SYSTEMATIC RISK, THE INVESTMENT HORIZON AND THE MARKET INDEX: AN ANALYTICAL EXAMINATION

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The effect of changes in the length of the return interval (the investment horizon) on the estimated value of a security's beta coefficient (systematic risk) has been investigated in several recent studies.

Theoretically, two results have been established. Levhari and Levy [6] demonstrate that under condition of time independence with multiplicative returns,[1] aggressive securities ($\beta > 1$) should exhibit a rising beta whereas defensive securities ($\beta < 1$) should display a declining beta as the investment horizon is increased. Using an alternative model, which assumes that returns are additive[2] and exhibit significant time interdependence, Hawawini [3,4] concludes that the behavior of beta in response to a change in the investment horizon depends on the magnitude of a security's q-ratio (defined in section I) relative to the autocorrelation coefficient of the market index. Securities with high (low) q-ratios relative to the market index autocorrelation coefficient will have rising (falling) betas as the investment horizon is lengthened.

Empirically, several results have been reported. Levhari and Levy [6] confirm their theoretical conclusion using a sample of 20 securities and investment horizons of one to 18 <u>months</u>. The Levhari and Levi proposition is also confirmed by Smith [8] using 200 securities and monthly reruns. Hawawini [3] presents evidence which supports his model using a sample of 50 securities and investment horizons of one to 20 days. There may be no inconsistency between these two sets of empirical results since Levhari and Levy's work as well as that of Smith are based on monthly investment horizons whereas Hawawini's work is based on submonthly returns; the independence assumption may hold over monthly returns but not over daily returns.

Recently, however, Hawawini and Vora (5), using a large sample of 1,115 securities, show that the Levhari-Levy proposition is not supported by the data even when investment horizons of a month and longer are employed. They present evidence that clearly indicates that changes in beta induced by a change in the investment horizon tend to follow a random-walk around an upward or downward market trend rather than display the systematic rise or fall predicted by Levhari and Levy. Finally, Saniga, McInish and Gouldey (7) report in this Journal that the sensitivity of beta to changes in the investment horizon is dependent upon the choice of the market index. They show that "for the value weighted and the Standard and Poor index, beta generally increases with differencing interval length; for the equally weighted index, beta generally decreases with interval length." Of all the empirical findings reported in the literature, these two most recent studies should be considered a reliable description of the actual behavior of securities' beta since they are both based on samples consisting of all continuously traded stocks listed on the CRSP tapes as opposed to previous studies which were based on small and selective samples of 20, 50 or 200 securities.

The purpose of this note is to clarify the current debate regarding the shifting behavior of beta, not by adding a new piece of empirical work to the list we have just surveyed but, rather, by presenting a theory based analysis of the empirical phenomena reported by Hawawini and Vora [5] and Saniga et al. [7]. Specifically, in section I, we present a model which predicts that individual securities' beta may exhibit a random-walk around a trend in response to changes in the investment horizon. In section II, we show how the choice of the market index affects the relationship between beta and the investment horizon exactly as reported by Saniga et al. [7]. Section III is a short conclusion.

I. Impact of the Investment Horizon

The following derivations are extensions of Hawawini's work [3,4]. Assuming that returns are measured as logarithm of investment relatives and allowing for the presence of market index autocorrelation as well as intertemporal cross correlations between securities' returns, we can write:[3]

$$\beta(T) = \beta(1) \left(\frac{T + \sum_{S=1}^{T-1}(T-S)q_S}{T + 2\sum_{S=1}^{T-1}(T-S)\rho_S}\right) \qquad (1)$$

where $\beta(T)$ is the beta coefficient of security i calculated over an investment horizons of T-day (or T-month if the shortest interval is a month), ρ_S is the daily market index autocorrelation coefficient of order (S) and q_S is the daily q-ratio of order(s) of security i. For the sake of clarity, the subscript i does not appear in equation (1).

The q-ratio is defined as $(\rho_{im}^{-S} + \rho_{im}^{+S})/\rho_{im}$, that is, the sum of the intertemporal cross-correlation of order (-S) between the returns of security i and those of a market index m [i lags m] and the intertemporal cross-correlation of order (+S) between the returns of security i and those of a market index m [i leads m], divided by the contemporaneous cross-correlation coefficient.

Now, for the purpose of illustration, consider the case where only first and second order correlations are non-zero.[4] Equation (1) can be rewritten as:

$$\beta(T) = \beta(1) \left[\frac{(1+q_1+q_2)T - (q_1+2q_2)}{(1+2\rho_1+2\rho_2)T - 2(\rho_1+2\rho_2)}\right] \equiv \beta(1)\frac{\alpha}{\gamma} \qquad (2)$$

where α and γ designate the numerator and the denominator of the bracketed term, respectively. Suppose that the investment horizon T is changed by ΔT. The resulting percentage change in $\beta(T)$ can be obtained by taking the logarithmic derivative of equation (2). As a first result, we have:

$$\frac{\Delta\beta(T)}{\beta(T)} = (\alpha\gamma)^{-1} \left[(q_1-2\rho_1) + 2(q_2-2\rho_2) + 2(q_2\rho_1-q_1\rho_2)\right]\Delta T \qquad (3)$$

Assuming that $\beta(T) > 0$ and hence $(\alpha\gamma) > 0$, then the sign of $\Delta\beta(T)/\beta(T)$ is that of the bracketed term in equation (3). To see how equation (3) describes the observed random-walk behavior of $\beta(T)$ reported by Hawawini and Vora [5], consider the following numerical example. Say $q_1 = .80$, $\rho_1 = .30$, $q_2 = .10$ and $\rho_2 = .15$.[5] When T varies from one to two we get:[6]

$$\frac{\Delta\beta(T)}{\beta(T)} = (\alpha\gamma)^{-1} (q_1-2\rho_1) = .20(\alpha\gamma)^{-1} > 0 \qquad (4)$$

The percentage change in $\beta(T)$ is positive, that is, $\beta(T)$ rises with T. Now, if we increase the investment by an additional day, from two to three days, we have

$$\frac{\Delta\beta(T)}{\beta(T)} = (\alpha\gamma)^{-1}[(q_1-2\rho_1)+2(q_2-2\rho_2)+2(q_2\rho_1-q_1\rho_2)] = -.30(\alpha\gamma)^{-1}<0 \quad (5)$$

The percentage change in $\beta(T)$ is now negative, that is, $\beta(T)$ falls after having previously risen. Of course, we can generalize the process by considering that nonzero correlation coefficients exist up to, say, the order $k < T$ and show that, by increasing T stepwise from $T = 1$ to $T = k+1$, $\beta(T)$ will either rise, remain the same or fall depending upon the magnitudes and signs of the pattern of correlation coefficients. We should add that although the model cannot show that the behavior of $\beta(T)$ conforms exactly to a random-walk, it nevertheless tells us that $\beta(T)$ can vary in any direction as T increases. One can expect, a priori, that upward and downward shift in beta would be equally probable as confirmed by empirical work.

II. Impact of the Market Index

Let us now turn to the analytical examination of the effect of the nature of the market index on the shifting behavior of beta. Samiga et al. [7] report that the choice of the market index affects the relationship between beta and the investment horizon. As the investment horizon increases, beta rises when the Standard and Poor index is used; it remains constant or rises slightly when the value-weighted CRSP composite index (all securities on tape) is employed; and it falls when the equally-weighted CRSP composite index is utilized.

Thus empirical phenomenon is predicted by the model presented in the previous section.

For the sake of illustration, assume that only first order daily correlations are present in the data. In this case, equation (4) holds:

$$\frac{\Delta\beta(T)}{\beta(T)} = (\alpha\gamma)^{-1} (q_1 - 2\rho_1) \tag{4}$$

The sign of equation (4) and consequently the shifting behavior of $\beta(T)$ will depend on the magnitude of ρ_1 relative to q_1. Now, q_1 is practically not affected by the nature of the index[7] whereas ρ_1 is very sensitive to the type of index employed. Specifically, it has been shown by Cohen et al. [1] and others [3,4] that: (1) Of two indexes constructed with the same weighing scheme, the broader index will generally exhibit a higher autocorrelation; and (2) Of two indexes constructed with the same number of securities the value-weighted index will generally exhibit a higher autocorrelation. Essentially, broader indexes tend to pick up securities with narrow markets and infrequent trading which do not move synchronously with the group of active stocks, hence inducing higher autocorrelation. Likewise, equally weighted indexes assign relatively higher weights to thinly traded securities with small market value whereas value-weighted indexes are heavily weighted by securities with frequent trading and large market value. This, again, induces higher autocorrelation in equally-weighted indexes than in value-weighted indexes.

What is the implication of the above for the shifting behavior of beta? The answer is found by referring to equation (4). To illustrate, suppose $q_1 = .60$ for security i and that the market autocorrelation coefficient is .20 for the Standard and Poor (the narrower index), $\rho_1 = .30$ for the value weighted index (the broader index), and $\rho_1 = .35$ for the equally-weighted index (also a broader index).[8] Applying these values to equation (4) we find that $\Delta\beta(T)/\beta(T)$ is positive (beta rises with T) in the case of the Standard and Poor, it is zero in the case of the value-weighted index (beta remains practically constant), and it is negative in the case of the equally-weighted index (beta drops where T rises). This is exactly what Saniga et al. [7] report. It is fully predicted by the model described in the previous section.

II. Conclusion

The purpose of this note was to shed some light on the underlying factors responsible for the reported behavior of beta in response to changes in the investment horizon and the nature of the market index. We have presented a model which can explain and predict the shift in beta resulting from a lengthening of the investment horizon and the choice of the market index.

Footnotes

[1] When rates of returns are measured in percentage returns, retruns measured over longer differencing intervals are equal to the product of returns measured over shorter differencing intervals.

[2] When rates of returns are measured as logarithm of investment relatives, returns measured over longer differencing intervals are equal to the sum of returns measured over shorter differencing intervals.

[3] For a derivation of equation (1), the reader is referred to [3] and [4].

[4] Empirical evidence presented in [4] indicates that this situation is typical of actual correlation patterns for securities.

[5] These numbers are consistent with observed values. See [3] and [4].

[6] Note that the second and third terms within brackets in equation (3) do not appear in equation (4). This is because when T varies from one to two days there are no second order correlations since second order correlations involve relationships between first and third days.

[7] If two indexes (x and y) are perfectly positively correlated then $q_1^y = q_1^x$ since q-ratios involve correlation coefficients. We can assume that the three indexes discussed in this study display close to perfect correlation coefficients.

[8] These numbers are consistent with observed values. See, for example, Francis [2], p. 680.

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