On Growth in Disequilibrium

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Abstract

This paper argues that models based on the assumption of growth along an optimal path *in static equilibrium* are inherently self-contradictory, since the mechanisms that drive economic growth are fundamentally dependent on disequilibrium. Moreover, there is compelling anecdotal evidence that the economy is not in equilibrium now. The new feature of the present analysis is that it is possible to formulate `endogenous growth' models for an economy with the usual constant returns to scale but without the equilibrium assumption that marginal productivities of capital, labor and resource inputs must be equal to factor shares in the national accounts. The reformulated model utilizes a more general production function with variable elasticities of output with respect to factors. Such a production function is consistent with many observed phenomena, including the prevalence of `double dividends'.

Competitive equilibrium: Concept vs. reality

The intellectual background for this paper is the idea of economic equilibrium. It is worth recapitulation because the purpose of this paper is to discuss an alternative. The standard notion goes back at least to Adam Smith's first articulation of the idea of an "invisible hand", which supposedly assures an efficient allocation of resources through the operation of a price mechanism in free competitive markets. This seminal idea was successively refined by Ricardo, Say, Walras, Jevons, Edgeworth, Wicksell, Pareto and others in the 19th century.

Walras postulated a unique *competitive equilibrium*, namely a set of prices such that supply would exactly balance demand in each market (i.e. for each good or service), including labor. Walras also postulated a kind of auction process, called *tâtonnement*, by means of which the equilibrium state would be reached spontaneously, assuming each market actor was always fully informed by a sort of super auctioneer about all transaction prices. Even though Walras was not able to prove this conjecture, it was widely accepted and had an enormous influence on subsequent developments in economic theory.

It was recognized long ago that one of the keys to the existence of competitive equilibrium is the universality of a property known as *declining marginal utility of consumption* for all consumers and *declining marginal productivity of capital* for all producers. The general acceptance of these properties became known as the "marginal revolution" in economics, dating back to Jevons and Walras c. 1870. It has enabled mathematicians to study the properties of markets and to derive general theorems about their behavior.

The first mathematical proof-of-existence of an equilibrium (in the above sense) depended on some restrictive simplifying assumptions.[1] Among them were the following:

1. The supply side of a competitive equilibrium consists of a large number of small independent producers (no **monopolies** or **oligopolies**); the demand side consists of a number of small independent consumers. None of the producers or consumers is able to influence prices or aggregate production levels.
2. Every commodity or service in the economy is produced from labor or from other commodities and services produced by the economy and sold in the market. By implication, no resources are taken from the environment and the environment provides no unpriced services (such as waste disposal). In other words, the economy is **closed**, and prices are unaffected by anything outside the market. Moreover, the closed economy produces only **goods**; there are no "bads" or **externalities**.
3. Each agent in the market is a perfectly rational utility maximizer. He is consciously aware of his own preferences and can instantaneously and consistently decide how he will re-allocate his income among all possible goods/service, given any

change in market prices.

4. Each agent in the market is perfectly informed about the prices and characteristics of all products and services offered for sale at all times. If any change *were* to occur (e.g., the introduction of a new product), it is assumed that information about it is instantaneous and automatically available to all agents.

The most important theorems about competitive equilibrium are as follows: (1) all product markets, and labor markets, "clear" in the sense that supply and demand are perfectly balanced and (2) once an equilibrium state has been reached, no transaction can improve the position of one actor without hurting that of another or others. The first attribute — market clearing — makes the competitive equilibrium *efficient*: there are no wasted or unutilized resources, either of capital or labor. The second important attribute of competitive equilibrium is known as "Pareto optimality", associated with the name of Vilfredo Pareto. In modern game-theoretic terms we would say that, in a Pareto optimum state, all economic contests are at best "zero sum", meaning that the sum of all gains and losses add up to zero (or less).[2]

The Pareto-optimal equilibrium state of an economy is, by definition, static. It is a state in which all agents in the system are as well-off as they can be, in the sense that no agent can improve its welfare/utility by voluntarily exchanging any goods or services with others. As a tool of analysis — a point of departure for theory — this concept is invaluable. It is, perhaps, the first of very few areas where truly rigorous analysis has been possible in economics (game theory being another), thereby differentiating economics from "softer" social sciences.

So much for the good news. The bad news is that the economy is never actually in equilibrium. In fact, to anticipate the conclusions of this paper, the real economy may be rather far from Walrasian equilibrium. (The analogy with Prigogine's non-equilibrium thermodynamics, which nevertheless permits self-organized stable states far from equilibrium, is irresistible.) Joan Robinson, one of the true philosophers of economics, has said:

> "The concept of equilibrium is, of course, an indispensable tool of analysis ... But to use the equilibrium concept, one has to keep it in its place, and its place is strictly in the preliminary stages of an analytical argument, *not in the framing of hypotheses to be tested against the facts, for we know perfectly well that we shall not find facts in a state of equilibrium*." [Robinson 1962 p. 78 italics added].

She goes on to note that:

> "Long run equilibrium is a slippery eel. Marshall evidently intended to mean by the long period a horizon which is always at a certain distance in the future, and this is a useful metaphor, but he slips into discussing a position of equilibrium *which is shifted by the very process of approaching it* ... No one would deny that to speak of a tendency toward equilibrium that

> itself shifts the position towards which it is tending is a contradiction in terms ..." [Ibid p.79].

The only production or exchange that occurs in an equilibrium state must be production of services that are "consumed" as they are produced (e.g. food), or to replace goods that are physically used up or depreciated. In a Pareto optimum nobody wants to buy more than he/she buys now, if it means giving up leisure time by working more hours to earn more money. Nobody wants to exchange the goods he has to buy others at the prices offered. In short, in equilibrium everybody is satisfied with the *status quo*, by assumption.

The same problem arises on the supply side. In a competitive static equilibrium system, all producers have perfect information about their own production possibilities and about consumer preferences under all conditions. They have, by assumption, already adopted the optimum technologies.

An important implication of a true competitive Walrasian equilibrium is that there would be no monopolies or oligopolies, so the cost of entry to any market is zero or negligible. Since all firms are assumed to have perfect information with regard to production possibilities, any firm that found an opportunity for making higher-than-average profits would immediately attract price-cutting competitors. Anyhow, no such opportunities for extraordinary profits could exist because each firm and sector has, by assumption, already selected the best available production technology for its product. There are no competitive advantages from scale. There can be no "excess" profits to be made by innovation. Innovation will not occur. In fact, competition among price-takers ensures that prices will inevitably fall to the *marginal* cost of production. Producers in a competitive equilibrium can expect to replace capital depreciation, but no more than that. Producers can never earn a return on equity capital sufficient to finance growth. Equilibrium is static, by definition. The real economy does not resemble this description.

There are many other facts about the real economy that are inconsistent with static equilibrium. For example, markets do not always clear. Both backlogs and shortages occur from time to time. Unemployment is, of course, inconsistent with equilibrium in labor markets.. Businesses do not set prices uniformly on the basis of cost. Enormous differences have been documented in the prices of brand name products, such as drugs, for instance, in different countries. Consumers do not insist on the lowest possible price, especially for domestic goods as compared to foreign imports. For instance, Japanese consumers cheerfully pay two to ten times world prices for commodities like rice, beef and fruit, produced domestically.

Moreover, firms often do not operate on or near the so-called "technology frontier" as they are assumed to do. There is ample evidence of this, some of which I note subsequently. (In fact, this is the source of many "double dividend" possibilities.) Some technologies are "locked in" by a combination of market power, institutional barriers (such as standards), and `returns to adoption' while others that might be potentially superior — can be "locked out" by the same mechanisms [Arthur 1988, 1988a]. The classic example is the QWERTY typewriter keyboard [David 1985]. The British system of weights and measures has been locked in by the US and Britain, while the metric system has been locked out. Driving on the left (or the right) is another case in point. These situations are inconsistent with the assumption of rational utility maximization with perfect information.

Neoclassical growth-in-equilibrium

The basic idea of growth theory since the 19th century is that the main drivers of economic growth are labor and capital accumulation. The labor force is assumed to be (roughly) proportional to population. Beyond this, the relationship is seldom explored more deeply. Land was the only form of capital considered at first, it being a surrogate for all natural resources. Later, the importance of man-made capital had to be recognized and included. Gross output, then, depends on "factors of production", usually identified as labor, capital and — in some models — resource inputs. The output is generated from inputs by a "production function", which may or may not be given explicit mathematical form.[3] The role of the competitive market (i.e. equilibrium) is to ensure that each factor of production is "paid" its appropriate share. This assumption has been nearly universal. However, as argued hereafter, it is probably wrong.

An obscure Russian economist was apparently the first (in 1928) to set forth the famous formula $g = s/v$, where g is the economic growth rate, s is the savings rate and v is the capital/output ratio [Fel'dman 1928]. The formula says that output is proportional to capital stock and that saving (= investment) is proportional to output (or income). It is justified by observed (relative) trendlessness of the two ratios in question. The formula was rediscovered independently, later, by Roy Harrod [Harrod 1939], and Evsey Domar [Domar 1946].[4] It was the basis of the so-called Harrod-Domar (H-D) growth models [Harrod 1948; Domar 1947]. But independent conditions on s and v must be satisfied, to reconcile labor productivity growth at a rate m and labor force growth at a rate r. The key implication of these models was that, to maintain full employment without exhausting the labor supply (and igniting inflation) $mr=s/v=g$.

This condition became known as "the razor's edge" and the implied problem for economic planners was to "walk" on it. This so-called "razor's edge" phenomenon gave rise to much talk of "structural disequilibria" (i.e. bottlenecks) — which was really a defense of central planning. The operation of the market in this model had almost no role, since savings could be voluntary or government enforced. The models also had nothing at all to say about the role of technological progress. An implication of the H-D models is that developing countries should oscillate between prolonged periods of increasing/decreasing unemployment and/or increasing/ decreasing capacity utilization rates. This behavior has not been observed. Kaldor (and others) tried to save the H-D model by endogenizing the relationship between s and v [Kaldor 1956]. Solow pointed out in 1956, however, that the "razor's edge" was actually an artifact of the H-D model structure. This observation, together with weak empirical support and increasingly strong criticism of central planning as a philosophy, finally cut the intellectual foundation from under the central planning approach.

In any case, empirical work in the 1950s drastically reduced the apparent role of savings and investment *per se* [e.g. Fabricant 1954; Abramovitz 1956; Schmookler 1966; Kendrick 1956]. By the mid 1950s it had became clear that savings and capital accumulation *per se* could not account for very much of the *per capita* growth in the US economy over the previous century. Fabricant's estimate of the capital contribution to growth was 10%, for instance. The remainder was an unexplained residual that was termed (for convenience) "technical progress".

In 1957 Solow showed a way to characterize "neutral technical progress" in an aggregate production function by introducing a multiplier that depends only on time.[5] (Usually this is done by introducing a simple exponential function of the form e^{rt}). He statistically confirmed the assumption of neutrality, meaning that the multiplier is independent of the capital-labor (K/L) ratio. Fitted to US time series data for 1909 through 1949, Solow's technical progress multiplier accounted for 87.5% of per capita non-farm growth per capita in the US. It grew annually at about 1% per annum during the first half of the period and 2% p. a. thereafter.[6]

To summarize briefly, the no-classical growth model as formalized by Solow and elaborated subsequently by others, includes many simplifying assumptions, of course. But three of its implications are crucial. First (1), the contribution of capital investment to growth will slow and finally cease due to saturation; this follows from the "law" of diminishing returns to (man-made) capital, that was mentioned earlier in connection with the proof of existence of competitive equilibrium. It follows (2) that the *only* source of growth thereafter must be technological progress — which the model does not explain and treats as exogenous. The third implication (3) is that poorer countries will grow faster than rich ones, other factors remaining the same. In other words, the neoclassical model of growth-in-equilibrium predicts that growth will slow for the rich nations, and that the poor nations will catch up (i.e. the "convergence" hypothesis).

Evidence of recent decades does not confirm either the slowdown (except for the US after 1972) or the convergence prediction. As for the post 1972 slowdown in the US, a number of plausible explanations have been offered that have little to do with the mechanisms in the model [OECD, 1991]. The most likely explanation is a combination of oil price shock, military expenditure and statistical problems of measuring productivity in the service sector.[7] In fact, while there has been a marked convergence within the small group of industrial countries since WW II, when the world as a whole is considered there is almost no correlation between the rate of growth of a country and its level of income [de Long 1988]. Some poor countries (the Asian "tigers", in particular) have grown much faster than the most industrialized countries (at least until 1998) but most other poor countries have lagged. This fact has motivated the recent flurry of interest in revising the theory, in particular by "endogenizing" technological progress [e.g. Romer 1986, 1987; Lucas 1988; Grossman & Helpman 1991; Aghion &

Howitt 1992].

The idea of *growth* in equilibrium (driven by continuous exogenous driving forces) can be explored mathematically. But in the real world it is an oxymoron. It is, as Robinson says, a contradiction in terms [Robinson 1962 op cit]. It makes no sense even in theory, because all of the inducements for economic actors to buy, sell, or produce are actually consequences of some kind of economic *disequilibrium*.

Economists have, for a long time, stuck with the assumption that technological change is exogenous — like "manna from heaven — despite both logic and overwhelming empirical evidence to the contrary. In effect, the assumption that technological change is exogenous amounts to a straightforward device which permits the construction of equilibrium models that have unique (however unrealistic) solutions. In particular, it permits the construction of so-called computable general equilibrium (CGE) models, which are currently enormously popular among macro-economists.

Nevertheless, it is important to acknowledge, here, that the simple assumption of exogenous technical change is not the only way of reconciling economic growth with the assumption of Walrasian equilibrium. The so-called "new" theory of endogenous growth that has blossomed since 1986, has uncovered several alternative (and more realistic) ways of linearizing the growth theory to accommodate more of the "stylized facts" of growth.[8]

If technological change were assumed to be truly endogenous (i.e. self-generating or "path dependent" in the jargon) the growth models would have to involve several feedback loops. For one example, it would have to accommodate the relationship between demand, scale of production, cumulative "learning by doing" effects, production costs, prices, and — again — demand.[9] A subsidiary loop is the negative impact on technological change of investment to increase capacity and exploit economies of scale [Abernathy 1978]. Another subsidiary loop is the relationship that links rising demand, increased profits, R&D spending and technical progress, which in turn affects product performance and/or manufacturing cost and, thence, demand [e.g. Ayres 1994]. The equations of dynamic models incorporating feedback loops of this kind become immensely complex and — more important — inherently non-linear.

A key point for purposes of the subsequent discussion is that *non-linear systems evolve in time, but they do not have equilibria*. This is a well-known fact about non-linear dynamics. Instead, the trajectories of non-linear systems are irregular. They move and unpredictably in regions, around "strange attractors", much the way a moth flutters around a candle. Non-linear models are not only difficult to solve (although modern computer software has greatly decreased the difficulties) but with certain parameter sets they also lead to irregular and complex behavior patterns, including the possibility of such phenomena as "deterministic chaos".[10] Be this as it may, the real economic system is certainly non-linear, mostly thanks to feedback loops involving technological change (but others as well). It is therefore also never in equilibrium.

Is the economy very close to Walrasian equilibrium?

This is a harder question. To begin with it is simpler to point out some empirical facts about growth that are inconsistent with the notion of moving equilibrium. First, the structure of the economy is changing over time. Some sectors are declining; others are growing. To take an obvious example, the coal and shipbuilding industries are declining. The semiconductor, computer and telecommunications industries are growing. This kind of structural change would not occur on a quasi-equilibrium (homothetic) growth path.

Growth rates vary enormously between countries and periods. This should not be the case given the conditions postulated for a competitive equilibrium. Thus, even the "endogenous" version of neoclassical growth theory can scarcely hope to explain the wide variations between countries except in terms of degrees of failure to satisfy the requirements of the theory (e.g. lack of "openness") If the market were very close to an equilibrium it would certainly have to be efficient. For instance, the stock market — acting as an information processor — would automatically and instantaneously ensure that all facts bearing on the value of a stock are incorporated in its price. Yet the underlying values cannot change much from one day to the next. How can this be consistent with violent day-to-day fluctuations in price?

Furthermore, profit opportunities vary enormously from sector to sector. A recent study of the 100 biggest firms in the world in 1912 has shown that the 14 petroleum firms in the sample outperformed the S&P 500 index by a factor of 3.7 in market valuation [Hannah 1996]. During the same period 5 electrical engineering firms outperformed the S&P by a factor of 2.7 (despite the laggard performance of Westinghouse); 10 chemical firms achieved a performance ratio of 2.4, and 18 "branded products" firms managed a combined ratio of 1.3, just above average. Meanwhile iron & steel and heavy engineering (18 firms), mechanical engineering (14 firms), non-ferrous metals (10 firms), coal mining (7 firms) and textile and leather goods (4 firms) performed below average, with the last category performing at only 0.1 (10%) of the average level [ibid]. It is scarcely necessary to note that the laggard sectors were already mature in 1912, while the best performers were all still relatively youthful in terms of the product life cycle[11].

Clearly the notion of moving equilibrium, in the Walrasian sense, is not strictly applicable. In a true equilibrium state there would be no better-than-average (or worse than average) investments, no growing (or declining) sectors. The "quasi-equilibrium" theories now being explored may accommodate some structural change, but only providing it is slow and gradual. In this sense, endogenous growth theory is only an incremental improvement over previous theories of growth-in-equilibrium. There is no room in the standard theory, however, for significant departures from equilibrium. There is no room in the conceptual scheme for hyper-inflation, "oil shocks", stock market "crashes" or "bubbles", not to mention wars or natural catastrophes.

The second reason for rejecting growth-in-equilibrium models is that the mechanisms that drive economic growth, whether savings and investment, or technological innovation in pursuit of monopoly profits, are essentially non-equilibrium phenomena. On close examination it can be seen that the strength and effectiveness of these mechanisms is a function of the "distance" of the system from equilibrium. In the Pareto-optimal equilibrium state, nothing changes because all economic agents have already maximized their utility. They are all as well off as they can possibly make themselves through economic transactions. Why save and invest? Why innovate? The "rainy day" explanation for savings is untenable: in a perfectly competitive market economy one simply buys insurance. Similarly, the "monkey curiosity" explanation of scientific research and technological development is untenable.

The classical explanation of savings and investment behavior is essentially the same as the classical explanation for discounting: consumers find it optimal to forego some current consumption in order to secure a higher level of consumption in the future through investment and consequent income growth. The tradeoff between current and future consumption is the discount rate, which is equal to the interest rate. (If the interest rate were higher than the discount rate, people would increase their savings; and conversely). This argument was articulated in mathematical terms long ago [Ramsey 1928]. Riskier investments should carry higher rates of return to compensate for the risk. But the incentive to invest actively (as opposed to putting money in the bank) is clearly higher in proportion to the difference between the expected rate of return and the bank interest rate. This difference is also a measure of disequilibrium, since in the postulated equilibrium state it would vanish.

Apart from this, it is not clear that savings and productive investment should be equated. Egyptian pharaohs built pyramids. Medieval towns built cathedrals. Keynes told the story of Alexander Pope's father, who retired from business to a Twickenham villa with a "chest full of guineas" from which he met household expenses thereafter [Keynes 1936 p. 221]. French peasants are notorious for keeping their savings as gold coins, hidden under the mattress, while Indian women traditionally keep their savings as gold bracelets and other jewelry. Many modern millionaires collect old masters. Modern nations build strategic nuclear forces. None of these activities can be regarded as productive investment in any meaningful sense, however much pyramids, cathedrals and old masters may have inadvertently added to the quality of life for people alive today. Surely investment in productive enterprise, or in R&D, must be explained otherwise — presumably in terms of expectations of future "supernormal" profits.

For that matter, why invest in R&D (formalized or not)? Simple human curiosity will not account for Bell Labs. Schumpeter's explanation (the search for monopoly profits) is far more plausible [Schumpeter 1912, 1934]. Mansfield has provided a wealth of empirical data to support this hypothesis [Mansfield *et al* 1977]. If the economy were truly in Walrasian equilibrium, either the opportunity to patent a new idea could not exist or — which is the same thing — it would be simultaneously available to all. Anyhow, monopoly profits are not possible by definition in a pure Walrasian model with perfect competition (no monopolistic price-makers). Thus, unequal distribution of intellectual property — knowledge, ideas, technology and skills — is another measure of disequilibrium and opportunity.Increasing returns to scale are not inconsistent with the neoclassical results (i.e. competitive equilibrium) but constant or increasing returns to capital is apparently a requirement of perpetual growth. Solow points out that to have constant returns to capital without negative marginal productivity for non-capital factors it is actually necessary to have increasing returns to scale [Solow 1994].

The "quasi-equilibrium" picture emerging from the new theory of endogenous growth does reflect some of the necessary motivational factors driving savings and investment.The neo-Schumpeterian endogenous growth models of Grossman & Helpman [1991] and Aghion & Howitt [1992] allows for a form of "creative destruction", called the "business-stealing effect". This refers to the fact that an innovation can destroy the monopoly rents of previous innovations. It also explicitly creates the possibility of earning private monopoly profits from innovation while society gains nothing from the innovation.[12] In effect, the "business stealing effect" can result in a difference between the private "laissez-faire" growth rate and the socially optimal growth rate. This is likely to be one of the principal reasons why investment in computers and software has done so little to improve productivity.

These are steps forward, but not yet a big enough one.Given that factors should reflect scarcity and that relative factor scarcity varies over time, thanks to capital accumulation and resource depletion, one can only conclude that factor shares must be variable in a realistic model [Ayres 1998]. Moreover, there is ample evidence that firms frequently do not operate on (or near) the efficiency frontier, which creates opportunities for "double dividends" (see below).

Finally, the notion that growth in equilibrium arises from increasing returns to capital, as all these `endogenous growth' models suggest, is an uncomfortable one. As Solow has pointed out, increasing returns results in infinite growth within a finite time and suffers from much the same `razor's edge' problem as the original H-D model, of which it is a straightforward elaboration with "sophisticated bells and whistles" [Solow 1994]. For several reasons, then, the standard neoclassical equilibrium assumption that factors of production are automatically paid their shares, which in turn remain constant, must be open to question.

Production functions and factor shares

It may surprise most economists to learn that variable factor shares are not inconsistent with Euler's theorem. But such is the case. This means that it may be possible to explain growth without assuming Let Y be total output and let K, L and E represent capital, labor and energy (or a more generalized resource, namely exergy), all of which are functions of time. Then the total logarithmic derivative of Y can be expressed as

$$\frac{dY}{Y} = \frac{a\,dK}{K} + \frac{b\,dL}{E} + \frac{c\,dE}{E} + \left(\frac{\partial \ln Y}{\partial t} \right) dt$$

where a, b, c can be interpreted as elasticities of production. These elasticities are not constants, in general. In fact

$$a(K, L, E) = \frac{K}{Y} \frac{\partial Y}{\partial K}$$

$$b(K, L, E) = \frac{L}{Y} \frac{\partial Y}{\partial L}$$

$$c(K, L, E) = \frac{L}{Y} \frac{\partial Y}{\partial E}$$

The elasticities reflect the marginal change in output resulting from an incremental change of the corresponding input (K, L, E). The time derivative reflects change in the production system not attributable to changes in K, L or E. In general this term is not zero, of course. It corresponds to a time-dependent multiplier in the production function itself.

It should be noted here that, in the special case where the elasticities are assumed to be constant, the corresponding productiion function is the neoclassical Cobb-Douglas form.In the standard case where only capital and labor are considered as factors of production, as Solow demonstrated, the time dependent term is dominant; it accounts for most of the economic growth [Solow 1956]. However, when the energy factor is included in a C-D production function it accounts for most of the growth, with a minimal contribution from the time dependent multiplier [Hannon & Joyce 1981]. But, in this case it turns out that the implied factor shares bear no resemblance to the shares in the national accounts.

There are, as it happens, more general mathematical functions that satisfy Euler's theorem: identically, for instance

$$Y = A\,K \exp\left(u_k \frac{K}{L} + v_k \frac{L}{E} + w_k \frac{E}{K} \right)$$

$$Y = B\,L \exp\left(u_l \frac{K}{L} + v_l \frac{L}{E} + w_l \frac{E}{K} \right)$$

$$Y = C\,E \exp\left(u_e \frac{K}{L} + v_e \frac{L}{E} + w_e \frac{E}{K} \right)$$

where the constants ($u_e \ldots w_e$) in the exponents must be determined by fitting the equations to historical data. The signs of these constants are determined by whether the accompanying ratios are increasing or decreasing over time. Thus if the capital/labor ratio is increasing, the coefficient of that term will be positive. If the labor/energy ratio is decreasing, the corresponding coefficient will be negative.

If labor is interpreted in the usual way, a Solow-type time-dependent autonomous technical progress multiplier (most likely of the usual form $\exp(kt)$ can be included at the cost of one additional parameter. This would reflect increased technological capability, or efficiency, resulting from "learning by doing" and/or technological spillovers. However the `endogenous' approach pioneered by Romer, Lucas and others is to try to account for technological progress by interpreting it as an increase in the stock of knowledge-as-capital. Here the problem is to measure knowledge capital.

Note that the first of the three production functions can be thought of as a modified Harrod-Domar (H-D) model; where growth is driven primarily by capital accumulation. The second production function corresponds to a neo-Marxist model, in which labor is the driving force. (Evidently if such a model were to be taken seriously, labor would have to be interpreted more broadly than in Marx's time). The third production function corresponds to a model in which energy (or a more general measure of natural resource) inputs are the primary engine of growth. It could perhaps be characterized as a Georgescu-Roegen (G-R) model.

It is interesting to note that the paper by Kummel *et al* (this issue, pp) is an example of the third production function. The parametric choices in the Kummel *et al* model are as follows: $u_e = a_0$; $v_e = a_0 c_t$; $w_e = -a_0$;. The model parameters have only been fitted, up to now, to the output of non-farm, non-service sectors. (Solow's model was fitted to entire non-farm economy.) The fits are remarkably good. The inescapable implication of this work, in conjunction with the earlier work of Hannon & Joyce [1981 op cit] is that the economy is not in equilibrium.

The three production functions can be expressed in reduced form by dividing both sides by L. Let $y = Y/L$, $k = K/L$ and $e = E/L$. Then we have a new set of simpler functions, viz.

$$y = Ak \exp\left[(u_k - w_k) k + (w_k - v_k) e \right]$$
$$y = B \exp\left[(u_l - w_l) k + (w_l - v_l) e \right]$$
$$y = Ce \exp\left[(u_e - w_e) k + (w_e - v_e) e \right]$$

The choice among the three functional forms should be made, *ceteris paribus* on the basis of explanatory power, i.e. determined by fitting to empirical (historical) data. Note that, absent a time dependent term, there are only three independent parameters to fit in each case.

It is interesting to consider implicit time-dependence. In earlier work I have suggested that the production function appropriate to a relatively primitive, low density agricultural (`cowboy') economy might be quite different from the form appropriate to an advanced `spaceship' economy [Ayres 1998]. For instance, in the first case the reduced variables k and e are both rather small (approaching zero) and production per worker is mainly attributable to natural factors such as soil fertility, and climate. In the spaceship case both reduced variables become large, although not necessarily at the same rate. (The capital/energy ratio probably increases also).The asymptotic forms of the three production functions obviously differ in the distant past and far future, although it is unclear whether this difference matters for purposes of analysis of the recent past or the near future.

Much more important for purposes of long-term economic analysis and forecasting, the assumption of growth-in-equilibrium can (and should) be dropped. This, in turn, means that the convenient assumption that future growth will follow an `optimal' trajectory (based on a number of assumptions that existing relationships will not change) must be discarded.

But relaxing those unrealistic assumptions opens the door to the possibility — indeed, the likelihood — of technical or policy innovations that generate social benefits less than, or greater than, the private benefits they bring to the innovators. The inevitable implication is that market forces cannot be relied upon to achieve an optimal result. This means that there is, after all, a significant role for government intervention, albeit not necessarily in the traditional ways.

Reprise

To summarize the several theses of this paper in a very few words, I have several specific propositions. They are as follows: First, (1) the idea of competitive equilibrium is an important tool of analysis, but it does not follow that the economy is actually in equilibrium. It is not. Indeed (2), if it were in equilibrium it could not grow, because the growth mechanisms all depend on some degree of departure from equilibrium. Indeed, (3) the economy is really a non-linear system and there is no such thing as an equilibrium state of a dynamic non-linear system.

It follows (4) that models of growth based on the assumption of moving equilibrium are inherently flawed. Such models cannot be used for long-range forecasting, whatever other uses they may have. Nor (5) can a model that assumes growth in equilibrium and treats technological change as exogenous be legitimately used to estimate the "costs" of hypothetical regulation by equating the effect of regulation to a constraint on growth. There are at least two reasons. The most obvious is (6) that this approach neglects gains that can result from the fact that firms rarely, if ever, optimize their own production potential (they do not sit on the textbook "production frontier"), whence there are almost always very large opportunities for `double dividends'.

Also, (7) it is highly plausible that well-designed regulation — like resource scarcity — may actually induce a more rapid rate of technological progress ("necessity breeds invention"). In fact, there is some empirical evidence for this hypothesis [e.g. Porter 1991; Porter and van der Linde 1995; MEB 1996]. It is important to note that neither the neoclassical growth theory nor the so-called `endogenous' growth theory now being developed, can accommodate this sort of non-linear effect. However (8) a new growth theory, that is not based on the standard equilibrium assumption, may have much greater explanatory power. This will be explored in future papers.

Endnotes

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1. The existence of a static general equilibrium in such a system was finally proved in the 1930's by Abraham Wald for some special cases. More general proofs were given in 1954 by Kenneth Arrow and Gerard Debreu [Arrow & Debreu 1954] and by McKenzie [McKenzie 1954]. This achievement has steered a generation of economists into the analysis of highly abstract mathematical models. Actually, the rather tight restrictions of the original Walras model have been significantly loosened. The general equilibrium has even been extended to the dynamic case, with exhaustible resources (e.g. [Solow 1974]), subject as before to the assumption of perfect futures markets for resources. A great deal of theoretical superstructure has been added on to this basic model in recent years, e.g.[Dasgupta & Heal 1979].

2. Game theory, along with the modern form of utility theory, were introduced to economics by John von Neumann and Oskar Morgenstern [Von Neumann & Morgenstern 1944].

3. The most modern models are known as KLEM, because they incorporate four factors, capital (K), labor (L), energy (E) and materials or mass (M). Production functions are generally expressed as separable functions of these variables. The earliest and simplest growth models used so-called Cobb-Douglas production functions, which were simple fractional powers of the factor inputs, subject to the assumption that the sum of the (2, 3 or 4) exponents must equal unity. This constraint expresses the mathematically convenient — and plausible — assumption that the economic system *as a whole* is characterized by constant (i.e. neither declining or increasing) returns to scale.

4. Joan Robinson has said: "The formula has made a great negative contribution to the development of economics. It marks, as it were, the watershed between Keynesian and modern analysis; but regarded as a positive contribution to thought it has not proved so useful" [Robinson 1962 p. 99].

5. Solow was careful to say that he used the phrase "technical progress" as shorthand for any shift in the production function, whether from slowdowns, speedups, educational improvements or whatever.

6. See Robert M Solow "Technical Change and the Aggregate Production Function" in *The Review of Economics and Statistics* August1957 pp. 312–320.

7. The importance of military expenditure is easily overlooked. The US has spent $8 trillion on strategic (nuclear) weapons alone, since 1946. There has been little or no spinoff in the civil sector, especially since the civilian nuclear power industry is being phased out. The Vietnam War also consumed several trillion dollars. Unquestionably these outlays reduced investment in other sectors.

8. The "stylized facts" in question are as follows: (1) many firms ;(2) discoveries are public knowledge (non-rival goods, in the jargon); (3) physical activities are replicable, whence the aggregate production function must be homogeneous of degree one in all inputs that can be owned and exchanged (i.e. rival goods); (4) technological progress is a consequence of human activity; (5) competition is imperfect [Romer 1994] . The "new" theory of endogenous growth was kicked off by Paul Romer, who simply discarded the neoclassical condition of diminishing returns to capital and defended that step by arguing for increasing returns to human capital, thanks to positive spillovers [Romer 1986]. Since then, Romer has also constructed models of sustained growth without spillovers, but with imperfect (monopolistic) competition [Romer 1987] and with both knowledge spillovers and imperfect competition [Romer 1990]. Many others have also contributed important insights. For useful surveys of this literature see [Andersen & Moene 1993] , especially the first article [Hammond & Rodriguez-Clare 1993], and the Winter 1994 issue of *Journal of Economic Perspectives*.

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9. This loop is sometimes known as the Salter cycle, after the economist who first considered it in detail [Salter 1960]

10. Weather and climate models, for example, are essentially non-linear. It is extremely difficult to forecast at long range with non-linear models because arbitrarily small differences in starting conditions can lead to arbitrarily large differences in results after the passage of some time. (This phenomenon has become known as the "butterfly effect", from the observation by one of the first non-linear climate modelers that a perturbing influence on the atmosphere as small as the fluttering of the wings of a butterfly in the Amazon forest could theoretically result in a change in the weather patterns over North America, years later).

11. The technology life cycle is an important feature of the landscape. It is widely accepted as a useful metaphor of the pattern of technological change. The basic idea of an aging process goes back to "Wolff's Law" of increasing marginal cost of improvement and Kuznets' work on industrial succession and the business cycle [Kuznets 1930]. It has been rediscovered and reformulated many times, especially by [Nelson 1962; Vernon 1966; Abernathy & Utterback 1975]. Also see [Ayres 1988 ("Barriers & Breakthroughs"); Ayres 1994 (Information, Entropy & Progress, Chapter 6].

12. The recent history of the Intel-Microsoft partnership is a case in point. Both companies have become enormously profitable and rich. But most consumers are scarcely better off from one software generation to the next. They are. forced to replace their PCs and softeware at roughly 3 year intervals, resulting in significant learning costs for them, despite notoriously little improvement in overall performance of most functions. Thanks to enormous increases in operating system complexity, word processors are no faster than they were a decade ago, and files using different systems are increasingly less inter-convertible.