INVENTORY CONTROL IN HYBRID SYSTEMS WITH REMANUFACTURING

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Abstract

This paper is on production planning and inventory control in systems where manufacturing and remanufacturing operations occur simultaneously. Typical for these *hybrid* systems is, that both the output of the manufacturing process and the output of the remanufacturing process can be used to fulfil customer demands. Here, we consider a relatively simple hybrid system, related to a single component durable product. For this system, we present a methodology to analyse a PUSH control strategy (in which all returned products are remanufactured as early as possible) and a PULL control strategy (in which all returned products are remanufactured as late as convenient). The main contributions of this paper are *(i)* to compare traditional systems without remanufacturing to PUSH and to PULL controlled systems with remanufacturing, and *(ii)* to derive managerial insights into the inventory related effects of remanufacturing.

Keywords: Production planning and inventory control, manufacturing, remanufacturing, statistical re-order point models, computational experiments.

1 Introduction

Our research in the area of production planning and inventory control with remanufacturing was initiated by a consulting project which we carried out for a large U.S. manufacturer of photocopiers (see Thierry et al., 1995). The manufacturer had developed a prototype of a new generation of photocopiers, which differed from older generations since some modules stemming from used photocopiers were re-usable in new photocopiers. This process, in which used components (modules) are processed to satisfy exactly the same quality and other standards as new components is named *remanufacturing*.

The main motivations for the manufacturer to develop copiers with remanufacturable modules were the (anticipation on) environmental laws that (will) apply in many European and other countries. These laws make product manufacturers responsible for the collection and further handling of their products and packaging materials after customer usage. Furthermore, in the near future it is expected that environmental laws will be tightened in many countries, forcing manufacturers to design products and production processes such that waste is limited and/or a significant percentage of product components and raw materials is re-used. Another incentive to remanufacture products is the existence of new technologies that enable manufacturers to design products and production processes such that remanufacturing becomes cost effective. Finally, a last but important motivation to apply remanufacturing is the opportunity to attract more customers due to the 'environment-friendly' image of remanufacturing companies.

The hybrid production and inventory system that has been implemented at the (re)manufacturer of photocopiers consists of four main processes. Upon return from the customer, used photocopiers first enter the *disassembly* process, in which inspection, cleaning and disassembly operations take place. After disassembly, the disassembled modules enter a quality test. Modules satisfying the quality requirements for remanufacturing enter the *remanufacturing* process, which consists of repair, upgrading and testing operations. Lower quality modules can be used as spare parts, or they can be recycled. However, if the quality is too low, they are disposed of. Unfortunately, the output of the remanufacturing process may be insufficient to cover all the demands for new modules. Therefore, a *manufacturing* process exists to produce new modules. Finally, in the *assembly* process new and remanufactured modules are assembled to obtain serviceables. The processes, goods-flows, and stocking points of this hybrid system are visualised in Figure 1.

It should be noted that the return flow of used photocopiers was quite uncertain, both in quality and in quantity. Crude estimates could only be made about the quantity of the reusable returns, but not about their timing. As a result, the underlying relation between return and demand flows could not be used to any advantage. In fact, we observed the same situation at a large German remanufacturer of car parts (Van der Laan, 1997).

In the beginning, the above system was controlled by a PUSH strategy, in which all returned modules were remanufactured almost immediately after disassembly and testing. However, the copier manufacturer had the impression that the operating costs of their system could be lowered by the introduction of a control strategy that offers a higher level of coordination between manufacturing and remanufacturing operations. Therefore, they decided to change to a PULL strategy, in which returned modules were remanufactured as late as convenient. This change at the copier (re)manufacturer motivated us to compare PUSH and PULL controlled systems more in detail.

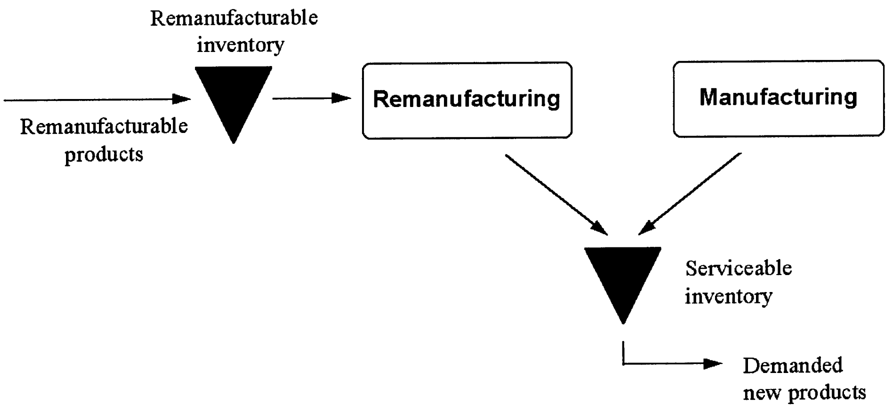


Figure 1. *A hybrid system with manufacturing and remanufacturing operations, and stocking points for remanufacturables and serviceables.*

Production planning and inventory control in hybrid systems is the central issue in this paper. The remainder of this paper is organised as follows. First, we present in Section 2 a literature review on this issue. Second, we introduce in Section 3 two control strategies, based on PUSH and PULL concepts respectively. The control strategies rely on the control strategy proposed by Muckstadt and Isaac (1981), but the system assumptions under which our strategies apply are more general. Third, we outline in Section 3 a new and exact methodology for the mathematical analysis of these control strategies. Fourth, we present in Section 4 a numerical study in which traditional systems without remanufacturing are compared to systems with remanufacturing, and in which PUSH control is compared to PULL control. Based on the results of the numerical study we also indicate in Section 4 some actions that management could take to reduce the operating costs in hybrid systems. Finally, Section 5 presents our conclusions and directions for future research.

2 Literature review

In the literature on production planning and inventory control many papers have appeared that consider the simultaneous occurrence of product returns and product demands. However, most of these articles relate to *repair* systems (see Nahmias (1981) and Cho and Parlar (1991)), in which repair centers repair defective products to working order.

Common assumptions in these systems are that product demands and product returns at the repair centers are completely dependent (i.e., every return of a defective product automatically generates a demand for a working order product), and that every defective product is repairable. Since the number of customers in the system is assumed to be constant over time, the total number of products in the system (i.e., the number of products at working order plus the number of products in repair) is constant. Consequently, there is no need to manufacture or to procure outside new products. Since in hybrid systems proper *coordination* between manufacturing operations and remanufacturing operations is one of the central issues, repair models are in these systems only of limited applicability.

Contrary to the number of publications involving repair systems, the number of publications on planning and control models for hybrid systems is rather limited. To structure the literature review, we distinguish between *periodic review models* in which the system status is reviewed at discrete time periods, and *continuous review models*, in which the system status is continuously reviewed. Furthermore, we distinguish between PUSH control and PULL control.

With PUSH control the timing of the *remanufacturing* operations is completely return driven: as soon as sufficient returned products are in remanufacturable inventory, these products are batched and *pushed* into the remanufacturing process. The timing of the remanufacturing operations under PULL control depends on a *composite* of returns, future expected demands, and inventory positions. Informally, under PUSH control remanufacturing operations are scheduled as early as possible, whereas under PULL control they are scheduled as late as convenient. In both strategies the timing of *manufacturing* operations is based on the serviceable inventory position.

Periodic review models

The first model in this category was proposed by Simpson (1978). It assumes stochastic and mutually dependent demands and returns. Remanufacturable products are either remanufactured or disposed of if they are not needed. Outside procurements satisfy the demands that can not be fulfilled from product returns. The timing and lotsizing of disposal, remanufacturing and outside procurements operations is controlled by a PULL-strategy. The cost function to be minimized consists of variable remanufacturing and outside procurement costs, inventory holding costs for remanufacturables and serviceables, backordering costs, and disposal costs. Limitations of this model are, that remanufacturing and outside procurement lead-times are assumed to be zero, and fixed manufacturing and outside procurement costs are not taken into account. Recently, an extension of this model that accounts for non-zero lead-times has been proposed by Inderfurth (1996).

Kelle and Silver (1989) formulate a model which differs from Simpson's model in that demand and return processes are totally independent, all remanufacturable products are remanufactured (i.e., no disposal occurs), and remanufacturing is controlled by a PUSH-strategy. Furthermore, the cost function includes fixed outside procurement costs, and service is modelled in terms of a service level constraint instead of backordering costs.

The above models all relate to a single component product. Brayman (1992) and Flapper (1994) discuss the difficulties that occur in more complex hybrid systems, where products consist of multiple components. Finally, Guide et al. (1996) present a simulation study to indicate some of the effects that occur in MRP systems with remanufacturing.

Continuous review models

The first continuous review model was proposed by Heyman (1977). It applies to a situation with stochastic uncorrelated demands and returns. Every returned product is either disposed of immediately, or immediately remanufactured. The control policy is a single-parameter (s_d) PUSH-strategy. The parameter s_d is the serviceable inventory level at which returned products are disposed of instead of being remanufactured. As in the discrete-time models, a limitation of this model is that remanufacturing and outside procurement lead-times are zero.

Muckstadt and Isaac (1981) consider a system which differs from that of Heyman. The most important differences are that it applies to a situation with uncertain remanufacturing lead-times, finite remanufacturing capacities, and non-zero outside procurement lead-times. Furthermore, fixed outside procurement costs and backordering costs are included in the cost function. On the other hand, fixed remanufacturing costs are disregarded, and the option of product disposal does not exist. The system is controlled by a two parameter (s_p, Q_p) PUSH-strategy, where s_p is the inventory level at which an outside procurement ordering of size Q_p is placed. Extensions of the Muckstadt and Isaac model to include the disposal of returned products have been studied by Van der Laan et al. (1994). A deterministic model that includes the disposal option has been studied by Richter (1994).

Alternative models that may serve as a starting point in hybrid systems are the cash-balancing models. These models consider a *local* cash of a bank with incoming money flows relating to customer deposits (returns), and outgoing money flows, relating to customer withdrawals (demands). To satisfy the customer demands adequately, the possibility exists to *increase* the cash-level of the local cash by ordering money from the central bank (outside procurement). If the cash-level of the local cash becomes too high, it can be *decreased* by transferring money to the central bank (disposals). The objective in these models is, to determine the timing and sizing of the cash transactions, such that the sum of fixed and variable transaction costs, backordering costs, and interest costs related to the local cash are minimized.

Constantinides and Richard (1978) pointed out that under particular conditions the optimal control policy has the following four parameter structure: if the inventory level at the local cash becomes less than s_p, a procurement order is placed at the central bank to raise the local cash level to S_p. If the local cash level exceeds s_d, the local cash level is reduced to S_d by transferring money back to the central bank. An important limitation of the cash-balancing models is the absence of a real remanufacturing process: every returned product (money) is instantaneously added to the serviceable inventory (local cash), i.e., remanufacturing costs and lead-times are zero. For an extensive overview of cash-balancing models we refer to Inderfurth (1982).

Finally, inventory models in which outside procurements may be placed at two different suppliers (see e.g. Moinzadeh and Nahmias, 1988) may serve as a starting point for further research in hybrid systems. In these models one supplier can be viewed as the manufacturing source, the other as the remanufacturing source. However, the traditional 'two supplier models' must be modified to account properly for the fact that only limited control exists over the remanufacturing source, since the ability to get orders delivered from the remanufacturing source depends on the availability of returned products in the remanufacturable inventory.

Positioning of our strategies

The two continuous review PUSH and PULL strategies that will be considered in the sequel of this paper rely on the (s_p, Q_p) PUSH-strategy proposed by Muckstadt and Isaac (1981). However, to analyse several aspects that we observed in practice as being relevant for hybrid systems, we extend the system assumptions of Muckstadt and Isaac.

First, to investigate the influence of demand and return variabilities and correlations between the timing of returns and demands, we consider correlated Coxian-2 distributed demand and return inter-occurrence times in addition to uncorrelated exponential ones. Note that the only correlation that we study in this paper is the one resulting from product replacements. Keeping the situation at the copier remanufacturer in mind, we assume that the dependence relation between a demand occurrence and its associated product return at some time in the future is unknown and therefore cannot be used to any advantage. Therefore, we assume the two processes to be independent, which in fact is a very common assumption in the remanufacturing literature.

Second, to investigate the influence of a more general cost structure, we allow for non-zero fixed remanufacturing costs and for separate holding costs for remanufacturables and serviceables. Third, to investigate the effect of lead-times we assume a deterministic manufacturing lead-time and a deterministic remanufacturing lead-time, rather than a deterministic manufacturing lead-time and stochastic remanufacturing lead-time resulting from limited remanufacturing capacity.

Finally, the procedure that Muckstadt and Isaac propose to calculate the total expected costs under the (s_p, Q_p) strategy is approximative. The procedures that we present here are exact.

3 System assumptions and control strategies

In the sequel we study a single-product hybrid system. Such systems occur in practice for copier modules, car parts, etc. For this system we define and numerically compare continuous review PUSH and PULL control strategies. As stated before, our main motivation to consider these strategies was, that variants of these strategies have been implemented at the (re)manufacturer of photocopiers. Other motivations to study these strategies are that they are not too complex, both from a practical (implementation) point of view and from a mathematical point of view[1]. The specific assumptions regarding the system that we consider are further outlined in Section 3.1. The notation and the cost function that we use in the remainder of this paper is further specified in Section 3.2. A new methodology for the analysis of the PUSH and PULL strategies and its relation to the existing literature is outlined in Sections 3.3 and 3.4 respectively. The need for an enumerative procedure to search for optimal strategy control parameters is briefly motivated in Section 3.5. Finally, to further investigate the effects of specific assumptions regarding process variables and system characteristics, we explain in Section 3.6 how the analysis

[1]Inderfurth (1996) derived for a class of periodic review models with manufacturing and remanufacturing operations the structure of the optimal control policies. Preliminary results indicate that even under more restrictive assumptions than ours this structure may become too complex to be implemented in practice.

is extended to deal with more general assumptions.

3.1 System assumptions

The system that we consider here is a simplification of the system implemented at the (re)manufacturer of photocopiers (see Section 1 and Figure 1), mainly because we assume that each end-product consist of a single module only. Consequently, assembly operations to assemble remanufactured components with new components need not be modelled. Regarding the processes, goods-flows, and stocking points we make the following assumptions:

- *The remanufacturing process.* All returned modules are remanufactured (i.e., disposals do not occur). The remanufacturing process has unlimited capacity and the remanufacturing lead-time is L_r. Fixed remanufacturing set-up costs are c_r^f per batch of remanufactured modules, and variable remanufacturing costs are c_r^v per module. After remanufacturing the remanufactured modules enter the serviceable inventory.

- *The manufacturing process.* New modules are manufactured. Raw materials are procured from suppliers and for simplicity we assume that raw materials arrive just-in-time, i.e., no raw material inventory is kept. The manufacturing costs consist of a fixed component of c_m^f per batch of manufactured modules, and a variable component of c_m^v per module. Manufacturing capacity is unlimited, and the manufacturing lead-time is L_m. Manufactured modules enter the serviceable inventory.

- *Stocking points.* There exist two infinite capacity stocking points in the system, one to keep remanufacturable inventory and one to keep serviceable inventory. The holding costs in the remanufacturable (serviceable) inventory are c_r^h (c_s^h) per module per time-unit. In most practical situations $c_r^h < c_s^h$, since remanufacturables represent a lower value than serviceables as no value has been added yet to modules stored in remanufacturable inventory. Furthermore, notice that in serviceable inventory *all* modules have identical inventory holding costs of c_s^h, independently of whether they were manufactured or remanufactured. Actually, this holding cost rate is some weighed average of the holding cost rates for manufactured and remanufactured products, depending on the return rate (Van der Laan, 1997). Here, for simplicity, we assume the holding cost rate for serviceables to be reflected by the single parameter c_s^h.

• *Demands, returns, and backorders.* To evaluate the influence of (uncertainties in) demands and returns on system performance, we assume that demands and returns have *Coxian-2* distributed inter-occurrence times. The average time between two subsequent module returns (demands) is $\frac{1}{\lambda_R}$ ($\frac{1}{\lambda_D}$). Furthermore, the return intensity λ_R is less than the demand intensity λ_D. The correlation between returns and demands due to product replacements is expressed by the parameter ρ_{RD}. The timing of a product return is assumed to be independent of the timing of its original demand. Finally, demands that can not be fulfilled immediately are backordered.

3.2 System notation and costs

Table 1 lists the notation that will be used in the remainder of this paper[2]. Moreover, for each of the time-dependent variables that appear in Table 1 (say $V_1(t)$ and $V_2(t, t+\delta)$) we define the long-run average ($\overline{V}_1$ and $\overline{V}_2$) as,

$$\overline{V}_1 = \lim_{t\to\infty} \int_0^t \frac{1}{t} V_1(u)d(u), \text{ and } \overline{V}_2(\delta) = \lim_{t\to\infty} \int_0^\delta \frac{1}{\delta} V_2(t, t+u)d(u), \text{ and } \overline{V}_2 = \lim_{\delta\to 0} \overline{V}_2(\delta).$$

The long-run average system costs per unit of time under control policy (.) are denoted by the function $\overline{C}(.)$. The function $\overline{C}(.)$ reads,

$$\overline{C}(.) = c_s^h \overline{I}_s^{OH} + c_r^h \overline{I}_r^{OH} + c_r^v \overline{E}_r + c_r^f \overline{O}_r + c_m^v \overline{E}_m + c_m^f \overline{O}_m + c_b \overline{B}. \qquad (1)$$

[2]Notation related to the policy parameters is defined throughout the text.

		Notation related to process (.)
$c^v_.$	=	variable processing and material costs per module
$c^f_.$	=	fixed set-up costs per batch
$L_.$	=	processing lead-time
$E_.(t_0, t_1)$	=	total number of modules that enter process (.) in the time-interval $(t_0, t_1]$
$W_.(t)$	=	total number of modules in work-in-process in (.) at time t
$O_.(t_0, t_1)$	=	total number of ordered batches from process (.) in time-interval $(t_0, t_1]$
		Notation related to stocking points (inventories)
c^h_r	=	inventory holding costs in remanufacturable inventory per module per time-unit
c^h_s	=	inventory holding costs in serviceable inventory per module per time-unit
$I_s(t)$	=	serviceable inventory position at time t. The serviceable inventory position is defined as the on-hand serviceable inventory, plus the number of modules in (re)manufacturing work-in-process, minus the number of modules in backorder at time t
$I^{OH}_s(t)$	=	number of modules in on-hand serviceable inventory at time t
$I^{net}_s(t)$	=	the net serviceable inventory at time t. The net serviceable inventory is defined as the number of modules in on-hand serviceable inventory minus the number of modules in backorder at time t
$I^{OH}_r(t)$	=	number of modules in remanufacturable on-hand inventory at time t
		Notation related to demands, returns, and backorders
λ_D	=	expected number of demanded modules per time-unit (demand intensity)
$D(t_0, t_1)$	=	number of demanded modules in the time-interval $(t_0, t_1]$
cv^2_D	=	squared coefficient of variation in the inter-arrival time of demanded modules per time-unit (demand uncertainty)
λ_R	=	expected number of returned modules per time-unit (return intensity)
cv^2_R	=	squared coefficient of variation in the inter-arrival time of remanufacturable returned modules per time-unit (return uncertainty)
ρ_{RD}	=	probability that a module return instantaneously induces a demand (return-demand correlation coefficient)
$B(t)$	=	number of modules in backorder at time t
c_b	=	backordering costs per module per time-unit

Table 1. *Definition of system notation.*

Remark 1. For ease of explanation the analysis of Section 3.3 and 3.4 applies to exponentially distributed demand and return inter-arrival times, and to uncorrelated demands and returns. Section 3.6 considers the generalisations.

3.3 Analysis of the (s_m, Q_m, Q_r) PUSH-strategy

The operating characteristics of the (s_m, Q_m, Q_r) PUSH-strategy are as follows: as soon as remanufacturable inventory contains Q_r modules, these modules are batched and pushed into the remanufacturing process, reducing remanufacturable inventory to zero, and increasing the serviceable inventory position by Q_r modules. Manufacturing starts whenever the serviceable inventory position $I_s(t)$ drops below the level $s_m + 1$. Manufacturing takes place in batches of Q_m modules. The strategy is visualised in Figure 2.

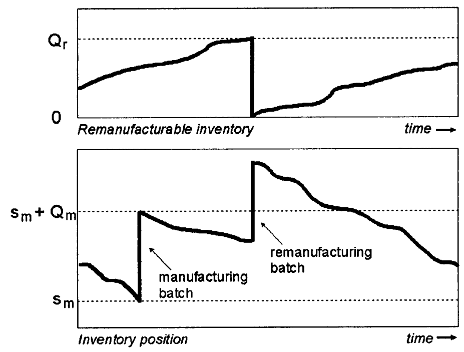


Figure 2. *A schematic representation of the PUSH-strategy.*

To calculate $\overline{C}(s_m, Q_m, Q_r)$ we first determine the long-run average on-hand inventory of serviceables per unit of time. To do so, we use the relation $\overline{I}_s^{OH} = \sum_{i_s^{net}>0} i_s^{net} \Pr\{I_s^{net} = i_s^{net}\}$, where $\Pr\{I_s^{net} = i_s^{net}\}$ is the probability that the net-inventory of serviceables equals i_s^{net} in the long-run steady-state situation. To calculate the probability distribution $\Pr\{I_s^{net} = i_s^{net}\}$, we use the following relationship (see also Table 1),

$$I_s^{net}(t) = I_s(t) - W_m(t) - W_r(t). \tag{2}$$

The distributions of $W_m(t)$ and $W_r(t)$ are difficult to evaluate, so we rewrite (2) as,

$$I_s^{net}(t) = \begin{cases} I_s(t-L_m) + E_r(t-L_m, t-L_r) - D(t-L_m, t), & L_r \leq L_m \\ I_s(t-L_r) + E_m(t-L_r, t-L_m) - D(t-L_r, t), & L_r > L_m \end{cases} . \tag{3}$$

To explain (3), we first consider the case $L_r \leq L_m$. In this case, the net serviceable inventory at time t equals *(i)* the net serviceable inventory at time $t - L_m$ plus *(ii)* the remanufacturing work-in-process at time $t - L_m$ (which arrive at or before t since $L_r \leq L_m$) plus *(iii)* the number of modules in manufacturing work-in-process at $t - L_m$ (which arrive in net serviceable inventory at or before time t) plus *(iv)* the number of modules that enter remanufacturing in the time-interval $(t - L_m, t - L_r]$ (which will have entered serviceable inventory at time t) minus *(v)* the demands in the interval $(t - L_m, t]$, i.e.,

$$\begin{aligned} I_s^{net}(t) &= I_s^{net}(t-L_m) + W_r(t-L_m) + W_m(t-L_m) \\ &\quad + E_r(t-L_m, t-L_r) - D(t-L_m, t) \end{aligned} \tag{4}$$

Substitution of the right-hand side of (2) at time $t - L_m$ in (4) then yields (3) for the case $L_r \leq L_m$. For the case $L_r > L_m$ analogous arguments can be used to derive (3).

Next, we further evaluate (3) to enable numerical analysis. First, we consider the case $L_r \leq L_m$. For this case it can be verified that $E_r(t-L_m, t-L_r)$ is negatively correlated with $I_s(t-L_m)$ since a low (high) number of modules that enter the remanufacturing process in the interval $(t-L_m, t-L_r]$ relates to a relatively high (low) serviceable inventory position at time $t - L_m$. Furthermore, the number of modules that may enter the remanufacturing process in the interval $(t - L_m, t - L_r]$, denoted by $E_r(t - L_m, t - L_r)$, is correlated with the number of modules that are available in remanufacturable on-hand inventory at the beginning of the interval, denoted by $I_r^{OH}(t-L_m)$. Taking into account these correlations, we obtain the limiting probability distribution $\Pr\{I_s^{net} = i_s^{net}\}$ using the relation,

$$\Pr\{I_s^{net} = i_s^{net}\} =$$

$$\lim_{t\to\infty} \sum_{\Omega_1} \Pr\{I_s(t-L_m) = i_s, I_r^{OH}(t-L_m) = i_r^{OH}, E_r(t-L_m, t-L_r) = e_r, D(t-L_m, t) = d\} =$$

$$\lim_{t\to\infty}\sum_{\Omega_1} \begin{array}{l}\Pr\{E_r(t-L_m,t-L_r)=e_r|I_s(t-L_m)=i_s, I_r^{OH}(t-L_m)=i_r^{OH}\}\times \\ \Pr\{I_s(t-L_m)=i_s, I_r^{OH}(t-L_m)=i_r^{OH}\}\times\Pr\{D(t-L_m,t)=d\},\end{array} \tag{5}$$

where $\Omega_1 = \{(i_s, i_r^{OH}, e_r, d)|i_s + e_r - d = i_s^{net}\}$.

The procedure to calculate the probability $\Pr\{E_r(t-L_m, t-L_r) = e_r|I_s(t-L_m) = i_s, I_r^{OH}(t-L_m) = i_r^{OH}\}$ is new in inventory theory as it requires the analysis of the *transient* system behaviour during the interval $(t-L_m, t-L_r]$. The technical aspects of this analysis are further outlined in Appendix A. The limiting joint probability distribution

$$\pi_1(i_s, i_r^{OH}) = \lim_{t\to\infty} \Pr\{I_s(t-L_m) = i_s, I_r^{OH}(t-L_m) = i_r^{OH}\}$$

is calculated using a continuous time Markov-chain model $\mathcal{M}_1$, with two-dimensional state variable $X_1(t) = \{I_s(t), I_r^{OH}(t)|t > 0\}$ and state-space $\mathcal{S}_1 = \{s_m+1, \ldots, \infty\} \times \{0, \ldots, Q_r - 1\}$. The transition rates $\nu_{s^{(1)},s^{(2)}}$ related to a transition from state $s^{(1)}$ to state $s^{(2)}$ is defined as,

$$\begin{array}{llll}
\nu_{(i_s,i_r^{OH}),(i_s,i_r^{OH}+1)} = \lambda_R, & i_r^{OH} < Q_r - 1, & \text{(occurrence of a product return)} \\
\nu_{(i_s,i_r^{OH}),(i_s+Q_r,0)} = \lambda_R, & i_r^{OH} = Q_r - 1, & \text{(occurrence of a remanufacturing ordering)} \\
\nu_{(i_s,i_r^{OH}),(i_s-1,i_r^{OH})} = \lambda_D, & i_s > s_m + 1, & \text{(occurrence of a product demand)} \\
\nu_{(i_s,i_r^{OH}),(s_m+Q_m,i_r^{OH})} = \lambda_D, & i_s = s_m + 1. & \text{(occurrence of a manufacturing ordering)}
\end{array}$$

Finally, the probability $\Pr\{D(t-L_m, t) = d\} = \exp^{-\lambda_D L_m} \dfrac{(\lambda_D L_m)^d}{d!}$.

Next, we consider (3) for the case $L_r > L_m$. Here, $E_m(t-L_r, t-L_m)$ depends on the serviceable inventory position $I_s(t-L_r)$, on the number of modules in on-hand remanufacturable inventory $I_r^{OH}(t-L_r)$, and on the demand $D(t-L_m, t)$. Taking into account these correlations we write analogously to (5),

$$\Pr\{I_s^{net} = i_s^{net}\} =$$
$$\lim_{t\to\infty}\sum_{\Omega_2} \pi_1(i_s, i_r^{OH}) \Pr\{E_m(t-L_r, t-L_m) = e_m, D(t-L_r, t) = d| I_s(t-L_r) = i_s, I_r^{OH}(t-L_r) = i_r^{OH}\}, \tag{6}$$

where $\Omega_2 = \{(i_s, i_r^{OH}, e_m, d) | i_s + e_m - d = i_s^{net}\}$. Again, $\Pr\{E_m(t-L_r, t-L_m) = e_m, D(t-L_r, t) = d | I_s(t-L_r) = i_s, I_r^{OH}(t-L_r) = i_r^{OH}\}$ is obtained by studying the *transient* behaviour of an appropriate continuous-time Markov chain (see Appendix A).

The long-run average number of backorders per unit of time follows from the relation $\overline{B} = -\sum_{i_s^{net}<0} i_s^{net} \Pr\{I_s^{net} = i_s^{net}\}$. The other cost components are straightforward to calculate: $\overline{I}_r^{OH} = \frac{Q_r - 1}{2}$; $\overline{E}_r = \lambda_R$; $\overline{O}_r = \frac{\lambda_R}{Q_r}$; $\overline{E}_m = \lambda_D - \lambda_R$; $\overline{O}_m = \frac{\lambda_D - \lambda_R}{Q_m}$.

3.4 Analysis of the (s_m, Q_m, s_r, S_r) PULL-strategy

We analyse the (s_m, Q_m, s_r, S_r) PULL-strategy in addition to the (s_m, Q_m, Q_r) PUSH-strategy is, that in the PULL-strategy the timing of remanufacturing operations is not based on product returns only, but on a composite of product returns, future expected demands, and inventory positions.

The PULL-strategy is implemented as follows (see Figure 3): as soon as the serviceable inventory position $I_s(t)$ drops below the level $s_r + 1$, it is continuously verified whether sufficient on-hand remanufacturable inventory, $I_r^{OH}(t)$, is available to increase the serviceable inventory position to the level S_r. If sufficient remanufacturable inventory is present, a batch of size $S_r - I_s(t)$ enters the remanufacturing process to be remanufactured. However, when the serviceable inventory position drops below $s_m + 1$ and still insufficient remanufacturable inventory is present to increase the serviceable inventory position to S_r, a manufacturing order of size Q_m is placed to increase the serviceable inventory position. Furthermore, to avoid an unlimited growth of remanufacturable inventory, it is assumed that the inventory position level at which continuous review of the remanufacturable inventory starts is not less than the inventory level at which a manufacturing order is placed, i.e. $s_r \geq s_m$.

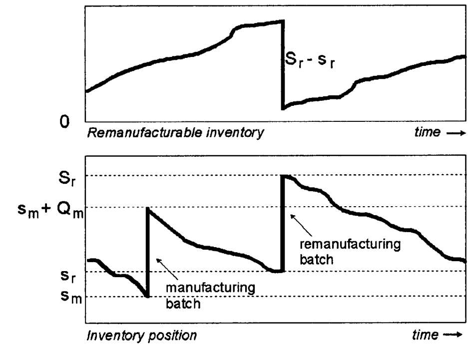


Figure 3. *A schematic representation of the PULL-strategy.*

Remark 2. As an alternative to the order upto level S_r we have also implemented the PULL-strategy with a fixed remanufacturing batch size Q_r. Computational results indicated that the differences between the two implementations are small.

To calculate the components of $\overline{C}(s_m, Q_m, s_r, S_r)$ we first determine the on hand serviceable inventory $\overline{I}_s^{OH}$. Here, we use again the relation $\overline{I}_s^{OH} = \sum_{i_s^{net}>0} i_s^{net} \Pr\{I_s^{net} = i_s^{net}\}$ and (3). The analysis of (3) for this strategy differs from the analysis of (3) for the PUSH-strategy, since other correlations are involved. First, we consider the case $L_r \leq L_m$. In this case, $E_r(t-L_m, t-L_r)$ is correlated with $I_s(t-L_m)$, with $I_r^{OH}(t-L_m)$, and with $D(t-L_m, t)$. Taking into account these correlations, $\Pr\{I_s^{net} = i_s^{net}\}$ is calculated using the relation,

$$\Pr\{I_s^{net} = i_s^{net}\} = \lim_{t\to\infty} \sum_{\Omega_1} \pi_2(i_s, i_r^{OH}) \Pr\{E_r(t-L_m, t-L_r) = e_r, D(t-L_m, t) = d | I_s(t-L_m) = i_s, I_r^{OH}(t-L_m) = i_r^{OH}\} \tag{7}$$

where $\pi_2(i_s, i_r^{OH}) = \lim_{t\to\infty} \Pr\{I_s(t-L_m) = i_s, I_r^{OH}(t-L_m) = i_r^{OH}\}$ is obtained from a continuous time Markov-chain model $\mathcal{M}_2$, with two dimensional state variable $X_2(t) = \{I_s(t), I_r^{OH}(t) | t > 0\}$ and state-space $\mathcal{S}_2 = \{s_m+1, \ldots, \max(s_m+Q_m, S_r)\} \times \{0, \ldots, \infty\}$. The transition rates $\nu_{s^{(1)}, s^{(2)}}$ related to a transition from state $s^{(1)}$ to state $s^{(2)}$ is defined as,

$$\begin{array}{lll}
\nu_{(i_s,i_r^{OH}),(i_s,i_r^{OH}+1)} = \lambda_R, & & i_s > s_r \text{ or } i_r^{OH} < S_r - i_s - 1, \\
 & & \text{(occurrence of a product return)} \\
\nu_{(i_s,i_r^{OH}),(S_r,0)} = \lambda_R, & & i_s \leq s_r \text{ and } i_r^{OH} = S_r - i_s - 1, \\
 & & \text{(occurrence of a remanufacturing order)} \\
\nu_{(i_s,i_r^{OH}),(i_s-1,i_r^{OH})} = \lambda_D, & & \left(i_s > s_m + 1 \text{ and } i_r^{OH} \leq S_r - i_s \right) \text{ or} \\
 & & i_s > s_r + 1, \\
 & & \text{(occurrence of a product demand)} \\
\nu_{(i_s,i_r^{OH}),(S_r,i_r^{OH}-(S_r-s_r))} = \lambda_D, & & i_s = s_r + 1 \text{ and } i_r^{OH} \geq S_r - s_r, \\
 & & \text{(occurrence of a remanufacturing order)} \\
\nu_{(i_s,i_r^{OH}),(S_r,i_r^{OH}-(S_r-s_m-Q_m)} = \lambda_D, & & i_s = s_m + 1 \text{ and } s_r \geq s_m + Q_m \text{ and} \\
 & & S_r - s_m - Q_m \leq i_r^{OH} \leq S_r - s_m, \\
 & \multicolumn{2}{l}{\text{(occurrence of a simultaneous remanufacturing and manufacturing order)}} \\
\nu_{(i_s,i_r^{OH}),(s_m+Q_m,i_r^{OH})} = \lambda_D, & & i_s = s_m + 1 \text{ and } i_r^{OH} < S_r - s_m \text{ and} \\
 & & \left(i_r^{OH} < S_r - s_m - Q_m \text{ or } s_r < s_m + Q_m\right). \\
 & & \text{(occurrence of a manufacturing order)}
\end{array}$$

The calculations required to obtain the probability $\Pr\{E_r(t - L_m, t - L_r) = e_r, D(t - L_m, t) = d | I_s(t - L_m) = i_s, I_r^{OH}(t - L_m) = i_r^{OH}\}$ of (7) are outlined in Appendix A.

Next, we consider (3) for the case $L_r > L_m$. Here, the term $E_m(t - L_r, t - L_m)$ is correlated with $I_s(t-L_r)$, with $I_r^{OH}(t-L_r)$, and with $D(t-L_r, t)$. Consequently, $\Pr\{I_s^{net} = i_s^{net}\}$ can be calculated analogous to (6). The long-run average on-hand remanufacturable inventory equals,

$$\overline{I}_r^{OH} = \sum_{i_s=s_m+1}^{\max(s_m+Q_m,S_r)} \sum_{i_r^{OH}=1}^{\infty} i_r^{OH} \pi_2(i_s, i_r^{OH}).$$

The long-run average number of batch set-ups per unit of time in the remanufacturing process can be obtained by using the Poisson Arrival See Time Averages (PASTA) property (see Wolff, 1982). The calculation of $\overline{O}_r$ proceeds then as follows:

$$\begin{aligned}
\overline{O}_r &= \sum_{i_r^{OH}=S_r-s_r}^{\infty} \pi_2(s_r + 1, i_r^{OH})\lambda_D + \sum_{i_s=s_m+1}^{s_r} \pi_2(i_s, S_r - i_s - 1)\lambda_R + \\
&+ \begin{cases} \displaystyle\sum_{i_r^{OH}=S_r-s_m-Q_m}^{S_r-s_m} \pi_2(s_m + 1, i_r^{OH})\lambda_D, & \text{if } s_r \geq s_m + Q_m \\ 0, & \text{otherwise} \end{cases}.
\end{aligned}$$

The components $\overline{E}_r$, $\overline{E}_m$, $\overline{O}_m$, and $\overline{B}$ are calculated analogous to the PUSH strategy.

3.5 Optimization of the control parameters

In the previous sections we outlined procedures to calculate the functions $\overline{C}(s_m, Q_m, Q_r)$ and $\overline{C}(s_m, Q_m, s_r, S_r)$ for *arbitrary* sets of control parameters. However, to analyze the system behaviour, we are interested in the *particular* set of control parameters under which the cost functions are *minimized*. Unfortunately, a *'nice'* structure of the cost function (such as convexity) which we could exploit to speed up the search procedure seems absent. Therefore, we were committed to an extensive enumerative search to find $\overline{C}^*_{PUSH} = \min_{(s_m,Q_m,Q_r)} \overline{C}(s_m, Q_m, Q_r)$ and $\overline{C}^*_{PULL} = \min_{(s_m,Q_m,s_r,S_r)} \overline{C}(s_m, Q_m, s_r, S_r)$.

3.6 Generalization of system assumptions

To further study the influences of process interactions and process uncertainties on system performance, we investigate the system defined in Section 4.1 under more general assumptions.

• *Uncertainties in returns and demands.* To model uncertainties in the timing of demands and returns more in detail, the assumption of exponentially distributed inter-occurrence times has been generalized to Coxian-2 distributed inter-occurrence times. The Coxian-2 distribution is often used to model inter-occurrence times and it enables to a first and second moment fit of processes, rather than a first moment fit only. The required modifications to calculate $\overline{C}(.)$ under Coxian-2 distributed demand and return inter-occurrence times are outlined in Appendix B.

• *Correlation between returns and demands.* The correlation between returns and demands is modelled by the coefficient ρ_{RD}, which indicates the fraction of module returns that instantaneously creates a demand for a new module to replace the returned module.

The introduction of correlations between returns and demands requires a modification in the calculation of $\overline{C}(s_m, Q_m, Q_r)$ and $\overline{C}(s_m, Q_m, s_r, S_r)$. For further details we refer to Appendix C.

Remark 3. Although the PUSH and the PULL-strategy can be evaluated in the presence of *lead-time uncertainty*, the mathematical analysis to calculate the cost function (1) tends to become very complex then. Therefore, the issue of lead-time uncertainty has been addressed in a separate paper by Van der Laan et al. (1996a). Quality uncertainty is also complex to model. Only in the special case that the return process is Poisson, the testing process has zero lead-time, and the testing outcome is Bernoulli (i.e., with probability p the returned product is remanufacturable, and with probability $1 - p$ it is not), the input distribution of the remanufacturing process is also Poisson (with rate $p\lambda_R$) and the analysis of Sections 3.3 and 3.4 remains applicable.

4 Numerical study

Since it seems technically infeasible to derive analytical results regarding the behaviour of the cost functions, we have set-up a numerical study. The numerical study starts out from a *base-case* scenario. Subsequently, additional scenarios have been generated in which elements from the base-case scenario, such as parameters related to the demand and return processes and cost structure, have been varied.

Base-case scenario

Regarding the characteristics of testing, remanufacturing and manufacturing processes we make the following assumptions:

Process	*fixed costs* ($c^f_.$)	*variable costs* ($c^v_.$)	*lead-times* ($L_.$)
remanufacturing	0	0	2
manufacturing	0	0	2

Inventory holding costs are $c_r^h = 0.5$. for remanufacturables, and $c_s^h = 1$ for serviceables. Backordering costs are $c_b = 50$. Demand and return processes are characterized as follows:

	Returns	*Demands*
inter-occurrence distribution	exponential	exponential
intensity	$\lambda_R = 0.7$	$\lambda_D = 1$
uncertainty	$cv_R^2 = 1$	$cv_D^2 = 1$
correlation	$\rho_{RD} = 0$	

Additional scenario's

Scenario 1 (Figures 4). Scenario 1 is to compare hybrid systems with remanufacturing to traditional systems without remanufacturing. We assume that the traditional systems are controlled by (s, S)-policies with associated costs $\overline{C}^*$.

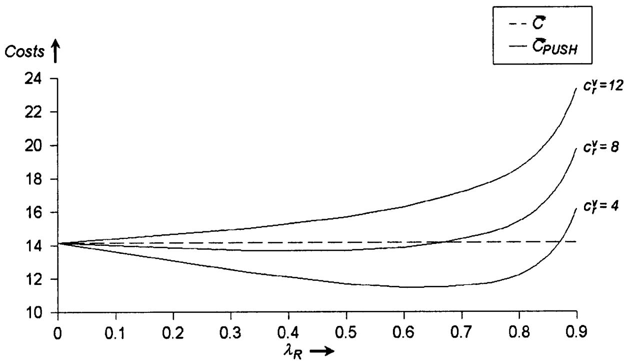
Scenario 2 (Figures 4). This scenario is to study costs at different stages of the product life cycle, represented by different ratios between λ_R and λ_D. The different ratios are obtained by keeping λ_D at the base-case level and varying λ_R between $[0, 0.9]$.

Scenario 3 (Figures 4). This scenario is to compare systems in which variable manufacturing costs ($c_m^v = 10$) are less than variable remanufacturing costs ($c_r^v = 12$) to systems having a reversed cost structure ($c_r^v = 4$ and $c_r^v = 8$).

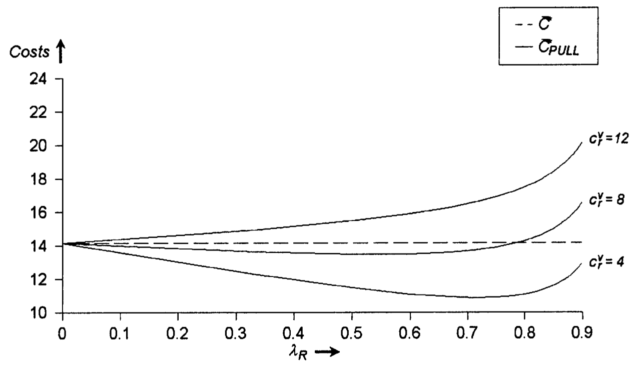
Scenario 4 (Figures 5). To indicate that the valuation of inventories is an important factor in deciding whether a PUSH strategy or a PULL strategy should be implemented, we have varied the remanufacturable inventory holding costs c_r^h between [0,1] at different fixed cost structures ($c_m^f = c_r^f = 0$ in Figure 5a, and $c_m^f = c_r^f = 10$ in Figure 5b).

Scenario 5 (Figures 6). This scenario is to investigate the influence of uncertainties in the timing of product returns (for some special case, the scenario also provides insight into the influence of quality uncertainty. See Remark 3 for further details). For this purpose the squared coefficient of variation of the Coxian-2 distributed return process (cv_R^2) has been varied between $[0.5, 3]$.

Scenario 6 (Figures 7). Here, the effect of correlations between returns and demands are investigated. For this purpose, the correlation coefficient ρ_{RD} has been varied over the interval [0,1]. Note that $\rho_{RD} = 0$ ($\rho_{RD} = 1$) corresponds to the extreme situation with zero (perfect) correlation between returns and demands.



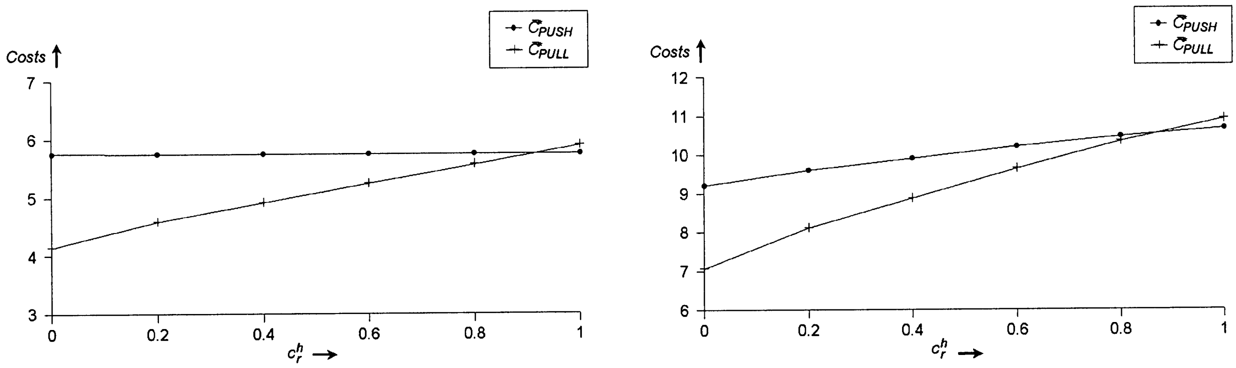
(a)



(b)

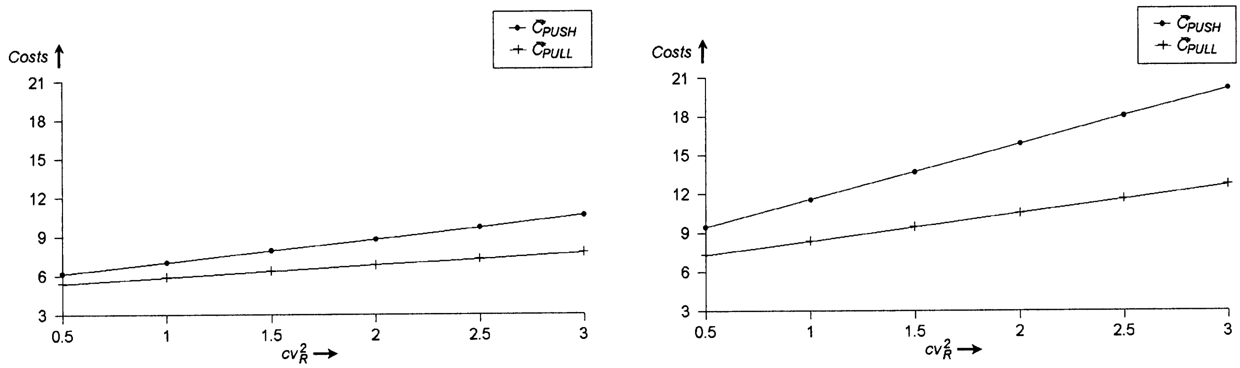
Figure 4a–b. *The relation between total expected costs and return intensity for the situation with remanufacturing and for the situation without remanufacturing. Figure 4a shows the relation for the PUSH-strategy, whereas Figure 4b shows the relation for the PULL-strategy.*

Remark 4. Due to space limitations we have limited our discussion in this paper to the above scenarios. Many alternative scenarios (related to e.g. backordering costs, fixed costs and demand uncertainties) are discussed in Van der Kruk (1995). The influence of lead-times and lead-time uncertainty is investigated in Van der Laan et al. (1998).



(a) *(b)*

Figure 5a–b. *The relation between total expected costs as function of the remanufacturing holding costs, with* $c_m^f = c_r^f = 0$ *(Figure 5a), and with* $c_m^f = c_r^f = 10$ *(Figure 5b).*

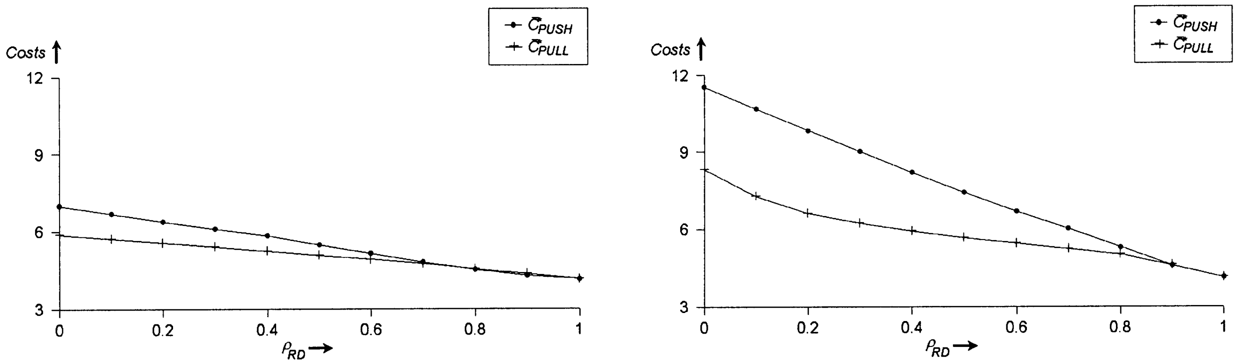


(a) *(b)*

Figure 6a–b. *The relation between total expected costs and return uncertainty for the return intensities* $\lambda_R = 0.8$ *(Figure 6a), and* $\lambda_R = 0.9$ *(Figure 6b).*

4.1 Hybrid vs. traditional systems

The total costs in the hybrid system implemented at the copier (re)manufacturer turned out to be *lower* than in their traditional system without remanufacturing, mainly because the re-use of modules saves material costs. These cost savings make it cheaper to remanufacture a used module than to manufacture a completely new module. However, alternative case-studies and this numerical study have shown that the *'opposite'* cost effect may also occur, i.e., the operating costs in PUSH and PULL controlled systems with remanufacturing may become *higher* than in traditional (s, S) controlled systems without



(a) *(b)*

Figure 7a–b. *The relation between total expected costs and correlation coefficient between returns and demand for the return intensities* $\lambda_R = 0.8$ *(Figure 6a), and* $\lambda_R = 0.9$ *(Figure 6b).*

remanufacturing, even when the variable costs to remanufacture a used module are lower than the variable costs to manufacture a completely new module (see Figures 4, in which all fixed costs are zero).

This effect occurs due to various sources of uncertainty which are absent in traditional manufacturing systems. These uncertainties, to be discussed in more detail in Section 4.3, induce a high variability in the output of the remanufacturing process, and cause in this way an increase in the sum of inventory holding costs and backordering costs. Apparently, the increase in inventory holding costs and backordering costs may dominate cost savings from material re-use.

4.2 PUSH vs. PULL control

In the beginning the hybrid system at the copier (re)manufacturer was controlled by a variant of the PUSH strategy. However, investigations showed that PULL control could be economically favourable, particularly due to savings in inventory holding costs. Therefore, the copier (re)manufacturer decided to change their PUSH controlled system into a PULL controlled system.[3]

The cost savings related to this change could have been expected in advance, since the copier (re)manufacturer values modules in remanufacturable inventory much lower than modules in serviceable inventory[4]. Under such an inventory holding cost structure a strategy in which the serviceable inventory is kept low at the expense of a somewhat higher remanufacturable inventory tends to perform better than a strategy in which most of the stock is kept as serviceable inventory. Indeed, Figures 5 show that in the situation where remanufacturable inventory is valued (almost) as high as serviceable inventory (i.e., $c_s^h \approx c_r^h$), a PUSH-strategy may be economically favourable over a PULL strategy, since a higher serviceable inventory enables to react faster on extreme demand situations, resulting in lower backordering costs.

Nevertheless, although the PULL-strategy may have *economical* advantages over the PUSH-strategy, the PUSH-strategy may from an *organisational* point of view still be preferable, since remanufacturable inventory and serviceable inventory can be controlled independently.

Remark 5. It should be noticed that the cost dominance relation between $\overline{C}_{PUSH}$ and $\overline{C}_{PULL}$ is independent of variable manufacturing or variable remanufacturing costs, since these variable costs are equal under both strategies. Furthermore, experiments have shown that the cost dominance relation is not much influenced by fixed costs (see Figure 5b for an example and Van der Kruk (1995) for a more extensive study) or by the backordering costs (except when the backordering costs become extremely low).

[3]Another advantage of the PULL-strategy occurs when remanufactured components can be used in different final products. Delaying the remanufacturing process then offers a higher flexibility.

[4]The inventory holding costs of serviceable modules were taken proportional to the manufacturing costs of a new module (independently of whether the serviceable module has been manufactured or remanufactured), whereas the inventory holding costs of a remanufacturable module were taken proportional to the difference between the manufacturing costs of a new module and the remanufacturing costs of a returned module.

4.3 Managerial insight

Based on the observations of the numerical study we further provide the following additional insights for managers of remanufacturing companies:

• *From an economic point of view it may be unwise to remanufacture all remanufacturables, even when the return intensity is lower than the demand intensity.* The cost decreases in Figures 4 occur since an increase in the return intensity implies that a larger fraction of the demands can be fulfilled by remanufacturing operations instead of by more expensive manufacturing operations. However, when the return intensity further increases, the variability in the output of the remanufacturing process increases, leading again to a higher sum of inventory holding costs and backordering costs (see also above). The return intensity at which $\overline{C}^*_{(.)}$ reaches its minimum depends on the cost structure and other factors, but most importantly, Figures 4 show that $\overline{C}^*_{(.)}$ may reach its minimum far before the average number of product returns equals the average number of demands. This indicates that remanufacturing companies should at *all* stages of the product life-cycle consider which remanufacturables should actually be remanufactured. If alternative options (such as disposal) exist to handle (part of the) remanufacturables, then these may be economically favourable (see Van der Laan et al. (1997) for an extension of the PUSH and PULL strategies in the situation of product disposals).

• *Remanufacturing companies should attempt to keep the uncertainty in the timing and quality of returned products as low as possible.* Figures 6 show that total system costs increase with increasing cv^2_R, in particular when the return intensity increases. Uncertainties in the number of remanufacturable products are mainly due to two components, i.e., uncertainties in the *timing* of product returns and uncertainties in the *quality* of returned products. A popular instrument to reduce the uncertainty in the timing of product returns are lease contracts with a fixed lease period.

• *Remanufacturing companies should keep track of correlations between product returns and product demands.* Figures 7 shows that when the correlation between product returns and product demands increases, the total system costs decrease. Furthermore, the magnitude of this effect increases with an increasing return intensity. The cost reduction occurs since correlations between returns and demands reduce total system uncertainty. In this way, the sum of inventory holding costs and backordering costs can be reduced. The observed effect stresses the importance of data collection to estimate ρ_{RD}. Both underestimates and overestimates of ρ_{RD} may lead to unnecessary high costs for remanufacturing companies.

5 Conclusions and directions for further research

This paper presents one of the first attempts to analyse the effects of remanufacturing in PUSH and PULL controlled production/inventory systems. An important conclusion is, that efficient planning and control in these systems tends to be more complex than in traditional systems without remanufacturing. Factors that we identified in practice at the manufacturer of photocopiers and in this study to be (partly) responsible for these complexities include *system interactions* (such as the interaction between the output of the manufacturing and remanufacturing processes, and the correlation between demands and returns) and *return uncertainties* (such as the uncertainty in the timing and quality of returned products). Clearly, these factors are not present in traditional systems.

This paper has also shown that management should take the decision to remanufacture only after thorough study, since total expected production and inventory related costs in systems with remanufacturing may become higher than in systems without remanufacturing. Once management has decided to remanufacture, the selection of a suitable control policy in combination with other efficiency improving actions is essential. Examples of such actions include the stimulation of lease contracts instead of regular purchasing contracts (to reduce the uncertainty in the timing of product returns), robust product design, maintenance contracts and diagnostic tools (to reduce the uncertainty in the quality of returned products), and the collection of data on correlations between demands and returns (to reduce total system uncertainty). Finally, the valuation of inventories turns out to be an important factor in deciding between PUSH or PULL control.

From a technical point of view, we conclude from this paper that the analysis of control policies in hybrid systems with stochastic demands and returns may become mathematically complex, even though the strategies are extensions of seemingly straightforward PUSH and PULL concepts. The existence of a 'simple' model and methodology by means of which the effects that have been observed in our numerical study can be proved analytically seem therefore highly questionable. Directions for future research include the search for further strategy improvements, the development of new strategies to include product disposal (see Van der Laan et al., 1994 and 1997), and the study of the complex dependence relation between the demand process and the return process. Finally, the insights obtained from this study will be applied to develop and test control policies for multi-echelon systems with product returns.

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Appendix A: Calculation of conditional probabilities in (5), (6), and (7)

In this appendix we show how the conditional probabilities,

- $\Pr\{E_r(t-L_m, t-L_r) = e_r | I_s(t-L_m) = i_s, I_r^{OH}(t-L_m) = i_r^{OH}\}$,
- $\Pr\{E_m(t-L_r, t-L_m) = e_m, D(t-L_r, t) = d | I_s(t-L_r) = i_s, I_r^{OH}(t-L_r) = i_r^{OH}\}$,
- $\Pr\{E_r(t-L_m, t-L_r) = e_r, D(t-L_m, t) = d | I_s(t-L_m) = i_s, I_r^{OH}(t-L_m) = i_r^{OH}\}$,

that appear respectively in (5), (6), and (7) are calculated.

To carry out the calculations we apply *transient analysis* to evaluate the system state of a Markov-Chain model at time $t = \tau$, given the initial state of the system at time $t = 0$. Transient analysis is based on the technique of uniformization, which enables to transform a continuous-time Markov-Chain model into an equivalent discrete-time Markov-Chain model (see Tijms, 1986).

The conditional probability that the system will be in state $s^{(1)}$ at time $t = \tau$, given that the system was in state $s^{(0)}$ at time $t = 0$ is denoted by $p_{s^{(1)}|s^{(0)}}(\tau)$. This probability is calculated as follows,

$$p_{s^{(1)}|s^{(0)}}(\tau) = \sum_{n=0}^{\infty} \exp^{-\nu\tau} \frac{(\nu\tau)^n}{n!} \overline{p}^{(n)}_{s^{(0)},s^{(1)}} \tag{8}$$

where $\overline{p}^{(n)}_{s^{(0)},s^{(1)}}$ is the n-step discretized transition probability from state s_0 into state s_1, and ν is a suitably chosen constant. The conditional probabilities (8) are then used to calculate the probabilities that appear in (5), (6), and (7).

As an example, we show how the conditional probability $\Pr\{E_r(t-L_m, t-L_r) = e_r | I_s(t-L_m) = i_s, I_r^{OH}(t-L_m) = i_r^{OH}\}$ that appears in (5) is calculated. The underlying Markov-Chain model $X(t) = \left\{I_s(t), I_r^{OH}(t), E_r(0,t) | t > 0\right\}$ has a three dimensional state space $\mathcal{S} = \{s_m+1,\ldots,\infty\} \times \{0,\ldots,Q_r-1\} \times \{0,\ldots,\infty\}$, where $X(t) = (i_s, i_r^{OH}, e_r)$ if at time t the serviceable inventory position is i_s, the number of products in on-hand remanufacturable inventory is i_r^{OH}, and the number of products entering the remanufacturing process in the interval $(0,t)$ is e_r. The transition rates for this model are as follows,

$$\begin{aligned}
\nu_{(i_s,i_r^{OH},e_r),(i_s+1,i_r^{OH}+1,e_r)} &= \lambda_R, \quad i_r^{OH} < Q_r - 1,\\
\nu_{(i_s,i_r^{OH},e_r),(i_s+Q_r,0,e_r+Q_r)} &= \lambda_R, \quad i_r^{OH} = Q_r - 1,\\
\nu_{(i_s,i_r^{OH},e_r),(i_s-1,i_r^{OH},e_r)} &= \lambda_D, \quad i_s > s_m + 1,\\
\nu_{(i_s,i_r^{OH},e_r),(s_m+Q_m,i_r^{OH},e_r)} &= \lambda_D, \quad i_s = s_m + 1.
\end{aligned}$$

Using the uniformization technique enables to calculate,

$$\lim_{t\to\infty} \Pr\left\{E_r(t-L_m,t-L_r) = e_r | I_s(t-L_m) = i_s, I_r^{OH}(t-L_m) = i_r^{OH}\right\} = \\ \sum_{k=s_m+1}^{Q_r-1} \sum_{\ell=0}^{\infty} p_{(k,\ell,e_r)|(i_s,i_r^{OH},0)}(L_m - L_r)$$

where the conditional probabilities $p_{(k,\ell,e_r)|(i_s,i_r^{OH},0)}(L_m - L_r)$ is obtained according to (8) with $\nu = \lambda_R + \lambda_D$.

The probabilities $\Pr\{E_m(t-L_r,t-L_m) = e_m, D(t-L_r,t) = d | I_s(t-L_r) = i_s, I_r^{OH}(t-L_r) = i_r^{OH}$ and $\Pr\{E_r(t-L_m,t-L_r) = e_r, D(t-L_m,t) = d | I_s(t-L_m) = i_s, I_r^{OH}(t-L_m) = i_r^{OH}\}$ are obtained analogously.

Appendix B: Coxian-2 distributed demands and returns

In this appendix we sketch how $\overline{C}(s_m,Q_m,Q_r)$ and $\overline{C}(s_m,Q_m,s_r,S_r)$ can be evaluated under Coxian-2 distributed demand and/or return inter-occurrence times.

A random variable X is Coxian-2 distributed if,

$$X = \begin{cases} X_1 & \text{with probability } p,\\ X_1 + X_2 & \text{with probability } 1-p. \end{cases}$$

where X_1 and X_2 are independent exponentially distributed random variables with parameters γ_1 and γ_2 respectively. Furthermore, $0 \le p \le 1$, and $\gamma_1, \gamma_2 > 0$. It should be noted that the Coxian-2 distribution reduces to an exponential distribution if $p = 1$ and to an Erlang-2 distribution if $p = 0$.

Under a Gamma normalization, an arbitrary distribution function with first moment $E(X)$ and squared coefficient of variation cv_X^2 can be approximated by a Coxian-2 distribution with,

$$\gamma_1 = \frac{2}{EX}\left(1 + \sqrt{\left(\frac{cv_X^2 - \frac{1}{2}}{cv_X^2 + 1}\right)}\right), \ \gamma_2 = \frac{4}{EX} - \gamma_1, \ p = (1 - \gamma_2 EX) + \frac{\gamma_2}{\gamma_1}. \tag{9}$$

and with a third moment equal to a Gamma distribution with first moment $E(X)$ and squared coefficient of variation $cv_X^2 \ge \frac{1}{2}$ (see Tijms 1986, pages 399-400).

The Coxian-2 arrival process can be formulated as a Markov-Chain model $\{Y(t)|t > 0\}$, with state space $\mathcal{S} = \{1, 2\}$. These states can be interpreted as being the states in a closed queueing network with two serial service stations and a single customer. The customer requires service from the first station only with probability p, and from both stations with probability $1-p$. The state $Y(t) = 1$ ($Y(t) = 2$) corresponds to the situation that the customer is being served by station one (two) at time t. The process is cyclical in that after service completion the customer enters the first service station again. The transition rates in this process are as follows,

$$\begin{aligned}
\nu_{1,1} &= p\gamma_1, \\
\nu_{1,2} &= (1-p)\gamma_1, \\
\nu_{2,1} &= \gamma_2.
\end{aligned}$$

The analysis of $\overline{C}(s_m, Q_m, Q_r)$ and $\overline{C}(s_m, Q_m, s_r, S_r)$ under Coxian-2 distributed demand and/or return inter-occurrence times solely requires a modification of the underlying Markov-Chain models $\mathcal{M}_1$ and $\mathcal{M}_2$ respectively. To demonstrate this modification, we adapt $\mathcal{M}_1$ to account for Coxian-2 distributed return inter-occurrence times, with $E(X) = \frac{1}{\lambda_R}$ and $cv_R^2 \geq \frac{1}{2}$. The adapted Markov-Chain model $\mathcal{M}_1^{'}$ has a three-dimensional state variable

$$X_1^{'}(t) = \{(I_s(t), I_r^{OH}(t), Y(t))|t > 0\},$$

with state space

$$\mathcal{S}_1^{'} = \{s_m + 1, \ldots, \infty\} \times \{0, \ldots, Q_r - 1\} \times \{1, 2\}.$$

Note that in $\mathcal{M}_1^{'}$ every return of a remanufacturable product corresponds to a service completion in the Markov-Chain model $\{Y(t)|t > 0\}$. Furthermore, the transition rates in $\mathcal{M}_1^{'}$ are as follows,

$$\begin{aligned}
\nu_{(i_s,i_r^{OH},1),(i_s,i_r^{OH}+1,1)} &= p\gamma_1, && i_r^{OH} < Q_r - 1, \\
\nu_{(i_s,i_r^{OH},1),(i_s,i_r^{OH},2)} &= (1-p)\gamma_1, && \\
\nu_{(i_s,i_r^{OH},2),(i_s,i_r^{OH}+1,1)} &= \gamma_2, && i_r^{OH} < Q_r - 1, \\
\nu_{(i_s,i_r^{OH},1),(i_s+Q_r,0,1)} &= p\gamma_1, && i_r^{OH} = Q_r - 1, \\
\nu_{(i_s,i_r^{OH},2),(i_s+Q_r,0,1)} &= \gamma_2, && i_r^{OH} = Q_r - 1, \\
\nu_{(i_s,i_r^{OH},y),(i_s-1,i_r^{OH},y)} &= \lambda_D, && i_s > s_m + 1, \\
\nu_{(i_s,i_r^{OH},y),(s_m+Q_m,i_r^{OH},y)} &= \lambda_D, && i_s = s_m + 1,
\end{aligned}$$

where γ_1, γ_2, and p are calculated according to (11). The modifications required to model Coxian-2 distributed demands are analogous. The long-run average number of batch set-ups in the remanufacturing process per unit of time is obtained as,

$$\begin{aligned}
\overline{O}_r &= \sum_{i_r^{OH}=S_r-s_r}^{\infty} \sum_{y=1}^{2} \pi_2(s_r + 1, i_r^{OH}, y)\lambda_D \\
&+ \sum_{i_s=s_m+1}^{s_r} \{\pi_2(i_s, S_r - i_s - 1, 1)p\gamma_1 + \pi_2(i_s, S_r - i_s - 1, 2)\gamma_2\} + \\
&+ \begin{cases} \sum_{i_r^{OH}=S_r-s_m-Q_m}^{S_r-s_m} \sum_{y=1}^{2} \pi_2(s_m + 1, i_r^{OH}, y)\lambda_D, & \text{if } s_r \geq s_m + Q_m \\ 0, & \text{otherwise} \end{cases}.
\end{aligned}$$

Appendix C: Correlation between returns and demands

Correlations between the timing of return and demand occurrences are modelled by modifying the Markov-Chain model $\mathcal{M}_1$ for the PUSH-strategy and the Markov-Chain model $\mathcal{M}_2$ for the PULL-strategy. As an example we extend the PUSH-strategy for the situation with correlated returns and demands. The modified Markov-Chain model $\mathcal{M}_1''$ has a two-dimensional state-variable $X_1''(t) = \{I_s(t), I_r^{OH}(t)\}$ and a state space $\mathcal{S}_1$. The transition rates of $\mathcal{M}_1''$ are as follows,

$$\begin{array}{llll}
\nu_{(i_s,i_r^{OH}),(i_s,i_r^{OH}+1)} & = (1-\rho_{RD})\lambda_R, & i_r^{OH} < Q_r - 1, & \\
\nu_{(i_s,i_r^{OH}),(i_s+Q_r,0)} & = (1-\rho_{RD})\lambda_R, & i_r^{OH} = Q_r - 1, & \\
\nu_{(i_s,i_r^{OH}),(i_s-1,i_r^{OH}+1)} & = \rho_{RD}\lambda_R, & i_s > s_m + 1, & i_r^{OH} < Q_r - 1, \\
\nu_{(i_s,i_r^{OH}),(i_s+Q_r-1,0)} & = \rho_{RD}\lambda_R, & i_s > s_m + 1, & i_r^{OH} = Q_r - 1, \\
\nu_{(i_s,i_r^{OH}),(s_m+Q_m,i_r^{OH}+1)} & = \rho_{RD}\lambda_R, & i_s = s_m + 1, & i_r^{OH} < Q_r - 1, \\
\nu_{(i_s,i_r^{OH}),(s_m+Q_r,0)} & = \rho_{RD}\lambda_R, & i_s = s_m + 1, & i_r^{OH} = Q_r - 1, \\
\nu_{(i_s,i_r^{OH}),(i_s-1,i_r^{OH})} & = \lambda_D - \rho_{RD}\lambda_R, & i_s > s_m + 1, & \\
\nu_{(i_s,i_r^{OH}),(s_m+Q_m,i_r^{OH})} & = \lambda_D - \rho_{RD}\lambda_R, & i_s = s_m + 1. &
\end{array}$$

Modification of $\mathcal{M}_2$ for correlations between returns and demands proceeds analogous.