the model are identical to those introduced in §3.1, except for the objective function of the firm. We use the subscript  $s$  to denote a start-up firm. The start-up seeks to time its capacity investment and set the precise capacity level in order to maximize the *probability of survival*, denoted  $\psi_s(K_s)$ . We assume that survival occurs for the start-up if, at the end of the selling season, total revenues are greater than debt, where debt is defined to be the sum of two components: fixed, capacity-independent debt  $\alpha$ , and variable, capacity-dependent debt, which is linear in the installed capacity.

The fixed component of debt,  $\alpha$ , is an exogenous parameter which may represent, for example, loans taken to fund initial start-up expenses, overhead, market research, or R&D costs. This aspect of the start-up's debt is pre-existing and fixed at the start of our model, and the terms of the loan are structured such that  $\alpha$  must be repaid after the start-up begins generating revenues. In other words, the start-up raises capital in multiple rounds; early rounds fund R&D and start-up expenses while late rounds fund capacity investment. We analyze the stage of the game after the early rounds but before the later rounds, i.e., after the start-up's initial capital structure, R&D expenses, etc. have been fixed, similar to the second stage of the two-stage capital structure and capacity games analyzed in Brander and Lewis (1986) and Brander and Lewis (1988).

The variable component of debt, linear in the capacity level, is only raised at the time that capacity is installed. Regardless of when the capacity investment is made (early or late) the terms of the loan state that repayment occurs after the start-up has generated revenues, i.e., at the end of the selling season. Consequently, the start-up must generate enough operating revenue during the selling season to pay both components of its debt; otherwise, it will fail. In other words, survival



**Figure 1.** The sequence of events in the monopolistic start-up model.

occurs if

$$\text{Operating Revenue} \geq \alpha + \text{Capacity Costs},$$

or, equivalently, if operating profit (revenues minus capacity costs) is greater than the fixed debt  $\alpha$ . The sequence of events is summarized in Figure 1.

In what follows, we assume that the start-up's capacity costs are identical to the established firm analyzed in the preceding section ( $c_1$  and  $c_2$  for early and late investment, respectively), with the understanding that, in general, the cost of capacity may be different for a start-up, particularly if the cost of capital differs from an established firm. §6.2 explores a generalization of our model with heterogeneous capacity costs.

The optimal survival probability from early investment is thus

$$\psi_s^* = \max_{K_s \geq 0} \Pr((A - K_s - c_1) K_s \geq \alpha), \quad (5)$$

while the optimal survival probability from late investment is

$$\psi_s^* = \Pr\left(\max_{K_s \geq 0} ((A - K_s - c_2) K_s) \geq \alpha\right). \quad (6)$$

Note that, in equation (6), we have assumed that a start-up investing late, no longer subject to any uncertainty in demand, chooses a capacity level to maximize profit; at this stage, the start-up does not maximize the probability of survival because the lack of uncertainty makes this quantity ill-defined. However, by maximizing profit after observing market size under late investment, the

start-up survives in the largest number of demand states of any possible alternative strategy, and hence this strategy is optimal in terms of maximizing the *ex-ante* survival probability. (Also, we observe that it's possible for  $A$  to be sufficiently low that survival is impossible. In this case, the start-up still enters the market and invests in the profit maximizing capacity despite the fact that it is doomed to failure. Because the start-up is already accountable for the initial debt,  $\alpha$ , it cannot avoid bankruptcy by investing in zero capacity. But building the profit maximizing capacity ensures that the start-up's lenders can be repaid to the greatest extent possible—as might be the case, e.g., if the start-up enters bankruptcy and its assets are managed to repay as much debt as possible before liquidation.)

The following theorem describes the optimal investment timing and capacity decisions, given equations (5) and (6).

**Theorem 2** *A monopolist start-up prefers early investment if and only if  $c_1 < c_2$ , yielding optimal capacity  $K_s^* = \sqrt{\alpha}$  and survival probability  $\psi_s^* = 1 - F(2\sqrt{\alpha} + c_1)$ . Otherwise, the firm prefers late investment, yielding optimal capacity  $K_s^* = (A - c_2)/2$  and survival probability  $\psi_s^* = 1 - F(2\sqrt{\alpha} + c_2)$ .*

Theorem 2 demonstrates that a start-up prefers early investment only if costs increase over time ( $c_1 < c_2$ ). If costs decrease over time, the start-up prefers late investment. While the latter result is identical to the established firm case, the former is not; Theorem 1 shows that the established firm can prefer late investment even if costs increase over time, so long as demand uncertainty is large enough. Thus, we conclude from Theorems 1 and 2 that, given any particular set of problem parameters, a monopolistic start-up is more likely to prefer early investment than an established firm.

It is somewhat counterintuitive that a start-up, prone to such serious consequences should failure occur, is more willing to invest in capacity early than an established firm (given that the two firms have equal capacity costs); moreover, the start-up's decision is curiously unaffected by the degree of demand uncertainty. The reason for the latter result is that the start-up maximizes the probability of survival by maximizing the range of demand outcomes in which it survives. To accomplish this, it chooses the capacity that leads to survival at the lowest possible demand threshold—with this capacity level, the firm will survive for all higher demand realizations. This

threshold demand level is independent of the demand variance, hence variance does not impact the start-up’s survival-maximizing capacity decision.

In addition, because the start-up chooses the capacity level which ensures survival over the largest range of demand outcomes, the ability to respond to demand via late investment is not valuable to the start-up; late investment does not change the minimum demand level which ensures survival, and hence does not increase the start-up’s survival probability. What *does* impact survival probability is capacity cost: lower capacity costs lead to a lower survival threshold and hence a greater survival probability. Consequently, as Theorem 2 shows, when capacity costs change over time, the survival probability will be greater in the lower cost period, which leads to the result that the start-up prefers to invest in the period with the lowest cost.

Lastly, we observe that the expression for the optimal capacity level under early investment,  $K_s^* = \sqrt{\alpha}$ , can lead to seemingly counterintuitive behavior. The fact that the optimal capacity is independent of both demand uncertainty and cost is a consequence of our stylized objective function; a more complicated (and realistic) objective function that incorporates both profit and bankruptcy risk will, in general, yield optimal capacities dependent on  $\alpha$ , demand uncertainty, and capacity costs.

Qualitatively, however, the insights generated by these stylized results are compelling. For instance, if  $\alpha$  is very small the optimal capacity is also very small, suggesting the start-up is very risk-averse for a small bankruptcy threshold; if  $\alpha$  is very high, the optimal capacity is also large, suggesting the start-up is very risk-seeking when the chance of bankruptcy is high. But a start-up maximizing the probability of survival is neither risk-averse *nor* risk-seeking: it is averse to bankruptcy. The optimal capacity  $K_s^* = \sqrt{\alpha}$  is entirely consistent with a notion of avoiding bankruptcy: if  $\alpha$  is small, bankruptcy can only occur if demand is very low relative to capacity, hence the optimal action (to minimize the chance of bankruptcy) is to set a very small capacity; similarly, if  $\alpha$  is very large, survival can only occur if demand is high and the firm can capitalize on this, so the optimal action is to set a high capacity and “hope for the best.” Thus, a start-up at high risk of bankruptcy (high  $\alpha$ ) can act in a seemingly aggressive manner, while a start-up with a low risk of bankruptcy (low  $\alpha$ ) has far greater incentive to be conservative in its capacity investment; this type of behavior will play a key role in determining the outcome of competition in the following sections.

## 4 Duopoly Model

We now move to the duopoly model. The details of the model are identical to the monopoly model addressed in the previous section, except there are now *two firms* competing with perfectly substitutable products in the new market. One firm is a start-up (denoted  $s$ ) and maximizes the probability of survival, while the other is an established firm (denoted  $e$ ) that maximizes expected profit. The quantity of the product released to the market by firm  $i$  is  $Q_i$ ,  $i \in \{s, e\}$ . The market price of the product is given by the linear demand curve  $p(Q_i, Q_j) = A - Q_i - Q_j$ . As before,  $A$  is a random variable with positive support, distribution function  $F$ , mean  $\mu$ , and variance  $\sigma^2$ . Firms have identical capacity costs, which, as in the monopoly case, may vary over time (heterogeneous costs are discussed in §6.2). Note that we implicitly assume that neither firm is an incumbent in the market, thus a typical nomenclature in the disruptive innovation literature—entrant vs. incumbent firms—does not exactly apply to our model. It might be natural, though, to assume that the established firm is an incumbent in a related market or industry. Examples of this scenario include: Amazon.com and Barnes & Noble, both of whom entered the online book space at roughly the same time, despite the fact that Barnes & Noble was an “incumbent” in the related market of brick-and-mortar book retailing; and Webvan, a start-up which competed with existing traditional grocery stores in the emergent online grocery market in the early 2000s.

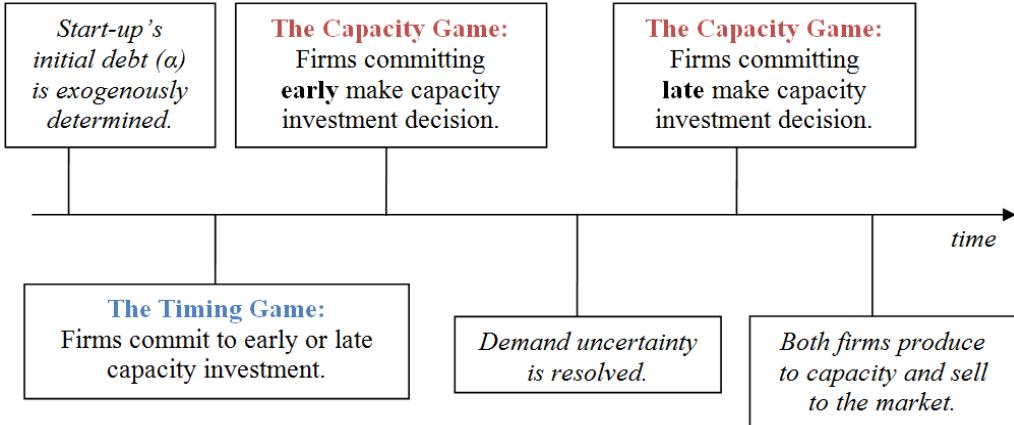
Before the early period (e.g., during an even earlier “decision period”), the firms simultaneously make their capacity timing decision. Each firm has two possible actions: either commit to invest in the early period, or commit to delay until the late period. We assume that these actions are credible and irreversible. This initial game is referred to as the investment timing game, or merely the *timing game*. There are four possible pure-strategy outcomes to the timing game: both firms invest early, both firms defer until the late period, and the two asymmetric outcomes in which one firm invests early and one firm invests late. The timing game, and the abbreviations used to refer to its outcomes, are depicted in Table 1.<sup>5</sup>

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<sup>5</sup>We note that while we consider a first stage investment timing game with embedded capacity subgames for its analytical convenience, this game is equivalent to a game in which firms do not first decide on an investment time, but rather simultaneously decide whether and how much to invest in the early period (i.e., whether to “invest now or wait”), under one key condition: if a firm unilaterally deviates from a particular equilibrium investment sequence, its competitor is allowed to optimally adjust capacity (but not investment timing) in response to this deviation. We believe this is a plausible scenario in reality, as capacity investment is a lengthy process and hence a firm sensing its competitor will deviate from a timing sequence (e.g., that the competitor will move from early to late investment) seems likely to modify its capacity level in the midst of the investment/construction process.

	Established Firm Early	Established Firm Late
Start-up Early	(E, E)	(E, L)
Start-up Late	(L, E)	(L, L)

**Table 1.** The four possible sequences of moves and their abbreviations.



**Figure 2.** The sequence of events in the duopoly model.

The *capacity subgame* then unfolds according to the sequence of moves determined by the timing game. In the late period, we assume all actions from the early period are publicly observable (e.g., if the established firm invests in the early period and the start-up defers, the start-up observes the precise capacity level of the established firm at the beginning of the late period before choosing its own capacity level). Thus, in addition to the informational and cost considerations from the monopoly model, there are strategic factors in play with the timing of capacity investment: if one firm moves early and the other moves late, the early-moving firm enjoys a leadership position in a sequential game while the late-moving firm is a sequential follower. As before, we assume that capacity investment is irreversible, and firms may invest in capacity in at most one period. The sequence of events is depicted in Figure 2.

In the following four lemmas, we analyze the equilibria to each of the four capacity subgames depicted in Table 1. Once we have derived these equilibria, we may in turn analyze the equilibrium to the investment timing game.

We first consider the case in which both firms invest in capacity late, i.e., after observing  $A$ . Because there is no randomness, as in the monopoly model, the start-up will choose capacity to maximize profit. The following lemma describes the equilibrium capacity investments for each firm in this game, in addition to providing the *ex-ante* survival probability of the start-up ( $\psi_s^*$ ) and the

*ex-ante* expected profit of the established firm ( $\mathbb{E}(\pi_e^*)$ ).

**Lemma 1** *If both firms invest in capacity late, then equilibrium capacities are  $K_e^* = K_s^* = \frac{A-c_2}{3}$ .*

*The ex-ante equilibrium expected profit of the established firm is*

$$\mathbb{E}(\pi_e^*) = \frac{\sigma^2 + (\mu - c_2)^2}{9}, \quad (7)$$

*while the ex-ante equilibrium survival probability of the start-up is*

$$\psi_s^* = 1 - F(3\sqrt{\alpha} + c_2). \quad (8)$$

We now move to the game in which the established firm invests late while the start-up invests in capacity early, i.e., prior to observing  $A$ . The following lemma describes the equilibrium capacity levels, expected profit, and survival probability.

**Lemma 2** *If the start-up invests early while the established firm invests late, equilibrium capacities are  $K_e^* = \frac{A-\sqrt{2\alpha}-c_2}{2}$  and  $K_s^* = \sqrt{2\alpha}$ . The ex-ante equilibrium expected profit of the established firm is*

$$\mathbb{E}(\pi_e^*) = \frac{\sigma^2 + (\mu - c_2 - \sqrt{2\alpha})^2}{4}, \quad (9)$$

*while the ex-ante equilibrium survival probability of the start-up is*

$$\psi_s^* = 1 - F(2\sqrt{2\alpha} + 2c_1 - c_2). \quad (10)$$

We next consider the case in which both firms invest in capacity early, i.e., before observing the value of  $A$ . Lemma 3 describes the equilibrium.

**Lemma 3** *If both firms invest early, equilibrium capacities are  $K_e^* = \frac{\mu-\sqrt{\alpha}-c_1}{2}$  and  $K_s^* = \sqrt{\alpha}$ .*

*The ex-ante equilibrium expected profit of the established firm is*

$$\mathbb{E}(\pi_e^*) = \frac{1}{4} (\mu - c_1 - \sqrt{\alpha})^2, \quad (11)$$

while the ex-ante equilibrium survival probability of the start-up is

$$\psi_s^* = 1 - F\left(2\sqrt{\alpha} + \frac{\mu - \sqrt{\alpha} + c_1}{2}\right). \quad (12)$$

Lastly, we address the case in which the start-up invests in capacity late while the established firm invests in capacity early. The following lemma describes the resulting equilibrium.

**Lemma 4** *If the established firm invests early while the start-up invests late, equilibrium capacities are  $K_e^* = \frac{\mu - 2c_1 + c_2}{2}$  and  $K_s^* = \frac{2A - \mu + 2c_1 - 3c_2}{4}$ . The ex-ante equilibrium expected profit of the established firm is*

$$\mathbb{E}(\pi_e^*) = \frac{(\mu + c_2 - 2c_1)^2}{8}, \quad (13)$$

while the ex-ante equilibrium survival probability of the start-up is

$$\psi_s^* = 1 - F\left(2\sqrt{\alpha} + \frac{\mu - 2c_1 + 3c_2}{2}\right). \quad (14)$$

## 5 Equilibrium to the Timing Game

Having derived equilibria to each of the capacity investment subgames, we may now derive the equilibrium to the investment timing game. The following theorem describes all of the possible equilibria to this game:

**Theorem 3** *Let  $\Delta c \equiv c_1 - c_2$ , let*

$$\Sigma_1 \equiv (\mu - c_1 - \sqrt{\alpha})^2 - (\mu - c_2 - \sqrt{2\alpha})^2 \quad (15)$$

and let

$$\Sigma_2 \equiv \frac{9}{8}(\mu + c_2 - 2c_1)^2 - (\mu - c_2)^2. \quad (16)$$

Then the following pure strategy equilibria to the investment timing game exist:

1. If  $\sigma^2 < \Sigma_1$  and  $\Delta c < \frac{1}{3}\sqrt{\alpha}$ , then both firms invest early.
2. If  $\sigma^2 > \Sigma_1$  and  $\Delta c < \frac{3-2\sqrt{2}}{2}\sqrt{\alpha}$ , then the start-up invests early while the established firm invests late.

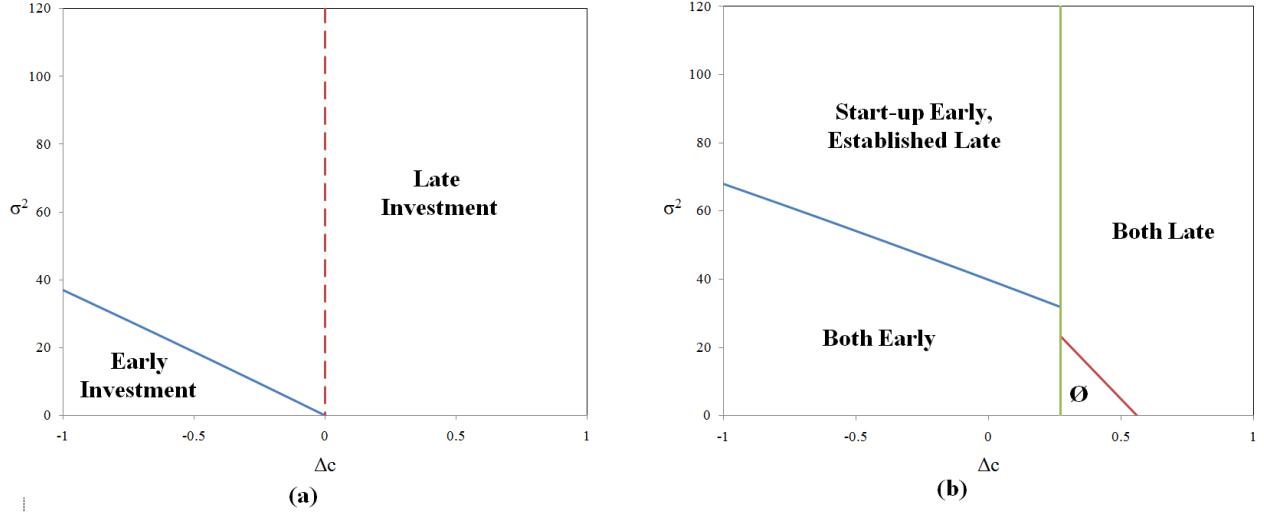
3. If  $\sigma^2 > \Sigma_2$  and  $\Delta c > \frac{3-2\sqrt{2}}{2}\sqrt{\alpha}$ , then both firms invest late.
4. If  $\sigma^2 < \Sigma_2$  and  $\Delta c > \frac{1}{2}\sqrt{\alpha}$ , then the start-up invests late while the established firm invests early.

There are several interesting consequences of these results. First, we note that the equilibrium regions are not exhaustive in covering the parameter space, nor are they mutually exclusive. As a result, regions of no (pure strategy) equilibria can occur, as can regions of multiple equilibria (in particular, regions in which late investment by both firms and early investment by both firms are both possible equilibria). In all, there are six potential equilibrium regions to the investment timing game: one region each for  $(L, L)$ ,  $(E, L)$ ,  $(L, E)$ , and  $(E, E)$ ; one region in which  $(E, E)$  and  $(L, L)$  are both possible; and one region in which no equilibrium exists. It may also be the case that the regions of  $(L, E)$  equilibrium existence, non-existence and multiple equilibria are empty, depending on the parameter values.

To help understand the behavior described in Theorem 3, it is useful to graphically compare possible equilibrium outcomes to the monopoly case. Figure 3 does this for a typical scenario. First, note that Figure 3a shows the optimal investment timing for a monopolist as a function of the variance of demand (vertical axis) and the cost differential  $\Delta c = c_1 - c_2$ : the solid line represents the boundary between early and late investment for a profit maximizing firm, while the dashed line represents the boundary for a survival maximizing start-up. As the figure shows, the start-up prefers early investment for a much larger portion of the parameter space.

Figure 3b depicts the timing equilibrium regions in the competitive model using the same parameter values as Figure 3a. The first observation one can make is that in the competitive case, early investment (for both firms) is far more likely. Moreover, if demand uncertainty is sufficiently high and costs do not decrease too much over time ( $\Delta c$  is not too large and  $\sigma^2$  is not too small, case (2) of the theorem), the unique equilibrium to the investment timing game is for the start-up to invest early and the established firm to invest late.

This equilibrium precisely describes the situation discussed by Bower and Christensen (1995): a new market enabled by disruptive technology with highly uncertain demand, in which a start-up plays the role of leader and the established firm the role of follower. This occurs because of three competing forces in the model. The first is that early investment is valuable due to first-mover



**Figure 3.** (a) Optimal investment timing for a monopolist. The solid line represents the boundary for an established firm and the dashed line for a start-up firm. (b) Equilibria to the investment timing game between a start-up and an established firm. In both examples,  $c_1 = 1$ ,  $\alpha = 10$ , and  $\mu = 20$ .

advantage in a sequential capacity game (if the competitor invests late). The second is that late investment is valuable due to the ability to exploit demand variance. The third is that the cheaper investment period is valuable due to cost savings, which can impact the value of either period. As we have already seen in the monopoly model, the second reason does not impact a start-up; hence, if costs do not decline severely over time, the start-up prefers early investment due to the leadership position in the capacity game. (Note that, unlike the monopoly model, a start-up facing competition from an established firm may invest early in capacity even if late investment is cheaper.)

By contrast, the established firm *does* value late investment due to the ability to exploit demand variance; hence, if variance is sufficiently high, the established firm prefers late investment even though it cedes a leadership position to the start-up. In particular, the start-up continues to choose the minimum capacity level that ensures survival over the widest range of demand outcomes, and hence does not exploit its leadership position to greatly increase capacity as a profit-maximizing firm might; consequently, it would appear that the established firm does not surrender as much by following a start-up as it might by following another established firm, a hypothesis that we verify in §6.1 by analyzing a model of two competing established firms.

We also observe that when costs decrease significantly over time, the picture can become complicated. In particular, a unique equilibrium may exist (either both early or both late, or the start-up

following the established firm), multiple equilibria may exist, or a pure strategy equilibrium may fail to exist. In the region of non-existence (denoted by the null symbol in Figure 3b), the start-up prefers to invest at the same time as the established firm (i.e., the start-up would like to exploit cost reduction and information but only if it does not mean giving up a leadership position), while the established firm prefers to invest at the opposite time of the start-up. As a result, the outcome of the game is unclear in this region (although, it should be noted, the region of non-existence typically covers a very small portion of the parameter space). Moreover, it is possible for an  $(L, E)$  equilibrium to exist if  $\Delta c$  is sufficiently large (or if  $\alpha$  is sufficiently small) and demand uncertainty is small—however, this equilibrium never exists for the parameter values used to generate Figure 3. Indeed, the equilibrium does not exist for most reasonable parameter values, since the decline in capacity costs over time must be very large relative to the mean demand and the bankruptcy threshold  $\alpha$ —for example, if  $c_1 = 1$  and  $c_2 = 0.8$ , representing a 20% cost reduction from period 1 to period 2, then for  $(L, E)$  to be an equilibrium it must be true that the bankruptcy threshold  $\alpha < 0.77$ .

## 6 Extensions

### 6.1 Competition with Two Established Firms

In this section, we analyze an investment timing game identical to the one discussed in §5, with one key difference: rather than competition between a start-up and an established firm, *both* firms are established, profit maximizing firms. We assume, as before, that the firms are *ex-ante* identical in all other respects. This allows us to compare the outcomes of the timing game with heterogeneous firms to an otherwise identical game with two mature firms, thus isolating the impact of bankruptcy risk on capacity investment timing. The following theorem presents the equilibrium to the timing game in this case.

**Theorem 4** *If two established firms compete in an investment timing game, then there exists some threshold  $\sigma^*$  such that, for all  $\sigma > \sigma^*$ , the unique equilibrium of the investment timing game is for both firms to invest late.*

As the preceding theorem demonstrates, a high degree of demand uncertainty leads to a unique equilibrium outcome when established, profit-maximizing firms compete: both firms invest in capacity late. This is in stark contrast to the investment timing equilibrium when a start-up competes with an established firm: in that case, we observed that high demand uncertainty can lead to equilibrium outcomes in which the start-up acts as a sequential leader in the investment game. We note that, in the game with two established firms, asymmetric outcomes *can* occur for lower demand variability; however, they can never occur if demand variability is sufficiently large, unlike the model with one start-up and one established firm. Hence, we conclude that a start-up's propensity to avoid bankruptcy can have a significant effect on the dynamics of competition, particularly when demand uncertainty is high in the context of new markets.

## 6.2 Firms with Heterogeneous Capacity Costs

In this extension, we return to the base model (one start-up and one established firm) and consider the impact of heterogeneous capacity costs. For the sake of simplicity, we will assume that costs are constant over time for both firms, since we have already explored the impact of time-varying costs. Let the cost of the established firm be  $c_e$  and the cost of the start-up be  $c_s$ . Our analysis of the asymmetric capacity games in fact already accommodates heterogeneous costs (since costs in the base model varied over time, when firms invest at different times, costs are by definition heterogeneous). Thus, we need only modify our analysis to account for heterogeneous costs in the symmetric investment games. The following lemma summarizes the equilibria to the capacity investment games:

**Lemma 5** *If firms have heterogeneous capacity costs that are constant over time, then:*

1. (L,L) *If both firms invest in capacity late, then equilibrium capacities are  $K_e^* = \frac{A+c_s-2c_e}{3}$  and  $K_s^* = \frac{A+c_e-2c_s}{3}$ . The ex-ante equilibrium expected profit of the established firm is*

$$\mathbb{E}(\pi_e^*) = \frac{\sigma^2}{9} + \left( \frac{\mu + c_s - 2c_e}{3} \right)^2,$$

*while the ex-ante equilibrium survival probability of the start-up is*

$$\psi_s^* = 1 - F(3\sqrt{\alpha} + 2c_s - c_e).$$

2. (E,L) If the start-up invests early while the established firm invests late, equilibrium capacities, profits, and survival probabilities are identical to those derived in Lemma 2, with  $c_e = c_2$  and  $c_s = c_1$ .

3. (E,E) If both firms invest early, equilibrium capacities are  $K_e^* = \frac{\mu - \sqrt{\alpha} - c_e}{2}$  and  $K_s^* = \sqrt{\alpha}$ .

The ex-ante equilibrium expected profit of the established firm is

$$\mathbb{E}(\pi_e^*) = \frac{1}{4} (\mu - c_e - \sqrt{\alpha})^2,$$

while the ex-ante equilibrium survival probability of the start-up is

$$\psi_s^* = 1 - F\left(\frac{\mu + 3\sqrt{\alpha} + 2c_s - c_e}{2}\right).$$

4. (L,E) If the established firm invests early while the start-up invests late, equilibrium capacities, profits, and survival probabilities are identical to those derived in Lemma 4, with  $c_e = c_1$  and  $c_s = c_2$ .

Armed with the equilibrium survival probabilities and expected profits, we may derive the equilibrium to the capacity investment timing game:

**Theorem 5** If firms have heterogeneous capacity costs that are constant over time, a unique equilibrium to the timing game exists. Let

$$\Sigma_1 \equiv (\mu - c_e - \sqrt{\alpha})^2 - (\mu - c_e - \sqrt{2\alpha})^2$$

and let

$$\Sigma_2 \equiv \frac{\mu + c_s - 2c_e}{2\sqrt{2}}.$$

Then the following pure strategy equilibria to the investment timing game exist:

1. If  $\sigma^2 > \Sigma_1$ , the start-up invests early while the established firm invests late.

2. If  $\sigma^2 < \Sigma_1$  and  $\sqrt{\alpha} > c_e - c_s$ , both firms invest early.

3. If  $\sigma^2 < \Sigma_2$  and  $\sqrt{\alpha} < c_e - c_s$ , the established firm invests early while the start-up invests late.

Intriguingly, when costs are constant over time but differ between the two firms, only one equilibrium is possible when demand uncertainty is high: the start-up is the leader. This preserves our main result—that bankruptcy risk leads to an increased frequency of equilibria in which start-ups lead established firms—and demonstrates that it is not sensitive to the homogenous cost assumption.

### 6.3 Holdback

Throughout our analysis, we have assumed that firms always produce to their maximum capacity—that is, both firms follow a *production clearance* strategy. From a modeling perspective this allows for a simple and relatively clean analysis of the capacity investment decision—in the absence of this assumption, closed form solutions for equilibrium capacities, profits, and survival probabilities cannot be obtained—and moreover the clearance assumption may be thought of as the outcome of selling the product at a series of different prices until capacity is exhausted or fully utilized, in which case the “market price” is actually an average price. Additionally, firms frequently produce at maximum capacity because of high fixed costs of starting and stopping the production process (e.g., in the chemical or semiconductor industries): see Goyal and Netessine (2007).

From a practical standpoint, though, it may be unwise for a firm to always produce at maximum capacity. Other papers (e.g., Chod and Rudi 2005) have demonstrated that a clearance assumption typically has a negligible impact on analytical outcomes, however, it is useful to verify this result in our setting. Hence, in this section, we discuss the impact of the alternative assumption: a *holdback* strategy, in which the firms may produce any *ex-post* profit maximizing quantity subject to their individual capacity constraints.

It is first useful to consider the qualitative impact of holdback. In fact, our previous analysis accommodates a holdback strategy whenever a firm invests *late*—this is because we assume that late investment occurs after the resolution of demand uncertainty, hence the firm would never invest in more capacity than necessary for maximizing profit (i.e., a firm investing in capacity late always produces to full capacity). Thus, the analysis for any firm investing late is unchanged if holdback is allowed. Furthermore, the analysis of a start-up investing early is also unchanged by the option of holdback. Recall that a start-up investing early chooses the minimum capacity level that supports survival—in other words, if a start-up invests in  $K_s$  units of capacity, it must sell all  $K_s$  units to survive. As a result, a start-up investing early will always produce up to its maximum capacity

level if it survives; holdback could only occur in demand states in which the start-up does not survive, which does not impact the start-up’s subsequent probability of survival.

It follows, then, that holdback only affects an established firm investing early. Intuitively, granting such a firm the option of producing less can only increase the value of early investment relative to late investment. Some incentive for late investment remains, though, particularly if capacity costs are significant relative to marginal production costs; in that case, there is still value to waiting for the resolution of demand uncertainty to avoid sinking excess money into costly capacity. Hence, we postulate that allowing holdback increases the established firm’s incentives for early investment without completely eliminating incentives for late investment.

While this thought experiment helps to understand the impact of holdback on firm profit, without further analysis, it’s unclear how holdback affects the competing start-up’s survival probability and the equilibrium of the timing game. To that end, we conducted a numerical study to explore precisely this issue. The model employed in the study is identical to the one analyzed in the rest of the paper, save for the fact that firms are allowed to engage in holdback. The additional complication is that a *quantity game* occurs at the start of the selling season: after observing the realized value of market size ( $A$ ), firms choose production quantities to maximize profit, subject to their individual capacity constraints. (As in the base model, production is assumed to be costless, though positive production costs do not qualitatively change any results).

To analyze this more complicated model, we must make an additional assumption concerning the order of moves in the quantity game. A variety of plausible options exist (e.g., the leader in the capacity game is the leader in the quantity game; the established firm is the leader in the quantity game due to greater market power; firms strategically time their quantity decisions just as they do their capacity decisions). We choose the simplest sequence: simultaneous quantity competition. Thus, the firms engage in capacitated Cournot competition in the quantity game—see Gabszewicz and Poddar (1997) for a proof of existence of an equilibrium in this subgame (as well as an analysis of a similar *ex-ante* capacity investment game, but with two profit-maximizing firms moving simultaneously).

We examined 200 parameter instances consisting of every combination in Table 2, which were selected to provide a wide range of possible scenarios (e.g., low to high demand variability, various product margins, etc.). In each case, we calculated the equilibrium to the investment timing game

Parameter	Values
Demand Distribution ( $A$ )	Gamma
$\mu$	10
$\sigma$	{2.5, 5, 7.5, 10, 12.5, 15, 17.5, 20}
$c_1$	1
$c_2$	{0.333, 0.667, 1, 1.333, 1.667}
$\alpha$	{10, 20, 30, 40, 50}

**Table 2.** Parameter values used in numerical experiments.

with holdback and with clearance. Comparing the incidence of specific equilibria between the two possible assumptions allows us to determine the impact of holdback on our theoretical results.

Our results are summarized in the first two rows of Table 3. As expected, since the value of early investment is higher with an option to produce less than capacity, early investment becomes a more attractive option for the established firm with holdback: the established firm invests early in only 10% of cases with clearance, but 63% of cases with holdback. Conversely, the impact on the start-up’s equilibrium investment timing is far less: the start-up invests early in 79% of cases with clearance and 86% of cases with holdback. Importantly, holdback never results in the reverse sequential outcome (i.e., the established firm leading and the start-up following). Moreover, even with the possibility of holdback,  $(E, L)$  equilibria in which the start-up leads still occur in roughly one quarter of our numerical examples (although at a more moderate frequency than under the clearance assumption).

#### 6.4 Alternative Objective Functions

In the preceding analysis, we assumed that the start-up chooses a capacity level and investment time to maximize its probability of survival. As we discussed in the introduction, if the probability of survival is low or the consequences of failure are severe, it is safe to assume that a start-up pays little attention to immediate profits and focuses more on simply avoiding bankruptcy. However, an interesting question is how the behavior of the start-up changes if it cares about both profit *and* the probability of survival. Moreover, start-ups financing their activities may be subject to limited liability should bankruptcy occur, which implies that while profit may be a factor in the objectives of start-ups, it is only the profit above the bankruptcy threshold which truly matters (Jensen and Meckling 1976, Brander and Lewis 1986). To that end, in this section we numerically examine

the impact of two alternative objective functions for a start-up. The first is referred to as the *integrated objective function*, and is equal to the expected operating profit ( $\pi$ ) minus an exogenous bankruptcy penalty ( $D$ ) times the probability of bankruptcy ( $1 - \psi$ )

$$\mathbb{E}(\pi_s) - D \times (1 - \psi_s). \quad (17)$$

As one might expect, since this objective is a linear combination of the previously analyzed survival probability and profit objectives, the behavior of a firm choosing capacity and investment time to maximize (17) lies somewhere between that a purely profit and a purely survival focused firm. In particular, the firm places more weight on the potential cost advantages of early investment (because this lowers the chance of bankruptcy) and less weight on the variance-exploiting advantages of late investment than a purely profit maximizing firm. Consequently, depending on the precise value of  $D$  (and hence the relative weight placed on each portion of the objective function), the equilibria to the timing game resembles a mixture of the cases previously analyzed (with a survival maximizing firm, and with two profit-maximizing firms).

The second alternative objective function is called the *limited liability objective function*. In this scenario, the start-up is assumed to lose all profit if bankruptcy occurs (e.g., any remaining funds are distributed to debtholders) while keeping any excess profit above the survival threshold; consequently, the start-up only cares about expected profit in excess of the survival threshold, i.e.,

$$\mathbb{E}(\pi_s - \alpha | \pi_s \geq \alpha) \times \psi_s. \quad (18)$$

Unlike the integrated objective function, (18) is not a linear combination of the profit maximizing and survival maximizing functions. As a result, how this objective function impacts the equilibrium to the investment timing game is, at first, not obvious.

While neither of these functions permits the relatively clean analytical treatment of a survival maximizing objective function, it is possible to analyze both using numerical methods. Table 3 presents the results of applying the same large-scale numerical study from the previous section (i.e., using the 200 parameter combinations depicted in Table 2) to models in which the start-up optimizes a limited liability or integrated objective function. For the sake of comparison, the first

row of the table lists equilibrium incidence for our base model (a survival maximizing start-up) and the last row lists results for a model with two profit maximizing firms. As the table shows, both the limited liability and integrated objective models yield results somewhere between the survival maximizing and profit maximizing cases.

The table nicely demonstrates a key feature of our model: that bankruptcy tends to shift equilibria toward the sequential outcome with the start-up as the leader. The intuition behind this result is clear in the case of the integrated objective function, as it is a linear combination of expected profit and survival probability: later investment allows the firm to exploit demand variance, which increases the value of the profit portion of the objective function, while earlier investment (particularly if it is less costly) allows the firm to reduce the chance of bankruptcy and hence reduce the impact of the bankruptcy penalty. Hence, depending on the value of the penalty parameter ( $D$ ), the frequency of equilibria occurrence is somewhere between that of the purely profit maximizing and purely survival maximizing cases.

As the table demonstrates, similar to the integrated objective, the incidence of equilibria under limited liability also lie somewhere between that of the base survival maximizing case and the profit maximizing case. Compared to the profit maximizing case, fewer  $(L, L)$  equilibria and more  $(E, L)$  equilibria occur; in other words, with limited liability, sequential outcomes (with the start-up as leader) are more likely than sequential outcomes in competition between two profit maximizing firms.

These numerical tests show a shift toward sequential outcomes persists regardless of the precise way in which bankruptcy risk is incorporated into the start-up's objective function. With a purely survival maximizing start-up, there is a very strong push towards sequential outcomes; with an objective function concerned with the upside of potential profit (such as the integrated objective or the limited liability objective) this effect is tempered somewhat, but not entirely eliminated. Consequently, we conclude that these results support our findings that the threat of bankruptcy—manifested in the start-up's objective function in a number of different ways—can lead to a greater chance of sequential outcomes in which the start-up takes a leadership role.

Model	Investment Sequence			
	(E, E)	(E, L)	(L, E)	(L, L)
Base Model	10%	69%	0%	24%
Base Model + Holdback	63%	23%	0%	18%
Limited Liability Startup	8%	29%	0%	66%
Integrated Objective Start-up, $D = 10$	12%	4.5%	0%	88%
Integrated Objective Start-up, $D = 100$	12%	11%	0%	81%
Integrated Objective Start-up, $D = 1000$	12%	32%	0%	59%
Integrated Objective Start-up, $D = 10000$	12%	56%	0%	35%
Two Profit Maximizing Firms	10%	2.5%	2.5%	90%

**Table 3.** Incidence of equilibria to the investment timing game under various models. Note that the total percentages of equilibrium incidence may sum to more or less than 100, due to regions of potential non-existence and multiple equilibria.

## 7 Conclusion

In this work, our chief goal was to analyze how the threat of bankruptcy impacts the capacity investment and timing decisions of firms entering new markets. We find that in monopoly markets, start-ups are more likely to prefer early capacity investment than profit-maximizing established firms. In competitive markets, when demand uncertainty is large, the outcome of a strategic investment timing game leads to an equilibrium in which the start-up invests early while the established firm invests late—starkly contrasting to a model with two established firms, which leads to simultaneous late investment under high demand uncertainty.

We arrived at these results despite invoking several assumptions intended to minimize the incidence of sequential equilibria. For example, in previous literature, one explanation offered for established firms failing to seize opportunities in disruptive markets is that their demand forecasts are too pessimistic or simply inaccurate. We have found, on the contrary, that even if both firms have *identical* demand forecasts (i.e., identical beliefs about the distribution of market size), sequential equilibria arise if a start-up is present. If we incorporated pessimistic forecasts by established firms into our model, this would have the effect of decreasing the expected market size in the established firm’s profit function, qualitatively preserving our results (and giving the established firm even more incentive to invest late). Similarly, we assumed that both firms have

access to the technology that enables the new market at the start of the strategic investment game—in other words, no firm is playing catch-up from a technological standpoint, and both are capable of capacity investment at any time.

In addition, because start-ups may face financial constraints that limit the maximum possible expenditure on capacity, one might reasonably suppose that it is appropriate to incorporate such a constraint into our formulation. Recall that the optimal capacity level of the start-up at either investment time is the minimum capacity level at which survival can occur—if the start-up has insufficient funds to support this capacity, then survival can never occur, and hence the survival probability is zero. Alternatively, if the start-up has more funds than necessary to support this minimum capacity level, the constraint is not binding and hence is irrelevant. Thus, at least in the survival maximizing case, such a constraint has a very “bang-bang” impact on the model: it is either irrelevant or it reduces the survival probability to zero. A financial constraint is more meaningful if the start-up considers some combination of profit and bankruptcy costs, e.g., as in §6.4. In this case, any constraint will likely limit the value of late investment as it reduces the ability of the start-up to react to high demand states with a high capacity level—consequently, though we do not explicitly include any financial constraints in our model, we anticipate that they would either have minimal impact on our results (in the case of survival probability maximization) or they would favor early investment even more than our current model (in the case of more complicated objective functions).

We also did not model a variety of other factors that may influence capacity investment timing. For example, greater sales may be enabled by earlier entry (e.g., if late entry results in slower time-to-market). Directionally, the impact of this effect is clear: it increases firm incentives to invest early. While this would likely change the equilibrium thresholds given in Theorem 3, the qualitative impact of the start-up’s survival maximizing objective function remains (as do the consequences of acting as a first- or second-mover in the capacity game), implying that the strategic investment game will have a similar structure and will yield similar results. Future work may investigate the robustness of our results with regards to a number of similar complications, including cost uncertainty, allowing firms to add capacity in multiple stages rather than only once, and the incorporation of the initial market entry and capital structure decisions that lead to the determination of  $\alpha$ .

We conclude that capacity competition involving start-ups subject to bankruptcy risk—in a variety of forms—is fundamentally different in nature from the competition between established firms, and our model offers a plausible explanation of some practically observed phenomena. Managerially, these results are important because they imply that the optimal strategic investment position differs depending on the nature of the competitor. Thus, blindly following a mantra of seizing the “first-mover advantage” can be a perilous strategy, as any such advantage (or disadvantage) depends critically on the characteristics of the firms in the market.

While our key findings relate to equilibrium capacity investment timing and investment, our results also relate to the literature on disruptive innovation, which has frequently observed that start-ups tend to pioneer new markets while established firms postpone investment. A variety of reasons for this phenomenon are offered: the established firms are said to be too close to and too trusting of their existing customers, who themselves are ill-equipped to articulate their own changing needs, therefore causing a failure to anticipate opportunities within the existing customer base; the established firms fail to recognize and cultivate entirely new markets; internal incentives at the established firms favor the development and implementation of incremental improvement over radical change. All of these explanations imply that established firms *fail* in some crucial way that newer firms do not. By controlling for these factors in our formulation, our results imply that, while it is certainly possible that managerial failures and other reasons cited in the disruptive innovation literature can lead to established firms detrimentally ceding a leadership role to start-ups in new markets, this need not be the case; the operational reality of capacity investment under demand uncertainty, coupled with facing competition from start-ups prone to failure, offers a purely rational explanation for these outcomes.

## A Appendix: Proofs

**Proof of Theorem 1.** *Early Investment:* the profit function implied by (1) is concave and yields a unique maximum at the Cournot monopoly point,  $K_e^* = (\mu - c_1)/2$ . Expected profit is thus  $\mathbb{E}(\pi_e^*) = (\mu - c_1)^2/4$ .

*Late Investment:* the profit function implied by (2) is concave and yields a unique maximum at the Cournot monopoly point,  $K_e^* = (A - c_2)/2$ . Expected profit is thus  $\mathbb{E}(\pi_e^*) = \mathbb{E}\left((A - c_2)^2/4\right) = (\mu - c_2)^2/4 + \sigma^2/4$ .

**Proof of Theorem 2.** *Early Investment:* maximizing the survival probability function in (5)

is equivalent to

$$\psi_s^* = \max_{K_s \geq 0} \Pr \left( A \geq \frac{\alpha}{K_s} + K_s + c_1 \right) = \max_{K_s \geq 0} \left( 1 - F \left( \frac{\alpha}{K_s} + K_s + c_1 \right) \right),$$

and, consequently, this is equivalent to minimizing  $\frac{\alpha}{K_s} + K_s + c_1$ . This expression is convex and yields a unique minimizing capacity of  $K_s^* = \sqrt{\alpha}$ . The corresponding optimal survival probability is thus  $\psi_s^* = 1 - F(2\sqrt{\alpha} + c_1)$ .

*Late Investment:* under late investment, the start-up maximizes profit after observing  $A$ . This implies the late investment capacity level is identical to the established firm's capacity level until late investment, i.e.,  $K_s^* = (A - c_2)/2$ . The survival probability is thus

$$\psi_s^* = \Pr \left( \frac{(A - c_2)^2}{4} \geq \alpha \right) = 1 - F(2\sqrt{\alpha} + c_2),$$

yielding the result.

**Proof of Lemma 1.** Because there is no randomness if both firms invest late, the capacity investment game is a Cournot duopoly with heterogeneous costs. Thus, the profit of each firm is given by

$$\begin{aligned} \pi_e(K_e) &= (A - K_e - K_s - c_2) K_e, \\ \pi_s(K_s) &= (A - K_e - K_s - c_2) K_s. \end{aligned}$$

Both profit functions are concave, yielding unique best replies

$$K_e^*(K_s) = \frac{A - K_s - c_2}{2} \text{ and } K_s^*(K_e) = \frac{A - K_e - c_2}{2}.$$

The equilibrium capacities are found by solving for the intersection of the best replies, which yields the unique equilibrium  $K_e^* = K_s^* = \frac{A-c_2}{3}$ . Equilibrium profit of each firm is

$$\mathbb{E}(\pi_e^*) = \mathbb{E}(\pi_s^*) = \mathbb{E} \left( \frac{A - c_2}{3} \right)^2 = \frac{\sigma^2 + (\mu - c_2)^2}{9}.$$

Recall that the start-up survives if the total profit level is above  $\alpha$ : in other words, if  $((A - c_2)/3)^2 \geq \alpha$ . Thus, the *ex-ante* survival probability (i.e., the probability of survival *before* learning market size, taking into account the competitive outcome of the capacity game that occurs *after* learning market size) is given by (8), while the (*ex-ante*) equilibrium expected profit of the established firm is given by (7).

**Proof of Lemma 2.** Recall that the best reply of the established firm is  $K_e^*(K_s) = \frac{A-K_s-c_2}{2}$  when both firms invest late: this continues to hold when the start-up invests early and the established firm invests late. The start-up's profit is thus

$$\begin{aligned} \pi_s(K_s) &= (A - K_e^*(K_s) - K_s - c_1) K_s \\ &= \frac{1}{2} (A - K_s - 2c_1 + c_2) K_s. \end{aligned}$$

The survival probability is the probability that  $\pi_s(K_s) \geq \alpha$ , i.e.,

$$\begin{aligned}\psi_s(K_s) &= \Pr\left(\frac{1}{2}(A - K_s - 2c_1 + c_2)K_s \geq \alpha\right) \\ &= 1 - F\left(\frac{2\alpha}{K_s} + K_s + 2c_1 - c_2\right).\end{aligned}$$

The maximizer of the survival probability is the minimizer of the argument of  $F$  in the above equation, i.e.,  $K_s^* = \sqrt{2\alpha}$ , yielding (10) when substituted into the expression for the start-up's survival probability. The established firm's profit is

$$\pi_e(K_e) = \frac{1}{4}(A - c_2 - \sqrt{2\alpha})^2,$$

and *ex-ante* expected profit is thus given by the expected value of this expression, yielding (9).

**Proof of Lemma 3.** Survival for the start-up occurs if  $A \geq \frac{\alpha}{K_s} + K_e + K_s + c_1$ , so the survival probability is thus

$$\psi_s(K_s, K_e) = 1 - F\left(\frac{\alpha}{K_s} + K_e + K_s + c_1\right).$$

Minimizing the the argument of  $F$  in the above expression is equivalent to maximizing the probability of survival. Thus, the start-up's optimal capacity investment is  $K_s^* = \sqrt{\alpha}$ , a dominant action that is independent of the established firm's capacity level. The established firm's expected profit is

$$\mathbb{E}(\pi_e(K_s, K_e)) = (\mu - K_e - K_s - c_1)K_e.$$

Substituting the equilibrium  $K_s^*$  and maximizing this concave function of  $K_e$  yields the established firm's optimal capacity,  $K_e^* = \frac{\mu - \sqrt{\alpha} - c_1}{2}$ . The associated expected profit is (11), and the equilibrium survival probability of the start-up is hence (12).

**Proof of Lemma 4.** The best reply of the start-up investing late is the same as in Lemma 1, i.e.,  $K_s^*(K_e) = \frac{A - K_e - c_2}{2}$ . Hence, the established firm's expected profit from early investment is

$$\mathbb{E}(\pi_e(K_e)) = \left(\mu - K_e - \frac{\mu - K_e - c_2}{2} - c_1\right)K_e$$

Maximizing this expression yields an optimal capacity level of  $K_e^* = \frac{\mu - 2c_1 + c_2}{2}$  for the established firm and hence

$$K_s^* = \frac{2A - \mu + 2c_1 - 3c_2}{4}$$

for the start-up. The equilibrium expected profit of the established firm is thus (13), and the start-up's equilibrium survival probability is (14).

**Proof of Theorem 3.** We will examine the viability of each subgame in Table 1 individually.

(i)  $(E, L)$ . First, let us consider the equilibrium in which the start-up invests early and the established firm follows:  $(E, L)$ . This is an equilibrium if no firm has incentive to unilaterally deviate: in other words, if the established firm enjoys greater expected profit than in  $(E, E)$ , and if the start-up enjoys a greater survival probability than in  $(L, L)$ . From Lemmas 1 and 2, comparing the arguments of the distribution function  $F$  in each of the equilibrium survival probabilities, we see that if the established firm invests late, the start-up enjoys a (strictly) greater survival probability by investing early if:

$$2\sqrt{2\alpha} + 2c_1 - c_2 < 3\sqrt{\alpha} + c_2.$$

Rearranging this expression, we see it reduces to

$$2\sqrt{2\alpha} < 3\sqrt{\alpha} + 2(c_2 - c_1).$$

If  $c_1 < c_2$ , the condition holds if  $\alpha > 0$ . If, on the other hand,  $c_1 > c_2$ , the start-up *may* unilaterally deviate from  $(E, L)$  for some  $\alpha > 0$ . Examining this expression, we see that the inequality is most likely to hold if  $\alpha$  is large—hence, the start-up will deviate from  $(E, L)$  if costs decrease over time, and  $\alpha$  is sufficiently small.

Next, consider the established firm, which, from Lemmas 2 and 3, will not deviate from  $(E, L)$  if

$$\frac{\sigma^2 + (\mu - c_2 - \sqrt{2\alpha})^2}{4} > \frac{(\mu - c_1 - \sqrt{\alpha})^2}{4}.$$

This expression reduces to

$$\sigma^2 > (\mu - c_1 - \sqrt{\alpha})^2 - (\mu - c_2 - \sqrt{2\alpha})^2. \quad (19)$$

In other words, the established firm will not unilaterally deviate from  $(E, L)$  if demand is variable enough, where the threshold variability is a function of the problem parameters. This demonstrates case (1) in the theorem.

(ii)  $(E, E)$ . We next consider the equilibrium in which both firms build capacity early. From Lemmas 2 and 3, the established firm will not deviate from this equilibrium precisely if (19) is violated. From Lemmas 3 and 4, the start-up will not deviate if

$$2\sqrt{\alpha} + \frac{1}{2}\mu - c_1 + \frac{3}{2}c_2 > 2\sqrt{\alpha} + \frac{\mu - \sqrt{\alpha} + c_1}{2}.$$

This inequality reduces to

$$\frac{1}{3}\sqrt{\alpha} > c_1 - c_2.$$

This demonstrates case (2) in the theorem.

(iii)  $(L, L)$ . We lastly consider the equilibrium with both firms building capacity late. In this case, part (i) of the proof demonstrated that the start-up prefers  $(L, L)$  to  $(E, L)$  if

$$2\sqrt{2\alpha} > 3\sqrt{\alpha} + 2(c_2 - c_1).$$

Similarly, from Lemmas 1 and 4, the established firm will not deviate from this equilibrium if

$$\frac{(\mu + c_2 - 2c_1)^2}{8} < \frac{\sigma^2 + (\mu - c_2)^2}{9}$$

This inequality reduces to

$$\frac{9}{8}(\mu + c_2 - 2c_1)^2 - (\mu - c_2)^2 < \sigma^2.$$

This proves case (3) of the theorem.

(iv)  $(L, E)$ . The start-up has incentive to deviate from  $(E, E)$  to  $(L, E)$  if  $\frac{1}{3}\sqrt{\alpha} < c_1 - c_2$ , and the established firm has incentive to deviate from  $(L, L)$  to  $(L, E)$  if

$$\frac{9}{8}(\mu + c_2 - 2c_1)^2 - (\mu - c_2)^2 > \sigma^2,$$

proving case (4) of the theorem.

**Proof of Theorem 4.** We must first analyze several additional aspects of the capacity subgames in order to analyze the investment timing game. First, consider the game in which both firms invest early. This is a Cournot duopoly, hence the equilibrium profits of the (symmetric) established firms are both

$$\mathbb{E}(\pi_e^*) = \frac{(\mu - c_1)^2}{9}.$$

Next, consider the game in which the firms invest sequentially. This is identical to the previously analyzed game in which the established firm invests early and the start-up invests late (because, in that case, the start-up maximized profit due to the elimination of uncertainty). Hence, the profit of the leader is

$$\mathbb{E}(\pi_e^*) = \frac{(\mu + c_2 - 2c_1)^2}{8}$$

while the profit of the follower is

$$\mathbb{E}(\pi_e^*) = \frac{4\sigma^2 + (\mu + 2c_1 - 3c_2)^2}{16}.$$

Finally, the game in which both firms invest late yields identical profits to both firms equal to

$$\mathbb{E}(\pi_e^*) = \frac{\sigma^2 + (\mu - c_2)^2}{9}.$$

Thus, the investment timing game in normal form has payoffs

	Firm 2 Early	Firm 2 Late
Firm 1 Early	$\left(\frac{(\mu - c_1)^2}{9}, \frac{(\mu - c_1)^2}{9}\right)$	$\left(\frac{(\mu + c_2 - 2c_1)^2}{8}, \frac{4\sigma^2 + (\mu + 2c_1 - 3c_2)^2}{16}\right)$
Firm 1 Late	$\left(\frac{4\sigma^2 + (\mu + 2c_1 - 3c_2)^2}{16}, \frac{(\mu + c_2 - 2c_1)^2}{8}\right)$	$\left(\frac{\sigma^2 + (\mu - c_2)^2}{9}, \frac{\sigma^2 + (\mu - c_2)^2}{9}\right)$

First, assume that firm 2 invests early. Firm 1 prefers *late* investment if

$$\frac{4\sigma^2 + (\mu + 2c_1 - 3c_2)^2}{16} > \frac{(\mu - c_1)^2}{9}. \quad (20)$$

Clearly, as  $\sigma^2$  increases, this inequality is more likely to hold. Similarly, if firm 2 invests late, firm 1 prefers late investment if

$$\frac{\sigma^2 + (\mu - c_2)^2}{9} > \frac{(\mu + c_2 - 2c_1)^2}{8}. \quad (21)$$

Again, as  $\sigma^2$  increases, this inequality is more likely to hold, thus for large enough  $\sigma^2$  (i.e.,  $\sigma^2$  above some threshold), late investment is the dominant strategy of both firms and  $(L, L)$  is the only possible equilibrium, proving the theorem.

**Proof of Lemma 5.** Omitted—similar to Lemmas 1–4.

**Proof of Theorem 5.** Similar to the proof in the base model, we will examine each possible equilibrium individually.

(i)  $(E, L)$ . This is an equilibrium if no firm has incentive to unilaterally deviate: from Lemma 5, the start-up will not deviate if

$$1 - F(2\sqrt{2\alpha} + 2c_s - c_e) > 1 - F(3\sqrt{\alpha} + 2c_s - c_e),$$

which always holds. The equilibrium is supportable if the established firm has no incentive to deviate, i.e., if

$$\frac{\sigma^2 + (\mu - c_e - \sqrt{2\alpha})^2}{4} > \frac{1}{4} (\mu - c_e - \sqrt{\alpha})^2,$$

which holds if  $\sigma^2 > (\mu - c_e - \sqrt{\alpha})^2 - (\mu - c_e - \sqrt{2\alpha})^2$ —thus, with constant, heterogeneous costs,  $(E, L)$  is an equilibrium if  $\sigma^2$  is sufficiently large.

(ii)  $(E, E)$ . This sequence is only an equilibrium if the established firm has no incentive to deviate, which the analysis of  $(E, L)$  showed occurs for low  $\sigma^2$ . It must also be the case that the start-up has no incentive to deviate, which holds if

$$1 - F \left( 2\sqrt{\alpha} + \frac{1}{2}\mu - c_e + \frac{3}{2}c_s \right) > 1 - F \left( \frac{\mu + 3\sqrt{\alpha} + 2c_s - c_e}{2} \right)$$

which is equivalent to  $\sqrt{\alpha} > c_e - c_s$ .

(iii)  $(L, E)$ . The start-up has incentive to remain in this equilibrium if  $\sqrt{\alpha} < c_e - c_s$ . The established firm has incentive to remain in this equilibrium if

$$\frac{\sigma^2}{9} + \left( \frac{\mu + c_s - 2c_e}{3} \right)^2 < \frac{(\mu + c_s - 2c_e)^2}{8}.$$

This reduces to

$$\sigma < \frac{\mu + c_s - 2c_e}{2\sqrt{2}}.$$

(iv)  $(L, L)$ . This sequence is only an equilibrium if the start-up has no incentive to deviate, which the analysis of  $(E, L)$  showed is never true. Hence,  $(L, L)$  cannot be an equilibrium.

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