WHY DOES BETA SHIFT WHEN THE LENGTH OF SECURITIES RETURNS VARIES?

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Here is a simple model that explains shifts in estimated beta in response to a change in the length of time interval over which securities' returns are measured. This model will also tell you if the shift is expected to be upward or downward.

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Modern Portfolio Theory (MPT) has produced several useful concepts which are today employed by portfolio managers and other practitioners in the field of finance. Central among these concepts is the systematic risk of a security, or its beta coefficient. In order to estimate a security's beta , one must use historical rates of return. MPT, however, does not tell us if these rates of return should be measured over daily, weekly, monthly or any other length of time. Several researchers have shown that it is not a trivial issue[1]. A monthly beta may be larger or smaller than a daily beta although both betas were estimated over the same fixed calendar period. This phenomemon is illustrated in Exhibit 1. The fixed estimation period is of 4 years (from January 1970 to December 1973). It can be broken down into either 1009 daily returns or 50 monthly returns, which in turn are used to estimate beta. In the case of Wayne Gossard (first firm on the list), daily beta is .459 and monthly beta is .976, an increase of 112%. In the case of Eastman Kodak (last firm on the list), daily beta is 1.251 and monthly beta is .932, a decrease of 34%.

In this article, I present a simple model which explains why estimates of beta may vary in response to a change in the length of the time interval over which securities' returns are measured. This model also predicts the direction and the strength of these variations in estimated betas. Specifically, I will show that securities that are "thinner" than the market average - those with a smaller market value of shares outstanding (MVSO) than the average of all securities outstanding - will generally have a rising beta, whereas securities that are "thicker"[2] than the market average - those with a larger MVSO than the average of all securities outstanding - will generally have a falling beta. In the concluding section, I examine the implications of this phenomenon for some practical aspects of MPT such as portfolio evaluation and the estimation of the Security Market Line and the market price of risk reduction.

What causes beta to shift when the return interval varies?

The major factor responsible for the observed shifts in estimated betas is the existence of intertemporal (non-contemporaneous) relationships between the daily returns of securities and those of the general market movement: securities' daily prices do not move in unison, some stocks may lag behind the general market movement and others may lead it. Evidence of this phenomenon is presented in Exhibit 2. The first column gives the contemporaneous correlation coefficient (ρ_{im}) between securities' daily returns and the daily returns of the S&P-500 which I used as the market index m. This is a measure of the simultaneous association between a security's daily price movements and those of the S&P-500. The second column gives the intertemporal correlation coefficient with a lag of one day (ρ^{-1}_{im}) which measures the strength of the association between today's S&P-500 returns and those of a security one day earlier, that is, the security's daily price movements lag behind those of the S&P-500 in this case. The third column gives the intertemporal correlation coefficient with a lead of one day (ρ^{+1}_{im}). Intertemporal correlation coefficients of order higher than the first - not reported in Exhibit 2 - are usually statistically insignificant[3]. The last column gives the ratio of the sum of the two intertemporal correlation coefficients to the contemporaneous correlation coefficient. It is a measure of the strength of intertemporal cross correlation relative to contemporaneous correlation. I call this ratio the q-ratio of a given security. We will see that this ratio will explain the described shifts in beta. As examples, the q-ratio of Wayne Gossard (WG) and that of Eastman Kodak (EK) are, respectively:

$$q_{WG} = \frac{\rho^{-1}_{WG,m} + \rho^{+1}_{WG,m}}{\rho_{WG,m}} = \frac{.106 + .060}{.143} = 1.161$$

$$q_{EK} = \frac{\rho^{-1}_{EK,m} + \rho^{+1}_{EK,m}}{\rho_{EK,m}} = \frac{.094 + .189}{.626} = .452$$

It is also useful to recognize that the S&P-500 has a measurable q-ratio:

$$q_m = \frac{\rho^{-1}_{mm} \quad \rho^{+1}_{mm}}{\rho_{mm}} = \frac{2\rho^{-1}_{m}}{1} = 2(.285) = .570$$

where $\rho^{-1}_{mm} = \rho^{+1}_{mm}$ = the autocorrelation coefficient with a lag of one day in the S&P-500 whose value was found to equal .285 over the period January 1970 - December 1973. Since the contemporaneous price movements of the S&P-500 are perfectly positively correlated with themselves, $\rho_{mm} = 1$ and hence the q-ratio of the S&P-500 is equal to twice its autocorrelation coefficient.

I will now show that the presence of these daily intertemporal correlations, whose relative strengths are measured by the q-ratio, causes beta to shift in response to changes in the length of the return interval. The beta coefficient of security i (β_i), estimated over time intervals of T days, is defined as the ratio of the T-day covariance ($\sigma_{im}(T)$) between the returns of i and those of the market index m to the T-day variance ($\sigma_m^2(T)$) of the returns of the market index:

$$\beta_i(T) = \frac{\sigma_{im}(T)}{\sigma_m^2(T)} \qquad (1)$$

I have shown elsewhere[4] that $\sigma_{im}(T)$ can be expressed as a function of the daily covariance $\sigma_{im}(1)$, the length T of the return interval in days, and the security's q-ratio (q_{im}) according to :

$$\sigma_{im}(T) = \sigma_{im}(1)[T + (T-1)\, q_{im}] \qquad (2)$$

An analogous expression can be derived to relate the T-day variance $\sigma_m^2(T)$, to the daily variance, $\sigma_m^2(1)$, as well as T and q_m such as:

$$\sigma_m^2(T) = \sigma_m^2(1)\ [T + (T-1)\, q_m] \qquad (3)$$

where $q_m = 2\rho_m^{-1}$, that is, twice the first order daily autocorrelation coefficient.

Substituting (2) and (3) in (1) gives:

$$\beta_i(T) = \frac{\sigma_{im}(1)\ [T + (T-1)q_{im}]}{\sigma_m{}^2(1)\ [T + (T-1)q_m]} = \beta_i(1)\ \frac{T + (T-1)q_{im}}{T + (T-1)q_m} \qquad (4)$$

Equation (4) clearly shows how intertemporal cross correlations affect beta as T varies. First, consider the case for which $q_{im} = q_m = 0$, then $\beta_i(T) = \beta_i(1)$ regardless of the value of T. In this highly unlikely situation, beta will be invariant to the length of the return interval. There is another case for which beta is invariant. This is when $q_{im} = q_m \neq 0$. A security's q-ratio is non-zero and equals that of the market index, a situation that may possibly arise. In general, q_{im} will be different from q_m causing $\beta_i(T)$ to be different from $\beta_i(1)$ when T varies.

Predicting the direction and the strength in a shift in beta

Which security will have a beta that rises, remains constant, or falls when the return interval is changed? How strong will this variation in beta be? To answer these questions we can take the derivative of (3) with respect to T. It will measure the response of $\beta_i(T)$ to a small change in T. We have:

$$\frac{d\beta_i(T)}{dT} = \frac{\beta_i(1)\ [q_{im} - q_m]}{[T + (T-1)\ q_m]^2}$$

For beta to rise, the derivative should be positive. This will occur whenever q_{im} is <u>larger</u> than q_m. For beta to fall, the derivative should be negative. This will occur whenever q_{im} is smaller than q_m. Furthermore, beta will rise faster the larger is q_{im} relative to q_m and it will fall faster the smaller q_{im} is relative to q_m.

To illustrate, consider Wayne Gossard and Eastman Kodak. The former has a q-ratio of 1.161, far in excess of that of the market index (.570), therefore its beta should rise with the return interval at a faster rate than for other securities in the sample for which the q-ratio exceeds .570 but is smaller than 1.161. This is indeed what we observe (see Exhibit 1). Eastman Kodak has a q-ratio of .452, below that of the market index, therefore its beta should fall as the return interval rises. This is again what we observe (see Exhibit 1). Take Aluminium Co. of America. Its q-ratio equals .569 (see Exhibit 1) which is approximately the value of the q-ratio of the market index. Therefore we should expect its beta to remain constant as the return interval varies. This is indeed the case as shown in Exhibit 1.

Is there a faster way to tell if beta will shift upward or downward? In other words, is it possible to predict the direction of the shift in beta without the knowledge of its q-ratio? The answer is yes. I have shown elsewhere[5] that a security's q-ratio is inversely related to that security's market value of shares outstanding (MVSO). This means that securities with large MVSO (relative to the market average) will have small q-ratios in comparison to the market. Those with small MVSO (relative to the market average) will have high q-ratios in comparison to the market. Therefore, we may use MVSO (a security's relative market thinness) to determine the direction of the shift in beta. Eastman Kodak with a large MVSO (relatively small q-ratio) will have a falling beta and Wayne Gossard with a small MVSO (relatively high q-ratio) will have a rising beta. Note also that in Exhibit 2, companies are listed in increasing order of MVSO. One can see that q-ratios have indeed a tendency to fall as MVSO rises.

Concluding remarks

The implications for portfolio evaluation of the shifts in beta should be obvious. Consider the Treynor (1965) index. It is the ratio of a portfolio's excess return to the beta of that portfolio. Since beta is sensitive to the return interval used, so will the index. Furthermore, since the direction of the shifts in betas are usually not the same, the ranking of portfolios according to the Treynor index may be altered for different length of the return interval.

Another implication concerns the estimation of the Security Market Lines of Sharpe (1964) and Lintner (1965). From an estimated SML one gets the value of the market price of risk reduction (MPR), that is, a measure of the reward offered by the market to offset an increase in risk by one more unit. I have shown elsewhere[6] that if one obtains a monthly estimate of MPR, then it will usually be incorrect to convert this number on an annual basis. One cannot get an annual estimate of the MPR by simply multiplying the monthly estimate by 12!

EXHIBIT 1

Estimated Betas over Return Intervals Varying from a Day to a Month: January 1970 - December 1973

	Daily Beta	Weekly Beta	Biweekly Beta	Triweekly Beta	Monthly Beta
Wayne Gossard	.459	.654	.986	.692	.976
Mich. Seamless Tube	.433	.784	.917	.883	.973
Publicker Inds.	1.006	1.277	1.513	1.491	1.521
Great West. United	1.442	1.911	2.122	2.311	2.496
Family Finance	.795	.821	1.212	1.324	1.268
Bobbie Brooks	1.405	1.592	1.818	1.889	1.874
Monogram Ind.	2.144	2.403	2.844	2.887	2.950
Faberge	1.449	1.416	1.511	1.511	1.882
Dillingham Corp.	.725	.750	.990	1.164	1.004
Vornado	1.765	1.823	2.170	1.628	2.329
Big Three Ind.	.712	.969	1.283	.970	1.339
Cabot Corp.	.756	.805	.844	.898	.752
General Development	1.358	1.382	1.657	1.628	1.423
Addresso-Multigraph	1.733	1.414	1.566	2.341	2.094
Great West. Finance	1.917	2.158	2.043	1.820	2.246
Colgate Palmolive	.850	.958	1.011	1.002	1.131
Aluminium Co. of America	1.118	1.150	1.118	1.221	1.115
Shell Oil	.742	.860	.827	1.093	.930
Kresge, S.S.	1.237	1.308	1.299	1.326	1.190
Eastman Kodak	1.251	1.166	.958	.859	.932

Notes: (1) Returns are measured as the logarithm of investment relatives.

(2) Market returns are those of the S&P-500.

(3) All betas are statistically significant at the 5% level.

EXHIBIT 2

Daily Correlation Coefficients and q-ratios

January 1970 - December 1973

Name	ρ_{im}	ρ_{im}^{-1}	ρ_{im}^{+1}	$q_{im} = \frac{\rho_{im}^{-1} + \rho_{im}^{+1}}{\rho_{im}}$
Wayne Gossard	.143	.106	.060*	1.161
Mich Seamless Tube	.165	.177	.044*	1.321
Publicker Inds.	.281	.150	.108	.981
Great West. United	.233	.131	.096	.974
Family Finance	.248	.200	.046*	.992
Bobbie Brooks	.339	.108	.100	.614
Monogram Ind.	.459	.119	.166	.621
Faberge	.337	.087	.134	.664
Dillingham Corp.	.192	.119	.047*	.865
Vornado	.391	.225	.124	.893
Big Three Ind.	.355	.219	.090	.870
Cabot Corp.	.324	.175	.037	.654
General Development	.328	.101	.132	.710
Addresso-Multigraph	.404	.074	.168	.599
Great West. Finance	.546	.176	.180	.652
Colgate Palmolive	.334	.164	.047*	.632
Aluminium Co. of America	.437	.148	.112	.569
Shell Oil	.394	.198	.110	.782
Kresge, S.S.	.502	.137	.168	.608
Eastman Kodak	.626	.094	.189	.452
S&P-500	1.000	.285	.785	.570

Asterisks indicate statistically insignificant correlation coefficients at the 5% level.

FOOTNOTES

1. See [1], [2], [5], [6], [8] and [10].

2. This may not be the best word to describe securities with large MVSO. Its advantage is its opposition to the word "thinness".

3. See [3].

4. See the appendix in [2].

5. See [3].

6. See [4].

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