High Quality Sellers and Warranties: Sorting Customers and Getting a Fair Price at the Same Time

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Comments Welcome

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Abstract

Warranties have become an increasingly important aspect of after-sales service in the marketing of durable goods. During a warranty period, consumers benefit from the comfort of knowing that if a product fails during the warranty period, it will be repaired free. However, beyond being an attribute that consumers are willing to pay for, warranties are interesting because of other roles they play for sellers.

Since consumers are invariably heterogeneous in their valuation of warranty coverage, sellers frequently sell different levels of coverage to different consumers and thereby increase profits (Kubo, 1986; Matthews and Moore, 1987; Padmanabhan and Rao, 1993). This use of warranties is analogous to the screening model developed by Mussa and Rosen (1978). A second role for warranties is to signal unobservable quality to consumers. A long warranty can be used to signal better quality because high quality sellers have a cost advantage over low quality sellers in offering warranty protection (Spence, 1977; Grossman, 1981; Emons, 1988; Lutz, 1989; Gal-Or, 1989).

This paper considers a situation where a high quality seller needs warranty policy to play these two roles simultaneously. There are many signalling mechanisms available to sellers including branding, reputation, and advertising. Yet all of these alternatives are expensive, take time and are relatively inflexible. In situations where sellers are local, under-capitalised, and need the flexibility to make different signals for multiple products, warranty policy is an efficient and effective option to provide credible information to buyers. Frequently, when a high quality seller needs warranty policy to play a signalling role, he faces a heterogeneous market where there are profits to be made by selling different lengths of coverage to different customers. Markets that fit this description include used durable goods, notably vehicles, appliances, and industrial equipment. This paper considers the decision process of a high quality seller when he wishes to use warranties in this manner and identifies his equilibrium strategies.

The model involves a seller who wishes to sell a product and an optional extended warranty to a heterogeneous market with two types of buyers. Specifically, the seller chooses a base price and warranty for his product and the duration and pricing for optional extended coverage. The seller knows the quality of the product and the buyer does not. The objective is to understand how sellers set warranty menus to maximise profit when the only signal of quality to buyers is the prices and warranty lengths chosen by sellers.

The findings of the analysis are first, that a high quality seller sells longer warranty coverage than a low quality seller to every buyer when quality is unobservable.
Nevertheless, a high quality seller’s optimal warranty policy is often unaffected by the unobservability of quality (when the “premium” that buyers are willing to pay for high quality is small). Second, when a high quality seller’s actions are affected by the “need to signal”, the equilibrium action involves lengthening the coverage sold to both types of consumers but shortening the length of the extended warranty. In fact, when the “premium” that buyers place on high quality is large, the high quality seller loses the ability to sell different warranty lengths to each buyer-type: signalling forces him to offer a maximum warranty to everybody. Finally, the model demonstrates that a high quality seller will not maximise profit by signalling solely with base warranties. A high quality seller maximises profit when all parts of his warranty menu are used to provide information to buyers.

To provide empirical support for the model, we discuss a survey of the used car market. The information collected provides evidence of significant unexplained variability in the length and prices of warranty coverage across very similar vehicles. Consistent with the model’s prediction that the entire menu is used to provide information to buyers, used car dealers are observed to provide complete details about warranty policy in a significant majority of selling encounters. Finally, as would be suggested by the model, we find preliminary evidence of a number of sellers attempting to shift consumers to longer warranty lengths.

Key Words: extended warranties, signalling, screening, unobservable quality, price discrimination, menu of contracts, used cars.
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1.0 Introduction

Warranties and extended service contracts have become an increasingly important element of post-sale service strategy (Lele, 1986). The benefit provided by warranties is important since the cost of repairs when products break can be prohibitive and warranties eliminate this worry. As long as the cost to provide warranty coverage is less than the price customers are willing to pay for it, we should expect to see warranties across a broad range of goods and this is in fact, the case.

However, warranties have another side for sellers and this comes from two functions they fulfil beyond just being another attribute valued by customers. First, as we discuss in the next section, the nature of warranties allows high quality sellers to use them as a vehicle to communicate (signal) unobservable information to the buying public. Second, because warranty coverage can be varied for each consumer, warranty policy is also a vehicle that sellers can use to discriminate (screen) between consumers who have different needs.

An issue on which the literature is silent is a situation where a seller needs warranty policy to perform both of the above functions – signalling and screening – simultaneously. This is a context where a seller has a need to communicate information to a buying public that is heterogeneous. The objective of this paper is to show that warranty policy is both an effective and efficient vehicle to address this need. In the following section, we consider the motivation for this work and its relevance to marketing.

2.0 Background and Literature Review

As long as merchants have been trading goods for money, high quality merchants have had to face the problem of how to get a fair price for their goods. In many categories, this problem is easily rectified by allowing customers to inspect goods (as in the case of clothing or vegetables) or sample them (as in the case of beverages). However, for some types of goods, quality cannot be evaluated through inspection or sampling (for example, artwork, the quality of pure-bred puppies, and medical services)\(^1\). In these situations, sellers of high quality face a problem of trying to prove the “higher quality” of their products to interested buyers.

As noted by Akerlof (1970), this situation sometimes leads to market failure in that only low quality goods are traded (high quality goods are not brought to market because their owners cannot prove their higher quality).

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\(^1\) With many goods, quality cannot be evaluated prior to purchase and becomes evident only after prolonged use or ownership. For example, the degree to which a puppy exemplifies a breed of dog only becomes clear as the dog matures. Nelson (1974) refers to these products as "experience goods".
cannot get a fair price for them). Interestingly, several market mechanisms have developed which do allow higher quality products to be traded even when the quality of the goods cannot be evaluated easily by a seller.

Amongst the most important of these mechanisms is reputation, where a merchant becomes known as a high quality seller broadly within the buying population. Kreps (1990) and Milgrom, North and Weingast (1990) model the reputation effect as a Repeated Prisoners' Dilemma game and show how reputation can be an effective vehicle for promoting honest trade. One can see how reputation plays a strong role in the prices paid and the quality of the work delivered in the case of local tradespeople (carpenters, plumbers and electricians).

Another mechanism, which facilitates the exchange of high quality products, is advertising. Klein and Leffler (1981) and later Milgrom and Roberts (1986) show how heavy advertising can be an unambiguous signal of high quality in a market when higher margins associated with high quality are necessary to finance the advertising. Advertising appears to play this role in many markets where well-financed sellers of high quality products spend large amounts of money on uninformative advertising (it is the amount and not the content of the advertising that matters). Klein and Leffler note that firms sometimes advertise that they advertise (“as seen on the Tonight Show”) versus providing informative data about their products.

A related mechanism is branding which, as noted by Landes and Posner (1991), provides a form of insurance or guarantee to the buyer about a product so branded. For example, the Intel Corporation has used aggressive branding coupled with advanced technology to be recognised by the majority of consumers as the top high quality microprocessor brand.

In addition to the three mechanisms mentioned above, other approaches that firms use to convey their higher quality to the public include “renting a reputation” by distributing a product through “reputable” channels (Chu and Chu, 1994), making expenditures in expensive outside certifications (as in the case of animal breeders), or by entertaining a highly visible clientele (like Sotheby’s auctioneers).

Four important attributes are common to all of these “market correction” mechanisms.

1. In order to function, all require considerable money and usually time. It can take years to develop a brand name, a reputation of being high quality or a perception of being highly advertised. As noted by Dierickx and Cool (1993), the fact that these processes generally

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2 Akerlof (1970) predicts that high quality products will not be traded when buyers cannot identify high quality. In this situation, buyers are not willing to pay higher prices to a seller claiming to be high quality, so high quality sellers do offer their products for sale.

3 Empirical support for this argument can be found in Mishra, Heide and Cort (1998).

4 This is discussed in “Sotheby’s, Amazon Team to Offer Online Auctions”, Wall Street Journal Europe, June 17, 1999.
occur over a long period makes it expensive to implement such strategies quickly\(^5\). In other words, a high quality seller may suffer for some time before he is able to benefit from the fruits of significant investments.

2. Reputation, advertising and branding all represent perceptual and not contractual relationships between the seller and the buyer. As a result, they cannot be gained or borrowed easily. They are also very specific to the companies that have them and cannot be transferred.

3. Reputation, advertising and branding all have a dichotomous character in the following sense: a seller either has a strong reputation (or a well-known brand) or does not. In fact, a seller of high quality depends on this reputation being broadly known and consistent (for the reputation mechanism to function). As a result, a seller is precluded from “varying” the amount of reputation for different customers (something that could conceivably be quite profitable).

4. Reputation, advertising and branding generally provide little of tangible value to consumers after the purchase. Their primary role is that of communicating something to the consumer \(\text{ex ante}\). If consumers could identify the quality of a product easily, it is unlikely that reputation or branding would be of great significance (nor is it likely that firms would make great efforts to develop a reputation).

A different mechanism that high quality sellers may use to trade their merchandise at higher prices, is warranty policy. As noted by Spence (1977), warranties are useful signalling mechanisms because for high quality sellers, they are cheaper to provide. As a result, a high quality seller can affordably distinguish himself from a low quality seller by providing longer warranty coverage\(^6\). But what makes warranty policy different?

Warranty policy is different from the mechanisms discussed earlier in four very important ways. First, reputation and branding involve significant \(\text{ex ante}\) expenditure (which are perhaps eventually recouped in terms of higher prices) whereas warranties involve \(\text{ex post}\) expenses for the seller. This difference means that high quality sellers who use mechanisms such as advertising and branding are invariably well funded “cash rich” companies. In contrast, warranties do not place such a requirement on sellers of high quality. Because high quality products break less often than low quality ones, they are relatively inexpensive. Moreover, the costs of providing warranty service accrue to the seller \(\text{after}\) the buyer has paid for the product. In contrast to the other mechanisms, warranties are largely self-financing.

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\(^5\) Williamson (1991) also highlights a number of problems with the reputation mechanism. Many of these problems relate to the process by which buyers and potential buyers become informed of a seller’s cheating (i.e. providing lower quality than expected by the buyer).

\(^6\) In related work, Grossman (1981) discusses the informational content of warranties and Gal-Or (1989) examines the signalling role of warranties in the context of differentiated products.
Second, warranties are explicit contracts between buyer and sellers that are enforceable in a court of law. This is an important difference because, in contrast to reputation or branding, it is both possible and common for the obligations of a seller under a repair warranty to be transferred to a third party. We discuss later why this important in the context of warranty policy’s use as a signal to consumers.

Third, in contrast to reputation or branding, warranties are not an all-or-nothing tool. They can be varied on at least two dimensions, one of which is continuous. The first dimension is the degree of coverage (what types of failure are covered, the percentage of repair costs that are covered, and how the warranty is exercised). The second is the length of coverage in terms of time. This aspect of warranties makes them a highly flexible tool for a high quality seller who is trying to use them as a mechanism for informing potential buyers about his quality.

Fourth, and perhaps most importantly, in contrast to branding, reputation or advertising; warranties are valuable in their own right and provide value to buyers long after their decision to purchase. As long as a high quality product sometimes fails, a warranty is of positive value to a buyer and the longer it is, the better. Of course, not all buyers place the same value on warranty coverage. If we consider points three and four together, we find a basis for one of the most common roles of warranty policy, “screening”. Because consumers place positive but different values on warranty coverage and because warranty coverage can be varied for different consumers, sellers can sell different lengths to different consumers and increase profit. As noted in recent articles this use of warranty policy is immensely profitable.

However, returning to our initial problem (that of a high quality seller trying to communicate the higher quality of his products to potential buyers), we can see that there may be a need for warranty policy to perform two roles simultaneously. The previous paragraph suggests that warranty policy will always have a role to play in terms of screening potential buyers to raise profit, but we can also see that under conditions where quality cannot be determined by inspection, a seller may want to use that same policy to send a message to potential buyers. Quite simply, the subject of this paper is how does this happen?

A natural question to ask is why is this important to marketers, especially when there are so many alternative mechanisms available to inform potential buyers about quality. For example, why would a high quality seller not restrict the use of warranty policy to screening and use branding or reputation to inform customers of his quality? The answer to this question follows from the first difference mentioned between warranties and the other mechanisms (reputation, advertising and branding) that allow high quality sellers to get a fair price.

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7 Kubo (1986) shows how a monopolist can increase its profits when consumers are heterogeneous through the use of an optional quality guarantee. Various aspects of this role for warranties are also considered by Matthews and Moore (1987) and Padmanabhan and Rao (1993).
Warranty policy is the *only marketing tool* available to a high quality seller to solve his endemic problem of getting a fair price that is effectively self financing (so a seller need not be a big cash-rich company). It can also be used when a seller does not have the benefit of time. Given that warranty policy is the *only* mechanism which provides these advantages and given its important role as a screening mechanism, its simultaneous use as both an informative signal and a profit-increasing screen is a topic of immense importance to marketers.

A problem that can interfere with this use of warranty policy is a buyer’s expectation about a seller’s future availability to honour a warranty if breakdown occurs (Boulding and Kirmani, 1993). For example, a buyer may worry that a small poorly-funded seller will close before the term of a warranty expires. However, when this is a problem, it can be solved (relatively) painlessly because the obligations of a warranty are explicit and contractual. When necessary, warranty obligations can be transferred to a better financed third party to provide assurance that they will be quickly honoured over the term of the warranty.

There is significant evidence that warranties are performing this simultaneous role in a number of important markets from used cars to cloned personal computers. There are certainly segments of each of these markets where other mechanisms are used to provide information to buyers (for example, in the used car market, a significant number of used cars are sold through well-known branded dealers). Nevertheless, a majority of the products sold in markets like used cars are sold by small poorly funded sellers and the buyer is faced with the problem of not being able to evaluate quality. Later in the paper, we provide evidence of warranty policy playing a dual role in the used car market.

Even if there is significant evidence that warranty policy plays a dual role, we also need to ask, why should this role should be interesting? Moreover, is the effect of asking warranty policy to perform these two roles simultaneously completely predictable? This leads us to three general questions that we might ask about any market where warranty policy is a major strategic decision.

The first question is: should high quality products always have longer warranties than low quality products? Warranties are cheaper to provide for high quality products. However, a buyer clearly places higher value on warranty protection for a product that is likely to break more frequently (i.e. a lower quality product). We will see that the answer to this question revolves

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8 *Business Week* (January 14, 1991) reports that ½ of the operating profits for big consumer-electronic chains come from extended warranties and more recently, an article in the *New York Times* (July 23, 1995) indicates that retailers may earn as much as 75% of their gross income from the sales of extended warranties.

9 The obligations of sellers in the used car market are transferred to third parties known as after-market warranty providers. In North America, Lubrico, Global, GE Capital, and Pafco are well known after-market warranty providers.

10 Three times as many used cars are sold as new cars and the used car market is twice the size of the new car market in value (“Detroit discovers the consumer: used cars”, *The Economist*, Vol. 338, Feb. 17, 1996, p.61 and Bennet, James, 1994). *Consumer Reports 1999 Buying Guide* underlines how important warranty sales and coverage are for used vehicles.
entirely around how well informed the buyer is about the unobservable quality of the product. Only when the buyer lacks information about product quality can we provide an unambiguous answer to this question.

The second question is will the need to communicate information with warranty policy lengthen or shorten extended warranties? Following from the work of Spence (1977), our initial answer might that extended warranties should be lengthened to communicate information to buyers. However, in the analysis that follows, we will see that precisely the opposite occurs: extended warranties are shorter when warranty policy is faced with the task of providing information to the buyer about quality.

The final question is should a high quality seller tell all buyers about the price of extended warranties even if a good percent of buyers are not interested? One might think that the best policy is to post a sticker price and base warranty and provide pricing on extended warranties to those consumers who are interested. In the analysis that follows, we will see that a policy which limits the amount of information provided to any buyer is strictly profit reducing for the seller. A high quality seller has strong incentive to communicate his entire warranty menu to each and every buyer.

In the next section, we present an analytical model that is used to provide insight into the warranty policy we are likely to see in a market where warranties have the burden of a dual role. In section 4.0, we analyse the equilibrium outcomes when warranty policy is asked to perform a dual role. In section 5.0, we present some empirical information and analysis in support of the model and we conclude in section 6.0.

3.0 Overview of the Model

As noted previously, the objective of this analysis is to identify the optimal warranty menu that a seller of high quality will offer to a buyer when i) the buyer is uncertain about the seller’s quality and cannot evaluate quality easily and ii) buyers have heterogeneous valuations for warranty coverage (that cannot be recognised by sellers). The market we consider is one in which a buyer purchases no more than 1 unit of product. We further assume that the seller has a degree of price setting ability and this follows from a situation in which buyers have positive search costs.\footnote{Diamond (1971) notes that a market which has subsequent search costs greater than zero results in monopoly-like pricing.} For example, when a buyer looks for a product (like a used car), she usually looks for a certain brand, with certain features (four door for example, standard transmission, CD player, and colour), in a certain price range. Once she finds a car that meets these criteria (at either the first or the second dealer or the $n$th dealer she visits), the dealer selling the car has a degree of price setting ability because the buyer would need more time to go to more dealers to find an alternative.
The price setting ability of the seller is unaffected by his having two similar products available (the pricing is a function of the buyer’s options outside the focal seller). In contrast, price setting ability is affected by the proximity of competing sellers and the closeness of their options to those offered by the focal seller. When competing sellers are nearby and known to have similar alternatives, the seller retains a degree of price setting ability but the overall pricing he offers will have to be more attractive because a buyer is more likely to investigate options with other sellers.\textsuperscript{12}

Several key assumptions underlie our analysis and we discuss these before proceeding with an exposition of the model.

Assumption 1. Sellers are risk neutral.

This assumption allows us to focus our analysis on the problem of signalling and screening without incorporating risk-sharing considerations.

Assumption 2. Producer and purchaser moral hazard are insignificant.

The primary reason for this assumption is to focus the analysis on simultaneous signalling and screening\textsuperscript{13}. However, the assumption is largely justified in markets where simultaneous screening and signalling take place because:

a) Sellers have limited ability to affect the performance of a product after its sale and,
b) Downstream moral hazard is frequently limited with deductibles and maintenance programmes that induce careful behaviour on the part of purchasers.

In addition, unless downstream moral hazard were systematically worse for higher or lower quality, it would not have a material effect on this analysis other than increasing the cost of warranty coverage.

Assumption 3. Consumers differ in their valuation for warranty coverage and sellers cannot observe these valuations.

Extended warranties are effectively “insurance” against uncertain repair costs. Consumers frequently place different values on these warranties and hence, we observe significant differences in the amount of warranty coverage purchased by consumers\textsuperscript{14}. In our simplified

\textsuperscript{12} We explain later how a situation where a buyer has more attractive options is reflected through a more restrictive individual rationality constraint.

\textsuperscript{13} Several papers consider warranties in the context of producer and purchaser moral hazard including Cooper and Ross (1985) and Dybvig and Lutz (1993).

\textsuperscript{14} Heterogeneous preference for warranty protection can arise from differences in income [Shaked and Sutton (1982)] or differences in risk aversion [Padmanabhan and Rao, 1993].
market, we assume there are two types of buyers: hedgers (type HE) place higher value on warranty coverage than gamblers (type G).

Assumption 4. The value of a product and hence warranty coverage declines over time.

Many durable goods depreciate over time due to wear and tear or obsolescence. Accordingly the value of a warranty which guarantees that broken products are repaired also declines over time

Assumption 5 and 6 deal specifically with how quality is represented. Following from the earlier discussion, the objective is to represent a dimension of quality that is known to sellers but uncertain to buyers.

Assumption 5. Products can be either high (H) or low (L) quality. Buyers prefer high to low quality but they cannot determine quality by inspection.

This implies that there are only two levels of product quality in the model. Buyers prefer high to low quality _ceteris paribus_ but will not pay higher prices for a product unless they are certain that the quality is high.

Assumption 6. A high quality product (H) fails more than a low quality product (L).

We do not model failure rates directly. Nevertheless, the assumption that a high quality product fails more than a low quality product has two important implications:

i) Because, low quality products are assumed to fail more than high quality products, the expected cost to provide an extended warranty for low quality products is higher. We assume linear repair costs with an expected repair cost per unit time that is higher for low quality products i.e. \( c_L > c_H \).

ii) Without warranty protection, a buyer needs to finance the repair of a breakdown herself. Since low quality products fail more frequently, the value of warranty coverage for a low quality product is higher (this is modelled by restricting the marginal value of warranty coverage for a low quality product to be higher than for a high quality product). Assumption 6 is summarised in Figure 1.

(Figure 1)

\[\text{An alternate assumption is that the cost of keeping a product operational becomes prohibitively expensive over time. A model based on this assumption would be structurally different but generate similar results. We thank a reviewer for having raised this issue.}\]
We now present a mathematical framework based on these assumptions to represent a high quality seller’s problem. We then use the model to derive equilibrium strategies. Obviously, real markets have more than two buyer types and more than two levels of unobservable quality. Yet reflecting both buyer preferences and product qualities with simple two-point distributions provides a parsimonious framework to gain insight about how a market with more levels would function.

The Buyer’s Decision

We assume that utility derived by a buyer from purchasing a product of known quality can be represented by a quasi-linear function in which there are three main components. The first component \((B_Q)\) is the benefit that a buyer obtains from a product of quality \(Q\) \((Q=H\) or \(L)\) without warranty protection. The second component is the benefit that the buyer obtains from warranty coverage on the product. The final component \((P)\) is the total price paid. The utility function for buyers (where \(x\) is the length of the warranty) is:

\[
U(\theta, Q, x, P) = B_Q + \gamma_Q \theta x V(x) - P
\]

The second term is the product of three items. Following from Assumption 6, the parameter \(\gamma_Q\) is used to capture the assumption that the marginal value of warranty coverage for a high quality product is less than the marginal value of warranty coverage for a low quality product i.e. \(\gamma_H < \gamma_L\).

The second item \(\theta_T\) is a valuation parameter, which is different for the two types of buyers in our model. Consistent with Assumption 4, the valuation parameter for buyers who place a higher value on warranty coverage (type HE) is larger \(\theta_{HE} > \theta_G\). We assume that a fraction \(\lambda\) of buyers are type HE and \(1-\lambda\) are type G.

\(V(x)\) is a function that allows us to reflect Assumption 4 (the value of warranty coverage declines over time). Mathematically, this implies that \(V(x)\) has the following properties: \(V'(x) > 0\), and \(V''(x) < 0\). We further assume that a warranty of zero length has no value i.e. \(V(0)=0\). To simplify the analysis, we utilise the following form for \(V(x)\):

\[
V(x) = \frac{1-(1-x)^2}{2}
\]

This function exhibits the required properties for \(x \in [0,1]\) and satiation at \(x=1\). Since most durable goods have a finite life, we model the warranty has having a limit beyond which it is of little value. Having chosen this functional form, we can now specify the constraint implied by Assumption 5.

When \(B_H > B_{min} = \frac{B_L + \gamma_L \theta_L}{\gamma_H \theta_G}\), both types of buyers (hedgers and gamblers) prefer high to low quality given equivalent prices and warranty coverage.
Two further comments are necessary to fully explain the buyer’s decision process. First, a buyer will not purchase unless the offering (i.e. the price, warranty coverage and quality) provides a minimum level of “reservation” utility. Reservation utility is used to represent the outside options of buyer. For example, when it is easy to find another seller with an acceptable alternative, the reservation utility will be high and this forces both high and low quality sellers to set lower prices. To simplify our analysis, we assume that reservation utility is zero\(^{16}\).

A second issue concerns the decision process of the buyer when she faces two options that both provide more than reservation utility i.e. that satisfy individual rationality. We assume that the buyer chooses the option that yields the maximum utility and this is captured mathematically through an Incentive Compatibility Constraint.

The Seller’s Problem

Our objective is to identify the equilibrium strategy for a high quality seller. This is linked to the action and profits realised by a low quality seller. Accordingly, we first characterise the problem faced by a seller of known quality. The expected profits of the seller are a function of the distribution of buyers, the cost of warranty provision and the marginal cost of the product. If we assume the marginal cost of the product is zero, a seller maximises the following\(^ {17}\):

\[
\pi_q(x_{HE}, P_{HE}, x_G, P_G) = \lambda(P_{HE} - c_Q x_{HE}) + (1 - \lambda)(P_G - c_Q x_G)
\]

(3)

The warranty/price bundles \((x_{HE}, P_{HE})\) and \((x_G, P_G)\) are purchased by hedgers and gamblers respectively and \(c_Q\) is the cost per unit time of providing warranty coverage for a seller of quality ‘Q’ (Q=H or L). The warranty length \(x_G\) can be thought of as the base warranty and \((x_{HE}-x_G)\) is the length of the extended warranty that can be purchased for \((P_{HE}-P_G)\). The two options available to consumers (the product without an extended warranty, the product with an extended warranty) are referred to as two points \((x_{HE}, P_{HE})\) and \((x_G, P_G)\) in warranty length/price space.

Two other conditions are worth mentioning in the context of the menus offered by sellers. In order for a seller to have an incentive to offer a positive warranty, the “most willing” buyer must be willing to pay more for the warranty than it costs the seller to provide it at \(x=0\) (where the marginal value of warranty protection is highest). This reduces to 2 simple conditions:

\[
\gamma_H \theta_{HE} > c_H, \quad \gamma_L \theta_{HE} > c_L
\]

(4)

For gamblers to purchase warranty protection, these conditions must also be satisfied for \(\theta_G\).

A second issue concerns the distribution of buyers in the marketplace. A key constraint faced by a seller in designing a menu for two types of buyers is that he cannot charge a hedger the maximum price she is willing to pay. Thus, when the percentage of gamblers becomes sufficiently

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\(16\) This simplification implies that both consumer types will purchase; however, situations where buyers “walk away” if quality is low can be represented with a positive reservation utility.

\(17\) The marginal cost of the product does not affect the optimal warranty menu because it drops out of the first order conditions.
low, it can be optimal for a buyer to offer warranty coverage to hedgers only (gamblers will be offered the product without warranty coverage). We allow for this possibility in our analysis but focus on the interesting case where both buyer-types are offered positive levels of warranty protection\textsuperscript{18}.

\textit{Informational Assumptions}

Sellers’ cost structures and buyers’ utility functions are assumed to be common knowledge. As previously mentioned, the buyer cannot tell \textit{a priori} whether the seller is high or low quality and the seller cannot tell \textit{a priori} whether the buyer has a low or high valuation for warranty coverage. In order for the buyer to figure out whether the seller is offering high or low quality, the buyer forms beliefs about the expected actions of a high quality seller. To identify, the equilibrium, we apply a refinement of the Perfect Bayesian Equilibrium (PBE) known as the Intuitive Criterion (Cho and Kreps, 1987 and Fudenberg and Tirole, 1991). The refinement allows the identification of a unique outcome in certain signalling games by arguing that certain types (of sellers) should not be expected to use certain strategies.

\textit{Extensive Form of the Game}

The game is a simultaneous single-shot game with an implied order of play:

\textit{Stage 1.} The seller chooses a menu of price/warranty bundles and announces them to any buyer who has interest in the particular product that the seller is offering.

\textit{Stage 2.} A buyer arrives at the seller’s place of business and shows interest in the product that the seller is offering.

\textit{Stage 3.} The buyer decides whether to purchase any of the bundles that the seller announces.

Following from our previous discussion, the seller’s price setting ability is captured by having him choose prices and warranty lengths first. We use the model to analyse the warranty policy of a high quality seller when quality is uncertain for buyers and the sellers have incentive to screen buyers because of their heterogeneity. The approach we use is to first describe the outcomes that obtain when quality is observable. We then describe the outcomes that obtain when quality is not observable (and buyers are uncertain about it). This allows us to isolate the impact that signalling has on the actions of a high quality seller.

\textsuperscript{18} As shown in the technical appendix, a necessary condition for gamblers to be offered positive warranty protection is $\lambda < (\gamma_Q \theta_Q - c_Q) / (\gamma_Q \theta_{HE} - c_Q)$. 

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4.0 Analysis and Discussion

4.1 Product Quality is Observable

Consider the action that a seller would take were product quality observable. Clearly the buyer’s valuation of a given warranty/price bundle depends on the quality of the product. When quality is observable, the seller knows the value that the buyer will place on a product with a given warranty length.

With observable quality, we solve a constrained optimisation problem with the objective function shown in equation 3. This objective function is based on buyers selecting the appropriate bundle: \((x_{HE}, p_{HE})\) or \((x_G, p_G)\) depending on their type (HE or G). Since all buyers have the option of buying both bundles, it is critical that buyers voluntarily select the bundle that is designed for them. This leads to two Incentive Compatibility Constraints, one for each type of buyer.

For hedgers
\[
B_q + \gamma_q \theta_{HE} V(x_{HE}) \cdot P_{HE} \geq B_q + \gamma_q \theta_{HE} V(x_G) \cdot P_G
\]
(5)

For gamblers
\[
B_q + \gamma_q \theta_G V(x_G) \cdot P_G \geq B_q + \gamma_q \theta_G V(x_{HE}) \cdot P_{HE}
\]
(6)

As previously discussed, we also assume that both buyer types realise positive surplus by purchasing and this is reflected through Individual Rationality Constraints.

For hedgers
\[
B_q + \gamma_q \theta_{HE} V(x_{HE}) \cdot P_{HE} \geq 0
\]
(7)

For gamblers
\[
B_q + \gamma_q \theta_G V(x_G) \cdot P_G \geq 0
\]
(8)

Finally, we constrain prices to be positive and warranty lengths to values between zero and one.

The key elements of the solution are that equations 5 and 8 bind with strict equality. This means that a hedger is indifferent between the bundle designed for her \((x_{HE}, p_{HE})\) and the bundle designed for a gambler \((x_G, p_G)\) and a gambler is indifferent between purchasing her bundle and not buying at all\(^{19}\). We summarise the nature of the solution to this problem in Proposition 1.

**Proposition 1**

When product quality is observable, the profit maximising action for the seller is to offer a menu of price/warranty combinations where each buyer-type self selects a bundle designed for her.

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\(^{19}\) In spite of the hedger’s indifference between the two bundles, she is assumed to choose the bundle designed for her. This is a typical assumption in self-selection problems justified because the hedger prefers her bundle strictly with an infinitesimal reduction in \(p_{HE}\).
Proposition 1 is consistent with the results of Mussa and Rosen (1978) and Maskin and Riley (1984) that relate to second-degree price discrimination. The basic challenge for the seller is that he wishes to serve two types of buyers, one of which is willing to pay more for warranty coverage than the other. If he only offers one warranty/price combination, the choices are simple. On the one hand, he could choose to serve only hedgers, the type which places the highest value on warranty coverage. But this means that he would only sell warranty coverage to a fraction \( \lambda \) of the market. On the other hand, he could choose a cheaper and shorter warranty/price bundle that both hedgers and gamblers would purchase. But here, hedgers would obtain a benefit that the seller could potentially charge for and the warranty coverage would be less than the optimal coverage that could be sold to hedgers. Essentially, the seller is looking for a way to sell hedgers more warranty coverage at a higher price and still sell warranty coverage to gamblers.

Not surprisingly, the solution is to offer different warranty/price bundles to each buyer-type and construct the menu in such a way that hedgers choose a more expensive bundle with more warranty coverage. If we restrict our attention to values of \( \lambda \) such that \( \lambda < \frac{\gamma_o \theta_g - c_o}{\gamma_o \theta_{he} - c_o} \), the seller has an incentive to offer positive warranty lengths to both types and the optimal menu for the seller is:

\[
(x_{HE}, p_{HE}) = \left(1 - \frac{c_o}{\gamma_o \theta_{HE}}, B_o + \frac{\gamma_o \theta_{HE}}{\gamma_o (\theta_g - \lambda \theta_{HE})^2} \left( \frac{(1-\lambda)^2 c_o^2}{\gamma_o (\theta_g - \lambda \theta_{HE})^2} - \frac{c_o^2}{\gamma_o \theta_{HE}^2} \right) + \frac{\gamma_o \theta_g}{2} \left[ 1 - \frac{(1-\lambda)^2 c_o^2}{\gamma_o (\theta_g - \lambda \theta_{HE})^2} \right] \right)
\]

(9)

\[
(x_g, p_g) = \left(1 - \frac{(1-\lambda)^2 c_o}{\gamma_o (\theta_g - \lambda \gamma_o \theta_{HE})}, B_o + \frac{\gamma_o \theta_c}{2} \left[ 1 - \frac{(1-\lambda)^2 c_o^2}{\gamma_o^2 (\theta_g - \lambda \theta_{HE})^2} \right] \right)
\]

(10)

Mathematical comparisons lead us to Proposition 2, which underlines the principal elements of the seller’s screening menu when product quality is observable.

**Proposition 2**

When quality is observable, the profit maximising menu for the seller has:

(a) A bundle designed for hedgers with warranty protection of efficient length and a price which leaves a hedger with strictly positive utility.

(b) A bundle for gamblers with a warranty that is shorter than the efficient length and a price which leaves a gambler indifferent between buying or not.

Proposition 2 underlines three key aspects of the optimal screening menu for a seller of known quality. First, offering two bundles to buyers certainly increases profit for the seller but it is not
ideal (theoretical “profit” is left downstream with buyers). In a sense, the seller needs to provide hedgers with a subsidy (or lower price) to buy the more expensive bundle. This situation obtains because regardless of how short the warranty is in the gambler’s bundle, it will be attractive to a hedger since she is willing to pay more for warranty coverage than a gambler. Any bundle designed for a hedger must provide her with the benefit she would obtain by not purchasing the extended coverage.

The second point is that this subsidy (or price reduction) to hedgers is effectively pre-determined by the characteristics of the gambler’s bundle. Accordingly, the seller maximises his profit on sales to hedgers by maximising the difference between the price he can charge for the high valuation buyer’s bundle and his expected costs i.e. where the marginal benefit of additional warranty coverage to hedgers, $\gamma_Q \theta_{HE}(1-x_{HE})$, equals the marginal cost of providing warranty coverage, $c_Q$. This explains why the warranty length offered to hedgers is efficient. Nonetheless, the price that hedgers pay is constrained by the gambler’s bundle: the longer is the warranty coverage in the gambler’s bundle, the greater is the price reduction needed in the hedger’s bundle. This explains why the optimal menu for the gambler involves a warranty that is strictly shorter than the efficient length (the seller sacrifices profit on gamblers in order to reduce the price reduction that he provides to hedgers). As the fraction of gamblers in the market increases (i.e. $\lambda$ decreases), the reduction in warranty coverage ($x_G$) from an efficient length gets smaller.

The Impact of Quality on the Screening Menu when Quality is Observable

When quality is observable, an example of optimal menus for a high and low quality seller is shown in Figure 2.

(Figure 2)

As discussed above, hedgers receive an efficient length of warranty (reflected by the tangency of the isoprofit lines and indifference curves at $x_{HE}$) regardless of whether the seller is of high or low quality. Conversely, a gambler is offered a bundle which lies on her reservation utility indifference curve but is strictly shorter than the surplus maximising bundle. In Figure 2, the high quality seller is observed to offer a menu where both buyer types pay higher prices and are offered longer warranty lengths. Assumption 5 implies that buyers always pay more for higher quality; however, the warranty coverage offered by a high quality seller can sometimes be less than the coverage offered by a low quality seller (consider the lengths $x_{HE}$ and $x_G$ when $\frac{c_L}{\gamma_L} < \frac{c_H}{\gamma_H}$). Because $c_H < c_L$, this is perhaps uncommon, yet if $\gamma_H$ is small, it is certainly possible$^{20}$. This highlights the importance

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$^{20}$ If high quality products rarely fail, the value of warranty coverage to buyers could be quite low. In the context of this model, such a situation implies a low value of $\gamma_H$. 

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of the ratio \( \frac{c_L}{\gamma_L} : \frac{c_H}{\gamma_H} \) in determining the relative location of the menus of high and low quality seller in warranty length/price space. When the ratio is large, the high quality seller’s menu will be well to the right of the low quality seller’s menu. In contrast, when the ratio is smaller, the appearance of the menus is similar to that shown in Figure 2. The difference between \( B_H \) and \( B_L \) (the fixed benefit difference between high and low quality) also has a significant effect on the relative location of the two menus. When it is large, the prices a high quality seller can charge are high and this tends to moves the high quality seller’s menu upward in warranty length/price space.

What would happen if quality were unobservable? A seller of low quality would love to obtain higher prices for his product/warranty combinations but on the other hand, he does not want to provide extra warranty coverage since his costs of doing so (\( c_L \)) are high. We can interpret this in the context of the Figure 2.

When the menu of the high quality seller lies significantly to the right (with long warranties), the low quality seller would be unlikely to mimic the high quality seller’s menu. He would get higher prices but he would also have to finance longer warranty coverage. As a result, a high quality seller can send an informative signal to potential buyers \textit{without altering his behaviour} from what he would do were quality observable.

In contrast, when the menu of the high quality seller lies above the menu of the low quality seller, the low quality seller would gladly offer the warranty/price combinations that are optimal for the high quality seller under observability. In this situation, a high quality seller needs to alter his menu when quality is not observable. A wary buyer will not pay high quality prices for a price/warranty combination unless she is sure that a low quality seller would be unwilling to offer her the same deal. The following section identifies the optimal action for a high quality seller when quality is unobservable.

4.1 The Optimal Menu for the High Quality Seller when Quality is Unobservable

A simple way of describing the problem created when product quality is unobservable is that we are looking for a market outcome in which sellers maximise profit and buyers actually obtain the quality that they think they are buying (a separating equilibrium). A situation in which a buyer thinks that she is purchasing high quality but actually obtains low quality is not an equilibrium. Based on the discussion in the preceding paragraph, there seem to be two situations. In the first, were quality observable, the high quality seller’s menu lies in a region where the low quality seller does not have an incentive to pretend to be a seller of high quality, i.e. the high quality product is offered with long warranties. In the second, were quality observable, the high quality seller’s menu lies in a region where a low quality seller does have an incentive to pretend to be a high quality seller i.e. the high quality seller’s menu has high prices. We now describe the solution to the game.
The Non-Mimic Constraint

The key problem when quality is unobservable rests with buyers who lack information about the quality of the products that they are buying. To help buyers make decisions, we assume that buyers form beliefs about product quality and these beliefs are based on prior information and actions (i.e. the warranty/price menus that are announced). This is the essence of signalling models in which uninformed players (i.e. the buyers) make inferences about informed players (i.e. sellers who know the quality they are selling) based on actions they take.

Consequently, solutions to signalling games involve specifying both the actions and beliefs (of uninformed players). As noted by Fudenberg and Tirole (1991), the PBE concept provides a basis for solving such games by imposing a rule of “logical consistency” on these beliefs. Specifically, these beliefs must be consistent with Bayes' Law given what has happened prior to the information set in question\(^21\). However, this concept only imposes logical consistency on the beliefs of players for actions that are on the equilibrium path i.e., there are no restrictions on the beliefs of players for actions that are off the equilibrium path. As a result, in signalling games, the PBE can generate multiple separating equilibria as well as pooling equilibria. In some cases, these equilibria are supported by off equilibrium beliefs that attribute positive probability to a seller-type choosing a strategy which strictly reduces profit. The Intuitive Criterion reduces the solution space for this type of problem by rejecting any putative equilibrium where a seller has a profit increasing deviation given that buyers attribute zero probability to a seller of quality \(Q\) playing a strategy which is equilibrium dominated (Cho and Kreps, 1987)\(^22\).

Let \(a_1\) be the menu offered by the seller i.e. \(a_1 = \{(x_G, P_G), (x_{HE}, P_{HE})\}\); let \(\mu(a_1)\) be the beliefs of buyers that a seller offering \(a_1\) is high quality; and let \(s_T\) be the strategy of buyer type \(T\)\(^23\). For each \(a_1\), define \(J(a_1)\) as the set of seller types for which: \(\pi^*(Q) > \max_{s_T} \pi^*_Q(a_1, s_G, s_{HE}, Q)\). With this definition, an equilibrium fails the Intuitive Criterion if there exists an \(a_1\) for:

A. The Seller of Quality (Q):
Such that: \(\pi^*_Q < \min_{s_G} \pi^*_Q(a_1, s_G, s_{HE}, Q)\). Let \(a_1^T\) be the set of menus where \(J(a_1)=L\). The equilibrium fails the criterion if high quality seller has any strategy (i.e. a warranty/price menu) that yields greater profit than the equilibrium profit with buyer beliefs and strategies as defined below:

\(^{21}\) Here, the PBE implies that the beliefs of the uninformed player (i.e. the buyer) must be logically consistent after she observes the menu offered by the seller.
\(^{22}\) Following Fudenberg and Tirole (1991), an action \(a_1\) for type \(Q\) seller is equilibrium dominated if the optimal profit \(\pi^*(a, Q)\) is greater than the expected profit \(\pi^*(a_1, Q)\) with buyers making optimal choices.
\(^{23}\) With two seller types, the beliefs \(\mu(\bullet)\) over all actions that a seller is high quality are complete because \(1-\mu(\bullet)\) are the beliefs that the buyer is low quality.
B. The Beliefs of Buyers:
\[ \mu(\bullet) \in \{ \mu : 1 - \mu(a^1_t) = 0 \} \]. The Intuitive Criterion requires checking the condition in point A, given that buyer’s beliefs assign zero probability to a low quality seller playing a strategy that is equilibrium dominated.

C. The Buyer’s Optimal Strategy
For buyer type \( T \), \( s_T^* = \arg \max_{s_T} U_T(a_1, \theta_T, \mu(\bullet)) \). This means that buyers are expected to choose the bundle \((x, P)\) that is optimal for them.

The solution to the game is the collection of optimal strategies for each seller-type \( \{(x^0_G, P^0_G, x^0_H, P^0_H)\} \), each buyer type \( s_T^* \) and buyer beliefs \( \mu(\bullet) \) that do not fail the Intuitive Criterion.

With only two sender-types (high and low quality) and a one-dimensional signal (warranty length), the only equilibrium not rejected by the Intuitive Criterion is a separating equilibrium with minimal inefficient signaling\(^{24}\). In fact, we show that the optimal action for the high quality seller is unique and minimizes the seller’s profit loss (versus a situation in which quality is observable).

We now discuss the mathematical construction of the problem outlined by points A, B, and C for the high quality seller.

Point C implies that any solution to the problem must involve each buyer-type choosing the price warranty bundle that optimises her surplus. Thus, we restrict our attention to warranty menus that are individually rational (both buyer types obtain positive surplus) and incentive compatible (both buyers prefer the bundle that is designed for them). Of course, without further restrictions this is the problem described in section 4.1.

Point B restricts the high quality seller’s strategies to a set of strategies \( a^1_l \) where \( J(a_1) = L \), i.e. \( a_1 \) where \( \pi^*(L) > \max_{s_T} \pi_L(a_1, s_G, s_H, L) \). This constraint implies that if a low quality seller offers a menu for which \( J(a_1) = L \), he will make less than the “guaranteed” level of profit, \( \pi^*(L) \) made by acting like a low quality seller and charging low quality prices. This is reflected in the high quality seller’s problem as a constraint which we call the ‘no mimic’ constraint:

\[ \pi^*(L) > \lambda(P^H_{HE} - x^H_{HE}c_L) + (1 - \lambda)(P^H_N - x^H_{HE}c_L) \] (11)

When the high quality seller’s optimal menu under observability is unattractive to the low quality seller, this constraint would be satisfied were the high quality seller to offer such a menu. In contrast, when the high quality seller’s menu under observability is attractive to the low quality seller, this constraint would be violated were the high quality seller to offer it.

\(^{24}\) As noted by Fudenberg and Tirole (1991), this equilibrium is known as the “Riley Outcome”.

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Point A obliges us to look for an equilibrium (with action $a_1^*$ for the high quality seller), where there exists no action $a_1$ (satisfying $J(a_1)=L$), that yields higher profit than the profit with action $a_1^*$. It is possible to identify the action $a_1^*$ by solving a constrained optimisation problem for the seller with self selection and individual rationality imposed on buyers (according to point A) and the no-mimic constraint imposed by point B. This is analogous to the high quality seller’s problem considered when quality is observable subject to the additional constraint (equation 11).

Before discussing the solution, we mention a further implication of equation 11. Given Assumption 5 and given that $B_H > B_{MIN}$, the prices for high quality products are always higher than the prices for low quality products. As a result, the only way that equation 11 can be satisfied is if the warranty lengths $x_{HE}^H$ and $x_{HE}^L$ are longer than $x_{HE}^L$ and $x_{HE}^L$. The solution to the constrained optimisation problem (in the technical appendix) leads to the following proposition:

**Proposition 3**

When product quality is unobservable:

(a) and the fixed benefit $B_H$ of a high quality product is less than $\hat{B}$, the warranty/price menu offered by a seller of high quality is identical to the one offered when product quality is observable.

(b) and the fixed benefit $B_H$ of a high quality product is greater than $\hat{B}$, the price/warranty menu offered by a seller of high quality is different from the one offered when product quality is observable.

Where: $\hat{B} = \pi^*(L) + c_L - \frac{\gamma_H \theta_G}{2} - \left(\frac{1 - \lambda}{2\gamma_H^2} \frac{c_H}{\theta} + \frac{\lambda c_H}{1 - \lambda H} \frac{\theta}{\gamma_H^2} \right)$, $\tilde{\theta} = \theta_G - \lambda \theta_{HE}$ and $\pi^*(L)$ is the profit realized by a low quality seller when quality is observable.

The expression for $\pi^*(L)$ is given by:

$$\pi^*(L) = B_L + \frac{\lambda \gamma_L \theta_{HE}}{2} \left[ \frac{(1 - \lambda) c_L^2}{\gamma_L^2 \theta_{HE}^2} + \frac{c_L^2}{\gamma_L^2 \theta_{HE}^2} \right] + \frac{\gamma_L \theta_G}{2} \left[ 1 - \frac{(1 - \lambda) c_L^2}{\gamma_L^2 \theta_{HE}^2} \right] - c_L + \frac{(1 - \lambda) c_L}{\gamma_L \theta_{HE}^2} + \frac{c_L \lambda}{\gamma_L \theta_{HE}^2}$$

Before providing the intuition for this result, we provide insight about the expression for $\hat{B}$, which appears very complex. $\hat{B}$ is a function of the relative costs of high and low quality sellers ($c_H$ and $c_L$), the relative values of warranty coverage for buyers of high and low quality respectively ($\gamma_H$ and $\gamma_L$), and $B_L$, the fixed benefit for low quality. It describes the $B_H$ above which the menu of a high quality seller under observability will be attractive to a low quality seller.

The intuition behind Proposition 3 is that when the fixed benefit for high quality is low i.e. $B_H \in \{B_{MIN}, \hat{B}\}$, the cost advantage of the high quality seller (i.e. the degree to which $c_H$ is less than $c_L$) overshadows higher prices that buyers are willing to pay for high quality (prices are a direct
function of the premium $B_H$\textsuperscript{25}. As a result, the optimal menu chosen by the high quality seller is too expensive for the low quality seller to mimic. In fact, when buyers have a high fixed benefit for low quality ($B_L$) or when the difference between the marginal costs of providing coverage is high ($c_L$ versus $c_H$), it is unlikely that a low quality seller will mimic the optimal menu of the high quality seller (under observability).

We should also discuss the role of the $\gamma_H$ and $\gamma_L$ (the marginal value of warranty protection parameters) in the equilibrium. First, the higher is $\gamma_L$ (the marginal value of warranty coverage on low quality products), the higher is $\hat{B}$ because a higher $\gamma_L$ strictly increases $\pi^*(L)$. The effect of an increase in $\gamma_H$ (the marginal value of warranty coverage on high quality products) on $\hat{B}$ is ambiguous because it has two effects. The first is to cause the high quality seller to offer more coverage making the menu (under observability) less attractive to a low quality seller. The other is to increase prices in the menu of the high quality seller and this of course, makes the menu attractive to the low quality seller. For most values of $\gamma_H$, an increase in $\gamma_H$ raises $\hat{B}$ and reduces the likelihood that a high quality seller will have to offer a menu that is different from what he would offer were quality observable. However, when either the difference between $c_H$ and $c_L$ is small or $\gamma_H$ is high, an increase in $\gamma_H$ can have the opposite effect on $\hat{B}$.

When $B_P > \hat{B}$, a low quality seller would gladly offer the menu that would be chosen by a high quality seller were quality observable. In this situation, the solution to the constrained optimisation problem for the high quality seller yields a positive Lagrangean multiplier on the no-mimic constraint (equation 11). This has two economic interpretations. First, the solution to the problem involves the high quality seller choosing a menu for which the low quality seller is indifferent between mimicking and not mimicking. This menu is different from the menu that the high quality seller would offer when quality is observable. Second, relaxing the no-mimic constraint by one unit (for example by increasing $B_L$ by one), the profit of the high quality seller would increase by an amount equal to the value of the multiplier. Thus, the profit of the high quality seller is strictly reduced due to the unobservability of quality (profit increases when the constraint is relaxed and the profit under observability is obtained by solving the problem with the constraint fully relaxed).

We now consider Proposition 4 which considers the specific actions of the high quality seller.

\textsuperscript{25} As noted earlier in the paper, $B_{MIN}$ is the minimum value of $B_H$ consistent with our basic assumptions.
**Proposition 4**

When product quality is unobservable and $B_H > B\hat{}$, the high quality seller’s menu involves more warranty protection and higher prices for all buyers (the high quality seller earns strictly greater profit by signalling with all elements of his warranty menu). Additionally, when $B_H > B\hat{}$, signalling causes a high quality seller to offer strictly shorter extended warranties ($x_{HE} - x_G$).

The intuition for Proposition 4 is that a high quality seller lengthens the warranties in his menu to make it ‘unaffordable’ to a low quality seller. Admittedly, he charges more for the bundles in question, but the higher prices do not compensate for the added cost of providing the coverage (this difference is the cost of signalling for the high quality seller). The reason that both $x_G$ and $x_{HE}$ are longer is that the premium quality seller minimises his cost of signalling by making as small movements as possible from the optimal screening menu under observability. By spreading the signalling between the two bundles in the menu, he minimises this cost. This has important implications for a high quality seller who wishes to both maximise profit (through sorting buyers) and inform buyers of his higher quality. A high quality seller will make strictly greater profit by signalling through all elements of his warranty menu and not just the base warranty. When signalling is done by the base warranty alone, the no-mimic constraint is:

$$\pi^*(L) > P_G^n - x_G^n c_L$$

(12)

It is straightforward to show that the profit for the high quality seller is strictly lower when his decisions are constrained by constraint 12 instead of constraint 11\(^{26}\).

A second implication of Proposition 4 is that extended warranties are shortened when a high quality seller has the need to signal. This occurs because the marginal value of warranty coverage is higher for gamblers than it is for hedgers under the screening menu described in section 4.1 (gamblers are obtaining less than efficient coverage). Thus, when a high quality seller is obliged to lengthen warranty coverage to distinguish himself from a low quality seller, increases in the base warranty generate greater price increases than increases in the extended warranty. Consequently, the optimal distortion involves greater increases in the base warranty ($x_G^H$) than in the coverage purchased by hedgers ($x_{HE}^H$) and extended warranties ($x_{HE}^H - x_G^H$) are shorter.

Interestingly, the shortening of extended warranties due to signalling is extreme when the fixed benefit for high quality is above a certain limit and this is outlined in Proposition 5.

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\(^{26}\) The solution to the problem subject to constraint 12 is a feasible solution to the problem subject to constraint 11. But $x_{HE}^H$ (constraint 12) $\neq$ $x_{HE}^H$ (constraint 11), therefore profit is strictly reduced.
Proposition 5

When product quality is unobservable and the fixed benefit for a high quality exceeds $B_{\text{MAX}} = \pi^*(L) + c_L - \frac{\gamma_H \theta_G}{2}$, signalling considerations eliminate the ability of a high quality seller to sort buyers and a maximum warranty of 1 is offered to all buyers.

When signalling requirements are severe, the inability of a high quality seller to sort buyers (by offering different levels of warranty coverage) occurs because of satiation in the buyers’ utility functions. As discussed earlier, we believe this to be a reasonable property for the utility function given that most durable products have a finite life. The intuition behind the result is that when $B_H > B_{\text{MAX}}$, both types of buyers are more than willing to pay $B_{\text{MAX}} + \frac{\gamma_H \theta_G}{2}$ for a high quality product with a full warranty of $x=1$. Here, the individual rationality constraints (equations 7 and 8) hold for neither buyer type (both obtain positive surplus by buying). Nonetheless, a high quality seller is prevented from charging more for his menu because of the ‘no mimic’ constraint. This contrasts with the menu that is chosen when product quality is unobservable and $B_H \in \{B_{\text{MIN}}, B_{\text{MAX}}\}$. When $B_H$ falls in this range, a hedger always receives positive surplus and a gambler’s individual rationality constraint is always binding (and hence she receives zero surplus). When $B_H > B_{\text{MAX}}$, the need to signal not only distorts the high quality seller’s menu, it also leaves a gambler strictly better off.

From the analysis it appears that there are three potential zones for $B_H$ given $B_L$, $\lambda$, $\theta_G$, $\theta_{HE}$, $\gamma_H$, $\gamma_L$, $c_H$ and $c_L$. The first is the zone $\{B_{\text{MIN}}, \hat{B}\}$ where the high quality seller’s action is unaffected by the unobservability of quality. The second is the zone $\{\hat{B}, B_{\text{MAX}}\}$ where the high quality seller needs to lengthen his warranties to signal higher quality. The third zone is where $B_p > B_{\text{MAX}}$ and the high quality seller offers one option with maximum warranty protection to everyone. If $B_H$ falls into one of these zones, then our findings certainly hold. However, there is no guarantee that all three zones exist given $B_L$, $\lambda$, $\theta_G$, $\theta_{HE}$, $\gamma_H$, $\gamma_L$, $c_H$ and $c_L$. If we examine the expression for $\hat{B}$, a low value of $\gamma_H$ has no effect on the first two terms, drives the third term to zero, and causes the final term to become large and negative. As a consequence, it is possible that $\hat{B}$ could be a value less than $B_{\text{MIN}}$ for a given set of parameters $B_L$, $\lambda$, $\theta_G$, $\theta_{HE}$, $\gamma_H$, $\gamma_L$, $c_H$ and $c_L$. When this happens, a high quality seller will be obliged to alter his menu to signal his quality for any feasible value of $B_H$. A similar argument can be used to show that for a sufficiently low value of $\gamma_H$, $B_{\text{MAX}}$ is also less than $B_{\text{MIN}}$. In this situation, only one zone exists (for all possible $B_H$), which implies that whenever quality is unobservable, a high quality seller will be obliged to offer a maximum warranty and one price to everybody.
Given parameters $B_L, \lambda, \theta_{HE}, \gamma_H, \gamma_L, c_H$ and $c_L$, it is possible to describe the outcome using Propositions 1-5 expected in a market where sellers have an incentive to sort buyers and high quality sellers wish to use warranty policy as a signal. Consider the interesting case where the high quality seller wishes to offer both buyer-types warranty protection i.e. $\lambda < \frac{\gamma_H \theta_{HE} - c_H}{\gamma_H \theta_{HE} - c_H}$ and $\gamma_H$ is high enough such that three zones described in the previous paragraph do exist. Then the equilibrium outcomes can be represented as 9 regimes in fixed benefit/cost ($B_H/c_L$) space (see Figure 3).

(Figure 3)

The legend in Figure 3 outlines the mathematical conditions that characterise each of the nine regimes. In all market conditions, a high quality seller must ensure that his menu is unattractive to a low quality seller (i.e. the menu offered by a seller of high quality seller must satisfy a no-mimic constraint).

The list of regimes in Figure 3 is complete for the assumptions that we have made. However, relaxing certain assumptions allows for even greater heterogeneity in the market outcomes. For example, in the absence of low quality sellers, a high quality seller might offer warranty protection to hedgers only (leaving the base warranty at zero). However, the need to signal can create a situation where a high quality seller not only lengthens the coverage sold to hedgers but also introduces a non-zero base warranty for gamblers.

The analysis suggests that when a high quality seller’s best signal is warranty policy, the availability of warranty coverage (both base warranties and extended warranties) will be higher than one might expect based purely on economics. Referring back to Figure 3, unless a product is of the poorest quality, a seller always offers some form of warranty coverage (even if optional) to potential buyers (the only sellers who do not offer any warranty coverage at all are low quality sellers in Regime 1).

5.0 Empirical Application

This section describes a study we conduct of warranty policy in a market where the following three conditions make the use of warranty policy, as a signal and screen, highly probable:

a) High quality sellers have a problem of trying to get a fair price because quality is variable and difficult to for buyers to evaluate i.e. a seller of high quality can easily be misidentified as a seller of low quality.

b) Buyers are heterogeneous in their valuations of warranty coverage. Similar to the model, the market must contain a spectrum of buyers from gamblers (who place little value on warranty coverage) to hedgers (who place high value on warranty coverage).
c) Alternate mechanisms used to signal high quality like reputation and advertising are not feasible. Consider how the used car market is perhaps the ideal market to satisfy these conditions.

First, quality is highly variable as evidenced by the work of Akerlof (1970) and Lacko (1986). Used car dealers obtain much of their inventory through wholesale auctions where they are given little time to inspect cars before the auction starts (Genesove, 1993). Cars at the auctions may be similar in mileage, age and appearance but are of variable quality due (primarily) to the manner in which previous owners drove and maintained them. As a result, there is significant variance in the unobservable quality of used cars both within and across dealers27. In addition, it is difficult for buyers to determine a car’s quality due to a lack of expertise, time and opportunity. Many sellers restrict potential buyers to a brief accompanied test drive. Even if a buyer is permitted to take the car for a longer period, significant time and costs are involved in obtaining an independent assessment of its quality.

Second, in the car market (new and used), buyers are heterogeneous in their valuations of warranty coverage, as evidenced by the pervasiveness of extended warranty menus that are offered to buyers (it is difficult to buy a vehicle of any kind without being offered optional levels of warranty protection)28. As mentioned earlier, when the cost of repairs is expensive (as is the case with automobiles), warranty policy become a critical element of post-sale strategy.

Third, mechanisms such as reputation and advertising are not feasible for many sellers in the used car market. Used cars are certainly sold through branded new-car dealerships and it is quite possible that these dealers rely on reputation or brandname to signal quality (as noted earlier, well-funded dealers may prune low quality vehicles from their inventory by sending them back to the auction). In addition, significant numbers of used cars are now being sold by AutoNation, Carmax and Internet services such as AutoTrader.com, where the uncertainty in buying used cars may be reduced by creating networks with standardised measures of a car’s condition29. Nevertheless, a majority of used cars are sold through poorly funded small- and medium-size dealers30. These dealers invariably have liquidity problems (for a small business, significant capital is tied up in inventory) and as previously discussed, a single dealer will likely have a variety of qualities available. Not only is such a dealer unlikely to have the funds to advertise and develop a reputation, he needs a mechanism which can be applied to some products

27 Theoretically a dealer could weed lower quality cars off his lot by sending them back to the auction, but this would entail significant transaction costs and is probably not an option for a small independent car lot.
28 We assume that the pervasiveness of optional warranty coverage in the automobile market is de facto evidence of buyer heterogeneity. It would not be plausible for sellers to offer optional coverage unless buyers in fact chose different levels.
30 In Toronto, more than 85% of the used car dealers listed in the yellow pages are both unbranded and do not engage in new car sales.
and not others (a dealer only faces the problem of getting a fair price for those cars on his lot which he knows are better quality). Warranty policy seems to be the only feasible tool available to these poorly funded dealers. In contrast to other signalling mechanisms, warranty coverage creates expenses after a car is sold (and has been paid for). In addition, because warranty policy can be varied for each car sold, it provides the opportunity for a used car salesperson (in a medium or small lot) to make an unambiguous statement about a specific vehicle. Buyers know that a dealer is more likely to offer attractive warranty policy on a car that is in good condition because the cost of honouring the policy is lower. Moreover, a used car dealer’s commitment to honour its warranty policy is credible because, as mentioned earlier, large well-funded companies underwrite the contracts.

In addition, downstream moral hazard, a factor that frequently complicates warranty provision, appears to be relatively low for used cars. As noted by Padmanabhan and Rao (1994), extended warranty contracts contain strict service schedule requirements for oil changes and tune-ups (at authorised dealers) that may even improve the regularity of buyers servicing their cars. Moreover, extended warranty contracts invariably have deductibles and exclude items that are susceptible to misuse by the buyer (like the tyres and the clutch).

Following from point c), the survey focuses on used car dealers which are unbranded and unassociated with a new car dealership. Information was collected by a “single prospective used-car buyer” over a period of two months in Metropolitan Toronto. Every data point in the survey was an independent buying situation in which the buyer looked for a car with specific features (brand, mileage, colour, number of doors, and technical features) in a distinct category. This approach was designed to mirror the reality of used car shopping and reflect the fact that when a buyer finds a car that meets her objectives, the seller gains a degree of price-setting ability. To allow grouping of data points, all cars surveyed were 5 years old and appeared to be in the same condition. We collected data from 109 dealers in five categories (we do not report data for the minivan category, because there were only seven usable observations).31

For the same reasons that a buyer cannot determine the quality of cars, neither could we. Nevertheless, the model allows us to make a number of predictions about the type of warranty policy that we should observe:

a) We should observe highly variable warranty policy for cars of a similar class, condition, age and mileage given that quality is unobservable.

b) We should observe a tendency of sellers to provide complete information about their extended warranties early in the buying process.

31 More than 109 dealers were visited but information was only gathered at dealers who had a vehicle close to the buyer’s search objectives.
c) Consistent with the model predictions, we should find evidence of a number of sellers trying to “move” buyers to longer warranty lengths\(^{32}\).

The survey’s objective is to gather data in the used car market to see if any of these predictions receive support. We should note that extended warranties are not available on cars that older than seven years and our buyer focussed on “power train” extended warranties with a standard deductible of $50. To evaluate our first prediction, we present summary statistics from the survey in Table 1.

<table>
<thead>
<tr>
<th>Car Class</th>
<th>base warranty</th>
<th>1 yr. extended warranty</th>
<th>2 yr. Extended warranty</th>
<th>3 yr. Extended warranty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Length (months)</td>
<td>Standard Deviation</td>
<td>Average Price</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Domestic Compact</td>
<td>5</td>
<td>4.5</td>
<td>371</td>
<td>183</td>
</tr>
<tr>
<td>Imported Compact</td>
<td>6.8</td>
<td>6.9</td>
<td>376</td>
<td>170</td>
</tr>
<tr>
<td>Domestic Midsize</td>
<td>4.3</td>
<td>4.2</td>
<td>399</td>
<td>168</td>
</tr>
<tr>
<td>Imported Midsize</td>
<td>4.6</td>
<td>4.4</td>
<td>362</td>
<td>129</td>
</tr>
</tbody>
</table>

Consistent with the model’s predictions, substantial variance in both the length of base warranties and the pricing of extended warranties is observed across all four categories. This variance cannot be explained by differences in the base price, the features, the options or the appearance of the cars. In addition, differences of up to 50% in the pricing of extended warranties were observed for identical cars of the same year.

Consistent with the model’s prediction that only the lowest quality sellers find themselves in a situation with no incentive to provide warranty coverage, more than 91% of all vehicles were offered with some form of base or optional warranty coverage. Figure 3 (Regimes 3, 6, and 9) suggests that high quality sellers will sometimes offer maximum coverage automatically (in this case that might be 36 months of coverage). However, no such examples were found in our survey. This may be because the unobservable difference in qualities of used cars may not be strong enough to drive high quality sellers to offer a maximum warranty.

We also recorded the content of the discussion between the prospective used car buyer and the salesperson. The objectives of this aspect of the survey were first, to see if salespeople at used car dealers spontaneously mention extended warranty options without “leading questions” from the

---

\(^{32}\) When high quality sellers need to signal, the model predicts that buyers will purchase more warranty protection and that extended warranties will be shorter. Across all dealers, extended coverage is sold at fixed points (12
prospective buyer early in the sales discussion. Second, in situations where salespeople do not mention the warranty menu spontaneously, we wanted to see if used car dealers provide details about a warranty menu with a simple question. Finally, we made a record of the dealers who had point-of-purchase materials or banners promoting various aspects of the extended warranty coverage offered on some of their cars. The results from this aspect of the survey are shown in Table 2.

Table 2
Selling Discussion Summary

<table>
<thead>
<tr>
<th>Car Class</th>
<th>% of Encounters where Full Warranty Menu Discussed*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full Warranty Menu raised automatically</td>
</tr>
<tr>
<td>Domestic Compact</td>
<td>59</td>
</tr>
<tr>
<td>Imported Compact</td>
<td>64</td>
</tr>
<tr>
<td>Domestic Midsize</td>
<td>46</td>
</tr>
<tr>
<td>Imported Midsize</td>
<td>48</td>
</tr>
</tbody>
</table>

* discussions where the dealer presented at least one optional length of warranty protection

As predicted by the model, salespeople across the 109 dealers exhibited a strong tendency to expose the complete details of their extended warranty policy early in the selling encounter. Once the buyer had shown interest in a specific car, 56% of the salespeople presented a complete warranty policy for the car without prompting. With prompting (but well before discussion of an actual sale), over 96% of the salespeople in the four categories provided complete information on extended warranty policy and pricing. In addition, 58% of all dealers surveyed actively promoted extended warranties with point of purchase materials and banners (this information is not shown in the table because it was dealer and not car specific). The survey suggests that dealers are fully aware of the ability of warranty policy to provide valuable information to buyers and they do not seem to rely on the base warranty alone to make a signal.

In order to examine whether some sellers try to “move” buyers to longer warranty lengths, we use the data collected on each car to estimate the following linear equation for each of the four categories using OLS:

\[ P_{i}^{3yr} = \beta_0 + \beta_1 BW_i + \beta_2 P_{i}^{1yr} + \beta_3 P_{i}^{2yr} + \epsilon_i \]  \hspace{1cm} (13)

months, 24 months and 36 months) so we cannot test this idea directly. There should however, be evidence of some sellers (at least) trying to shift buyers to longer warranty lengths.

33 We have restricted our analysis to extended warranties menus with 3 choices (12, 24 and 36 months) that cover the powertrain. The standard deductible in the survey was $50 per claim.
The dependent variable is $P_{3 \ yr}$, the price quoted for a 36 month extended warranty. The independent variables are the length of the base warranty $BW$, the price for the 12 month extended warranty $P_{1 \ yr}$, and the price for a 24 month extended warranty, $P_{2 \ yr}$.

We expect that the coefficient of $BW$ would be negative even in the absence of signalling because of how sellers quote prices for extended warranties: a 36 month extended warranty provides coverage for 36 months from the date of purchase. Therefore, we would expect the incremental cost for such a contract to be lower, the longer is the base coverage which a buyer obtains automatically. In the absence of signalling, we would expect the coefficients of $P_{1 \ yr}$ and $P_{2 \ yr}$ to be positive, since lower/higher quality should increase/decrease the prices of all extended warranty lengths proportionally (however, we might expect the relation to the price of the 24 month warranty to be somewhat stronger given its greater similarity to 36 months of coverage).

Evidence of signalling on the other hand, would be suggested by negative coefficients on $P_{1 \ yr}$ or $P_{2 \ yr}$. Before we consider the basis for this suggestion, we should mention that common practice amongst more than 70% of used car dealers is to offer menus with 3 choices: 12, 24 and 36 months. Thus, even though a high quality seller may wish to lengthen the warranty policy he sells to his customers (to signal higher quality), he may nonetheless feel constrained to quote menus with the three “standard” choices. To “lengthen” the warranty coverage that customers actually purchase, he might purposefully “over-price” shorter warranty coverage e.g. 12 month and reduce the price of longer coverage.

### Table 3

Model Results to Predict Price of 3 Year Extended Warranty

<table>
<thead>
<tr>
<th>Car Class</th>
<th>Intercept</th>
<th>base warranty length</th>
<th>1 yr. extended warranty price</th>
<th>2 yr. extended warranty price</th>
<th>Model Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_0$</td>
<td>T-stat p-value</td>
<td>$\beta_1$ T-stat p-value</td>
<td>$\beta_2$ T-stat p-value</td>
<td>$\beta_3$ T-stat p-value</td>
</tr>
<tr>
<td>Domestic Compact</td>
<td>-18.49</td>
<td>-1.189 ns</td>
<td>0.597 ns</td>
<td>-0.462 ns</td>
<td>1.739 ns</td>
</tr>
<tr>
<td>Imported Compact</td>
<td>-5.222</td>
<td>-0.291 ns</td>
<td>-0.267 ns</td>
<td>-0.364 ns</td>
<td>1.686 ns</td>
</tr>
<tr>
<td>Domestic Midsize</td>
<td>16.68</td>
<td>0.431 ns</td>
<td>-2.495 ns</td>
<td>-0.707 ns</td>
<td>1.833 ns</td>
</tr>
<tr>
<td>Imported Midsize</td>
<td>-783.4</td>
<td>-0.494 ns</td>
<td>-211.6 ns</td>
<td>-14.62 ns</td>
<td>13.28 ns</td>
</tr>
</tbody>
</table>

The results of the regression for each car category are shown in Table 3. First, we find that the impact of base warranty on the price of the extended warranties is insignificant in all categories. This finding is not consistent with the predictions made earlier. However, a potential explanation...
for this unusual finding is that the variation in the base warranty is relatively small (several months) compared to the length of coverage associated with the dependent variable $P^{3\ yr}$, i.e. 36 months. Second, in all four categories, we find that the coefficient of $P^{2\ yr}$ (pricing of 24-month coverage) is significant and positive. This finding is consistent with our expectations about the direction of the relationship between the pricing of different lengths of extended warranty coverage in the absence of signalling. Finally, we find that in three of the four categories, the coefficient of on $P^{1\ yr}$ is significant and negative (in the fourth category, the coefficient is also negative but not significant). This finding suggests that in three of the four categories, when dealers raise the price of 12 month extended coverage, they make reductions in the price of 36-month coverage. One explanation for this is that dealers with higher quality are purposefully trying to push people from shorter extended warranties to longer ones (by making the shorter length more expensive and the longer one cheaper). In the absence of signalling, there would be no reason for such a relationship. This finding is not incontrovertible evidence of signalling. Nevertheless, the results suggest that sellers are manipulating the prices of warranty policy in a way that is inconsistent with the simple predictions of a screening model.

6.0 Conclusion

The objective of this paper has been to understand how a high quality seller can use warranty policy to screen potential buyers and also inform these buyers about the quality at the same time. We argue that this is an important role for warranty policy because it is the only signal available to a seller that is effectively self-financing. The costs of servicing a warranty contract are only incurred after a sale has been made and funds have been received from a buyer. Moreover, warranty policy also offers two further advantages over other possible signals:

1. Warranty policy is flexible because it can be varied for any product that a seller has. When a seller has multiple products of different qualities, this allows a seller to make a signal that is product (or even item) specific.

2. Because warranties are contracts that can be underwritten by a third party, they can be credible signals for sellers who are completely unknown or who appear to be in financial difficulty.

We contend that this can make warranty policy the signalling mechanism of choice in a market where product quality is variable and difficult for buyers to evaluate and sellers are small, independent, and relatively local. Many markets fit this description, most notably any market for used durable goods. This includes large markets like used cars (the largest durable goods market in the U.S.) and trucks, but also includes smaller markets such as used pleasure craft (sail and motor boats), used industrial equipment and used appliances.

In addition, whenever buyers are heterogeneous in their preferences for warranty coverage and they are not able to determine the quality of components in a product that is manufactured by a small business, warranty policy may have a dual role. The market for unbranded personal
computers, where many small manufacturers import components from the Far East and assemble them, is such a market. Warranty policy is an ideal signal because it is clearly more expensive for a manufacturer which installs low quality components in its computers to offer the same length of warranty as a firm which uses better components. At the same time, all manufacturers (regardless of the quality of computer they are selling) have an incentive to offer an extended warranty option to extract additional profit from those purchasers who are more risk averse.

The model in this paper provides insight into the decisions faced by a high quality seller that is attempting to use warranty to both signal and screen. In fact, the model makes several predictions about the impact of signalling with warranty policy that are somewhat counterintuitive. These predictions relate specifically to the three questions we suggested in section 1.0.

1. *Should high quality products always have longer warranties than low quality products?*

The model shows that when warranty policy is being used to signal as well as to screen consumers, the warranty coverage offered by high quality consumers is unequivocally longer. This is so because a high quality seller needs to choose a menu that places an additional cost on a low quality seller who considers pretending to be high quality. A high quality seller does this by offering warranty lengths that are longer than those that a low quality seller would offer when he “acts” like a low quality seller. In contrast, without signalling requirements on warranty policy, it is quite possible for the warranty coverage on a high quality product to be shorter than the coverage offered on a low quality product (since the value of warranty coverage is lower on a product which fails less).

2. *Will the need to communicate information with warranty policy lengthen or shorten extended warranties?*

The analysis shows that signalling either shortens or has no effect on the length of extended warranties offered by high quality sellers. When a high quality seller is forced to alter his warranty menu to signal, all buyer types will purchase more coverage. When buyers are offered more coverage, the lengthening of the base warranty is greater than the lengthening of the extended coverage. As a result, the extended warranties (or optional coverage) purchased by buyers who desire more coverage are *strictly shorter*. In fact, the model predicts that when the difference in buyers’ valuations between high and low quality is large, a high quality seller may offer the same warranty coverage to all buyers (warranty policy’s only role in such a situation is to signal).
3. Should a high quality seller tell all buyers about the price of extended warranties even if a good percent of buyers are not interested?

As noted earlier, it is not clear how a high quality seller might use his warranty policy to signal. It is plausible that a high quality seller might choose to have the base warranty “do all the work” in terms of sending a message to potential buyers. The model demonstrates that this strategy is sub-optimal. A high quality seller can earn strictly greater profits by using all elements of his warranty policy to signal. In fact, the high quality seller should inform all buyers about the pricing of his entire menu even if a minority of buyers actually buys the extension.

To provide empirical support for the model developed in this paper, we first identify several conditions necessary for warranty policy to be used simultaneously to both screen and signal. Second, we explain how the used car market is an ideal market to meet these conditions. Finally, we conduct a survey of the warranty policy observed in the Toronto used car market. The purpose of the survey was to collect information that provides insight into the major predictions made in the analytical model.

The survey shows that there is considerable variance in the warranty policy offered across similar cars in both the length of base warranties provided and the pricing of extended coverage. This variance cannot be explained by differences between the cars in terms of options, the brand, or the car’s apparent conditions.

Second, the survey provides strong evidence that dealers in the used car market understand both the potential of warranty policy to act as a signal and the advantage of using the entire menu to signal (and not just a piece of it). In 56% of encounters, the salesperson presented the complete details of warranty policy and a further 40% (96% total) provided these same details with minimal encouragement. Their conduct suggests that sellers are well aware of how warranty policy can be used to fulfil two roles simultaneously.

Finally, as might be suggested by the need to signal, the survey provides some evidence that a number of dealers seem to be pushing buyers to longer warranty lengths. Even in the context of a rather rigid framework (where buyers generally offer 12, 24 and 36 months of optional coverage), we find that more expensive 12 month coverage seems to associated with cheaper “more attractive” pricing on longer coverage. Further investigation of this preliminary finding would be a fruitful area for further research.

We recognise that our research suggests that whenever sellers are poorly funded or cannot use other “more expensive” signalling mechanisms, warranties seem to be the perfect answer. Accordingly, it is also important to highlight limitations of the research i.e. situations that argue against the usefulness of warranty policy. There are many durable products where misuse cannot be distinguished from defective manufacturing. As a result, warranties are limited on sporting
equipment such as skis, tennis racquets and hockey sticks or garden products such as weed killer. Second, there are many products where the lack of a repair network makes the concept of extended warranty coverage unrealistic. What makes used cars such an ideal product is that the repair network is amongst the most sophisticated of any product class. Third, there are many products which are used so quickly that a warranty is somewhat meaningless (for example, fireworks). Finally, there are many low priced items from children’s toys to lighters to toothbrushes that are certainly durable yet extended warranties are not economic: the transaction costs of administering a warranty probably exceed the product’s value. In spite of these limitations, we see that there are nonetheless a great many categories where warranty policy has the potential to play a dual role.

In fact, the relevance of the model developed in this paper may be broader than that of explaining when and how warranty policy may play a dual role. The reason that warranty policy can be used to screen and signal simultaneously is that it is a valuable attribute (to buyers) and can be metered. There are other markets where a) buyers desire an attribute which can be metered and b) the quality of a good or service is variable and difficult for buyers to evaluate in advance. Several notable examples include the service contracts offered by firms on industrial equipment, service contracts offered to home owners on heating or air conditioning systems, different coverage plans offered by health maintenance organisations (HMO’s) in the United States and different redemption provisions on financial assets of unknown riskiness. In all of these cases, the seller of the product or service knows more about the quality of the product or service than the buyer. In addition, the seller can offer variable amounts of positive attribute (degree/extent of service contracts; the size of the HMO’s network and the number of medical procedures covered, or the penalty associated with the early redemption of an asset) and these attributes are generally cheaper for a high quality seller to provide. An interesting extension to this study would be to examine the phenomenon of simultaneous signalling and screening in a broader context across several markets.
References


Technical Appendix for High Quality Sellers and Warranties: Sorting Customers and getting a Fair Price at the Same Time

Proof of Proposition 1

When product quality is observable, the profit maximising action for the seller is to offer a menu of price/warranty combination where each buyer-type self selects to a bundle designed for her.

To prove this proposition, we find the optimal arguments for equation 3 subject to the constraints of equations 5, 6, 7, and 8 (all in the main text). With the assumptions, we have made equation 7 is redundant. The logic is as follows: if equation 8 is satisfied then \( B_Q + \theta_{HE} V(x_{HE}) - P_{HE} > 0 \) since \( \theta_{HE} > \theta_G \). Thus, equation 5 implies that \( B_Q + \theta_{HE} V(x_{HE}) - P_{HE} > 0 \) so we can drop equation 7. We now write the problem as a Lagrangean function for equation 3 subject to equations 5, 6, and 8 (\( B_Q \) drops out of the third and fourth terms of this expression).

\[
L = \lambda \left( P_{HE} - c_Q x_{HE} \right) + \lambda \left( P_G - c_Q x_G \right) + \mu_1 \left( \gamma_Q \theta_{HE} V(x_{HE}) - P_{HE} - \gamma_Q \theta_{HE} V(x_G) + P_G \right) + \mu_2 \left( \gamma_Q \theta_G V(x_G) - P_G - \gamma_Q \theta_G V(x_{HE}) + P_{HE} \right) + \mu_3 \left( B_Q + \gamma_Q \theta_G V(x_G) - P_G \right)
\]

(1)

The Kuhn Tucker conditions for this problem in terms of \( x_{HE}, x_G, P_{HE} \) and \( P_G \) are:

\[
\frac{\partial L}{\partial x_{HE}} = \lambda (c_Q) + \mu_1 \gamma_Q \theta_{HE} V'(x_{HE}) - \mu_2 \gamma_Q \theta_G V'(x_{HE}) \leq 0
\]

with complementary slackness condition \( \frac{\partial L}{\partial x_{HE}} x_{HE} = 0 \)

(2)

\[
\frac{\partial L}{\partial x_G} = (1 - \lambda)(-c_Q) - \mu_1 \gamma_Q \theta_{HE} V'(x_G) + \mu_2 \gamma_Q \theta_G V'(x_G) + \mu_3 \gamma_Q \theta_G V'(x_G) \leq 0
\]

with complementary slackness condition \( \frac{\partial L}{\partial x_G} x_G = 0 \)

(3)

\[
\frac{\partial L}{\partial P_{HE}} = \lambda - \mu_1 + \mu_2 \leq 0
\]

with complementary slackness condition \( \frac{\partial L}{\partial P_{HE}} P_{HE} = 0 \)

(4)

\[
\frac{\partial L}{\partial P_G} = 1 - \lambda + \mu_1 - \mu_2 - \mu_3 \leq 0
\]

with complementary slackness condition \( \frac{\partial L}{\partial P_G} P_G = 0 \)

(5)

If positive warranty coverage is sold to the hedger, Equation 2 implies that \( \mu_1 > 0 \), because \( \gamma_Q \theta_{HE} V'(x_{HE}) = \gamma_Q \theta_{HE} (1 - x_{HE}) > 0 \) for the allowable range of \( x_{HE} \). Assuming non-zero prices, I add conditions 3 and 4 to obtain:

\[
\lambda - \mu_1 + \mu_2 + 1 - \lambda + \mu_1 - \mu_2 - \mu_3 = 0 \quad \therefore \quad \mu_3 = 1
\]

(6)
The next step in this solution is to prove that equation 6 in the main text is not binding (i.e. \( \mu_2 = 0 \)). First, add equations 5 and 6 in the main text:

\[
\gamma_Q \theta_{HE} V(x_{HE}) - \gamma_Q \theta_{HE} V(x_G) \geq \gamma_Q \theta_G V(x_{HE}) - \gamma_Q \theta_G V(x_G)
\]  

(7)

Evaluating the buyer utility function at \( x \), we know that:

\[
\gamma_Q \theta_f V(x) = \gamma_Q \theta_f \left[ \frac{1-(1-x)^2}{2} \right]
\]

(8)

Substituting into equation 7 and simplifying, we obtain:

\[
(\theta_{HE} - \theta_G) \left[ \frac{1-(1-x_{HE})^2}{2} \cdot \frac{1-(1-x_G)^2}{2} \right] \geq 0
\]

(9)

\[
(\theta_{HE} - \theta_G) \left[ \frac{(1-x_G)^2 - (1-x_{HE})^2}{2} \right] \geq 0
\]

(10)

Because \( \theta_{HE} - \theta_G > 0 \), equation 10 implies that \( x_{HE} \) is greater than or equal to \( x_G \). We now rewrite constraint 5 from the main text:

\[
P_{HE} - P_G \geq \gamma_Q \theta_{HE} V(x_{HE}) - \gamma_Q \theta_{HE} V(x_G)
\]

(11)

\[
\therefore P_{HE} - P_G \geq \gamma_Q \theta_{HE} \int_{x_G}^{x_{HE}} V'(x) \, dx
\]

(12)

But \( V'(x) = 1-x \) which implies that: \( \therefore P_{HE} - P_G \geq \gamma_Q \theta_{HE} \int_{x_G}^{x_{HE}} 1-x \, dx \). Therefore,

\[
P_{HE} - P_G > \gamma_Q \theta_{HE} \int_{x_G}^{x_{HE}} 1-x \, dx \quad \text{strictly. This implies that}
\]

\[
(P_{HE} - P_G) > \gamma_Q \theta_G V(x_{HE}) - \gamma_Q \theta_G V(x_G) \Rightarrow \gamma_Q \theta_G V(x_G) - P_G > \gamma_Q \theta_G V(x_{HE}) - P_{HE}
\]

(13)

Thus, \((x_G, P_G) \neq (x_{HE}, P_{HE})\) and it is optimal to offer a different bundle to each type. Q.E.D.

**Proof of Proposition 2**

When quality is observable, the profit maximising menu for the seller has:

(c) A bundle designed for the hedger with warranty protection of efficient length and a price which leaves her with strictly positive utility.

(d) A bundle for the gambler with a warranty that is shorter than the efficient length and a price which leaves her indifferent between buying or not.

Equation 13 of this technical appendix implies that \( \mu_2 = 0 \). Therefore equation 4 implies that \( \mu_1 = \lambda \). Substituting these values into equation 2 we obtain:

\[
0 = \lambda(-c_Q) + \lambda \gamma_Q \theta_{HE} V(x_{HE}) \quad \therefore V'(x_{HE}) = 1-x_{HE} \Rightarrow x_{HE} = 1 - \frac{c_Q}{\gamma_Q \theta_{HE}}
\]

(14)

This is the socially optimal length of warranty protection for the hedger (this can be checked by setting the marginal benefit of warranty protection equal to the marginal cost of providing it). Substituting into equation 3 we obtain:

\[
0 = (1-\lambda)(-c_Q) - \lambda \gamma_Q \theta_{HE} V(x_G) + \theta_G \gamma_Q V'(x_G) \quad \therefore V'(x_G) = 1-x_G \Rightarrow x_G = 1 - \frac{(1-\lambda)c_Q}{\gamma_Q \theta_G - \lambda \gamma_Q \theta_{HE}}
\]

(15)
It is easy to show that this is less than the socially optimal length for the gambler $x_G^*$ where: $x_G^* = 1 - \frac{c_Q}{\gamma_Q \theta_G}$. (Note: using equation 15, $x_G > 0$ when $\lambda < (\gamma_Q \theta_G - c_Q)/(\gamma_Q \theta_{HE} - c_Q)$. See footnote 10 in the main text). Because $\mu_3 = 1$, equation 8 in the main text binds. Thus, the gambler is indifferent between buying and not buying. Using equation 8 in the main text:

$B_Q + \gamma_Q \theta_G V(x_G) \cdot P_G = 0 \Rightarrow B_Q + \gamma_Q \theta_{HE} V(x_G) \cdot P_G > 0$ strictly but equation 5 implies that $B_Q + \gamma_Q \theta_{HE} V(x_G) \cdot P_G \geq B_Q + \gamma_Q \theta_{HE} V(x_G) \cdot P_G : B_Q + \gamma_Q \theta_{HE} V(x_G) \cdot P_G > 0$ strictly. This means that the hedger is left with strictly positive surplus. Q.E.D.

**Explicit Expression for Prices**

Substituting the value for $x_L$ in equation 15 into equation 8 in the main text, we obtain:

$$P_L = B_Q + \frac{\gamma_Q \theta_G}{2} \left[ 1 - \frac{(1 - \lambda)^2 c_Q^2}{\gamma_Q^2 (\theta_G - \lambda \theta_{HE})^2} \right]$$  \hspace{1cm} (16)

Because $\mu_1 = \lambda$, equation 5 in the main text holds with strict equality i.e.

$B_Q + \gamma_Q \theta_{HE} V(x_{HE}) \cdot P_{HE} = B_Q + \gamma_Q \theta_{HE} V(x_G) \cdot P_G$. Substituting the values of $x_{HE}$, $x_G$ and $P_G$ that obtain from equations 14, 15, and 16 respectively, we can use this equation to derive:

$$P_{HE} = B_Q + \frac{\gamma_Q \theta_{HE}}{2} \left[ \frac{(1 - \lambda)^2 c_Q^2}{\gamma_Q^2 (\theta_G - \lambda \theta_{HE})^2} - \frac{c_Q^2}{\gamma_Q^2 \theta_{HE}^2} \right] + \frac{\gamma_Q \theta_G}{2} \left[ 1 - \frac{(1 - \lambda)^2 c_Q^2}{\gamma_Q^2 (\theta_G - \lambda \theta_{HE})^2} \right]$$  \hspace{1cm} (17)

To obtain the explicit expression for seller profit, the values of $x_{HE}$, $x_G$, $P_G$, and $P_{HE}$ are substituted into the objective function given by equation 3 in the main text.

**Proof of Proposition 3**

When product quality is unobservable:

(c) and the fixed benefit $B_H$ of a high quality product is less than $\hat{B}$, the warranty/price menu offered by a seller of high quality is identical to the one offered when product quality is observable.

(d) and the fixed benefit $B_H$ of a high quality product is greater than $\hat{B}$, the price/warranty menu offered by a seller of high quality is different from the one offered when product quality is observable.

The problem for a high quality seller when his quality is unobservable is analogous to the constrained optimisation problem of equations 3,5,6,7 and 8 with the added constraint of equation 11 (all in the main text). The Lagrangean function for this problem is:

$$L = \lambda [ P_{HE} - c_H x_{HE} ] + \lambda [ P_G - c_H x_G ] + \mu_1 [ \gamma_H \theta_{HE} V(x_{HE}) \cdot P_{HE} \cdot \gamma_H \theta_{HE} V(x_G) + P_G ] + \mu_2 [ \gamma_H \theta_G V(x_G) \cdot P_G - \gamma_H \theta_G V(x_{HE}) + P_{HE} ] + \mu_3 [ \pi^*(L) - \lambda (P_{HE} - x_{HE} c_L) - (1 - \lambda)(P_G - x_G c_L) ]$$  \hspace{1cm} (18)

where: $\pi^*(L) = B_L + \frac{\lambda \gamma_H \theta_{HE}^2}{2} \left[ \frac{(1 - \lambda)^2 c_L^2}{\gamma_H \theta_{HE}^2} - \frac{c_L^2}{\gamma_H \theta_{HE}^2} \right] + \gamma_H \theta_G \left[ 1 - \frac{(1 - \lambda)^2 c_L^2}{\gamma_H \theta_{HE}^2} \right] - c_L + \frac{(1 - \lambda)^2 c_L^2}{\gamma_H \theta_{HE}^2} + \frac{c_L^2 \lambda}{\gamma_H \theta_{HE}^2}$

The Kuhn Tucker conditions for this problem in terms of $x_{HE}$, $x_G$, $P_{HE}$ and $P_G$ are:
\[
\frac{\partial L}{\partial x_{HE}} = \lambda (-c_{H}) + \mu_{H} \gamma_{H} \theta_{HE} V(x_{HE}) - \mu_{2} \gamma_{H} \theta_{G} V(x_{HE}) + \mu_{4} \lambda c_{L} \leq 0
\]

with complementary slackness condition \( \frac{\partial L}{\partial x_{HE}} x_{HE} = 0 \) \hspace{1cm} (19)

\[
\frac{\partial L}{\partial x_{G}} = (1 - \lambda)(-c_{H}) - \mu_{H} \gamma_{H} \theta_{HE} V(x_{G}) + \mu_{2} \gamma_{H} \theta_{G} V(x_{G}) + \mu_{4}(1 - \lambda)c_{L} \leq 0
\]

with complementary slackness condition \( \frac{\partial L}{\partial x_{G}} x_{G} = 0 \) \hspace{1cm} (20)

\[
\frac{\partial L}{\partial P_{HE}} = \lambda - \mu_{1} + \mu_{2} - \mu_{3} \lambda \leq 0
\]

with complementary slackness condition \( \frac{\partial L}{\partial P_{HE}} P_{HE} = 0 \) \hspace{1cm} (21)

\[
\frac{\partial L}{\partial P_{G}} = 1 - \lambda + \mu_{1} - \mu_{2} - \mu_{3} (1 - \lambda) \leq 0
\]

with complementary slackness condition \( \frac{\partial L}{\partial P_{G}} P_{G} = 0 \) \hspace{1cm} (22)

For any \( \lambda, c_{H}, \gamma_{H}, \theta_{HE}, \) and \( \theta_{G} \), there are unique warranty lengths \( (x_{G} \text{ and } x_{HE}) \) in the optimal menu under observability (equation 14 and 15). Only the prices \( (P_{G} \text{ and } P_{HE}) \) under observability (equation 16 and 17) depend on \( B_{H} \). Using the no-mimic condition, we find a unique \( B_{H} (\hat{B}) \) given \( \lambda, c_{H}, \gamma_{H}, \theta_{HE}, \) and \( \theta_{G} \) and \( \pi_{L}^{\text{max}} \) that solves \( \pi^{*}(L) = \lambda(P_{HE} - x_{HE} c_{L}) + (1 - \lambda)(P_{G} - x_{G} c_{L}) \) where \( x_{G}, x_{HE}, P_{G}, \) and \( P_{HE} \) are given by equations 14, 15, 16 and 17.

\[
\hat{B} = \pi^{*}(L) + c_{L} - \frac{\gamma_{G} \theta_{G}}{2} - (2c_{L} - c_{H}) \left( \frac{(1 - \lambda)^{2} c_{H} - \lambda c_{H}}{2 \gamma_{H} \tilde{\theta}} \right) \text{ where } \tilde{\theta} = \theta_{G} - \lambda \theta_{HE}
\]

(23)

Whenever \( B_{H} \) exceeds \( \hat{B} \), we know that the no mimic constraint is violated if the high quality seller chooses \( x_{G}, x_{HE}, P_{G}, \) and \( P_{HE} \) as given by equations 14, 15, 16 and 17 because \( P_{G} \) and \( P_{HE} \) are strictly increasing functions of \( B_{H} \). Conversely, when \( B_{H} \) is less than \( \hat{B} \), the no mimic constraint will not be violated and \( \mu_{4} = 0 \). In this situation, the no-mimic constraint does not bind and the high quality seller’s menu is unaffected by the presence of low quality.

When \( B_{H} > \hat{B}, \mu_{4} > 0 \) and \( \pi^{*}(L) = \lambda(P_{HE}^{H} - x_{HE}^{H} c_{S}) + (1 - \lambda)(P_{G}^{H} - x_{G}^{H} c_{S}) \). Note in this case, the superscript \( H \) indicates that the high quality seller chooses a menu that is different from the menu he chooses when quality is observable. Q.E.D.

**Proof of Proposition 4**

*When product quality is unobservable and \( B_{H} > \hat{B}, \) the high quality seller’s menu involves more warranty protection and higher prices for all buyers (the high quality seller is earns strictly greater profit by signalling with all elements of his warranty menu). Additionally, when \( B_{H} > \hat{B}, \) signalling causes a high quality seller to offer strictly shorter extended warranties ( \( x_{HE} - x_{G} \)).*
To simplify notation, we dispense with the superscript H introduced above. We now derive the optimal menu assuming that quality is unobservable. We add equation 21 and 22 to obtain:

$$1 - \mu_2 \cdot \mu_4 = 0 \Rightarrow \mu_4 = 1 - \mu_3$$  \hspace{1cm} (24)

As with all Kuhn Tucker problems, the Lagrangean multipliers are restricted to non-negative values. Therefore, equation 24 implies that $$\mu_4 \in [0,1]$$ and we rewrite equation 21,

$$\{ \lambda(1 - \mu_4) + \mu_2 \cdot \mu_1 = 0 \}.$$  \hspace{1cm} The term in braces is clearly non-negative, therefore $$\mu_1 \geq 0$$. Unless $$\mu_4 = 1$$, $$\mu_1 > 0$$ strictly (we will show later that the maximum value for $$\mu_4$$ is $$c_H/c_L < 1$$).

Assuming $$\mu_1 > 0$$, we now prove that $$\mu_2 = 0$$. We add the incentive compatibility constraints (equation 5 and 6 in the main text) to obtain:

$$B_H + \gamma_H \theta_{HE} V(x_{HE}) - P_{HE} + B_H + \gamma_H \theta_G V(x_G) - P_G \geq B_H + \gamma_H \theta_{HE} V(x_{HE}) - P_{HE} + \gamma_H \theta_G V(x_G) - P_{G} \Rightarrow \gamma_H (\theta_{HE} - \theta_G) [V(x_{HE}) - V(x_G)] > 0.$$  \hspace{1cm} Since $$\gamma_H > 0$$ and $$\theta_{HE} - \theta_G > 0$$, $$V(x_{HE}) - V(x_G) > 0$$. This implies that $$x_{HE} \geq x_G$$. Since $$\mu_1 > 0$$, equation 5 in the main text holds with strict equality.

Simplifying this equation and rearranging we obtain: $$\gamma_H \theta_{HE} [V(x_{HE}) - V(x_G)] = P_{HE} - P_G.$$  Therefore, $$\gamma_H \theta_G (V(x_{HE}) - V(x_G)) < P_{HE} - P_G$$ strictly $$\Rightarrow$$ $$B_H + \gamma_H \theta_G V(x_G) - P_G > B_H + \gamma_H \theta_G V(x_{HE}) - P_{HE}$$ strictly and this proves that $$\mu_2 = 0$$.

Since $$\mu_2 = 0$$ and $$\mu_4 = 1 - \mu_3$$, equation 22 implies that $$\mu_1 = \lambda \mu_3$$. We can now substitute these values into equation 19 to obtain an expression for $$\mu_3$$ in terms of $$x_{HE}$$:

$$\mu_3 = \frac{c_H - c_L}{\gamma_H \theta_{HE} (1 - x_{HE}) - c_L}$$  \hspace{1cm} (25)

We can now use equation 23 and the identity $$\mu_1 = \lambda \mu_3$$, to write expressions for $$\mu_1$$ and $$\mu_4$$ in terms of $$x_{HE}$$:

$$\mu_1 = \frac{\lambda (c_H - c_L)}{\gamma_H \theta_{HE} (1 - x_{HE}) - c_L}, \mu_4 = \frac{\gamma_H \theta_{HE} (1 - x_{HE}) - c_L}{\gamma_H \theta_{HE} (1 - x_{HE}) - c_l}$$  \hspace{1cm} (26)

Because $$\mu_1 > 0$$, $$\gamma_H \theta_{HE} (1 - x_{HE}) - c_L < 0$$ because $$\lambda (c_H - c_L) < 0$$ by definition. Therefore,

$$\therefore \mu_4 > 0 \Rightarrow \gamma_H \theta_{HE} (1 - x_{HE}) - c_L < 0 \Rightarrow x_{HE} > 1 - \frac{c_H}{\gamma_H \theta_{HE}}$$  \hspace{1cm} (27)

Thus, $$x_{HE}$$ is strictly longer than the efficient length when $$\mu_4 > 0$$. Equations 20 can be used to derive an expression for $$x_G$$ as a function of $$x_{HE}$$:

$$x_G = 1 - \frac{(1 - x_{HE})(1 - \lambda) \theta_{HE}}{\theta}$$  \hspace{1cm} (28)

Note that when the efficient length of $$x_{HE}$$ (equation 14) is substituted into equation 25, the value of $$x_G$$ is equivalent to the value of $$x_G$$ in the menu when quality is observable (equation 15). Since $$x_{HE}$$ is longer than the efficient length (equation 27), $$x_G$$ is longer than the warranty length offered to the low type when quality is observable. Because $$\mu_1$$ and $$\mu_3$$ are greater than zero equations 5 and 8 in the main text hold strictly and we use these to express $$P_{HE}$$ and $$P_G$$ as functions of $$x_{HE}$$.

$$P_G = B_H + \frac{\gamma_H \theta_G}{2} - \frac{\gamma_H \theta_G \theta_{HE}^2}{2 \theta} (1 - \lambda)^2 (1 - x_{HE})^2$$  \hspace{1cm} (29)

$$P_{HE} = B_H + \frac{\gamma_H \theta_G}{2} + \left[ (\theta_{HE} - \theta_G) \frac{\gamma_H \theta_{HE}^2}{2 \theta} (1 - \lambda)^2 - \frac{\gamma_H \theta_{HE}}{2} \right] (1 - x_{HE})^2$$  \hspace{1cm} (30)
Both $P_G$ and $P_{HE}$ are decreasing functions of $x_{HE}$ in the range (0,1). When the efficient length of $x_{HE}$ is substituted into equations 29 and 30, the expressions reduce to equations 16 and 17 (the prices when quality is observable). Since $x_{HE}$ is longer than the efficient length, $P_G$ and $P_{HE}$ are greater than the prices observed when quality is observable.

Using equation 28, $x_{HE} - x_G = (\theta_{HE} - \theta_G)(1-x_G)/(1-\lambda)\theta_{HE}$. If we take the derivative of this expression with respect to $x_G$, we obtain $\frac{\partial(x_{HE} - x_G)}{\partial x_G} = \frac{\theta_{HE} - \theta_G}{(\lambda-1)\theta_{HE}} < 0$. Hence, since $x_G$ is longer than the warranty length offered to gamblers when quality is observable, the extended warranty is shorter. Q.E.D.

Note: to solve for $x_G$, $x_{HE}$, $P_G$, and $P_{HE}$ explicitly, substitute equations 28, 29, and 30 into the no-mimic constraint (equation 11 in the main text) to obtain a quadratic expression in $x_{HE}$.
Proof of Proposition 5
When product quality is unobservable and the fixed benefit for high quality exceeds \( B_{\text{MAX}} = \pi^*(L) + c_L - \frac{\gamma_H \theta_G}{2} \), signaling considerations dominate screening considerations and a high quality seller offers the maximum warranty length to all buyers.

Step 1 When \( x_{HE} = 1 \), show that \( x_G = 1 \) and \( P_{HE} = P_G = B_H + \frac{\gamma_H \theta_G}{2} \). When \( \mu_4 > 0 \), then equation 28 implies that \( x_G = 1 \). Substituting \( x_{HE} = 1 \) into equations 29 and 30 produces the desired result.

Step 2 The maximal mimicking cost is placed on a low quality seller by choosing a warranty length of 1.

Step 3 The value of \( B_H \) which necessitates maximal signalling can be determined by substituting equations 28, 29, and 30 into the no-mimic constraint (equation 11 in the main text), setting \( x_{HE} = 1 \) and solving for \( B_H \). We call this expression \( B_{\text{MAX}} = \pi^*(L) + c_L - \frac{\gamma_H \theta_G}{2} \). Q.E.D.

Note: When \( B_H = B_{\text{MAX}} \), the values of the Lagrangean multipliers are \( \mu_1 = \lambda (c_L - c_H) \),

\( \mu_2 = (1 - \lambda) \frac{c_H}{c_L} \), \( \mu_3 = \frac{c_L - c_H}{c_L} \), \( \mu_4 = \frac{c_H}{c_L} \). Thus, the maximum value of \( \mu_4 \) where the constrained optimization problem yields an interior solution is \( c_H/c_L \).

When \( B_H > B_{\text{MAX}} \), the individual rationality constraint does not bind for either consumer type and the optimal price is found by solving \( \pi^*(L) = P - c_L \). This is simply the reduced form of equation 11 in the main text when \( (x_G, P_G) = (x_{HE}, P_{HE}) \).