THE ZERO-ROOT PROBLEM : DYNAMIC DETERMINATION OF THE STATIONARY EQUILIBRIUM IN LINEAR MODELS

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FRANCESCO GIAVAZZI AND CHARLES WYPLOSZ

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Directeur de la Publication :

Jean-Claude THOENIG
Associate Dean: Research and Development
INSEAD

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DYNAMIC DETERMINATION OF THE STATIONARY EQUILIBRIUM

IN LINEAR MODELS

Francesco Giavazzi
University of Essex and Università di Venezia

Charles Wyplosz
INSEAD, France

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Introduction

It is a well known problem in the theory of optimal growth with many, infinitely lived consumers that the convenient assumption of recursive and time-additive preferences of the form $u(\mathbf{c}) = \Sigma_t \delta^t u(c_t)$ generates, in the steady state, either an indeterminacy or a corner solution, depending on whether the positive, constant rates of time preference are all identical or differ across consumers. If they differ, the consumption of all but the most patient consumer will converge to zero. If they are identical, the distribution of utility in the steady state is indeterminate because agents will consume the permanent income generated by whatever wealth level they reach, independently of the level of utility.

The same problem arises in the open economy framework, where the choice of the optimal consumption path leads to an indeterminacy of the stationary equilibrium in two situations. In the "small-country" case - a single agent who can borrow or lend to the rest of the world at an exogenously fixed interest rate - if the domestic rate of time preference is equal to the world interest rate, the level of external debt in the stationary equilibrium is indeterminate; otherwise we have a corner solution. On the other hand, models incorporating more than one country, are really reproductions of the case of many consumers. Hence, if the rates of time preference are identical across countries, in the stationary equilibrium the distribution of wealth across countries is indeterminate.

As a consequence these models, which typically exhibit the strict saddle point property, seem to admit an infinity of convergent paths. This indeterminacy is different from the well known problem associated with saddle-path stability, namely that one needs additional conditions, such as ruling out explosive solutions, in order to identify a unique convergent path. Here the problem arises from the fact that the models are short of at least one stationary state condition: for example in the two-country case, in the stationary equilibrium current account and goods markets equilibrium conditions reduce to a single equation.

The purpose of this paper is to show that this indeterminancy is only apparent. We show that if explosive paths are ruled out, the stationary equilibrium of this class of models is unique and depends on both the initial conditions and the approach path, so that parameters which characterize the dynamics of adjustment have a permanent effect on the economy.[1]

The paper gives a formal proof of this property when the dynamics of the model can be cast in the form of a linear differential equations system of the type $\dot{x} = Ax - Z$. In this case the apparent indeterminacy of the stationary equilibrium arises from the singularity of the state transition matrix A: i.e., since A^{-1} does not exist, it is impossible to solve directly for the stationary equilibrium $\bar{x}$. The essence of the proof is a generalization of the solution of linear differential equations systems under perfect foresight given in Blanchard and Kahn (1980) and Buiter (1981).

In models derived under the assumption of time additive preferences, the dynamic dependence of the stationary equilibrium is a transparent consequence of time additivity: in this case the paper makes the point that time additivity, often and perhaps correctly criticized as an assumption of convenience with little economic rationale, (see for example Lucas and Stockey [1982]) not only should not be dismissed on the grounds of indeterminacy, but yields an interesting and intuitive property. Two examples of this class of models, both taken from the open economy framework, are described in section 2.

Our result, however, is more general and may apply to a wider class of models, as shown for instance in the third example of section 2. There we show that the time honored remark by Wicksell (1907) that the price level is indeterminate if monetary policy pegs the nominal rate of interest is another manifestation of the phenomenon described here.

2. Examples

2.1 Intertemporal Optimization in a Small Open Economy

The problem has been described by Blanchard (1981). The example is as simple as can be to illustrate the role of intertemporal optimization, and essentially extends Ramsey's problem to the case of an open economy. The representative agent consumes and produces a unique good which is also used for investment. This good is traded and its price is fixed at the world level. Spending is the sum of consumption, E_t, and investment I_t. Following Abel (1979), investment spending is the sum of investment, $\dot{K}_t$, plus "installation costs" $\Psi(\dot{K}_t)$:

$$(2.1) \qquad I_t = \dot{K}_t[1+\psi(\dot{K}_t)], \ \Psi' > 0, \ \Psi'' \geq 0.$$

The representative agent can borrow from, or lend to, the rest of the world at a fixed interest rate δ, and the excess of spending plus interest on debt, δZ_t, over output, $y(K_t\bar{L})$, is equal to the current account deficit, itself equal to the change in debt $\dot{Z}_t$. The intertemporal utility function is time-additive and the subjective rate of time discount is equal to the world interest rate. Accordingly, the problem is:

$$(2.2) \qquad \max_{E,\dot{K}} \int_0^\infty e^{-\delta t} u(E_t)dt$$

subject to (2.1) and

$$(2.3) \qquad \dot{Z}_t = \delta Z_t + E_t + I_t - y(K_t,\bar{L})$$

The solution is best described by defining net wealth as (the time index is omitted whenever possible):

$$W = A - Z$$

where gross wealth is:

$$(2.4) \qquad A_t = \int_t^\infty e^{-\delta(s-t)}(y_s - I_s)ds, \text{ or } \qquad \dot{A} = \delta A - (y-I)$$

The solution implies a constant stream of spending:

$$E_t = \bar{E} = \delta W$$

so that net wealth W is also maintained constant through borrowing or lending abroad, as domestic gross wealth fluctuates.

The complete law of motion can be described as follows:

$$\begin{aligned}
\dot{q} &= \delta q - y_K(K,\bar{L}) - (q-1)^2/2\phi \\
\dot{K} &= (q-1)/\phi \\
\dot{A} &= \delta A - y(K,\bar{L}) + (q^2-1)/2\phi \\
\dot{Z} &= \delta Z + \delta(A-Z) + (q^2-1)/2\phi - y(K,\bar{L}) = \dot{A}
\end{aligned} \tag{2.5}$$

where q is the shadow price of installed capital, and where we choose to describe the cost of investment $\Psi(\dot{K}) = \phi\dot{K}/2$. Implicit in (2-5) is the assumption that the return on domestic capital is equal to the foreign interest rate δ.

Upon inspection of (2.5), we see that the stationary equilibrium is well defined for q, K and A (a bar denotes the stationary equilibrium value):

$$\bar{q} = 1$$

$$y_K(\bar{K},\bar{L}) = \delta, \qquad \text{which determines } \bar{K}$$

$$\bar{A} = y(\bar{K},\bar{L})/\delta$$

while there is no definite value for $\bar{Z}$: the steady state level of debt appears to be arbitrary. The reason for this indeterminacy is easily understood. In the steady state, optimal consumption is equal to output less interest payment on debt, so that constraint (2.3) is automatically satisfied. However, as will be shown formally in section 3, following a disturbance, Z will move to a well defined steady state value, which will depend upon the initial debt level Z_0, and the speed of adjustment of the economy, as characterized by installation costs parameter ϕ.

This can be seen in figure 1, where we present the projection of the adjustment path in the (q,Z) space. The SS line is the set of <u>a priori</u> possible positions in the steady state. With initial debt Z_0, we start at point A, where

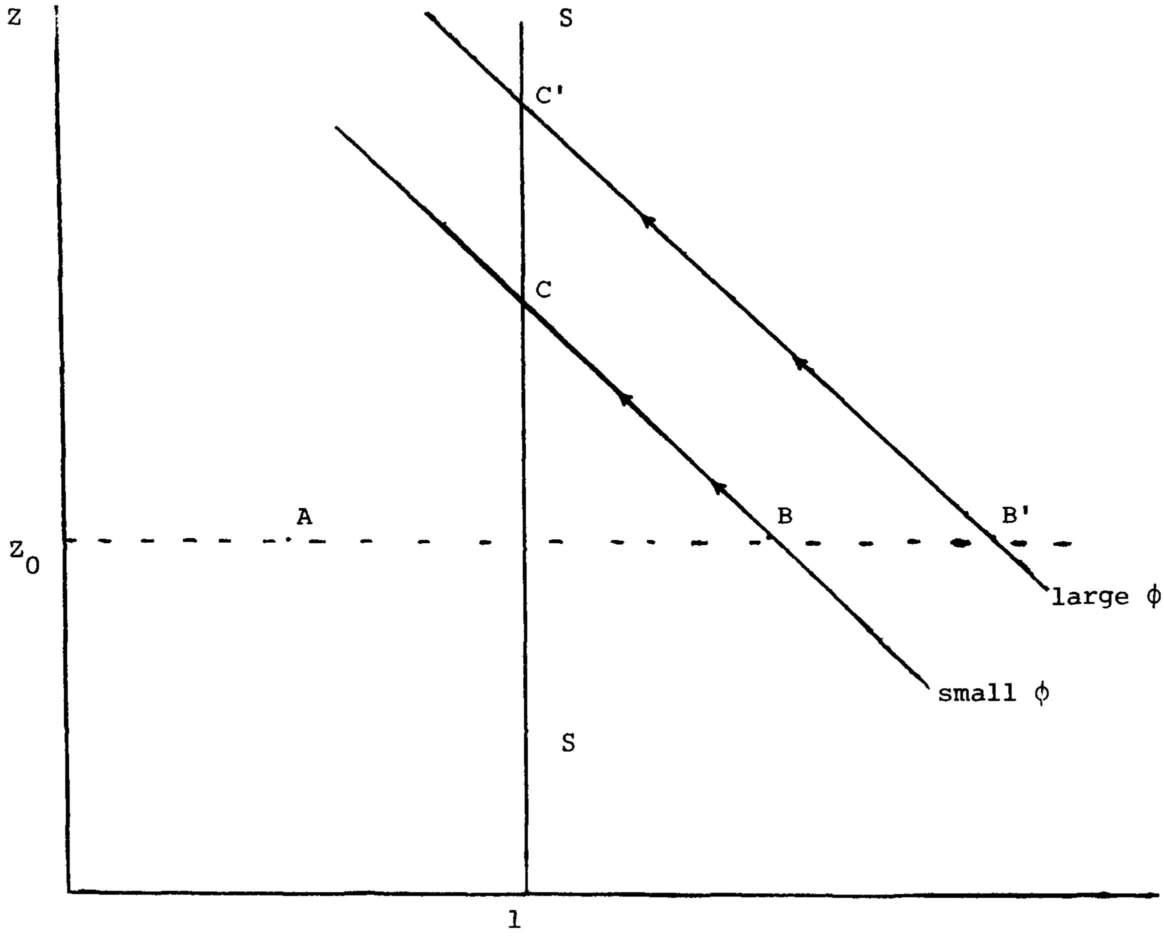

Figure 1

the capital stock is below its steady state level, and depending on ϕ, we jump to B or B', and then proceed along the convergent path of capital accumulation. If we "consume" a large amount of goods in the process of investing (large ϕ), we will incur a larger, and more lasting, current account deficit, leading to a larger steady state debt, as is shown by point C'.

It is interesting to note that the apparent indeterminacy does not occur in a closed economy, where both Z_0 and $\dot{Z}$ are, by assumption, equal to zero. On the other hand, in a two-country model, the indeterminacy extends to the exchange rate, as is shown in the next sub-section.

2.2 Intertemporal Optimization in a Two-Country Model

The problem has been described by Giavazzi and Wyplosz (1982). There are two countries which trade in goods and assets with each other. Each country faces the same intertemporal problem characterized in the previous section, but now there are two goods, consumption preferences differ across countries, and, in order to contrast the previous example, we neglect capital accumulation, concentrating on the labor market. Real wages (w) are rigid and slowly adjust to restore full employment, $(\bar{L})$, while actual employment (L) is demand determined:

$$(2.6) \qquad w = \gamma(L-\bar{L}) \quad ; \quad \dot{w}^* = \gamma^*(L^*-\bar{L}^*)$$

$$(2.7) \qquad wL = y_L(\bar{K},L) \quad ; \quad w^*L^* = y^*_{L^*}(\bar{K}^*,L^*)$$

where γ and γ^* capture the speed of adjustment of w and w* (a star refers to the foreign economy). With a Cobb-Douglas instantaneous utility function, in each period spending on local goods (respectively, C and C*) is a constant share (respectively, a and $(1-a^*)$) of total consumption (respectively, E and E^*)

$$C = aE \qquad C^* = (1-a^*)E^* \tag{2.8}$$

$$\lambda C_m = (1-a)E \qquad \lambda^{-1}C_m^* = a^*E^*$$

where C_m and C_m^* represent consumption of imported goods and λ is the relative price of the two goods, the real exchange rate. As in the previous example, we have:[2]

$$E = \delta W \qquad E^* = \delta W^* \tag{2.9}$$

Trade in assets takes the form of indexed bonds, claims to units of the issuing country's output. The yields on the two bonds are equalized and their flow $\dot{Z}$ reflects current account imbalances:

$$r = r^* + \dot{\lambda}/\lambda \tag{2.10}$$

$$\dot{Z} = rZ + E - y \tag{2.11}$$

where Z represents net domestic indebtness.

Finally, gross wealth, A and A^*, is defined as in (2.4) (with zero investment) and we assume continuous goods market equilibrium.

In the steady state, $r = r^* = \delta$ and A and A^* are well defined:

$$\bar{A} = y(\bar{K},\bar{L})/\delta \quad , \quad \bar{A}^* = y^*(\bar{K}^*,\bar{L}^*)/\delta$$

while the net domestic debt and the real exchange rate are linked by the condition:

$$\bar{\lambda} = \frac{(1-a)\bar{A} + (a-a^*)\bar{Z}}{a^*\bar{A}^*} \tag{2.12}$$

so that, given different consumption preferences ($a \neq a^*$), any pair $\bar{\lambda},\bar{Z}$, satisfying (2.12) is consistent with a steady state equilibrium . For example, following an increase in domestic real wages, employment, and therefore output, will fall. If spending decreases by less than output, the adjustment path will be characterized by falling real wages and a current account deficit. A very rigid labor market will imply a prolonged current account deficit and hence, in the new steady state, a redistribution of wealth towards the foreign country. The real exchange rate will then reflect the new pattern of world demand.

2.3 The Indeterminacy of the Price Level

The problem goes back at least to Wicksell (1907). It can be presented with the Cagan model:

$$(2.13) \quad m - p = -\alpha i + \beta\bar{y}$$

$$(2.14) \quad i = \bar{r} + \dot{p}$$

where m and p are the logarithms of the money stock and the price level, i the nominal interest rate and $\bar{r}$ the real interest rate, set to be constant and equal to the productivity of capital as we assume full employment at the natural level of output $\bar{y}$ (in logarithm). As is well known, if in this model the monetary authorities follow a constant interest rate policy, $i = \bar{i}$, the price level is indeterminate unless they also set the money stock m_0 at time $t = t_0$ and announce that the future growth rate $\dot{m}$ will be that one implied by m_0 and $\bar{i}$. This exactly corresponds to what we show in the next section: because the dynamic system described by (2.14) has a singular transition matrix (actually the matrix here is identically equal to zero) its transition path and steady state is not unique and depends upon the initial position of the economy (the value of m_0 and hence of p_0).

3. General Formulation and Solution

In this section we state the problem in its general form and give a formal solution. We first note that while the zero-root property means that the variables share among themselves one degree of freedom, a subset of variables may actually have a uniquely determined stationary state value, so that the non-uniqueness is confined to the remaining subset. We shall refer to the former variables as stationary state invariant. This is in line with the examples presented in section 2 where Tobin's q, the capital stock and gross wealths are stationary state invariant.

The general form of a first-order linear system is $\dot{x} = Ax - Z$, where x is the n-vector of endogenous variables, Z an n-vector of exogenous variables and A is the (nxn) transition matrix. The indeterminacy of the stationary state obtains when A is singular. Accordingly we assume that its rank is n-1.[4] As stated above, we partition x into two subsets: the k first elements, which we call x_1, are stationary state invariant, the last (n-k) ones, called x_2, are those which share the degree of freedom. We shall first characterize the restrictions on A for this to happen, and then proceed to the solution. The model is, therefore, rewritten as:

$$(3.1) \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}$$

3.1 Conditions for x_1 to be Uniquely Determined in the Stationary Equilibrium

In the stationary equilibrium when, $\dot{x} = 0$, the system (3.1) becomes:

$$(3.2) \quad \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_1 \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}$$

As A is singular, we know that (3.2) admits an infinity of solutions, or none at all. We want to find conditions such that $\bar{x}_1$ is uniquely determined. We assume that A can be diagonalized, so that there exists a matrix V of right eigenvectors of A such that $V^{-1}AV = \Lambda$ with:

$$\Lambda = \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_1 \end{bmatrix} \quad \text{and } V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

where Λ_1 is an (n-1) x (n-1) diagonal matrix, where the diagonal terms are the non-zero eigenvalues of A. We order V and Λ such that the first term along the diagonal of Λ is zero, followed by all the eigenvalues with negative real parts, then by all the eigenvalues with positive real parts. Then the column vector $\begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix}$ is the eigenvector associated with the null eigenvalue. Using the transformation:

$$x = Vy \qquad \text{with } y = \begin{bmatrix} y_0 \\ y_1 \end{bmatrix},$$

(3.2) becomes:

$$\Lambda\bar{y} = V^{-1}z \qquad \text{with } V^{-1} = \begin{bmatrix} \bar{V}_{11} & \bar{V}_{12} \\ \bar{V}_{21} & \bar{V}_{22} \end{bmatrix}, \quad \bar{y} = \begin{bmatrix} \bar{y}_o \\ \bar{y}_1 \end{bmatrix}$$

where $[\bar{V}_{11} \ \bar{V}_{12}]$ is the row left-eigenvector corresponding to the zero eigenvalue. This system is decomposed as:

$$\text{(3.3)} \qquad \begin{aligned} 0 \cdot \bar{y}_0 &= \bar{V}_{11}z_1 + \bar{V}_{12}z_2 \\ \Lambda_1\bar{y}_1 &= \bar{V}_{21}z_1 + \bar{V}_{22}z_2 \end{aligned}$$

and will admit no solution unless $\bar{V}_{11}z_1 + \bar{V}_{12}z_2 = 0$. We suppose that this version of the rank condition (see, e.g. Hadley (1961), p. 168) is fulfilled. Then the first equation in (3.3) is satisfied for any value of the scalar $\bar{y}_0$. The (n-1) last equations are readily solved and $\bar{y}_1$ is entirely determined:

$$\bar{y}_1 = \Lambda_1^{-1}(\bar{V}_{21}z_1 + \bar{V}_{22}z_2) \quad .$$

Using the transformation x = Vy we can go back, and compute $\bar{x}_1$ and $\bar{x}_2$:

$$\text{(3.4)} \qquad \begin{aligned} \bar{x}_1 &= V_{11}\bar{y}_0 + V_{12}\bar{y}_1 \\ \bar{x}_2 &= V_{21}\bar{y}_0 + V_{22}\bar{y}_1 \quad . \end{aligned}$$

as $\bar{y}_0$ is undetermined, $\bar{x}_1$ will be a unique solution to (3.2) if, and only if, all the k elements of V_{11} are null. Correspondingly, it must be that the (n-k) elements of $\bar{V}_{21}$ are all nonzero, so that all elements of $\bar{x}_2$ admit an infinity of solutions, parametrized by $\bar{y}_0$. As $\begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix}$ is the eigenvector for the zero eigenvalue, $V_{11} = 0$ implies:

$$A \begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix} = \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} V_{21} = 0$$

so that the last n - k columns of A are linearly dependent. This then is the restriction to be imposed on A for the last (n-k) elements of x to be stationary state invariant.

3.2. The System in Speed-of-Adjustment Form

We rewrite (3.1) in the following form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 - \bar{x}_1 \\ x_2 - \bar{x}_2 \end{bmatrix}$$

such that the coefficients of matrix A can be interpreted as speeds of adjustments. Note that we know that $\bar{x}_2$ is not unique. Given the solutions of (3.2) as shown in (3.4), this system can now be written as:

$$(3.5) \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{22} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 - \bar{x}_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} A_{12}\,u \\ A_{22}\,u \end{bmatrix}$$

where $u = V_{22}\,\Lambda_1^{-1}(\bar{V}_{21}Z_1 + \bar{V}_{22}Z_2)$. To reach (3.5) we use the fact that $AV = V\Lambda$ and $V_{11} = 0$.

3.3. Solution of the System

Using, as before, the transformation $x = Vy$, (3.5) becomes:

$$\dot{y}_0 = -(\bar{V}_{11}A_{12} + \bar{V}_{12}A_{22})u - (\bar{V}_{11}A_{11} + \bar{V}_{12}A_{21})\bar{x}_1$$

$$\dot{y}_1 = \Lambda_1 y_1 - (\bar{V}_{21}A_{12} + \bar{V}_{22}A_{22})u - (\bar{V}_{21}A_{11} + \bar{V}_{22}A_{21})\bar{x}_1 \quad .$$

Using the fact that $V_{11} = 0$ and $V^{-1}A = \Lambda V^{-1}$, these equations simplify to:

$$\dot{y}_0 = 0$$

$$\dot{y}_1 = \Lambda_1(y_1 - \bar{y}_1) \quad \text{where } \bar{y}_1 = \bar{V}_{21}\bar{x}_1 + \bar{V}_{22}u \; .$$

We note that $\bar{y}_1$ is well defined as a function of the exogenous variables of the system, x_1 and u. The solution of this system is straightforward:

$$y_0 = k_0$$

$$(3.6) \qquad y_1 = e^{\Lambda_1 t} K_1 + \bar{y}_1$$

where k_0 and K_1 are a set of n arbitrary constants of integration. Stability of the system requires that the last (n-p) elements of K_1, called K_1^n, be all null, as they match the eigenvalues with positive real parts. This leaves p constants, k_0 and K_1^p, to be determined: they are related to the initial position of the system at time zero, as represented by the values taken by the pre-determined endogenous variables x^p. (On all this, see Blanchard and Kahn (1980) and Buiter (1981)). Then, (3.6) can be further detailed as:

$$y_0 = k_0$$

$$(3.6') \qquad y_1^p = e^{\Lambda_1^p t} K_1^p + \bar{y}_1{}^p$$ where Λ_1^p is a (p-1) x (p-1) disgonal matrix with negative terms

$$y_1^n = \bar{y}_1{}^n \; .$$

These p predetermined variables are drawn from both sets x_1 and x_2: separating them implies extracting a submatrix V^p from V as follows:

$$x = Vy \Rightarrow x^p = V^p_y \; .$$

We can further partition V^p, which has p rows, into its first p and last n-p columns:

$$V^p = [V_1^p \;\; V_2^p],$$

and we will assume that the square (pxp) matrix V_1^P is invertible (which is not an additional assumption as we already posed that V is nonsingular).

Then, the p unknown constants (k_0, K_1^P) are the solution to the system of p linear equations:

$$x^P(0) = V^P y(0)$$

which, using (3.6') gives the solution:

$$\begin{bmatrix} k_0 \\ K_1^P \end{bmatrix} = (V_1^P)^{-1} \left(x^P(0) - V^P \begin{bmatrix} 0 \\ \bar{y}_1 \end{bmatrix} \right) .$$

This, then, entirely determines y_0 and y_1 in (3.6), and, using $x = Vy$, we can obtain the solution to (3.6), which is:

$$(3.7) \quad \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & V_{12} e^{\Lambda_1^P t} \\ V_{21} & V_{22} e^{\Lambda_1^P t} \end{bmatrix} \begin{bmatrix} (V_1^P)^{-1} \\ 0 \end{bmatrix} \left(x^P(0) - V^P \cdot \begin{bmatrix} 0 \\ \bar{y}_1 \end{bmatrix} \right) + V \begin{bmatrix} 0 \\ \bar{y}_1 \end{bmatrix}$$

To obtain the stationary state value $\bar{x}$, we take the limit of (3.7) as t goes to infinity. The k first rows of the right hand side reduce to $\bar{x}_1$ as required. Of interest are the last (n-k) rows:

$$(3.8) \quad \bar{x}_2 = [V_{21} \quad 0] (V_1^P)^{-1} \left(x^P(0) - V^P \begin{bmatrix} 0 \\ \bar{y}_1 \end{bmatrix} \right) + V_{22} \bar{y}_1$$

This is the central result of the paper. It appears that the *a priori* indeterminacy of the stationary state has been eliminated through the solution of the system. Still, while $\bar{x}_2$ is unique, it exhibits some properties that do not usually appear:

- $\bar{x}_2$ depends upon the initial position of the system at time $t = 0$, as described by the values of the predetermined variables $x^P(0)$.

- The term in front of $x^P(0)$, $[V_{21}\ 0](V_1^P)^{-1}$ is an $(n-k) \times p$ matrix that does not seem to be amenable to further simplification. It collects elements of the eigenvectors associated with the eigenvalues which are negative or null. In general, this expression will not be independent of the particular values of the eigenvalues in Λ_1^p, which play the role of speeds of adjustment for the dynamic system, as is clear from (3.8). For the same reason, V_{22} may usually include some parameters which are a function of the speeds of adjustment.

4. Conclusion

We have derived an explicit solution for linear differential equations models characterized by a singular state transition matrix. Interest in this class of models is justified by the fact that they follow from intertemporal utility maximization with a constant subjective rate of time preference. It is our belief that this property is in fact more general and can be obtained in other models. For example, Drazen (1980) describes a model of optimal factor accumulation, characterized by a continuum of steady states, which crucially depend on initial conditions and on the approach path: the heterogeneous labor force adjusts - through learning-by-doing - to the existing stock of heterogeneous capital, while at the same time the composition of the capital stock adjusts to the existing skill composition of labor. This dual adjustment makes the structure of production in the steady state dependent upon the employment history of the economy. The phenomenon is identical to the one described in the present paper, although Drazen's investment function cannot be easily described with differential equations, making it apparently impossible to show that the problem is formally identical to the one solved here.

The common feature of these models, which makes their mathematical structure identical, is the existence of speed of adjustment factors together with inter-temporal optimization. Yet, recently, Drazen (1982), following earlier work by Kempand Wan (1976), has re-cast his model of choice of heterogeneous factors of production with explicit costs of adjustment. Interestingly, here again he obtains the indeterminacy of stationary equilibrium property. But it now follows from the assumption that adjustment costs do not disappear even for infinitesimal shifts of these factors, so that there exist "bands" of factor allocations for which further adjustments are too costly to undertake. Apparently then the formal nature of the phenomenon is different from the one discussed here, but further research is clearly necessary to clarify this point.

Finally, we note that although the examples of section 2 are cast in models with perfect foresight, the property is more general, and could presumably be found in models where all the variables are pre-determined, which is a particular case of the general form of section 3, as can be easily verified.

Footnotes:

1 - Note that this property differs from that of standard saddle point models where the impact effect of an exogenous disturbance depends on the speed at which some "slow-moving" variable responds, while the stationary equilibrium to which the economy will eventially converge is independent of both the initial conditions and the transitory path of adjustment.

2 - The simple expressions (2.8) obtain here only under the assumption that the intertemporal elasticity of substitution in consumption is unity. This is without loss of generality for the problem at hand.

3 - Note that in the present example, W (and W^*) is not constant: $\dot{W} = (r-\delta)W$. In the previous case, with only one good and perfect asset substitutability, the domestic interest rate had to be equal to the rest of the world's interest rate, which we assumed equal to the rate of time preference. Here, with two goods, perfect asset substitutability implies (2.10). Only in the steady state do we need $r = r^* = \delta$

4 - The problem can, in principle, be generalized to lower ranks. We only consider the case of one degree of freedom as this is the only case obtained in models with intertemporal optimization.

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