"DYNAMIC DECISION CALCULUS: FORMULATING LONG-RUN STRATEGIES"

by

Philip M. PARKER*
and
Miklos SARVARY**

94/15/MKT

* Associate Professor at INSEAD, Boulevard de Constance, Fontainebleau, 77305 Cedex, France.

** PhD Candidate INSEAD, Boulevard de Constance, Fontainebleau, 77305 Cedex, France.

Printed at INSEAD, Fontainebleau, France
Dynamic Decision Calculus: Formulating Long-Run Strategies

by Philip M. Parker and Miklos Sarvary

May, 1994

Philip M. Parker is Associate Professor of Marketing and Miklos Sarvary is a Ph.D. candidate at INSEAD, Fontainebleau, France. Authors are listed alphabetically. They would like to thank Eva Szekeres for her excellent research support, Bart Bronnenberg for comments on an earlier draft, Phil Caminez, Director of Nynex Corporation, Claes Tadne, Strategic Planner, of Ericsson Radio Systems, Nicky Bishop, Head of Marketing, Mobile International Division, of British Telecom and Andrew Roscoe, President of EMCI Inc., for their participation in this research. Please send correspondence to INSEAD Boulevard de Constance 77305 Fontainebleau France, Tel: (33-1) 60 72 40 00.
Dynamic Decision Calculus: Formulating Long-Run Strategies

Abstract

This paper presents a general approach to assist managers in strategically setting prices and allocating resources over the product, brand, or adoption (diffusion) life cycle. While substantial theoretical work has been achieved in this area in the management science and operations research disciplines, approaches which can be implemented as managerial tools are generally lacking. Our methodology, which has been applied as a PC-based decision support tool in the telecommunications industry, marries the optimal control literature with decision calculus approaches offering sufficient flexibility for applied contexts. We discuss the approach and describe its use to derive optimal price and advertising policies. Strategies not commonly suggested using traditional formulations are also presented.

Keywords: decision calculus, optimal control, decision support, marketing strategy
1 Introduction

Management science has long recognized the importance of strategic thinking, as opposed to myopic management, in the setting of long-run policies. While the importance of dynamic strategies for new categories or brands facing a life cycle or diffusion process is unquestioned, managers invariably are faced with the following dilemma. At the pre-launch or early phase of product introduction, there is greatest need for strategic thinking. At this phase, however, there is a basic lack of hard data or market experience from which to derive requisite strategies. In this paper we propose a method, called dynamic decision calculus, to assist managers in deriving broad strategic guidelines as well as precise pricing and budgetary policies prior to the launch of new products or services. Dynamic decision calculus marries optimal control theory with the concept of decision calculus. The basic idea is to formulate optimal control problems in a way that allows managers to easily model life cycle dynamics for their particular industry. This is achieved by modeling changes in consumer response to marketing variables, as well as dynamic competitive effects, with the evolution of scalar measures over time: the elasticity paths of each strategic marketing variable. We propose that elasticity paths be generated using decision calculus, relying on historical data when possible, and/or managerial judgement (delphi procedures). The basic advantage of this approach is its ability to reasonably compromise between the need for mathematical tractability in deriving solutions, and the managerial necessity to incorporate complex
(non-tractable) phenomena. In other words, the method allows managers to benefit from systematic (analytic) approaches when solving their decision problems, yet remaining implementable i.e. useful in applied settings.

In the next section we briefly summarize the relevant literature. Next, we discuss in detail a class of dynamic decision calculus models which are well suited to applied settings of new product launch. This is followed by simulations of the method with empirically tested functional forms and parameter values which vary over known plausible ranges. The purpose of this discussion is to illustrate the implementation and relate the results to existing solutions proposed by the optimal control literature. In Section 5 we briefly present a case where dynamic decision calculus has been successfully applied as a decision support tool in order to gain insights into long-run marketing strategies for a mobile telecommunications service in Europe. The paper ends with concluding remarks and suggestions for further research. To improve readability, certain mathematical derivations are given in the Appendix.

2 Contributing Literature

Dynamic decision calculus builds heavily on two separate streams of literature. The first stream relies on optimal control theory to derive time-dynamic pricing and marketing mix strategies (advertising, sales force, etc.). Optimal control methodologies allow the analyst to specify a gen-
eral objective function (profit maximization over time) subject to a variety of constraints, including dynamic demand or diffusion processes. An early example of this approach in marketing is reported in Dolan and Jeu-land (1981) which illustrates conditions when price skimming strategies are likely to be more profitable for monopolies than penetration strategies. This stream of work has been extended to a variety of other marketing areas including distribution and advertising while often incorporating many complicating factors including the extent to which firms discount future cash flows, possible learning effects on unit costs generated from increases in cumulative sales or production, the extent to which early purchasers or opinion leaders influence future purchases (diffusion effects), the effect of differing competitive structures as the industry takes off or matures, etc. For extensive reviews of this literature, the reader is referred to Hanssens, Parsons and Schultz (1990, Chapter 10), and Lilien, Kotler and Moorthy (1992, pp. 197-202).

While substantial academic insight has been generated from optimal control methodologies, applied use of this technique is rare in the literature. In particular, to our knowledge, no decision support system exists which relies on optimal control techniques. The main reason for this, we believe, is that applied settings will often lead to mathematical intractabilities which prevent either meaningful optimal control formulations or the derivation of robust/reasonable solutions. The mathematical requirements of these techniques and the resulting need to stylize formulations has made optimal con-
trol theoretic studies mostly applicable to interesting yet largely academic problems. Our paper proposes that the second stream of research, decision calculus calibration, be used to lessen the gap between the academic appeal of optimal control theory and the highly complex yet "nuts-and-bolts" requirements of strategic planners. The use of decision calculus approaches, as introduced in Little (1970), has been reported for a number of applied, yet generally static or short-run (from one to three years) resource allocation problems including advertising (Little 1970), media planning (Little and Lodish 1969), sales force allocation (Lodish 1971, Lodish, Curtis, Ness and Simpson 1988), retailing (Lodish 1982), store repositioning (Corstjens and Doyle 1989), and shelf-space allocation (Singh, Cook and Corstjens 1988), among others (see Lodish 1981 for a review). The philosophy behind decision calculus approaches is that managerial insight, based on historical data or generated via delphi procedures, can be used to calibrate model parameters which themselves reflect highly complicated phenomena. Little (1970, p. 470) defines decision calculus "as a model-based set of procedures for processing data and judgements to assist a manager in his decision making." Based on applications of such approaches, Little (1970, pp. 470-471) recommends that decision calculus models be (1) simple - easy to understand and parsimonious formulations, (2) robust - always generating solutions within reasonable ranges, (3) easy to control - ability to generate a wide variety of solutions, (4) adaptive - ability to incorporate new information or market structures, (5) complete on important issues - handling key phenomenon yet remaining simple, (6) easy to communicate
with - allowing simple, quick and/or on-line conversational use permitting easy changes in inputs and outputs. In this paper, we propose dynamic decision calculus models which generally meet these requirements, while also integrating insights which can be derived from optimal control theoretic formulations. In doing so we must make explicit, yet reasonable tradeoffs between academic appeal, and managerial usefulness.

3 A Dynamic Decision Calculus Model

3.1 Motivation

A number of factors makes a decision calculus approach useful in determining optimal long-run or life cycle strategies, especially prior to product introduction. First, the derivation of reasonable optimal long-run strategies ultimately requires managerially accepted scenarios of future industry (or category) behavior. This entails some understanding (or at least opinion) of the future behaviors of players in the market including competitors, consumers, distributors, suppliers, government officials, or other social system agents. Second, these beliefs in future dynamics, may represent a variety of conflicting "plausible" scenarios; for each of these scenarios managers can gain insights from the various optimal solutions (even though only one might be implemented). Third, we must accept that many of the scenarios may not be empirically supported based on previous experience of the product in question, or that extrapolations of historical experience may not be appropriate in some circumstances. For many new products
previous demand curves may not exist, or if they do, they may not incorporate changes in the level of competition likely to be seen in the future. In his review of marketing decision models, Rangaswamy (1993, p. 742) concurs that "in contexts such as new products where market experience is limited, the subjective estimation approach is better" than empirical approaches. This last point implies that many of the parameters of the dynamic response functions must be capable of being calibrated using historical analogies which have undergone similar processes, informed guesses, or similar non-statistical sources (what Little 1979, p. 18, calls "historical norms or managerial judgements"). While great care needs to be exercised when managerial judgements are used to calibrate models, the literature clearly supports this approach where such complications prevent alternative methods (Chakravarti, Mitchell and Staelin 1979, 1981; Little and Lodish 1981, and McIntyre 1982).

3.2 Objective function

We begin by verbally stating the general problem: firms must set prices, and marketing budgets over time in order to maximize discounted profits over a fixed, but perhaps very long, time horizon (e.g. the product life cycle, the trial curve, or the diffusion process). The unit or marginal cost of the product (mc) may decline over time based on cumulative production experience. The cost of different marketing approaches may differ (e.g. direct mail, sales force, television advertising), and these alternatives may generate different sales responses. The firm's objective can be mathematically
written as follows:

\[
\max_{P(t), A_i(t)} \int_0^T e^{-rt} [(P(t) - kQ^{-\alpha}(t))q(t) - \sum_{i=1}^I A_i(t)] dt \tag{1}
\]

where \( r \) is the firm's discount rate, \( k \) is a start-up cost level, \( \alpha \) is the learning rate, \( Q(t) \) is cumulative sales up to, but not including \( t \), \( A_i(t) \) is expenditure over time on marketing effort \( i \) (advertising, direct mail, etc.) and \( T \) is the length of the planning horizon. The objective function, (1), combines most aspects of objective functions used in optimal control theoretic studies.

### 3.3 Sales-response function

Given an objective, we must formulate the marketing constraints in the form of a response function. In order to simplify the complexity associated with life cycle phenomena, our approach assumes atomistic competition and concentrates on the dynamics of market responses to changes in strategic variables (the marketing mix). Atomistic competition involves firms competing as "imperfect monopolies" whereby aggregate market trends are not affected by, or do not react to a single firm’s strategies, yet consumer demand is affected by both. This form of competition is likely to occur when individual players can not dominate the behavior within an industry. In highly concentrated industries, therefore, our assumption should be viewed with caution. Atomistic competition has been used in modeling a number of industrial settings (see, for an early example, Phelps and Winter 1970, in the economics literature, and Nascimento and Vanhonacker 1990, in the marketing literature).
From a modelling point of view, atomistic competition means that competitors do not directly react to a single firm's actions. The most appealing alternative to this, and the one suggested by the optimal control literature, would be a game theoretic approach. Even in a simple problem including one control variable, however, models quickly become intractable with the inclusion of more than two firms. Another drawback of game theoretic formulations in applied contexts is that a small modification of the problem (e.g. the inclusion of an additional competitor) requires the modeler to completely reformulate the model. In other words, such models are not flexible and adaptive enough to handle changes in the manager's forecasts or to allow him to explore different possible scenarios. By assuming an atomistic environment we can handle (or approximate) the impact of both direct and indirect forms of competition while also avoiding game theoretic limitations that make the optimal control methodology extremely cumbersome, if not unworkable.

Our formulation proposes the following market response model to complement the objective function given in (1):

\[ q = f(.)P(t)^{n(t)}A_i(t)^{e_i(t)} \quad i = 1, \ldots, I \]  

(2)

where \( f(.) \) is some baseline demand function (e.g. a constant term, a diffusion curve, or some base-line path); in studies of diffusion processes, \( f(.) \) is typically assumed to be a function of cumulative industry sales or first
purchases up to time \( t: f(Q(t)) \). \( P(t) \) is price over time; \( n(t) \) is the dynamic price elasticity of demand \( (n(t) \leq -1) \); \( A_i(t) \) is a positive marketing effort associated with mix element \( i \); and \( \epsilon_i(t) \) is a dynamic elasticity associated with \( i \) \( (0 \leq \epsilon_i(t) \leq 1) \).

Assuming atomistic competition allows us to model the response to each marketing mix element (elasticities) as an *exogenous* function of time. The elasticity function is thus assumed to reflect any *relevant* change in market structures and/or consumer responsiveness. For example, as competition is foreseen to increase over the time horizon, price elasticities will likely increase in absolute value accordingly. While many optimal control formulations assume a fixed elasticity over the life cycle (or one which is endogenous to the strategic variable in question), both theoretical and empirical studies in marketing argue that price elasticities often vary in an exogenous manner over the product, brand or adoption (diffusion) life cycle: Lilien and Yoon (1988), Liu and Hanssens (1981), Mickwitz (1959), Nagle (1987, pp. 152-153), Parker (1992), Parsons (1975), Simon (1979), Tellis (1988), and Wildt (1976). Similar conclusions are made for advertising and mass media communication elasticities (see, for example, Kotler 1971, Parsons 1975, Arora 1979, and Lieberman and Montgomery 1988). These elasticity dynamics have been hypothesized to be generated by many of the factors previously discussed (changes in competition, changes in segments targeted, free-riding effects due to sequential entry, etc.).
Three further features are noteworthy concerning response function (2). First, the response function is separable in the underlying demand effect and the effects associated with marketing mix. In this respect it is similar to both empirical studies of life cycle marketing (Kamakura and Balasubramanian 1988, Parker 1992, Jain and Rao 1990) and normative studies of the marketing mix (Bass 1980, Bass and Bultez 1982, Dolan and Jeuland 1981 and Robinson and Lakhani 1975). Second, it mirrors the classic constant elasticity (multiplicative) demand function with the important exception that elasticities are allowed to vary over time. Finally, assuming that $e(t)$ is constrained between 0 and 1, advertising (or other expenditure) response curves are concave functions which also exhibit a constant elastic response in the short run. Concave marketing response functions have been used in a variety of empirical studies of advertising, for example, including Hanssens and Levien (1983), Simon and Arndt (1980), Aaker and Carman (1982), and Lambin (1976), as well as in a number of theoretical optimal control and static optimization studies; see, for example, Horsky and Simon (1983).

Using the generalized form of the constant elasticity response function is useful for two reasons closely related to decision calculus. First, more so than most other static measures, the constant elasticity formulation is unitless and allows for cross industry-product-brand comparisons which becomes an important issue for model calibration within decision calculus frameworks (see Lilien, Kotler and Moorthy, 1992, p. 174). There exists a considerable body of empirical evidence on the likely ranges as well as
dynamics of price and advertising elasticities. Tellis (1988) reports a meta-
analysis on price elasticity estimates whereas Assmus, Farley and Lehmann
(1984), summarize previous econometric results on advertising elasticities.
This body of evidence allows analysts to begin understanding reasonable
ranges for elasticities and their dynamics over time. The second advantage
of this formulation is that it captures the essence of marketing response at
every moment in time with a simple number (elasticity or responsiveness).
If we want managers to gauge the response to a certain marketing variable
over time, then we have to make sure that this response can be described
with a single scalar function of time and in unitless measures. This also
allows the manager to easily visualize, interpret and communicate plausible
scenarios. Thus the proposed model is parsimonious and flexible, is the-
oretically appealing for many industry settings, is empirically tested, and
allows for direct (or external) interpretation and calibration of its parame-
ters as required in new product situations and decision calculus approaches
(Little, 1970).

3.4 Implementation

The final step to complete the dynamic decision calculus model is to specify
the elasticity paths of the marketing mix elements for which the manager
wishes to obtain optimal strategies. As mentioned earlier, this has to be
done by a careful evaluation of historical data if available, and/or manage-
rial judgement. If such data are not available, managers can generate these
via delphi procedures. Given our specification of the response function any
functional form can be used provided that its values lie within reasonable ranges. Once the elasticity paths are entered they can be fitted by simple polynomials; the resulting models are optimized using a simulation program.

In practice, we suggest that the specification of the elasticity paths be simple and integrated within a menu driven interface that allows the manager to change the relevant parameters of the model, to plot existing data as well as to visualize his own input and the output of the simulations. Besides allowing the manager to use the model iteratively, exploring several scenarios, such systems make possible the pooling of inputs from different managers. As in standard decision calculus, this would allow for *ex ante* consensus building around accepted scenarios (for further discussions on delphi and related procedures see for example Makridakis et al. 1983).

4 Illustration

This section provides an illustration for the technical implementation of the model described in the previous section. In what follows we will distinguish between two special cases of the general model given by (1) and (2). First, we explore the case when only price is the decision variable. Next, we discuss the situation when price is fixed and an optimal advertising path
is sought. In both cases we will specify two exogenous elasticity paths. The choice of these functions is arbitrary and we considered quadratic response functions to illustrate convex and concave, increasing and decreasing elasticity paths.

4.1 Optimal dynamic pricing

Under the first scenario only price, $P(t)$, is incorporated as a decision variable. The general objective function, given by (1) becomes:

$$\max_{P(t)} \int_0^T e^{-rt} [(P(t) - kQ^{-\alpha}(t))q(t)]dt.$$  (3)

We assume that the manager is facing a new product diffusion process as suggested by Bass (1969). The general response function given in (2) can be written:

$$q(t) = (a + b \frac{Q(t)}{cM})(cM - Q(t))P(t)^{n(t)}$$  (4)

where $a$ represents the first year penetration level for the product, generated from influences external to the diffusion process, $b$ reflects the speed of diffusion due to internal influences such as word-of-mouth, and $c$ represents the ultimate penetration ceiling of the innovation in the long run; $M$ is the size of the social system, or target market. The term specified for $f(Q)$ is the well-known Bass-model (Bass 1969) which has proved to be efficient in describing first purchase diffusion processes for a large variety

---

1The analytic solution to the general model is complex and does not add substantial insight. It is available from the authors.
of products (see Mahajan, Muller and Bass, 1990, for a recent review on extensions and applications of new product diffusion models).\textsuperscript{2} We use this formulation as an illustration; any dynamic demand function separable in price or marketing effort could be used. One advantage of the Bass model in addition to its wide empirical applicability and interpretability of parameters, is its known behavior across various industry settings. Sultan, Farley and Lehmann (1990) in a meta-analysis of estimated Bass models report average values for the coefficient of external influence \( (a) \), and for the coefficient of internal influence \( (b) \). The separable form of the demand function allows price to affect both coefficients of the Bass model \( (a \text{ and } b) \). Similar separable formulations have been suggested in the optimal control literature in Bass (1980), Bass and Bultez (1982), Dolan and Jeuland (1981) and Robinson and Lakhani (1975), and Kalish (1983) and in the empirical literature: Kamakura and Balasubramanian (1988), Jain and Rao (1990), and Parker (1992). The time varying elastic response allows for differing influences of price on external and internal influences. Mahajan, Muller and Bass (1990) note that during the early phases of the diffusion process, the external influence parameter (reflecting mostly influences from early adopters and innovators) dominates the diffusion process, while the internal influence parameter becomes important later in the life cycle. If

\textsuperscript{2}Note that this diffusion model would not be an appropriate one in the case of low ticket items that are frequently purchased. In such cases the formulation would be modified for this difference, using, say, the Fourt and Woodlock (1960) model for the trial process of frequently purchased goods launched into a mature category, etc.
we were to fix the price elasticity over the life cycle, the separable function would lose its reasonableness. However, as the elasticity is allowed to vary over time (or as the category matures), it can be allowed to have differential impact on the two influences.

In order to solve for optimal pricing strategies we follow the general methodology of optimal control theory using the Hamiltonian formulation (see Kamien and Schwartz 1981, p. 151.). Details of the derivations are provided in the Appendix. The optimal price path is the solution of the following differential equation:

\[ \dot{P} = (n(n + 1))^{-1}[r(Pn + Pn^2 - n^2kQ^{-\alpha}) + \dot{n}P - nP^{(n+1)}f_Q] \]  \hspace{1cm} (5)

where \( f_Q \) denotes the partial derivative of \( f(Q) \) with respect to \( Q \); the time subscripts have been dropped. Equation (5) does not have a closed form solution but we can derive solutions with simulations for different elasticity functions. One has to be careful, however, to use plausible parameter ranges in order to derive reasonable solutions. Before discussing these issues we briefly summarize the analytic solution to the second case.

### 4.2 Optimal dynamic advertising

In the case of advertising, when price is fixed, the general objective function, (1) becomes:

\[ \max_{A(t)} \int_0^T e^{-rt}[(P - kQ^{-\alpha}(t))q(t) - A(t)]dt. \]  \hspace{1cm} (6)
and the response function takes the form:

\[ q(t) = (a + b \frac{Q(t)}{cM})(cM - Q(t))A(t)^{\epsilon(t)}. \]  \hspace{1cm} (7)

Somewhat different formulations have been proposed in the literature for the constraint in the optimal advertising contexts. In most models, advertising affects one or both of the parameters of the diffusion model in a linear or log-linear fashion (for example Deal 1979, Teng and Thompson 1983, Horsky and Simon 1983). In this paper the separable functional form is inherently non-linear and affects both parameters of the diffusion model. Had we fixed \( \epsilon \) over time, equation (7) would be a highly questionable formulation as this would imply a similar effect of advertising on both external and internal influence parameters (\( a \) and \( b \)). Again, allowing \( \epsilon \) to vary over time allows these effects to be relatively more or less important as the relative impact of each coefficient changes as the category matures. Setting a higher elasticity early on will principally affect the external influence process; the elasticity can decrease over time if it is believed that advertising has no or little effect on later adoptions. The time dynamic specification of \( \epsilon(t) \) provides, therefore, a flexible yet simple description of demand interacting with the marketing mix.

Following the same steps as in the previous section the differential equation for the optimal advertising path is:

\[ \dot{A} = \frac{A}{1 - \epsilon} [r + \dot{\epsilon}(\ln A + \frac{1}{\epsilon})] - \frac{r \epsilon A^\epsilon}{1 - \epsilon} (P - kQ^{-a})f(Q) \]  \hspace{1cm} (8)
Again, (8) does not have a closed form solution so we have to derive it using simulation. The next section describes the parameter ranges used to simulate the solutions of both scenarios given in (5) and (8).

4.3 Plausible Scenarios

Besides underlying functional forms, one needs empirically plausible parameter ranges to evaluate realistic scenarios. In the present study we rely on previous empirical work to define parameter ranges for learning, diffusion, price and advertising elasticities. In the simulations three levels were chosen for the parameters of the diffusion model based on a meta analysis by Sultan, Farley and Lehmann (1990) of empirical studies using the Bass model. They report an average value of 0.03 for the coefficient of innovation, \( a \), and a mean of 0.38 for that of imitation, \( b \). In practice the external influence coefficient, \( a \), rarely ranges above 0.10 (or 10 percent initial penetration) and below 0. The internal influence coefficient, \( b \), rarely ranges below 0.10 and above 0.90. Two levels were chosen for the learning rate based on Simon (1989), who reports learning rates for various innovations: he found a minimum of 0.097 and a maximum of 0.377.\(^3\) Discount rates take on three levels varying from 0.001 to 0.05 in the case of advertising.

\(^3\)This maximum is an extreme value and learning parameters practically range between 0.10 and 0.25. Several simulations in the case of both price and advertising indicate that higher learning rates do not give qualitatively different solutions from the ones reported here.
and 0.01 to 0.25 in the case of price.\textsuperscript{4}

In both price and advertising formulations $n(t)$ and $\epsilon(t)$ have to be generated by the manager. As discussed earlier, for the illustration we will use the following quadratic function to describe elasticity dynamics:

$$n(t) = n_2 t^2 + n_1 t + n_0$$  \hspace{1cm} (9)

and

$$\epsilon(t) = \epsilon_2 t^2 + \epsilon_1 t + \epsilon_0$$  \hspace{1cm} (10)

It is clear that any other - preferably polynomial - form could have been used. We have chosen the quadratic form for its simplicity and its relative flexibility; it has been previously used to describe increasing decreasing, or increasing and decreasing elasticity patterns (Parker 1992, and Shoemaker 1986). Elasticities are allowed to follow, therefore, two types of patterns: a bell-shaped, and a u-shaped pattern with extreme values at the middle of the time horizon. For example, a bell-shaped price elasticity pattern might exist for a new product which is first perceived by the market as a high risk (high absolute elasticities), followed by a period of general acceptance (low elasticities), followed by a period where, perhaps, extensive competition drives elasticities up in absolute value. The parameters of the function $\epsilon(t)$ were chosen so that elasticities move between -4.0 and -1.0 for price, and

\textsuperscript{4}Higher discount rates in the case of advertising gave "front-loaded" solutions similar to those reported in the next section.
0.05 and 0.3 for advertising. These values are based on the cited studies of elasticity dynamics and the empirical work of Tellis (1988), who finds that the average price elasticity of demand for consumer durables is equal to -2.0 and Assmus, Farley and Lehmann (1984) who find that short term advertising elasticities have a mean value of 0.221. Studies of elasticity dynamics, cited earlier, generally find elasticities to vary within these proposed ranges for a variety of categories. Table 1 summarizes the parameter values used in the simulations. Any combination of parameter values in the table corresponds to two “cells” for the simulations of (5) and (8), each belonging to one of the two (u-shaped or bell-shaped) elasticity paths.

**INSERT TABLE 1 ABOUT HERE**

Certain parameters in each model were chosen arbitrarily since they do not influence the dynamics of the model. In the case of optimal advertising, for example, price and initial cost were both set to $10, M was set to be equal to 100,000 units and c was set to 1.5 Similar values are used in the price simulations, except that advertising is assumed to be absent and price is not fixed. The simulations correspond to the solution of the problem over an infinite time horizon. Practically, the market has saturated after 30 periods in the case of price and after 20 periods in the case of advertising. Optimal strategies were generated in a two step procedure. For each parameter combination or “cell” and a given starting value, the differential equations

---

5The starting margin is thus 0 but increases rapidly due to learning on cost.
(5) and (8) were solved numerically using SIMGAUSS. This routine was embedded in a grid search on the starting values to find the solution that gives the highest cumulative profit. The solutions corresponding to the starting values giving the highest cumulative profits over the time horizon was kept as the solutions of the optimization problems.

4.4 Solutions

First we examine the effects of the model parameters on the “opening” strategies for both price and advertising. After, we evaluate the shape of the long-run strategy. Rather than reporting the actual solution corresponding to each “cell” we only report typical solution patterns. These results are summarized in Tables 2 and 3 for price and advertising respectively and should only be considered for the cases simulated; each industry situation generates unique strategies.

Optimal Opening Strategies:
As previously reported in the optimal control literature, we find the optimal introductory price to be negatively related to diffusion rates (coefficients $a$ and $b$), the learning rate on costs, $\alpha$, and the initial price elasticity. The introductory price is positively related to the discount rate. In the case of advertising, diffusion rates ($a$ and $b$) were found to have a different effect on the optimal starting expenditure of advertising depending on the shape of the exogenous elasticity function. If the later is bell-shaped (first increasing then decreasing) then larger values of the diffusion coefficients
increase the optimal opening expenditure, whereas in the case of a u-shaped elasticity path, larger values of $a$ and $b$ results in smaller optimal opening expenditures. Managers must have, therefore, a clear view of both the likely elasticity dynamics and diffusion process, in order to set opening advertising strategies. The learning rate as well as the discount rate generates intuitive effects in all cases: as they increase the optimal initial expenditure also increases.

**Optimal Long-Run Strategies:**
The optimal price paths are found to follow three basic patterns. Table 2 shows these patterns in different regions of the parameter space. Two of these patterns basically follow the trends of the exogenous price elasticity function although their exact shape is refined by the interplay of the parameters. Under Pattern 2, which is valid when discount rates and learning curve effects are sufficiently and simultaneously large (see Table 2), price follows a "saw tooth" pattern. Interestingly, over the values of discount rates, diffusion rates, and learning rates, elasticity dynamics generally dominate all of these other effects. Again, a pre-launch opinion of elasticity dynamics appears to be critical to formulating long-run strategies.

\[6\text{ It is worthwhile to mention here that in the case of bell-shaped elasticity dynamics, the maximum reached by the optimal advertising path decreases with increasing diffusion effects.}\]
Table 3 shows typical advertising paths for the various parameter combinations. As in the case of price, three basic patterns were identified, two of which approximately follow the shape of the exogenous elasticity path. A third pattern however is very different from the previous two and exhibits two local maxima. The first variant of all these patterns can be considered to be a “full-horizon solution” in the sense that advertising expenditures are always positive for the whole length of the time horizon. The other variants are solutions where the total market potential is captured before reaching the end of the time horizon.

Sometimes, especially in cases with strong diffusion effects (high imitation and/or innovation parameters), this can happen after 4 or 5 periods. These solutions can be interpreted as first-year (front loaded) spikes reported previously in the literature on optimal advertising. Here, the optimal advertising strategy is a large initial advertising campaign followed by no advertising at all.

In cases where the model generalizes previous optimal control formulations the simulations replicate previous findings or follow intuition. In other cases, the solutions obtained by simulations are within reasonable ranges: the models thus satisfy Little’s (1970) robustness criterion. In some complicated situations, when several effects are important, we obtain solutions

---

Read Pattern A/B as variant B of pattern A. Higher B means that the total market potential is captured earlier.
that would be difficult to foresee. The family of curves with two maxima in the case of both price and advertising shows that the complex interplay of parameters can lead to solutions that do not necessarily follow rule-of-thumb intuition, nor theoretical solutions previously reported in the literature. Thus the model is useful for managers who have to explore the optimal price and/or advertising path in real life settings when a number of complicated effects have to be taken into account in a parsimonious way.

5 Case application

To illustrate the use of dynamic decision calculus, we will briefly describe an applied case. Given its illustrative purpose and certain confidentialities, we will only sketch the basic issues involved and the insights gained. Traditionally, European telephone companies have acted as monopolists for fixed (wireline) telephone services, and have, over the last two decades, evaluated pricing strategies for mobile communications services which have faced both direct and indirect competition; these include a variety of services based on competing technologies: improved mobile telephone service (IMTS), analogue cellular telephone services, telepoint or CT2 (micro-cellular) services, personal communications services (PCS), wide-area and networked private or specialized interconnected mobile radio services (PMR and SMR), and, since July 1991, digital cellular telephone services (GSM mobile telephone). While competition has traditionally been limited to firms within the same country (e.g. France Telecom versus SFR in France for cellular telephone
services), foreign consortiums have recently entered the arena (including regional Bell operating companies and various Asian, European and North American equipment manufacturers). In these new markets, the firm in question was to start as a "near" monopolist having to face only indirect competitive services. In the near term (one or two year's out) a second or third entrant into the market was foreseen in addition to stronger competition from indirect competitors. In the mid term (two to three years out), two to perhaps four additional entrants with near, but not perfect substitute services were likely to come on the market. In addition, a general recession was foreseen for the coming three to five years. The existing reasoning of management was that consumers would become more and more price sensitive as the category matured; related cross-sectional survey research and circumstantial evidence appeared to support this belief. Management felt that it was critical to "get people signed up" before various direct and indirect competing services would enter the market. An initial evaluation of the situation suggested a strategic decline in service prices along the same lines that terminal equipment (telephone handsets) prices were foreseen to decline ("along the learning curve"). Figure 1 describes the "pre-analysis" managerial disposition at the time: since price elasticities of demand are increasing over time (in absolute value), prices therefore should decrease over time, say from $100 to $50 (as a monthly bill) over the next three to six years. Price elasticities were foreseen to increase in absolute value due to differences in tastes and preferences (later adopters will be more price sensitive) while the foreseen recession, and the eventual entry of competing
products would reinforce this trend. Communication strategies were considered, but only as a secondary issue.

**INSERT FIGURE 1 ABOUT HERE**

This case illustrates why the application of traditional optimal control theoretic models might quickly generate a number of analytic problems. First, applied problems are often too "messy" to model from both theoretical and mathematical (operational) perspectives. Competitive structures foreseen in the case are fuzzy, dynamic and, uncertain: from quasi-monopoly, to perhaps quasi-duopoly or oligopoly. Second, even if all "relevant" phenomena over the product life cycle could be modeled rigorously it is likely that the model will be very specific to the situation at hand. The danger in creating such a model is that as soon as an unanticipated change in the market occurs (e.g. the entry/exit of a competitor) the whole model might loose its relevance and need a complete reformulation. Finally, the theoretical complexity of such models make their results difficult to interpret, calibrate, and communicate to all concerned members of the organization. As a result they would appear as "black-boxes" to managers who would be either reluctant to rely on them or likely to misuse them. Little (1979, p. 17) warns against such models which risk becoming "so extraordinarily elaborate that they collapse of their own weight as data and calibration requirements become so enormous that testing and calibration are infeasible". In such cases, the resulting formulation, even if estimable, may
generate implausible or distrusted solutions to a particular manager's complex and unique problem.

The application of dynamic decision calculus to the telecommunications service in question provided what Lodish (1974) calls, in reference to implementable decision support systems, a "vaguely right approach". The implementation followed decision calculus-based methodologies previously published in the literature (see, for example, Lodish, Curtis, Ness and Simpson 1988 who allocated sales forces in the pharmaceutical industry). The process involved (1) reviewing of problem and planning objective, (2) reviewing of decision calculus approach, (3) reviewing of pricing/advertising issues, (4) reviewing the literature on innovation diffusion, (5) discussing the mobile communications industry, (6) evaluating various scenarios for elasticity dynamics, (7) collecting data on analogies, calibrating various models and evaluating scenarios using an interactive PC-based computer program (8) developing pricing plans, and (9) validating recommendations using external data. Before the decision calculus approach was introduced to management, they were sent various readings on new product diffusion and pricing. The principal persons involved included a marketing manager, a corporate planner, and a manager from an independent firm responsible for selling equipment (terminal units).

Plausible ranges of key model inputs were derived as follows: (1) discount rates were calculated based on internal corporate rates of return, and vari-
ous cost of capital measures, (2) diffusion parameter estimates and certain end-user equipment costs were based on existing diffusion curves for similar services launched in various countries with similar levels of development and telecommunications policies, (3) elasticity dynamics were generated via subjective estimates extrapolated from elasticity measures for other telecommunications products/services, and those previously reported in the literature. Discussions of the market, within the modelling context, revealed two critical elasticity dynamics: (1) the elasticity with respect to the fixed monthly service charge and, (2) the air time usage elasticity. The discussion, data collection and resulting simulations indicated that the pre-analysis assumption of service price elasticities following those for terminal equipment were unfounded. Furthermore, the treatment of the industry as a single segment, as opposed to multiple segments, was also found to be unjustified.

Instead of detailing each aspect of the application, Figure 2 summarizes management's post-analysis understanding of optimal pricing strategies. The critical understanding became that elasticities for fixed monthly fees (and equipment) were likely to increase (with a certain pattern) over time across segments reflecting the differences in end-user usage patterns (e.g. heavy versus light users), but that price elasticities within segments were likely to remain flat or even decrease in absolute value over time due in part to declining equipment prices. Furthermore, the price elasticity of demand for per minute usage time was also foreseen to remain flat or even decrease
over time, once a customer signed up for the service; the extent to which this elasticity changed depended on the type of customer involved (e.g. that person’s usage level). Instead of generating a single optimal strategy, the analysis indicated that a form of dynamic price discrimination policy be implemented. Rather than one price plan with decreasing fixed and variable prices over time, the analysis indicated that multiple plans be simultaneously offered with more plans being added as the life cycle matured. Each new plan would have a lower fixed monthly charge (tailored to each segment). The per minute usage fees, within each plan, would, however, remain constant or even increase over time (especially for the low usage segments) as the category matures. A post-analysis study of pricing policies in the United States revealed that similar price discrimination strategies had been implemented whereby segments self select their optimal usage plans.

**INSERT FIGURE 2 ABOUT HERE**

Derived optimal advertising strategies indicated that differences in media effectiveness and segment size would results in a “snake-like” wave pattern of expenditures over time. Given some level of pent-up demand, communications were foreseen to start low, then build up over time until the industry matured at which point expenditures would decline; within each segment, however, communications expenditures decline over time. In or-

---

8 Though the persons involved in this application were not be responsible for the service’s communications strategy, this question was briefly discussed.
der to test the sensitivity of the recommendations, a simple (PC-based) user interface was developed that allowed managers to modify any input assumption (learning rates, elasticity dynamics which were approximated using a quadratic function, diffusion patterns, discount rates, etc.); optimal solutions were generated on a fast personal computer in roughly 5 minutes per scenario.

6 Concluding Remarks

Using a combination of optimal control techniques and decision calculus calibration, this paper proposes a parsimonious as well as flexible approach to finding optimal marketing strategies over the product, brand or adoption life cycle. We have seen that such an approach is useful in the management context where a variety of complex situations (including competition, cost learning and discounting) must be considered. While the simulations reported earlier reveal the robustness of the proposed model, the case presented illustrates that the process of strategy evaluation was as important as the modelling aspects of the exercise. On this later point, besides reproducing existing solutions to classic optimal control (strategic allocation) problems, the approach also generates original solutions and approaches which were not readily apparent from the outset (e.g. waves). Additionally, results on the effects of various factors (e.g. the discount rate versus the learning rate, versus diffusion rates, versus elasticities) also proves valu-
able in understanding the primary forces underlying specific recommendations. Finally, one of the major benefits of dynamic decision calculus is its ability to model competitive forces without burdening the processes to such an extent that solutions can not be obtained. The flexibility of the proposed approach allows managers to effectively use historical data or intuitive judgements based on previously launched products in order to design optimal strategies for new innovations. The value in the dynamic decision calculus approach in the case presented above was three-fold: (1) it structured the problem away from traditional rules-of-thumb, (2) it forced managers to put into concrete terms their future views of industry structure and (3) it helped to formulate and generate strategic recommendations.

Further research in this area could follow two directions. One would be to explore other functional forms that simultaneously capture the essence of complex marketing situations while retaining simplicity to be useful in management contexts. Such models should be adaptable to different real-life situations by adjusting the parameters rather than by reformulating the entire problem. Another research direction is suggested by the empirical aspect of the model. In order for the proposed method to be used in practice more widely, one needs more empirical generalizations, as suggested by Bass (1993) on various aspects of the problem: (1) what are typical learning rates on production costs and why are they different across categories, (2) how do elasticities change over time, and why, and (3) what are the reasons behind some categories or brands having faster diffusion patterns,
versus those having slower ones? Answers to these questions will allow managers to calibrate optimal marketing strategies when these are most critical: during the pre-launch or launch phases of the new product process. These patterns may be unique for each application but may show similarities for products within the same product category. A more organized knowledge of such patterns would be invaluable for future research. In their summary review of analytic models, Lilien, Kotler and Moorthy (1992, p. 213) note that a major challenge to marketers is "the development of operational models that integrate competitive interactions with market dynamics and measurements procedures to make these models operational." Dynamic decision calculus, which captures atomistic competition over the product life cycle, represents but a first step in this direction.
Appendix

Derivation of equation (5)

Under the first scenario, where the firm sets only prices and discounts future profits, marginal costs decline over time due to learning effects, and the effects of other marketing variables are ignored, equation (3) takes the form:

$$\max_{P(t)} \int_0^\infty e^{-rt}(P(t) - kX^{-\alpha}(t))X(t)dt.$$  \hspace{1cm} (11)

where $X(t)$ corresponds to cumulative sales up to time $t$ (denoted $Q(t)$ in (3)) and $\dot{X}(t) = dX(t)/dt$ is the per period sales at time $t$ (denoted $q(t)$ earlier). The constraint associated with the above objective function is assumed to be similar to (4). To derive the solution we formulate the Hamiltonian (see Kamien and Schwartz, 1981, p. 151) which is, after dropping the time subscript:

$$H(P, X, \lambda) = e^{-rt}(P - kX^{-\alpha})\dot{X} + \lambda f(X)P^n$$  \hspace{1cm} (12)

where $\lambda = \lambda(t)$ is the adjoint variable with $\lambda(\infty) = 0$ ($\lambda(T) = 0$ in cases with finite time horizons). The first order conditions are given by (see Kamien and Schwartz, 1981):

$$\frac{\partial H}{\partial P} = 0$$  \hspace{1cm} (13)

$$\frac{\partial H}{\partial X} = -\dot{\lambda}$$  \hspace{1cm} (14)

$$\frac{\partial H}{\partial \lambda} = \dot{X}$$  \hspace{1cm} (15)
and the second order condition is:

\[
\frac{\partial^2 H}{\partial P^2} \leq 0. \tag{16}
\]

To solve the above system we first express \( \lambda \) from condition (13) which gives after simplification:

\[
\lambda = e^{-rt}(kX^{-\alpha} - \frac{n+1}{n} P) \tag{17}
\]

Differentiating this expression and equating it with the expression of \( \dot{\lambda} \) obtained from (14) we get (using (17)) equation (5) of the paper.

**Derivation of equation (8)**

Under this second scenario price is constant, we assume learning on cost, discounting and the control variable is advertising. The optimal control problem can be stated as:

\[
\max_{A(t)} \int_0^\infty e^{-rt}[(P - kX(t)^{-\alpha})\dot{X}(t) - A(t)]dt \tag{18}
\]

subject to

\[
\frac{dX(t)}{dt} = \dot{X}(t) = f(X(t))A(t)^{\epsilon(t)} \tag{19}
\]

where (19) is identical to (7). Now the Hamiltonian is:

\[
H(A, X, \lambda) = e^{-rt}[(P - kX^{-\alpha})f(X)A^{\epsilon} - A] + \lambda f(X)A^{\epsilon} \tag{20}
\]

and the first and second order conditions are similar to (13)-(15). Using the same steps as for price we can express \( \lambda \) to get:

\[
\lambda = e^{-rt}\left[ A^{1-\epsilon} \left( \frac{A^{1-\epsilon}}{\epsilon(a + b \frac{X}{cM})(cM - X)} - (P - kX^{-\alpha}) \right) \right] \tag{21}
\]
after dropping the time subscript. Differentiating (21) with respect to $t$ and equating it with the expression of $\dot{\lambda}$ obtained from the derivative of the Hamiltonian with respect to $X$ gives equation (8) in the paper.
Table 1: Parameter values

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Diffusion effects</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>external influence (a)</td>
<td>0.01</td>
<td>0.03</td>
<td>0.09</td>
</tr>
<tr>
<td>internal influence (b)</td>
<td>0.1</td>
<td>0.38</td>
<td>0.9</td>
</tr>
<tr>
<td><strong>Learning effects</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Discount factor (price)</strong></td>
<td>0.01</td>
<td>0.1</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Discount factor (advertising)</strong></td>
<td>0.001</td>
<td>0.01</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Price elasticity</strong></td>
<td>-4</td>
<td>-1.1</td>
</tr>
<tr>
<td><strong>Advertising elasticity</strong></td>
<td>0.05</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Table 2: Summary of optimal pricing strategies

<table>
<thead>
<tr>
<th>Diffusion effects</th>
<th>Learning effects on cost</th>
<th>Bell-shaped elasticity path ((-4, -1.1, -4))</th>
<th>U-shaped elasticity path ((-1.1, -4, -1.1))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>External Influence</strong></td>
<td><strong>Internal Influence</strong></td>
<td><strong>Discount rate = 0.01</strong></td>
<td><strong>Discount rate = 0.10, 0.25</strong></td>
</tr>
<tr>
<td>Low, or Medium, or High ((0.01, 0.03, 0.09))</td>
<td>Low, or Medium, or High ((0.01, 0.03, 0.09))</td>
<td>Low ((0.10))</td>
<td>Pattern 1</td>
</tr>
<tr>
<td>High ((0.25))</td>
<td></td>
<td></td>
<td>Pattern 1</td>
</tr>
</tbody>
</table>

Table 2 presents the summary of optimal pricing strategies under different diffusion effects and learning effects on cost. The table compares the Bell-shaped elasticity path with the U-shaped elasticity path for discount rates of 0.01, 0.10, and 0.25, as well as for discount rates of 0.01, 0.10, and 0.25.
Table 3: Summary of optimal advertising strategies

<table>
<thead>
<tr>
<th>Diffusion effects</th>
<th>Bell-shaped elasticity path (0.05, 0.30, 0.05)</th>
<th>U-shaped elasticity path (0.30, 0.05, 0.30)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal Influence</td>
<td>Learning effects on cost</td>
<td>Discount rate = 0.001</td>
</tr>
<tr>
<td>Low (0.10)</td>
<td>Low or Medium (0.01, 0.03)</td>
<td>Pattern 1/1 or 1/2</td>
</tr>
<tr>
<td></td>
<td>Low or High (0.10, 0.25)</td>
<td></td>
</tr>
<tr>
<td>High (0.09)</td>
<td>Low (0.10)</td>
<td>Pattern 1/3</td>
</tr>
<tr>
<td></td>
<td>Low or High (0.10)</td>
<td></td>
</tr>
<tr>
<td>Medium or High (0.38, 0.90)</td>
<td>Low, Medium or High (0.01, 0.03, 0.09)</td>
<td>Pattern 2/3</td>
</tr>
</tbody>
</table>

Pattern A/B means variant B of Pattern A
References


Little, J. D. C. and L. M. Lodish, “A Media Planning Calculus,” Oper-


Parker P., "Price Elasticity Dynamics Over the Adoption Life Cycle," *Jour-


