THE DYNAMICS OF TECHNOLOGICAL ADOPTION IN HARDWARE/SOFTWARE SYSTEMS: THE CASE OF COMPACT DISC PLAYERS

by

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Abstract

In this paper we examine the dynamic resolution of technological adoption in "hardware/software" systems. We are interested in determining to what extent software availability affects hardware sales and/or vice-versa. We first develop a dynamic model for estimating demand when costs (and hence prices) are declining over time. We then estimate it empirically for the case of compact disc players.

We find that there is "two-way" feedback between hardware and software for compact disc players. The result that the availability of compatible software (the CDs) plays a significant role in determining the adoption of compact disc players is likely due in part to the fact that compact disc players were not compatible with any existing audio standard.

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Keywords: Technological Adoption, Complementary Products, Empirical Study.
1 Introduction

1.1 Objective of Paper

In this paper we examine the dynamic resolution of technological adoption in "hardware/software" systems. Such systems abound in consumer electronics; examples include television sets and programming, CD players and CD's, and video-game systems and video games. Our objective is to formulate a model of the evolution of such systems, focusing on two features: (i) the interdependence between the hardware and software components of the system, and (ii) the presence of technological progress. We then estimate the model, using data on the CD industry.

For many systems, software and hardware products are produced by different firms.\(^1\) Hence when a new hardware technology becomes available, consumers and software developers play a "waiting game". Consumers hesitate to adopt the new hardware (which involves a sunk investment) until it is clear that there will indeed be a sufficient variety of software. Similarly, software firms hesitate to develop compatible software (which also involves a sunk investment) until a sufficient number of consumers have adopted the new technology. This creates interdependence between the timing decisions of consumers and software developers: When (or whether) a consumer decides to purchase hardware depends on when (or whether) software developers make their products available for sale and vice versa.

Another important feature of hardware/software systems is the presence of technological progress. In the CD industry this is reflected in declining product prices and in increasing product quality. Given this fact, an important determinant of the timing decision of consumers is how much progress has already been made and how much more progress is expected to be made in the future. Likewise an important determinant of the timing decision of software developers is how costly it is to put a new variety on the market now vs. how costly it will be to do so in the future.

Our formulation is based on these two considerations: We view the diffusion of a new system as driven by the rate of technological progress of its components and by the interdependence between them. This formulation raises several empirical issues: One issue is to

\(^1\)Even in cases in which a hardware firm produces software (e.g., Nintendo), independent software producers are critical sources of software variety.
disentangle the effect of technological progress from the interdependence between software availability and hardware adoption. A second issue is to quantify these effects, determining thereby whether increases in software availability lead to more hardware sales, whether increases in hardware sales lead to greater software availability, or whether there is two-way feedback. The empirical part of our paper addresses these issues in the context of CD systems.

The CD industry provides a good "case study" for our purposes because when CD's were first introduced they were not "backward compatible" with any existing audio technology. In other words, a consumer could not play a vinyl record or a cassette on a CD player (or—vice versa—play a CD on a turntable). Consequently, in estimating the evolution of the CD industry, we are able to focus on the effect of technological factors and the interaction between the two sides of the market, while avoiding the effect of a pre-existing installed base of software.2

Our main empirical finding is that there is two-way feedback between hardware and software. This result is due—in part—to the fact that pre-existing software was nonexistent, so that software was indeed as much of a "bottleneck" to the diffusion of hardware as the diffusion of hardware was to the development of software.

Interestingly, this result no longer holds if we ignore the theoretical structure and try to establish the direction of the causality links by running a "Granger Causality" test. Using that approach, software availability causes hardware adoption, but not the other way around. The "Granger Causality" result, that increases in hardware sales do not lead to increases in future software supply, is not consistent with casual empiricism.

Two related contributions of the paper are as follows. First, the methodology we develop might be useful for public-policy analysis regarding the benefits of backward compatibility for other systems. A topical example is high-definition televisions (HDTV). Recently, the FCC

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2This can be contrasted with the television industry. Shortly after color-television technology was introduced, the FCC imposed the adoption of a standard which enabled Black and White (B/W) television-set owners to receive color programming (albeit in black and white) and, symmetrically, color-television owners to receive B/W programming. Consequently, the software side of the market (television programming) was not as crucial. In such a setting, we would expect the purchase of hardware to cause the development of software, but not vice versa. Indeed, one article in the popular press (TV Guide, September 9, 1961) noted that consumers "broke the logjam" (page 20) and adopted color television sets, spurring the networks to begin producing color programming.
set down the guidelines for a new digital television standard, which is essentially incompatible with the existing analog (NTSC) standard. There is, however, the possibility of adapters in both directions; NTSC televisions will be able to view new broadcasts with a "down-converter" box, which will provide a somewhat improved image at an estimated cost of 300 dollars. New HDTVs will be able to watch old NTSC programs if they have a second (analog) tuner built-in; these televisions will begin with at least a 1000 dollar premium over similar sized NTSC television sets. Our methodology (suitably applied to the TV industry) could be useful in predicting the pace of adoption of the new digital technology. The speed of adoption has some ramifications, as the FCC has scheduled an end to NTSC broadcasts by the year 2006.\footnote{See "HDTV: How the Picture Looks Now," \textit{Business Week}, May 26, 1997, and "Should you Roll Out the Welcome Mat for HDTV?" \textit{The New York Times}, April 27, 1997.}

Another contribution of the paper is in the area of firm strategy. It is often claimed that high-tech firms can enhance their profits by subsidizing the adoption of their technology—\textit{when it is first introduced}. In this spirit, the Netscape browser and other Internet software is or was disseminated free of charge to early adopters; similarly, Sony entered into the production of software although its comparative advantage is in hardware. Our results illuminate the logic behind these business strategies. Given our estimates we are able to determine the elasticities of hardware sales with respect to hardware prices and with respect to software availability. We show that these elasticities decline over time. Thus, for firms introducing new system standards, subsidies would indeed be most profitable at early stages of the industry.

The paper also makes a methodological contribution; our dynamic model for estimating demand (technology adoption) is applicable even when there is no complementary software industry as long as costs (and hence prices) are declining over time. In our setting, consumers explicitly trade-off the lower prices which will result from waiting one period to purchase with the loss of one period's services from the durable product. We believe that this methodology can be employed in order to estimate dynamic models of technological adoption for computers and other high-tech products where quality adjusted prices are falling over time.

The paper proceeds as follows. In the next subsection we provide a brief review of related
literature. In section 2 we set up the theoretical model. In section 3 we describe the data. In section 4 we estimate the model; this section includes a comparison between the actual and the estimated evolution of the industry and a comparison with Granger-Causality tests. In section 5 we determine the elasticities of hardware sales with respect to software availability and hardware prices. Section 6 concludes.

1.2 Brief Literature Review

The idea that the diffusion of a system depends on the interaction between its components and on technological factors is not new. Katz and Shapiro (1985), (1986) and Farrell and Saloner (1985) were the first to introduce these ideas in a static context using a single product paradigm. Chou and Shy (1990) and Church and Gandal (1992) extended these ideas to the context of systems, consisting of hardware and a variety of software, but their framework also employed static models. Dynamic formulations include Katz and Shapiro (1992) and Kandori and Rob (1997); these models, however, focus exclusively on the adoption decision of consumers and not on the software industry.

Several papers have empirically examined technological adoption of hardware/software systems. For example, Greenstein (1992), Gandal (1994) and Saloner and Shepard (1995) all provide evidence that the value of the hardware depends on the variety of complementary software. Economides and Himmelberg (1995) estimate a dynamic model of network growth for fax machines; in this case there is no complementary product for that industry. Gandal, Greenstein, and Salant (1995) examine the adoption of a hardware/software system and focus on causation between components. However, they employ Granger causality tests, and not a structural model.

The novelty of the present paper is that it formulates a structural model and then estimates it. In doing so it ties together theory and empirical estimates, and provides a framework for quantitative evaluation of public policy and firm strategy.
2 Model Formulation

2.1 Generalities

In our model, hardware is a homogenous, infinitely-durable product. The market for hardware is competitive, so hardware is provided at marginal cost. We denote the marginal cost of hardware in period \( t \) by \( c^h_t \), where \( c^h_t \) is assumed to be (strictly) decreasing in \( t \) as a result of (exogenous) technological progress.

Software firms are infinitely lived and maximize total profit, which is the discounted stream of per-period profits. A software firm that enters the market at time \( t \) incurs a fixed cost of capacity installation denoted \( F_t \) and sells its software product in each period beginning with \( t + 1 \). We let \( N_t \) denote the number of software-producing firms in the market in period \( t \); this gives the amount of software which will be produced in period \( t + 1 \). Software is assumed to provide service for only a single period.\(^4\) Reductions in \( F_t \) (which are exogenous to the system) and increases in the size of the hardware installed base induce more software firms to enter over time.

Consumers are also infinitely lived and are differentiated by a taste parameter, \( \theta \), which measures their eagerness to own the system. The distribution of \( \theta \) across consumers is denoted \( F(\theta) \), with support \([0, \bar{\theta}]\), \( \bar{\theta} < \infty \), and \( F(\bar{\theta}) = M < \infty \). Consumers maximize lifetime utility, which is the discounted stream of per-period utilities. Each consumer who buys a unit of hardware at time \( t \) has a demand for software varieties (specified below) in each period beginning with \( t + 1 \).

\( Y_t \) is the "installed base" of hardware in period \( t \), that is, the number of consumers who have purchased hardware by the end of period \( t \); this gives the size of the software market in period \( t + 1 \). Different individuals buy the system at different dates depending on how strong their taste is for the system (how large is their \( \theta \)). \( Y_t \) increases over time because the price of hardware, \( c^h_t \), decreases and because of increases in software availability \( (N_t) \) and decreases in software price.

The discussion above indicates that the adoption decision of consumers depends not only on the existing variety of software available for a system but also on expectations regarding

\(^4\)Nothing qualitative changes in the case in which software is fully or partially durable.
software availability in the future. Similarly, the attractiveness of providing software depends on both the existing installed base of hardware as well as expectations about future increases in the installed base. The model we develop in this section makes the assumption that both consumers and software firms have rational expectations.

The timing of the game is as follows. In each period, (1) some consumers make initial hardware purchases, (2) consumers with hardware purchase software, (3) some software firms enter the software market and install capacity, and (4) established software firms sell their software products to consumers. We assume that all these actions occur simultaneously. Then we go to the next period (with new values of $N_t$, $Y_t$, $F_t$, and $c_i^t$) and the same set of actions is repeated.

In the following subsections, we first describe competition in the software industry and the software entry decision. We then describe consumer preferences over hardware/software systems and the consumer adoption decision.

2.2 Software Market

Within a period, $t$, the per-consumer demand for software variety $i$ is $D_i(p_1, ..., p_N)$, where $N$ is the number of software varieties available in that period and $p_j$ is the price of variety $j$, $j = 1, ..., N$. We assume that demands are symmetric: $D_j(p'_1, ..., p'_j, ..., p'_N) = D_i(p_1, ..., p_i, ..., p_N)$, whenever $p'_j = p_i$ and $(p'_k)_{k \neq j}$ is a permutation of $(p_k)_{k \neq i}$. We also assume a constant marginal cost of software production, $c$, and quasi-concavity of the per-consumer profit function:

$$(p_i - c)D_i(p_1, ..., p_N).$$

Given the symmetry of demands and the quasi-concavity of the profit functions, there exists an equilibrium in which all firms charge the same price, $p$. This equilibrium is characterized by:

$$p = c - \frac{D(p,...p)}{\partial D(p,...p) / \partial p}.$$

Denote the equilibrium price by $p = \varphi(N)$ and let $\varphi'(N) < 0$, so that the equilibrium software price is declining in the number of available software varieties; this is consistent with the properties of common spatial competition models. Furthermore, let $f(N) \equiv (\varphi(N) - \varphi'(N)N)/N$. 

The period $t+1$ profit of a software firm is then

$$\pi_{t+1} = Y_t f(N_t),$$

since (by symmetry) each software firm has an equal share ($Y_t/N_t$) of the market.

Consider now the entry decision of software firms. If a firm enters in period $t$ it pays the entry fee $F_t$ and earns the profit stream $(\pi_{t+1}, \pi_{t+2}, ...)$, generating a lifetime profit of

$$-F_t + \delta \pi_{t+1} + \delta^2 \pi_{t+2} + ....$$  \hspace{1cm} (1)

Likewise, if it enters in period $t+1$ it generates a lifetime profit (evaluated as of period $t$) of

$$-\delta F_{t+1} + \delta^2 \pi_{t+2} + \delta^3 \pi_{t+3} + ....$$  \hspace{1cm} (2)

In a free-entry equilibrium firms must be indifferent between these two options. This implies:

$$F_t - \delta F_{t+1} = \delta \pi_{t+1} = \delta Y_t f(N_t).$$  \hspace{1cm} (3)

We assume $F_t - \delta F_{t+1}$ is decreasing over time (insuring that more software firm enter). Taking the natural logarithms of both sides of (3) we obtain:

$$\log f(N_t) = -\log \delta - \log Y_t + \log (F_t - \delta F_{t+1}).$$  \hspace{1cm} (4)

We return to this equation below.

### 2.3 Hardware Market

There is no standalone value to either hardware or software. Consider consumer $\theta$’s hardware purchasing decision. If he purchases in period $t$, his outlay is $c^h_t$ and he enjoys the stream of utility $(CS(p_{t+1}), CS(p_{t+2}), ...)$, where $CS(p)$ is the consumer surplus when he pays $p$ for a variety of software. This generates a net benefit of

$$-c^h_t + \theta [\delta CS(p_{t+1}) + \delta^2 CS(p_{t+2}) + ...].$$  \hspace{1cm} (5)

Likewise, if he purchases in period $t+1$ he generates the benefit (evaluated as of period $t$)

$$-\delta c^h_{t+1} + \theta [\delta^2 CS(p_{t+2}) + \delta^3 CS(p_{t+3}) + ...].$$  \hspace{1cm} (6)
Let \( \theta_t \) be the consumer indifferent between these two. Then, subtracting (5) from (6), we obtain
\[
c_t^h - \delta c_{t+1}^h = \theta_t \delta CS(p_{t+1}) = \theta_t \delta CS(\varphi(N_t)) \equiv \theta_t \delta g(N_t).
\] (7)

Taking the natural logarithms of the two sides we obtain
\[
\log(\theta_t) = \log(c_t^h - \delta c_{t+1}^h) - \log \delta - \log g(N_t).
\] (8)

We assume that \( c_t^h - \delta c_{t+1}^h \) is decreasing in \( t \); this insures that the installed base keeps increasing. We now turn to the econometric specification of the model.

2.4 Econometric Specification

We assume \( F(\theta) = \tau \theta^{\beta_1} \). Taking the logarithm we obtain
\[
\log F(\theta_t) = \log \tau + \beta_1 \log \theta_t = \log \tau / \delta^{\beta_1} + \beta_1 [\log(c_t^h - \delta c_{t+1}^h) - \log g(N_t)],
\] (9)

where the second equality follows by substitution from (8). Cumulative sales up to period \( t \) equal \( Y_t = M - F(\theta_t) \). We can substitute this into (9), and obtain:
\[
\log(M - Y_t) = \log(M - Y_t) = \log \tau / \delta^{\beta_1} + \beta_1 [\log(c_t^h - \delta c_{t+1}^h) - \log g(N_t)].
\] (10)

We assume, for tractability, that \( g \) is a power functions of \( N \), \( g(N) = N^\rho \). This allows us to estimate, in log-linear form, the consumer adoption equation:
\[
\log(M - Y_t) = \beta_0 + \beta_1 \log(c_t^h - \delta c_{t+1}^h) + \beta_2 \log N_t + \epsilon_{1,t},
\] (11)

where \( \beta_0 = \log \tau / \delta^{\beta_1} \), \( \beta_1 \) is the parameter in \( F \), \( \beta_2 = -\rho \beta_1 \) and \( \epsilon_{1,t} \) is a noise term.

From equation (4), using \( f(N) = N^\gamma \) as above, we derive the software entry equation to be estimated:
\[
\log(N_t) = \alpha_0 + \alpha_1 \log Y_t + \alpha_2 \log(F_t - \delta F_{t+1}) + \epsilon_{2,t},
\] (12)

where \( \alpha_0 = \log(1/\delta^{1/\gamma}) \), \( \alpha_1 = -1/\gamma \), \( \alpha_2 = 1/\gamma \) and \( \epsilon_{2,t} \) is a noise term. Note that \( \alpha_1 = -\alpha_2 \).

In section 4 we shall estimate the coefficients \( (\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2) \).
3 Data

Compact-disc technology was developed by Sony and Philips and introduced to the United States in 1983. In order to encourage adoption, Sony and Philips licensed their technology quite liberally. We now discuss our data:

- We obtained quarterly data on compact-disc player sales for the 1985-1992 period from the Electronics Industries Association. For reasons of confidentiality, this series was given as an index. The series of cumulative CD player sales is shown in Figure 1. The index series LINSTALLED BASE is the natural log of the cumulative compact disc player sales index.

- Price and sales series on CD players by model and feature were not available from the above source. Such data is necessary to construct a quality-adjusted price series. We were fortunate to receive data on prices and characteristics for all CD players sold in the U.S. for the 1983-1992 period from Glenn MacDonald. Using these data, we ran a hedonic price regression; a description of the data and the results are reported in the appendix. We then constructed yearly (third quarter) quality-adjusted prices and averaged over the other quarters to get the series for 1985-1992. We found that quality adjusted prices fell by 36 percent during this period. We converted these to real quality adjusted prices using the Consumer Price Index.

Since we assume that the "hardware" market for compact disc players is competitive, we use the real quality adjusted price series as the marginal cost series; we denote

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5See Grindley and McBryde (1992) for more details.
6We have data on CD player sales through the 1994 period. However, during the 1993-1994 period many of the purchases were "repeat" purchases of portable and "Walkman" CD players. Since our model only applies for initial purchases and since our data does not distinguish among the types of models purchased, we restrict analysis to the 1985-1992 period, which is the critical period for the adoption of the technology. By the end of 1991, 28 percent of households had a CD player. Another compelling reason to examine this period is that we only have data on hedonic prices though the third quarter of 1992.
7We are extremely grateful to Glenn MacDonald for providing us with these data. As reported in Horstmann and MacDonald (1995), these data come from Audio magazine and are based on third quarter prices.
8This series runs through the third quarter of 1992 and limits our analysis to the following time period: first quarter 1985 through third quarter 1992; hence there are 31 observations.
9The consumer price index rose by approximately 33 percent during the same period.
this series as MARGINAL. For the case when $\delta = .75$ the variables MARGINAL and 
$LDMARGINAL = \log(c^h_t - \delta c^h_{t+1})$ are shown in Figure 2.\(^{10}\) These plots show that real
quality adjusted CD player prices (MARGINAL) fell continuously during the period,
but that they fell slowly from 1985 until 1988 and then began falling more quickly.\(^{11}\)

- Our data on compact disc availability from 1985 to 1992 comes from a series of Schwann
publications. Schwann guides, which are published quarterly, list all compact discs
available by major music category: classical, popular, and jazz. Compact disc avail-
ability is calculated by multiplying the average number of titles per page in the Schwann
guides by the number of pages in the relevant category. We then aggregate across the
categories to obtain total compact disc availability. The series VARIETY, which is
the total compact disc availability in each quarter, is shown in Figure 3. The series
$LVARIETY$ is the natural log of total compact disc availability.

- Our data on the fixed cost of capacity installation for producing compact discs (soft-
ware) for the 1985 to 1992 period comes from a Harvard Business School study on
Compact Discs that was prepared by M. McGahan (1993). In her study, she cites
industry estimates of the one-time cost (per unit of disc-pressing capacity per year) of
installing disc pressing capacity. She provides yearly estimates for the first four years
and a "long run" estimate. We use the long run cost estimate as the cost estimate for
"year ten" and average over the "year four" and the "year ten" estimate in order to fill
in the missing years. We then fill in the quarterly observations by averaging over the
quarters. We denote the series on fixed costs as FIXED. For the case when $\delta = .75$,
the variables FIXED and $LDFIXED = \log(F_t - \delta F_{t+1})$ are shown in Figure 4.

4 Estimation

The two equation system to be estimated consists of the consumer dynamics equation, (11)
and the software entry equation, (12). Since this is a simultaneous equations system ($N_t$
and $Y_t$ are endogenous), Ordinary Least Squares (OLS) estimation will lead to biased and

\(^{10}\)We discuss this choice of $\delta$ and also consider the case $\delta = .9$ below.

\(^{11}\)This means that the series LDMARGINAL has a small spike at the point where prices began falling quickly. We return to this point when we discuss our results.
inconsistent estimates. In order to identify our two system equation, we need to find instru-
ments for \( N_t \) and \( Y_t \). Since we do not estimate \( \delta \), we employ the cost shifter \( F_t - \delta F_{t+1} \) for 
\( N_t \) in (11) and the cost shifter \( C_t^h - \delta C_{t+1}^h \) for \( Y_t \) in (12). Additionally, we use three quarterly 
dummies as instruments. We estimate the full system via the general method of moments 
\((GMM)\).\(^{12, 13}\)

Before we present the empirical results, note that from the theoretical model, the sign of 
\( \alpha_1 \) should be positive and the sign of \( \alpha_2 \) should be negative. Further, the sign of \( \beta_1 \) should be 
positive and the sign of \( \beta_2 \) should be negative. If \( \alpha_1 \) is positive and significantly different from 
zero while \( \beta_2 \) is not significantly different from zero, this would provide evidence that only 
software firms play a waiting game, i.e., that hardware adoption causes software availability. 
On the other hand, if \( \alpha_1 \) is not significantly different from zero while \( \beta_2 \) is negative and 
significantly different from zero, this would provide evidence that only consumers play a 
waiting game, i.e., that software availability causes hardware sales. Finally, if \( \alpha_1 \) is positive 
and significantly different from zero AND \( \beta_2 \) is negative and significantly different from zero, 
this would provide evidence that both consumers and software firms play a waiting game, 
i.e., that there is “two-way feedback” between hardware adoption and software availability.

In order to perform the estimation, we chose values for \( M \) and \( \delta \). We set \( M = 140,000 \) 
and \( \delta = .75 \). We chose \( M \) in the following fashion. By the end of 1991, 28 percent of all 
households had adopted a CD player. Our index of cumulative CD sales stood at 50,000. 
Assuming that eventually between 75 and 80 percent of households would have a CD player, 
the size of the potential market (in index terms) is approximately 140,000. Our preferred 
choice of \( \delta \) is .75, rather than \( \delta = .9 \), (which might seem more natural) because it provided 
better results in the sense of minimizing the difference between the actual and predicted 
varieties and actual and predicted sales. Our qualitative results, however, regarding two-
way feedback are unchanged regardless if we use \( \delta = .75 \) or \( \delta = .9 \). Hence we report results 
and predictions using both of these values. Note that higher values of \( \delta \) will magnify the 
feedback effects and reduce the cost effects.

\(^{12}\)The software we use was written by Hansen, Heaton, and Ogaki. See Hansen and Singleton (1982) for 
theoretical foundations.

\(^{13}\)Our GMM estimates are similar to the TSLS estimates when we do not use the quarterly dummies, i.e., 
when the system is exactly identified.
4.1 Results

Despite the fact that OLS estimates are expected to be biased, we first present these regressions in Table 1. In the case of OLS, the “cost” coefficients have the expected signs and are statistically significant; the feedback from hardware to software is also significant, but the feedback from software to hardware is not significant and has the wrong sign.\textsuperscript{14}

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{CONSTANT} ($\alpha_0$)</td>
<td>6.85</td>
<td>0.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\text{LINSTALLED BASE} ($\alpha_1$)</td>
<td>0.316</td>
<td>0.047</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\text{LDIVIDED (a_2)}</td>
<td>-0.76</td>
<td>0.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\text{CONSTANT} ($\beta_0$)</td>
<td></td>
<td>13.10</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>\text{LDIVARITY (\beta_1)}</td>
<td>0.578</td>
<td>0.075</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\text{LVARIETY (\beta_2)}</td>
<td>0.002</td>
<td>0.020</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Adj. $R^2$: 0.99 | Adj. $R^2$: 0.88 |

Table 1: OLS Results: Unconstrained Model with $\delta = .75$

Tables 2 and 3 present the results using GMM estimation. The difference between the tables is that in Table 2 we employ the constraint (from the theoretical model) that $\alpha_1 = -\alpha_2$. Table 3 shows the results using GMM estimation without employing the constraint that $\alpha_1 = -\alpha_2$.

Comparing Tables 2 and 3, the GMM estimation without the constraints yields much better results than the GMM estimation with the constraint. This is not surprising, given that in the unconstrained model, the magnitudes of the estimates of $\alpha_1$ and $\alpha_2$ are quite different. Hence, in the following analysis we focus on the unconstrained model. Table 3, provides evidence that in the case of CDs, there is two-way feedback between hardware and software, although the feedback from hardware to software is more significant. This is shown by the signs and magnitudes of the estimates of $\alpha_1$ and $\beta_2$. This is despite the fact that the value of $\delta$ is relatively low.

With $\delta = .9$, the feedback from software to hardware is much more significant. Table 4

\textsuperscript{14}When $\delta = .9$, the feedback from software to hardware is significant with the correct sign in the case of OLS. See Table 8 in the appendix.
has GMM estimates for the case when $\delta = .9$. This table shows that changes in $\delta$ modify the results in a predictable way: higher values of $\delta$, the variety and installed base factors are more important and more significant; this is particularly the case for variety, i.e., the feedback from software to hardware.

Note that the "cost" coefficients also have the expected signs and are statistically significant in all of the tables. With the higher value of $\delta$ in Table 4, the cost factors are less important, but still significant. Table 7 in the appendix shows that our results are quite robust to the choice of the size of the potential market by setting $M = 280,000$, or twice the estimated potential market.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Software Entry Equation: Coefficient</th>
<th>Standard Error</th>
<th>Consumer Purchase Equation: Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT ($\alpha_0$)</td>
<td>5.51</td>
<td>0.11</td>
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</tr>
<tr>
<td>LINSTALLED BASE ($\alpha_1$)</td>
<td>0.46</td>
<td>0.01</td>
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</tr>
<tr>
<td>LDFIXED ($\alpha_2$)</td>
<td>-0.46</td>
<td>0.01</td>
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<tr>
<td>CONSTANT ($\beta_0$)</td>
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<td></td>
<td>13.43</td>
<td>0.15</td>
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<td>LDMARGINAL ($\beta_1$)</td>
<td></td>
<td></td>
<td>0.62</td>
<td>0.008</td>
</tr>
<tr>
<td>LVARIETY ($\beta_2$)</td>
<td></td>
<td></td>
<td>-0.019</td>
<td>0.018</td>
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<td></td>
<td></td>
<td></td>
<td>GMM OBJ 6.70 (p value=0.46)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: GMM Results: Constrained Model with $\delta = .75$

4.2 Multiple Equilibria and Rational Expectations

Multiple equilibria are typical when there are network effects or when there are complementary products.\textsuperscript{15} Our model is no exception. Substituting the value for $\log N_t$, from (12), into the consumer dynamics equation, (11), and rewriting yields the following expression.

\textsuperscript{15}The appendix has TSLS estimates for the case in which the quarterly dummies are not used as instruments, i.e., when the system is exactly identified. These estimates are in Table 9 for the case when $\delta = .75$ and in Table 10 for the case when $\delta = .9$.

### Table 3: GMM Results: Unconstrained Model with $\delta = .75$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CONSTANT (}a_0)$</td>
<td>7.50</td>
<td>0.79</td>
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<tr>
<td>$\text{LINSTALLED BASE (}a_1)$</td>
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</tr>
<tr>
<td>$\text{LDFIXED (}a_2)$</td>
<td>-0.96</td>
<td>0.21</td>
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</tr>
<tr>
<td>$\text{CONSTANT (}\beta_0)$</td>
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<td>13.12</td>
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<tr>
<td>$\text{LDMARGINAL (}\beta_1)$</td>
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<td>0.06</td>
</tr>
<tr>
<td>$\text{LVARIELTY (}\beta_2)$</td>
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<td></td>
<td>-0.019</td>
<td>0.014</td>
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</tbody>
</table>

GMM OBJ 3.03 (p value=0.80)

### Table 4: GMM Results with $\delta = .9$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
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<td>$\text{CONSTANT (}a_0)$</td>
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<td>0.36</td>
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<tr>
<td>$\text{LINSTALLED BASE (}a_1)$</td>
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<td>0.046</td>
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<td>$\text{LDFIXED (}a_2)$</td>
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<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{CONSTANT (}\beta_0)$</td>
<td></td>
<td></td>
<td>13.37</td>
<td>0.28</td>
</tr>
<tr>
<td>$\text{LDMARGINAL (}\beta_1)$</td>
<td></td>
<td></td>
<td>0.27</td>
<td>0.080</td>
</tr>
<tr>
<td>$\text{LVARIELTY (}\beta_2)$</td>
<td></td>
<td></td>
<td>-0.080</td>
<td>0.01</td>
</tr>
</tbody>
</table>

GMM OBJ 3.19 (p value=0.78)

\[
\log(M - Y_t)Y_t^{-\beta_2a_0} = (\beta_0 + \beta_2 a_0) + \beta_1 \log(c_t^h - \delta c_{t+1}^h) + \alpha_2 \beta_2 \log(F_t - \delta F_{t+1}) + \beta_2 \varepsilon_{2,t} + \varepsilon_{1,t}. \quad (13)
\]

Since our estimate for $a_1$ is positive and our estimate for $\beta_2$ is negative, the exponent on $Y_t$ on the left-hand side of (13) is positive. The left-hand side of the equation is a "hump-shaped" curve in $Y_t$, while the right-hand side is constant in $Y_t$. This suggests that for a range of $Y_t$ there can be two "non-trivial" solutions or equilibria to (13). The solution to (13) with the lower value of $Y_t$, which we denote $Y_t^l$, is unstable, because a small increase in the number of consumers adopting the technology makes the hardware more attractive, inducing all consumers in the range $Y_t^l$ to $Y_t^h$, which is the higher solution to equation (13), to adopt the technology. Hence, only the solution $Y_t^h$ is a stable equilibrium. Despite the
theoretical possibility of obtaining an unstable solution, this never arose in practice.\footnote{One additional stable equilibrium is that the system never gets adopted. However, since CD were adopted we focus on the positive, high solution only.}

4.3 Predicted Values and Other Post Estimation Analysis

We use equation (13) with our estimated coefficients to obtain predicted values for both installed base and variety.\footnote{We are of course using this equation without the error terms. To get the predicted variety, we substitute the result for predicted installed base from equation (13) into (12).} Figures 5 and 6 are respectively plots of (i) actual vs. predicted installed base and (ii) actual vs. predicted variety for $\delta = .75$. These series begin only in the fourth quarter of 1985, for reasons we discuss below. Note from these Figures that our model does reasonably well despite the exponential growth in the installed base.

Note from the predicted series in Figures 5 and 6 that there is a dip corresponding to the third quarter of 1988; i.e., the predicted installed base and the predicted variety actually shrink in this quarter, which is clearly not reflected in the actual numbers. The cause of this can be readily seen in Figure 2, showing the marginal cost and the variable $LDMARGINAL = log(c_i^h - \delta c_{i+1}^h)$. From the higher curve, representing the marginal cost of compact disc players, one can see a kink in 1988; prices are falling faster in the 1988-1992 period than they were in the 1985-1988 period. Hence in the lower curve in Figure 2, the series $LDMARGINAL$, rises at the kink. Sales should decline near the kink, since prices are expected to fall faster; hence some consumers will hold off purchasing hardware, waiting for the lower prices. Empirically, our results show that the right hand side (RHS) of equation (13) actually rises due to the increase in $LDMARGINAL$. This leads to a predicted decrease in the installed base for one period, rather than simply a decrease in sales; the same effect also leads to a decrease in predicted variety. If we had smoothed the price (MARGINAL) series, these dips would not have occurred; for obvious reasons we prefer to work with the actual data.

Figures 5 and 6 can be compared with Figures 7 and 8, which show the actual and predicted series for variety and installed base for $\delta = .9$. Note that the predictions are better for $\delta = .75$, especially in the earliest years. Note also that the dip in 1988 is more pronounced for $\delta = .9$; the explanation is simply that the increase in $LDMARGINAL$ is larger for larger
Figure 9 shows the left-hand side (LHS) and right-hand sides (RHS) of Equation (13) for $\delta = .75$. The LHS of equation (13) is concave in $Y_t$, with a maximum in the third quarter of 1985. In order for an equilibrium with positive sales to exist in a given period, the RHS of equation (13) must be less than the maximum of the LHS of equation (13). Otherwise, the model predicts that no adoption takes place. From Figure 9, the RHS of equation (13) exceeds the maximum value of the LHS of equation (13) for the first three quarters of 1985. Hence, the model predicts that no consumer would have adopted CD players before the fourth period of 1985.

It is not surprising that the model predicts that no consumers would have adopted CD players before the fourth period of 1985. By the end of the third quarter of 1985, our index of CD sales stood at 700. By the end of the period for which we have data (the third quarter of 1992), this index stood at nearly 65,000. A trivially small number of CD players were purchased in the first three quarters of 1985. Our model simply estimates this number to be zero for the first three quarters of 1985.

4.4 A Comparison with Granger Causality Tests

We now briefly discuss the results of Granger-causality tests. Recall that a variable $x_t$ causes $y_t$ in the spirit of Granger if lagged values of $x_t$ are significant in a regression of $y_t$ on lagged values of $y_t$ and lagged values of $x_t$. Granger-causality tests were conducted by running ordinary least squares regressions. The first regression in the following table has the natural log of installed base of CD players (LINSTALLED BASE) as the dependent variable and lagged values of the natural log of INSTALLED BASE and the natural log of VARIETY (LVARIETY) as well lagged values of the natural log of MARGINAL (denoted LMARGINAL) and the natural log of FIXED (denoted LFIXED) on the right hand side of the equation, while the second regression in the table has the natural log of VARIETY as the dependent variable with the same right hand side variables as the installed base equation. A single lag is employed.

---

19The way to find the equilibrium value of installed base in Figure 9 for any period is to draw a straight horizontal line from the value for the RHS of equation(13) to the curve. This gives the value of the LHS of the same equation and then the value for installed base can be backed out.
From Table 5, Granger causality tests conclude that software causes hardware, but hardware does not cause software. The result from the Granger causality tests, that increases in hardware sales do not lead to increases in future software supply is not consistent with casual empiricism, nor with the results from the structural model, in which there is strong feedback from hardware to software.\(^2\) This suggests that there are indeed benefits from estimating a structural model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dept. var: LINSTALLED BASE</th>
<th>Dept. var: LVARIETY</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>-1.26</td>
<td>1.58</td>
</tr>
<tr>
<td>LINSTALLED BASE(-1)</td>
<td>0.60</td>
<td>0.033</td>
</tr>
<tr>
<td>LVAIETY (-1)</td>
<td>0.45</td>
<td>0.83</td>
</tr>
<tr>
<td>LMARGINAL(-1)</td>
<td>-0.57</td>
<td>0.027</td>
</tr>
<tr>
<td>LFIXED (-1)</td>
<td>0.18</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

\(R^2: .99\) \(R^2: .99\)

Table 5: Granger Causality Tests

5 Elasticities

One question of interest to firms introducing new hardware/software systems is how to increase profits by taking advantage of the complementarities between hardware/software systems. When adoption is not as certain as in the case of compact disc technology, an overriding concern for both hardware and software firms is implementing strategies to ensure that the technology is widely adopted, with an eye towards creating a de facto standard. Even under the assumption that adoption is a certainty, strategies may exist for increasing system profits. These considerations drive a variety of firm strategies.

Many firms introducing new systems severely discount or give away the "hardware" portion in order to ensure future sales of hardware; vertically integrated firms may also discount hardware in order to stimulate software sales. Netscape Communications gave its early World Wide Web browsers away for free to increase its installed base and stimulate sales.\(^2\)

\(^2\) Of course, a different model is being estimated.
sales of complementary products. Nintendo sells its video game consoles at low prices in
order to sell more software, for which it receives royalties from all producers.

On the other hand, hardware firms may stimulate hardware sales by vertically integrating
into software production in order to ensure a greater variety of software. This is a strategy
which is attributed to Sony, which purchased Columbia Pictures in part to avoid another
Betamax debacle.

We use the estimated parameters of Table 3 to show the sensitivity of hardware sales to
two variables; the variety of software and the price of hardware. Using the implicit function
theorem, from equation (11) one can derive the elasticity of hardware sales with respect to
the variety of software:

$$
\epsilon_{Y_t,N_t} = \frac{dY_t}{dN_t} \frac{N_t}{Y_t} = -\frac{\beta_2(M - Y_t)}{Y_t},
$$

This series is presented in Figure 10. Figure 10 shows that this elasticity is always positive
and decreasing over time. Therefore, hardware sales are more sensitive to the variety of
software in the early years, implying that an investment by a hardware firm in increasing
software availability is going to have the greatest impact on hardware sales when the system
is first introduced.

Also, the effect of hardware price reductions on hardware sales can be determined. We
examine two different ways of discounting hardware. First, consider a one-time reduction of
the price of hardware, with no promise to lower future prices. This should have a large effect
on sales, as many consumers will shift their purchases forward in order to profit from this
limited offer. The elasticity of hardware demand with respect to a one-time reduction in the
price of hardware is calculated from equation (13) using the implicit function theorem.21

$$
\epsilon_{Y_t,c_t} = \frac{dY_t}{dc_t} \frac{c_t}{Y_t} = \frac{\beta_1(M - Y_t)c_t}{(-\beta_2\alpha_1(M - Y_t) - Y_t)(c_t - \delta c_{t+1})}
$$

The second way of lowering prices is a price cut which consumers expect to last for the
foreseeable future. This permanent price cut will have a smaller effect on current sales, as
there is less incentive to shift purchases forward, but would of course still increase sales.

---

21Recall that hardware price is represented by $c_t$, as we assumed perfect competition in hardware sales.
In this case, we assume that hardware prices are shifted down by a constant proportion represented by $a$. Again using equation (13), suitably modified with the factor $a$, we derive the elasticity of hardware sales with respect to $a$ as:

$$
\varepsilon_{Y_t,a} = \frac{dY_t}{dY_t} \frac{a}{Y_t} = \frac{\beta_1(M - Y_t)}{-\beta_2\alpha_1(M - Y_t) + Y_t}.
$$

The elasticity of hardware sales with respect to these price measures is given in Figure 11. In both cases, the elasticities are negative and decreasing in absolute value. The implication is that price cuts have the greatest relative effect in the early stages of system introduction, when the system is relatively new. Note that, as expected, the elasticity with respect to a one-time price reduction is greater in absolute value than the elasticity with respect to a general decrease in the price level; in the former case consumers shift purchases forward, while in the latter case there is simply a general increase in sales due to the lower price level.

6 Conclusion

In this paper we examined the dynamic resolution of the adoption of CD systems. We found that there was two-way feedback between hardware adoption and software availability. We estimated the elasticity of hardware sales with respect to both hardware price and software availability; these elasticities show that subsidizing the price of hardware or integrating into software production can be an effective strategy for a firm introducing a new hardware standard; this is particularly true when a system is new, as the elasticities decline in absolute value over time. These results confirm the intuition driving business strategies commonly used by firms introducing new systems.

Our results also suggest that had the compact disc players been compatible with an existing audio standard, the adoption process would have occurred more quickly because from the beginning the quantity of available software would have been large, increasing the sales of hardware. In the case of CD players this was not feasible. The question of backward compatibility arises, however, for many if not most new technologies. The digital video disc prototype,\textsuperscript{22} and the high-definition television (HDTV) standard will both be backward

\textsuperscript{22}The digital video disc is a new standard for compact discs which will be used for audio as well as video and computer applications.
compatible with the existing similar technologies. Our results suggest that compatibility will quicken the adoption of these technologies relative to the case if the new technologies had been incompatible. Our methodology could be potentially important in estimating the benefits that this quickening would bring about.
References


8 Appendix

Hedonic Price Regression

In order to derive a quality adjusted price index for compact disc players, we employed quality and price data gathered by Horstmann and MacDonald for the 1983-1992 period. The data on price and product characteristics were gathered from an annual survey contained in the October issues of *Audio* magazine. Their data set contains 1700 observations. As sales data for each model were not available, we were unable to produce a quantity-adjusted price index. In order to not give too much weight to models for which few sales could be expected, we restricted the set to compact disc players costing less than 1000 dollars; this reduced the number of observations to 1291.

The results of the hedonic price regression are in Table 6. We use all variables for which there were no missing observations; hence the following variables are used:

- The variable Lsnratio is the log of the signal-to-noise ratio of the compact disc player; the higher this ratio, the less extraneous noise is introduced.

- The variable Losrate is the log of the oversampling rate of the compact disc player; it gives an indication of how rigorously the player translates the digits contained on the disc into sound.

- The variable Lthd is the log of the total harmonic distortion which the player introduces when reproducing music.

- The variable Lfrelo gives the log of the lower limit of the frequency response of the compact disc player.\textsuperscript{23}

- Lweight is the log of the weight of the compact disc player - it provides a proxy for portable players as opposed to stereo components.

- Finally, the year 19xx variables are the time dummy variables.

\textsuperscript{23}A variable for the upper limit of the frequency response was not available in every year of the sample.
From the regression, all variables were significant, and all characteristics except Lfrelo had the correct sign. All of the time dummy variables had a negative sign. The hedonic price index was calculated by taking the exponentiated estimated coefficients on the time dummy variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
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<tr>
<td>CONSTANT</td>
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<tr>
<td>Lsnratio</td>
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<tr>
<td>Losrate</td>
<td>0.67</td>
<td>0.11</td>
</tr>
<tr>
<td>Lthd</td>
<td>-0.10</td>
<td>0.012</td>
</tr>
<tr>
<td>Lfrelo</td>
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<tr>
<td>Lweight</td>
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<td>year1984</td>
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<td>year1985</td>
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<td>year1986</td>
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<tbody>
<tr>
<td>Adjusted $R^2$: .39</td>
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Table 6: Hedonic Price Regression: Dept. Var. Log(nominal price)
### Software Entry Equation: Consumer Purchase Equation:

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<tr>
<th>Variable</th>
<th>Coefficient</th>
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<tbody>
<tr>
<td>CONSTANT ($\alpha_0$)</td>
<td>7.47</td>
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<tr>
<td>LINSTALLED BASE ($\alpha_1$)</td>
<td>0.25</td>
<td>0.085</td>
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<tr>
<td>LDFIXED ($\alpha_2$)</td>
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<td></td>
<td></td>
<td>13.12</td>
<td>0.064</td>
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<td>0.21</td>
<td>0.030</td>
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<tr>
<td>LVARIETY ($\beta_2$)</td>
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</table>

GMM OBJ: 3.09 (p value=0.80)

Table 7: GMM Results with $\delta = .75$ and $M = 280000$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
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<tr>
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<td>5.64</td>
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<tr>
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Table 8: OLS Results with $\delta = .9$
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<th>Coefficient</th>
<th>Standard Error</th>
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</thead>
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<td>0.15</td>
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<td>0.55</td>
<td>0.084</td>
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<tr>
<td>LVARIETY ($\gamma_2$)</td>
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<td>0.019</td>
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</table>

Table 9: TSLS Exact Identification with $\delta = .75$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
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<td>0.060</td>
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</tr>
<tr>
<td>LDFIXED ($\alpha_2$)</td>
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<tr>
<td>CONSTANT ($\gamma_0$)</td>
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<tr>
<td>LDMARGINAL ($\gamma_1$)</td>
<td></td>
<td></td>
<td>0.27</td>
<td>0.080</td>
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<tr>
<td>LVARIETY ($\gamma_2$)</td>
<td></td>
<td></td>
<td>-0.076</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 10: TSLS Exact Identification with $\delta = .9$
Figure 2: Marginal and LD marginal, δ=76
Figure 5: Actual Installed Base vs. Predicted 0-75
Figure 7: Actual Installed Base vs. Predicted, $\delta = 9$
Figure 8: Actual Variety vs. Predicted, 0.79
Figure 10: Elasticity of installed base with respect to variety
Figure 11: Elasticity of Installed Base with Respect to CD Player Prices