THE IMPACT OF INFLATION ON REAL VARIABLES: A GENERAL EQUILIBRIUM APPROACH

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ABSTRACT

A general equilibrium approach is used to analyze the impact of inflation on the real sector of the economy. A two-sector model of the economy with assets markets and specific labor supply assumptions is developed. The economy produces consumption and investment goods with different technologies exhibiting constant returns to scale. The consumption goods industry is assumed to be relatively more capital intensive. Rybczinski's theorem is used to show that, under these circumstances and for a given price of capital relative to the price of consumption goods, an increase in labor supply leads to an increase in the output of the investment goods industry and a decrease in the output of the consumption goods industry. The exact magnitude of these effects on the composition of output is determined.

In the assets markets, consumers allocate their wealth among money, bonds and physical capital, according to their respective rates of return. It is shown that the rate of return on capital decreases as the relative price of capital increases.

The model is closed with a consumption function, based on consumer's disposable income, including expected capital gains and losses.

The impact of inflation is analyzed: in assets markets, the increased demand for physical capital drives up its relative price; in the labor market, an increased rate of inflation induces more workers to look for a job. As the labor supply increases, the output of consumption goods tends to decrease, because of the Rybczinski effect. The

higher relative price of capital also reduces the supply of consumption goods. Excess demand can occur if the losses due to the decrease in the real value of the government debt do not induce consumers to increase their savings sufficiently.

Some preliminary data for the U. S. economy is presented, showing that under fairly plausible circumstances an increase in inflation can indeed lead to excess demand for consumption goods, fueling additional price increases.

Further results are obtained, to account for different assumptions on consumer behavior and labor supply. In all cases, the impact of inflation on the supply side of the economy is shown to have potentially the consequence of making prices increase persistently, at least in the short term.

1. INTRODUCTION

"Inflation is always and everywhere a monetary phenomenon".[1] From this basic and undisputed proposition the monetarist approach to the theory of inflation derives most of its strength. It is indeed impossible to explain inflation without studying carefully the supply of and the demand for money and their rates of growth.

Many economists, however, believe that, in spite of the power inherent to a simple and robust hypothesis, the monetarist view of inflation is insufficient for a thorough understanding of the causes of the continuous rise in prices; and in practice anti-inflation policy almost always includes and often relies on instruments not related to the rate of growth of money supply. Moreover, the importance of inflation results from its adverse consequences for the welfare of current and future generations, on which it must necessarily have some impact. All this indicates that the study of inflation should include work on the real side of the economy instead of being confined to the monetary aspects of the inflationary process.

The assumption that the money supply is entirely exogenous, or that the government prints more bills essentially for the purpose of supplementing tax revenues has in fact been questioned often. The monetary authorities will in many cases expand the money supply in response to very strong pressures - often of a political nature - the roots of which are to be found in impending disruptions in the real sector of the econ-

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[1] Friedman (1970)

omy. In those cases an anti-inflation policy relying solely on a tight control of money supply overlooks the fact that the capacity of the government to resist those pressures is limited and underestimates the economic - and often political - costs of a rigid monetary policy.

The early Keynesian models of demand pull inflation were the first attempt to find non-monetary causes for inflation. The rise in prices became a rationing mechanism among competing excessive demands. Shortly afterwards the cost-push approach asserted that inflation results from the struggle among the owners of productive factors to increase their share of total income. Both views seem now rather outdated. The sociological hypothesis that inflation is caused by excessive union or corporate power does not survive most empirical tests; and sustained inflation persists in the face of rather large unemployment rates. Attention has again shifted to simple models based on refined versions of the quantity theory of money. Simultaneously, a large effort was directed to another area, where the real effects of inflation can be very significant: the Phillips curve and the importance of expectations about inflation for labor supply.

More recently, inflation theorists have pointed out the need for a general equilibrium approach leading to the development of models capable of providing an integrated treatment of the real and monetary aspects of an inflationary process.[2] In fact the interrelationships and feedback effects among what are conventionally called causes and conse-

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[2]For a comprehensive survey of the various approaches to inflation see Laidler and Parkin (1975).

quences of price rises, if modeled appropriately, can lead to results and conclusions otherwise unattainable, providing new insights on inflation and helping to explain empirical observations much more satisfactorily. The richness of the results and the practical usefulness of the models is greatly enhanced if it is possible to go beyond the traditional highly aggregated approaches. An exploration of the real effects of inflation and of the impact of changes in the supply of labor is much more realistic if more than one sector of production is identified. It is also plausible to assume that the decisions related to labor supply are based on expectations about the prices of consumer goods, rather than on the GNP deflator; which would indicate the need to treat separately consumer and investment goods. Finally, an adequate treatment of assets markets, capable of explaining the demand for cash balances, the behavior of the interest rate and the level of investment in inflationary periods requires that money, bonds and capital be dealt with individually.

A disaggregated approach to inflation has been used previously with considerable success in the so-called Scandinavian model.[3] Assuming two sectors with different productivity growth and stating that the rate of change in wages is determined by the changes in prices in the world markets plus the rapid productivity increase in the tradables sector, this model predicts inflation or inflationary pressures leading to conclusions that cannot be attained using more aggregative models.

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[3]See Aukrust (1970) and Edgren, Faxen, and Odhner (1973).

The interdependence between real and monetary phenomena has also been modeled in a two sector general equilibrium framework in a well known work by D. Foley and M. Sidrauski.[4] Although it can be used to study some of the real effects of inflation - in particular on resource allocation and growth - the Foley-Sidrauski model does not incorporate the impact of inflation expectations on labor supply and therefore misses one of the main links between monetary and real phenomena.

It is precisely the purpose of this work to show that by modeling the interdependence between real and monetary phenomena in a disaggregated general equilibrium framework and by specifically accounting for the impact of inflation on labor markets it is possible to reach new conclusions about the reallocation of resources caused by inflation and specially about the inflationary process itself.

Because the Foley-Sidrauski model is well known and provides a convenient framework, the analysis will be conducted as an extension to that model. However, the results will not depend on some of the more specific assumptions made by Foley and Sidrauski and, where appropriate, will deal with different specifications.

2. THE MODEL

General equilibrium in a two sector model of production with assets markets requires that careful attention be paid to the interdependence

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[4]Foley and Sidrauski (1971).

between stocks and flows. The production sectors are identified by the output of consumer and investment goods, each according to a different technology. In the assets markets the prices - and therefore the rates of return - of bonds, capital and money are adjusted so that private wealth is allocated to each asset according to the amount supplied. The model is closed by specifying a demand function for consumer goods - or consumption function. The investment function becomes redundant in a general equilibrium framework.

2.1 PRODUCTION SECTOR

The technologies for the production of consumption and investment goods can be represented by linearly homogeneous functions on capital and labor.

$$C = F_c(N_c, K_c) \qquad (1)$$

$$I = F_i(N_i, K_i) \qquad (2)$$

where N stands for labor and K for capital, and C and I denote consumption and investment goods respectively.

Under the usual assumptions of perfect competition with full employment, factor mobility and price flexibility, the allocation of resources - the amount of capital and labor used in each sector - must meet the following conditions:

$$r = f_c'(k_c) = p_k f_i'(k_i) \qquad (3)$$

$$w = f_c(k_c) - k_c f_c'(k_c) = p_k[f_i(k_i) - k_i f_i'(k_i)] \qquad (4)$$

$$N_c + N_i = N \qquad (5)$$

$$K_c + K_i = K \qquad (6)$$

where f_c and f_i are the functions F_c and F_i written in factor intensity form, k_c and k_i are the capital labor ratios in the two sectors, r is the rate of return on capital, w is the wage rate, and p_k is the price of capital goods relative to the price of consumer goods, the latter being the numeraire. N and K are the total supply of labor and capital, respectively.

The allocation of capital and labor, output in each sector and the prices of factors are determined from the total endowments - N and K - if the relative price of capital goods is given. When demand equations are specified the model is closed and p_k is determined endogenously.

In the following analysis it will be assumed that the capital goods technology is more labor intensive, that is $k_c > k_i$. This assumption, which can be documented empirically, is common and has become the usualy intensity case.

Two straightforward and well known results can be derived from the equilibrium conditions in the production sectors, and will be necessary

at later stages: first, the output of investment goods increases and the output of consumption goods decreases if the relative price of capital goes up; equivalently, there is an economy wide convex production possibility frontier on which the equilibrium point is determined by the relative price of capital. Second, factor intensities in each sector and factor prices depend only on the relative price of capital, assuming that the economy does not specialize in the production of only one of the two goods. The choice of technique in each sector is not altered if p_k does not change even if there is a change in endowments. Therefore the capital labor ratio for the whole economy may change without a change in k_c or k_i, which can only happen by shifting both capital and labor from one sector to another. This leads directly to Rybczinski's theorem[5] which states that if the supply of one factor increases and the relative prices of the goods produced remain fixed, total output for the economy will increase, but there will be a reduction in the output of the sector relatively less intensive in the now more abundant factor. The implications of this result will be explored in the following analysis.

A further extension of the production model deals with consumption goods - or services - supplied by the government. These will be subtracted from the total output of the consumption goods sector, to obtain private production. If the production functions are redefined in terms of the endowments and the relative price of capital, it is possible to write

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[5]Rybczinski (1955).

$$C_p = Q_c(K,N,p_k) - E \qquad (7)$$

where E represents the supply of goods and services by the government and C_p is private production. Q_c and Q_i will now denote the optimal levels of output for each sector, given p_k; in per capita terms

$$\frac{C_p}{N} = q_c(k,p_k) - e \qquad (8)$$

and

$$\frac{I}{N} = q_i(k,p_k) \qquad (9)$$

2.2 ASSETS MARKETS

The model deals with money, bonds and physical capital. Given a certain amount of private wealth, the demands for these assets depend on their prices and on their rates of return. No specific treatment of risk bearing is included.

As traditionally, real income is also an argument of asset demand functions. Since consumer goods are the numeraire, real income is defined in per capita terms as

$$q = q_c(k,p_k) + p_k q_i(k,p_k) \qquad (10)$$

It should be noted that, because of the units of measurement chosen, income increases as p_k increases.

The rates of return are defined to include gains and losses due to changes in the real value of the assets. For physical capital it will be

$$\rho_k = \frac{r(p_k)}{p_k} + \pi_k \qquad (11)$$

where r is the rate of return per unit of capital or the value in consumption units of the marginal product of capital; and π_k is the rate of capital gains or

$$\rho_k = \frac{\dot{p}_k}{p_k}$$

ρ_k will therefore be the rate of return per unit of value of capital.

It is easy to show that, for a given π_k, the rate of return on capital decreases as the relative price of capital increases. In fact

$$\frac{dr(p_k)}{dp_k} < 0 \qquad (12)$$

because, as p_k increases, more investment and less consumption goods are produced, which raises the capital labor ratio, and therefore lowers the

marginal productivity of capital, in both sectors. By differentiating the first order conditions for profit maximization

$$f_c'(k_c) = p_k f_i'(k_i) \qquad (13)$$

$$f_c(k_c) - k_c f_c'(k_c) = p_k[f_i(k_i) - k_i f_i'(k_i)] \qquad (14)$$

a system of two equations in dk_c, dk_i and dp_k is obtained:

$$\begin{aligned} f_c''dk_c - p_k f_i''dk_i &= f_i'dp_k \\ - f_c''k_c dk_c + p_k f_i''k_i dk_i &= [f_i - k_i f_i']dp_k \end{aligned} \qquad (15)$$

where, for simplicity of notation, the arguments of f_c and f_i are omitted. Defining

$$\begin{aligned} \Delta &= f_c''p_k f_i''k_i - f_c''k_c p_k f_i'' \\ &= f_c''p_k f_i''(k_i - k_c) \end{aligned} \qquad (16)$$

which is negative, Cramer's rule provides the solutions for dk_c and dk_i in terms of dp_k

$$dk_c = \frac{1}{\Delta}[f_i'p_k f_i''k_i dp_k + (f_i - k_i f_i')p_k f_i''dp_k] \qquad (17)$$

and

$$dk_i = \frac{1}{\Delta}[f_c''(f_i - k_i f_i')dp_k + f_c'' k_c f_i' dp_k] \quad (18)$$

which are both positive when dp_k is positive. This result, however, depends on the normal intensity case - $k_c > k_i$ - which is assumed here.[6]

The rate of return on money corresponds to the capital gains or losses incurred by holding money:

$$\rho_m = \pi_m \quad (19)$$

where

$$\pi_m = \frac{\dot{p}_m}{p_m}$$

and p_m is the price of money or the inverse of the price of consumption goods. According to this definition in periods of inflation $\pi_m < 0$ and naturally the rate of return on money will be negative. Alternatively π_m can be viewed as the rate of deflation.

For simplicity bonds are defined as having a fixed nominal value and paying interest at a variable rate - much like savings accounts. Then the rate of return on bonds includes the interest rate but also the real appreciation or depreciation of the bonds.

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[6]This result is known as the Stolper-Samuelson Theorem. See W.F.Stolper and P.A.Samuelson (1941).

$$\rho_b = i + \pi_m = i + \rho_m \qquad (20)$$

All these rates of return, as well as total private wealth and real income are arguments in each demand function. If m, b and k are the per capita values of the stocks of money, bonds and physical capital, total per capita wealth in real terms will be

$$a = mp_m + bp_m + kp_k \qquad (21)$$

or

$$a = gp_m + kp_k \qquad (22)$$

where g=m+b is the total debt of the government in nominal terms. Then equilibrium will consist of equality between supply and demand for each asset which, again in per capita terms, can be written as

$$mp_m = L(a, q, \pi_m, i+\pi_m, \frac{r(p_k)}{p_k}+\pi_k) \qquad (23)$$

$$bp_m = H(a, q, \pi_m, i+\pi_m, \frac{r(p_k)}{p_k}+\pi_k) \qquad (24)$$

$$kp_k = J(a, q, \pi_m, i+\pi_m, \frac{r(p_k)}{p_k}+\pi_k) \qquad (25)$$

where L, H and J are the demand functions for respectively money, bonds and capital, per capita, in real terms.

The partial derivatives of these demand functions with respect to wealth are naturally positive; the derivative with respect to income, however, will be positive only in the demand for money function. All assets are substitutes: the demands respond positively to increases in the own rate of return and negatively to increases in the rates of return of the other assets.

With the help of Walras' law it is possible to analyze equilibrium in the assets markets by studying two markets only. For given expectations about the rate of inflation and capital gains, π_m and π_k, the equality of supply and demand for all assets is obtained by movements of p_m, p_k, and i. The price of capital determines the rate of return to capital and the interest rate determines the rate of return to bonds; both contribute to equilibrium by acting on the demand side. The price of capital and the price of money (p_m) also affect the supply side, since they determine the real value of the existing stocks of capital and money and bonds, respectively. Finally, total wealth and real output also depend on p_m and p_k. Under these circumstances, an increase in the price of capital has to be matched by a simultaneous increase in the price of money to keep all assets markets in equilibrium. In fact, the increase in p_k reduces the demand for capital - since it lowers the rate of return, and increases the value of output - while at the same time increasing the value of the stock of capital; it therefore generates excess supply of capital. Simultaneoysly, the lower rate of return on capital and the

higher values of wealth and output lead to excess demand for money. Clearly, equilibrium can only be restored by increasing the price of money.

Since equilibrium in the bond market will follow from the equality of supply and demand in the money and capital markets, this positive relationship between p_m and p_k will be sufficient to characterize asset market equilibrium. Schedule aa in Figure 1 represents the values of p_m and p_k that clear assets markets.

2.3 THE CONSUMPTION GOODS MARKET

The supply of consumer goods was specified above as:

$$c_s = q_c(k,p_k) - e \qquad (26)$$

Demand, also in per capita terms, can be represented as proportional to disposable income

$$c_d = (1-s)\, y \qquad (27)$$

where s is the propensity to save. As usual, disposable icome is net of taxes but includes transfers and interest received on the government debt. Moreover, it is assumed that consumers adjust their disposable income to account for capital gains and losses due to the change in value of the government debt - implying that consumers will save more when their holdings of money and bonds lose value, which is in line with post- Keynesian consumption theory.

$$y = q_c(k,p_k) + p_k q_i(k,p_k) + p_m ib + p_m(v-t) + \pi_m p_m g \quad (28)$$

where the first two terms correspond to real income, the third term is interest received, the fourth net transfer - v is per capita transfers and t per capita taxes, both lump sum and in nominal terms - and the last component is capital gains or losses on the government debt.

To simplify the equilibrium condition this consumption function can be rewritten by defining d as the nominal value of the government deficit. Then

$$dp_m = e + ibp_m + (v-t)p_m \quad (29)$$

and

$$y = q_c(k,p_k) + p_k q_i(k,p_k) - e + p_m(d+\pi_m g) \quad (30)$$

Equilibrium in the consumption goods market - and also in the market for investment goods - will be achieved when

$$q_c(k,p_k) - e = \quad (31)$$

$$(1-s)[q_c(k,p_k) + p_k q_i(k,p_k) - e + p_m(d+\pi_m g)]$$

Given the endowments of capital and labor and the government deficit and expenditures, and assuming again that expectations about the rate of inflation are predetermined, equilibrium requires that p_m and p_k vary in opposite directions. If the general price level increases - p_m decreases - the real value of the deficit and of capital gains on the go-

vernment debt drops - since $d+\pi_m g$ is usually positive - and disposable income is reduced. The resulting excess supply of consumption goods can only be eliminated by increasing the relative price of capital, which not only reduces the supply of consumption goods, but also increases real income. Figure 2 presents the schedule cc with the values of p_k and p_m that are consistent with equilibrium in the goods markets.

2.4 THE COMPLETE MODEL: IMPACT OF CHANGES IN THE INFLATION RATE

General equilibrium in goods and assets markets can be achieved only with certain unique values of p_m and p_k. These correspond to the intersection of the aa and cc schedules, as in Figure 3. The equilibrium values however will naturally be altered if endowments are modified, if expectations change or it the government alters any of its policy variables.

Innumerous exercises can be performed with a model of this type, to illustrate the responses to changes of scenarios or of policies. For the purpose of this work the most interesting will be an analysis of the implications of a change in the inflation rate. This is presented graphically in Figure 4. If inflation increases - or π_m decreases - wealth holders will attempt to change their portfolios by decreasing their holdings of money and bonds. The resulting excess demand for capital can only be eliminated by an increase in the price of capital. This is equivalent to a shift upwards of the aa schedule. In the goods market, an increase in the inflation rate decreases consumers perceived disposable income, as they try to absorb the losses in their holdings of

government debt. The excess supply of consumer goods thus generated, leads to an increase in the relative price of capital - the cc schedule shifts up.

The final outcome is an increase in p_k with uncertain results for p_m. As a result, more investment will be forthcoming, which leads to the conclusion that an increase in the rate of inflation may actually favor growth.

3. INTRODUCTION OF A PHILLIPS CURVE

The many results obtainable with this model, emphasizing the interrelationships between real and monetary phenomena, are greatly enhanced if another important link is added to it: a positive relationship between the rate of inflation and the supply of labor. Even if this link is only temporary - since labor supply will return to its natural level as soon as the new inflation rate is fully anticipated - its relevance cannot be denied because of its implications for the short term behavior of employment and, as will be shown, prices.[7]

To be consistent with the notation used above, the Phillips curve relationship will be modeled as an increase in the participation rate of a fixed total population, when the inflation rate increases

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[7]The most important papers dealing with the impact of inflation on labor supply are Lucas and Rapping (1969) and Lucas (1972). For a recent discussion of the empirical evidence see Hall (1979).

$$N = \eta(\pi_m)L \qquad (32)$$

where L is total population, π_m is the deflation rate, η is the participation rate - with

$$\frac{d\eta}{d\pi_m} < 0$$

- and N is total employment, as before.

From the first order conditions for profit maximization, it is clear that the choice of technique in each sector is independent of the total supply of labor, since the capital labor ratios k_c and k_i depend only on the relative price of capital goods. Then, the impact of a change in the participation rate can be studied by looking only at the full employment equations

$$N_c + N_i = \eta(\pi_m)L \qquad (33)$$

$$K_c + K_i = K \qquad (34)$$

These can be written as

$$k_c\frac{N_c}{N_c+N_i} + k_i\frac{N_i}{N_c+N_i} = \frac{K}{\eta(\pi_m)L} \qquad (35)$$

If it is assumed, as a first approximation, that the capital labor ratios do not change, an increase in the inflation rate ($d\pi_m<0$), will change the levels of employment in each sector in the following way:

$$k_c d\left[\frac{N_c}{\eta(\pi_m)L}\right] + k_i d\left[\frac{N_i}{\eta(\pi_m)L}\right] = \frac{K\eta' d\pi_m}{[\eta(\pi_m)]^2 L} \quad (36)$$

where

$$\eta' = \frac{d\eta(\pi_m)}{d\pi_m} \quad (37)$$

As

$$\frac{N_c}{\eta(\pi_m)L} = 1 - \frac{N_i}{\eta(\pi_m)L}, \quad (38)$$

$$d\left[\frac{N_c}{\eta(\pi_m)L}\right] = - d\left[\frac{N_i}{\eta(\pi_m)L}\right] \quad (39)$$

and

$$(k_c - k_i)\, d\left[\frac{N_c}{\eta(\pi_m)L}\right] = - \frac{K\eta' d\pi_m}{[\eta(\pi_m)]^2 L} \quad (40)$$

Finally,

$$(k_c - k_i)\frac{dN_c}{\eta(\pi_m)L} = \frac{[(k_c - k_i)N_c - K]}{[\eta(\pi_m)]^2 L}\eta' d\pi_m \quad (41)$$

since

$$d\left[\frac{N_c}{\eta(\pi_m)L}\right] = \frac{dN_c}{\eta(\pi_m)L} - \frac{N_c\eta' d\pi_m}{[\eta(\pi_m)L]^2} \quad (42)$$

As $\eta' < 0$, if $d\pi_m$ is negative dN_c will also be negative because

$$(k_c - k_i)N_c - K = K_c - \frac{K_i}{N_i}N_c - K_c - K_i \quad (43)$$

which is obviously negative, in the normal intensity case $(k_c > k_i)$.

Thus, an increase in the supply of labor - as a result of an increase in the inflation rate - leads to a reduction in the level of employment in the consumer goods sector, which is capital intensive, and to a more than proportional increase in the level of employment in the investment goods sector, which is labor intensive. This is simply a restatement of Rybczynski s theorem, here presented in a mathematical form convenient for future results.

The changes in the allocation of labor to each sector will naturally lead to changes in the composition of output. Assuming still that capital labor ratios remain fixed, the levels will change proportionately to the changes in employment, or

$$\frac{dq_c}{q_c} = \frac{dQ_c}{Q_c} = \frac{dN_c}{N_c} \quad (44)$$

and

$$\frac{dq_i}{q_i} = \frac{dQ_i}{Q_i} = \frac{dN_i}{N_i} \qquad (45)$$

From

$$(k_c - k_i)dN_c = \frac{[(k_c - k_i)N_c - K]\eta' d\pi_m}{\eta(\pi_m)} \qquad (46)$$

it is possible to determine the proportional change in the level of employment in the consumption goods sector as

$$\frac{dN_c}{N_c} = \frac{(k_c - k_i)N_c - K}{(k_c - k_i)N_c} \varepsilon \, d\pi_m \qquad (47)$$

where

$$\varepsilon = \frac{\eta'}{\eta(\pi_m)} \qquad (48)$$

is the semielasticity of the Phillips curve, here defined as the percentage increase in the participation rate when inflation increases one unit.

Noting that

$$\begin{aligned}(k_c - k_i)N_c - K &= K_c - k_i N_c - K_c - K_i \qquad (49)\\ &= - k_i N_c - k_i N_i\\ &= - (N_c + N_i)k_i\end{aligned}$$

the changes in the composition of output can be determined from

$$\frac{dq_c}{q_c} = \frac{dN_c}{N_c} = - \frac{N_c+N_i}{N_c} \frac{k_i}{k_c-k_i} \varepsilon \, d\pi_m \qquad (50)$$

and

$$\frac{dq_i}{q_i} = \frac{dN_i}{N_i} = \frac{N_c+N_i}{N_i} \frac{k_c}{k_c-k_i} \varepsilon \, d\pi_m \qquad (51)$$

the second result being obtained from

$$\frac{dN}{N} = \frac{dN_c}{N_c} \frac{N_c}{N} + \frac{dN_i}{N_i} \frac{N_i}{N} = \varepsilon \, d\pi_m \qquad (52)$$

As stated above, the increase in the level of employment in the labor intensive sector will be more than proportional to the total increase in employment, since

$$\frac{N_i+N_c}{N_i} \frac{k_c}{k_c-k_i} > 1 \qquad (53)$$

always.

However, and more surprisingly, the change in the capital intensive sector can also be more than proportional than the overall change in total employment. This requires that

$$\frac{N_i+N_c}{N_i} \frac{k_i}{k_c-k_i} > 1 \qquad (54)$$

or that

$$\frac{k_c - k_i}{k_i} < \frac{N_c + N_i}{N_c} \qquad (55)$$

which will happen if the differences between the capital labor ratios in the two sectors are not too large - in particular if

$$k_i\left(2+\frac{N_i}{N_c}\right) > k_c \qquad (56)$$

These results indicate that the Rybczynski effects can be very significant. In fact, when studying the impact of a change in the inflation rate, it may be much more important to look at the shift in the supply functions for each sector than at the overall effect on labor supply.

The relevance of the disaggregate approach can also be perceived from the result that fluctuations in the output of investment goods will tend to be proportionately larger than for consumption goods. In fact,

$$\frac{dq_i}{q_i} > \left|\frac{dq_c}{q_c}\right| \qquad (57)$$

if

$$\frac{k_c}{k_i} > \frac{N_i}{N_c} \qquad (58)$$

which will clearly be true in most cases, since N_i will usually be less than N_c. This difference in the behavior of the two sectors agrees with empirical observations and can help understand a large part of the

"unexplainable" changes in investment. It should be emphasized that the Rybczynski effects work only on the supply side of the economy. Equilibrium will be determined in conjunction with demand, and naturally a large proportion of the change in the composition of output will never occur but will rather translate into changes in relative prices - here in p_k. Then, by returning to the full model, the exploration of these effects should lead to new conclusions about changes in prices, an investigation to be pursued next.

4. THE EXTENDED MODEL

When describing some of the basic results of the Foley-Sidrauski model it was shown how inflation affects the demand for consumer goods, by inducing consumers to save more. In the previous section, the potentially large impact of inflation on the supply side of the economy was seen to lead to a reduction in the output of consumption goods. The two effects seem to work together towards a new allocation of resources, characterized by a larger amount of investment. The changes in the supply side, in particular, will tend to reduce the rise in the relative price of capital that would normally follow from the increased level of saving. It can be argued, then, that inflation reallocates resources without drastic changes in relative prices.

One other issue, however, remains to be studied: can the impact of inflation on the supply side of the economy dominate its effect on demand, that is, can inflation generate excess demand for consumption goods, the final outcome being a further increase in the overall price level. Or, in other words, can inflation lead to more inflation

To answer this question, it is necessary to determine the conditions under which an increase in the rate of inflation leads to excess demand for consumption goods.

Let Z denote excess demand for consumption goods.

$$Z = (1-s)[q_c(k_c) + p_k q_i(k_i) - e + p_m (d+\pi_m g)] - q_c(k_c) + e \quad (59)$$

and the change in Z can be expressed in terms of a change in π_m as

$$dZ = (1-s)\left[p_k \frac{dq_i}{q_i} q_i + p_m g d\pi_m\right] - s\frac{dq_c}{q_c} q_c \quad (60)$$

$$= [(1-s)\left[p_k q_i \frac{N_i+N_c}{N_i} \frac{k_c}{k_c-k_i}\right.$$

$$\left. + s\frac{N_i+N_c}{N_c} \frac{k_i}{k_c-k_i} q_c\right] \varepsilon d\pi_m + (1-s)p_m g d\pi_m$$

Excess demand will increase if $dZ>0$ or

$$[(1-s)p_k q_i \frac{N_i+N_c}{N_i} \frac{k_c}{k_c-k_i} + \quad (61)$$

$$s\frac{N_i+N_c}{N_c} \frac{k_i}{k_c-k_i} q_c] \varepsilon < p_m g(s-1)$$

This condition will not be satisfied always, since it depends critically on the semielasticity of the Phillips curve, as defined here, as well as on the discrepancy between the capital labor ratios in the two sectors.

Some recent data from the U. S. economy,[8] however, will show that it is not implausible to assume that excess demand for consumption goods will often increase as a result of inflation.

Using data from 1976 - a year of relatively moderate inflation, by recent standards - and assuming a long term propensity to save of 7%, the following conclusions can be drawn:

- The total government debt held by the public amounted to 480 billion dollars; after multiplying by s-1 this value becomes -446.4.
- The value of net investment was about 64 billion dollars; assuming that the capital labor ratio in the consumption goods sector is twice the ratio of the investment goods sector, and that the labor force employed in the latter sector is about 8.5% of the total, the expression within the square brackets is valued at about 1540.
- The inequality, then, will be satisfied for any value of ε less than -.28987; or, in other words, if an increase of one percentage point in the inflation rate $d\pi_m$=-0.01 leads to an increase in the participation rate of .29% or more.
- For a total labor force of 95 million and a total population of 215 million, an increase of .29% in the participation rate would amount roughly to an additional

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[8]The data were obtained from the Survey of Current Business. There is no presumption of exact measurement of any variable. The numbers are used here only as an illustration of a plausible case where excess demand for consumer goods would be a result of an increase in the inflation rate.

275 thousand people entering the labor force, which is a quite plausible number according to recent experience.

Graphically, if inflation leads to positive excess demand for consumption goods, the cc curve shifts to the left, as in Figure 5, with the aa curve moving up, as before. Equilibrium will be restored at a lower value of p_m, or a higher general price level, and the relative price of capital may or may not increase.

The main implication of these results, therefore, is that if inflation increases, under fairly plausible values for the variables describing the structure of the economy, and for relatively elastic short run Phillips curves, the reallocation of resources due to the increase in the labor supply can generate excess demand for consumption goods, which will only be eliminated if prices rise further. Even without further empirical work it is clear that this mechanism will not lead to self sustaining inflation. The elasticity of the Phillips curve will tend to zero, as inflation proceeds and the supply of labor will stop increasing. But, under the circumstances defined above, this result reveals that some of the real effects of inflation work towards making it persist for a while, even if no other inflationary events occur - such as increases in the money supply. It can therefore be another contribution to the explanation of why inflation takes so long to control. And if it is assumed that the monetary authorities are reluctant to stop the growth of the money supply when prices are increasing rapidly - because of the inevitable costs of such a policy - the results presented here acquire additional relevance. The interaction of a policy of monetary

expansion which to a certain extent is a consequence of the increase in prices, with a reallocation of the resources leading to further growth in prices goes a long way to explain the persistence of inflation

5. A MORE DETAILED TREATMENT OF EXPECTATIONS

Up to this moment no distinction was made between the actual and the expected rate of inflation. It was assumed that the current observed rate of inflation was the appropriate argument in the demand for assets functions and the correct variable to determine changes in labor supply. It was also assumed that consumers included in disposable income any capital gains or losses resulting from inflation, again measured by the actual rate of increase in prices. This corresponds to a large extent to the neo-Keynesian view of some degree of money illusion on the part of workers, in contrast with perfect adjustment of expectations in the savings behavior of consumers and in portfolio choice. Even if there are empirical grounds to accept this asymmetry,[9] it will be useful to explore the implications of some alternative assumptions about expectations.

5.1 CAPITAL GAINS NOT INCLUDED IN DISPOSABLE INCOME

A first alternative treatment of expectations consists in assuming that money illusion is present not only in labor markets but also in

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[9]Using data for the U.S. post war economy Hall (1979) finds that an increase in the money stock shifts the labor supply curve.

consumers perception of their disposable income. Capital gains and losses in the real value of the government debt held by the public will not influence the demand for consumption goods. Then, equilibrium in the goods market will be given by

$$q_c(k,p_k) - e = (1-s)\,[q_c(k,p_k) + p_k q_i(k,p_k) - e + p_m d] \qquad (62)$$

and excess demand for consumption goods will always be a result of an increase in the rate of inflation since

$$dZ = \left[(1-s)p_k q_i \frac{N_i+N_c}{N_i}\frac{k_c}{k_c-k_i} + s\frac{N_i+N_c}{N_c}\frac{k_i}{k_c-k_i}q_c\right] \varepsilon\, d\pi_m > 0 \qquad (63)$$

will always be satisfied - unless $\varepsilon=0$, implying a vertical Phillips curve.

In this case, increases in the rate of inflation decrease the supply of consumption goods, because of the Rybczinski effect, while increasing the demand, since total income goes up because of the increase in the labor force. The potentially offsetting effect, consisting in increasing saving to compensate for the additional losses in the real value of the government debt, is not present. The economy will be less stable since any increases in inflation will lead to further incentives for price rises, which will be stronger than before and which will occur always, up to the point where the short run Phillips curve becomes vertical.

5.2 THE INFLATION TAX AND THE DEFICIT

An alternative specification of consumers behavior consists in including in disposable income some measure of the impact on the government s position of the inflation tax. In contrast to the previous assumption of money illusion with respect to capital gains and losses this assumes that consumers are quite aware that their losses are the government's gains which must in some way be reflected in reduced tax liabilities, or increased spending. Then consumers would naturally expect the deficit to increase with inflation, and a possible way of modeling expectations to reflect this attitude consists in assuming that the real value of the deficit less any capital gains made by the government is constant, or

$$\delta = p_m(d+g\pi_m) \qquad (64)$$

where δ is a constant.

In fact, in the long run the steady state requires that the government debt increase just enough to match population growth - so that its real value per capita remain constant. But if the inflation tax is duly taken into account the steady state value of the deficit must be such that

$$p_m(d+g\pi_m) = p_m gn \qquad (65)$$

where n is the rate of growth of the population. And an inverse relationship between d and π_m - or a direct relationship between the deficit

and the rate of inflation - can be established as an adequate long run basis for consumers expectations.

In this context, consumers disposable income will consist of

$$y = q_c(k,p_k) + p_k q_i(k,p_k) - e + \delta \qquad (66)$$

where δ is independent of the rate of inflation. Capital gains and losses due to inflation are again excluded from disposable income, not because of money illusion, but because consumers adequately perceive their symmetric impact on the government position.

The consequences are similar to those obtained above: inflation does not contribute to increase saving, while at the same time reducing the supply of consumer goods. Equilibrium requires higher p_m and more inflation follows.

5.3 A "NATURAL RATE" LABOR MARKET

A different result is obtained when the assumption of money illusion in the labor market is ruled out. If the increase in the labor supply as a result of an increase in the rate of inflation is only temporary - remaining only as long as the higher rate of inflation is not anticipated - and if employment tends to return to its natural level, the impact on the excess demand for consumption goods will also vanish as soon as expectations adjust. The economy becomes more stable, not only in the long run but also in the short run since the impact of inflation in the goods market through the Rybczinski effect will cease before the as-

sets markets adjust. This is presented graphically in Figures 6 and 7. In the short run - Figure 6 - the higher rate of inflation is not anticipated, leading to an increase in labor supply, but no changes in the expected rates of return on assets. The cc curve shifts to the left always - since consumers have not adjusted their disposable incomes to account for the higher inflation - but the aa curve remains in the same position. As a result, p_k drops and so does p_m, but less than in the case where the aa curve also shifts.

In the long run - Figure 7 - the Rybczinski effect has vanished and consumers and wealth owners have adjusted to the new rate of inflation. Both curves shift up, as in the analysis done with the original Foley-Sidrauski model. The impact on p_m is indeterminate as before.

In this case the results obtained here will be relevant to the short term only. But since expectations adjust slowly, the increase in the excess demand for consumption goods can be an important component of the behavior of the economy after an increase in the rate of inflation.

6. CONCLUSIONS

The issue addressed in this work is the non-neutrality of inflation and the exploration of some of the effects inflation has on the real side of the economy. The results obtained show that these effects contribute to an explanation of the persistence of inflation, even beyond the point when monetary expansion is no longer pursued.

The non-neutrality of inflation has been ascribed to its impact on assets demand. If prices rise, wealth owners attempt to move away from cash balances and into physical assets, leading to increases in the price of capital. Often, inflation has been considered favorable to growth, because of the increased investment resulting from the higher price of capital.

Another basis for the non-neutrality of inflation is its influence on the labor market. Even though workers will sooner or later adjust their expectations to the higher rate of price increase, increases in inflation usually lead to an expansion of labor supply in the short run. In a multisector framework such an increase in the supply of a production factor will have an asymmetric impact on the different sectors of production. In particular, if the production of consumer goods is assumed to be relatively more capital intensive - the so-called normal intensity case - a direct application of Rybczinski's theorem shows that the supply of consumer goods will shrink.

Either in the short run only, until it becomes perfectly anticipated, or in the long run, if workers money illusion persists, inflation may lead to excess demand for consumption goods, because of its impact on the supply side of the economy. If such a hypothesis is confirmed by empirical work, an additional explanation of the strength and continuity of inflationary pressures may become available.

REFERENCES

Aukrust, Odd (1970), "PRIM I: A Model of the Price and Income Distribution Mechanism of an Open Economy", Review of Income and Wealth, Vol. 16, pp. 51 - 78.

Edgren, G.; Faxen, K-O.; and Odhner, C-E. (1973), Wage Formation and the Economy, London: Allen and Unwin.

Foley, D. K. and Sidrauski, M. (1971), Monetary and Fiscal Policy in a Growing Economy, New York: Macmillan.

Friedman, Milton (1970), The Counter Revolution in Monetary Theory, London, IEA Occasional Paper, No. 33.

Hall, R. E. (1979), "Labor Supply and Aggregate Fluctuations", NBER Working Paper No. 385.

Laidler, D. and Parkin, M. (1975), "Inflation: A Survey", The Economic Journal, Vol. 85, pp. 741 - 809.

Lucas, R. E. (1972), "Expectations and the Neutrality of Money", The Journal of Economic Theory, Vol. 4, pp. 103 - 124.

Lucas, R. E. and Rapping, L. A. (1969), "Real Wages, Employment and Inflation" The Journal of Political Economy, Vol. 77, No. 5.

Rybczinski, T. M. (1955), "Factor Endowment and Relative Commodity Prices", Economica, pp. 336 - 341.

Stolper, W. F. and Samuelson, P. A. (1941), "Protection and Real Wages", Review of Economic Studies, November 1941.

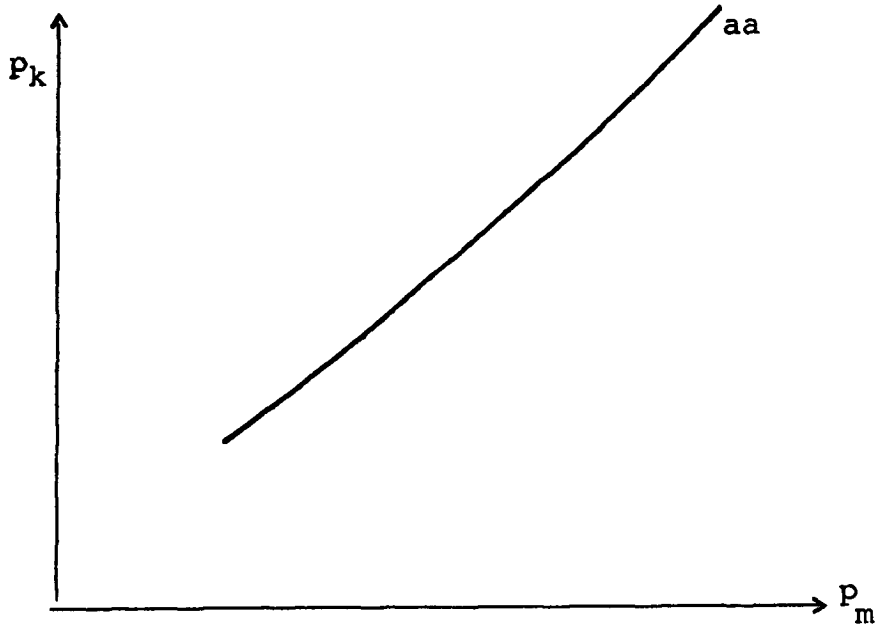

Figure 1 - Equilibrium in assets market

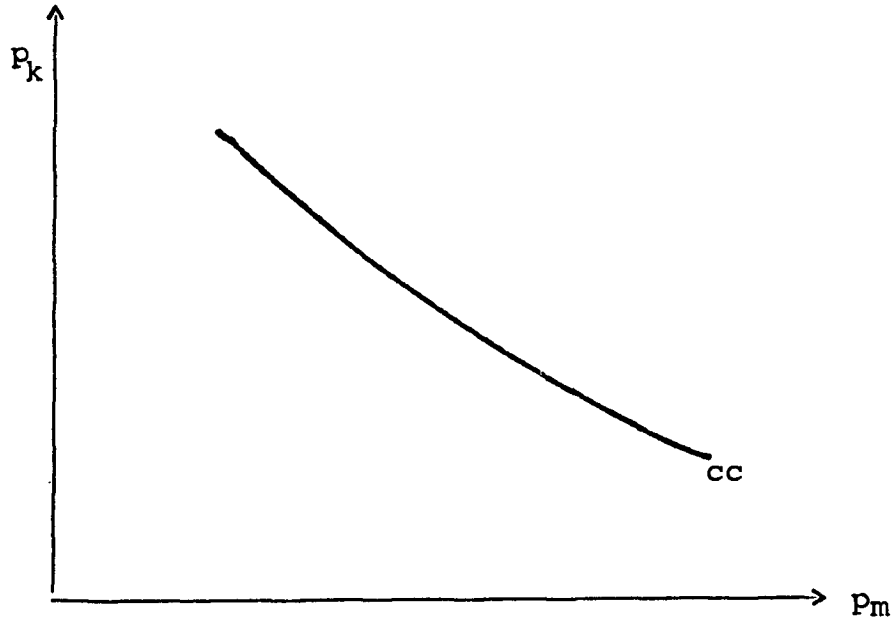

Figure 2 - Equilibrium in goods market

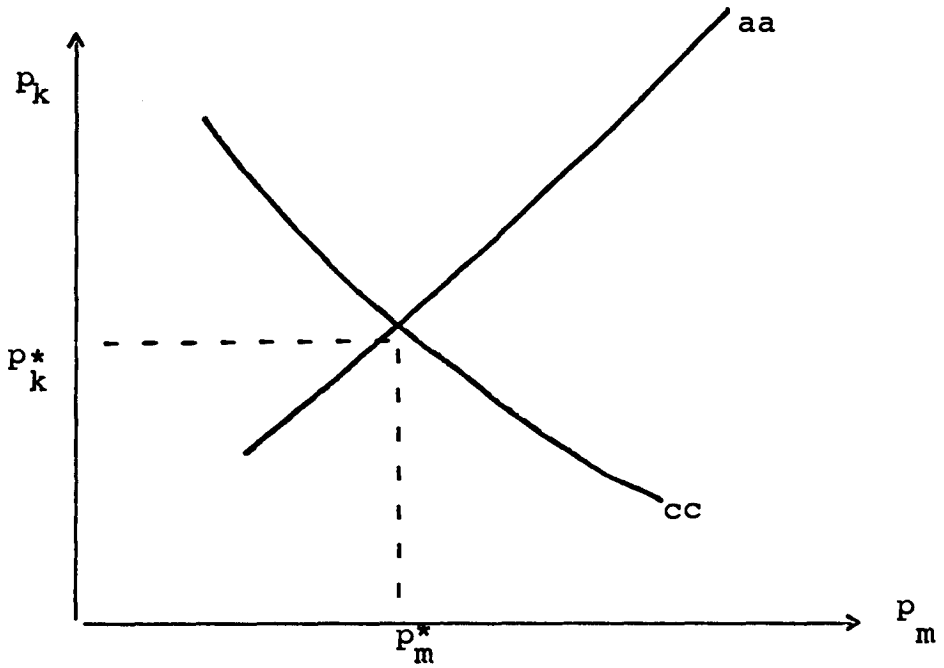

Figure 3 - The equilibrium values of p_k and p_m

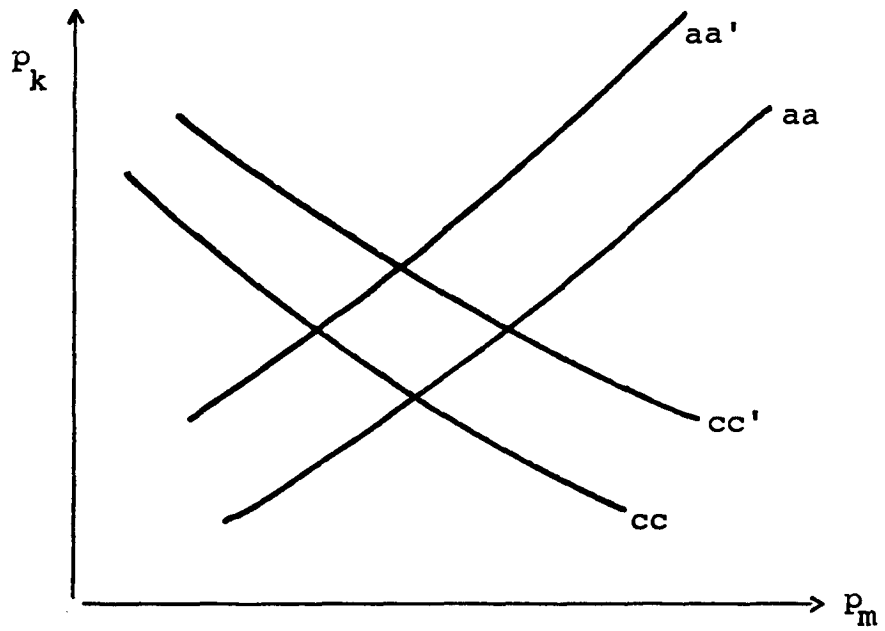

Figure 4 - The impact of an increase in the rate of inflation

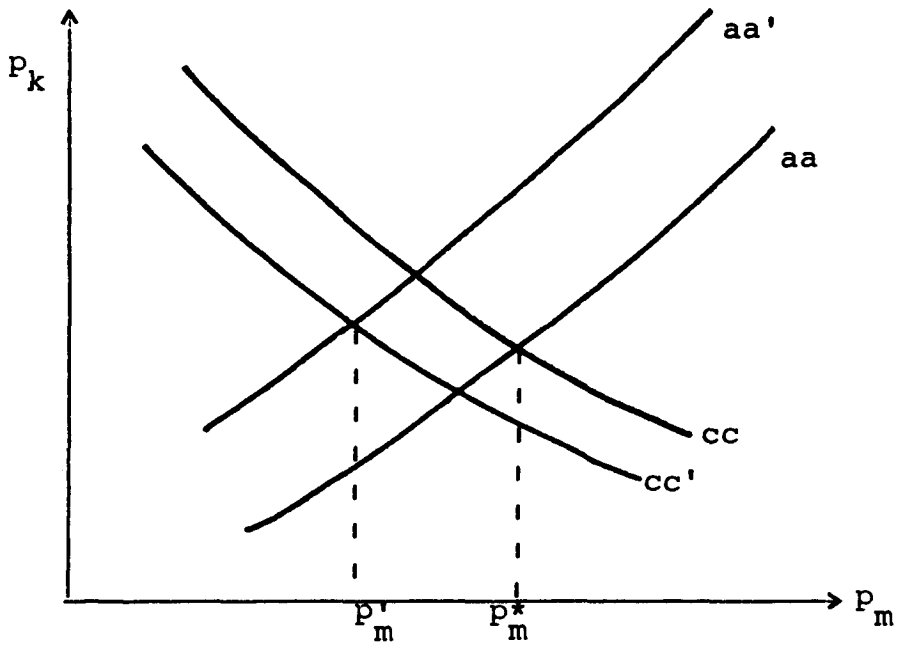

Figure 5 - An increase in the rate of inflation generating excess demands for consumption goods

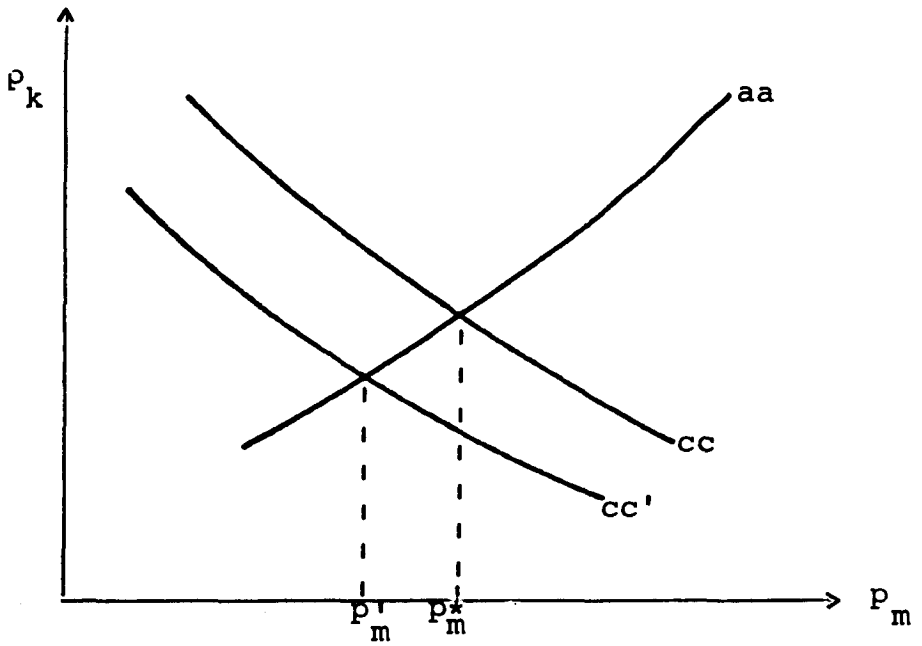

Figure 6 - The impact of a higher and unanticipated rate of inflation

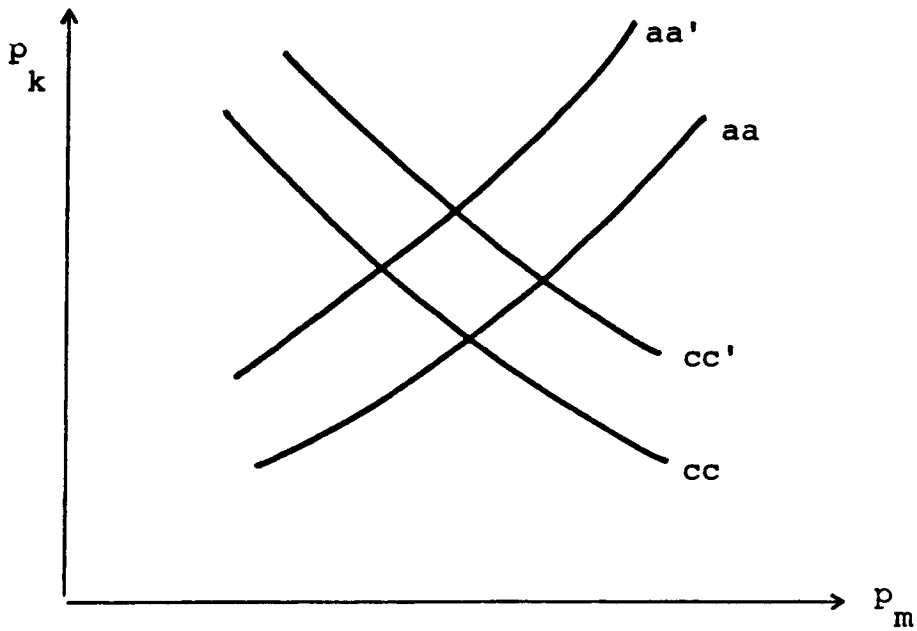

Figure 7 - The impact of a fully anticipated increase in the rate of inflation

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