TO BE A RISK SEEKER OR A RISK AVOIDER: A SALE'S PERSPECTIVE IN RESPONSE TO QUOTAS AND RANK-ORDER CONTESTS

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To be a Risk Seeker or a Risk Avoider: A Sales Manager's Perspective in Response to Quotas and Rank-Order Contests*

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Abstract

This paper examines how decisions on risk taking on the part of sales managers are influenced by compensation schemes. We show theoretically and with experimental data that such decisions are sensitive to the payoff structure of the quota-based and rank-order-contest-based compensation. More specifically, we argue that a high quota or a low proportion of winners in a rank-order contest induce risk seeking, while a low quota or a high proportion of winners in a rank-order contest promote risk aversion. Such behavior does not stem from any violation of standard expected utility, but from jumps in the payoffs. Managerial implications for sponsors of quota-based and contest-based compensation schemes are discussed. The results extend beyond just salesforce management, to many situations where payoffs are based on reaching a certain threshold level in performance or are based on relative performance.
1. Introduction

Salesforce compensation is a crucial issue in management and, not surprisingly, has received much attention in the marketing literature. The commonly used compensation schemes are piece-rate (salary as a direct function of sales output), quota-based (a fixed salary and a flat bonus or a piece-rate commission beyond a certain level of sales), and sales contests (a fixed salary and a bonus if the sales are among the top few). Much of the focus in marketing research has been on designing optimal compensation schemes, driven primarily by an agency theory perspective (see, for example, Basu, Lal, Srinivasan and Staelin 1985; Lal and Staelin 1986; Raju and Srinivasan 1996). The problem is formulated as one of optimization of the firm's profit taking into account the likely behavior of the sales managers in response to the various compensation schemes. Such analysis, however, is limited to piece-rate and quota-based schemes. Marketing research regarding rank-order contests has been mainly in the form of surveys substantiating their widespread use. Chrapek (1989), for example, reports that the expenditure on sales contests increased from $1.6 billion in 1971 to over $8 billion in the 1980s, and that 83% of the firms in a survey reported usage of contests in some format or other. Contests in general have been widely analysed in the economics literature (see, for example, Lazear and Rosen 1981; O'Keeffe, Viscusi, and Zeckhauser 1984), and Frank and Cook (1995) provide evidence for the ubiquity of contests in all aspects of real life.

In contrast to piece-rate compensation, quota-based and contest-based compensation schemes are often considered conceptually similar. While in contests salespeople compete against each other, quotas are also viewed as a type of contest where employees compete with themselves (Churchill et al. 1993). Quotas and contests are extensively used as short term incentive programs (Sales and Marketing Management
Also, several different formats are observed. Churchill et al. (1993) report that 35% of the quotas and contests are designed such that salespeople have 1 in 5 odds of succeeding, 31% are designed with odds of success at about 2 in 5, 21% have the odds of 3 in 5, and 13% have the odds of 4 in 5.

Almost all the analysis on salesforce compensation schemes (e.g., BLSS 1985) and on contests (e.g., Lazear and Rosen 1981) focuses on effort as the key decision variable for a participant, and the participant's ability is considered as a critical variable for his/her success. Greater effort increases the chances of success for a participant, as does higher ability. Of course, at a given level of ability and effort, random fluctuations do occur in the participant's performance. However, it is assumed that the random fluctuations are part of exogenous noise over which the participant has no control. For example, in a typical model, a salesperson (contestant) might be able to shift the probability distribution of sales (output) to a higher mean by expending greater effort but the dispersion of the distribution is considered as a given. In this paper, we reexamine this assumption by considering another key decision variable for a salesperson which is how much risk to undertake in the selling task with a given level of effort. In other words, we consider the fact that a salesperson often has a choice of "riskiness" of the probability distribution (e.g., the dispersion of the distribution) for her sales. Consider, for example, a salesperson who has the option of pursuing two different customers, one whose order will be worth $250,000 with the probability of obtaining the order at 0.8 and another whose order will be worth $1 million with the probability of success at 0.2. Both options have the same expected value, but the second option is presumably more risky. We examine the impact of the availability of such a choice on the risk behavior of sales managers under quota-based and contest-based compensation schemes.
We develop a parsimonious model and provide empirical evidence from laboratory experiments to show that the payoff structure of compensation schemes can have substantive effect on the risk behavior of sales managers independent of the shape of their utility functions. For example, it is common to assume that all salespeople are risk averse or risk neutral (see, for example, Coughlan and Sen 1989) and hence cannot be expected to perform risky sales activities. Such risk-aversion on the part of the sales managers implies preference for the less risky (for example, lower variance with the same expected value) option. In this paper, we show that such an assumption needs closer scrutiny. The propensity to take risk on the part of the sales managers is not independent of the payoff structure of the compensation scheme, but is largely driven by it. So, it is possible that a salesperson with a concave utility function will opt for a risky option even in the absence of any special risk premium over and above the regular compensation. Similarly, a risk-seeking individual (one with a convex utility function) might choose a less risky option when facing some of the commonly used compensation schemes. It is shown, for example, in Gaba (1997) that a rank-order contest with a small proportion of winners leads to risk-seeking behavior whereas the same with a high proportion of winners induces risk-averse behavior. This does not necessarily stem from any kind of violation of standard expected utility theory, as it has been noted in some studies (see, for example, Ross 1991), but arises from the specific structure of the payoffs (e.g., jumps in payoffs). This is consistent with the view that risk aversion or risk seeking in any one context does not necessarily translate into the same in a different context.

The remainder of the paper is organized as follows. In Section 2, we outline the characterization of comparative risk. In Section 3, we consider the choice of risk for a salesperson under a quota-based compensation scheme and in Section 4 we present
similar results for rank-order contests. The experimental studies are discussed in section 5. Section 6 concludes the paper and discusses some managerial implications of the results.

2. Comparative Risk

Before we begin to discuss the choice of riskiness in performance on the part of a salesperson, we must clearly define what we mean by a more risky or a less risky prospect for a salesperson. Suppose that a salesperson, for her performance level, has a choice between two uncertain prospects (i.e., two probability distributions) with the same expected value but a higher variance in one case. It is then common to assume that the prospect with the higher variance is more risky. However, comparison of uncertain prospects by a mean-variance analysis is often considered inadequate. For example, for any nonquadratic concave utility function, there exist rankings of uncertain prospects by expected utility and by variance that are different (Rothschild and Stiglitz 1970). In other words, it is often possible to find a case where a risk-averse expected utility maximizer would prefer an uncertain prospect which has higher variance relative to another even though both have the same expected value. On the other hand, consider two cumulative distribution functions (c.d.f.s) $G$ and $F$ with the same finite mean such that all risk-averse (those with concave utility functions) expected utility maximizers prefer $F$ to $G$. Then it would be reasonable to say that $G$ is more risky than $F$. Rothschild and Stiglitz (1970) provide such a characterization of comparative risk in terms of a mean preserving spread, defined below.
Definition 1. A c.d.f. $G$ differs from a c.d.f. $F$ by a mean preserving spread (MPS) if there exists an interval $I$ such that $G$ assigns no greater probability than $F$ to any open subinterval of $I$ and $G$ assigns at least as much probability as $F$ to any open interval either to the left or to the right of $I$.

Rothschild and Stiglitz link an MPS to the idea of an increase in risk by demonstrating that the following four conditions for a pair of univariate c.d.f.s $G$ and $F$, with the same finite mean, are equivalent:

(a) $G$ can be obtained from $F$ by a sequence of one or more mean preserving spreads.

(b) There exists a joint distribution of random variables $(X, Z)$, such that the marginal distribution of $X$ is $F$, $E(Z|X) = 0$ for all $X$, and $Y = X + Z$ has distribution $G$.

(c) $G$ and $F$ satisfy

$$T(y) = \int_{-\infty}^{y} [G(x) - F(x)] dx \geq 0, \forall y,$$

and

$T(y)$ converges to zero as $y \to \infty$.

(d) $\int U(x) dF(x) \geq \int U(x) dG(x)$

for every (not necessarily increasing) concave function $U$.

In the case above, we say that $G$ is more risky than $F$. The intuition behind the Rothschild-Stiglitz definition of comparative risk is that an MPS is an operation which moves probability mass in a distribution ($F$) from some central region to the tail regions.

1 This more general definition, which does not require a distinction between the absolutely continuous and
and, hence, leads to a new distribution \( G \) with "heavier tails." Figure 1 shows two distributions \( G \) and \( F \) which differ by an MPS.

**Figure 1: Two Probability Distributions that Differ by an MPS**

If \( G \) and \( F \) differ by an MPS then \( G \) and \( F \) have a single crossing, i.e., there exists some outcome level \( c \) such that \( G(x) \geq F(x) \) for \( x < c \) and \( G(x) \leq F(x) \) for \( x \geq c \).\(^2\) It is clear from the figure that \( G \) has more probability mass nearer to the tails relative to \( F \) and, if \( G \) and \( F \) have the same mean, then \( G \) must have higher variance than \( F \).\(^3\) Condition (b) above states that performing an MPS is similar to obtaining \( G \) by adding a zero-conditional-mean-noise to \( F \). The equivalent condition (c) states that \( F \) second order stochastically dominates \( G \), and that \( F \) and \( G \) have the same mean. The last equivalent condition (d) states that all risk-averse expected utility maximizers will prefer \( F \) to \( G \). This appears intuitively reasonable since \( G \) has "heavier tails" than \( F \) and all risk-averse individuals would want to avoid that.

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\(^2\) Even if \( G \) and \( F \) cross only once, the value of \( x \) at which they cross need not be unique. Also, a mean preserving single crossing does not necessarily imply an MPS (Pratt and Machina 1997).

\(^3\) Ordering of distributions using the concept of MPS is only partial, while the ordering by a mean-variance analysis is complete.
The Rothschild-Stiglitz characterization of comparative risk remains dominant in the standard economics and finance literature (Scarsini 1994). In this paper, we use the definition of Rothschild-Stiglitz MPS to characterize comparative risk, however, with an additional specification. The point of single crossing between $G$ and $F$ (which differ by an MPS) plays an important role in the analysis in the paper. Accordingly, the following definition is adopted.

**Definition 2.** $G$ is said to differ from $F$ by a *mean preserving spread about $c$* (MPS about $c$), and $F$ is said to differ from $G$ by a *mean preserving contraction about $c$* (MPC about $c$), if $G$ differs from $F$ by an MPS and $c$ is a point of crossing between $G$ and $F$.4

Consider, for example, a mean preserving increase (decrease) in variance for a symmetric distribution. The increase (decrease) in variance implies an MPS (MPC) about the mean. The class of MPS about $c$ forms a proper subset of the class of MPS.

### 3. Quota-based Compensation

Consider a salesperson who gets a fixed salary $A$ and a bonus $B$ if his sales equal or exceed a certain level $q$. Of course, $A$, $B$, and $q$ are assumed to be strictly positive. Suppose that the salesperson has a choice between two c.d.f.s, $F$ and $G$, for her dollar (or unit) sales. Let the corresponding probability density function (p.d.f.) of $F$ be uniform on $[10, 20]$ and that of $G$ be uniform on $[0, 30]$. Clearly, $G$ differs from $F$ by an MPS about the mean and, hence, we can say that $G$ is more risky than $F$. First, consider $q = 17$. If the salesperson opts for $F$ his probability of getting the bonus is $(20-17)/(20-10) = 0.3,$
whereas his probability of obtaining the bonus with $G$ is $(30-17)/(30-0) = 0.43$. Next, consider $q = 13$. Now, the salesperson’s probability of getting the bonus is 0.7 with $F$ and 0.57 with $G$. In the first case, it appears that the salesperson is better off by increasing his risk and in the second case the salesperson is better off by choosing lower risk. It is easy to verify that higher risk is strictly preferable as long as $q > 15$ (the mean of $F$ and $G$), and lower risk is strictly preferable if $q < 15$. This seems reasonable since the salesperson will choose the option that maximizes her chances of getting the bonus.

If the quota is high, then the salesperson is better off with a more risky distribution. Intuitively, the greater volatility of sales increases the upside potential by increasing the chances of exceeding the set quota and the greater downside risk is inconsequential since the payoff is bounded from below by the fixed salary. We discuss this phenomenon below in a more general form for all distributions.

**Theorem 1.** Let a random variable $x$ be the dollar sales of a salesperson whose compensation plan is given by

$$S(x) = \begin{cases} 
A & \text{if } x < q, \\
A + B & \text{if } x \geq q,
\end{cases}$$

with $A > 0$, $B > 0$, and $q > 0$. Let $G$ and $F$ be two c.d.f.s (absolutely continuous or discrete) such that $G$ is more risky than $F$ in the sense that $G$ differs from $F$ by an MPS about $c$. Suppose that the salesperson can choose $G$ or $F$ as a distribution for $x$. Then, given that the salesperson has a nondecreasing utility function, she will (weakly) prefer $G$ to $F$ if and only if the quota is “high,” i.e., $q > c$.

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4 Recall that the value of $c$ need not be unique (Footnote 2). The term “mean preserving spread about $v$,” is also used in Landsberger and Meilijson (1990). Their definition, however, does not necessarily imply a crossing at $v$. 

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**Example:** Consider, for example, a mean preserving increase (decrease) in variance in a symmetric distribution, which would be an MPS (MPC) about the mean. The increase (decrease) in variance will be strictly preferred if and only if the quota is above (below) the mean.

**Proof:** The salesperson will choose $G$ over $F$ if and only if her probability of getting the bonus $B$ is at least as much with $G$ as with $F$, i.e., iff

$$
\Phi(x) = \int_{q}^{\infty} dG(x) - \int_{q}^{\infty} dF(x) \geq 0.
$$

But $\Phi(x) = [1 - G(q-)] - [1 - F(q-)] = F(q-) - G(q-)$, (1)

Where $F(q-) = P_F \{x < q\}$ and $G(q-) = P_G \{x < q\}$.

Recall that, if $G$ differs from $F$ by an MPS about $c$, then $G(x) \geq F(x)$ for $x < c$ and $G(x) \leq F(x)$ for $x \geq c$. Hence, $\Phi(x)$ in (1) is nonnegative iff $q > c$.

**Remarks:**

1) The preferences in Theorem 1 will be strict preferences if and only if $G$ and $F$ are not identical.

2) For a higher $q$ the class of acceptable MPSs is larger since the interval of values for $c$ which satisfy $c < q$ is larger. Loosely speaking, higher the value of $q$ the more likely it is that a salesperson will be risk seeking, and lower the value of $q$ the more likely it is that a salesperson will be risk averse.

3) Note that the equal-means condition for $G$ and $F$ is not necessary for the above result in the asymmetric case. If $G$ has lower mean than $F$ (in which case, we may think of $G$ as a mean reducing increase in risk with respect to $F$) and $q > c$, a salesperson will
still prefer \( G \) to \( F \). It is plausible in certain situations that an increase is risk might be obtained only at some cost, for example, a reduction in the mean.

In the next theorem, we show that a similar result holds for all possible variations of a quota-based compensation scheme and for all arbitrary distributions.

**Theorem 2.** Let a random variable \( x \) be the dollar sales of a salesperson whose compensation plan is given by

\[
S(x) = \begin{cases} 
A & \text{if } x < q, \\
A + \phi(x) & \text{if } x \geq q,
\end{cases}
\]

where \( A > 0 \), \( q > 0 \), and \( \phi \) is nonnegative and nondecreasing in \( x \). Let \( G \) and \( F \) be two c.d.f.s (absolutely continuous or discrete) such that \( G \) is more risky than \( F \) in the sense that \( G \) differs from \( F \) by an MPS about \( c \). Suppose that the salesperson can choose \( G \) or \( F \) as a distribution for \( x \). Assume that the salesperson has a nondecreasing utility function \( U \) with a continuous first derivative. Then, if the quota is "high," i.e., if \( q > c \), she will (weakly) prefer \( G \) to \( F \).

**Proof:** Without loss of generality, let \( U: t \rightarrow [0, \infty), t \in \mathbb{R} \), with \( U(t) = 0 \) for \( t = A \). Further, define \( U'(A + \phi(x)) = dU(A + \phi(x))/dx \).

The salesperson will (weakly) prefer \( G \) to \( F \) if and only if her expected utility is at least as high with \( G \) as with \( F \), i.e., iff

\[
\Delta U(x) = E_G[U(S(x))] - E_F[U(S(x))] = \int_q^\infty [U(A + \phi(x))d[G(x) - F(x)] \geq 0.
\]

Integrating by parts yields
\[
\Delta U(x) = U(A + \phi(\infty)) \left[ \int_q^\infty d[G(y) - F(y)] \right] - \int_q^\infty U'(A + \phi(x)) \left[ \int_q^x d[G(y) - F(y)] \right] dx
\]

\[
= U(A + \phi(\infty))(F(q-) - G(q-)) - \int_q^\infty U'(A + \phi(x))(G(x) - G(q-) - F(x) + F(q-)) dx
\]

\[
= U(A + \phi(\infty))(F(q-) - G(q-)) - [F(q-) - G(q-)][U(A + \phi(\infty)) - U(A + \phi(q))]
\]

\[
- \int_q^\infty U'(A + \phi(x))(G(x) - F(x)) dx
\]

\[
= [F(q-) - G(q-)]U(A + \phi(q)) - \int_q^\infty U'(A + \phi(x))(G(x) - F(x)) dx .
\] 

(2)

Note that since \( U \) is nondecreasing in its argument, \( U(A) = 0 \), and \( \phi \) is nonnegative, \( U(A + \phi(q)) \) is nonnegative. Adding the assumption that \( \phi \) is nondecreasing in \( x \), \( U' \) is also nonnegative. Recall that if \( c < q \), then \( G(x) - F(x) \leq 0 \) for \( \forall x \in [q, \infty) \) and \( [F(q-) - G(q-)] \geq 0 \). Combining all these observation in (2) shows that if \( c < q \), \( \Delta(x) \) is nonnegative.

**Remarks:**

1) Note that all the remarks for Theorem 1 are also valid here.

2) The necessary and sufficient condition in Theorem 1 for taking greater risk is only a sufficient condition (in a minimal sense) here in Theorem 2. In some cases of this more general setup, even if \( c \geq q \), \( G \) might be preferable to \( F \). This seems reasonable since the bonus is not fixed but is larger for higher values of \( x \) beyond the quota. Thus the primary objective is not to just cross the quota but also to try and go as far beyond the quota as possible. Intuitively, this should increase the attractiveness of a more risky distribution vis-à-vis the case where the bonus is fixed for all values of \( x \) beyond the quota. Also, note that if \( \phi \) is linear in \( x \), one can view the compensation...
scheme as a financial option (with $q$ as its strike price) which increases in its value with greater variability in $x$.

4. Contest-based Compensation

Consider 10 salespeople who are involved in a rank-order contest. Any one salesperson gets a fixed salary $A$ and a bonus $B$ ($A$ and $B$ are strictly positive) if her sales are among the top 3 sales. Let the dollar sales of the 10 salespeople be independently and identically distributed with c.d.f $F$ with corresponding p.d.f. that is uniform on [400, 600]. Let $G$ be another c.d.f. with corresponding p.d.f. that is uniform on [0, 1000]. Clearly $F$ and $G$ have the same mean and $G$ differs from $F$ by an MPS about the mean. Suppose that one of the 10 salespeople, say Cindy, obtains an opportunity to switch her sales distribution from $F$ to the more risky distribution $G$. Cindy will switch to $G$ from $F$ if and only if her probability of “winning” (achieving a sales level among the top 3) with $G$ is at least as much as that with $F$. Given the symmetrical nature of the contest, Cindy’s probability of winning with $F$ is simply $3/10 = 0.3$. If Cindy switches her sales distribution to $G$, her probability of winning becomes $P\{\text{Cindy sells between 400 and 600 units and wins}\} + P\{\text{Cindy sells between 600 and 1000 units}\} = 3/10(200/1000) + 400/1000 = 0.46$. Here, it appears that Cindy is better off by increasing the risk in her performance. Of course, in doing so, she will also increase the probability of getting the lowest sales level among the ten contestants. However, whether she obtains the lowest sales or the fourth highest is of no consequence and what matters is whether she can increase her probability of obtaining a sales level that is among the top three.

Now, suppose that salespeople with the top 7, and not top 3, sales levels get a bonus. In this case, Cindy’s probability of winning (achieving a sales level among the top 7) with $F$ is 0.7, and if she switches to $G$ the probability becomes 0.54. In this case, it
appears that Cindy is better off by remaining with the less risky distribution. Table 1 below shows the difference between probability of winning with $G$ and probability of winning with $F$ for Cindy for different values of $k$, where $k$ is the number of winners out of 10 who get the bonus $B$.

Table 1: The Impact of Adopting an Increase in Risk in a Rank-Order Contest with 10 Contestants for Different Number of Winners $k$

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_G(\text{Win}) - P_F(\text{Win})$</td>
<td>0.32</td>
<td>0.24</td>
<td>0.16</td>
<td>0.08</td>
<td>0</td>
<td>-0.08</td>
<td>-0.16</td>
<td>-0.24</td>
<td>-0.32</td>
</tr>
</tbody>
</table>

Note that adopting $G$ is beneficial for Cindy if and only if $k < 5$. Also, given $k < 5$, the gain from adopting a more risky distribution is higher for a lower $k$. On the other hand, given $k > 5$, the loss in probability of winning from an increase in risk is higher as $k$ gets higher. To sum up, the above example suggests that if the proportion of winners is low, an MPS works to one’s advantage; on the other hand, if the proportion of winners is high, an MPS reduces the probability of winning. We explore this phenomenon below in a more general form.

Suppose that $n$ equally able salespeople are engaged in a rank-order contest and all face i.i.d. sales levels $X_i$, $i = 1, 2, ..., n$, with common c.d.f. $F$. Let $X_{1:n} \leq X_{2:n} \leq \ldots \leq X_{n:n}$ be the order statistics obtained by arranging the performance levels in an increasing order of magnitude. Contestants with the highest $k$ performance levels ($X_{n-k+1:n}$ to $X_{n:n}$) are to be the winners and each gets payoff $A+B$, and the remaining $n-k$ contestants get payoff $A$ each, where $A$ and $B$ are strictly positive. Given the symmetrical conditions in the contest, the probability of winning for any contestant is simply $k/n$. 
Now, suppose that one of the salespeople (Cindy) obtains an opportunity to shift her performance distribution from $F$ to $G$ where $G$ differs from $F$ by an MPS about $c$. The question then is whether Cindy would prefer $G$ to $F$ — whether she would prefer adopting an increase in risk in her performance while maintaining the expected value of her performance. Assuming that Sandy has a nondecreasing utility function, she will prefer $G$ at least as much as $F$ if and only if her probability of winning (achieving one of the top $k$ performance levels) is at least as high with $G$ as with $F$, i.e., iff

$$\Omega_{G,F}(k,n) = P_G\{\text{Win}\} - P_F\{\text{Win}\}$$

$$= \int_{\infty}^{\infty} F_{n-k:n-1}(x) d\{G(x) - F(x)\} \geq 0,$$  \hspace{1cm} (4)

where

$$F_{n-k:n-1}(x) = \frac{(n-1)!}{(n-k-1)!(k-1)!} \int_0^{F(x)} \frac{u^{n-k-1}}{(1-u)^{k-1}} du,$$  \hspace{1cm} (5)

is the c.d.f. for $X_{n-k:n-1}$, the $(n-k)$th highest performance level among the $n-1$ contestants who still face $F$.\footnote{For distributions of order statistics, see, for example, Arnold, Balakrishnan and Nagaraja (1992).}

For ease of exposition, we confine our results in this paper to uniform distributions (more general cases are discussed in Gaba 1997). Consider the case where $F$ and $G$ are c.d.f.s of continuous uniform distributions, with supports in $[m-v, m+v]$ and $[m-v-\Delta v, m+v+\Delta v]$, respectively, where $v > 0$ and $\Delta v > 0$. Clearly, $F$ and $G$ have the same mean and $G$ differs from $F$ by an MPS about $m$. Cindy’s probability of winning the contest with $G$ is given by

$$P_G\{\text{Win}\} = \int_{m-v}^{m+v+\Delta v} F_{n-k:n-1}(x) dG(x)$$

Integrating the right-hand side in (6), by parts, and letting $t = F(x)$, yields
\[
P_G\{\text{Win}\} = 1 - \int_{0}^{1} G[F^{-1}(t)] f_\beta(t \mid n-k,n) dt, \tag{7}
\]

where

\[
G[F^{-1}(t)] = G[m - v + t(2v)] = (2vt + \Delta v)/(2v + \Delta v), \tag{8}
\]

and

\[
f_\beta(t \mid n-k,n) = \frac{(n-1)!}{(n-k-1)!(k-1)!} t^{n-k-1}(1-t)^{k-1} \tag{9}
\]

is a beta probability density function. Hence, we get

\[
P_G\{\text{Win}\} = 1 - \frac{2vt[(n-k)/n] + \Delta v}{2(v + \Delta v)}, \tag{10}
\]

and, noting that \(P_F\{\text{Win}\} = k/n\),

\[
\Omega_{G,F}(k,n) = P_G\{\text{Win}\} - k/n = \frac{\Delta v}{v + \Delta v} \left( \frac{1}{2} - \frac{k}{n} \right). \tag{11}
\]

Note in (12) that, since \(v > 0\) and \(\Delta v > 0\), \(\Omega_{G,F}(k,n)\) is nonnegative if and only if \(k/n \leq 1/2\), which implies that Cindy will switch from \(F\) to \(G\) if and only if \(k/n \leq 1/2\). Also, note that the net gain from switching to \(G\) from \(F\) is greater for a lower \(k/n\). Similarly, it is easy to verify that if \(H\) is a less risky distribution than \(F\) in the sense that \(H\) differs from \(F\) by an MPC about \(m\), then Cindy will switch to \(H\) from \(F\) if and only if \(k/n \geq 1/2\) and the benefit of switching to \(H\) will be greater for a higher \(k/n\).

The analysis above is incomplete without also considering the possibility that all contestants might have an equal opportunity to choose the riskiness of their performance distributions. Suppose that all \(n\) contestants face the same expected value \(m\) for their sales and beyond which each has a choice between c.d.f.s \(G\) and \(F\) as a distribution for her sales, where \(G\) and \(F\) are c.d.f.s of uniform distributions and \(G\) differs from \(F\) by an MPS about \(m\). Let \(P_i(d_1, d_2, \ldots, d_n)\), where \(d_i = G\) or \(F\) for all \(i\), be the probability of
winning (achieving a sales level among the top $k$) for contestant $i$ as a function of everyone's strategies including her own. Then, a Nash equilibrium point is any vector of strategies $(d_1^*, d_2^*, ..., d_n^*)$ such that for each $i = 1, 2, ..., n$,

$$P_i(d_1^*, d_2^*, ..., d_i^*, ..., d_n^*) = \max_{d_i} P_i(d_1^*, d_2^*, ..., d_i, ..., d_n^*).$$

Since all contestants are identical, all contestants will choose the same strategy in equilibrium. If $k/n < 1/2$, then as shown earlier

$$\max_{d_i} P_i(F, F, ..., d_i, ..., F) = P_i(F, F, ..., G, ..., F), \forall i,$$

and

$$\max_{d_i} P_i(G, G, ..., d_i, ..., G) = P_i(G, G, ..., G, ..., G), \forall i,$$

since $d_i = F$ would be equivalent to choosing an MPC about $m$ which is preferred if and only if $k/n > 1/2$. So, the vector of strategies $(G, G, ..., G, ..., G)$, where all contestants choose the more risky distribution will be the unique pure strategy equilibrium point. Similarly, it can be easily seen that if $k/n > 1/2$, the vector of strategies $(F, F, ..., F, ..., F)$, will be the unique pure strategy equilibrium point. Finally, if $k/n = 1/2$, any vector of strategies $(d_1, d_2, ..., d_n)$ would be an equilibrium point.

The results of this section have serious implications for sales contests and for contests in general. Any contestant who is able to obtain a special opportunity to manipulate the riskiness of his or her performance would have a clear substantive advantage. Such special opportunities for a contestant may arise if, for example, the contestant is a protégé of the manager organizing the contest and is able to obtain a greater sales budget relative to others. If all contestants have an equal opportunity to manipulate the riskiness of their performances, a contest that ends with a small proportion of winners might induce a collective behavior that is risk seeking and, on the
other hand, a contest with a high proportion of winners might lead to collective behavior that is risk averse. These implications are relevant, of course, for all contests and not just sales contests, other examples being R&D races, promotion tournaments in organizations, contests among financial fund managers to be in the “top ten” list, and so on.

5. Experimental Studies

Three experiments were conducted to test the predictions of the analyses in Sections 3 and 4. The subjects assumed the role of sales managers who had to either meet some sales quota targets or participate in a rank-order sales contest. The task required them to select from alternative sales distributions that varied in riskiness. Real monetary incentives were provided to subjects.

Experiment 1

The objective of this study was to test whether higher quota levels resulted in the selection of more risky distributions. A single factor between subjects design with two levels (two quota levels) was used. The subjects were 66 MBA students enrolled in an introductory marketing management class. A $50 lottery was used as an incentive to volunteer for the experiment. Subjects were told that they were participating in a decision-making experiment. Before the main task, subjects’ inherent risk aversion was assessed by responses to three questions (see, for example, Clemen 1996). In the first question, the subjects were asked to choose between receiving $500 for certain or participating in a lottery that would give them $1000 with probability 0.5 and $0 with probability 0.5. In the next two questions, subjects were asked to state the most they would be willing to pay to play two lotteries. The first lottery had a 50% chance of
winning $1000 and a 50% chance of getting $0. The second lottery had a 80% chance of winning $1000 and a 20% chance of winning $0.

In the main task, the subjects were asked to play the role of sales managers selling an industrial product Beta. They were told that the firm employing them was implementing an incentive scheme for the upcoming quarter. In a memo from senior management, they were informed that they would get a bonus if they equaled or exceeded their individual sales quota set for the quarter. They were provided with the sales quota design which was manipulated with the sales quota being either 350 units or 650 units. Subjects were told that they would be paid $10 if they met the sales quota. Next, the market structure was described. The market for product Beta consisted of four types of customer segments (called Type A to Type D) which varied in terms of the responsiveness to sales effort. The subjects were instructed that they could focus their effort on only one segment and that their task was to make a decision on the segment type. They were provided with the characteristics of each segment, which were essentially the sales potential for the 4 segments. The sales estimate for each segment was a discrete uniform distribution: 300 to 700 units for Segment A, 200 to 800 units for Segment B, 100 to 900 units for Segment C, and 0 to 1000 units for Segment D. Thus, if a subject, for example, selected Segment A, his/her sales for the upcoming quarter was bounded from below by 300 units and bounded from above by 700 units. It was repeatedly emphasized to the subjects that all sales numbers in a given range were equally likely. Note that the expected value of sales is 500 for all segments but the variance increases from Segment A to Segment D. All four distributions for the four segments differ from each other by an MPS about the mean such that Segment D is the most risky while Segment A is the least risky.
Subjects were told that, after selecting the segment type, they would draw a card from a box corresponding to the segment type selected. Box A contained cards for each number between 300 and 700, Box B contained cards for numbers ranging between 200 and 800, and so on. The number drawn by a subject represented his/her sales level for the quarter. If the number drawn equaled or exceeded the sales quota, the subject was entitled to $10. Finally, following a debriefing session, the winning subjects were paid.

Results and Discussion of Experiment I

First, responses to the three risk aversion questions were analyzed to ensure that there were no significant differences in the inherent risk preferences across the two conditions (the two quota levels). In terms of the first risk preference question, there was no significant difference between the two conditions: the percentage of subjects who preferred the $500 for certain versus the equivalent fair lottery was 82.86% in the low quota group and 80.65% in the high quota group. Regarding the next two risk preference questions, the mean willingness to pay for Lottery 1 in the low-quota condition ($X_{350} = $86.02) was not significantly different from the mean willingness to pay in the high-quota condition ($X_{650} = 90.16; F < 1, p < 0.89$). Similarly, the mean willingness to pay for Lottery 2 did not vary across the two conditions ($X_{350} = 181.02, X_{650} = 202.96; F = 0.29, p < 0.96$). These checks indicate that the subjects across the two conditions did not significantly differ in terms of risk aversion.

Table 2a provides the probabilities for equaling or exceeding the quota levels for the different distributions. The results of the main task are given in Table 2b. The results indicate that most subjects tended to select the Segment type with the least risky distribution (62.86 %) when the sales quota was set low. However, when the sales quota was set high, most subjects (74.19%) selected the Segment type associated with the most
risky distribution. These results clearly provide strong support for the analysis of quota-based compensation in Section 3.

Table 2a: Probabilities of Equaling or Exceeding Quotas with Different Distributions

<table>
<thead>
<tr>
<th>Segment</th>
<th>Segment A (300-700)</th>
<th>Segment B (200-800)</th>
<th>Segment C (100-900)</th>
<th>Segment D (0-1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quota: 350</td>
<td>0.88</td>
<td>0.75</td>
<td>0.69</td>
<td>0.65</td>
</tr>
<tr>
<td>Quota: 650</td>
<td>0.13</td>
<td>0.25</td>
<td>0.31</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 2b: Choice of Riskiness in Response to the Two Quotas

<table>
<thead>
<tr>
<th>Probability Distributions</th>
<th>Segment A (300-700)</th>
<th>Segment B (200-800)</th>
<th>Segment C (100-900)</th>
<th>Segment D (0-1000)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quota: 350</td>
<td>22 (0.63)</td>
<td>7 (0.20)</td>
<td>4 (0.11)</td>
<td>2 (0.06)</td>
<td>35</td>
</tr>
<tr>
<td>Quota: 650</td>
<td>0 (0.00)</td>
<td>1 (0.03)</td>
<td>7 (0.23)</td>
<td>23 (0.74)</td>
<td>31</td>
</tr>
</tbody>
</table>

Experiment 2

The results of Section 4 regarding contest-based compensation imply that the decision to take risky actions on the part of the sales managers is contingent on the design of the contest. If the proportion of winners in a contest is small, the contestants are likely to select more risky distributions relative to a contest where the proportion of
Experiment 2 was designed to investigate the empirical validity of this result.

A single factor between subjects design with two levels (proportion of winners of 10% and of 90%) was used. Fifty-six students enrolled in a marketing elective course participated in the experiment. Approximately 40% of the students were executives enrolled in an evening MBA program while the remaining 60% were full-time MBA students. The average work experience was 5.23 years. A $50 lottery was used as an incentive to volunteer for the experiment. As in Experiment 1, the subjects were asked to play the role of sales managers and were told that the firm employing them was conducting a sales contest for the upcoming quarter, and the task began with elicitation of the inherent risk preferences.

For the main task, subjects were instructed that they would be competing against the other participants in the class session, where all participants were said to be employed by the same organization. The subjects were also to assume that all participants were equal in terms of selling ability and selling effort (defined as the number of hours spent on the job). Next, the contest design was explained. The subjects were told that their sales would be compared to those of the other participants in the experiment. In the low-proportion-of-winners condition, a subject had to obtain sales which was among the highest 10% in the group to get the sales bonus. The instructions, for example, were "..if your sales are in the highest 10% in your group, you will get the bonus of $10. If you are not in the top 10% in sales (i.e., you are in the bottom 90%), you will get nothing." In the high-proportion-of-winners condition, the proportion of winners was 90%.

The description of the market structure was similar to the setup in Experiment 1. However, the subjects could choose between only two segment types. The sales estimate for Segment A ranged between 400 and 600 units while that of Segment B ranged
between 0 and 1000 units, with each number in a given range being equally likely. After all the subjects had drawn cards from the boxes corresponding to their choices of segment type, the results were tabulated and the winners were paid $10 each.

Results and Discussion of Experiment 2

As in Experiment 1, responses to the three risk aversion questions were analyzed to ensure that there were no significant differences in the inherent risk preferences across the two conditions. In the low-proportion-of-winners condition, 81.48% of the subjects preferred the option of $500 for certain, while 72.41% preferred the same option in the high-proportion-of-winners condition (the difference was not significant). The mean willingness to pay for Lottery 1 in the low-proportion-of-winners condition ($X_{10\%} = $106.00) was not significantly different from the same in the high-proportion-of-winners condition ($X_{90\%} = 86.75; F = 0.22, p < .64$). Also, the mean willingness to pay for Lottery 2 did not vary across the two conditions ($X_{10\%} = 203.15, X_{90\%} = 242.96; F = 0.29, p < .59$). The checks here indicate that any differences in the choice of the distributions in the main task are not due to any inherent significant differences in terms of risk aversion. The results of the main choice task in Experiment 2 are provided below in Table 2. Approximately 85% of the subjects in the low-proportion-of-winners condition selected the segment with the high-risk distribution. Also, as expected, 72% of the subjects in the high-proportion-of-winners conditions selected the low risk distribution. These results provide strong support for the implications suggested in Section 4, i.e., a low proportion of winners in a contest induces behavior that is risk seeking while a high proportion of winners leads to risk-averse behavior.
Table 3: Choice of Riskiness in Response to Proportion of Winners in the Contest Number (Proportion) of Respondents

<table>
<thead>
<tr>
<th>Probability Distributions</th>
<th>Segment A (400 – 600)</th>
<th>Segment B (0-1000)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Proportion of Winners: 10%</strong></td>
<td>4 (0.15)</td>
<td>23 (0.85)</td>
<td>27</td>
</tr>
<tr>
<td><strong>Proportion of Winners: 90%</strong></td>
<td>21 (0.72)</td>
<td>8 (0.28)</td>
<td>29</td>
</tr>
</tbody>
</table>

*Experiment 3*

Recall that, in Section 4, the Nash equilibrium for a sales contest is discussed for the case where choices entail only two distributions. A similar model does not appear to be mathematically tractable for choices between a larger set of distributions. However, this additional experiment was designed to explore whether the intuition from the model is also valid when subjects select from more than two distributions.

This study was identical to Experiment 2 in all aspects except that the choice of distributions on the part of a subject was among five distributions, rather than just two, and four levels for proportion of winners were used. The subject pool was first and second year MBA students. A total of 182 subjects participated in the experiment. A single factor between subjects design with 4 levels (proportion of winners equal to 10%, 40%, 60% and 90%) was employed. As in the earlier experiments, measures of risk preferences were first obtained. For the main task, in contrast to Experiment 2, the subjects could select from 5 discrete uniform distributions (400 to 600, 300 to 700, 200 to...
800, 100 to 900, and 0 to 1000). All the other procedures were identical to those in Experiment 2.

As in the earlier experiments, there were no significant differences in the inherent risk preferences across the different conditions. In question 1, the percentages of the subjects who preferred the option of $500 for certain were 71.74%, 75.56%, 78.26%, and 77.78% for the 10%, 40%, 60%, and 90% proportion-of-winners conditions, respectively. Regarding the next two questions about risk preferences, the means for willingness to pay for Lottery 1 ($X_{10\%} = 135.60$, $X_{40\%} = 128.18$, $X_{60\%} = 106.93$, $X_{90\%} = 105.16$; $F = 0.38, p < 0.77$) and for Lottery 2 ($X_{10\%} = 251.5$, $X_{40\%} = 242.93$, $X_{60\%} = 233.56$, $X_{90\%} = 192.48$; $F = 0.44, p < 0.73$) were also not significantly different. The results of the main task are provided in Table 4. The data indicate that the intuition of the model in Section 4 still holds in more complex setups of contest-based compensation. In the case with 10% winners, 74% of the respondents chose Segment E and 15% chose D, the two most risky segments, whereas no one chose the least risky Segment A. On the other hand, with 90% winners, most subjects choose Segment A (22%) or Segment B (49%), the two least risky segments, and only two respondents (4%) chose the most risky Segment E. Such effects held also for the cases of 40% and 60% winners, though were a bit muted. In the 40% winners case, 62% still chose Segments D or E and only 15% chose Segments A or B; in the 60% case, 41% chose Segments D or E and 48% chose Segments A or B. The effects are not as strong in these cases, perhaps due to the fact that the benefit of adopting greater risk or less risk is less when the proportion of winners is closer to 50%.
Table 4: Choice of Riskiness in Response to Proportion of Winners in the Contest Number (Proportion) of Respondents*

<table>
<thead>
<tr>
<th>Proportion Of Winners</th>
<th>Probability Distributions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Segment A  (400-600)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Segment B  (300-700)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Segment C  (200-800)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Segment D  (100-900)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Segment E  (0-1000)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>0 (0.00)</td>
<td>3 (0.07)</td>
</tr>
<tr>
<td>40%</td>
<td>2 (0.04)</td>
<td>5 (0.11)</td>
</tr>
<tr>
<td>60%</td>
<td>9 (0.20)</td>
<td>13 (0.28)</td>
</tr>
<tr>
<td>90%</td>
<td>10 (0.22)</td>
<td>22 (0.49)</td>
</tr>
</tbody>
</table>

*The proportions in rows 2 and 4 do not add to 1 due to rounding.

6. Conclusion and Managerial Implications

The existing literature on compensation plans has primarily focused on one component of decision making on the part of the sales managers: the amount of effort to be expended in the selling task. A major point of this research is that, in many situations, salespeople may also make decisions on how much risk to undertake in the selling task. One scenario where this is likely to occur is in the selection of specific customers by salespeople to concentrate effort on. Typically, salespeople have to choose a subset of consumers from the total market potential. The selection decision is often observed when the product is new or when the firm is attempting to grow in new markets. However, even in relatively mature product categories, salespeople often have to decide whether to
increase sales by allocating a given amount of effort on the low risk approach of retaining existing customers or by incurring the effort on the high risk option of new prospects. New customers may be more risky particularly if they have been brand loyal to competing products in the past. Similarly, the sales managers may make decisions on spending effort on retaining a small but satisfied customer segment or a more risky and large but dissatisfied segment.

One of the critical assumptions included in most of the normative models on salesforce compensation concerns the risk preferences of the firm’s employees. Often it is suggested that the employees are either risk averse (e.g., BLSS 1985; Lal and Staelin 1986) or risk neutral (Rao 1990; Mantrala, Raman and Desiraju 1997). Alternatively, it is recommended that a firm elicit risk preferences of its employees (Raju and Srinivasan 1996). Based on the knowledge of the risk preferences, the models are used to design incentive schemes so that the interests of the salespeople are aligned with the objectives of the firm. One of the important implications of this paper is that this approach could lead to employee behavior that is not consistent with the firm’s goals. We demonstrate that the payoff structure of the incentive scheme itself can influence the risk behavior of the sales managers. In other words, risk behavior is not exogenous but endogenous to the compensation plans.

More specifically, we have shown that the risk behavior of salespeople is greatly influenced by the payoff structure of the quota-based and contest-based compensation schemes. Higher quotas induce greater risk seeking. Low quotas, on the other hand, lead to risk-averse behavior. The result is not limited to the case where a salesperson just gets a flat bonus on exceeding a set quota, but applies also to other commonly used variations of the quota-based compensation plan (which might include, for example, an increasing
or a decreasing rate of commission for every unit sold beyond the quota level). The theoretical results are further supported by data from experimental studies.

Quotas are typically set taking into consideration the salesperson and territory characteristics (Stanton, Buskirk and Sprio 1991). An insight from this paper is that the firm should also take into account the strategic objectives of the product line/brand. Setting a quota in terms of a basic sales level could induce the salesperson to meet the short term quota goal by either adopting a high risk or low risk strategy. However, the customer base obtained by the salesperson may not be the most desirable (for example, the firm might end up with price-sensitive or brand-switchers). The results suggest that the firm could be better off by differentiating quota goals by segment types (e.g., new customers versus retaining existing customers). Such an approach would align the focus of salespeople to be more in line with the long-term goals of the firm.

This paper also presents implications for contest-based compensation schemes. For instance, a firm that employs a severe contest where there are only very few winners is likely to face a collective behavior that is risk seeking. On the other hand, in a case where most contestants are declared as winners, the firm sponsoring the contest would face behavior that is risk averse. Whether the resulting behavior is favorable or detrimental to the objectives of the firm would further depend on the context in which the firm is operating. For example, the firm might either be engaged in an aggressive contest over market share with other firms or it might be engaged in protecting its presence in a market.

While this paper discusses the context of salesforce compensation, the implications of contest designs can be generalized to many other situations where the payoffs are based on relative performance. For example, in the new product development process, some market characteristics such as the length of the product life cycle or
competitive behavior may require portfolios of high-risk R&D projects. The firm should set up incentive structures so that the decisions made by the product managers are compatible with the objectives of the firm. At a broader level, similar implications hold for tournaments for promotion to a limited number of higher management positions in organizations, management of portfolio managers who compete for the substantial rewards associated with being in list of top few mutual funds, and so on.

It is important to point out that our results do not derive from any violation of standard expected utility theory as suggested in some studies (Ross 1991). Our results show that the source of such risk behavior is the jumps in the payoffs rather than a violation of the standard expected utility paradigm.

Finally, we acknowledge that situations may exist where the fraction of compensation that is quota-based or contest-based might be small relative to the total compensation. Our model would still predict the results shown in this paper, although we did not test for this specifically in the experimental studies. It might be worth noting that, for an individual participant, the utility associated with exceeding a quota or winning in a contest might be dependent not just on the monetary rewards (which, in some cases, might be insignificant relative to total compensation) but also on whether one is regarded as a “loser” or a “winner” in an organization (thus enhancing the impact of exceeding a quota or winning in a contest).

Also, the question of how high or low the quota levels or the proportion of winners in a contest should be set remains to be explored. It appears to us that there can be no general rule, since different firms may have different objectives (i.e., collective risk seeking or collective risk aversion may be desirable). Clearly, if a firm would like to dampen the part of risk behavior that is induced by the compensation schemes, setting a
quota level or the proportion of winners in such a way so that roughly 50% of the participants are likely to succeed might do so.

Another interesting issue to be explored concerns the use of quotas versus rank-order contests. Determining the proportion of participants likely to succeed in a quota-based plan requires an assessment of the performance distributions by the firm even if all participants are considered to be equally able, whereas that is not necessary with rank-order contests. In this respect, contests are likely to reduce the uncertainty regarding the total compensation costs faced by the firm. However, a quota-based scheme might have other advantages. We leave this issue for future research.
References


