"L O C A T I O N"

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LOCATION*

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1 Introduction

Space, by its very nature, is a source of market power. Indeed, most markets operate over intricate networks of scattered buyers and sellers. Because the market activities are performed at dispersed points in space, each firm finds only a few rivals in its immediate neighborhood; further away there might be more competitors, but their influence is weakened by the existence of transportation costs. Similarly, not all consumers are alike to the firm; those who are far away will not buy from the firm because they have to pay too high a transportation cost. Accordingly, competition in space occurs "among the few", thus leading to an analysis of the problem as a game of strategy.

The model designed for describing that situation has come to be known as the model of spatial competition. In this model, a population of consumers is spread out over a geographical area, while firms selling a homogeneous product are (to be) located in the same space. Consumers have specific preferences regarding the commodity made available by the sellers either at the firms' or consumers' place (depending on who controls the transport). Since the product is homogeneous, a basic feature of consumers' behavior is that they buy from the firm charging the lowest full price, i.e. the price gross of the transportation costs. As a result, the number of customers patronizing a particular firm depends on its location and price policy, as well as on locations and price policies of competing firms established in the relevant area. This situation typically involves the basic ingredients of a noncooperative game in which the players are firms, strategies prices and/or locations, and the payoffs are profit functions.

The economic relevance of location games does not exclusively stem from their initial geographical set-up. Indeed, location problems are fundamentally related to many aspects of business competition in modern economies. Firstly, the spatially dispersed nature of markets has a direct analog in industrial economies under the form of an industry with differentiated products. In that set-up, product substitutes are dispersed in a space of characteristics à la Lancaster, and the seller of a particular variant enjoys a quasi-monopolistic position relative to the consumers who most prefer it. Moreover, the counterpart of the transportation costs is the utility loss incurred by a consumer who does not find his "ideal product" on the market. (In the geographical setting, this means that transport is under the control of the consumer.) Thus, the interest in modelling spatial competition extends immediately to the process of competition amongst firms producing differentiated commodities. In this domain, it was found useful to distinguish between market competition under horizontal versus vertical product differentiation. Two variants of a product are said to be horizontally differentiated whenever, sold at the same price, some consumers choose one variant while the others buy the alternative variant. Two variants are vertically differentiated whenever, sold at the same price, all consumers purchase the same variant (like in the case of a "standard" and "luxury" product). Along several dimensions, the nature of competition turns out to be different under the two types of differentiation. Interestingly enough, these two forms of competition have precise counterparts in spatial competition. To horizontal product differentiation corresponds a process of spatial competition with firms locating within the sub-space where the consumers themselves are located. The typical case is provided by shops installed inside the residential area : the "Main Street" model of Hotelling (1929). In what follows, we call such games inside location games. The analog of vertical product differentiation in spatial competition corresponds to a situation where the sellers locate outside the residential area, like shopping centers set up along a road at the outskirts of a city. At the same price, all consumers prefer to buy from the shopping center which is the closest to the city. These games are called outside location games.

Secondly, another important issue in industrial economics is related to the practice of price discrimination. As some sort of market segmentation is inherent to price discrimination, the spatial competition model offers a natural framework for the study of oligopolistic markets with price discriminating firms. Of course, price discrimination is possible only when firms can discern among customers. To this effect, we suppose that transport is under the firms' control, thus enabling discrimination with respect to location. If the difference between delivered prices at two different locations is not larger than the transportation costs between these points, arbitrage is never profitable and firms may exercise price discrimination. When two or more firms price discriminate in a spatial economy, the resulting game typically involves, as strategic variables, price schedules specifying the delivered prices at which each firm is willing to supply the customers located at each point. This gives rise to a new class of location games in which firms' decision variables are price functions instead of price scalars, i.e. discriminatory versus mill pricing.

Finally, the location model is also well suited for analyzing non-price competition. In other words, firms are assumed to compete on other variables than prices; in particular products specification appears as a basic decision variable in such a competitive environment. Marketers view the product sold by a firm as a mix of goods in conjunction with an array of services. The spatial analog of a firm choosing the attributes of a product defined as such, given some competitive brands, is the choice by a shop-keeper of a location for his store, given some competing facilities. It is worth noting that this model may also be useful for dealing with collective decision-making processes, like voting or competition between political parties.

The remaining of this chapter is organized as follows. In Section 2, we study inside and outside location games assuming mill price competition; we first assume that firms' locations are given and subsequently allow for variable locations. Section 3 deals with the same games under discriminatory pricing. Location under non-price competition is taken up in Section 4. In each section, we concentrate on the basic models and results, mentioning in footnotes several recent extensions and reinterpretations. Regarding historical details, we refer the reader to Ponsard

(1983). Finally, in Section 5, we draw some conclusions and suggest some topics for future research.

2 Location Under Mill Price Competition

2.1 Variable prices and parametric locations

2.1.1 The inside location game

The prototype model of spatial competition for the inside location game has been introduced by Hotelling (1929). On a line of length, L normalized to one by an adequate choice of the unit of length, two sellers A and B of a homogeneous product with zero production cost, are installed at respective distances a and b from the endpoints of the line $(a+b \leq 1; a \geq 0, b \geq 0)$. Customers are uniformly distributed along the line, and each customer consumes exactly one unit of the commodity. Since the product is homogeneous, a consumer will buy from the seller who quotes the lower full price, namely the mill price plus transportation cost. It is supposed that the transport is under the customers' control. We denote by $c(x)$ the transportation cost function, i.e. the cost in terms of a given numéraire of shipping one unit of the product over a distance of length x . The transportation cost function is assumed continuous, increasing and convex in x, with $c(0) = 0$. Let p_1 and p_2 denote, respectively, the mill price of A and B and denote by $x(p_1, p_2)$ the "marginal consumer" $y \in [0,1]$ satisfying

$$p_1 + c(| y - a |) = p_2 + c(| 1 - b - y |); \tag{1}$$

whenever it exists, it is unique. If $x(p_1, p_2)$ does not exist, then either, for all $y \in [0,1]$,

$$p_1 + c(| y - a |) < p_2 + c(| 1 - b - y |),$$

or, for all $y \in [0,1]$,

$$p_1 + c(| y - a |) > p_2 + c(| 1 - b - y |).$$

In the first case, the market is segmented at $x(p_1, p_2)$: customers located in $[0, x(p_1, p_2)]$ buy from seller A , those in $]x(p_1, p_2), 1]$ from seller B . In the second case, the whole market is served by seller A at prices (p_1, p_2) while the converse holds in the third one. Figures 1a, 1b, 1c illustrate the three possible cases.

[Insert here Figures 1a, 1b, 1c]

The situation described above gives rise to a two-person game with players A and B, strategies $p_1 \in [0, \infty[$ and $p_2 \in [0, \infty[$, and payoff functions

$$
\begin{aligned}
\pi_1(p_1,p_2;a,b) = \quad & p_1 x(p_1,p_2) && \text{if there exists } y \in [0,1] \text{ solving } (1)\ , \\
= \quad & p_1 && \text{if, for all } y \in [0,1], p_1 + c(\mid y-a \mid) < p_2 + c(\mid 1-b-y \mid), \\
= \quad & 0 && \text{if, for all } y \in [0,1], p_1 + c(\mid y-a \mid) > p_2 + c(\mid 1-b-y \mid),
\end{aligned}
$$

and

$$
\begin{aligned}
\pi_2(p_1,p_2;a,b) = \quad & p_2[1-x(p_1,p_2)] && \text{if there exists } y \in [0,1] \text{ solving } (1)\ , \\
= \quad & p_2 && \text{if, for all } y \in [0,1], p_1 + c(\mid y-a \mid) > p_2 + c(\mid 1-b-y \mid), \\
= \quad & 0 && \text{if, for all } y \in [0,1], p_1 + c(\mid y-a \mid) < p_2 + c(\mid 1-b-y \mid).
\end{aligned}
$$

Now we consider the question of existence of a noncooperative price equilibrium in pure strategies for the class of inside location games described above, i.e. a pair of prices (p_1^*, p_2^*) such that $\pi_i(p_i^*, p_j^*; a, b) \geq \pi_i(p_i, p_j^*; a, b)$, $\forall p_i \geq 0, i = 1,2$ and $i \neq j$. The original case, treated by Hotelling in his celebrated paper, assumes linear transportation costs :

$$c(x) = tx,$$

where the scalar $t > 0$ denotes the transportation rate. Note that, in this case, $x(p_1,p_2)$ exists only if $x(p_1,p_2) \geq a$; otherwise there would exist y solution of (1) such that $p_1 + t(a-y) = p_2 + t(1-b-y)$ which is impossible (except if firms locate back-to-back) . Similarly $x(p_1,p_2)$ exists only if $x(p_1,p_2) \leq 1-b$. Hence, $x(p_1,p_2)$ exists only if $a \leq x(p_1,p_2) \leq 1-b$ so that $x(p_1,p_2)$ must be the solution of the equation

$$p_1 + t(y-a) = p_2 + t(1-b-y),$$

that is,

$$x(p_1,p_2) = \frac{p_2 - p_1}{2t} + \frac{1-b+a}{2}.$$

It is easily seen that $x(p_1,p_2) \in [a, 1-b]$ if and only if $\mid p_1 - p_2 \mid \leq t(1-a-b)$. Furthermore, since $p_1 < p_2 - t(1-a-b)$ implies $p_1 + t \mid y-a \mid < p_2 + t \mid 1-b-y \mid$ for all $y \in [0,1]$, and $p_2 < p_1 - t(1-a-b)$ implies $p_2 + t \mid 1-b-y \mid < p_1 + t \mid y-a \mid$ for all y , the payoff functions in the linear case are given by

$$
\begin{aligned}
\pi_1(p_1,p_2;a,b) = \quad & \left(\tfrac{1-b+a}{2}\right) p_1 + \tfrac{1}{2t}(p_1p_2 - p_1^2) && , \text{ if } \mid p_1 - p_2 \mid \leq t(1-a-b). \\
= \quad & p_1 && , \text{ if } p_1 < p_2 - t(1-a-b), \\
= \quad & 0 && , \text{ if } p_1 > p_2 + t(1-a-b);
\end{aligned}
$$

$$
\begin{aligned}
\pi_2(p_1,p_2;a,b) = \quad & \left(\tfrac{1-b+a}{2}\right) p_2 + \tfrac{1}{2t}(p_1p_2 - p_2^2) && , \text{ if } \mid p_1 - p_2 \mid \leq t(1-a-b). \\
= \quad & p_2 && , \text{ if } p_2 < p_1 - t(1-a-b), \\
= \quad & 0 && , \text{ if } p_2 > p_1 + t(1-a-b).
\end{aligned}
$$

The profit function of player A is illustrated in Figure 2 for a fixed value $\bar{p}_2$: it exhibits two discontinuities at the prices where a whole group of buyers is indifferent between the two sellers.[1]

[Insert here Figure 2]

The following proposition, proved in d'Aspremont et al. (1979) provides the necessary and sufficient conditions on the location parameters a and b guaranteeing the existence of a price equilibrium $(p_1^\star, p_2^\star)$ in pure strategies for the above game. Furthermore, the equilibrium strategies are computed as functions of the parameters a and b.

Proposition 1 *For $a+b=1$, the unique price equilibrium is given by $p_1^\star = p_2^\star = 0$. For $a+b<1$, there is a price equilibrium if and only if*

$$\left(1+\frac{a-b}{3}\right)^2 \geq \frac{4}{3}\,(a+2b),$$

$$\left(1+\frac{b-a}{3}\right)^2 \geq \frac{4}{3}\,(b+2a).$$

Whenever it exists, the price equilibrium is uniquely determined by

$$p_1^\star = t\left(1+\frac{a-b}{3}\right) \quad , \quad p_2^\star = t\left(1-\frac{a-b}{3}\right).$$

Accordingly, in the linear transportation case, price equilibria in pure strategies fail to exist in a wide domain of location parameters. This may suggest that the discontinuities in the payoff functions, observed under linear transportation costs, are responsible for the absence of equilibrium.[2] A reasonable conjecture, then, would be that the assumption of strictly convex transportation cost functions - which guarantees the continuity of the payoff functions - would reinstore the existence property in the whole domain of (a,b)-locations. This point of view is reinforced when the quadratic transportation cost case is examined, i.e. when $c(x)$ is defined by

$$c(x) = sx^2 \; ; \; s > 0.$$

Under this alternative specification, some easy computations lead to the following expressions for the payoffs :

$$\begin{aligned}
\pi_1(p_1, p_2; a, b) = \; & p_1\left[a+\frac{p_2-p_1}{2s(1-a-b)}+\frac{1-a-b}{2}\right] && , \text{if } 0 \leq a+\frac{p_2-p_1}{2s(1-a-b)}+\frac{1-a-b}{2} \leq 1, \\
= \; & p_1 && , \text{if } a+\frac{p_2-p_1}{2s(1-a-b)}+\frac{1-a-b}{2} > 1. \qquad (2) \\
= \; & 0 && , \text{if } a+\frac{p_2-p_1}{2s(1-a-b)}+\frac{1-a-b}{2} < 0;
\end{aligned}$$

$$\pi_2(p_1,p_2;a,b) = p_2\left[b+\frac{p_1-p_2}{2s(1-a-b)}+\frac{1-a-b}{2}\right] \quad , \text{if } 0 \le b+\frac{p_1-p_2}{2s(1-a-b)}+\frac{1-a-b}{2} \le 1,$$
$$= p_2 \quad , \text{if } b+\frac{p_1-p_2}{2s(1-a-b)}+\frac{1-a-b}{2} > 1, \quad (3)$$
$$= 0 \quad , \text{if } b+\frac{p_1-p_2}{2s(1-a-b)}+\frac{1-a-b}{2} < 0.$$

These payoff functions are not only continuous, but also quasiconcave. Accordingly, under quadratic transportation costs, there exists a price equilibrium in pure strategies wherever the locations a and b. Furthermore it is easily checked that the pair of prices (p_1^*, p_2^*) defined by

$$p_1^* = s(1-a-b)\left(1+\frac{a-b}{3}\right) \quad , \quad p_2^* = s(1-a-b)\left(1+\frac{b-a}{2}\right) \qquad (4)$$

is the unique Nash equilibrium for fixed a and b .

Unfortunately, as shown by the following example, even if strictly convex transportation cost functions imply the continuity of the payoff functions, they are not sufficient to imply the existence of an equilibrium for every location pair (a,b) . Assume, indeed, that the transportation cost function is of the "linear-quadratic" type, i.e.

$$c(x) = sx^2 + tx \quad , s > 0 \text{ and } t > 0.$$

Anderson (1986) has showed that, wherever seller A's location, there is always a location for seller B such that no price equilibrium in pure strategies exists for the corresponding location pair. To illustrate, suppose that both firms are symmetrically located around the market center ($a = b$). For a fixed value of $p_2, \bar{p}_2$ says, a typical example of player A's payoff function is provided in Figure 3.

[Insert here Figure 3]

We notice that, at the price $p_1''' = \bar{p}_2 - s(1-2a)^2 - t(1-2a)$ where the market boundary appears exactly at $\frac{1}{2}+a$, the quasiconcavity of π_1 is destroyed. That a significant departure from quasiconcavity may lead to the nonexistence of an equilibrium in pure strategies is shown here by the following result (see Gabszewicz and Thisse (1986)) : if $a > \frac{1}{4}$ and $\frac{t}{s} > \frac{(1-2a)^2}{a^2}$, then there exists no price equilibrium in pure strategies. As in the Hotelling's case, nonexistence is more likely when firms are close to each other (a is "large") and/or when the linear therm of the transportation cost function is high relative to the quadratic term (t/s is "large"). When $t = 0$ (quadratic case), payoff functions are quasiconcave and existence is guaranteed. When $s = 0$ (linear case), we fall back on Hotelling's model : for $a > \frac{1}{4}$, no price equilibrium exists, as confirmed by Proposition 1 applied to the symmetric case.

Accordingly, and contrary to wides pread opinion, the nonexistence of a price equilibrium for the inside location game is not related to the discontinuities in the payoffs. Instead, it is the quasiconcavity of the payoffs which poses problem, thus confirming an observation made by Roberts and Sonnenschein (1977) in a different, but related, context. Hence, it is worth noting that Champsaur and Rochet (1987a) have been able to show the quasiconcavity of the profit funcitons for a certain class of transportation costs. Surprisingly enough, their result depends on the length L of the market segment (that was so far normalized to one). Furthermore, they assume that the function c is three times continuously differentiable in an open interval of $\mathbf{R}$ including $[0, L]$.

Proposition 2 *If*

$$\left|\frac{d^3c}{dx^3}\right| \leq \frac{2}{L}\frac{d^2c}{dx^2} \ , \textit{ for all } x \in [0,1] \ , \tag{5}$$

then there exists a price equilibrium in pure strategies.

Clearly, condition (5) is always satisfied for quadratic costs. By contrast, Proposition 2 does not apply to the linear-quadratic case because c is not differentiable at $x = 0$. Condition (5) imposes an upper bound on the "degree of convexity" of c which is inversely related to the length L. When L is large enough, we are very close to the quadratic case. Consequently, the class of admissible transportation cost functions has some extension only if L is small.[3]

The standard game-theoretic solution to the nonexistence of a noncooperative equilibrium in pure strategies is to resort to mixed strategies. When transportation costs are strictly convex, the payoff functions are continuous and, by the theorem of Glicksberg (1952), a price equilibrium in mixed strategies exists. When transportation costs are linear, the payoff functions are discontinuous, but the theorem of Dasgupta-Maskin (1986) applies and an equilibrium also exists. In this special case, Osborne and Pitchik (1987) show that in equilibrium firms randomize over two disjoints price intervals. Each firm chooses its price either from an interval just below a relatively high price, or from an interval just below the price that undercuts its rival's highest price. In view of the complexity of Osborne and Pitchik's analysis, characterizing the equilibrium in mixed strategies for more general models of spatial price competition seems to be a formidable task.[4,5,6,7,8]

2.1.2 The outside location game

In the outside location game, firms are no longer established in the residential area, still represented by the interval $[0, 1]$, but on the right side of this interval at locations a and $b \in [1, +\infty[$ with $a \leq b$.

Let us denote again by $x(p_1, p_2)$ the customer $y \in [0,1]$ satisfying

$$p_1 + c(a-y) = p_2 + c(y-b),$$

whenever firm A (resp. B) quotes price p_1 (resp. p_2).

First, we consider the Hotelling's case of a linear transportation cost function $c(x) = tx$. In Figure 4, it is seen that the whole market is served by seller A at prices $\bar{p}_1$ and $\bar{p}_2$. Clearly, such a situation generates a series of price cuts that results in a price tie with seller B quoting a zero price. We assume that price ties are broken in favor of the nearer firm. This is so because this firm can always price ϵ-below the price quoted by its rival. Then, it is straightforward that the unique price equilibrium in pure strategies obtains at the pair of prices $(p_1^*, p_2^*) = (t(b-a), 0)$: any higher price p_1 of seller$A$ could be advantageously undercut by seller B, who would then attract all the customers.

[Insert here Figure 4]

Now, as for the inside location game, we study the problem of existence of a price equilibrium in pure strategies for the linear-quadratic transportation cost case. The market boundary $x(p_1, p_2)$ easily obtains as the solution of the equation

$$p_1 + s(a-x)^2 + t(a-x) = p_2 + s(b-x)^2 + t(b-x),$$

i.e.

$$x(p_1, p_2) = \frac{p_2 - p_1 + s(b^2 - a^2) + t(b-a)}{2s(b-a)} .$$

Accordingly the payoff functions of the two firms are given by

$$\begin{aligned}
\pi_1(p_1, p_2; a, b) &= 0 &&, \text{ if } p_1 \geq p_2 + s(b^2 - a^2) + t(b-a) \stackrel{\text{def}}{=} p_1'; \\
&= \left[\frac{p_2 - p_1}{2s(b-a)} + \frac{t + s(b+a)}{2s}\right] &&, \text{ if } p_1' > p_1 \geq p_1'' \stackrel{\text{def}}{=} p_2 + (t-2s)(b-a) + s(b^2 - a^2); \\
&= p_1 &&, \text{ if } p_1'' > p_1 \geq 0;
\end{aligned}$$

and

$$\begin{aligned}
\pi_2(p_1, p_2; a, b) &= 0 &&, \text{ if } p_2 > p_1 + (2s-t)(b-a) - s(b^2 - a^2) \stackrel{\text{def}}{=} p_2'; \\
&= \left[\frac{p_1 - p_2}{2s(b-a)} + \frac{t + 2s - t - s(b+a)}{2s}\right] &&, \text{ if } p_2' > p_2 \geq p_2'' = p_1 - t(b-a) - s(b^2 - a^2); \\
&= p_2 &&, \text{ if } p_2'' > p_2 \geq \cdot
\end{aligned}$$

In Gabszewicz and Thisse (1986), we prove that, given linear-quadratic costs, there exists a unique price equilibrium in pure strategies given by

$$p_1^\star = (b-a)\frac{s(a+b+2)+t}{3} , \quad (6)$$

$$p_2^\star = (b-a)\frac{s(4-a-b)-t}{3} , \quad (7)$$

when $\frac{t}{s} < 4-a-b$, and by

$$p_1^\star = (b-a)[s(a+b-2)+t] , \quad (8)$$

$$p_2^\star = 0 , \quad (9)$$

when $\frac{t}{s} \geq 4-a-b$.

In contrast with the inside location game, we see that for both linear and linear-quadratic cost functions, a price equilibrium exists for all location pairs (a,b) when firms are located outside the segment in which consumers are located. Consequently, it seems that more stability in noncooperative price behaviour is to be expected when one of the two players is endowed with a strict exogenous advantage over the other one, as in the outside location game. The fact that, in this game, seller A's location is viewed as strictly better by all consumers than seller B's location, prevents the latter from using price strategies which would attract to him the whole market. This privilege is reserved to firm A. This asymmetry between sellers no longer exists when firms are located inside the consumers' area. In that case, both firms may use "undercutting" price strategies, leading possibly to price instability.

The foregoing analysis about the existence of a noncooperative price equilibrium is rather disappointing. Our inquiry has shown that, even in the simplest case of two firms and a uniform density of consumers located along the line, a price equilibrium in pure strategies may fail to exist for some reasonable transportation cost functions in the inside location game. Furthermore, solving the price game by using mixed strategies proves to be very difficult from the technical standpoint. No counterexample has been found for the outside location game : on the contrary, existence has been proved for the linear and linear-quadratic cost functions. Notwithstanding, it is our feeling that no general results can be expected to hold in spatial price competition under mill pricing.

2.2 Variable prices and locations

2.2.1 The simultaneous game

In the foregoing subsection, firms (players) were supposed to control their price strategy. Now we consider the more general case where firms are allowed to choose simultaneously both price and location. While in the above two games, locations a and b were viewed as parameters in the payoff functions π_1 and π_2 , they are now considered, as well as prices p_1 and p_2 , as strategic variables available to the players. More specifically, a strategy is a pair (p_1, a) (resp: (p_2, b))

for seller A (resp. seller B) with $p_1 \in [0,\infty[$ and $a \in S_1 = [0,1]$ in the inside location game or $a \in S_1 = [1,\infty[$ in the outside location game (resp. $p_2 \in [0,\infty[$ and $b \in S_2 \equiv S_1$) , and payoff functions $\tilde{\pi}_1$ and $\tilde{\pi}_2$ in the simultaneous game are given by

$$\begin{aligned}\tilde{\pi}_1((p_1,a),(p_2,b)) &= \pi_1(p_1,p_2;a,b)\\ \tilde{\pi}_2((p_1,a),(p_2,b)) &= \pi_2(p_1,p_2;a,b),\end{aligned}$$

with π_1 and π_2 as defined in 2.1. A noncooperative price-location equilibrium in pure strategies for the simultaneous game consists of a pair of strategies $[(p_1^*,a^*),(p_2^*,b^*)]$ such that

$$\tilde{\pi}_1((p_1,a),(p_2^*,b^*)) \leq \tilde{\pi}_1((p_1^*,a^*),(p_2^*,b^*)), \forall (p_1,a) \in [0,\infty[\times S_1$$

and

$$\tilde{\pi}_2((p_1^*,a^*),(p_2,b)) \leq \tilde{\pi}_2((p_1^*,a^*),(p_2^*,b^*)), \forall (p_2,b) \in [0,\infty[\times S_2.$$

It is well known that, regardless of the transportation cost function $c(x)$, or whether an inside or outside location game is considered, no price-location equilibrium in pure strategies can exist in the simultaneous game. To see this, consider the inside location game and assume that a price-location equilibrium $[(p_1^*,a^*),(p_2^*,b^*)]$ exists. Clearly, at this equilibrium, both firms must have strictly positive payoffs (profits), which implies that $p_1^* > 0$ and $p_2^* > 0$. Two cases may then arise. In the first one we have $a^* \neq 1-b^*$. Without loss of generality we may assume that seller B's payoffs exceed or equal seller A's payoffs. Then firm A can increase its profits by locating at $\tilde{a} = 1-b^*$ and by charging a price $\tilde{p}_1 = p_1^* - \epsilon$, with $\epsilon > 0$ arbitrarily small. Indeed

$$\tilde{\pi}_1((\tilde{p}_1,\tilde{a}),(p_2^*,b^*)) > \tilde{\pi}_2((p_1^*,a^*),(p_2^*,b^*)),$$

since firm A now captures the whole market at price $p_2^*-\epsilon$. Since we have assumed $\tilde{\pi}_2((p_1^*,a^*),(p_2^*,b^*)) \geq \tilde{\pi}_1((p_1^*,a^*),(p_2^*,b^*))$, we get the desired contradiction. In the second case, $a^* = 1-b^*$. But then, each player has an incentive to undercut its competitor and, as in Bertrand, to capture the whole market, again a contradiction.[9],[10]

2.2.2 The Sequential game

As we have just seen, the simultaneous game approach looks like a blind alley to determine prices and locations when both these strategic variables are under the control of the players. Consequently, let us turn to the alternative formulation in terms of a sequential game. There, price and location strategies are assumed to be played one at a time in a two-stage process. The choice about location is viewed as prior to the decision on price, so that locations are chosen in the first stage of the sequential game, while prices are decided in the second stage. Assuming that prices p_1 and p_2 are chosen at a noncooperative price equilibrium (in pure strategies) in the subgame consisting of the second stage, the corresponding equilibrium payoffs are well defined whenever this price equilibrium exists and is unique. Furthermore, they depend only upon the location choice made in the first stage. Accordingly these payoffs can be used as payoff functions in the first-stage game in which strategies are locations a and b .

We now proceed to a formal definition of a subgame perfect equilibrium for this sequential game setting. A subgame perfect price-location equilibrium is a pair of locations $(a^*, b^*) \in S_1 \times S_2$ and a pair of price functions $[p_1^*(a,b), p_2^*(a,b)]$ such that

(i) for any $(a,b) \in S_1 \times S_2, \pi_i[p_i^*(a,b), p_j^*(a,b); a, b] \geq \pi[p_i, p_j^*(a,b); a, b]$, $\forall p_i \geq 0, i = 1,2$ and $i \neq j$,

(ii) $\pi_1[p_1^*(a^*,b^*), p_2^*(a^*,b^*); a^*, b^*] \geq \pi_1[p_1^*(a,b^*), p_2^*(a^*,b^*); a, b^*]$, $\forall a \in S_1$,

(iii) $\pi_2[p_1^*(a^*,b^*), p_2^*(a^*,b^*); a^*, b^*] \geq \pi_1[p_1^*(a^*,b), p_2^*(a^*,b); a^*, b]$, $\forall b \in S_2$,

where π_1 and π_2 are defined as in 2.1.

The concept of subgame perfect equilibrium captures the idea that, when firms choose their locations, they both anticipate the consequences of their choice on price competition. In particular, they are aware that this competition will be more severe if they locate close to each other, rather than far apart. Unfortunately, this concept is meaningful only if, for any location choices by firms, there exists one, and only one, corresponding price equilibrium - otherwise, either payoffs would be undefined or multivalued. From 2.1, we know how demanding these existence, and a fortiori uniqueness, conditions are.

To illustrate, let us first consider the inside location game with $c(x) = sx^2$ (quadratic transportation cost function). Then, we know that $[p_1^*(a,b), p_2^*(a,b)]$ exists and is unique for all pairs $(a,b) \in [0,1]^2$. Substituting $p_1^*(a,b)$ and $p_2^*(a,b)$, given by (4), in π_1 and π_2 given by (2) and (3), routine calculations show that regardless of the location of the other player, the payoffs of firm A decrease when a increases whereas the payoffs of firm B is a decreasing function of b . Consequently, each firm gains by moving away as far as possible from its competitor. Hence, the equilibrium of the first stage is given by $(0,1)$ and the resulting prices are $p_1^*(0,1) = p_1^*(0,1) = s$. Clearly, the so-obtained locations differ from the socially optimal locations that minimize total transportation costs, i.e. $a = b = \frac{1}{4}$.[11],[12],[13]

It is also interesting to characterize the subgame perfect price-location equilibrium in the case of an outside location game with $c(x) = sx^2 + tx$ (linear-quadratic transportation costs). For this case, the existence of a unique price equilibrium has been established above, for any location pair (a,b). Furthermore, the corresponding equilibrium pair of prices $[p_1^*(a,b), p_2^*(a,b)]$ are given by (6) and (7) whenever $\frac{t}{s} < 4 - a - b$, and by (8) and (9) if $\frac{t}{s} \geq 4 - a - b$. Clearly, if $2s < t$, the payoffs $\pi_2[p_1^*(a,b), p_2^*(a,b)\ ;\ a, b]$ of player B are necessarily equal to zero, while the payoffs $\pi_1[p_1^*(a,b), p_2^*(a,b)\ ;\ a, b]$ of player A decreases when a increases, whatever $b \in [1, \infty[$. In consequence, for any b in this interval, seller A locates at $a^* = 1$ and prices at $p_1^*(1,b) = (b-1)[s(b-1)+t]$ whereas $p_2^*(1,b) = 0$. By contrast, when $2s > t$, seller B can always choose a location b in $[1,\infty[$ large enough so as to verify the condition $\frac{t}{s} < 4 - a - b$, guaranteeing himself strictly positive payoffs. Furthermore, seller A's payoffs still decrease with a for any b in $[1,\infty[$, so that seller A still locates at $a^\star = 1$. The corresponding value of b which maximizes seller B's payoffs then obtains from the first-order condition

$$\frac{d}{db}\ \pi_2[p_1^\star(1,b), p_2^\star(1,b)\ ;\ 1,b] = 0,$$

i.e. $2s > t$. Hence, if $2s > t$, the equilibrium locations are unique and such that $a^\star = 1$ and $b^\star = \frac{2s-t}{3t}$, while the resulting prices are $p_1^\star(1\ ,\ \frac{2s-t}{3t}) = \frac{2s-t}{9s}(4s + t + \frac{2s-t}{3})$ and $p_2^\star(1\ ,\ \frac{2s-t}{3t}) = \frac{2s-t}{9s}(2s - t - \frac{2s-t}{3})$.

The equilibrium analysis of games with both prices and locations as strategic variables is as disappointing as the approach with variable prices, but parametric locations. Equilibrium never exists in the simultaneous game; and the existence of a subgame perfect equilibrium in the sequential game relies heavily on the existence and uniqueness of a price equilibrium in the second stage subgame, conditions which are hardly met, as shown in 2.1.[14],[15],[16] Nonetheless, the sequential game approach sheds some light on an important issue in the economics of imperfect competition : do firms selling substitute products prefer to "copy" each other when selecting their products, or, on the contrary, do they differentiate them in some optimal manner ? It was Hotelling's belief that a "Principle of Minimum Differentiation" must hold. However, we must conclude from our analysis that, both in the inside and outside location games, firms tend to relax price competition, at the subgame perfect price-location equilibrium, by locating apart from each other.[17]

3 Location Under Discriminatory Price Competition

3.1 Variables prices and parametric locations

3.1.1 The inside location game

Let us consider a model similar to the one described in 2.1.1, but in which firms are no longer constrained to sell at the same mill prices. Instead, we suppose that firms deliver the product to the customers and can, therefore, exercise price discrimination. Since firms know the customers' locations, they can charge location-specific prices. In general, the difference between delivered prices quoted by the same firm at two distinct locations does not equal the transportation cost between these points. Thus, firms price discriminate, setting different mill prices at the firms' door.[18] As in Hotelling, we also assume that transportation costs are linear in distance.

Given sellers A and B located respectively at distances a and b from the extremities of the segment $[0,1]$, a strategy for seller A is a price schedule $p_1(\cdot)$ that specifies for each location $y \in [0,1]$ the delivered price at which A is willing to sell its product to the customers at y.

Formally, we suppose that $p_1(\cdot)$ belongs to the class $\mathbf{P}_1$ of measurable functions defined over $[0,1]$ which satisfy a.e. the inequality $p_1(y) \geq t \mid a - y \mid$. If this latter condition were not verified, then seller A could do at least as well, for any given price of B at y, by pricing at cost $t \mid y - a \mid$. Similarly, a strategy for seller B is a measurable function $p_2(\cdot)$ defined on $[0,1]$ for which $p_2(y) \geq t \mid 1 - b - y \mid$ holds a.e. We denote by $\mathbf{P}_2$ the strategy set of seller B.

Since the product is homogeneous, customers buy from the seller quoting the lower delivered price. In the event of a price tie, we suppose that customers choose to buy from the nearer firm. This can be justified by the fact that this firm can always price ϵ-below its rival. When customers are equidistant from both firms, any allocation of the local demand is acceptable. Indeed, we will see that, in equilibrium, no seller makes positive profit at such a point. Hence, the payoff functions are

$$\pi_1[p_1(\cdot), p_2(\cdot); a, b] = \int_{M_1} [p_1(y) - t \mid y - a \mid] dy$$

where $M_1 = \{y \in [0,1]; p_1(y) < p_2(y) \text{ or } (p_1(y) = p_2(y) \text{ and } \mid y - a \mid < \mid 1 - b - y \mid)\}$, and

$$\pi_2[p_1(\cdot), p_2(\cdot); a, b] = \int_{M_2} [p_2(y) - t \mid 1 - b - y \mid] dy$$

where $M_2 = \{y \in [0,1]; p_2(y) < p_1(y) \text{ or } (p_1(y) = p_2(y) \text{ and } \mid 1 - b - y \mid < \mid y - a \mid)\}$.

A noncooperative price schedule equilibrium in pure strategies of the above game is a pair $[p_1^\star(\cdot), p_2^\star(\cdot)]$ of price schedules such that

$$\pi_i[p_i^\star(\cdot), p_j^\star(\cdot); a, b] \geq \pi_i[p_i(\cdot), p_j^\star(\cdot); a, b] \ , \ \forall p_i(\cdot) \in \mathbf{P}_i, i = 1,2 \text{ and } i \neq j.$$

As transportation costs to a point are unaffected by transport to other points and as marginal production costs are constant (zero), delivered prices charged at different points by the same seller are strategically independent. In other words, there is a separate Bertrand game at every point y . (Of course, arbitrage could link local markets through possible resales among consumers located at different points. However, we will see that arbitrage is not binding in equilibrium.) A standard Bertrand-like argument then runs as follows. Assume that for customers at y , A is the nearer firm. Despite the assumption of zero marginal production costs, seller A has a (transport) cost advantage which allows him to undercut any price set by seller B. The price undercutting process will stop when B can no longer reduce its price, i.e. when price is equal to $t \mid 1 - b - y \mid$, the transportation cost incurred by the second-nearer firm. Returning to the allocation rule introduced above, customers at y buy from the nearer firm, i.e. seller A. The set of customers equidistant from sellers A and B has a zero measure provided only that the two firms are not coincidentally located $(a \neq 1 - b)$. Thus, we have :

Proposition 3 *There exists a unique price schedule equilibrium; it is given by*

$$p_1^\star(y) = p_2^\star(y) = \max\{t \mid y - a \mid, t \mid 1 - b - y \mid\} \tag{10}$$

for almost all $y \in [0,1]$.

The resulting market price schedule is represented by the heavy line in Figure 5. It is seen there that the market is segmented at x where customers are equidistant from both sellers : $x = \frac{1-b+a}{2}$. Furthermore, arbitrage is never profitable since the difference between two delivered prices is smaller than or equal to the corresponding transportation cost. This equilibrium was first identified by Hoover (1937) and formally investigated by Lederer and Hurter (1986).

[Insert here Figure 5]

Two remarks are in order. First, Proposition 3 guarantees the existence of an equilibrium for <u>any</u> location pair (a, b). This is to be contrasted with the mill pricing case where an equilibrium exists only when sellers A and B are sufficiently far apart (see Proposition 1).[19] Second, the existence property is <u>general</u> and extends to the cases of : (i) multidimensional space; (ii) nonuniform or atomic distributions of customers; (iii) continuous, decreasing and location-specific demand functions; and (iv) increasing and firm-specific transportation cost functions in distance (see Thisse and Vives (1988)). Essentially, the argument is similar to that used in the above example. But $p_i^\star(y)$ may now differ from (10), thus reflecting the properties of the local demand and the sellers' costs. The key-assumptions are the constant marginal production costs and constant returns w.r.t. the volume hauled.[20] Moreover, since the transportation cost functions are general, the possibility of arbitrage may arise. To obviate this potential difficulty, we suppose that customers have access to a less efficient transportation technology than firms do. (The same assumption is made below.)

3.1.2 The outside location game

Using the model presented in 2.1.2 as well as the allocation rule described there, we immediately see that, for linear transportation costs in distance, the equilibrium price schedules are given by

$$p_1^\star(y) = p_2^\star(y) = t(b-y)\ ,\ \forall y \in [0,1].$$

The firm nearer to the residential area, i.e. seller A , supplies the whole market because it has the transportation cost advantage everywhere. Thus, discriminatory pricing endows this firm with enough flexibility in its pricing decisions to prevent seller B to enter the market. This still holds in the case of a general transportation cost function (which is continuous and increasing in distance) when the two firms have access to the same transportation technology. It is then readily verified that

$$p_1^\star(y) = p_2^\star(y) = c(b-y)\ ,\ \forall\ y \in [0,1].$$

More interesting is the case where seller B has access to a more efficient transportation technology than his rival, i.e.

$$c_2(x) < c_1(x) \text{ , for all } x \geq 0.$$

Let us also assume that $y \in [0,1]$ exists such that $c_1(a-y) = c_2(b-y)$ holds. (Otherwise, the firm which has the transportation cost advantage would capture the whole market.) Then, transposing the argument given in 3.1.1 yields the following result : there exists a unique price schedule equilibrium; it is given by

$$p_1^\star(y) = p_2^\star(y) = \max\{c_1(a-y), c_2(b-y)\} \tag{11}$$

for almost all $y \in [0,1]$.

Each seller serves the customers for whom he has the transportation cost advantage. Figure 6 illustrates the case where the two curves $c_1(a-y)$ and $c_2(b-y)$ intersect only once.[21] The market price schedule is represented by the heavy line. Seller A supplies the customers in $[\bar{x}, 1]$, while seller B supplies those in $[0, \bar{x}[$. In the segments where the firms are active, each seller exercises price discrimination as shown by (11).

[Insert here Figure 6]

3.1.3 Comparison of price policies

Before studying the price-location games, it is worthwhile to make a digression concerning the debate on the social desirability of mill vs. discriminatory pricing. Antitrust authorities often take it for granted that the former is socially superior to the latter.[22] The analysis developed in 2.1 and 3.1 provides us with some useful tools to compare the two price policies in an oligopolistic environment. To this end, we assume that the transport is under the firms' control and that transportation costs are linear in distance.

Consider, first, the inside location game and suppose that firms are located in such a way that a mill price equilibrium in pure strategies exists (see Proposition 1). From (10), we have $p_1^\star(a) = p_2^\star(1-b) = 1-a-b$. Straightforward calculations show that $p_1^\star(a) < p_1^\star(1-b) < p_2^\star$, where $p_1^\star$ and $p_2^\star$ are the equilibrium mill prices given by Proposition 1. Consequently, since transportation costs are linear, the delivered price under mill pricing is higher everywhere than the delivered price under discriminatory pricing (see Figure 7 for an illustration). Of course, the payoff functions are also higher under mill pricing than under discriminatory pricing. There is a twist here. As shown by Thisse and Vives (1988), in a two-stage game in which firms' strategies are, first, to commit to mill pricing or to price discriminate and, then, to compete in price under the price policy chosen, both firms choose to price discriminate in equilibrium. Thus, in keeping freedom in their price decisions, firms get trapped into a Prisoner's Dilemma situation and end up with lower profits.

[Insert here Figure 7]

Let us now come to the outside location game. It has been shown in 2.1.2 that seller A supplies the whole market and charges the mill price $p_1^* = t(b-a)$. The corresponding delivered price at y is, therefore, equal to $t(b-y)$. In the discriminatory pricing case, we have just seen that seller A also serves the entire market and chooses the delivered price schedule $t(b-y)$. Here, the two price policies result in identical delivered prices for the customers. This shows, once more, that price competition may yield very different outcomes in the inside and outside location games.

3.1.4 Variable prices and locations

In the discriminatory pricing case, there exists no simultaneous price schedule-location equilibrium (the argument is similar to that developed in 2.2.1). We therefore concentrate on the sequential equilibria only. Because of the lack of space, we will limit ourselves to the inside location game.[23]

Consider the model described in 3.1.1. A diagrammatic argument will be sufficient to prove the existence of a location equilibrium. Assume first that seller B, located at distance b from the right endpoint of the unit interval, is the only firm on the market. He then supplies all the customers and the corresponding total transport costs are given by the area of the triangles $BC0$ and $BD1$ in Figure 10. Now let seller A be located at distance a from the left endpoint. Given the resulting equilibrium price schedules (see Proposition 3), seller A supplies the customers located in $[0, \bar{x}[$ and receives a payoff equal to the area of the quadrilateral shaded horizontally. Then, it is readily verified that this area is precisely the difference between the total transportation costs borne by seller B when he is alone on the market and the total transportation costs borne by sellers A and B when they are both on the market. Hence, in order to maximize his profits, A must choose to locate at a point generating the largest decrease in total transportation costs. Consequently, if both firms locate at the transportation cost-minimizing points, i.e. $a = b = 1/4$, no firm can increase its profits by changing unilaterally its location. Thus, we have :

Proposition 4 *The socially optimal location pair is an equilibrium of the first-stage game.*[24]

[Insert here Figure 8]

The above argument, developed formally by Lederer and Hurter (1986), can be generalized to the case of : (i) multidimensional space, (ii) nonuniform or atomic distributions of customers; and (iii) increasing and firm-specific transportation cost functions. The critical assumptions for the proof turn out to be the constant marginal production and transportation costs and the (perfectly) inelastic demand.[25] Furthermore, the socially optimal location pair may not be the only location equilibrium in a n-dimensional space, with $n \geq 2$ (see Lederer and Hurter (1986) for a counter-example).

The conclusions of the analysis of competition under discriminatory pricing are more encouraging than those derived under mill pricing. There exists a price schedule equilibrium for a wide class of problems. The fact that each firm has more flexibility in its response to its rivals helps in restoring existence.[26] To gain the customers at one point, a firm has only to change its local price. By contrast, under mill pricing a price cut affects the whole set of the firm's customers, thus generating more potential instability in the competitive process. Furthermore, the existence of a subgame perfect price schedule-location equilibrium has been established for a significant class of problems. In particular, provided that firms have access to the same transportation technology, they never locate coincidentally in equilibrium. The reason is identical to that found in the mill pricing case : firms want to avoid the damage of price competition by separating from each other in space.

4 Location Under Non-Price Competition

In some industries firms do not exert any control over their price because of either cartel agreements or price regulation by public authorities. Hence, competition among firms must take alternative forms. In particular, firms may compete by choosing location in such a way that they obtain the largest possible sales (which amounts here to profit maximization since prices are parametric and marginal production costs are zero).[27] The locational process may imply that either all firms locate simultaneously (4.1) or sequentially (4.2).

4.1 Simultaneous Locations

As in the previous sections, let us assume that customers are distributed uniformly over the segment $[0, 1]$ and that each consumer buys exactly one unit of the product. Since the product is homogeneous, we know that each consumer wants to purchase from the firm with the lowest full price. In this section, the assumption is made that the mill price is given and equal for all firms. Consequently, consumers will choose to patronize the nearest firm. (When several firms are equidistant from a customer, we assume that each has an equal probability to sell.) Finally, it is supposed that transport is under the customers' control and that the cost of carrying one unit of the product is a continuous and increasing function of the distance.

In the present class of games, firms' strategies are given by locations only. Furthermore, it is readily verified that a firm's payoff is given by the measure of the set of consumers for whom this firm is the nearest one. (If several firms are located at the same point, they equally share the corresponding market segment.) To start with, let us consider the case of two firms. If firms A and B are located respectively at distances a and b from the extremities of $[0,1]$, their payoffs are given by

$$S_1(a,b) = \frac{a+b}{2} \quad \text{and} \quad S_2(a,b) = 1 - \frac{a+b}{2} \quad \text{if} \quad a \neq b,$$

$$S_1(a,b) = S_2(a,b) = \frac{1}{2} \quad \text{if} \quad a = b\,.$$

Clearly, the payoff functions exhibit a discontinuity when the two firms cross each other outside the market center.

A <u>noncooperative location equilibrium</u> in pure strategies is a pair $(a^\star, b^\star) \in [0,1]^2$ such that

$$S_1(a^\star, b^\star) \geq S_1(a, b^\star)\ ,\ \forall a \in [0,1]$$

and

$$S_2(a^\star, b^\star) \geq S_2(a^\star, b)\ ,\ \forall b \in [0,1].$$

Interestingly, in spite of the discontinuity of the payoffs, a single equilibrium can be shown to exist. Indeed, let firm 1, say, be located outside the center. In this case, firm 2 maximizes its sales by establishing itself near to firm 1 on the larger side of the market. But then firm 1 has an incentive to leapfrog firm 2 since this allows it to increase its sales. This prevents any pair of noncentral locations to be an equilibrium. Assume now that both firms are placed at the center. Then, each of them gets half of the market and any unilateral move of a firm away from the center leads to a decrease in its sales. In other words, <u>the clustering of the two firms at the market center is the only location equilibrium in pure strategies.</u>[28]

The case of n firms, with $n \geq 3$, has been studied by Lerner and Singer (1937) and, more recently, by Eaton and Lipsey (1975). The concept of location equilibrium, introduced for two firms, can easily be extended to cope with n firms and, therefore, is not defined here.

For $n = 3$, there exists no location equilibrium. The argument runs as follows. Assume that an equilibrium exists where the three firms are separated. Then the two peripheral firms have an incentive to sandwich the interior firm which finds itself with an infinitesimal volume of sales. As a result, this firm wants to leapfrog one of its rivals in order to obtain a positive markets share, thus generating instability. Suppose, now, that two firms are clustered and the third isolated. Then the latter can increase its sales by selecting a location next to the clustering on the same side of the market. Finally, if the three firms are agglomerated, each one gets one third of the market. By choosing a location close to the clustering on the larger side of the market, any firm can gain a larger fraction of the market.

Somewhat surprisingly, existence is restored for $n \geq 4$. Let us briefly describe the main results (see Eaton and Lipsey (1975) for more details). When $n = 4$, there exists a unique equilibrium for which two firms are located at the first quartile and the two others at the third one. For $n = 5$, the equilibrium is unique and such that two firms are located at the first sextile, two other at the fifth one and one firm is isolated at the market center. If $n \geq 6$, there exists an infinite number of equilibrium configurations, characterized as follows : peripheral firms occupy the same location as their neighbor and have identical hinterlands. Each peripheral pair of firms is separated from the nearest competitor by a distance twice as long as the hinterland. The interior firms are paired or single.[29,30]

At first glance, it seems that we have obtained for the inside location game more positive results than those derived in Section 2. However, as noticed by Eaton and Lipsey (1975), they are not very robust to the specification of the model. In particular, they turn out to be very sensitive to the assumption of a uniform customer distribution. To show this, let us assume that consumers are continuously distributed over $[0, 1]$ according to the cumulative function $F(x)$. Then, in the 2-firm case, we have :

Proposition 5 *If $n = 2$, there exists a unique location equilibrium in pure strategies for which the two firms are located at the median of the cumulative function F.*

In contrast, there are no equilibria in pure strategies when $n \geq 5$ and when the customer density is strictly convex or strictly concave, however close it is to the uniform one. This has led Osborne and Pitchik (1986) to investigate the existence problem for arbitrary distributions by resorting to mixed strategies. Here also, the Dasgupta-Maskin theorem applies and a location equilibrium in mixed strategies does exist. Osborne and Pitchik then show that, for $n \geq 3$, the game has a symmetric equilibrium $(M, \cdots, M)$ where M is the equilibrium mixed strategy. As observed by the authors themselves, an explicit characterization of M appears to be impossible. Yet, when n becomes large, M approaches the customer distribution F. In this case, one can say that firms' location choices mirror the customer distribution.

Finally, returning to the 3-firm case with a uniform distribution, Shaked (1982) has shown that firms randomize uniformly over $[1/4, 3/4]$, which suggests some tendency towards agglomeration. Osborne and Pitchik have identified an asymmetric equilibrium for the same problem in which two firms randomize, putting most weight near the first and third quartiles, while the third firm locate at the market center with probability one.[31,32,33]

4.2 Sequential locations

In practice, it is probably quite realistic to think of firms entering the market sequentially according to some dynamic process. If firms are perfectly mobile, then the problem associated with the entry of a new firm is equivalent to the one treated in 4.1 since the incumbents can freely make new location decisions. However, one often observes that location decisions are not easily modified. At the limit, they can be considered as irre vocable.

When entry is sequential and when location decisions are made once and for all, it seems reasonable to expect that an entrant also anticipates subsequent entry by future competitors. Accordingly, at each stage of the entry process, the entrant must consider as given the locations of firms entered at earlier stages, but can treat the locations of firms entering at later stages as conditional upon his own choice. In other words, the entrant is a follower with respect to the incumbents, and a leader with respect to future competitors. The location chosen by each firm is then obtained by backward induction from the optimal solution of the location problem faced by the ultimate entrant, to the firm itself. This is the essence of the solution concept proposed independently by Hay (1976), Prescott and Visscher (1977), and Rothschild (1976). The main contribution of this approach is to reveal that entry can be deterred if firms are sophisticated enough to locate in such a way as to leave an insufficient market for a new firm. Of course, at the resulting long-run equilibrium, firms earn supranormal profits.[34]

5 Concluding Remarks

Spatial competition is an expanding field lying at the interface of game theory and economics. It is still in its infancy but attracts more and more scholars' interest because the competitive location problem emerges as a prototype of many economic situations involving interacting decision-makers. In this chapter, we have restricted ourselves to the most game-theoretic elements of location theory. In so doing, we hope to have conveyed the message that space can be used as a "label" to deal with various problems encountered in imperfect competition. The situations considered in this chapter do not exhaust the list of possible applications in that domain. Such a list would include intertemporal price discrimination and the supply of storage (see Phlips (1980), Phlips and Thisse (1981)), competition between multiproduct firms (see Champsaur and Rochet (1987b), Martinez-Giralt and Neven (1988)), the incentive to innovate for imperfectly informed firms (see Jovacic and Rob (1987)) and the techniques of vertical restraints (see Mathewson and Winter (1984)).

As a last remark, we would like to say that the location paradigm has also served as a useful vehicle for analyzing economic games in several other contexts, like international trade (see Kierzkowski (1984)), fiscal federalism (see Mintz and Tulkens (1984)), and exchange rate policies between countries (see Dornbusch (1987)). Most probably, Hotelling was not aware that game theory would so successfully promote the ingenious idea he had in 1929.

Footnotes

1 These discontinuities vanish when the characteristics space is n-dimensional, with $n \geq 2$, and when a ℓ_p-metric is used, with $p \in]1, \infty[$ (see Economides, 1986). However, even in the simple case of the Euclidean metric ($p = 2$), there is still a lack of quasiconcavity in the payoff functions. Economides (1986) has shown that a price equilibrium exists in the special case of two firms located symmetrically on an axis passing through the center of a disk over which consumers are evenly distributed.

2 Economides (1984) shows that the introduction of a reservation price, i.e. the maximum full price that a customer is willing to pay to obtain the product, reduces the (a, b)-segment of nonexistence but does not suppress it (except in the limit case of two separated monopolies). Shilony (1981) reaches similar conclusions by considering symmetric, single-peaked distributions of customers.

3 Champsaur and Rochet also provides a generalization of (5) dealing with nonuniform consumer distributions.

4 Another approach, which has considerable intuitive appeal, is to assume that the products sold by the firms are not homogeneous. As a result, consumers also take variables other than full price into account. Because of the (frequent) non-observability of these variables, firms can at best determine the shopping behavior of a particular consumer up to a probability function. Describing the purchasing probabilities by the logit model (see, e.g. McFadden (1984)), de Palma et al. (1986) show that a price equilibirum in pure strategies exists when products are sufficiently heterogeneous.

5 Caplin and Nalebuff (1986) have tackled the problem in a completely different way. Dealing with the case of a multidimensional characteristics space, they propose to separate the space of consumers' preferences and the characteristics space. They investigate the case of consumers having Cobb-Douglas utility functions in the characteristics and show that with two characteristics, there exists a unique price equilibrium if the distribution of consumers in the Cobb-Douglas utility-parameters space is concave and twice continuously differentiable. This result seems to heavily rely on the fact that the market boundary is given by a straight line, as in the case of quadratic transportation costs (see Hanjoul and Thill (1987)). Interestingly, the concavity condition supposed by Caplin and Nalebuff is then identical to that used by Neven (1986) in his study of the Hotelling model with nonuniform distributions and quadratic transportation costs. Both approaches, then, suggest that when the distribution is very concentrated, a price equilibrium may not exist for some location pairs (see also Shilony (1981)).

6 Gabszewicz and Garella (1986) give up the assumption that consumers are perfectly informed about firms' prices. Instead, they suppose that consumers have subjective probabilistic

beliefs about these prices. Gabszewicz and Garella then show that a price equilibrium exists provided that the two firms lie sufficiently far apart. This is to be compared to Proposition 1.

7 In a recent paper, Eaton and Engers (1987) show that existence of a price equilibrium in pure strategies can be restored in a discrete dynamic structure in which the two firms take turns setting price, with each firm's price remaining in effect until that firm's next turn. In this context, a firm's strategy is a reaction function R mapping the price history to a current price for each time period that this firm chooses a price. Furthermore, the payoffs of a firm are given by the discounted value of its profits. Concentrating on symmetric solutions, one says that $\bar{p}$ is a symmetric steady-state price if there exists a reaction function R^* which is a best reply to itself and which satisfies the condition $\bar{p} = R^*(\bar{p})$. Two types of equilibrium may then emerge, depending upon the ratio reservation price/transportation costs. In the first one, $\bar{p}$ is sufficiently low that no firm wants to undercut its rival. In the second, $\bar{p}$ can take quite high values. These results are interesting because they suggest that taking the time dimension into account may help in solving existence problems in spatial price competition.

8 In the horizontally differentiated product context, Klemperer (1987) studies a two-period model in which consumers have to bear a start-up cost when they buy a new product. He then shows that prices are higher in both periods than in an otherwise identical market without start-up (and therefore switching) costs. Klemperer also discusses the cases where (i) consumers may relocate between the two periods, and (ii) consumers leave or enter the market after the first period.

9 To cope with the nonexistence of a simultaneous price-location equilibrium, Lerner and Singer (1937) have proposed modifying the concept of noncooperative equilibrium. It is assumed that firms, anticipating their competitors' reaction, do not consider strategies that would eliminate these competitors from the market. This amounts to restricting the strategy sets to prices and locations yielding positive payoffs for all players. In the case of linear transportation costs, such a "modified" simultaneous equilibrium does indeed exist under reasonable assumptions (see Eaton (1972), Novshek (1988), Kohlberg and Novshek (1982)). However, a sufficient departure from the linear case may invalidate equilibrium (see , Gabszewicz and Thisse (1986), MacLeod (1985)).

10 Note that a simultaneous price-location equilibrium can be shown to exist if the product is heterogeneous enough (see de Palma et al. (1985)).

11 Neven(1986) shows that, when the (concave) distribution of customers becomes more concentrated, the equilibrium locations move inside the market.

12 In the case of linear transportation costs, Osborne and Pitchick (1987) show numerically that firms locate symmetrically inside the first and third quartiles, i.e. in the region when they randomize their pricing decision. On the other hand, when the space is given by a circle, Kats (1987) proves that firms choose to set up in the subregion for which a price equilibrium in pure strategies exists. This suggests that results about the randomization of pricing are very sensitive to the particular specification of the location space.

13 In the characteristics interpretation à la Lancaster, Anderson and Neven (1988) consider an amended version of the above model in which customers can "produce" commodities by combining fractional amounts of the firms' products. Formally, the commodities that can be obtained are given by the convex combinations of a and $1-b$. Anderson and Neven then show that the payoff functions remain quasiconcave in prices, thus guaranteeing the existence of a price equilibrium for all a and b. Furthermore, the solution of the first-stage game is still given by $a^\star = b^\star = 0$. Interestingly, these locations are now socially efficient for they allow each customer to produce his ideal commodity.

14 An alternative approach is taken up by Anderson (1987). Within the original Hotelling model, he assumes that firms A and B enter sequentially and choose their prices following the rules of a Stackelberg game. The outcome is such that the first firm to enter the market locates at the market center and the second close to one of the market endpoints. The second firm to enter will prefer to be the price leader and the first one the price follower. It is worth noting that the introduction of space into a price duopoly allows one to endogenize the price leadership.

15 Bester (1986) tackles the spatial competition problem in a completely different way. He assumes that prices are no longer set by firms but determined by a noncooperative bargaining game between sellers and buyers. More specifically, the bargaining procedure is taken to be a modified version of Rubinstein's (1982) model in which an outside option (purchasing from a competitor) is introduced. Bester shows that there exists a unique perfect equilibrium which, in turn, leads to a unique price system (given firms' locations). Introducing these prices into the payoff functions, he then proves the existence of a noncooperative Nash equilibrium in location. At this solution. firms locate symmetrically but not coincidentally.

16 A radically different approach has recently been proposed by Hamilton et al. (1987). In the second stage, a core allocation involving sellers and customers, who are free to form any coalition, determines the terms of trade, i.e. the prices at both firms. As the core does not contain a unique allocation, the firms exercise their market power by selecting the core outcome which maximizes their joint profits. By contrast, the location choice in the first stage is still modelled as a noncooperative game. At the Nash equilibrium of this game, firms choose to locate at the first and third quartiles, i.e. the socially optimal locations.

The corresponding prices are identical to those of the spatial price equilibrium given in Proposition 1.

17 See d'Aspremont et al. (1983) for a general argument. However, firms may want to agglomerate when the product is heterogeneous (see de Palma et al. (1985)).

18 Note that price discrimination may also result from Cournot competition over spatially separated markets (see, e.g. Anderson and Neven (1986)). An overview of spatial discrimination under Bertrand and Cournot competition is contained in Anderson and Thisse (1988).

19 Kats (1988) considers price discrimination with m tiers in which each firm charges the same mill price per tier. When firms' strategies are given by m mill prices and m scalars describing the size of each tier, he shows that $m \geq 2$ is already sufficient to restore existence. Furthermore, when $m \rightarrow \infty$, the m tier equilibrium converges to the equilibrium identified in Proposition 3.

20 When marginal costs are nonconstant, the maximization problem of a player becomes an optimal control problem, thus leading to a game which has the flavor of a differential game. Lederer (1986) shows that existence of an equilibrium can still be proved in a two-stage game in which firms choose, first, their price schedules and, then, how much to produce and where to ship the output.

21 Empirical evidence supports the assumption that the transportation cost of a commodity is concave in distance (see, e.g. Isard (1956)), thus justifying the shape of the curves represented in Figure 7. This does not contradict the assumption of convexity made in Section 2 where the transport is supposed to be made by the consumers. The convexity of the cost function may then follow, for example, from the value of the customers' travel time.

22 In the monopoly case, most analysis suggests that mill pricing is indeed socially superior to discriminatory pricing. See Holahan (1975), however, for a counter-example in the spatial context.

23 See Gabszewicz and Thisse (1988) for a price-quality duopoly with discriminating firms akin to the outside location game. As in the inside location game, they show that firms choose the socially optimal quality at the subgame perfect price-quality equilibrium.

24 Kats (1988) obtains a similar result in the case of price discrimination with m tiers, provided that m is larger than or equal to 2.

25 The case of differentiated products is dealt with by Anderson and de Palma (1986).

26 In some sense, price discrimination operates with respect to mill pricing as mixed strategies operate with respect to pure strategies by enlarging the space of strategies.

27 Hotelling has suggested reinterpreting that model to explain the choice of political platforms in party competition, when parties aim at maximizing their constituency. This idea has been elaborated by Downs (1957) and developed further by many others. A recent surveys of this literature is provided by Enelow and Hinich (1984).

28 Osborne and Pitchick (1986) show that this location pair is also the only equilibrium in mixed strategies.

29 The above results may significantly be affected when a reservation price is introduced. For example, with $n = 2$, the two firms do not necessarily locate coincidentally, while for $n = 3$ an infinity of equilibria may exist. For more details, the reader is referred to Graitson (1982).

30 One conclusion of the foregoing analysis is that Hotelling's Principle of Minimum Differentiation is valid only for $n = 2$. Nevertheless, de Palma et al. (1985) show that the Principle holds when the products are sufficiently heterogeneous.

31 Another limitation of the model is the use of a linear space. Attempts made to generalize the model to other strategy spaces are rather disappointing. In the 2-dimensional 2-firm case, when the support X of the consumer distribution is compact and has a nonempty interior, there exists a location equilibrium in pure strategies if and only if X contains a point $\bar{x}$ such that the mass of consumers on each side of any straight line passing through $\bar{x}$ is equal to half the total mass, a very restrictive condition (see, e.g. Demange (1982)). Shaked (1975) proves the nonexistence of an equilibrium in pure strategies with three firms when customers are uniformly distributed on a disc. There are some positive results, however. Okabe and Aoyagi (1987) show that the hexagonal lattice is an equilibrium configuration in the case of a uniform distribution of customers over $\mathbf{R}^2$. Simon (1987) has provided a general existence theorem of an equilibrium in mixed strategies for consumers distributed in $\mathbf{R}^n$, whatever n is. A completely different approach, in which a finite number of customers are located on a network, is taken up by Wendell and McKelvey (1981). They show that an equilibrium with two firms exists when the network contains no cycles. For more general networks, they establish the existence of local equilibria.

32 Palfrey (1984) has studied an interesting game in which two established firms compete in location to maximize sales but, at the same time, strive to reduce the market share of an entrant. More specifically, the incumbents are engaged in a noncooperative Nash game with each other, whereas both are Stackelberg leaders with respect to the entrant who behaves like the follower. The result is that the incumbents choose sharply differentiated, but not extreme, locations (in the special case of a uniform distribution, they set up at the first and third quartiles). The third firm always gets less than the two others.

33 In contrast to the standard assumption of a fixed, given distribution of consumers, Fujita and Thisse (1986) introduce the possibility of consumers' relocation in response to firms' location decisions. Thus, the spatial distribution of consumers is treated as endogenous, and a land market is introduced on which consumers compete for land-use. The game can be described as follows. Given a configuration of firms, consumers choose their location at the corresponding residential equilibrium, which is of the competitive type. With respect to firms, consumers are the followers of a Stackelberg game in which firms are the leaders. Finally, firms choose their location at the Nash equilibrium of a noncooperative game whose players are the firms. The results obtained within this more general framework prove to be very different from the standard ones. For example, in the 2- and 3- firm case, the optimal configuration can be sustained as a location equilibrium if the transport costs are high enough or if the amount of vacant land is large enough.

34 Although the sequential location models discussed here have been developed in the case of parametric prices, the approach can be extended to deal with price competition too.

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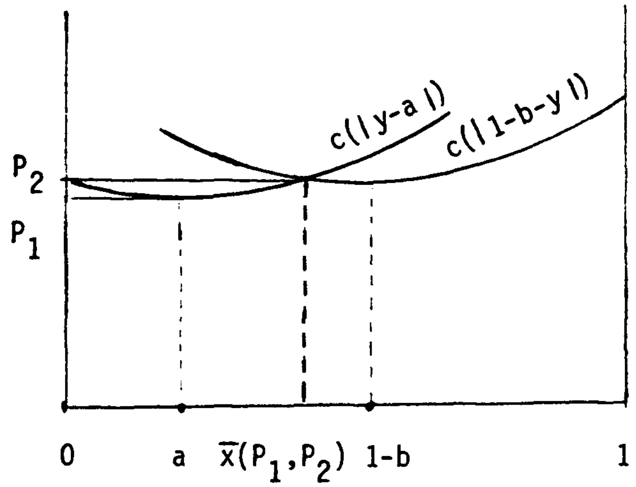
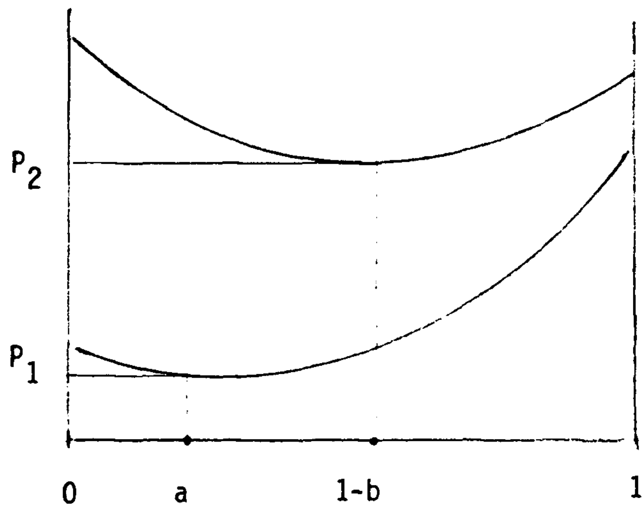


FIGURE 1a



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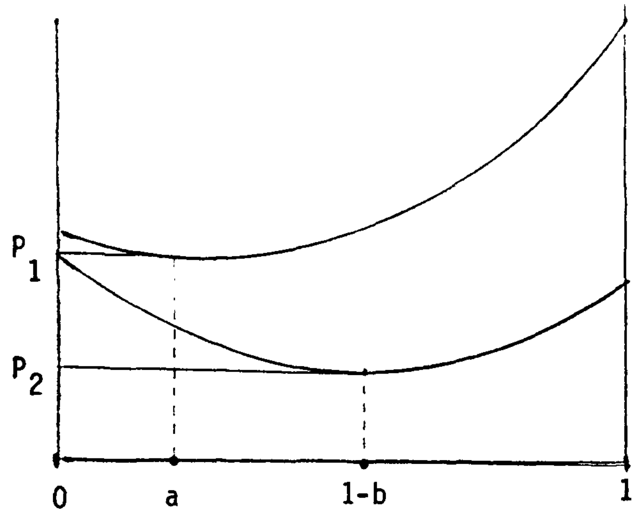

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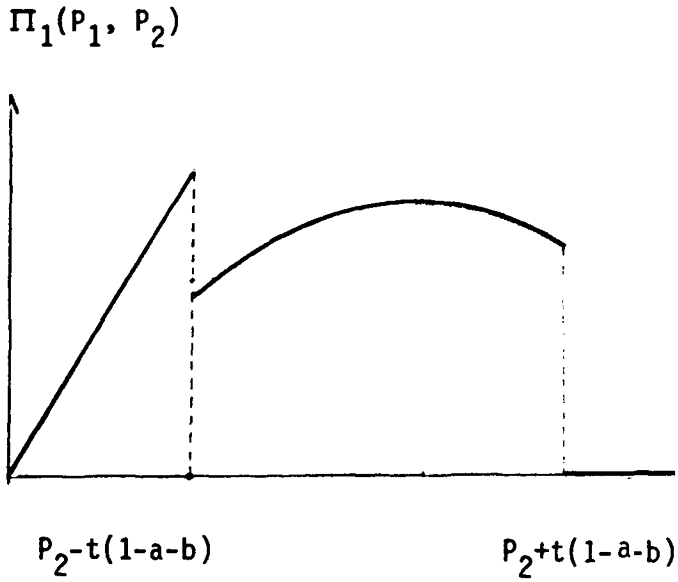

FIGURE 2

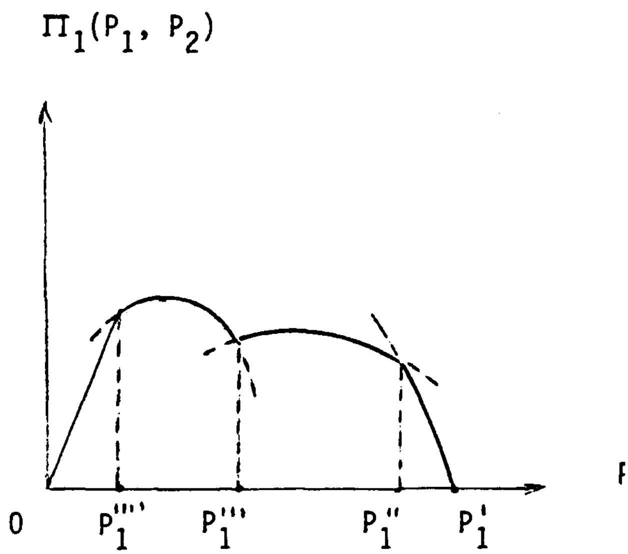

FIGURE 3

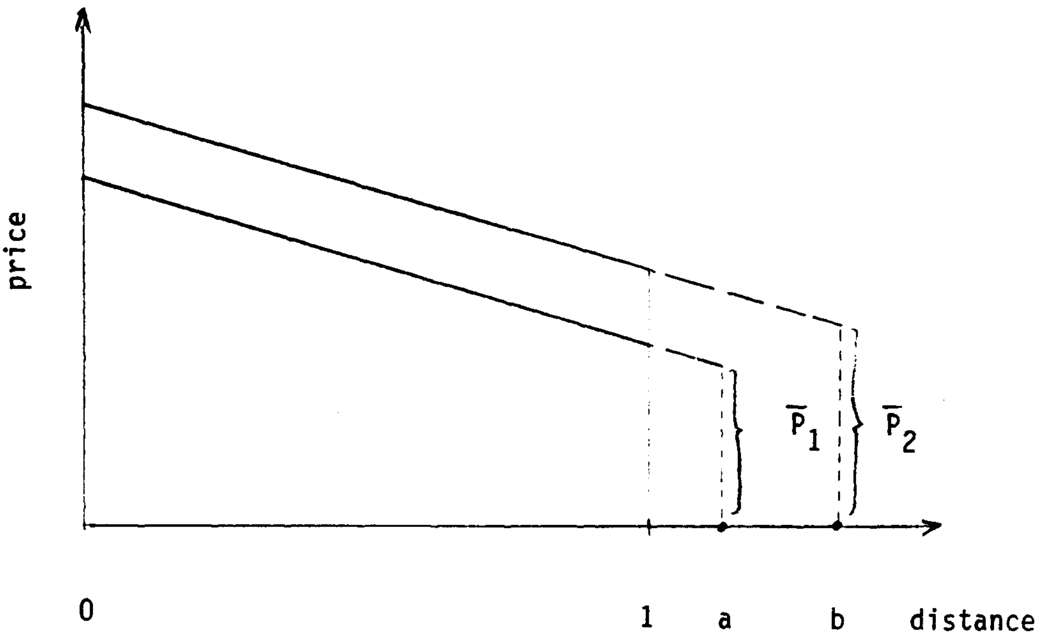

FIGURE 4

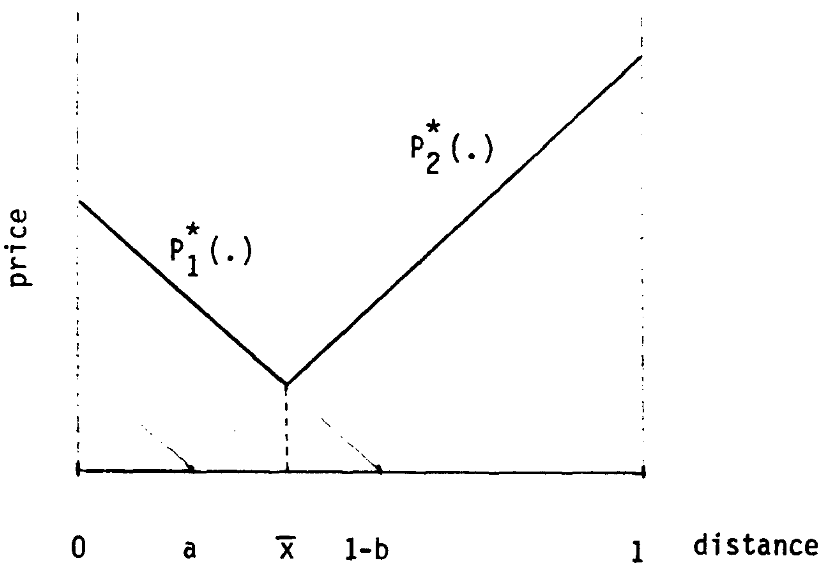

FIGURE 5

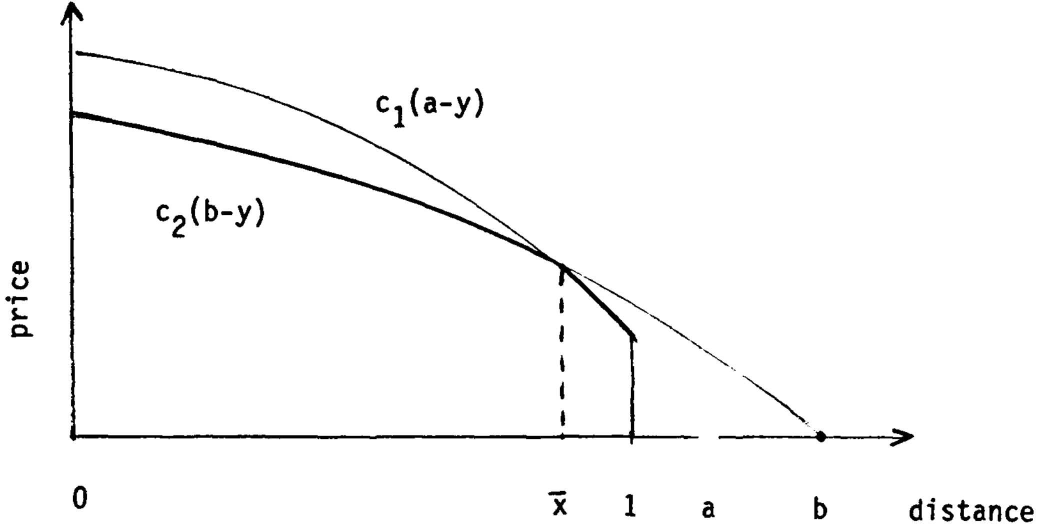

FIGURE 6

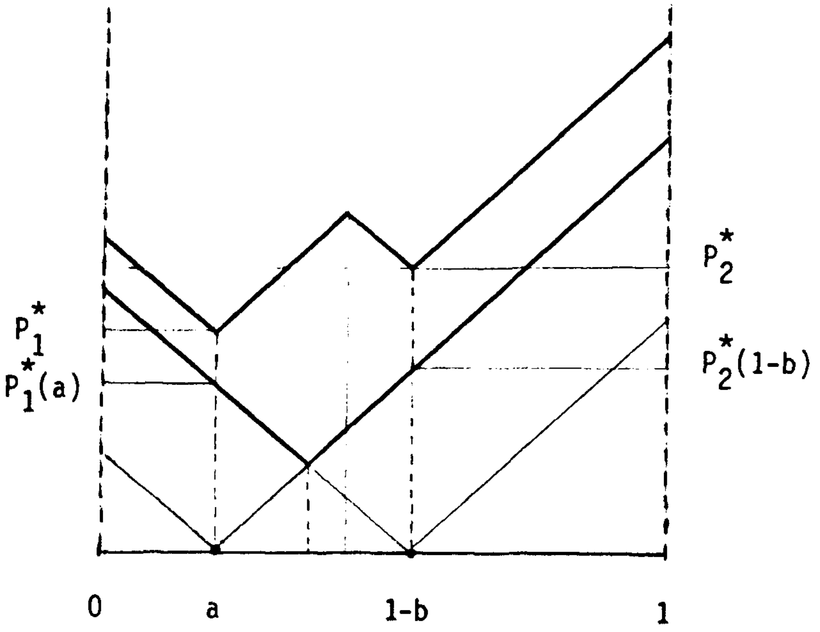

FIGURE 7

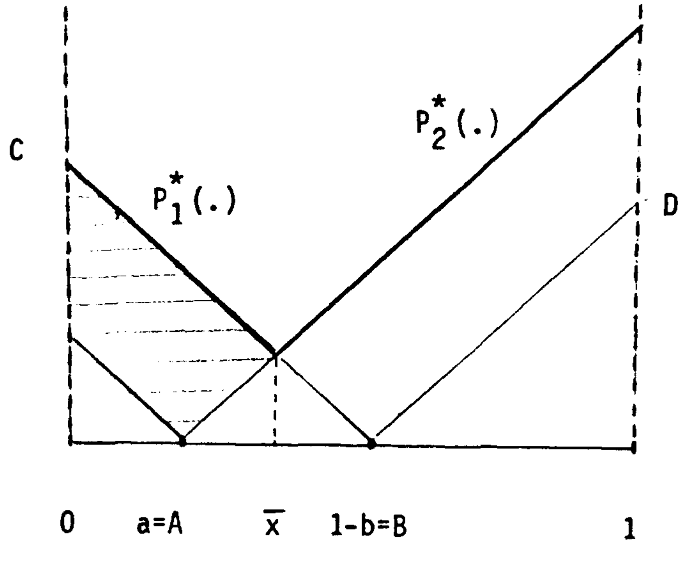

FIGURE 8

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