CAPITAL MARKETS, FINANCIAL INTERMEDIARIES, AND LIQUIDITY SUPPLY

by

P. FULGHERI*

and

R. ROVELLI**

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* Associate Professor of Finance at INSEAD, Boulevard de Constance, 77305 Fontainebleau Cedex, France.

** Department of Economics, University of Bologna, Milano, Italy.

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CAPITAL MARKETS, FINANCIAL INTERMEDIARIES, AND LIQUIDITY SUPPLY

Paolo Fulghieri

Riccardo Rovelli
Dipartimento di Economia, Università di Bologna, Strada Maggiore 45, 40125 Bologna, Italy and IGIER, Milano, Italy.

Abstract. We study a dynamic economy endowed with a sequence of overlapping generations of consumers and production processes, and where productive assets are illiquid and consumption preferences are subject to uninsurable demand for liquidity. We characterize the steady states that can be achieved with alternative financial systems. We show that infinitely lived financial intermediaries offering a liability with age-dependent restrictions may implement a social optimum with full insurance. If, instead, they offer anonymous, unrestricted contracts, then only second-best consumption allocations with partial insurance obtain. We also examine the consumption allocations available when agents can trade shares in competitive stock markets. While allowing for trade across generations may or may not improve upon generational autarky, we show that this competitive equilibrium is not a social optimum, and is dominated by a system of infinitely lived, unrestricted intermediaries.

Keywords: Overlapping generations, financial intermediaries, stock markets.
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1. Introduction

The role of the banking system as a supplier of liquidity in an economy has been recognized long ago in the literature. For instance, Gurley and Shaw (1960) argue that one of the primary functions of financial intermediaries is to transform illiquid liabilities issued by the corporate sector into more liquid instruments held by consumers. Even before that, Edgeworth (1888) suggested that, because of the diversification in the pool of depositors, financial intermediaries are efficient producers of liquid assets. Starting with the seminal work in Diamond and Dybvig (1983, henceforth DD), this problem has been systematically re-examined in the recent literature.

The desirability of the provision of liquidity in an economy derives ultimately from the combination of the uncertainty on cash-flow needs of individuals and the presence of production processes with returns increasing with the investment horizon. In particular, in the absence of a financial system, random events (such as shocks on income or preferences) may force individuals to finance consumption with the premature liquidation of productive assets, leading to a welfare loss. In this case, the presence of a financial system may increase efficiency by preventing unnecessary liquidation of assets, leading to higher levels of consumption and welfare.¹

Liquidity to an economy may be supplied either directly through trade on capital markets, or indirectly through financial intermediaries. In the first case, liquidity is created by allowing agents with different cash flow needs to trade among themselves claims on productive assets. In the second case, specialized intermediaries interposed between producers and consumers hold in their portfolios claims on productive assets and issue to individual investors secondary instruments which are better suited to match their consumption needs.

The case of a depository institution is examined in DD. In that paper it is shown that an

¹ A different notion of liquidity derives from the desire of uninformed agents to minimize trading losses associated with asymmetric information, and it will not be considered here. The role of financial intermediaries as suppliers of liquidity in this different sense is examined in Gorton and Pennacchi (1990).
intermediary, by creating a non-negotiable liability redeemable on demand (interpreted as "demand deposit"), may provide optimal insurance against privately observed shocks on consumption preferences.\footnote{This possibility was also explored in Bryant (1980). Hellwig (1994) discusses a version of the DD model with a stochastic production technology with aggregate risk, and examines the optimal social allocations of this risk.} This is possible because the intermediary, by issuing a liability with a return pattern smoother than the underlying productive technology, is able to offer to individuals a more desirable consumption pattern, thus increasing their welfare.\footnote{In DD it is also shown that the divergence between the structure of rates of return on assets and liabilities of the intermediary may create the possibility of bank runs. In this paper we will not directly analyze this and the related issues.}

The possibility of duplicating the consumption allocation available with a depository intermediary by trading securities in an anonymous market is addressed in Jacklin (1987). In that paper it is shown that, if consumers have preferences similar to the ones assumed in DD, the allocation that may be obtained with a depository intermediary may also be achieved by direct trade of shares of firms which hold the productive assets of the economy, and follow a predetermined dividend payout policy. In a related paper, Haubrich and King (1990) distinguish between the \textit{liquidity} and \textit{insurance} role of banks,\footnote{In this framework, "liquidity role" refers to ability to smooth income over time, while "insurance role" refers to the ability to smooth income across states of nature.} and show that the role of financial intermediaries in providing liquidity to an economy may be duplicated by trading on security markets. Depository institutions may instead be superior to markets in providing insurance to privately observable risks, if some trading restrictions are imposed on their liabilities. In absence of these restrictions, it is shown that other financial intermediaries issuing tradeable securities, such as mutual funds, may provide the same consumption allocation as depository institutions.

In this paper, we re-examine in a more general, intertemporal framework the respective roles of capital markets and financial intermediaries in providing liquidity and consumption insurance to an
economy. In particular, we analyze a dynamic version of the DD model in the context of overlapping generations, where insurance and trade may occur across generations as well as within generations, and where the DD case would correspond to a situation of generational autarky.

The focus of our paper is on the welfare properties of the consumption allocations that emerge under alternative configurations of the financial sector of an economy. We consider first the case in which liquidity is supplied by a system of infinitely lived financial intermediaries. We show that if these intermediaries issue a liability with age-dependent restrictions, they may support the socially optimal consumption allocation, which provides for full insurance. If, instead, these intermediaries issue liabilities that are fully anonymous and are not subject to any restriction, they are constrained to offer second-best consumption plans, which provide for partial insurance only.

We consider next the case in which members of each generation act as individual entrepreneurs and trade securities in capital markets. In this configuration of the financial system, agents incorporate the technological process they own into firms and trade in a competitive capital market rights to fractions of future output. By interpreting these rights as shares of firms, we denote an economy endowed with this arrangement of the financial system as a stock market economy. We show that while the steady state equilibrium of this economy improves on the case of complete autarky, the equilibrium consumption allocation is not a social optimum. In this setting, trade in shares of firms is desirable because it allows agents with a preference for early consumption to avoid costly premature liquidation of physical capital, improving efficiency. However, we find that in this economy members of a new generation invest a fraction of their endowment in shares of existing firms. This reduces their investment in new physical capital and leads to a steady state level of capital which is lower than the social optimum.

We then compare the consumption allocations available with a stock market and with financial intermediaries. We first show that, depending on the proportion of agents with a preference for early consumption, the equilibrium consumption allocation available with an autarkic intermediary à la DD may
(or may not) dominate the one available in a stock market economy. Second, we find that, differently from the static framework, the consumption allocation available in a competitive stock market economy is dominated by the one available in financial system endowed with infinitely lived intermediaries issuing fully anonymous liabilities. We argue, however, that if intermediaries are allowed to hold assets with each other (earning the prevailing competitive interest rate) then the allocation that intermediaries are able to offer duplicates again the one available in a stock market economy.

Our paper is related to Bencivenga and Smith (1991), who also consider an overlapping generations version of the DD model. That paper, however, studies intermediaries engaged in intra-generational transfers only, and it does not analyze stock market equilibria. Qi (1994) examines infinitely lived depository intermediaries and shows how they may emerge endogenously in a setting similar to ours. The main differences with our paper are two-fold. First, unlike Qi, we explicitly characterize the equilibria emerging in a stock market economy, and analyze the role of an intergenerational stock market as a source of liquidity in a dynamic economy. We then compare the consumption allocations available in this dynamic stock market economy with the ones available in economies that are instead dominated by a system of financial intermediaries, in which a variety of restrictions are imposed on the trading opportunities of individuals. Dutta and Kapur (1994) also examine the role of financial intermediaries as liquidity suppliers in an overlapping generation structure. Allen and Gale (1994) discusses the role of intermediaries in the intertemporal smoothing of aggregate risk. Differently from ours, their paper assumes that the stock of long-term capital is given, and focuses instead on the impact of the financial structure of an economy on the incentives to accumulate short-term safe assets. Finally, our paper is also related to the recent literature exploring the relationships between financial markets and growth. Among them, Levine (1990) compares allocations obtained with a stock market and with financial intermediaries.

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5 For a related model, see also Bernanke and Gertler (1987).

6 Recent surveys of this literature may be found in Pagano (1993) and Levine (1997).
He finds that the presence of financial intermediaries improves on a stock market economy to the extent that transaction costs are present. Our work shows that this result obtains in more general settings. In this context, our paper provides an additional channel by which the structure of the financial system may affect long term growth of an economy.

The remainder of the paper is organized as follows. In section 2 we present the basic model. In section 3 we characterize the optimal consumption allocations available in our economy under alternative sets of constraints. In section 4 we discuss the equilibria which are sustainable with financial intermediaries. The steady state equilibrium in a competitive stock market economy is examined in section 5, where we establish our main results. In section 6 we discuss our main results and present the main implications of our analysis. Section 7 concludes. All proofs are in the appendix.

2. The model

We consider an infinite-horizon economy with an infinite sequence of overlapping generations of consumers of the type studied in Samuelson (1958) and Bryant (1980). A new generation of consumers, whose size is normalized to 1, is born in each period. Consumers are endowed with 1 unit of a homogeneous good, which can be used for either consumption or productive investment. There are two types of consumers in this economy. Consumers of the first type live for one period only, and are identified as "early consumers." Those of the second type live instead for two periods (three dates total) and are identified as "late consumers." Agents of both types do not derive any utility from consumption at the beginning of their lives, making the initial savings decision trivial. Early consumers born at $t$ consume at $t + 1$ and die immediately after. Late consumers live until $t + 2$ when, for simplicity, all their consumption takes place. Consumers born at $t$ learn their own type privately at $t + 1$, that is one period.

7 The structure of preferences assumed in this section is similar to the one adopted in Bhattacharya and Gale (1987) and Jacklin and Bhattacharya (1988). We develop our analysis in this simplified framework in order to make the exposition as transparent as possible. Jacklin (1987) studies the single-generation welfare properties of financial intermediaries versus stock-market contracts for more general
after their birth. Every individual of a new generation is an early consumer with probability $e$, $0 < e < 1$, which is common knowledge. A new born then has a lifetime expected utility given by

$$W = eU(C_1) + (1 - e)U(C_2),$$

(1)

where $U(C)$ is a strictly increasing, concave function, $C_1$ is the level of consumption of early consumer, and $C_2$ the one of a late consumer.

We assume also that the population is large enough so that there is no uncertainty on the aggregate distribution between early and late consumers: with probability one, every generation has a fraction $e$ of early consumers. The demographic structure of the population is then constant through time, and in each period is formed by four groups: (i) new born individuals born at $t$ (identified with "n"), of measure 1; (ii) early consumers (identified with "e"), of measure $e$; these individuals were born in the previous period ($t - 1$), consume now and die immediately after; (iii) late consumers (born at $t - 1$ as well) who are in the intermediate stage of their life, of measure $1 - e$; these individuals, identified as survivors "s," have just learnt that they survive for another period, when they consume and die; finally (iv) late consumers who are at the end of their life-cycle, of measure $1 - e$; they were born two periods earlier (at $t - 2$) and die right after consumption (identified with "d"). Thus, at any one time, the population is composed of $3 - e$ individuals, of whom 1 are consumers and the remainder wish to invest their wealth in the assets available in the economy.

The economy is endowed with a technology to produce output over time. This technology is illiquid in the sense that output is greater the longer the productive assets remain employed in the production process. Following Diamond and Dybvig (1983), we model this illiquidity by assuming that production lasts for two periods, but that it can be interrupted in the intermediate stage at the cost of a lower return. Letting $I_t$ denote aggregate investment in a production round which is started at time $t$, in
the following period an amount \( L_{t+1} \), with \( 0 \leq L_{t+1} \leq I_t \), may be liquidated and made available for consumption. Assets remaining in place yield in the following period an amount of output given by \((I_t - L_{t+1})R\), with \( R > 1 \). To simplify the exposition, we assume that the production technology allows a maximum investment level equal to the unit endowment, that is \( I_t \leq 1 \), for all \( t \).\(^8\)

A new round of production may be started in every period. Given the dynamic structure of our model, this results in a sequence of overlapping technologies which are at different stages of the production process. In every period there is one round of start up production processes, one round at the intermediate stage, and one which comes to maturity. Aggregate resources available for consumption in any period are then given by

\[
Y_t = 1 - I_t + L_t + (I_{t-2} - L_{t-1})R ,
\]

which is the sum of the periodic endowment, net of current investment, plus current liquidations \( L_t \) from production processes started one period earlier and output from technologies currently maturing.

3. Constrained optimal allocations

In this section we characterize the stationary consumption allocations that satisfy alternative optimality criteria.\(^9\) As a benchmark, we consider first the case of individual autarky in which new born individuals invest directly all their endowment in the production technology. Individuals who, in the following period, turn out to be early consumers terminate production prematurely and consume only one unit of the commodity which may either be used for immediate consumption or as input in production, while firms can only produce consumption goods (we thank an anonymous referee for pointing this out to us).

\(^8\) This assumption allows us to capture in the simplest possible way the property of diminishing marginal rates of return on long term investment. An alternative interpretation of this assumption is that consumers are endowed with one unit of a commodity which may either be used for immediate consumption or as input in production, while firms can only produce consumption goods (we thank an anonymous referee for pointing this out to us).

\(^9\) We focus on the steady state allocations of our economy because they offer a natural benchmark for welfare and policy analysis. A similar emphasis may be found in the classical works by Diamond (1965), Gale (1973); see also the discussion in Blanchard and Fischer (1989). The issues concerned with the dynamic transition from one steady state to another are examined explicitly in Bhattacharya, Fulghieri and Rovelli (1997).
unit of consumption good. If instead an individual turns out to be a late consumer, she is able to continue production until the following period, and to consume $R > 1$. In this case, individuals face at birth an uncertain stream of consumption given by the pair $C^A = (C^A_1, C^A_2) = (1, R)$, yielding a lifetime expected utility of $W^A = eU(1) + (1 - e)U(R)$. This consumption allocation is marked as point A in Figure 1.

Risk averse individuals would like to obtain insurance against the consumption risk they face at birth in the case of individual autarky. First, we consider the case of a social planner who provides insurance against consumption risk to each generation of consumers separately and independently. In this case, individuals of each generation are offered a consumption plan maximizing their ex-ante expected utility (1) subject to a constraint limiting the aggregate of the consumption plans to be no greater that the resources available to each generation individually and separately. We denote this constraint as the \textit{generational feasibility constraint}. This leads to the problem:\footnote{Since we focus only on stationary consumption allocations, from now on we drop the index $t$.}

\begin{align}
W^D &= \max_{\{C_1, C_2, L\}} eU(C_1) + (1 - e)U(C_2) \tag{3a} \\
\text{s.t.} \quad (1 - e)C_2 &\leq R(I - L) \tag{3b} \\
eC_1 &\leq L \leq L, \quad I \leq 1. \tag{3c}
\end{align}

The consumption plan $C^D = (C^D_1, C^D_2)$ solving (3) has been characterized by Diamond and Dybvig (1983). There it is shown that all the endowment of each generation is invested in the technology, so that $I^D = 1$, and that if the coefficient of relative risk aversion for $U(C)$ is greater than one everywhere, then the optimal consumption plan $C^D$ provides for smoothing of consumption over the two dates: $1 < C^D_1 < C^D_2 < R$. Diamond and Dybvig have also shown that, since $C^D_1 < C^D_2$, the consumption allocation $C^D$ is individually incentive-compatible, that is it may be implemented by the social planner without identifying individual consumption preferences directly. This allocation, that we denote as one of \textit{generational autarky}, is marked as point D in Figure 1.
In the case of generational autarky individuals are insured against consumption risk by forming insurance pools composed only of agents within their generations. If we allow for risk sharing among individuals in different generations in addition to the ones in the same generation, we are led to the following problem:

$$W^p = \max_{(C_1, C_2, I, L)} \varepsilon U(C_1) + (1 - \varepsilon) U(C_2)$$  \hspace{1cm} (4a)

subject to:

$$\text{s.t. } (1 - \varepsilon)C_2 + \varepsilon C_1 + I \leq 1 + R(I - L) + L$$  \hspace{1cm} (4b)

$$L \leq I, \ I \leq 1.$$  \hspace{1cm} (4c)

In problem (4) the social planner maximizes individual expected utility (1) subject to an aggregate feasibility constraint (4b) that reflects the dynamic nature of the problem and limits the total consumption across individuals of different ages and generations to be no greater than aggregate resources available in the overall economy at any given time. We denote the consumption allocation $C^p = (C_1^p, C_2^p)$ that solves problem (4) as a social optimum.11 We obtain the following.

**Proposition 1:** In a socially optimal allocation the entire periodic endowment is invested, there are no liquidations, and consumption is fully ensured:

$$I^p = 1, \ L^p = 0, \text{ and } C_1^p = C_2^p = R.$$  

By allowing for intergenerational transfers, in a social optimum consumption by early consumers may be provided by the output from production processes started two periods earlier. Differently from the case of generational autarky, now premature liquidation of physical capital is not necessary. The dynamic structure of the economy makes effectively liquid a production technology that would otherwise be illiquid. Complete insurance from consumption risk is now feasible, and it is the preferred allocation for

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11 These steady state allocations are also those that would emerge from the optimization problem of a social planner giving an equal weight to each generation (see, for instance, Blanchard and Fischer, 1989) and it coincides with the Golden Rule of Phelps (1967).
risk averse agents. The social optimum allocation is represented as point P in Figure 1. We have:

**Corollary 1.1:** The following welfare comparisons hold:

\[ W^p > W^D \geq W^A, \]

where the last inequality is strict when the coefficient of relative risk aversion is everywhere greater than one.

4. **Financial intermediaries**

In their seminal paper DD suggest that even if individual consumption preferences are not directly observable (so that explicit markets for insurance against consumption risk are not feasible), the allocation \( C^D \) can still be implemented by each generation separately as an incentive-compatible mechanism. This task may be accomplished within each generation by creating an institution that offers the following contract. At the beginning of every period each member of a generation exchanges her own endowment with a non-tradeable liability of the institution that is redeemable on demand for \( C_0^D \), if redeemed after one period, and for \( C_2^D \), if redeemed after two periods. By interpreting this liability as a demand deposit, DD identify this institution as an intragenerational bank which offers periodic rates of return on their deposits equal to \( r_1^D = C_0^D \), for deposits lasting one period, and \( r_2^D = \sqrt{C_2^D} \), for deposits lasting two periods.\(^{12}\)

It is important to note that a crucial condition for the allocation \( C^D \) to be implementable by an intragenerational bank is that the liabilities of the bank are not tradeable in a secondary market. In fact, if demand deposits are freely tradeable by depositors, the allocation \( C^D \) is no longer incentive-compatible. As Jacklin (1987) shows, an individual may invest directly her own endowment in the technological

\(^{12}\) Note that Jacklin (1987) shows that, under the "corner" preferences assumed in DD, an intragenerational stock market, in which individuals trade shares of a firm paying a pre-determined dividend, can also achieve the consumption allocation offered by the intragenerational bank, \((C_0^D, C_2^D)\).
process and adopt the following trading strategy: if she turns out to be a late consumer, she can hold the technology until maturity and consume $R$; if instead she turns out to be an early consumer, she may *trade* the claim on the technology with another consumer who turns out to be a late consumer in exchange for a claim on the intermediary, which is immediately redeemable for $C^p_t$. Hence, by following this strategy, an individual would be able to secure a consumption plan equal to $(C^p_t, R)$, which is strictly better than the social contract $C^p$. As a result, nobody would be willing to deposit the endowment with the intragenerational bank, producing a collapse of the social allocation mechanism. The only viable allocation remaining is the autarkic consumption plan $C^A = (1, R)$.

In a spirit similar to DD, the socially optimal allocation $C^p$ can be implemented as an incentive-compatible mechanism by an institution that offers the following contract. Individuals in each generation exchange at birth their endowment for a liability redeemable on demand at any future date and at the same redemption value $R$. The main difference with the case of generational autarky is that now the institution that implements this social contract starts new rounds of production over time, and insures against consumption risk by pooling together individuals of different generations. In the same spirit as DD, we denote this institution as an *intergenerational bank.*

Note that we should interpret these institutions as a mutual organizations, such as thrift institutions or credit unions, rather than as true profit-maximizing entities. These considerations lead to the following.

**Proposition 2:** The socially optimal allocation $C^p$ may be implemented by a system of intergenerational banks that offers to individuals of every new-born generation a (non-negotiable) contract redeemable on demand at a constant redemption value $R$.

The redemption value of the contract supporting the social optimum is constant, and independent from

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13 A possible interpretation is as a *universal bank*, selling a variety of products, including disability insurance and variable annuities.
the time of redemption. The periodic rates of return implied by this investment is given by \( r_1^p = R \), if it is redeemed after one period, and \( r_2^p = \sqrt{R} \), if it is redeemed after two periods. There are two possible interpretations of this type of contract. The first one is a puttable bond, with a fixed, pre-determined exercise price equal to \( R \). The second interpretation, which is closer to the spirit of our model, is a retirement contract (such as "variable annuity") that investors purchase in young age. This contract allows investors to retire either in their middle or in old age, and to receive upon retirement a fixed redemption value \( R \).

A necessary condition to ensure that the social optimum \( C^p \) is incentive-compatible and therefore implementable by a system of intergenerational banks is that a contract is proposed only to new born individuals. In the absence of such a requirement, a late consumer would redeem her contract in the middle age for \( R \), and start a new contract (possibly at another intermediary) so as to attain a total consumption of \( R^2 \) when in old age. The adoption of this policy by all late consumers would violate the feasibility constraint of the intergenerational bank, and it cannot be admissible. To ensure incentive-compatibility (and feasibility) of the social optimum it is therefore necessary to make these contracts age-dependent, by introducing a no-restart restriction.\(^{14}\)

Alternatively, it is also of interest to investigate which consumption allocations are feasible when no restrictions are imposed on the trading opportunities of individuals. Therefore, we characterize the consumption allocations that may be achieved when contracts are fully anonymous and no restrictions are imposed. An intermediary is able to offer a fully anonymous contracts only if the implied periodic rates of return for redemption of contracts after one period, \( r_1 \), and for redemption after two periods, \( r_2 \), satisfy the no arbitrage condition \( r_1 = r_2 \). We denote this constraint as the anonymity constraint. Since, in this case, the contract offered by the intermediary is a fully anonymous security redeemable on demand, it

\(^{14}\) Note that this trading restriction complements the ones discussed in Jacklin (1987), and depends crucially on the dynamic structure of our model.
may be identified as a deposit contract, and the intermediary that offers it as a depository institution. Competitive intermediaries now choose a pair \((I, L)\), and offer deposit contracts yielding a constant total return \(r^u\) which solves:

\[
W^u = \max_{(C_1, C_2, I, L, r^u)} \quad \epsilon U(C_1) + (1 - \epsilon) U(C_2) 
\]

\[\begin{align*}
\text{s.t.} \quad (1 - \epsilon)(r^u)^2 + \epsilon r^u &\leq 1 - I + L + R(I - L), \\
C_1 &= r^u, \quad C_2 = (r^u)^2, \\
L &\leq I, \quad I \leq 1.
\end{align*}\]  

In problem (5), expected utility (1) is maximized subject to constraints (5b), requiring that the profits of a representative intergenerational bank are non negative in a steady state, and (5c), ensuring that the anonymity constraint is satisfied. We denote intermediaries offering this class of unrestricted, fully anonymous contracts as unrestricted intermediaries.\(^{15}\) This problem has been characterized in Qi (1994), giving:

**Proposition 3:** *Qi, 1994* An unrestricted intermediary invests the entire periodic endowment, \(I^u = 1\), makes no liquidations, \(L^u = 0\), and offers an (age- and time-independent) one period total return \(r^u\) given by:

\[
r^u = \frac{\sqrt{e^2 + 4(1 - \epsilon)R} - \epsilon}{2(1 - \epsilon)},
\]

with \((r^u)^2 > R > r^u > \sqrt{R}\). Furthermore, the expected utility that a representative consumer may achieve over her lifetime is strictly less than the social optimum:

\[W^u < W^p.\]

---

\(^{15}\) We will refer to this type of intermediaries as unrestricted, even if in Section 5 we will show that a restriction on the asset allocation of the intermediary is necessary to sustain the consumption plan described in Proposition 3.
Unrestricted intermediaries optimally invest the entire endowment of every generation and make no premature liquidations. Given the periodic return $r^U$ that intermediaries offer, consumers are able to support a consumption allocation $C^U = (r^U, (r^U)^2)$, which is represented in Figure 1 as the point $U$. Since the consumption allocation that it is able to offer to its depositors is constrained by the no arbitrage condition $r^U_1 = r^U_2$, this allocation is strictly dominated by the one available with an intermediary that issues age-dependent liabilities. Removal of the no-restart restriction has lowered overall welfare.

5. Equilibrium with a competitive stock market

In the previous section we have characterized the consumption allocations implementable with a system of financial intermediaries. In this section we investigate the consumption allocations that may be obtained by society when there are no financial intermediaries, and liquidity to the economy is supplied by allowing agents to directly trade among themselves on anonymous capital markets.

In this configuration of the financial structure, agents trade shares of the production process in a competitive capital market. Although for simplicity we do not introduce uncertainty in the production process, we refer to an economy endowed with this arrangement of the financial system as a stock market economy. The driving force to set up an exchange arises from early consumers' incentives to avoid premature liquidation of their production process (which is at the intermediate stage) by trading its ownership for a fraction of a new born's endowment. New born individuals are willing to buy shares of a production process that has a payoff $R$ in the following period. In this way, trade in securities may be a substitute for costly liquidation of physical capital: the presence of a market for shares of firms makes long term, illiquid production processes effectively liquid.

5.1 The stock market economy

Individuals in a new born generation start every period a new round of production by investing an amount $1 \leq 1$ in the technological process. We assume that they incorporate the production unit into
a firm, and that they issue one share for every unit of investment. At any point in time, there are three
types of firms: new firms, which have just started production, denoted by "f"; firms in the intermediate
stage, "intermediate firms" denoted by "i," and "mature firms" ready to pay a liquidating dividend.

Three markets are open each period in this economy: (1) a market for physical goods, with their
unit price normalized to one; (2) a market for shares in newly formed firms, with unit price $P_f$; and (3)
a market for shares in firms at the intermediate stage, with unit price $P_i$.

Given the ability to trade goods and shares in the above markets, agents now face a richer set of
opportunities. Letting $I^J_k$ denote the share of firm $J$, $J \in \{f, i\}$, owned by an individual at stage, $K \in
\{n, e, s, d\}$, individuals face at different periods of their lives the following set of budget constraints
(beginning of period resources on the r.h.s., consumption and end of period investments on the l.h.s):

stage n : \[ P_f I^n_f + P_i I^n_i = 1 + (P_f - 1)I, \]  
(7)

stage e : \[ C_1 = R I^n_f + P_i I^n_i, \]  
(8)

stage s : \[ P_f I^s_f + P_i I^s_i = R I^n_f + P_i I^n_i, \]  
(9)

stage d : \[ C_2 = R I^n_f + P_i I^n_i. \]  
(10)

Consider a new born individual. She may devote an amount of resources $I$ to a new round of production
and/or invest in shares for new, $I^n_f$, and intermediate firms, $I^n_i$. Resources for investment are given by
the unit endowment plus the sale of shares in the new firm $P_f$. Note that the last term on the r.h.s. of
(7) is the capital gain that new born individuals make as they sell the new technology at the market price
for new shares, $P_f$, and invest the proceed in production. This gain represents the net present value from
operating a new firm and it is proportional to the level $I$ of initial investment.

In the following period, those individuals who turn out to be early consumers finance
consumption with the liquidating dividends of maturing firms, and with the sale of their holdings of
intermediate shares, leading to (8). Buyers in this market are provided by new born individuals from the generation that follows, and by survivors. Note that the presence of a secondary market for shares allows early consumers to avoid inefficient liquidation of physical capital. Survivors are active only on the markets for shares to rebalance their portfolios. They reinvest their share of the liquidating dividends of mature firms by buying additional shares of the intermediate firms and/or shares in the new firms started by the new generation, giving (9). Finally, late consumers consume their accumulated wealth, giving (10).

5.2 Steady state equilibrium

We now proceed to characterize the steady state equilibrium for this economy by considering first the equilibrium conditions in the market for assets. In each period individuals have the opportunity to invest in shares of new or intermediate firms. No arbitrage conditions between these markets require then that the rate of return on an investment in these two assets must be equalized, that is

\[ \frac{R}{P_i} = \frac{P_i}{P_f} = \sqrt{\frac{R}{P_f}}. \]

Note that (11) requires the one period rate of return on the shares of new and intermediate firms to equal the implied one-period rate of return from investing in new shares and holding them until the production process is over. We may state the following.

**Proposition 4:** The steady state competitive equilibrium with a stock market is characterized by

\[ P_t^M = 1, \quad P_i^M = \sqrt{R}, \quad C_1^M = \sqrt{R}, \quad C_2^M = R, \]

\[ I^M = 1 - \varepsilon \frac{R - \sqrt{R}}{R - 1}, \quad L_1^M = 0, \]

where \( 0 < P^M < 1 \).

The consumption allocation that can be achieved with a competitive stock market is represented as point
M in Figure 1. For concreteness, note that a possible specification of equilibrium asset demands over individual life-cycles is the following. New born individuals invest $I^M$ in a new production process, keep all the shares of the new firm in their portfolio and invest the remainder in shares of intermediate firms. Early consumers sell their holdings of intermediate firms to individuals in the next generation and to the survivors in their same age group. Survivors keep their holdings of intermediate firms and use the liquidating dividend of maturing firms to acquire additional shares of the intermediate technology.

Proposition 4 implies that the competitive allocation $C^M = (\sqrt{R};R)$ dominates the autarkic solution $C^A = (1;R)$. The presence of a stock market where shares of the intermediate firms are traded improves welfare by making premature liquidation of physical capital in intermediate firms by early consumers unnecessary. This result is obtained by making intergenerational trade, between early consumers and new-born individuals possible. The purchase of already existing capital on the stock market by new born individuals, however, diverts some of their resources away from investment in new production processes. The opening of a stock market increases the opportunity cost of capital for new born individuals, with the effect that financial investment in the stock market partially crowds out investment in new physical capital. The result is a steady state level of investment, $I^M$, which is strictly less then the one in a steady state social optimum, $I^* = 1$, determining underinvestment. The net effect on output available each period is however positive. This can be easily seen by comparing the amount of aggregate consumption in this case, given by $Y^M = \varepsilon \sqrt{R} + (1 - \varepsilon)R$, with the equivalent aggregate consumption under individual autarky, given by $Y^A = \varepsilon + (1 - \varepsilon)R$.

5.3 Stock markets and financial intermediation

We are now able to compare the welfare properties of the steady state consumption allocations available in a stock market economy with the allocations available with financial intermediaries.

5.3.1 Stock market and intragenerational banks
While it is clear that the equilibrium with a competitive stock market is superior to individual autarky, it is not obvious whether it is also superior to the case of intra-generational risk-sharing studied by DD (see Figure 1). In general, inter-generational risk sharing achieved through a competitive stock market may or may not be superior to intra-generational risk sharing achieved through an intermediary of the DD type. This depends on the degree of individual risk aversion and on the fraction of early consumers in the population. In particular we have the following:

**Proposition 5:** (a) The steady state competitive stock market equilibrium $M$ dominates the DD intra-generational risk-sharing if $\varepsilon > \bar{\varepsilon}$, for some $\bar{\varepsilon} > 0$.

(b) Conversely, if $\varepsilon < 1/2$ and individuals are sufficiently risk averse, then a DD intermediary dominates the stock market for $R \in (1, \bar{R})$, with $\bar{R} > 1$.

If the proportion of early consumers in the economy is low, the aggregate demand for liquidity is small. Thus a DD type intermediary is able to maintain a low level of liquidations, while offering a time distribution of consumption more desirable than the one available in a stock market economy. In this case, financial intermediaries may provide a structure of returns which is better tailored to individual consumption preferences than the one rigidly determined in the capital markets by the no-arbitrage condition (11). Instead, if the demand for liquidity is sufficiently large, the intermediary may be compelled to a substantial premature liquidation of physical capital, producing an overall welfare lower than the one available with a stock market. Hence, while a financial intermediary may be better equipped at addressing the insurance need of an economy, capital markets are superior for satisfying its liquidity needs by preventing un-necessary liquidation of physical capital.

Finally, Proposition 5 sheds some light on the effects of imposing trading restrictions, such as a generational participation constraint, in a stock market economy. From Jacklin (1987) we know that the consumption allocation supported by an intragenerational bank à la DD can be duplicated by an
intragenerational stock market, where individuals of each generation are restricted to trade securities only with members of their own generation. In this setting, again firms are able to pay dividends only if they liquidate prematurely a fraction of their assets, thus leading to the same welfare trade-offs that we examined in the case of intragenerational banks and discussed in Proposition 5.

5.3.2 Stock market and unrestricted intergenerational banks

We may now compare the consumption allocation available with unrestricted, infinitely lived intermediaries with the one available in a stock market economy. We have the following:

**Proposition 6:** The consumption allocation available with unrestricted financial intermediaries strictly dominates the one available with a competitive stock market:

\[ W^M < W^U. \]

The reason for this result lies in the greater flexibility of financial intermediaries in setting the rates of returns on their liabilities (i.e. deposits) relative to the technological rate of return on their assets. In a stock market economy, rates of return on traded financial assets are determined by the no-arbitrage condition (11), and hence by the characteristics of the underlying production technology. Since, in our simplified case, the technology shows a two period total return \( R \), this implies a per period constant return \( r^M = \sqrt{R} \). Furthermore, as we discussed in section 5.2, in a stock market economy trade of existing capital between new-born individuals in one generation and late consumers of the previous generation "displaces" investment in new assets, resulting in an aggregate stock of capital below the level that is socially optimal.

In an intermediated economy, on the contrary, an intergenerational bank has more flexibility in setting the rate of return \( r^U \) it offers to investors, and in choosing the appropriate level of investment in physical capital. In this economy intermediaries hold the productive assets of the economy and compete for depositors' funds. The return offered on the liabilities of an intergenerational bank, \( r^U \), is no longer
constrained by the no-arbitrage condition (11) to equate the implied one-period marginal productivity of capital \( \sqrt{R} \). Hence, in this institutional setting, intermediaries are induced by competition to invest the entire endowment of the economy into new physical capital, avoiding underinvestment, and to offer a rate of return \( r^U \) which is greater than the technological rate of return \( \sqrt{R} \). The result is a level of investment which is equal to the social optimum, and a level of consumption which dominates the one available in a stock market economy.\(^{16}\)

Finally, it is useful to note that if financial intermediaries are allowed to hold deposits with each other, the consumption allocation \( C^U \) that they are able to offer would coincide again with the one available with a competitive stock market. This may be seen by noting that as long as \( (r^U)^2 > R \) (as established in Proposition 2), intermediaries find it optimal to make no investment in physical capital, but rather to redeposit the funds they receive at another intermediary. Investment in physical capital then occurs only if \( (r^U)^2 = R \). This would result in a steady state consumption allocation that duplicates the one available in a stock market economy.\(^{17}\) Hence, restrictions on the asset holding of intermediaries are needed in an intermediated economy in order to produce a steady state consumption allocation that dominates the one available in a competitive stock market economy.

6. Discussion and implications

Our paper contributes to the now expanding literature on the effects that the configuration of the financial system of an economy has on macroeconomic behavior and individual welfare. The first implication of our analysis is that, due to the dynamic structure of the economy, the desirable supply of liquidity and risk-sharing may be achieved at no cost, and full insurance against consumption risk is

\(^{16}\) The dynamic issues regarding the transition to an intergenerational bank are examined in Bhattacharya, Fulghieri and Rovelli (1997).

\(^{17}\) Note that a similar conclusion may be obtained if we introduce a stock market in an economy endowed with unrestricted intermediaries.
feasible (unlike what may be achieved in a static setting). The overlapping nature of the aggregate technology ensures that in equilibrium no premature liquidation of assets occurs, and endowments are fully invested in the most productive technology.

We have shown that an intergenerational stock market, where the stock of capital is passed through generations by anonymous trades of shares, supplies liquidity to an economy by eliminating the necessity of inefficient liquidation of productive assets. Such an intergenerational stock market, however, is shown to be, in a steady state, an inferior vehicle for risk-sharing. The reason is that optimal risk-sharing in our economy requires the presence of a security redeemable on demand and paying a constant payoff at redemption. Also, incentive compatibility requires that such security is offered to new-born individuals only. It may be noted that the payoff of this security implies that all generations face at birth a downward sloping yield curve. This is inconsistent with the constant periodic rate of return which prevails in a stock market equilibrium.\textsuperscript{18}

The foregoing discussion suggest that in this economy there is a demand for "insurance" products that display a downward sloping yield curve. These products, however, would be arbitraged away in a stock market economy and could not survive in equilibrium in the absence of re-trading restrictions. This implies that, in our economy with incomplete markets, the presence of financial products that are not fully negotiable and are instead subject to trading restrictions may be desirable and that they can improve consumers' welfare.

Our model allows us to comment also on the relative advantages of pure market based, fully funded and voluntary pension systems, versus partially funded (or unfunded) pay-as-you-go mandatory systems. Our analysis implies that market based, funded retirement systems, while they may dominate intragenerational insurance, are always dominated by partially funded, pay-as-you-go intergenerational pension systems. This is true whether or not the latter systems have restricted or unrestricted liabilities

\textsuperscript{18} We thank an anonymous referee for point this out to us.
(that is whether they are offered by a restricted or unrestricted intermediary). Furthermore, programs limiting the access of individuals to their retirement funds, or restricting the terms at which they may generate retirement benefits, may relax the incentive-compatibility constraint for their participants and increase overall individual welfare.

Finally, our model sheds some light into how different arrangements of the financial structure may affect the rate of growth of the economy. In particular, we have shown that economies endowed with a financial system consisting of either a stock market or a set of intermediaries dominate economies without a financial sector at all. The presence of a financial sector improves economic efficiency by making premature liquidations not necessary. We have also shown that, while this goal may be achieved either by allowing trade in a stock market or introducing long lived financial intermediaries, the latter solution dominates in the steady state from a welfare point of view. Economies in which liquidity is supplied by a system of financial intermediaries have an aggregate steady state level of output, consumption, and stock of productive capital which is greater than the levels characterizing stock market economies.

7. Summary and conclusions

In this paper we have studied a dynamic economy where the production process has increasing returns over time and individuals have an uncertain, uninsurable demand for liquidity of the type analyzed in DD. We extend that analysis by embedding the basic DD model into a dynamic sequence of overlapping generations of consumers and production processes, and we study the steady state allocations that may be achieved through alternative arrangements of the financial system.

We first examine the steady state allocation that may be obtained with a system of financial intermediaries. We show that if intermediaries offer a liability with an age-dependent restriction, then they may implement a social optimum with full insurance. If, instead, they offer an unrestricted, fully anonymous deposit contracts, then only second-best consumption allocation with partial insurance are
feasible.

We then examine the consumption allocations available when agents are allowed to trade shares in an economy endowed with a competitive stock market. We find that allowing for trade across generations may or may not lead to an improvement over the generational autarky analyzed by DD. Further we show that this competitive equilibrium is not a social optimum, and it is dominated also by the equilibrium consumption allocations available with a system of infinitely lived intermediaries offering anonymous deposit contracts. We show that this requires, however, a restriction on the assets holdings of intermediaries.

A number of problems remain open. The first one is to extend the analysis from steady states to the issue of the transition from one steady state to another. This question is addressed in Bhattacharya, Fulghieri and Rovelli (1998). A second problem relates to the examination of the effect of the presence of aggregate uncertainty, such as a random proportion $c$ of early consumers in the economy. This issue is examined in the original DD paper, where it is suggested that government insurance along with a head tax, restores first best allocations. We suspect that in the overlapping generation context, the presence of aggregate uncertainty may provide an additional reason why restricted, government sponsored insurance-retirement programs may dominate market based ones.

Finally, we would like to conclude by emphasizing that there are a number of other factors determining the existence of financial intermediaries (e.g. asymmetric information) that are not explicitly considered in our paper (for a review of this literature, see Bhattacharya and Thakor, 1993). Furthermore, the restrictions on asset trading needed to implement some of the intermediary equilibria may not be compatible with a fully competitive, profit maximizing financial intermediary sector. Thus, if the intermediary contracts proposed in our paper must be implemented through government regulation, agency problems or imperfect competition may arise as a consequence. Hence, the welfare costs stemming from these problems may outweigh the risk-sharing benefits discussed in our paper. In this
case, competitive markets may still provide an overall higher economic welfare.
APPENDIX

Proof of Proposition 1. Maximizing lifetime expected utility (1) subject to constraint (4), requires maximization of resources, that is setting $I = 1$, and $L = 0$. First order conditions for consumption require also that $U'(C_1) = U'(C_2)$, which implies $C_1^* = C_2^* = R$. QED

Proof of Proposition 2. From Proposition 1, the proposed consumption plan is budget feasible. We need to check that it is also incentive-compatible. This may be established by noting that a late consumer is indifferent between revealing herself, and consume $R$ in old age, or pretend to be an early consumer, redeem the deposit certificate for $R$, and invest directly in the technology, liquidate it the period after, and consume again $R$ in old age. QED

Proof of Proposition 3. From budget constraint (5b), it can immediately be seen that an intermediary will optimally invest all its deposits in the technology, so that $I^u = 1$, and will not liquidate prematurely any physical capital, and $L^u = 0$. Given this, equation (5b) must be met as an equality, and it may be solved for $r^u$. The greatest root is given by (6), which may be shown to satisfy $r^u > \sqrt{R}$. QED

Proof of proposition 4. Given the proposed equilibrium prices $P_t = 1$ and $P_i = \sqrt{R}$, we must establish that consumption and asset markets clear, for the equilibrium consumption and asset demands, and that no liquidation is necessary. First, notice that $P_t = 1$ implies that the r.h.s. of (7) is equal to unity. This, with the equilibrium asset prices, implies that, independently of portfolio allocation decisions, each new born individual is worth $\sqrt{R}$ after the first period of life, and $R$ after two periods. Hence, sustainable consumption levels are $C_1 = \sqrt{R}$ and $C_2 = R$. Market clearing condition between aggregate demand and supply (the "material balance" equation) requires

$$(1-\varepsilon)C_2 + \varepsilon C_1 + I = 1 + RI.$$  \hfill (a1)

Substituting equilibrium consumption levels, we obtain that good markets clear iff $I = I^M$, with clearly $0 < I^M < 1$. We need now to check that this solution is compatible with equilibrium in the market for
shares of intermediate firms. Since in equilibrium asset holdings are not unique, it will suffice to show that there is a specification of asset demands at which markets clear. Consider then the case in which new born individuals held all the shares of the new technology: \( I^n_i = I \). Note that this implies also that survivors' demand for new shares will be zero, \( I^n_s = 0 \). In this case, shares of intermediate firms will be sold next period by early consumers only, with an aggregate supply of \( eI^n_i = eI \). Demand for intermediate shares will be given by the new born individuals, plus the net demand by survivors, after portfolio rebalancing. From the budget constraints of new born individuals (7), we obtain

\[
I^n_i = \frac{1 - I}{\sqrt{R}}.
\]  

(a2)

For survivors, substituting (a2) into the budget constraint (9), we obtain

\[
I^n_s = \frac{\sqrt{R} (1 - I) + P_s I_i}{P_i} = 1,
\]  

(a3)

since in equilibrium \( P_i = \sqrt{R} \). We may now compute the market clearing condition for intermediate shares. Equating net aggregate demand (on the l.h.s.) with aggregate supply we have

\[
(1 - e)(I^n_i - I) + I^n_s = eI.
\]  

(a4)

Substituting demands for assets, we can check that the market for intermediate shares clears for \( I = I^M \). Therefore, the steady state level of investment is financially sustainable, that is the demand for liquidity arising from early consumers may be satisfied by selling intermediate shares, without the need to liquidate prematurely the underlying physical investment. Hence \( L^M = 0 \). QED

Proof of Proposition 5. Let \( U^M \) and \( U^D \) be the levels of expected utility under a stock market and DD allocation. From Proposition 4, we have that

\[
U^M = eU(\sqrt{R}) + (1-e)U(R).
\]

From the budget constraint (3) for a DD intermediary, we have that
Hence, for $\epsilon \to 1$, $U^M = U(\sqrt{R}) > U(C^D_1) = U(1) = U^D$, which implies part (a). Consider now $C^D$. As individuals become more risk averse, $C^D_2 \to C^D_1$ and at the limit $C^D_2 = C^D_1 = \bar{C}^D = R/[1+\epsilon(R-1)]$, and $U^D \to U(\bar{C}^D)$. Similarly, as individuals become more risk averse, the certainty equivalent of the consumption allocation available in a stock market $(\sqrt{R}, R)$ tends to $\sqrt{R}$, and $U^M \to U(\sqrt{R})$. Hence, if individuals are sufficiently risk averse, $U^M < U^D$ if $\bar{C}^D > \sqrt{R}$. It may be immediately verified that $\epsilon < 1/2$ implies that there is a $\bar{R} > 1$ such that $\bar{C}^D > \sqrt{R}$ for all $R \in (1, \bar{R})$, proving (b). Q.E.D.

**Proof of Proposition 6.** No-arbitrage conditions (11) for the competitive equilibrium, and for unrestricted intermediation, are easily seen to be equivalent. Thus the only difference between the two allocations is that an intermediary is able to set the steady state level of investment equal to 1. Hence the aggregate resource constraint in the intermediated economy dominates the one for the competitive unintermediated one (where, as shown in Proposition 4, $I^M < 1$). Since both allocations must satisfy $C_2 = (C_1)^2$, the consumption plan in a competitive stock market is strictly dominated by the one available with a financial intermediary. Finally, since we have that $C^U_2 = (C^U_1)^2 > R > C^*_2$, the consumption allocation available with an unrestricted intermediary is strictly dominated by the social optimum. Q.E.D.
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Figure 1: A Comparison of Five Allocations. The steady state consumption allocations that are available in the five possible equilibria are represented here as follows: autarky A, Diamond-Dybvig D, competitive stock market M, unrestricted (depository) intermediary U, and efficient P. In particular, for a utility function with a coefficient of constant relative risk aversion equal to 2, and with $\epsilon = .5$, $R = 2$, we obtain that: $A = (1;2)$; $D = (1.2; 1.7)$; $M = (1.4; 2)$ and $I = .7$; $U = (1.6; 2.5)$ and $P = (2;2)$. 

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