THE IMPACT OF INCORPORATING THE COST OF ERRORS INTO BANKRUPTCY PREDICTION MODELS

by

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ABSTRACT

This study examines the ability of a bankruptcy prediction model to outperform a naive model, in the context of a bank lending funds, after incorporating the costs of incorrect forecasts into both models. The study uses a sample of bankrupt versus non-bankrupt firms that is representative of the true population proportions, compares the prediction and naive models based on the net profit each would generate on a subsequent time period, and examines the sensitivity of the results to alternative sample time periods. The results indicate that a bankruptcy prediction model will outperform a naive model of lending to all firms only when Type I errors (lending to firms which go bankrupt) are costly (25 times as large for the sample) relative to Type II errors (failing to lend to firms which do not go bankrupt). This implies the usefulness of bankruptcy prediction models cannot be assessed independently of the costs of forecast errors.

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1. INTRODUCTION

The ability to predict whether a corporation will meet its future obligations has obvious benefits to a wide variety of groups and individuals. The use of a model, based on financial ratios, which yields probability estimates of the likelihood of firms defaulting on their obligations is often a first step in lending decisions. Prior research, starting with the work of Beaver [1966], has demonstrated the ability of accounting variables to discriminate between bankrupt and non-bankrupt firms. Most of the subsequent research in this area has focused on the appropriate statistical methods used to develop the model, finding the accounting variables which best discriminate between the bankrupt and non-bankrupt firms, and demonstrated the ability of a prediction model to outperform a naive model by examining the percentage of firms predicted correctly.[1]

The purpose of this paper is to demonstrate the necessity of incorporating the decision context in the determination of a prediction model's ability to add value to a potential user. To properly evaluate whether a bankruptcy prediction model can be used to enhance a lender's profitability, the evaluation must simulate the lending environment and incorporate both the cost of lending to firms which go bankrupt (a Type I error) and the opportunity cost of not lending to firms which do not go bankrupt (a Type II error).

This paper considers these costs of incorrect forecasts in the context of a bank lending funds. Specifically, the profit a bank would have made using the model is compared with the bank's profit had a naive decision rule of lending to all firms in the sample been followed.[2] The model assumes that lenders calculate a probability of bankruptcy for every firm requesting a loan, and then lend to all firms which have a lower probability of bankruptcy than some predetermined cutoff point.

To overcome the lack of empirical evidence on Type I (loss from lending to firms that declare bankruptcy) and Type II (opportunity cost of not lending to firms that do not declare bankruptcy) error costs, costs are varied over a wide range and weighted to loan size. Logistic regression is used to estimate probabilities of bankruptcy from company financial data. The model's predictive ability is tested on data from a subsequent time period in a fashion that replicates how the model could be used in practice. The study also updates previous research by using data from a more recent time period. The data used in this study (1979-1986) reflect changes in the economic environment and the Bankruptcy Act of 1978.

The results demonstrate that the bankruptcy prediction model used in this study did not outperform naive models unless Type I errors are costly (25 times) relative to Type II errors. The following section presents some background for this paper. This is followed by sections on methodology, sample selection, the model, the results, and a summary and conclusion.

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[1] For a review of the literature see Zavgren [1983] and Jones [1987].

[2] The profit maximization function may be written as:

$$\text{Max } [(aX_i - bX_j) \mid P(B_i) < P(B^*)], \text{ where}$$

a = the % profit on a loan to a firm that does not declare bankruptcy
b = the % loss on a loan to a firm that does declare bankruptcy
X_i is the total amount lent to firms that did not declare bankruptcy
X_j is the total lent to firms that did declare bankruptcy
$P(B_i)$ is the probability of bankruptcy for each firm, and
$P(B^*)$ is the cutoff probability

II. BACKGROUND

Empirical attempts to utilize financial information to predict bankruptcy began with studies by Beaver [1966] and Altman [1968] which used discriminant analysis. More current research (e.g. Olson [1980] and Zmijewski [1986]) has used logit and probit models. The current bankruptcy prediction studies assess a model's ability to predict by counting the total errors and generally correctly classify 95% or more of a sample into bankrupt and nonbankrupt categories. However, as noted by Palepu [1986] in a study on predicting takeover targets, Type I and Type II errors are likely to be quite different. Taking into account these differential costs, it may be more profitable to lend to all firms (or no firms) than to base lending decisions on a bankruptcy prediction model.

Altman et al. [1977] argue that a reasonable approximate cost for Type I errors is 70% of the amount lent, and for Type II errors is 2% of the amount that could have been lent, and then concludes that 35 loans to firms which do not go bankrupt can be foregone for each loan to a bankrupt firm.[3] However, this reckoning does not take into account the heterogeneity of firm size among the sample population. Incorrectly foregoing loans to 35 large firms which do not go bankrupt is not equivalent to incorrectly lending to one small firm which does go bankrupt. Conversely, many loans to firms that do not go bankrupt may be foregone if a loan to a Penn Central is avoided. The size of the firm (or loan amount) and the relative cost of errors should be included in the bankruptcy prediction model.

The sample of bankrupt and non bankrupt firms in current research contains a bankruptcy rate which is substantially higher than the true population proportion. Dun and Bradstreet report that the frequency rate of business failures in the United States has never exceeded 0.75% since 1934 (Business Failure Record [1982]), yet no study to date has a failure rate below 2%. Finally, this study tests the model's predictive ability out of sample with a more realistic test.[4] Since lenders typically build the model on one time period and use it in later time periods in practice, an evaluation of a model's predictive ability should be tested in the same fashion.

III. METHODOLOGY

This study considers the costs of errors in a specific decision making context and uses a sample that is representative of the true population proportions. The decision making context is that of a bank faced with the choice of whether to make a loan for a fixed percentage of the customer's total assets (book value). The bank estimates a probability of bankruptcy using a bankruptcy prediction model and firm-specific data from prior periods.[5]

My research method is to:

1) Develop parameter estimates for the predictive model.
2) Compute the probability of bankruptcy for each firm in the sample (by applying the model with its estimated parameters to the specific values of each company in the sample).
3) Rank firms in the sample by their probability of bankruptcy from lowest to highest.
4) Determine the expected profit, at each probability level, if loans were made only to firms with a lower probability of bankruptcy. This step requires estimates of Type I and Type II error costs.
5) Select the probability level that maximizes expect profit as the cutoff probability.
6) Compare the lender's ex-post profit using the cutoff point with the profit that would have been realized if loans had been made to all firms.

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[3] These estimates appear reasonable. However, the empirical evidence to support these costs is limited.

[4] Prior studies use a holdout sample from the same time period.

[5] Assuming that the loan will be a fixed percentage of the firm's assets is made in the belief that the size of a loan is positively correlated with firm size. Discussions with lending officers indicated that a prime consideration in determining the amount a bank is willing to lend is the size of a firm's assets.

The appropriate cutoff point is determined in two different ways. One is to empirically find the cutoff probability that maximized the lender's profit in a prior period. The other is to calculate the point where the lender no longer expects to make a profit on additional loans given specific profit and loss percentages, which can be determined for any level of Type I and Type II error costs as follows:

1. $P^* = B/(B+A)$, where[6]
   P^* = the cut off point where no further loans should be made
   A = the percentage of the loan that is expected to be lost if the borrower declares bankruptcy-- Type I error
   B = the percentage profit lost if the loan is not given and the borrower does not declare bankruptcy-- Type II error

To overcome the lack of empirical evidence on the magnitude of Type I and Type II error costs, costs were varied from 100/1 to 1/1.[7] As the ratio of Type I to Type II costs decreases, the optimal lending cut-off point will increase. To illustrate, if a Type I cost is 100 times the size of a Type II cost (i.e. if a firm declares bankruptcy the percentage of the loan lost is 100%, while if the firm does not declare bankruptcy the amount of profit made on the loan is 1%) the optimal cut-off is 0.0099 (i.e. loans should be made to all firms with a probability of bankruptcy less than 0.99%), at 50 to 1 (i.e. 50% and 1% respectively) the optimal cut-off is 0.0196, while at 1 to 1 the optimal cut off is 0.5.

To examine any changes in the model across different time periods, the above procedure was repeated for 3 different time periods. Each estimated version was tested on the year following the estimation sample period. The periods used were as follows:

	Estimation Period	Prediction Period
i)	1979-1982	1983
ii)	1980-1983	1984
iii)	1981-1984	1985

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[6] Proof: The expected % profit on any individual loan is:

1. $(1-P)B - PA = B - PB - PA = B - P(B + A)$, where

P = the probability that a firm will declare bankruptcy

If loans are made when the expected value is positive (a rational assumption), then they are made where:

2. $B - P(B + A) > 0$

Our cut-off may then be written as:

3. $B - P(B + A) = 0$, which is restated as:
4. $B = P(B + A)$, and
5. $P = B/(B + A)$

[7] The ratio of Type I to Type II error costs can be used in the place of varying Type I and Type II error costs individually (assuming that they are constant in % term across firms of different sizes and probabilities of bankruptcy). This reduces the number of runs required and simplifies the paper. The algebra is as follows:

$\underset{p}{\mathrm{ArgMin}}\,[A^*X(p) + B^*Y(p)] = \underset{p}{\mathrm{ArgMin}}\,[(A/B)^*X(p) + Y(p)]$, where

A = the Type I error cost, B = the Type II error cost,
X = LoanB = the loan to a bankrupt firm, and
Y = LoanNB = the loan to a non-bankrupt firm.

One limitation of the methodology is that it assumes fixed error costs; i.e. the potential loss/profit on a loan does not change as the probability of bankruptcy increases. Actually, the interest rate charged to a borrower is likely to increase as the probability of bankruptcy increases thus changing the Type I and Type II error cost relation as the credit rating of the firm changes. The impact of this simplification is to understate the predictive ability of the model. In principle, armed with a bankruptcy prediction model, lenders could adjust the rate to equalize expected profit across all loans. They do not appear to do this. Stiglitz and Weiss (1981) suggest that banks resort to credit rationing (as assumed in this model) to avoid problems of adverse selection that arise when borrowers know more about their own ability to repay than lenders do.

IV. SAMPLE SELECTION

The sample of firms that declared bankruptcy, and the sample of firms that did not declare bankruptcy were collected separately. All financial service companies were excluded, by dropping companies with SIC codes greater than 6,000, to avoid any reduction in the predictive ability of the model occurring due to differences between the financial statements of financial services companies and those of industrial firms.

A) The Bankrupt Firm Sample:

The bankrupt firm sample consists of all New York and American Stock Exchange listed firms that went bankrupt between Oct. 1, 1979 and Dec. 31, 1986. The post Oct. 1, 1979 period is used since that is when the new bankruptcy code came into effect. The final sample consisted of 97 firms that filed for bankruptcy.[8]

The list of firms that declared bankruptcy over the period studied was compiled from (1) The SEC's annual report to Congress (which contains a listing of all presentations the SEC made to bankruptcy courts), (2) the Wall Street Journal Index ("WSJI") General Listings, (3) Compustat's Research File listing of all firms that were dropped due to bankruptcy filings; and (4) the Center for Research into Security Prices ("CRSP") files for firms delisted or suspended by the Exchange. Each firm was traced to either the WSJI Corporate Listings, its 10K filing, or to Bankruptcy court records to ensure that the firm had in fact declared bankruptcy and to obtain the date of filing.

Financial data were then obtained from the 10K reports. The 10K reports indicate the date they were filed with and processed by the Securities and Exchange Commission. This allows the timing of the release of the financial information to be explicitly considered, and reduces any overstatement of the predictive ability of the model by using information released after the bankruptcy filing as noted by Ohlson [1980]. The last 10K report filed prior to the date of a bankruptcy filing was used to obtain financial data. For example, the first 10K report to be examined for a firm which had a Dec. 31 year end and which filed for bankruptcy April 14, 1983 would be Dec. 31, 1982. If the report was filed prior to April 14, 1983 then that report would be used. If not then the Dec. 31, 1981 report would be examined, and so on until a report filed prior to April 14, 1983 was found.

Table 1 provides a summary of the sample selection criteria. Note that while many of the firms were listed in several of the sources, no one source provided a complete listing. Table 2 provides a breakdown of the sample by year, exchange, and SIC code. As shown, there were three times more American Stock Exchange filings than New York Stock Exchange filings, and a fair representation from each of the major SIC codes. Table 3 provides the distribution of the time from the date of the 10K reports, to the date they were filed with the SEC, to the date of the bankruptcy filing. On average, the last 10K report is filed 3.5 months prior to the bankruptcy filing, and approximately 9 months after year end. Overall, the average 10K reports reflect information just over a year old for the firms which declared bankruptcy. The predictive ability of the model would be expected to increase if more recent data were used.

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[8] Manville Corporation and Continental Airlines were healthy companies that declared bankruptcy (due to contingent lawsuits and severe union problems respectively) during the sample period. They were excluded from the final sample due to the unusual nature of their bankruptcies. Their exclusion had virtually no impact on the final results and did not affect the conclusion of the paper.

The Non Bankrupt Firm Sample:

The sample of non-bankrupt firms and their related financial information was obtained from Standard and Poor's Compustat service. If a firm listed on Compustat did not file for bankruptcy in a given year or in the prior year, financial information on the firm from its previous year end was included in the sample. A separate listing was obtained for each year, resulting in most of the non-bankrupt firms being included repeatedly.

Unfortunately, the study still has a higher sample proportion of bankrupt firms (though closer than previous studies) than the true population proportions because the Compustat database does not include every firm listed on the New York and American Stock Exchange. A bias may exist if inclusion of the missing firms would alter either the parameters or the cutoff point. If the excluded firms are similar in nature and size to the included firms, then the model can easily be adjusted to reflect the true population proportions as demonstrated by Palepu [1986]. To the extent the excluded firms are different from the included firms, the impact on the model is unknown. The higher proportion of bankrupt firms in the sample versus the population may be partially mitigated because the omitted non bankrupt firms are smaller, on average, than those included (i.e. Compustat, on average, includes the larger firms).

V. THE MODEL

This study uses the Logit model, and is estimated using the maximum likelihood procedure in the statistical package SAS. The probability estimates can be readily interpreted as the probability of the firm's entering bankruptcy. The model has the following formulation:

2. $Pi = F(Z_i) = 1/[1 + \exp(-Z_i)]$, where
   $Zi = B_1 + B_2 * \text{Variable1} + B_3 * \text{Variable2}...$

In this context, the explanatory variables were chosen from the prior literature based on their ability to consistently discriminate between bankrupt and non bankrupt firm.[9] Included are:

1. Leverage (Lev) = Total Debt/Total Assets. Lev proxies for the indebtedness of a firm. As the percentage of the firm's financing by debt increases, its ability to meet the interest and capital repayments becomes less likely, and the probability of bankruptcy is predicted to increase.

2. Liquidity (Liq) = Current Assets/Current Liabilities. Liq proxies for the ability of a firm to meet liabilities as they come due. As liquidity increases, the ability of the firm to meet its current obligations increases, and the probability of bankruptcy is predicted to decrease.

3. Log of Total Assets (LTA). LTA is included to control for firm size. As the size of a firm increases, lenders may be more willing to renegotiate lending terms, and it is predicted that the probability of bankruptcy will decrease. Standardizing Total Assets by its log is done to normalize the distribution.

4. Return on assets (ROA) = Net Income/Total Assets. ROA proxies for the true profitability of a firm. As profitability increases, the firm will have less difficulties meeting its obligations, and it is predicted that the probability of bankruptcy will decrease. Also, large firms may be better able to withstand short-term downturns in their operations.

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[9] See Ohlson [1980] and Zmijewski [1984].

The variables and the predicted signs of their coefficients are the following:

Variable	Predicted Sign
ROA (return on assets)	-
Lev (financial leverage)	+
LIQ (liquidity)	-
LTA (log of total assets)	-

A profile analysis of the data indicates marked differences between the sample of bankrupt and non bankrupt firms, as shown in Table 4. Leverage for the bankrupt sample averages 50% more than for the non-bankrupt sample, and the first quartile of the bankrupt sample is higher than the fourth quartile of the solvent sample. Even greater differences hold for the other variables. The average size of a bankrupt firm is $88 million, while for non bankrupt firms it is $237 million (more than 2.5 times as much). Perhaps more interesting is the change in the ratios over time, illustrated in Table 5. Most of the variables show erratic changes from year to year, while some show a strong trend (like the negative trend in ROA for the non bankrupt sample).

A direct comparison of these variables to previous studies cannot be made because the earlier studies either do not provide a profile analysis, or include non New York and American Stock Exchange firms in their sample. Additionally, any differences may reflect changes due to the time period under study rather than the impact on the nature of bankrupt versus non-bankrupt firms.

VI. RESULTS

Table 6 lists the estimated model parameters. The signs of the coefficients are as predicted over all periods. The likelihood ratio index (which is analogous to R^2 for the logit model) is over 90% for all time periods, indicating the model is able to discriminate between bankrupt and non bankrupt firms. The variables Liq and LTA are statistically different from zero[10] in all time periods, but not Leverage (for which the null can not be rejected in 2 of the 4 time periods) or ROA (for which the null can not be rejected in 1 of the 4 time periods). Lev and ROA fluctuate widely (more than 50%) from time period to time period. This reflects the changes in financial ratios caused by the changing economy, and illustrates the sensitivity of these variables to the time period from which they are developed.

Table 7 compares the optimal cutoff calculated by expected values and determined empirically for each time period. It is clear, from their large variations, that the empirically determined cutoffs are sensitive to the time period. Table 8 reveals the ability of the model to predict bankruptcy. The most dramatic observation is the positive correlation between the ratio of Type I to Type II error costs and the percentage saved by using the model. The percentage saved by using the model versus a naive model increases dramatically as the ratio increases. When the ratio is less than 25 to 1 (i.e. a loss of 50% on a loan if a firm declares bankruptcy, and a 1% profit if the firm does not declare bankruptcy) the model cannot beat the naive model--so those results are omitted from the presentation. Comparing the results between the two different methods of determining cutoffs reveals that no conclusive statements can be made about which method is superior. The percentage increase in profit from using a prediction model versus a naive decision model of lending to all firms, varies with the ratio of Type I to Type II error costs, the time period, and the method of selecting the cutoff used. This illustrates the importance of determining the relationship of Type I to Type II error costs before making any statements about a model's value added ability. It also underlines the sensitivity of the model to the underlying assumptions, especially the likelihood of the Type I to Type II error ratio being 25 to 1 or greater. If, as Altman et al. [1977] argues, the ratio is 35 to 1 (a reasonable approximate cost for Type I errors is 70% of the amount lent, and for Type II errors is 2% of the amount that could have been lent) then the models provide value. Otherwise, they do not. This illustrates the importance of knowing the ratio.

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[10] The null hypothesis that the estimated coefficient is equal to zero can be rejected at the 0.025 levels in a one-tailed test.

VII. SUMMARY AND CONCLUSION

This study develops a bankruptcy prediction model in a decision context that allows the economic usefulness of a bankruptcy prediction model to be examined. The costs of incorrect forecasts are incorporated, in the context of a bank lending funds, into a bankruptcy prediction model. The study also replicates how the model could be used in practice, uses a sample that is representative of the true population proportion of bankrupt versus non-bankrupt firms, and examines differences due to alternative time periods.

This framework for bankruptcy prediction highlights the sensitivity of all bankruptcy prediction models to the Type I / Type II error cost relationship, the method used to determine the cutoff values, and the time period. The results indicate that bankruptcy prediction models add little information value unless Type I errors are costly (at least 25 times) relative to Type II errors.

Discovering the actual amount of Type I and Type II error costs will allow further refinements in our estimates of the true abilities of bankruptcy prediction models. Without these estimates, no statement can be made on whether using a bankruptcy prediction model will increase a lending firm's profitability.

TABLE 1
SAMPLE SELECTION CRITERIA

Data Source	Total	NYSE&AMEX	SIC > 6000	SIC <= 6000
SEC Report to Congress	211	71	7	64
Compustat	161	26	5	21
WSJI	157	54	5	49
CRSP-5[11]	176	86	13	73
CRSP-6[12]	11	4	0	4
Overlap[13]	-127		-15	-112
Net	114		15	99
Manville & Cont. Air[14]	-2			-2
Total	112			97

TABLE 2
NYSE AND AMEX INDUSTRIAL FIRM BANKRUPTCIES[15]

By Exchange and Year:

Exchange/Year	1980	1981	1982	1983	1984	1985	1986	Total
NYSE	2	2	8	0	3	6	3	24
AMEX	9	9	10	9	13	10	13	73
Total	11	11	18	9	16	16	16	97

By 2 Digit SIC Code:

Code	Number	Code	Number
10-19	17	40-49	7
20-29	20	50-59	23
30-39	30	Total	97

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[11] These are all firms that were delisted from the NYSE and AMEX between Nov. 1, 1979 and Dec. 31, 1986 as per the CRSP tapes.

[12] These are all firms that were suspended from the NYSE and AMEX between Nov. 1, 1979 and Dec. 31, 1986 as per the CRSP tapes.

[13] This represent the number of repeat listing of firms.

[14] Manville & Continental Airlines are excluded from the sample because they represent bankruptcies due to tort liabilities which could not have been predicted from financial information.

[15] Manville and Continental Airlines are excluded above. They filed in 1982 and 1983, their SIC codes were 329 and 451, and both firms were listed on the NYSE.

TABLE 3
DISTRIBUTION OF THE TIMING OF BANKRUPTCY
(Days)

Statistic	10K to SEC[16]	SEC to Distress[17]	10K to Distress
Mean	265	109	375
Min	17	63	111
Max	904	448	996
25th Centile	131	92	234
75th Centile	365	108	463

TABLE 4
PROFILE ANALYSIS OF THE SAMPLE 1979-1985

6928 Non Bankrupt Firms

Variable	Mean	Std. Dev.	First Quartile	Fourth Quartile
ROA	0.043	0.108	0.024	0.082
Lev	0.509	0.176	0.402	0.603
Liq	2.256	1.113	1.525	2.696
LTA	5.466	1.781	4.158	6.739

97 Bankrupt Firms

Variable	Mean	Std. Dev.	First Quartile	Fourth Quartile
ROA	-0.140	0.195	-0.209	-0.152
Lev	0.801	0.239	0.646	0.925
Liq	1.381	0.641	1.050	1.687
LTA	4.476	1.431	3.421	5.401

ROA (return on assets)
Lev (financial leverage)
LIQ (liquidity)
LTA (log of total assets)

[16] Number of days between the date of the 10K reports (i.e. year-end date) and the date they were processed by the SEC.

[17] Number of days between the date the 10K reports were processed by the SEC and the date firms filed for bankruptcy.

TABLE 5
PROFILE ANALYSIS OF THE SAMPLE MEANS BY YEAR

Non Bankrupt Sample

Year	1979	1980	1981	1982	1983	1984	1985
# Firms	1079	1042	1010	984	982	909	922
ROA	0.06	0.056	0.048	0.034	0.037	0.040	0.019
	(0.740)	(0.166)	(0.099)	(0.094)	(0.088)	(0.091)	(0.113)
Lev	0.521	0.511	0.506	0.505	0.492	0.506	0.524
	(0.163)	(0.154)	(0.172)	(0.186)	(0.174)	(0.171)	(0.206)
Liq	2.204	2.223	2.246	2.323	2.322	2.20	2.274
	(0.992)	(0.952)	(1.076)	(1.198)	(1.164)	(1.103)	(1.295)
LTA	5.192	5.316	5.403	5.460	5.558	5.679	5.724
	(1.780)	(1.789)	(1.784)	(1.794)	(1.757)	(1.729)	(1.773)

Bankrupt Sample

Year	1979	1980	1981	1982	1983	1984	1985
# Firms	11	11	19	8	16	16	16
ROA	-0.12	-0.141	-0.138	-0.061	-0.119	-0.156	-0.197
	(0.152)	(0.193)	(0.194)	(0.071)	(0.169)	(0.214)	(0.273)
Lev	0.609	0.867	0.799	0.807	0.739	0.823	0.925
	(0.182)	(0.165)	(0.202)	(0.106)	(0.176)	(0.222)	(0.375)
Liq	1.691	1.177	1.545	1.266	1.307	1.387	1.239
	(0.648)	(0.427)	(0.531)	(0.344)	(0.759)	(0.692)	(0.780)
LTA	3.91	4.236	4.759	4.471	4.351	4.813	4.486
	(1.324)	(1.572)	(1.42)	(0.777)	(1.400)	(1.459)	(1.729)

Items in brackets represent the standard deviation.

ROA (return on assets)
Lev (financial leverage)
LIQ (liquidity)
LTA (log of total assets)

TABLE 6
ESTIMATED MODEL PARAMETERS

Time	Constant	Leverage	Liquidity	LTA	ROA	LLR[18]
79-82	-1.98	0.72	-1.02	-0.20	-2.74	92.3%
	(-1.95)	(0.87)	(-3.69)	(-2.16)	(-2.47)	
80-83	-2.33	1.91	-1.17	-0.20	-1.48	91.8%
	(-2.18)	(2.28)	(-4.15)	(-2.31)	(-1.60)	
81-84	-2.46	1.84	-1.0	-0.20	-1.90	90.8%
	(-2.43)	(2.29)	(-2.95)	(-2.33)	(-2.09)	

Items in brackets under the parameters are the t-statistics

TABLE 7
COMPARISON OF PROBABILITY CUTOFFS

Ratio[19]	Expected[20] A/(A+B)[22]	Empirical[21] 79-82[23]	Empirical 80-83	Empirical 81-84
100/1	0.0099	0.0226	0.0172	0.0163
50/1	0.0196	0.0226	0.0306	0.0186
25/1	0.0385	0.0353	0.0561	0.0573
10/1	0.0909	0.0353	0.0586	0.0573
5/1	0.1667	0.0655	0.0631	0.0970
2/1	0.3333	0.0753	0.0888	0.1354
1/1	0.5000	0.9503	0.9551	0.3143

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[18] LLR = Log Likelihood Ratio which is analogous to R^2

$$LLR = 1 - \frac{\text{log likelihood at convergence}}{\text{log likelihood at 0}}$$

[19] Ratio is the relation of Type I to Type II error costs.

[20] The expected cutoff point is calculated given the profit and loss percentages. It is calculated as P* = B/(B+A), where
P* = the cut off point where no further loans should be made
A = the percentage of the loan that is expected to be lost if the borrower declares bankruptcy--Type I error
B = the percentage profit lost if the loan is not given and the borrower does not declare bankruptcy--Type II error

[21] The empirical cutoff point is determined by finding the point which maximized the lender's profit in the period.

[22] A/(A+B) is the theoretical cutoff determined by expected values.

[23] The dates represent the time period used to estimate the model.

TABLE 8
COMPARISON OF % SAVED BY USING THE MODEL VS. NAIVE LENDING POLICY USING EMPIRICALLY DETERMINED CUTOFFS

	1979-1982[24] Emp v Exp[25]		1980-1983 Emp v Exp		1981-1984 Emp v Exp	
100/1[26]	6.14	7.02	21.49	3.35	136.92	78.84
50/1	1.77	0.98	4.96	7.65	33.19	35.43
25/1	0.76	1.02	2.17	1.63	0.25	0.41

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[24] Time Period is the time period used to estimate the model. The model is then tested on the following year.

[25] Emp represent cutoffs determined empirically by choosing the cutoff point which maximized profit in the estimation period. Exp represents the cutoff determined mathematically based on expected profits and the costs of errors. All cutoffs are indicated in Table VII.

[26] The ratio is the relation of Type I to Type II error costs.

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