"BATCHING DECISIONS: STRUCTURE AND MODELS"

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Batching decisions: structure and models[1]

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Abstract

Batching decisions are one of management's instruments to impact performance of goods-flow systems. There is a vast body of literature on analysis and modeling of batching. This paper aims to provide a structure for batching decisions that can help in positioning batching research and models with respect to issues pertinent to goods-flow management. The basis of the structure is a distinction of batching issues as related to three decision levels: (i) process choice/design, (ii) activity planning (aggregate planning and activity programming), and (iii) activity control.
Furthermore, the paper discusses some often heard criticisms of batching analysis. The paper concludes with a little speculation of the authors on the future directions of batching research.

Key words: batching and lotsizing, planning and control, modeling, decision making.

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1 Introduction

Product and service availability and reliability have become key to the competitive strength of firms. Availability and reliability are the accomplishments of the firm's primary process. This process consists of those systems and activities that add value to the products and services provided to customers. For many firms, materials and product flows are central to the primary process. Consequently, effective and efficient goods flow management has drawn much emphasis.

Batching is the clustering of items for transportation or manufacturing processing at the same time. Batching is also called lotsizing. It is a mechanism that induces time-phased production that is usually nonsynchronized with the actual consumption or demand pattern. In this way, batching typically generates cycle inventory or backorders. Still, batching can be advantageous for economic reasons. Production in larger quantities requires fewer setups or changeovers. Economic benefits can ensue from the reduction of the number of setups through a more efficient use of production and other resources.
The trade-off of inventory against setups under consideration of cost and service effectiveness is basic in lotsizing. The trade-off is not new and neither is its modeling. The Economic Order Quantity (EOQ) model developed by Harris [Harris; 1913] is an early attempt in this field.

Lotsizing decisions are taken, consciously or not, throughout organizations. In a complex way, they heavily affect key areas of an organization's performance with respect to flow and storage of material and information. Not surprisingly, there therefore exists a vast amount of research on lotsizing. Hence, an exhaustive overview is undoable in a limited space such as this article. This paper will therefore make reference to only a tiny portion of the mass of lotsizing research. Also, the ubiquity of lotsizing decisions in production and distribution means that decisions on lotsizing are made in widely varying circumstances under different, multidimensional, objectives. Consequently, any selection of research referred to, as is the case in this article, will necessarily be personally biased.
Despite these provisos, the paper aims and hopes to establish a tantalizing presentation of the structure and models of lotsizing decisions. Papers that have endeavored reviewing and structuring part of lotsizing are [De Bodt, Gelders, and Van Wassenhove; 1984], [Bahl, Ritzman, and Gupta; 1987], [Hackman and Leachman; 1989], [Salomon; 1991], [Goyal and Deshmukh; 1992], and [Potts and Van Wassenhove; 1992].

The paper is organized into five sections. The next section introduces a categorization of lotsizing decisions and actions based on the decision making horizon. Section 3 discusses the components found in models of lotsizing. A fourth section discusses often heard criticism of lotsizing research and the fifth and last section presents conclusions and outlooks on lotsizing research.

2 The handles of lotsizing

The primary goods-flow process is sustained by a network consisting of input-transformation-output subsystems, which together form the total goods-flow system[2] of the firm. The behavior of each of the input-transformation-output systems, and hence the behavior of the total, is governed by process parameters and by the levels of activity that are applied to the input-transformation-output systems.
Often inventory buffers exist between different input-transformation-output subsystems. In fact, a buffer can be seen as an input-transformation-output system itself. A general model formulation for such a structure of input-transformation-output systems has been presented in [Hackman and Leachman; 1989].

The performance of a goods-flow system can be measured along several dimensions, such as delays, flow times, conformance to due dates, work-in-process, and (end) product inventory. Performance on each of these dimensions can ultimately be translated into monetary values as costs or revenues. Several difficulties are encountered when trying to determine good activity levels for a given system. These difficulties refer to (i) the problem of translating physical accomplishments into monetary value, (ii) the problem of intricate interaction between the processes of the different subsystems due to capacity and material availability, (iii) the problem of the different time scales on which different decisions and actions take effect on performance, and (iv) the problem of the multidimensional character of the performance measures.
An established method for dealing with a complex problem is to decompose it into smaller, more tractable, possibly hierarchically related, subproblems. Following Anthony [Anthony; 1965], many researchers have proposed decomposing (lotsizing) problems in production and distribution into three levels: strategic, tactical, and operational. A system's performance can therefore be influenced by hierarchically interrelated lotsizing decisions at the level of:

1. process choice/design,

2. activity planning, consisting of two sublevels: aggregate production planning and the programming of time-phased production quantities,

3. activity control.

The sequence of Levels 1-3 reflects a decrease of the cycle length of the decisions involved. The decrease is parallelled by an increase of the importance of consideration of feedback as a regulating mechanism of a system: to some extent, the sequence of Levels 1-3 also reflects a transition from examination of open-loop to closed-loop systems.
Part of lotsizing research has been directed towards the development of models re-integrating the three levels of decision making, see e.g. [Hax and Candea; 1984], [Afentakis; 1985], [Lasserre; 1992], and [Dauzere-Peres and Lasserre; (This Issue, 1994)].

Process choice/design concerns the arrangement of machines and facilities in anticipation of future demands. Major concerns at this level are the choice of technology, product development and establishing appropriate levels of capacities for handling material/product and information

[2] A system is a structure of entities having attributes interacting, under certain conditions, with activities creating events that change the state of the system, cf [Shannon, 1975]. The state of the system is a set of values for the attributes of its entities. The system's process is the time-sequence of states of the system.

flows. Process choice/design includes deciding on the preferred type of planning and control concept for marshaling the system's activities. Since decisions at the level of process choice/design frequently involve major investments, these decisions are usually long-term decisions. Decisions made at this level set constraints on the feasible decisions regarding activity planning and control: they set some of the parameters and capabilities for activity planning and control. One choice, for example, in planning and control can run between the use of a synchronous, centrally controlled (also called indexing) planning and control method versus an asynchronous, locally controlled method. A choice for one of these methods for managing a production line will set conditions for the discretion that remains in arranging activity planning and control.
One of the parameters determined at the level of process choice/design is the unit production quantity, i.e., the smallest quantity in which a good is produced at an operation. This unit quantity can, for instance, be the size of a tank, oven, conveyor cart, or truck. Batch sizes determined at the level of activity planning and control are usually integer multiples of the unit production quantity. When environmental conditions are stable, other examples of policy parameters set at this level are the length of the production cycle, the planning horizon and the frequency of runs per item. These quantities, *in principle*, are not directly dictated by the "hardware" of the production-distribution system and can therefore, again *in principle*, be changed more often. Models for this type of decision making usually assume stationary or even constant conditions. Examples are given in [Elmaghraby; 1978], [Vergin and Lee; 1978], [Zipkin; 1986], and [Roundy; 1986]. Examples of recent papers addressing these lotsizing issues in process choice/design are [Zipkin; 1991] and [Golhar and Sarker; 1992].

Aggregate production planning constitutes the starting point for activity planning. Concerns in aggregate planning are, among others, workforce levels, overtime, and subcontracting. Requirements are determined through some aggregate measure of demand, like production hours. The time horizon used in aggregate planning is typically one to two years. Aggregate production planning is discussed in e.g. [Land; 1958], [Hax and Candea; 1984 (Chapter 3)], [Silver and Peterson; 1985 (Chapter 14)], [Aardal and Ari; 1987], [Erenguc and Tufekci; 1988]. An extension including learning effects is discussed in [Kroll and Kumar; 1989]. A review on aggregate production planning is given in [Nam and Logendran; 1992]. A critical assessment of the practicality of aggregate planning modeling has appeared in [Buxey; 1988].

The phases activity programming and activity control together refer to the decision stages in which production quantities and schedules are determined over some rolling planning horizon on the basis of the aggregate production plan and more detailed information concerning actual demand. The time horizon used in production programming is shorter than that of aggregate production planning: it usually is three to six months. Available capacities, like workforce levels or available machining time, are often considered fixed over this horizon. When programming production activities, one leaves some latitude for the exact timing of operations in the sense that planned production quantities are assigned to a so-called time bucket, e.g. days or weeks.

Manufacturing resource planning (MRP) is a classic example of such a production programming system. Throughout the years, starting as early as the fifties, MRP-like systems have drawn much attention, [Manne, 1958], [Zangwill; 1966], [Veinott; 1969], [Love; 1972], [Afentakis, Gavish, and Karmarkar; 1984], [Billington, McClain, Thomas; 1986], [Afentakis and Gavish; 1986]. These references consider multilevel systems, i.e., primary processes that are sustained by a system that is comprised of a number of input-transformation-output systems coupled by material flows. Naturally, also the single-level system has drawn much attention. The classic paper, considering a single-item unlimited-capacity system, in this area is [Wagner and Whitin; 1958]. In recent years more involved extensions, including multiple items and limited capacity, of the system considered by Wagner and Whitin have continued to attract much attention, e.g. [Dixon and Silver; 1981], [Maes and Van Wassenhove; 1986], [Zangwill; 1987]. See also [Jain and Siver; (This Issue, 1994)]

Activity control refines and extends the programming of production activity with dispatching decisions [Hill, 1991]: loading (assignment of jobs to machines), sequencing (determination of the order in which jobs should be completed), and scheduling (determination of start and finish time of jobs). Through production activity control, production units or departments are given some discretion to establish desired loadings, sequences and schedules.
The starting point for activity control is the release of production orders. Although programmed production orders may not have been released for certain items, these planned orders may trigger release of component production orders in order to meet future (planned) requirements. In this way activity programming leads to initiation of activity control for component production. In principle, in closed-loop systems, information on capacity problems encountered in activity control can be fed back to the activity programming function to reprogram activities such that capacity overload is avoided. There thus is an evidently strong interaction between production programming and activity control, see e.g. [Dobson, Karmarkar, and Rummel; 1987], [Monma and Potts; 1989], [Dobson, Karmarkar, and Rummel; 1992], [Potts and Van Wassenhove; 1992], and [Dillenberger et al; (This Issue, 1994)]. Detailed scheduling for overlapping operations by lot splitting allows for an optimal use of time [Glass, Gupta, Potts; (This Issue, 1994)]. When setup times are involved, scheduling becomes very hard [Bruno and Downey; 1978], [Monma and Potts; 1989].

Production activity control can also be viewed as a process for which process parameters have to be determined. An example is the accepted level of work load: new orders may only be released to the shop floor if the current work load drops below the acceptable level. Research along this line is reported in e.g. [Wiendahl; 1987]. Much of the research on dispatching rules (such as shortest-processing-time or earliest-due-date) can also be seen as containing attempts to determine process parameters for activity control. An interesting paper, combining process choice/design issues in activity control with decision making, is [Jacobs and Bragg; 1988].

A further distinguishing feature of planning and control of a process is its temporal relation with process activity. Two broad categories can be recognized

- ⋆ hands-on (on-line) planning and control,
- ⋆ hands-off (off-line) planning and control.

In hands-on planning and control, the process obtains directions during its execution, before its completion. In contrast, in hands-off planning and control provisions are made prior to the start of the process and no action is taken until after process completion. In a slightly different form the dichotomy in temporal relation is sometimes referred to as the periodic management model versus the responsive service model [Davis and Manrodt; 1992] or as precontrols/postcontrols versus concurrent controls [Baird, Post, and Mahon; 1989 (Part Five)].

Activity planning and control tends to relate more to hands-on planning, whereas process choice/design tends to relate to hands-off planning.

At each of the above levels and in each type of planning and control, lotsizing is an important handle by which management can affect work-in-process, safety and cycle inventory, and leadtime (throughput-time) performance. In turn, these dimensions of performance will affect operating cost as well as availability and reliability of product supply and, therefore, operations profitability.

3 Modeling elements of system behavior

Cost (efficiency) and service (effectiveness) are two main measures of performance for an input-transformation-output system. Their attainable values are largely dependent on system capacity, since this constrains system behavior. System behavior is further constrained by (non)availability of required materials. In general, in modeling the behavior of a network of input-transformation-output subsystems, one has to include three types of constraints,

- material flow-transformation constraints,
- material and product availability (service) constraints,
- capacity constraints.

Within these constraints, lotsizing decisions aim to achieve optimal values for the performance measures.

A more detailed account of the modeling approaches to the types of constraints and other model components, is given in the following subsections.

Model components

A popular distinction in modeling concerns the use of prescriptive versus descriptive models. Prescriptive models focus on decision making whereas descriptive models focus on the evaluation of the performance of a specified system [Dietrich; 1991]. While lotsizing analysis may use a descriptive model, like a simulation model, the focal point of lotsizing models is predominantly on decision making and action taking.
The type of decision level considered, and the system/technology concerned, determine the components and aspects that are relevant to the decision making. These components and aspects relate to

1. choice/selection and value of model parameters,
2. type and detailed structure of constraints included in the model,
3. the criterion by which (model) solutions are evaluated: the objective(s).

The examples of components/aspects discussed below largely follow those given by Gavish and Johnson in [Gavish and Johnson; 1990].

Model parameters are quantities that are predetermined exogenously to the model and remain fixed during the model's solution. In contrast, variables are determined endogenously during model solution: their values are output of the model. One first therefore has to decide which quantities are parameters and which quantities are variables. Preference towards consideration of a quantity as parameter reflect conditions that are perceived stationary or predetermined during the decision routine. The exact choice whether a quantity, for instance capacity, is considered a parameter or a variable depends on the decision level. For example, capacity that is considered fixed at an activity control level might be considered variable in a process choice/design level.

Parameters. The following elements are usually considered as parameters.

- Planning horizon and time scale. The planning horizon is the time interval on which assumptions are made with respect to demand and on which performance will be measured. The planning horizon may be *finite* or *infinite*. An infinite planning horizon is usually accompanied by stationary demand and a finite planning horizon by dynamic demand.
  The time scale is either *continuous*, i.e., a(n in)finite interval of the reals, or *discrete*, i.e., a(n in)finite interval of the integers. In the latter case, one thinks of an integer as an index to a time interval. Thus the discrete scale represents a partitioning of time into a number of time buckets or planning periods. In case of a discrete time scale, the "real-world" events and decisions, which occur in continuous time, have to be translated into events and decisions occurring according to the discrete time scale. To make this translation reasonably valid and accurate one has to carefully select the size of the time bucket.
  For example, by using "small" time buckets, it may be natural to build a model that allows for at most one item to be produced per planning period, allowing setups to be carried from one period to the next. On the other hand, the use of larger time buckets may lead to models with multiple items produced per period, and therefore setups that cannot easily be carried over from one period to the next when production of an item occurs in two, consecutive, periods.

- Demand rate. Demand is nearly always considered as an exogenous quantity, i.e., it is considered input to the model. Two broad lines of approach can be distinguished

  - models in which demand is considered stationary or even constant
  - models in which demand is considered dynamic.

  The first line characteristically addresses the determination of a policy in continuous time. In the second line of approach, time-dependent demand is specified on a period-by-period base.

Type and detailed structure of variables, parameters, and constraints included in the model. An obviously important decision in model building concerns the type and detail of quantities and constraints that will be incorporated in the model. Once again, such decisions are contingent on the decision level. Several types of quantities and constraints are presented below.

- Service threshold. Service thresholds or minimum service levels, can be specified through constraints stating minimum material and product availability. A 100% service threshold would mean that all demand, over the planning horizon, is met in time by the production-distribution system. Thus, the net inventory would at no time be negative for any item. When demand has to be modeled as a stochastic parameter, it may be very costly to set a service threshold close to 100%. Therefore the service threshold is often expressed as a fill rate or maximum number of stockout occasions allowed per year. When demand cannot be met by its due date one needs to specify whether and to which extent this demand can be *backlogged* and delivered later and to which extent this demand should be considered as *lost sales.*

- Resource constraints. When capacity restrictions on resources are not restrictive, or modeled as costs components in the objective (as will be discussed below), the model is said to be an *uncapacitated* model. When capacity constraints are explicitly stated the model is said to be *capacitated.* When active, capacity can be considered as exogenous or endogenous to the model: the available capacity of a resource is either given, or to be determined (e.g. when overtime is allowed). System capacity is usually determined by decisions at the level of process choice/design or aggregate planning. As lotsizing analysis is often concerned with activity programming and control, available capacities are frequently considered to be parameters, not decision variables.
  Capacity is an important directly accessible channel by which system behavior and performance can be altered, but frequently in a nontransparant way. For this reason, a subsection below contains an expanded discussion with more details on the way models deal with capacity.

- Leadtimes. Lotsizing models can have: *(i)* exogenous (or nominal) leadtimes, or *(ii)* endogenous leadtimes.
  Exogenous leadtimes can be due to the nature of a transformation process itself, e.g. the drying of paint during some fixed time interval, or to external factors such as purchasing leadtime. Many statistical inventory control models, such as (s, Q) and (s, S), see e.g. [Silver and Peterson; 1985], assume exogenous, fixed, leadtimes. Exogenous leadtimes are also often used in models of MRP-systems. Endogenous leadtimes, on the other hand, are an outcome of the schedule of operations and thus of the batching decisions.
  In practice and in many models, of course, leadtimes are a composite of exogenous and endogenous elements.

- Production structure. The input-transformation-output subsystems that constitute the primary process interact in two principal manners. One mode of interaction is through the use of shared resources of limited capacity. The other principal manner of interaction originates from the work flow: material output of one input-transformation-output system serves as input to another input-transformation-output system.
  The work flow interactions are mathematically represented in material flow-transformation constraints. These constraints specify the stages through which input materials are converted to output materials and how materials and products are accounted for (as work-in-process inventory).
  Pictorially, the work-flow dependencies can be represented in a network. In this directed network input-transformation-output systems are the nodes. The output-input relations, corresponding to the process sheets or routings, determine the arcs in the network. The network thus constructed represents the so-called production network structure of the goods flow system.
  The structure is termed *single level* when all input materials are supplied exogenously to the model, i.e., the goods flow network consists of isolated nodes only. (The nodes may interact due to limited capacity though, i.e., several input-transformation-output systems

may use the same resource (machine). This, for example, is the situation modeled by the (single-level, multiitem) Capacitated Lotsizing Problem CLSP, see e.g. [Dixon and Silver; 1981].) Other structures, with a goods flow network in which at least one pair of nodes is connected by an output-input arc, are termed *multilevel* structures. Multilevel structures are frequently further distinguished on the type of network: (i) a serial structure has a (connected) network in which each node has at most one incoming and one outgoing arc, (ii) an assembly structure has a (connected) network in which each node has at most one outgoing arc, and (iii) an arborescent structure has a (connected) network in which each node has at most one incoming arc. If a structure does not fit into any of the above categories it is termed a general structure.
When items are reparable the production structure can have cycles: items can return after use. A review addressing the issues pertinent to such production structures is [Nahmias; 1981].

The criterion by which (model) solutions are evaluated: the objective(s).

- Objective function. The generic objective is the minimization of costs per unit of time. However, the way in which lotsizing models translate activity and system accomplishments into costs differs substantially between models. In fact, the approaches to assign costs to activity and accomplishments parallels the ways in which capacity absorption is linked to levels of activity and accomplishments.
  Thus, the function that accounts (expected) cost, models a cost response linear *(i)* in the level of capacity absorption or in the level of activity (production rate and inventory position), and/or *(ii)* in the setup/changeover intensity.
  In some lotsizing models physical accomplishments are not rephrased into monetary value, but are directly stated as objectives. Examples are objectives that seek to minimize the maximum completion time, maximum lateness, total completion time, or total tardiness [Monma and Potts; 1988], [Potts and Van Wassenhove; 1992].

Coping with limited capacity

Although the formal format of material flow and availability constraints varies between lotsizing models, there are no fundamental differences in these types of constraints as far as model formulation is concerned.
However, in modeling capacity constraints some fundamentally different choices need to be made.

Increasing activity levels, under sufficient material availability, increases the output rate of an input-transformation-output system until some part of the system becomes saturated.
Modeling such behavior may require some dexterity but causes no fundamental problems. Capacity supply is usually modeled as a function of time, stating the number of resource units available per unit of time. Modeling capacity demand is more involved. One needs to state how the level of transformation activity translates into capacity (resource) consumption per unit of time, in order to determine the activity levels that can be supported. Prevalently, demand for capacity is modeled according to the following two principles:

1. capacity demand is linear in the level of activity,

2. capacity demand is linear in activity setup/changeover intensity per item.

Although the model formulation as such may not be that difficult, the picture often changes when one wishes to run the model for purposes of analysis and planning. Models with capacity consumption from activity setup/changeover are hard to solve. Consequently, this route to modeling system behavior may encounter two drawbacks expressed in the following points.

1. The behavior of the model is very hard to understand (the model's behavior is nearly as hard to understand as the behavior of the system itself); the robustness of the results obtained is hard to assess.

2. Large amounts of time are required to solve a model; comparison of model behavior under alternate circumstances (demand patterns, service requirements) becomes very time consuming and this diminishes the model's usefulness in practical decision-making processes.

Apart from meeting the difficulties posed by the direct modeling of capacity consumption from activity setup/changeover head on, two main routes have been pursued.
The first route consists of giving up the requirement that the model should be solved exactly. Instead, intelligent rules of thumb, called heuristics, are proposed that hopefully yield good, but not necessarily optimal solutions to the model.
The second major route consists of changing the model: one attempts to emulate the system's behavior in response to the constraints imposed by limited capacity. For instance, instead of modeling capacity consumption due to setup/changeover decisions, one will typically add a cost penalty to the objective function, i.e., setup costs become a surrogate for setup times.
Note that lotsizing decisions heavily alter operations intensity and schedule, while the latter are often poorly translated into capacity consumption and resource utilization due to the replacement of setup times by setup costs. Thus, the validity of the approach comes under question, especially when the system is heavily loaded and setup times are nonnegligible. A critique to the setup costing approach is given in e.g. [Karmarkar, 1987], cf [Karmarkar, Kekre and Kekre; 1992].

A rough-cut layout of lotsizing models

The dimension capacitated/uncapacitated constitutes one of two main axes along which lotsizing models can be categorized. The other axis relates to the way demand is modeled: models can be distinguished on assumed knowledge of future demand. Is demand modeled as a stationary stochastic (or even constant) parameter or as a dynamic (time-dependent but known) parameter? The two axes lead to the following typology of lotsizing models.

\ *capacity* *demand* \	infinite	finite
stationary (& constant)	• EOQ • SIC	• ELSP • Queuing/Batching
dynamic	• (multilevel)-WW	• (multilevel)-CLSP • DLSP • Batching/Scheduling

EOQ:	Economic Order Quantity. This is the model introduced in [Harris; 1913]
SIC:	Statistical Inventory Control. Examples of this type of model are given in [Hadley and Whitin; 1963], see also e.g. [Silver and Peterson; 1985].
ELSP:	Economic Lot Scheduling Problem. See e.g. [Elmaghraby; 1978].
Queuing/Batching:	Models based on queuing theory. See e.g. [Karmarkar; 1987]
multilevel WW:	multilevel Wagner-Whitin type of models. See e.g. [Afentakis and Gavish; 1986].
(multilevel)-CLSP	multilevel Capacitated Lotsizing Problem. See e.g. [Dixon and Silver; 1981] and [Billington, McClain, and Thomas; 1986].
DLSP:	Discrete Lotsizing and Scheduling Problem. See e.g. [Fleischmann; 1990] and [Salomon et al.; 1991]
Batching/Scheduling:	See e.g. [Potts and Van Wassenhove; 1992].

An expanded table categorizing lotsizing models is given in appendix.

Note that in the rough-cut categorization, models with a priori leadtimes, i.e., with leadtimes that do not depend on the operations schedule, are considered as models with infinite capacity. Of course, leadtimes often signal finite capacity at some process. However, here the point is that the model's behavior concerning leadtimes is independent of the level of activity, i.e., independent of the decisions taken. In particular the places and variables for which capacity plays a role are considered exogenous to the theatre of decision making.
Finite capacity models, on the other hand, consider capacity as active, be it given or to be determined, in the decision making theatre: in this way, leadtimes are always considered endogenously.

Models from queuing theory, (stochastic) dynamic programming, and mixed integer linear programming are the most prominent in the area of lotsizing. Batching analysis that makes use of models with stochastic elements, such as queuing theory, as a rule builds on the assumption of stationarity of the conditions under which the system operates: although actual conditions at time instances may vary, conditions are statistically time-invariant. Only statistical information (e.g. averages and variances) is assumed known. So, only (constant) statistical information can be used in decision making. Regularly, the analysis yields stationary timing and sizing of batches as best solution. For these reasons, queuing models foremost relate to analysis at the level of process choice/design in which decisions on batching (e.g. the unit size) are made on the basis of hands-off planning and control.
Queuing model are capacitated models. The finiteness of processing (service) times limits the output rate of the models. Limited capacity effects manifest themselves prominently when the system's utilization approaches 100%: inventory (work-in-process) rises sharply as utilization nears 100% and the cost per unit of output rises sharply accordingly.
Deterministic models can be uncapacitated or capacitated. In uncapacitated models one frequently finds, in pointed contrast with queuing models, an economy of scale effect in that the cost per unit of output decreases with the volume of output (demand).
Being deterministic, mixed integer linear models are based on knowledge of values (realizations) for model parameters such as capacity and demand in order to determine lotsizes. These models are suitable in situations in which the state of and requirements on the system can be set to specific numerical values. Deterministic models have been employed at all three levels of lotsizing analysis: process choice/design, activity planning, and activity control.

4 Lotsizing's expediency

Lotsizing research has always been criticised but in recent years the criticism has intensified (to the extent that one can almost speak of a lotsizing-criticism tradition). Changing conditions in technology and competition in the business environment have put the expediency of lotsizing analysis to solve management problems under new scrutiny.

As more and more products shipped to customers are customized, end product variety has increased considerably [Shapiro and Heskett, 1985], [Council of Logistics Management, 1988], [Persson, 1991], and product life cycles have shortened [Magee, Copacino, and Rosenfield, 1985], [Persson, 1991]. The risk of supply from 'grey' products (i.e. inventory), has grown accordingly. Emphasis on low inventory levels and short leadtimes has created new goals and objectives for management in production planning and control, cf [Stalk, 1988].
In addition, progress in production and information technology has influenced the economics underlying transformation processes and their coordination. Many companies have succeeded in increasing flexibility, enabling them to do more production changeovers. Other companies have resorted to a new approach to management control like Just-In-Time in which emphasis lies on eliminating the causes for inventory and lot production. Companies that have adequately mastered the use of new management approaches and tools built on new technologies have been able to reduce inventories while suppressing negative consequences, such as increased costs and lost productivity resulting from additional setups. One of the factors towards success is a critical reassessment of the use of lotsizes and inventories.

The experience that low inventory levels and short production runs can be achieved without increasing total costs has led some practitioners and academics to believe that the inventory-setup tradeoff is arguable, see e.g. [St. John; 1984], [Woolsey; 1988], and [Weiss; 1990], and that lotsizing analysis has lost its relevance. As may be apparent from the discussion thus far we disagree with this view.
We have listed and commented on some of the interrelated criticism on lotsizing research in more detail below:

- *'Lotsizing is not the real issue. The real issue is to design production processes that are so flexible that production quantities (batches) equal customer demand quantities, and timing of production is such that inventory positions are almost zero.'*
The theme of this criticism is that the issue is not *how* to cope with the inventory-setup problem but to *eliminate* the need to deal with it. Thus, this criticism states that the process choice/design should eliminate the need to deal with lotsizing in activity planning and control. The accelerations in technology, alluded to above, have created some potential for such elimination. The observation that flexibility is key to a firm's success carries some substance.
Lotsizing does not argue with this line of reasoning. Rather, lotsizing tries to take advantage of flexibility, by encouraging company operations to take place at the cost-service frontier (e.g. to operate at minimum cost under a specified service level). The fact that flexibility is dynamic causes the cost-service frontier to shift. Understandably, this change of the basis for the models will change the nature of the formulation and use of models. E.g., even if setups require little capacity or time due to technological progress, performing frequent setups may still negatively effect output quality and slow down processing. Thus,

models are required to formulate a relation between lotsizing and quality. The point is that, although the cost-service frontier is shifting, the dynamics does not erase this frontier altogether.

As a consequence, at a specific point in time, when decisions on process choice/design or activity planning and control have to be taken, the setup-inventory (plus other factors) trade-off remains manifest.

For instance, in activity planning and control, one frequently has to deal with production processes in which 100% flexibility with zero inventory is still a long way off. In some production processes (e.g. production of paints, or tubes) it is simply too expensive or even technically infeasible to make just-in-time arrangements with suppliers and/or to design flexible manufacturing lines that provide the ability to switch instantaneously without loss of production time between different products. In such circumstances, batching remains an important issue.

Alternatively, customer demand may have a *large variability* over time (short term and seasonal demand fluctuations), whereas available capacity only allows for *small variation* over time (limited capacity of overtime work and/or subcontracting). This may cause demand in some periods to exceed available production capacity. In order to guarantee timely delivery (customer service), demand in such periods must be fulfilled from production in earlier periods, again leading to batching and inventory decisions.

- *'Classical lotsizing models are "single issue" models, which neglect interactions between relevant process components to a large extent.'*

  To be of any use, models always need to be an abstraction of real-life problems. The key is to retain the relevant components and interactions. For example, in some classical lotsizing models (Economic Order Quantity related models, Wagner-Whitin related models) production quantities are determined based on a trade-off between fixed production costs, variable production costs, and inventory holding costs under a threshold customer service. So, capacity, for example, is ignored as a factor. In some production environments this may be an adequate and relevant model to support decision making. In other settings the model may be of no use at all.

Nonetheless, it remains an intricate problem in lotsizing that lotsizing strategy, at all levels of decision making and action taking, is part of total strategy and that it is enclosed in the management process. New approaches to lotsizing are often accompanied by new work and management processes. A review of lotsizing policy can occur as part of a project that investigates a transition from a push to a pull planning and control strategy. For example, Kanban driven production can be looked upon as a lotsizing strategy, but it is more than just that. It also entails a new pull management approach towards control that encompasses issues of product/process quality and worker involvement. In fact, when ignoring that lotsizing is embedded in the global management process it is hard to infer benefits of alternative strategies. It follows that recent lotsizing research that indicates that MRP logic is superior to Kanban and other reorder-point logics [Axsäter and Rosling; this issue of EJOR] and that MRP logic outperforms the other methods when the production scheduler does a good job in setting gross requirements, i.e., when there is a positive correlation between forecast gross requirements and realized demand [Jacobs and Whybark; 1992], has to be supplemented by the consideration that lotsizing is only part of a larger production environment picture.

In any case, the imputation of benefits such as cost savings or increased revenue effects to lotsizing strategy is in general arduous. Here lies a challenge.

- *'Lotsizing models are built on quicksand. The (cost) parameters they use cannot be validated.'*
  Traditional accounting methods strongly aim at recording the flow of assets through a firm. Consequently, information on costs and other inputs are often not measured in a form or detail appropriated for lotsizing decisions. Lotsizing shares such limitation with many practical studies on planning and control, or mathematical modeling in general for that matter.
  In the optimization process one has, as a result, to take care when trying to capture the influence of batching decisions on monetary performance. However, one can start modeling quantitatively, regardless of adequacy of input formats and detail. In the process of modeling and upon obtaining model results, one has to deal with the question of what is sufficient input adequacy and detail. The increasing practicality of this approach comes with the increasing responsiveness of decision support tools which use batching models. Nowadays, one has for example on-line data capture and analysis, and on-line process monitoring and control. One can thus establish a more immediate and direct link between action and physical result and one therefore experiences less of a need to go through an indirect "monetary" comparison of pros and cons of certain actions. In addition, the shortened response cycles can in some situations help in establishing the sensitivity of the model output to parameter values. The limitation of inputs, therefore, although challenging, does not inhibit the completion of successful lotsizing studies and applications. On the contrary, lotsizing studies can point out the critical dimensions of performance that require careful consideration and measurement. We refer to [Karmarkar and Rummel; 1990] for a more extensive discussion of the relation between lotsizing and cost accounting.

- *'Lotsizing models and their solution procedures are too complicated and/or not effective for use in practice.'*
  It is certainly true that some of the models and algorithms, developed to solve lotsizing problems, are too complicated to be understood by non-experts in the field of optimization. But, in many instances the details of internal processes need not be known to users or decision makers. Indeed, how much does the modern manager understand about the inside of the PC he uses, the telecommunication network he thrives on, the plane he takes or even the plain copier he utilizes? The issue, therefore, is not how to get production planners to understand the details of models and algorithms used. The real issue is to *express* the function, the added value, and the robustness of lotsizing procedures to practitioners, and to establish responsive interaction between planning practice and research on lotsizing.
  In fact, increased automation and time pressure forces the trend to built-in decision making, i.e., hardwired blackbox routines. Since one needs routines anyhow, because the system needs them to function automatically, one may as well use good ones, i.e., lotsizing becomes *more* important because mistakes can no longer be easily repaired by human interaction.

5 On the road: lotsizing's vitality

Lotsizing applies techniques and theory from operations research in order to develop systems that support decision making in the management of production and distribution operations. The systems developed are largely computer based. So, it is natural that lotsizing will participate in and react to developments in operations research theory, to developments in computing and communication capacity (computers), and to developments in operations management. For example, in the last decade research on mixed integer-linear programming spends increasing effort on obtaining polyhedral results such as "valid inequalities" or cutting planes with which one can obtain tight representations of problems. This effort and its results extend to models of lotsizing problems, see e.g. [Pochet; 1991] and [Pochet and Wolsey; 1991]. At the same time, classic problems have continued to attract attention and surprising new results have been obtained. For example, the classic Wagner-Whitin problem is now known to be solvable in linear time, [Aggarwal and Park; 1990], [Federgruen and Tzur; 1991] and [Wagelmans, Van Hoesel, and Kolen; 1992]. Also, very good bounds coming from relatively easy to compute solutions, have been obtained for the multilevel EOQ problem, [Roundy; 1986].
However, the future of lotsizing as a practical theory will be most profoundly impacted by its interaction with events and changes in its domain of application.

Under the increasing pressure for fast response in production and distribution management, we believe that traditional lotsizing analysis and modeling will increasingly expand and be supplemented by analysis and modeling addressing issues in the following directions.

- *More emphasis on physical accomplishments and responsiveness.* The direct accomplishments of a goods-flow system are physical: flow times and inventory levels. Financial performance is a consequence of physical performance. However, the temporal and rational relation between the two may be very hard to uncover, especially in a dynamic environment.
  On the other hand, there is now widespread recognition of the direct link between physical accomplishments and a firm's competitive position. This has increased the demand for direct responsive planning and control of physical accomplishments. It will re-emphasize the building and analysis of models in which feedback is used to regulate and direct the primary system. Lotsizing will reflect this trend, see [Dobson, Karmarkar and Rummel; 1992]. Activity control already has a tradition of considering planning and control in which feedback is an important aspect. This can be a further stimulus to develop models that re-integrate activity control with other levels of decision making and will draw new attention to aggregation/disaggregation issues in modeling.
  Also, emphasis on responsiveness and feedback may create interest in the interaction between system stability (reliability) or transient system behavior (under dynamic conditions) and lotsizing. Early work in logistics on system dynamics already appeared in [Forrester; 1961], cf [Axsäter, 1985].

- *More analysis on (changing) the constraints.* Many system constraints and parameter values derive from the assumption of permanence of certain conditions in the system's environment. The dynamics of the environment will force a closer examination and comparison of contingencies in analysis and results ("optimal solutions") with respect to constraints and parameter values.
  For example, one can research the effect of a reduction in fixed (exogenous) leadtimes or

a reduction of the variability in purchasing leadtimes on inventory levels. Research along these lines is not new, but we expect it to attract more attention in the future, see e.g. [Silver; 1992].

- *Enlargement of the classic setup-inventory tradeoff.* When an input-transformation-output process does not keep pace with its demand process(es) the result is a stockout or (output) inventory. Thus, inventory is a result of a mismatch between a supply and demand process. Many things can be the cause of such mismatch, for example, (i) failure of the supply process (machine breakdown) requiring curative maintenance, (ii) preventive maintenance that had not been included in the activity planning process, (iii) rejection by the demand process of certain output products because of low quality, and (iv) changed requirements of the demanding process. The number of setups influences each of the four above factors: failure and poor (run-in) product quality can be consequence of production changeovers and requirements may change on a time scale that is shorter than the one set by the setup schedule (few setups result in low responsiveness).
  In closer accordance with reasons for inventory, i.e., with the physical processes, we expect the volume of research factoring, for instance, a setup versus maintenance and quality tradeoff, into the model, to grow.
  Although certainly not all relevant interactions that occur in practical settings are captured by the current body of models and solutions procedures, substantial progress has been made in modelling relevant (non-classical) components that may influence lotsizing decisions. A sample from literature considering some of these interactions is given in the following table.

model feature	*references (exempli gratia)*
setup times	[Dobson; 1992]
leadtimes, delays, and work in process	[Lee and Zipkin; 1992]
machine maintenance	[Groenevelt, Pintelon, and Seidman; 1992]
quality and learning	[Chand; 1989], [Kroll and Kumar; 1989], [Chand and Sethi; 1990], [Dolinsky, Vollman, and Maggard; 1990], [Porteus; 1990]
product mix & flexibility	[Bahl, Taj, and Corcoran; 1991], [Hum and Sarin; 1991], [De Groote; (This Issue, 1994)]
job scheduling	[Coffman et al.; 1990]

Progress in modeling interactions

The inclusion of more elements and interactions in lotsizing models will make the models more complex to analyze. The complexity will almost invariably make models unamenable to exact, rigorous, analysis. There is no escape. Although rigorous analysis of relatively simple models remains interesting, the potency of lotsizing theory as a field of practical research demands that it copes with, and not abstracts from, the complexities posed by real-life situations. One may expect, therefore, that the body of lotsizing research will grow fastest with building of complex models which will be analyzed heuristically. Next to establishing "good" solutions, an important objective in developing heuristics is the achievement of their robustness and the ease with which their logic (not their computational detail) can be conveyed, for example by use of a decision support system, to decision makers.

Lotsizing research has been around for quite some time. This has established a research tradition that can be both a strength and a weakness. Being around for a long time has enabled the gathering of much practical experience with implementing theoretical results. Lotsizing is susceptible to criticism because promises and expectations can be compared to performance in practice. Thus the limitations of lotsizing can be made clear. Demonstrated limitations can tempt one to abandon lotsizing and to resort to other new and promising approaches. This, however, blatantly ignores the many successful contributions and applications of lotsizing research. Nevertheless, the times do change and, as stated in previous sections, when conditions change, new approaches carry substance. Lotsizing's relevance has not faded. Instead, lotsizing is vital and swiftly adapting to changing circumstances.

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Appendix

In the tables below we present an overview of the lotsizing models most commonly discussed in literature. Given the large number of papers on lotsizing, this overview is far from complete. However, we hope that it serves as a starting point to anyone interested in lotsizing research.

Table. A taxonomy of lotsizing models.

Modeling stationary demand processes, and uncapacitated production resources

problem or model name	planning horizon	demand rate	service policy	objective function	resource constraints	leadtimes	product structure	literature selection
Economic Order Quantity (EOQ)	infinite, continuous	continuous, deterministic	usually no backlogging allowed; backlogging is in some model variants addressed in terms of backlogging costs	minimization of setup costs and inventory holding costs	–	deterministic	single level, single product	Harris (1913) [OM], Hax & Candea (1984) [ES, PE, RA], Silver & Peterson (1985) [ES, PE, RA], Erlenkotter (1989) [RA]
Economic Order Quantity with joint replenishment costs (EOQJR)	infinite, continuous	continuous, deterministic	usually no backlogging allowed; backlogging is in some model variants addressed in terms of backlogging costs	minimization of setup costs, inventory holding costs, and joint replenishment costs	–	deterministic	single level, multiple products	Jackson, Maxwell & Muckstadt (1985) [HS], Akroy & Erenguk (1988) [RA], Federgruen & Zheng (1992) [HS]
Economic Order Quantity in multilevel product structure (EOQML)	infinite, continuous	continuous, deterministic	usually no backlogging allowed; backlogging is in some model variants addressed in terms of backlogging costs	minimization of setup costs and inventory holding costs	–	deterministic	multilevel, multiple products	Schwarz & Schrage (1973) [ES], Moily (1986) [ES], Roundy (1986) [PE, HS, RA], Atkins et al. (1992) [HS, PE]

The classification with respect to the literature is as follows: [CR] = paper on complexity result(s); [CS] = paper containing extensive computational study; [ES] = paper on exact solution procedure(s); [HS] = paper on heuristic solution procedure(s); [OM] = paper containing original model formulation(s); [PE] = paper on problem extension(s); [RA] = review article with references.

Table. A taxonomy of lotsizing models (continued).

problem or model name	planning horizon	demand rate	service policy	objective function	resource constraints	leadtimes	product structure	literature selection
Modeling stationary demand processes, and uncapacitated production resources (continued)								
Statistical inventory control models (Newsboy problems, (s, Q), (s, S), (R, S), (R, s, S), and (s, c, S)-policies)	infinite discrete or continuous	stochastic (stationary)	service defined by service level constraint and/or costs related to imperfect service	minimization of setup costs, inventory holding costs, and costs related to imperfect service	-	deterministic or stochastic	single or multilevel, one or multiple products	Tijms & Groenevelt (1984) [CS, HS], Federgruen, Groenevelt & Tijms (1984) [CS, HS], De Bodt & Graves (1985) [CS, HS], Silver & Peterson (1985) [PE, HS, RA], Porteus (1990) [PE, HS, RA], van Houtem & Zijm (1991) [CS, HS], Zheng & Federgruen (1991) [ES], Badinelli (1992) [HS], Iyer & Schrage (1992) [CS, ES, PE]
Modeling dynamic demand processes, and uncapacitated production resources								
problem or model name	**planning horizon**	**demand rate**	**service policy**	**objective function**	**resource constraints**	**leadtimes**	**product structure**	**literature selection**
Wagner-Whitin (WW)	finite, discrete	deterministic, dynamic	usually no backlogging allowed; backlogging is in some model variants addressed in terms of backlogging costs	minimization of setup costs and inventory holding costs	–	deterministic	single level, single product	Wagner & Whitin (1958) [OM], Aggarwal & Park (1990) [CR, ES, PE], Federgruen & Tzur (1991) [ES, CR], Baker (1990) [HS, RA, CS], Wagelmans, Van Hoesel & Kolen (1992) [CR, ES, PE], Van Hoesel, Wagelmans & Moerman (1994) [CR, CS, ES]
Wagner-Whitin with change-over costs (WWCC)	finite, discrete	deterministic, dynamic	no backlogging allowed	minimization of setup costs and inventory holding costs	–	deterministic	single level, multiple products	Gascon & Leachmann (1988) [ES], Wolsey (1989) [CS, ES, PE]
Wagner-Whitin with joint replenishment costs (WWJR)	finite, discrete	deterministic, dynamic	no backlogging allowed	minimization of setup costs, inventory holding costs, and joint replenishment costs	–	deterministic	single level, multiple products	Silver (1979) [ES], Erenguk & Aksoy (1988) [RA], Joneja (1990) [HS]

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Table. A taxonomy of lotsizing models (continued).

Modeling dynamic demand processes, and uncapacitated production resources (continued)

problem or model name	planning horizon	demand rate	service policy	objective function	resource constraints	leadtimes	product structure	literature selection
Wagner-Whitin with stochastic demands (WWS)	finite, discrete	stochastic, dynamic	service defined in terms of service level constraint	minimization of setup costs and inventory holding costs	–	deterministic	single level, single product	Silver (1978) [HS, CS], Askin (1981) [HS, CS], Bookbinder & Tan (1988) [HS, CS]
Multilevel Lot-sizing Problem (MLLP)	finite, discrete	deterministic, dynamic	usually no back-logging allowed; backlogging is in some model variants addressed in terms of backlogging costs	minimization of setup costs and inventory holding costs	–	deterministic	serial	Love (1971) [ES]
							assembly	Afentakis et al. (1984) [ES, HS], Rosling (1985) [ES, HS], Joneja (1991) [HS], Atkins (1994) [HS]
							general	Afentakis & Gavish (1986) [HS, ES], Arkin et al. (1989) [CR], Pochet & Wolsey (1991) [ES].

Modeling stationary demand processes, and capacitated production resources

problem or model name	planning horizon	demand rate	service policy	objective function	resource constraints	leadtimes	product structure	literature selection
Economic Lot-sizing and Scheduling Problem (ELSP)	infinite, continuous	continuous, deterministic	no backlogging allowed	minimization of setup costs and inventory holding costs	single resource with finite capacity	deterministic	single level, multiple products	Elmaghraby (1978) [CS, HS, RA], Hsu (1983) [CR], Anderson (1990) [CR, PE], Dobson (1987) [HS], Inman & Jones (1989) [HS], Roundy (1989) [HS], Carreno (1990) [HS, PE], Zipkin (1991) [HS], Glass (1992) [HS], Dobson (1992) [HS, PE]
					multiple resources with finite capacity		serial	Hsu & El-Najdawi (1990) [HS], El-Najdawi & Kleindorfer (1991) [HS], Dobson & Yano (1992) [HS], El-Najdawi (1994) [HS]

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Table. A taxonomy of lotsizing models (continued).

Modeling stationary demand processes, and capacitated production resources (continued)

problem or model name	planning horizon	demand rate	service policy	objective function	resource constraints	leadtimes	product structure	literature selection
Economic Lotsizing and Scheduling Problem (ELSP) (cont.)	infinite, continuous	continuous, deterministic	no backlogging allowed	minimization of setup costs and inventory holding costs	multiple resources with finite capacity	deterministic	general	Roundy (1989) [HS]
Stochastic Economic Lotsizing and Scheduling Problem (SELSP)	infinite, continuous	stochastic, dynamic	service defined in terms of backlogging costs	minimization of setup costs, inventory holding costs, and costs related to imperfect service	single resource with finite capacity	deterministic	single level, multiple products	Vergin & Lee (1978) [HS], Graves (1980) [ES], Leachman & Gascon (1988) [HS], Leachman et al. (1991) [HS]
Queuing type of models	infinite, continuous	stochastic, stationary	service defined in terms of leadtimes	minimization of setup costs and inventory holding costs *or* minimization of queueing delays	single resource with finite capacity	stochastic	single level, one or multiple products	Jönsson & Silver (1985) [CS, HS], Zipkin (1986) [PE, HS], Karmarkar (1987) [CS, PE, HS], Karmarkar et al. (1992) [CS, PE, HS]

Modeling dynamic demand processes, and capacitated production resources

problem or model name	planning horizon	demand rate	service policy	objective function	resource constraints	leadtimes	product structure	literature selection
Capacitated Lot Sizing Problem (CLSP)	finite, discrete (big time buckets)	deterministic, dynamic	usually no backlogging allowed; backlogging is in some model variants addressed in terms of backlogging costs	minimization of setup costs and inventory holding costs	one or multiple resources with finite capacity	deterministic	single level, one or multiple products	Bitran & Yanasse (1982) [CR], Eppen & Martin (1987) [ES, CS], Maes & Van Wassenhove (1988) [RA, HS, CS], Pochet (1988) [ES, CS], Leung et al. (1989) [ES, CS], Trigeiro et al. (1989) [CS, HS, PE], Dixon & Poh (1990) [HS, CS], Campbell & Mabert (1991) [CS, HS, PE], Kirca & Kökten (1994) [CS, HS], Chung et al. (1994) [ES]

The classification with respect to the literature is as follows: [CR] = paper on complexity result(s); [CS] = paper containing extensive computational study; [ES] = paper on exact solution procedure(s); [HS] = paper on heuristic solution procedure(s); [OM] = paper containing original model formulation(s); [PE] = paper on problem extension(s); [RA] = review article with references.

Table. A taxonomy of lotsizing models (continued).

Modeling dynamic demand processes, and capacitated production resources (continued)

problem or model name	planning horizon	demand rate	service policy	objective function	resource constraints	leadtimes	product structure	literature selection
Multilevel Capacitated Lotsizing Problem (MLCLSP)	finite, discrete (big time buckets)	deterministic, dynamic	usually no backlogging allowed; backlogging is in some model variants addressed in terms of backlogging costs	minimization of setup costs and inventory holding costs	one or multiple resources with finite capacity	deterministic	general	Billington et al. (1983) [OM], Billington et al. (1986) [HS, CS], Maes et al. (1991) [CS, CR, HS], Roll & Karni (1991) [CS, HS], Tempelmeier & Helber (1994) [CS, HS], Kuik et al. (1993) [CS, HS]
Continuous Setup Lotsizing Problem (CSLP)	finite, discrete (small time buckets)	deterministic, dynamic	usually no backlogging allowed; backlogging is in some model variants addressed in terms of backlogging costs	minimization of setup costs and inventory holding costs	single resource with finite capacity (at most one product can be produced per planning period)	deterministic	single level, one or multiple products	Karmarkar & Schrage (1985) [CS, HS], Karmarkar, Kekre & Kekre (1987) [ES], De Matta & Guignard (1989) [CS, HS, PE], Pochet & Wolsey (1991) [CS, ES]
Discrete Lotsizing and Scheduling Problem (DLSP)	finite, discrete (small time buckets)	deterministic, dynamic	no backlogging allowed	minimization of setup costs and inventory holding costs	single resource with finite capacity (at most one product can be produced per planning period; production per period is either zero or equal to available capacity (all or nothing production))	deterministic	single level, one or multiple products	Fleischmann (1990) [CS, ES], Magnanti & Vachani (1990) [CS, ES], Salomon et al. (1991) [CR, PE, RA], Fleischmann (1994) [CS, HS, PE], Cattrysse et al. (1992) [CS, HS, PE], Kuik et al. (1992) [ES], van Hoesel & Kolen (1994) [ES]

The classification with respect to the literature is as follows: [CR] = paper on complexity result(s); [CS] = paper containing extensive computational study; [ES] = paper on exact solution procedure(s); [HS] = paper on heuristic solution procedure(s); [OM] = paper containing original model formulation(s); [PE] = paper on problem extension(s); [RA] = review article with references.

Table. A taxonomy of lotsizing models (continued).

Modeling dynamic demand processes, and capacitated production resources (continued).								
problem or model name	**planning horizon**	**demand rate**	**service policy**	**objective function**	**resource constraints**	**leadtimes**	**product structure**	**literature selection**
Batching type of models	finite, continuous	deterministic, dynamic (in terms of jobs with release and due dates)	service defined in terms of lateness, tardiness, or number of late jobs	minimize maximum lateness or tardiness, or minimize the number of late jobs	single or multiple resources with finite capacity	deterministic	single or multilevel with multiple job classes (products)	Bruno & Downey (1978) [CR, ES], Monma & Potts (1989) [CR], Potts & Van Wassenhove (1992) [CR, PE, RA].
Unbatching (lot streaming) type of models	finite, continuous	deterministic, dynamic	service defined in terms of makespan	minimize makespan	multiple resources with finite capacity	deterministic	serial	Baker & Pyke (1990) [ES, HS, OM], Glass, Gupta & Potts (1994) [CR, ES]

The classification with respect to the literature is as follows: [CR] = paper on complexity result(s); [CS] = paper containing extensive computational study; [ES] = paper on exact solution procedure(s); [HS] = paper on heuristic solution procedure(s); [OM] = paper containing original model formulation(s); [PE] = paper on problem extension(s); [RA] = review article with references.