INVESTMENT HORIZON, DIVERSIFICATION, AND THE EFFICIENCY OF ALTERNATIVE BETA FORECASTS

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I Introduction

When we estimate the systematic risk of a security we normally use the observed beta from the characteristic line as the best estimator of true beta. In fact, this is only one of many different predictors that can be constructed. The standard construction involves adjusting the observed beta through the Bayes-Vasicek [17], Blume [1, 2], or Merrill Lynch [10] methods. Recently, Lavely, Wakefield, and Barrett (L-W-B) [11] have used the James-Stein [4] method to adjust betas.

In this study we examine the joint effect of changes in (i) the investment horizon (holding period or return interval), (ii) the size of portfolio (number of securities in a portfolio), and (iii) the forecasting method on the beta forecasts of securities and portfolios.

Klemkosky and Martin (K-M) [10] investigate the sources of forecast errors of beta coefficients extrapolated with three alternative adjustment techniques. They consider the methods developed by Blume [1, 2] and Vasicek [17], and the method used by Merrill Lynch (MLPFS). Using the monthly returns of about 800 securities, grouped in portfolios of one to ten securities, they conclude, "the accuracy of a simple no-change extrapolative beta forecast can be improved. A combination of the Bayesian (Vasicek) predictor and a reasonable portfolio size would appear to make the beta coefficient a highly predictable risk surrogate." Eubank and Zumwalt (E-Z) [7] confirm K-M's results over estimation periods of 12 to 120 months.

Elgers, Haltiner, and Hawthorne [5], and Elton, Gruber, and Urich [6] confirm the general results with updated data. In this paper we broaden the investigation through four avenues.

First, we re-examine K-M's and E-Z's conclusions for investment horizons shorter than a month. We use monthly, biweekly, weekly and daily returns. This is motivated by the increasing use of and interest in short-term betas[1]. The associated problems and advantages of short-term betas are noted in Hawawini [8], and Scholes and Williams [14]. We also broaden the universe of securities to 1115 common stocks listed continuously from 1962 to 1976 on either the NYSE or the AMEX. Previous studies restricted the analysis to NYSE issues.

Second, K-M and E-Z do not investigate in detail the effect of diversification of beta forecasts since their largest portfolios contain only ten securities. One does not know if larger, and hence more diversified, portfolios can further improve the predictability of the beta coefficient. We analyze that predictability by considering portfolios of up to forty securities, the size at which the unsystematic risk almost disappears.

Third, we examine the effect of changes in the length of the investment horizon on the predictive ability of various forecasting techniques. The question addressed is: Does the ability to forecast beta improve or deteriorate in any systematic fashion as the length of the investment horizon is shortened from a month to a day?

Last, we test a new adjustment procedure based on the James-Stein estimator (Effron and Morris [4], Thomson [15], and Zellner and Vandaele [18]). This estimator minimizes the quadratic loss function defined cross-sectionally over all the securities' betas and *not* just over an individual beta as in the case of the classical statistics. We examine the properties of this new estimator and its relationship with the other estimators (Judge and Bock [9]).

The remainder of this paper is organized as follows. The next section describes the data and discusses the methodology. In section III we examine the comparative behavior of the alternative estimators and in section IV we evaluate empirically the predictive ability of the alternative beta forecasts. The last section contains concluding remarks.

II Data and Methodology

Data

The data are from the Center for Research in Security Prices. The sample consists of 1115 securities - 828 from the NYSE and 287 from the AMEX - traded continuously from 3 July 1962 to 12 November 1976. Thus, each security has 3600 observations of daily returns. The proxy for the market portfolio is a value-weighted index of 3950 securities appearing on the CRSP tape. The data used in this study cover a wider range of securities compared to those employed by Blume [1, 2, 3], Elgers, Haltiner, and Hawthorne [5], Eubank and Zumwalt [7], and Klemkosky and Martin [10].

Beta Estimates for Securities and Portfolios

The 3600 daily returns are divided into 6 non-overlapping subperiods of 600 observations each (approximately 30 months). Within each subperiod we use investment horizons (H) of 1 day (H=1, 600 obs.), 1 week (H=5, 120 obs.), 2 weeks (H=10, 60 obs.), and 1 month (H=20, 30 obs.). For each subperiod, betas for securities (and portfolios) are estimated using the market model:

$$R_{jt}(H) = \alpha_j(H) + \beta_j(H)R_{mt}(H) + e_{jt}(H) \qquad (1)$$

$H = 1, 5, 10,$ or 20 days, and $t = 1, 2, \ldots, 600/H$

where

$R_{jt}(H)$ = the return of security j (or portfolio j) for investment horizons of H trading days,

$R_{mt}(H)$ = the corresponding H-day returns of the market index,

$\alpha_j(H)$ = the H-day intercept term,

$\beta_j(H)$ = the H-day systematic risk, and

$e_{jt}(H)$ = the residual term.

All the usual assumptions of the market model are taken to hold and the beta coefficient is given by[2]:

$$\hat{\beta}_j(H) = \frac{\text{cov}[R_j(H), R_m(H)]}{\text{var}[R_m(H)]}$$

The portfolios needed for the analysis are formed as follows. Over each of the first five subperiods for each of the four investment horizons (H) estimated betas are ordered in a descending order of magnitude. Portfolios of size N are obtained by assigning the first of N securities to the first portfolio of the next subperiod, the second group of N securities to the second portfolio of the next subperiod,and so on until all securities are assigned. (N = 1, 3, 5, 10, 20, 30, and 40 securities). Portfolio returns are then computed assuming an equal investment in all the securities within those portfolios. This produces portfolios for five subperiods - the second to the sixth - with new allocation every subperiod[3]. The beta coefficients are estimated for the portfolios using equation (1).

Adjustment Techniques

As we have more than one estimator, a decision rule based on the "goodness" of the estimators is necessary for selecting the "best" estimator. According to Tiao and Box [16], "one early attempt (arbitrarily) limited the estimators to be (a) *linear* functions of data which were (b) unbiased. Among these, the estimator which had (c) *minimum variance* was selected." (p. 622)

The decision rule is useful if we have only one security. But if we have three or more betas to be estimated, and if we have a quadratic loss function, the classical sampling theory estimator is no longer admissable. That is, the classical estimator does not minimize loss and there exist Bayes and James-Stein estimators that are better than the classical estimators (Effron and Morris [4], Zellner and Vandaele [18]). These estimators are *not* linear, *not* unbiased, and do not have, individually, minimum variance.

The standard adjustment techniques we employ are the Bayes-Vasicek method, the Blume method, and the MLPFS method. Consider the following notations:

b_e = beta estimated according to equation (1)

b_p = predicted beta

s_t = standard error of estimation of beta from time-series analysis based on equation (1)

$\bar{b}_e$ = cross-sectional average for the estimated betas of portfolios

s_c = cross-sectional standard deviation for the estimated betas of portfolios

1, 2, 3 = *any* three consecutive subperiods from the six subperiods under examination.

Based on this notation, the Bayes-Vasicek estimator is given by:

$$b_{3p} = \frac{(b_{2e}/s_{2t}^2) + (\bar{b}_{2e}/s_{2c}^2)}{(1/s_{2t}^2) + (1/s_{2c}^2)} \qquad (2)$$

The Blume estimator is given by:

$$b_{3p} = \hat{d} + \hat{d}' b_{2e} \qquad (3)$$

where $\hat{d}$ and $\hat{d}'$ are derived from the regression $b_{2e} = d + d'_{1e}$ + error.

The MLPFS estimator is given by:

$$b_{3p} = 1 + k(b_{2e} - 1) \qquad (4)$$

where k is common to all securities (or portfolios) and is equal to the product moment correlation between b_{1e} and b_{2e}. Note that while 2 subperiods are required to obtain the Blume and MLPFS estimators, only one subperiod is necessary for the Bayes-Vasicek estimator.

If the beta estimates are not adjusted, then next period betas are predicted by betas estimated over the preceding period:

$$b_{3p} = b_{2e} \qquad (5)$$

i.e., b_{2e} is the classical or unadjusted estimator of b_3.

We now introduce a new estimator, the James Stein estimator[4,5]. It is given by:

$$b_{3p} = \bar{b}_{2e} + h(b_{2e} - \bar{b}_{2e}) \qquad (6)$$

where $h = 1 - \dfrac{(n-3)s^2_{2t}}{(n-1)s^2_{2c}}$

and n = the number of cross-sectional observations (number of individual securities or portfolios).

III Comparative Behavior of Alternative Estimators

The classical, the Bayes-Vasicek, and the James-Stein estimators

These three estimators can be expressed in the form:

$$b_{3p} = b_{2e} + \lambda(b_{2e} - b_{2e}) \qquad (7)$$

where λ depends on the particular estimator under consideration. For $\lambda = 1$, equation (7) reduces to $b_{3p} = b_{2e}$, the case of classical unadjusted estimator. For $\lambda = 1 - (s^2_{2t}/s^2_{2c})$ and n sufficiently large, equation (7) gives the James-Stein estimator. For $\lambda = 1 + \; 1 + (s^2_{2t}/s^2_{2c})^{-1}$, equation (7) collapses to the Bayes-Vasicek estimator.

Suppose that the ratio (s^2_{2t}/s^2_{2c}) is small relative to one. In this case:

$$\frac{1}{1 + (s^2_{2t}/s^2_{2c})} \simeq 1 - (s^2_{2t}/s^2_{2c}) \leq 1$$

and the Bayes-Vasicek and James-Stein estimators will be approximately equal. If the ratio for a particular security is close to zero, implying a high reliability of the unadjusted beta, both the estimators will be approximately equal to the classical unadjusted estimator. This is what we find later in our empirical analysis. The ratios (s^2_t/s^2_c) are calculated for 1115 securities, for varying lengths of the investment horizon (daily, weekly, biweekly, and monthly), and for different portfolio sizes (N = 1, 2, 3, 5, 10, 20, 30, and 40) over the different subperiods. These ratios are averaged across securities and subperiods and are presented in Table 1.

We see from Table 1 that the average value of the ratios is inversely related to portfolio size and directly related to horizon length. If a large portfolio is coupled with a very short investment horizon, the beta forecast adjusted with either the Bayes-Vasicek method or the James-Stein method will not differ significantly from the classical unadjusted forecast.

The Blume and MLPFS Estimators

Referring to equation (7), we see that for $\lambda = k = cor(b_{1e}, b_{2e})$ and $\bar{b}_{2e} = 1$, we get the MLPFS estimator (4). The Blume estimator (3) can be rewritten:

$$b_{3p} = \bar{b}_{2e} + k(s_{2c}/s_{1c})(b_{2e} - \bar{b}_{1e})$$

In this case, $\lambda = k(s_{2c}/s_{1c})$, but unless $\bar{b}_{2e} = \bar{b}_{1e}$ we cannot express the Blume estimator as a special case of equation (7). The MLPFS estimator and the Blume estimator will be identical only if the mean and the variance of the cross-sectional sample of estimated betas are the same[6].

IV Predictive Ability of Alternative Beta Forecasts

Evaluation techniques

The predictive ability of alternative beta forecasts is first measured by the product moment correlation coefficient between betas of a given subperiod and their predicted values based on the immediately preceding subperiod betas. The six subperiods available produced five estimated correlation coefficients in the case of the classical (unadjusted), the bayes-Vasicek, and the James-Stein estimators and only four correlation coefficients in the case of the Blume and MLPFS estimators. The correlation coefficients were then averaged over four common subperiods to yield a single value of the correlation coefficient for each type of estimator analyzed[7]. The method was repeated for varying lengths of the investment horizon H(H = day, week, 2-weeks, month) and varying portfolio size N(N = 1, 3, 5, 10, 20, 30, and 40).

An alternative and more general method of evaluating the predictive ability of beta forecasts is to compute the mean square error (MSE) between predicted (b_p) and estimated (b_e) beta coefficients and to examine its components[8]. In this case we have:

$$MSE = \frac{1}{n}\sum_{j=1}^{n}(b_e - b_p)^2$$

$$MSE = (\bar{b}_e - \bar{b}_p)^2 + (1 - \delta_{ep})[s_c^2]_p + (1 - k_{ep}^2)[s_c^2]_e \qquad (8)$$

where δ_{ep} is the slope coefficient of the regression of b_e on b_p, $[s_c^2]_p$ and $[s_c^2]_e$ are the cross sectional sample variance of b_p and b_e, respectively; and k_{ep} is the correlation coefficient between b_e and b_p.

The first term in equation (8) is the bias component which indicates the portion of the MSE due to over- or underestimation of the mean of estimated betas ($\bar{b}_e$) over the mean of predicted betas ($\bar{b}_p$). The second term is the inefficiency component which captures the tendency of the forecast errors to be positive for low values of b_p and negative for high values of b_p. The last term represents the random component of the MSE. Note that a perfect correlation between b_e and b_p (k_{ep} = 1) will reduce the random component to zero but will not yield a zero MSE. The MSE will be zero only if the predicted values of betas (b_p) are identical to their estimated values (b_e). As in the case of the correlation coefficient, the six subperiods available produced five estimates of the MSE for the classical (unadjusted), the Bayes-Vasicek, and the James Stein estimators and only four estimates of the MSE for the Blume and the MLPFS estimators. These MSE were then averaged over four common subperiods to yield a single value of the MSE for each type of estimator analyzed.

Correlation Coefficients and MSE-Random[9]

It can be observed from the third term of equation (8) that the correlation between b_e and b_p, and the MSE-random will be inversely related. If the correlation equals unity, then the MSE-random will equal zero. Tables 2(a)-2(c) present the average values for these two variables[10]. The reported values are for four investment horizons: daily, weekly, biweekly, and monthly; and for seven portfolio sizes: 1, 3, 5, 10, 20, 30, and 40 securities. (To aid the interpretation of data, we plot these correlations in Figure 1).

We observe that: first, for a given investment horizon and portfolio size, all five estimators have statistically the same correlations and MSE-random[11,12]. For their limited tests, K-M [10] and E-Z [7] present results indicating the same behavior. *This implies that none of the estimators can be considered superior to the other four in predicting the unanticipated shift in next period's beta. Also, none of the estimators can be considered superior in predicting the risk-ordering of securities.* This result is not surprising for the Blume and MLPFS methods as these constructions preserve the risk-order generated by the unadjusted betas.

Second, *for each of the estimators at a given investment horizon, the correlation increases (and MSE-random decreases) as the portfolio size increases.* This increase is numerically and statistically significant as the portfolio size increases from 1 to 20 securities. For the portfolios of 30 and 40 securities this increase is statistically significant but numerically very small. The largest portfolio examined by K-M [10] and E-Z [7] was with 10 securities - with similar results for MSE-random[13].

This increase in the correlation (decrease in the MSE-random) is smallest for the daily returns and largest for the monthly returns. It should be noted that the correlation (MSE-random) for the monthly return betas is the lowest (highest) to begin with.

Last, *for each of the estimators at a given portfolio size, the correlation increases (and at an "average" portfolio size the MSE-random decreases) as the investment horizon decreases from a month to a day*[14]. This increase in the correlation (decrease in the MSE-random) is numerically and statistically significant as the investment horizon decreases from a month to a week, but then the increase is small and statistically insignificant. This increase in the correlation (decrease in the MSE-random) is largest for the 1-security portfolios and smallest for the 40-security portfolios. We should note that the correlation (MSE-random) for the 40-security portfolios is the highest (lowest) to begin with.

MSE-Bias

In the preceding subsection, we saw that the MSE-random is "well-behaved" with respect to the portfolio size and the investment horizon. In this and the next subsection we see that the MSE-bias and MSE-inefficiency are "ill-behaved" and that permits only a few weak generalizations.

First, *for a given investment horizon and portfolio size, the unadjusted, James-Stein, Bayes-Vasicek, and MLPFS betas have statistically same MSE-bias.* On average, at every comparison Blume's beta has significantly higher MSE-bias; though this is not the case for one of the four subperiods examined. K-M's [10] results show that the MSE-bias of Blume's method is generally higher than the values for unadjsuted betas and Bayes-Vasicek method, but lower than MLPFS method in two out of three periods. E-Z's [7] results show that the MSE-bias of Blume's method is generally lower than that for the unadjusted betas.

As discussed earlier, Bayes-Vasicek [2] and James-Stein [6] estimators shrink the unadjusted betas towards the grand mean, and MLPFS [4] estimators shrink them towards one. But Blume's [3] method shrinks them around the grand mean augmented by the difference of grand means of the two preceding periods. If the grand mean of the sample shifts through time, the MSE-bias for Blume's method will be the worst[15]. The results presented in this paper confirm, rigorously and in detail, E-G-U's [6] conclusion that the Blume method uses past relationships of betas to predict future betas and in the process introduces biases and inefficiencies that otherwise did not exist.

Second, *for each of the estimators at a given investment horizon, the MSE-bias is not influenced by the size of the portfolio.* This is an expected result because the grand mean of the sample betas does not depend on how many portfolios the sample securities are distributed.

Last, *for each of the estimators, on average, the MSE-bias is significantly low for the monthly betas.* This implies that the grand mean of betas based on monthly returns has more interperiod stationarity. For submonthly returns, the autocorrelations of the market index and the intertemporal cross-correlations (between the securities and the market index) are highly significant[16]. These correlations bias the estimated betas, and this bias is not constant through time. However, these correlations are not significant in monthly returns. Hence, we obtain unbiased estimates of monthly betas and consequently the lowest MSE-bias.

MSE-Inefficiency

The MSE-inefficiency represents the second component of equation (8) and captures the tendency of forecast errors - after adjusting for the bias of changing grand mean - to be of the same sign for betas less than one, and of the opposite sign for betas greater than one. For this component of the MSE we observe the following.

First, *for a given investment horizon and portfolio size, on average, the MSE-inefficiency is the lowest for MLPFS method and the highest for Blume's method.* James-Stein and Bayes-Vasicek estimators have statistically same MSE-inefficiency which is lower than that of the unadjusted betas. This result is in contrast to the results of K-M [10] and E-Z [7]. K-M found that the MLPFS and Blume estimators produced similar MSE-inefficiency and it was lower than the corresponding value for the unadjusted betas. E-Z found that the values for the Blume estimators were lower than those for the unadjusted betas.

Second, *for each of the estimators - except Blume's - at a given investment horizon, the MSE-inefficiency decreases as the portfolio size is increased from 1 to 20 securities.* For portfolios of 30 to 40 securities the values are similar to that of 20-security portfolios. This result is expected as the betas of the larger portfolios are more stationary. K-M found that "the inefficiency component was unchanged in moving from single to ten security portfolios", a result contrary to ours. But, we form the portfolios first and then derive the beta to make an adjustment, while K-M explain the behavior of their result as, "This can be explained by the fact that the adjustments were made in single security betas before aggregating them into portfolios" [10, p 1128].

Last, *for each of the estimators, on average, the MSE-inefficiency was lowest for the weekly-return betas.*

MSE-Total

With such varied behavior of MSE-bias, MSE-inefficiency, and MSE-random it is highly unlikely that we can generalize the behavior of MSE-total.

V Conclusions

In this paper, we extend some recent works on the predictive ability of extrapolated beta coefficients. We examine five techniques over four investment horizons and seven sizes of portfolio. *We show that the size of the portfolio and the investment horizon are more important determinants of the MSE-total than the adjustment techniques themselves.*

The predictive ability of beta forecasts as measured by the correlation coefficient and the MSE-random is the same for all four adjustment techniques and the unadjusted betas themselves. *On average, this predictive ability is inversely related to the investment horizon, and is directly related to the portfolio size. This finding suggests that although daily betas may not be reliable estimates of true betas, their unanticipated shifts are the most predictable*[17]. Also, increasing the size of the portfolio, but only up to 20 securities, will significantly improve the predictive ability.

The difference between the means of the predicted and realized betas is reflected in the MSE-bias. The Blume method has significantly higher MSE-bias than the other four techniques analyzed here. The unadjusted betas, and the James-Stein, Bayes-Vasicek, and MLPFS methods have same MSE-bias. As expected, the size of portfolios has no effect on the MSE-bias; and the monthly betas have the lowest error of this type.

While the MLPFS technique has the lowest MSE-inefficiency, the Blume method has the highest. Also, except for the Blume technique, as the size of the portfolio increases, MSE-inefficiency decreases.

In the light of these findings, once the portfolio size and the investment horizon are selected (or given), there appears to be statistically insignificant gain from adjusting betas even with the "best" technique and major loss if a "bad" tachnique is used. The investors should rely on simple no-change unadjusted beta forecasts to select securities for their portfolios.

TABLE 1

Average Values of (s_t^2/s_c^2)

(Averaged across securities and subperiods)

Investment Horizon	Portfolio Size							
	1	2	3	5	10	20	30	40
One Day	.0709	.0329	.0214	.0122	.0062	.0033	.0020	.0018
Two Days	.1203	.0593	.0393	.0324	.0117	.0061	.0038	.0032
One Week	.2187	.1193	.0820	.0493	.0258	.0136	.0086	.0071
Two Weeks	.2966	.1821	.1302	.0809	.0434	.0230	.0147	.0110
One Month	.3530	.2486	.1876	.1239	.0707	.0389	.0249	.0229

TABLE 2 (a)

Correlation and Forecast Errors: Unadjusted Betas

	Portfolio Size (Number of Securities)						
	1	3	5	10	20	30	40
Daily Returns							
Correlation Coefficient	.72947	.86896	.90647	.94208	.96511	.96964	.97596
Mean Square Error (MSE)	.18529	.08453	.06798	.05133	.04092	.04143	.03241
Portion of MSE due to:							
Bias	.01632	.01619	.01632	.01597	.01562	.01597	.01487
Inefficiency	.03625	.02276	.02113	.01812	.01580	.01701	.01152
Random error	.13272	.04558	.03053	.01724	.00950	.00845	.00602
Weekly Returns							
Correlation Coefficient	.66362	.82626	.88060	.92780	.96063	.96682	.97362
Mean Square Error (MSE)	.20540	.08149	.06008	.04197	.03058	.02949	.02553
Portion of MSE due to:							
Bias	.01294	.01156	.01169	.01132	.01093	.01132	.01022
Inefficiency	.04250	.01938	.01590	.01286	.01089	.01136	.00957
Random error	.14996	.05055	.03249	.01779	.00876	.00681	.00574
Biweekly Returns							
Correlation Coefficient	.56945	.76096	.83106	.90038	.94403	.95798	.96354
Mean Square Error (MSE)	.26738	.10548	.07657	.05123	.03770	.03608	.03124
Portion of MSE due to:							
Bias	.01685	.01659	.01684	.01625	.01570	.01625	.01437
Inefficiency	.06119	.02328	.01759	.01322	.01103	.01121	.00970
Random error	.18934	.06561	.04214	.02176	.01097	.00862	.00717
Monthly Returns							
Correlation Coefficient	.50612	.68219	.76984	.84681	.90836	.94078	.95360
Mean Square Error (MSE)	.35308	.12924	.08467	.05085	.03160	.02806	.02150
Portion of MSE due to:							
Bias	.00645	.00624	.00645	.00581	.00531	.00581	.00420
Inefficiency	.09688	.03387	.02321	.01551	.01204	.01148	.00971
Random error	.24975	.08913	.05501	.02953	.01425	.01077	.00759

TABLE 2 (b)

Correlation and Forecast Errors: Stein's Adjustment

	Portfolio Size (Number of Securities)						
	1	3	5	10	20	30	40
Daily Returns							
Correlation Coefficient	.73411	.87013	.90709	.94237	.96524	.96972	.97601
Mean Square Error (MSE)	.17262	.08168	.06640	.05062	.04061	.04122	.03431
Portion of MSE due to:							
Bias	.01554	.01589	.01613	.01589	.01559	.01594	.01486
Inefficiency	.02613	.02058	.01994	.01758	.01555	.01685	.01344
Random error	.13095	.04521	.03033	.01715	.00947	.00843	.00601
Weekly Returns							
Correlation Coefficient	.64731	.82731	.88113	.92819	.96091	.96699	.97373
Mean Square Error (MSE)	.17472	.07443	.05853	.04432	.02978	.02976	.02517
Portion of MSE due to:							
Bias	.01064	.01080	.01340	.01106	.01081	.01122	.01017
Inefficiency	.01348	.01310	.01256	.01139	.01023	.01092	.00926
Random error	.15060	.05053	.03257	.02187	.00874	.00762	.00574
Biweekly Returns							
Correlation Coefficient	.55856	.76193	.83126	.90083	.94445	.95812	.96373
Mean Square Error (MSE)	.23079	.09790	.08397	.04955	.03692	.03485	.03087
Portion of MSE due to:							
Bias	.01588	.01588	.02772	.01596	.01556	.01615	.01431
Inefficiency	.02048	.01576	.01372	.01172	.01037	.01081	.00938
Random error	.19443	.06626	.04253	.02187	.01099	.00789	.00718
Monthly Returns							
Correlation Coefficient	.49775	.68188	.77143	.84846	.90973	.94107	.95387
Mean Square Error (MSE)	.28800	.11530	.07629	.04514	.03000	.02714	.02064
Portion of MSE due to:							
Bias	.00319	.00354	.00445	.00474	.00474	.00544	.00380
Inefficiency	.02981	.01914	.01596	.01291	.01100	.01084	.00923
Random error	.25500	.09262	.05588	.02749	.01426	.01086	.00761

TABLE 2 (c)

Correlation and Forecast Errors: Bayes/Vasicek's Adjustment

	Portfolio Size (Number of Securities)						
	1	3	5	10	20	30	40
Daily Returns							
Correlation Coefficient	.73360	.88214	.90707	.94236	.96524	.96972	.97601
Mean Square Error (MSE)	.17389	.08178	.06644	.05063	.04061	.04122	.03436
Portion of MSE due to:							
Bias	.01557	.01590	.01613	.01589	.01559	.01594	.01486
Inefficiency	.02718	.02065	.01997	.01759	.01555	.01685	.01349
Random error	.13114	.04523	.03034	.01715	.00947	.00843	.00601
Weekly Returns							
Correlation Coefficient	.65454	.82760	.88119	.92968	.96090	.96698	.97373
Mean Square Error (MSE)	.17651	.07694	.05647	.04027	.02979	.02960	.02517
Portion of MSE due to:							
Bias	.01018	.01281	.01116	.01106	.01081	.01123	.01017
Inefficiency	.01873	.01374	.01278	.01144	.01024	.01092	.00926
Random error	.14760	.05039	.03253	.01777	.00874	.00745	.00574
Biweekly Returns							
Correlation Coefficient	.57930	.76332	.83169	.90085	.94445	.95812	.96373
Mean Square Error (MSE)	.22845	.09776	.07256	.05167	.03693	.03560	.03112
Portion of MSE due to:							
Bias	.01479	.01572	.01624	.01804	.01556	.01615	.01431
Inefficiency	.02736	.01650	.01401	.01179	.01039	.01081	.00964
Random error	.18630	.06554	.04231	.02184	.01098	.00864	.00718
Monthly Returns							
Correlation Coefficient	.52029	.68634	.77228	.84847	.90967	.94108	.95462
Mean Square Error (MSE)	.29168	.05587	.04685	.04777	.03006	.02712	.02081
Portion of MSE due to:							
Bias	.00259	.00392	.00473	.00484	.00477	.00542	.00396
Inefficiency	.04340	.02158	.01686	.01310	.01104	.01086	.00924
Random error	.24569	.03037	.02526	.02983	.01425	.01085	.00761

TABLE 2 (d)

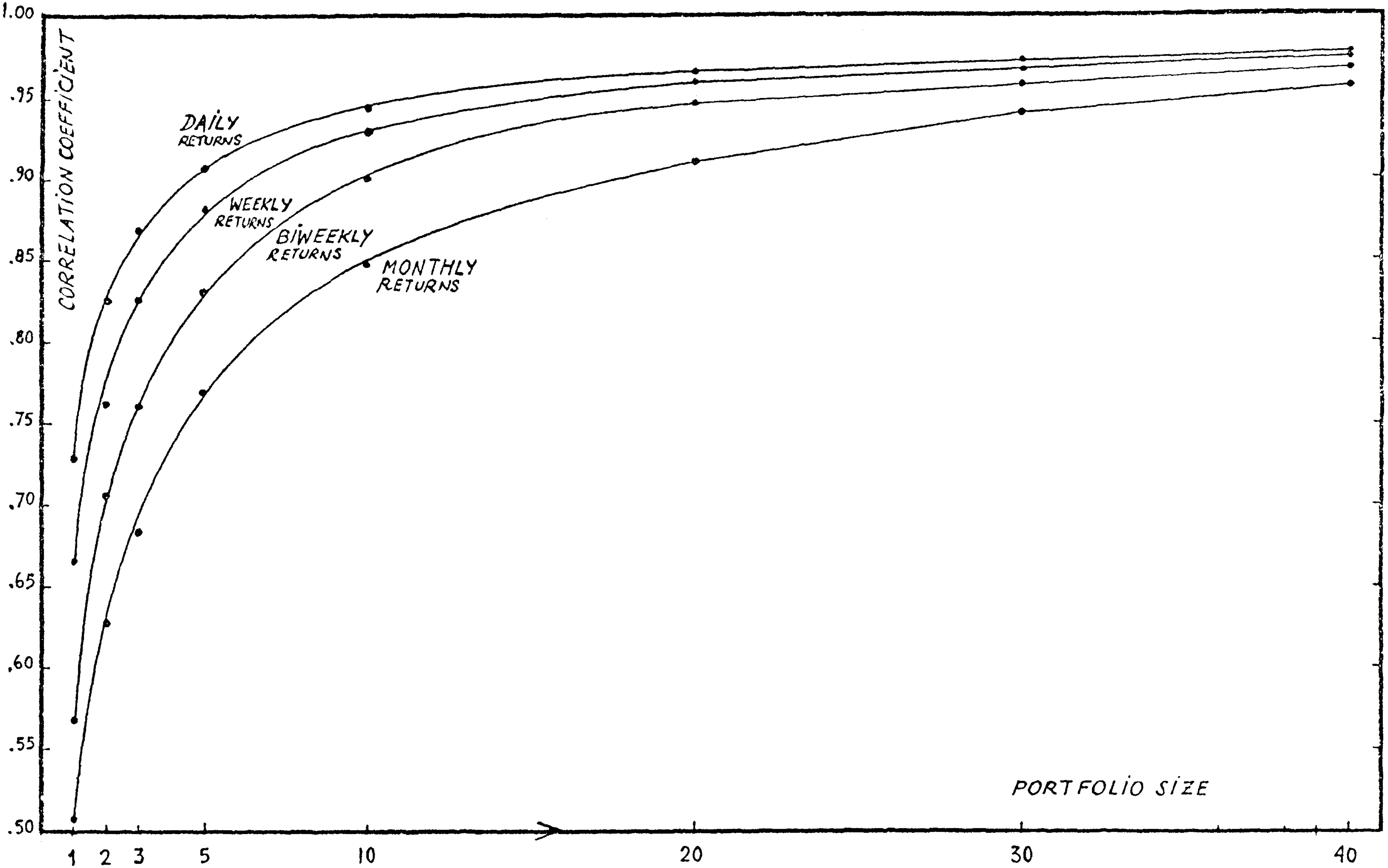
Correlation and Forecast Errors: Blume's Adjustment

	Portfolio Size (Number of Securities)						
	1	3	5	10	20	30	40
Daily Returns							
Correlation Coefficient	.72946	.86896	.90648	.94208	.96511	.96965	.97596
Mean Square Error (MSE)	.17409	.08846	.06478	.05022	.03964	.04171	.03358
Portion of MSE due to:							
Bias	.01823	.01832	.01824	.01826	.01778	.01827	.01699
Inefficiency	.02314	.02451	.01601	.01472	.01236	.01499	.01056
Random error	.13272	.04563	.03053	.01724	.00950	.00845	.00603
Weekly Returns							
Correlation Coefficient	.64693	.82626	.88061	.92780	.96063	.96682	.97362
Mean Square Error (MSE)	.17427	.07759	.06057	.04593	.03513	.03545	.02927
Portion of MSE due to:							
Bias	.01472	.01562	.01594	.01610	.01596	.01618	.01529
Inefficiency	.00958	.01142	.01209	.01204	.01041	.01164	.00824
Random error	.14997	.05055	.03254	.01779	.00876	.00763	.00574
Biweekly Returns							
Correlation Coefficient	.56943	.76097	.83106	.90038	.94403	.95798	.96354
Mean Square Error (MSE)	.24498	.13172	.10606	.08690	.07393	.07627	.06617
Portion of MSE due to:							
Bias	.03211	.03598	.03727	.03791	.03762	.03873	.03635
Inefficiency	.02352	.03013	.02665	.02723	.02534	.02894	.02265
Random error	.18935	.06561	.04214	.02176	.01097	.00860	.00717
Monthly Returns							
Correlation Coefficient	.50763	.68219	.76984	.84681	.90836	.94078	.95360
Mean Square Error (MSE)	.28521	.14907	.11825	.10635	.09145	.09194	.07573
Portion of MSE due to:							
Bias	.01242	.01512	.01744	.01841	.01951	.01971	.02229
Inefficiency	.02303	.04357	.04580	.05841	.05769	.06146	.04571
Random error	.24976	.09038	.05501	.02953	.01425	.01077	.00757

TABLE 2 (e)

Correlation and Forecast Errors: MLPFS's Adjustment

	Portfolio Size (Number of Securities)						
	1	3	5	10	20	30	40
Daily Returns							
Correlation Coefficient	.72946	.86896	.90647	.94208	.96511	.96964	.97597
Mean Square Error (MSE)	.16510	.07390	.06046	.04728	.03866	.03954	.03226
Portion of MSE due to:							
Bias	.01750	.01680	.01671	.01629	.01582	.01621	.01506
Inefficiency	.01488	.01152	.01322	.01375	.01334	.01488	.01217
Random error	.13272	.04563	.03053	.01724	.00950	.00845	.00603
Weekly Returns							
Correlation Coefficient	.64693	.82626	.88060	.92780	.96063	.96682	.97362
Mean Square Error (MSE)	.17061	.06960	.05236	.03792	.02852	.02860	.02470
Portion of MSE due to:							
Bias	.01114	.01130	.01138	.01119	.01089	.01129	.01016
Inefficiency	.00950	.00775	.00852	.00894	.00887	.00968	.00836
Random error	.14997	.05055	.03254	.01779	.00876	.00763	.00574
Biweekly Returns							
Correlation Coefficient	.56943	.76097	.83106	.90038	.94403	.95798	.96353
Mean Square Error (MSE)	.22162	.09372	.06366	.04670	.03503	.03426	.02946
Portion of MSE due to:							
Bias	.01387	.01497	.01076	.01512	.01496	.01579	.01382
Inefficiency	.01840	.01314	.01076	.00982	.00910	.00987	.00847
Random error	.18935	.06561	.04214	.02176	.01097	.00860	.00717
Monthly Returns							
Correlation Coefficient	.50763	.68219	.76984	.84681	.90836	.94078	.95360
Mean Square Error (MSE)	.27142	.10774	.07059	.04386	.02745	.02497	.01923
Portion of MSE due to:							
Bias	.00862	.00422	.00340	.00316	.00340	.00418	.00319
Inefficiency	.01304	.01314	.01218	.01117	.00980	.01002	.00847
Random error	.24976	.09038	.05501	.02953	.01425	.01077	.00757



CORRELATION COEFFICIENT
1.00
.95
.90
.85
.80
.75
.70
.65
.60
.55
.50
DAILY RETURNS
WEEKLY RETURNS
BIWEEKLY RETURNS
MONTHLY RETURNS
PORTFOLIO SIZE
1
2
3
5
10
20
30
40

FIGURE 1

FOOTNOTES

1. The studies using the daily betas for event testing etc. are too numerous to be listed here.

2. Note that $\hat{\beta}_j(H)$ is not invariant to the length H of the investment horizon. The time behavoir of the beta coefficient is examined in detail by Levhari and Levy [10] and Hawawini [7].

3. In the second subperiod we have only one set of portfolios, but in the third subperiod we have two parallel sets of portfolios. In the third subperiod we have the portfolios based on the rankings of securities' betas of the second subperiod. But in the third subperiod we also have the portfolios based on the rankings of the first subperiod. It is only this second set of the third subperiod that is comparable to the second period portfolios.

4. "Stein's estimator is obviously a nonlinear, biased estimator. Stein has thus gone outside the class of linear, unbiased estimators to produce an estimator with lower risk. That the risk associated with Stein's estimator is lower over the entire parameter space, given $n \geq 3$ and finite variance, is indeed surprising." Zellner and Vandaele [16, p. 629]. This argument will apply to Bayes estimators also. The main difference between the Bayes estimator and the James-Stein estimator is the assumption about the nature of the prior distribution.

5. While the classical estimator minimizes the error of sampling, the James-Stein estimator minimizes the combined loss due to misestimation if $n \geq 3$. In spite of this, the James-Stein estimators are consistent, i.e., the probability limit of the estimator is the true parameter. The important point in this calculation is that the expected error is

added up for all the securities. Any particular beta might be wrong, but the sum of all such errors would be smaller for the James- Stein estimators than for the observed betas. But, there is one purpose for which the observed beta may well be superior to the James-Stein estimator; when a single beta is considered in isolation.

6. It should be noted that all the four estimators we are using are nonlinear, biased, and by the standards of the classical sampling theory, they are inefficient. But still, James-Stein and Bayes-Vasicek estimators are consistent in probability limit. Vasicek [17] explicitly assumes the grand mean to be the mean of the prior distribution, and thus starts with an "informative" prior while James-Stein analysis assumes a "diffused" prior. All four estimators induce shrinkage (Thomson [15]). Bayes-Vasicek shrinks the estimators around a priori mean which in this case is the grand mean. James-Stein shrinks them around the grand mean, but allows for the possibility of an estimator to shift from one side of the grand mean to the other. Blume shrinks them around the grand mean using the cross-sectional information only without the help of any time-series statistics. MLPFS shrinks them around 1. Blume's and MLPFS's methods are most restrictive because the raw estimates and the predictors maintain exactly the same ranks.

7. The individual values of the subperiods are not very different from the average. Also, this method condenses an unyieldy set of results and makes comparisons that are not time-dependent.

8. For details see Mincer and Zarnowitz [13].

9. The detailed results of the four subperiods from which Tables 2(a)-2(c) are derived, as well as the results obtained by aggregation within each of the tables, are available from the authors.

10. As expected, for all the portfolio sizes and investment horizons, Blume's and MLPFS's methods generate identical correlations and MSE-random. Still, their MSE-bias and MSE-inefficiency are significantly different.

11. This statistical similarity is based on the pairwise F-test applied to MSE-random.

12. The MSE-random for the Vasicek method, on average, is minutely lower; but this difference is not statistically significant.

13. K-M and E-Z did not examine correlations.

14. For portfolios with 1, 3, 5, or 10 securities the MSE-random is minimum for the daily-returns betas. For portfolios with 20, 30 and 40 securities, it is minimum for the weekly-returns betas, but this MSE-random is not statistically different from the corresponding value for the daily-returns betas.

15. If the sample equals the population generating the market index then MSE-bias will be zero. This is because each of the grand means will equal unity and the difference among them will equal zero.

16. For a theoretical and empirical analysis of this phenomenon, see Hawawini [8].

17. For the deficiencies of daily betas, see Scholes and Williams [14], and Hawawini [8].

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