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EXECUTIVE STOCK OPTIONS: EARLY EXERCISE PROVISIONS AND RISK-TAKING INCENTIVES

by

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Abstract

Traditional executive stock option plans vest gradually over a period of several years and may climb deep In-The-Money long before the manager is allowed to exercise them. We analyse the risk-taking incentives this may create and show when these may become counter-productive. We show how vesting can be used to efficiently re-optimise incentives and we derive optimal vesting schedules as a function of stock price gains. When firms face risky-but-profitable Growth Opportunities we find that they should allow more options to vest the deeper they move In-The-Money, a result we term 'Progressive' Performance Vesting. We examine the implications for commonly observed option schemes and make normative predictions as to the design and improvement of effective Executive Stock Option compensation plans.

JEL classification: G31, J33.

Keywords: Executive compensation, stock options, vesting, early exercise, corporate investment decisions, risk taking.

1 Introduction

In this paper we address the question "Why are executives allowed, if not encouraged, to exercise options immediately upon vesting rather than holding them until expiration?" (Murphy (1999)). We offer an explanation based on the dynamic risk-taking incentives induced by stock options. Furthermore, we pose and answer our own complementary question, "*How many* options should the executive be allowed to exercise early?"

During the course of the 1990's, the single largest component of CEO compensation has become executive stock options. These options have a long-term (ten year) expiration date, yet typically 'vest' gradually in fixed proportions over the first few years. Once an option has vested the manager is permitted freely to exercise at his own discretion and it is the design of these vesting terms which interests us here. Finance theory suggests that firms grant stock option incentives as the carefully considered contractual solution to agency problems such as managerial effort aversion, short-termism, or aversion to the selection of risky projects, yet it is remarkable how little variation is found in the vesting terms of option plans. It also seems puzzling that the firm effectively 'hands over' to the manager substantial control over his own incentives, allowing him to unilaterally modify them by exercising options at a time of his own choosing and in his own interests.

In this paper we attempt to reconcile the practice with the theory by showing how early vesting can work in the firm's favour. A large theoretical and empirical literature finds that the convex payoff profile provided by stock options can be used to induce risk averse managers to take more risky-but-profitable projects. However, the risk-taking incentives provided by options *decrease* as the options move In-The-Money and, perversely, may subsequently make a risk-averse manager even more reluctant to select risky-but-profitable projects. We claim that permitting early exercise may mitigate this problem, removing those incentives which have become most counter-productive and rendering more effective the firm's attempts to re-establish correct incentives via the issue of new options. We apply this intuition to solve a Risk Avoidance Agency Problem where the risk neutral Principal (The 'Firm'[1]) faces risky-but-profitable Growth Opportunities and yet the risk averse Agent (The 'Manager') may underinvest in such projects due to his exposure to performance based pay. We derive optimal vesting schedules as a function of stock price gains achieved and show how the vesting profile of options can be used to help the firm manage efficiently the convexity of

[1] We shall usually refer to the Principal as 'The Firm', understanding that this means the shareholders, perhaps represented by a Board of Directors or by a Compensation Committee and implicitly abstracting from any agency problems or conflicts of interests between such parties.

executive compensation.

Whilst previous literature has documented variation in vesting terms of options (Aboody (1996)) and sought to explain it in terms of investment horizon (Kole (1997)), we believe ours to be the first paper which explains vesting in terms of risk-taking incentives. And whereas theoretical work (Lambert, Larcker and Verrechia (1991), Carpenter (2000)) has recognised the potentially counterproductive incentives offered by In-The-Money options, these papers have considered European options only and so have not analysed the possibility of rectifying incentives through an early exercise provision. Our research question could be interpreted as conceptually analogous to the problem of option 'repricing'. When stock prices suffer large decreases leaving executive options deep Out-of-The-Money, firms face the problem that managerial incentives may become suboptimal or perverse and one controversial 'solution' is to reduce the exercise price of these options back downwards to re-establish incentives. We consider the converse problem when stock prices have enjoyed large increases and our main results are as follows:

-Holding 'old' unexercised, deep-In-The-Money options can make a manager more risk averse and cause him to pass up risky-but-profitable projects. This problem is more likely to occur the more options he has outstanding and the deeper they are In-The-Money.

-In the absence of early exercise, this problem can be solved by granting large amounts of 'new' options. This solution becomes rapidly more expensive to the firm (requires many more new options), the deeper In-The-Money are the unexercised old options and the more of them that are outstanding.

-Allowing early exercise of some of these old options mitigates the risk avoidance problem, necessitating the granting of *fewer* new options than would otherwise be required and so making its solution cheaper for the firm. Early exercise also reduces the cost of old options to the firm, since Black-Scholes 'time value' is dissipated by premature exercise.

-We derive the optimal vesting schedule for the options and show that it is an *increasing* function of stock price. We interpret this as implying a variant of Performance Vesting options, whereby options vest early when a stock price target is attained and we introduce the term 'Progressive Performance Vesting' to describe our normative prediction that vesting should be a gradually increasing function of stock price attained. Interestingly, even with this vesting schedule, the absolute number of new options granted is still *increasing* in the stock price gain achieved, yet not as high as it would be without early exercise.

-When the cost of making the manager supply effort in the job is high relative to the cost of making the manager remain in the job, the optimal vesting schedule is still increasing in stock price but 'capped' once a certain level of stock price gain has been achieved.

-We note that the most commonly observed contracts allow fixed numbers of options to vest at pre-determined calendar dates. We term this practice 'Calendar Vesting' and note that our results imply the potential sub-optimality of such a vesting schedule which risks allowing 'too many' or 'not enough' options to be exercised early. Fortunately, risk-averse managers rationally exercise early more options the deeper they are In-The-Money. In some circumstances this exercise behaviour may go some way towards approximating the outcomes of Progressive Performance Vesting and we interpret this as a possible explanation for why the firm is prepared to allow the manager discretion over the timing of his exercise decisions - paradoxically it can be his own risk aversion which acts to remove those incentives which would otherwise stop him taking risks. However, when this coincidence of objectives does not occur, particularly when fewer options have vested than would be optimal to exercise from the manager's and firm's points of view, the firm may be vulnerable to missing out on valuable Growth Opportunities. This has several practical implications for the design and improvement of effective stock option compensation schemes.

The remainder of the paper is organized as follows: In Section 2 we offer an extended literature review which outlines many of the stylized facts about Executive Stock Options. We give institutional detail and empirical evidence as to the design characteristics of stock option plans, from the widely used 'Calendar Vesting' options, to the increasingly popular 'Performance Vesting' plans. We examine theoretical and empirical evidence on how option design is related to the riskiness and growth opportunities available to firms and their risk averse managers and how their use affects the investments made by these agents. We explain the contribution of the present paper and note that we have found no other paper in the literature which addresses the impact of potential early exercise upon the subsequent risk-taking characteristics of Executive Stock Options, nor which attempts to explain the theoretically optimal or empirically observed vesting characteristics of this most common of compensation vehicles.

In Sections 3 and 4 we describe and then solve a model in which a risk averse manager holds options from an anterior grant and faces a 'Risky versus Safe' project decision. We show how In-The-Money options may offer counter-productive incentives, necessitating re-optimization of incentives by the firm via the issue of costly new options. We then examine the role of early vesting provisions, deriving explicitly how many options the firm should allow the manager to exercise early in order to facilitate re-optimization of risk-taking incentives whilst maintaining sufficient participation incentives for the second period. We also show that the manager will rationally exercise these vested options early, in his own interests. We derive implications for the vesting profile of executive stock options and interpret our results as indicating that our innovative *'Progressive Performance Vesting' options may offer significant advantages over the widespread practice of 'Calendar Vesting' options.*

In Section 5 we extend the model by introducing costly effort and in Section 6 we interpret our results in the light of several variations of option plans, outline suggestions for further research and make empirical predictions. In Section 7 we draw conclusions for those who design or regulate Executive Compensation Schemes. A substantial Appendix contains a summary of notation, consideration of technical issues, proofs of all lemmas and propositions and can be safely omitted by all but the most dedicated of readers.

2 Literature Review and some facts about Executive Stock Options (ESO's)

Murphy (1999) puzzles "Why is there so little variation in option parameters (e.g., exercise prices, expiration terms) across companies?" In this section we provide practitioner and institutional detail backed up by stylized facts on the design of stock options and their use in the firm, concentrating on their risk incentive characteristics. Theory predicts that firms can use ESO's to encourage managerial risk taking, particularly when there exist valuable-but-risky Growth Opportunities which may otherwise be left on the table by risk averse managers. Empirical work confirms that these are the sorts of firms that most use options and that options do indeed appear to encourage risk-taking. There is also evidence of widespread early exercise of ESO's by management, involving the dissipation of substantial proportions of the Black-Scholes value of the option.

Risk-taking Incentives From a finance incentives perspective, a fundamental difference between the use of ESO's and Restricted Stock as a compensation vehicle is the convex payoff profile of the former. This immediately suggests the potential for ESO's to affect the manager's preferences towards the riskiness of the projects he undertakes and a large theoretical and empirical literature addresses the incentives that ESO's create.

Since Jensen and Meckling (1976), the separation of ownership and control has been known to cause potential agency problems between shareholders and management. Amihud and Lev (1981) find that manager controlled firms make more risk-reducing conglomerate mergers in order to decrease their own 'employment risk'. Smith and Stulz (1985) show that managerial risk aversion may lead them to pursue risk reduction strategies such as hedging or avoiding risky projects. They predict that managers compensated with option-like features are less likely to hedge than those who receive stock-like remuneration. Hirshleifer and Suh (1992) show how a convex compensation schedule can motivate a manager to choose risky projects when the firm cannot monitor project selection.

Empirically, Defusco, Johnson and Zorn (1990) find that the announcement of a stock option plan leads to a wealth transfer away from bondholders and towards shareholders, consistent with the market anticipating increased managerial risk taking. This anticipation is subsequently confirmed by a significant increase in stock price volatility. The distinction between incentive effects of stock and options is illustrated by Tufano (1996). He studies corporate risk management activity in the North American gold mining industry and finds that "firms whose managers hold more options manage less gold price risk, and firms whose managers hold more stock manage more gold price risk, suggesting that managerial risk aversion may affect corporate risk management policy". Rajgopal and Shevlin (1998) find that ESO's motivate oil and gas industry managers to take on exploration risk. Williams and Rao (2000) find that "stock options are useful in moderating the CEO's inherent preference for engaging in lower risk activity than preferred by outside shareholders". For NASDAQ firms, they document that variance increasing mergers (and mergers accompanied by leverage increases) are associated with greater CEO stock option risk incentives in the year prior to merger announcement. Guay (1999) measures the sensitivity of CEO wealth to equity risk. He finds that cross-sectionally this sensitivity is positively related to firms' investment opportunities, firms providing managers with convex incentives when the potential loss from under-investment in valuable risk-increasing projects is greatest. He further confirms that firms' stock return volatility does indeed increase with the convexity provided to managers.

Several theoretical studies show that stock options may not always motivate a manager to take on more risk. Depending on the manager's degree of risk aversion and the background risk emanating from his investment in stock, Lambert, Larcker and Verrecchia (1991) show that options may actually *decrease* a manager's propensity to take risks if the probability that the options will expire In-The-Money is high enough. Their paper provides our driving intuition that deep In-The-Money options may provide counter-productive incentives. Without solving any particular agency problem, Carpenter (2000) shows that an option-compensated fund manager may dramatically reduce the volatility of his portfolio selection when the option is deep In-The-Money. Empirical evidence is supplied by Orphanides (1996) who finds that mutual fund managers paid on the basis of calendar year performance tend to reduce risk (lock-in) when they are winning and increase risks (bold strategy) when losing. He concludes that "the agency problem in the industry with regard to risk taking is not a mere theoretical possibility but is of practical significance".

The implications of this body of work are clear. Carefully designed, options can help to ensure profitable risk-taking. However, the convexity of the contract must be carefully managed in order to ensure that incentives which are optimal *ex ante* do not become counter-productive *ex post*. Our contribution is to show how vesting provisions can play a critical role in this issue.

Exercise Price Murphy (1999) finds that "exercise price equals the grant-date fair market value in 95% of the regular option grants". Kole's (1997) corresponding statistic is 93%. This means that the vast majority of ESO's are issued At-The-Money.[2] In our model we are interested in the risk-taking incentives induced by the convex payoffs of options and so we assume that the options are issued At-The-Money, positioning the kink in payoffs at the current share price. However, we show that this is a benign assumption in our model and that Out-of-The-money options would give us just the same results.

Duration At first sight, ESO's appear to provide very Long Term Incentives. 83% of Murphy's sample had a maturity of exactly 10 years, and a further 11% a term of 5-10 years. ESO's are similar to American options insofar as they can be exercised early. However, unlike straight American options, they cannot be exercised until an initial vesting period has elapsed. To this extent they are technically 'Bermudan' options, exercisable during a particular window of time.

Vesting Dates The Vesting Date can be defined as the date on which the option becomes unconditionally the property of the manager. Before an option has vested, exercise is not permitted and ownership is contingent upon continued employment - the manager forfeits the option if his employment terminates. Once an option has vested, the manager can exercise whenever he wishes.

[2] Favourable accounting treatment is lost if options are issued In-The-Money, perhaps explaining the rarity of so-called 'Discount options' and in the context of our model, discount options do not offer very strong risk-taking incentives since they have downside risk exposure. Conversely, 'Premium options' (issued Out-of-The-Money) may have very low incentive value to risk-averse managers. Hall and Murphy (2000) offer a theoretical explanation for setting exercise price equal to grant date price: they show that At-The-Money options may offer the most 'bang-per-buck' in terms of value-for-performance based managerial incentives per dollar of option cost to the firm.

Remarkably little theoretical or empirical research has addressed the issue of vesting. Aboody (1996) documents the vesting schedules of a sample of 478 firms from the period 1983-1990. Typically a grant of options vests in equal tranches over a period of 3 years (33% vesting per annum), 4 years (25% p.a.) or 5 years (20% p.a.). By far the most common period is 4 years meaning that a typical grant of ten-year options would vest 25% after 1 year, 25% after 2 years, ... until all had vested after 4 years. The gradual 'staggered' vesting schedule over 3-5 years appears to be such standard plain vanilla practice that we have seen no technical term by which to call it, so in the present paper we introduce the term 'Calendar Vesting' to emphasize the fact that it is the mere passage of time which causes the option to vest[3].

The choice of this vesting period is of interest in itself. Kole (1997) analyses the variation in vesting dates for stock options and restricted stock, recorded in individual company plans. While her sample also exhibits very little variation in option dates, she relates longer vesting dates to firms for which specialized knowledge is an important asset and for which the horizon for resolution of uncertainty is longer (high R&D, innovation).

Performance Vesting In response to criticisms that calendar vesting may reward executives for mediocre performance, an increasingly popular alternative has been the use of Performance Vesting Options. Such options vest when a pre-determined performance target has been achieved such as a manager-specific accounting target or, more usually, a stock price attainment. For example, vesting may be triggered when the stock price reaches 150% of its option grant date value. Johnson and Tian (2000) treat these as 'up-and-in' options and use the barrier option literature to derive closed form solutions for the Black-Scholes value, to calculate the 'greeks' and to evaluate the *ex ante* incentives offered by such options. Our paper takes a different perspective - that of vesting as a mechanism to help modify incentives when they have become counter-productive *ex post*.

Theoretically, the performance vesting terms of options could be much more finely tuned to stock price performance and the optimal vesting schedules we derive here do just that.

ESO's are a non-traded asset The efficacity of Executive Stock Options as an incentive tool depends on their ability to expose the manager to the risks of the outcomes that his actions will provoke. If the manager were able to diversify or could somehow negate this risk then ESO's utility as performance-based-pay would be undermined. For this reason, ESO's are inalienable,

[3] As an alternative we could call it 'straight-line' vesting, by analogy to the straight-line method of accounting for the depreciation of Fixed Assets.

they cannot be sold nor transferred nor assigned to a third party, even when they have vested. Nor can they be pledged as collateral for a loan.

Insiders are prohibited (by Section 16-c of the Securities Exchange Act) from short selling their firm's stock and so dynamic hedging strategies are dangerous as well as difficult and expensive to implement. Some business press articles[4] may leave the reader with the impression that legitimate hedging is widespread, however firms are free to follow the advice of Bettis, Bizjak and Lemmon[5] (2001) and explicitly bar the manager from hedging. We content ourselves with the claim that even if, theoretically, options may be hedged, in practice there is limited evidence that managers do hedge and very much evidence that they exercise options early, thereby dissipating substantial Black-Scholes option value.

Early exercise The argument in our paper depends on the manager's inability or unwillingness to hedge his option exposure and it is well understood that in such circumstances a risk averse manager may find it rational to exercise early an ESO, even on a non-dividend paying stock[6]. The full Black-Scholes value of a call option can be decomposed into its 'intrinsic value' (its immediate exercise value, when non-negative) and its 'time value'. Exercise prior to expiration involves sacrificing the time value, but this sacrifice is smaller the deeper In-The-Money (the more 'stock-like') is the option and so more likely to be worthwhile in terms of manager utility. Lambert, Larcker and Verrecchia (1991) use a certainty equivalent approach to show that a risk averse manager may privately value his option package much lower than traditional Black-Scholes analysis would suggest[7]. Huddart

[4] "Undermining Pay for Performance" by Louis Lavelle, Business Week, 15 January 2001.

[5] Bettis, Bizjak and Lemmon (2001) document the use of hedging transactions, mainly zero-cost collars, on the executive's *stock* position. They identify 15 transactions in 1996, 39 in 1997, 35 in 1998 and 67 in 1999, typically covering 36% of the executives gross stock position. These transactions should be reported in Table II of Form 4, filed to the SEC by insiders of the firm, however, the authors explain that these filings may receive little scrutiny from the market and may be under-reported. The use or abuse of these instruments may be growing and may offer a regulatory challenge however there is limited evidence of widespread and systematic hedging across the full population of CEO's. This may be explained by reputational and transaction cost reasons and Schizer (2000) details the "potentially chilling tax consequences" of hedging options, including the risks of paying taxes on profits without being able to deduct the offsetting losses.

[6] Even a risk neutral manager might rationally exercise an option early if the underlying stock is about to pay a dividend. Empirically, early exercise is pervasive even for non-dividend paying stocks and our model assumes no dividends.

[7] They implicitly assume a package of European rather than American options.

(1994) works backwards along a binomial tree model to show that when the stock price gets 'high enough', a risk averse manager will prefer the certain payoffs available to early exercise rather than the uncertain payoffs associated with waiting for one more period. Detemple and Sundaresan (1999) extend this intuition and derive numerically the optimal exercise policy, allowing the manager an endogenous investment policy for his non-option wealth. Other papers building on the rationality of the early exercise decision include Hall and Murphy (2001) and Meulbroek (2001).

It follows from this analysis that an option which can be exercised early is worth more than a European option, to a risk averse manager. Moreover, the early vesting option costs shareholders less, because the potential premature exercise reduces the expected dilution. This alone might provide rationale for why options vest early and this is the view taken by Hall and Murphy (2001). However, if we believe that options have an incentive role, then we must recognize that early exercise of a ten year option after, say, three years, leaves the firm with the imperative of re-installing incentives for the remaining period. 'Cheaper' they may be, but American options are likely to be much shorter lived than European options of the same maturity. It is exactly the contention of the present paper that this trade-off may be worthwhile to the granting firm because managers tend to end the life of an American option when it is deep In-The-Money and has lost its risk inducing properties. And when the option remains close to the money, its risk inducing properties remain intact and it lives to creat incentives for another day.

Aboody (1996) finds that 80% of 10 year options exercised were exercised before the 5 year halfway stage. Huddart and Lang (1996) and Heath, Huddart and Lang (1999) examine the exercise decisions of 50,000 employees at eight corporations and find that "Employees typically exercise options years before expiration, commonly sacrificing half of the Black-Scholes value". They also find that options are more likely to be exercised the deeper they are In-The-Money. Hemmer, Matsunaga and Shevlin (1994) examine option exercises by top executives and find that high variance of ESO returns is related to earlier exercise by the executive, indicating that managers do indeed exercise early in order to diversify their risk.

There are of course other non-mutually exclusive potential motivations for early exercise, including the need for liquidity and the existence of private information. Whilst these no doubt play some part, the Black-Scholes option value dissipated would often imply an enormously high discount rate 'paid' in order to obtain liquid funds. Carpenter and Remmers (2001) find "no evidence that insiders in the current regime use private information to exercise in advance of adverse stock price performance". Our paper concentrates on the risk aversion motive for rational early exercise.

'Cashless Exercise' Heath, Huddart and Lang (1999) note that early exercise almost invariably involves immediate sale of the purchased shares. Indeed a 'cashless exercise' service is offered by many firms via brokerage houses and this removes the need for the employee to find the cash otherwise necessary to exercise his options. Some firms 'encourage' managers to hold the stock, indeed they make their option plans part of the mechanism by which the manager is expected to fulfill his stock ownership target. Such a policy may be inconsistent with generating the risk incentives which we wish to study in the present paper. We note that early exercise with the intention of holding on to the underlying stock is inconsistent with a risk reduction motive for exercise and so whenever we write 'exercise', we shall mean 'exercise and resell underlying' or 'cashless exercise'.

Grants Yermack (1995) examines option award practices and finds that newly appointed CEO's receive particularly large grants. This accords with Hall's (1999) description of an initial 'mega-grant' of options. Subsequent grants are relatively smaller and may be on a contractual or discretionary basis. Whatever the granting practice it is clear that managers may develop very complex dynamic options (and stock) portfolio incentives induced by holdings of options granted at different dates, with different exercise prices, unvested, vested or vested and exercised.

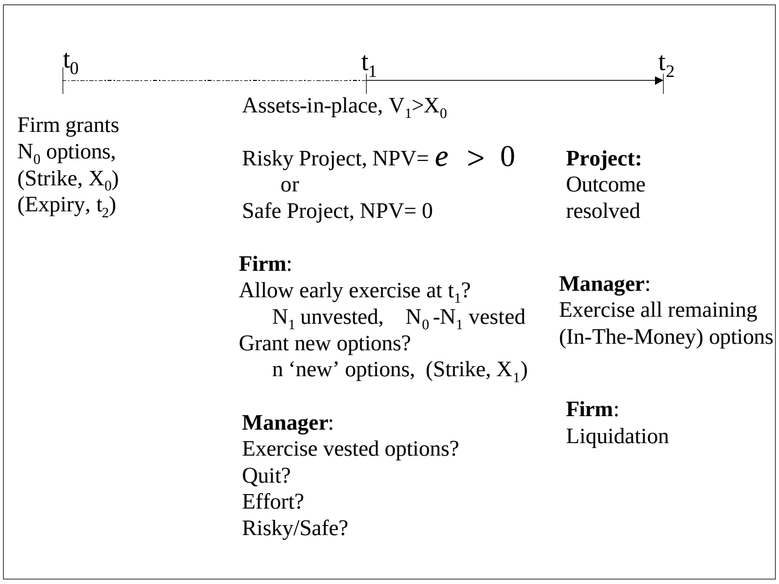
The implications of this body of work are clear. Carefully designed, options can help to ensure profitable risk-taking. However, the convexity of the contract must be carefully managed in order to ensure that incentives which are optimal *ex ante* do not become counter-productive *ex post*. Our contribution is to show how vesting provisions can play a critical role in this issue.

3 The Model

A reference summary of all notation can be found in Appendix A.

We study a model with three dates, t_0, t_1, t_2 and the reader is referred to Figure 1 for a timeline of events and decisions. At t_0 the manager is granted N_0 call options with exercise price X_0 and expiration date t_2. N_0 and X_0 are exogenously given and we do not model the t_0 managerial decisions which presumably led to their granting. These options are still 'in force' later at t_1 when the value of Assets-In-Place has evolved to V_1 and when the firm faces a risky-but-profitable growth opportunity with Net Present Value (NPV) $\varepsilon > 0$. The manager must decide whether or not to take this project and his option incentives will determine his choice. To optimise these incentives, the

Figure 1: Timeline



firm chooses N_1, the number of options that remain unvested until t_2, allowing $N_0 - N_1$ options to vest early at t_1. The firm also chooses n, the number of new options to grant. We proceed to specify in detail the preferences of the agents, the projects available, the option remuneration structure and the resulting objectives of the agents.

3.1 Preferences

3.1.1 The Manager

The manager is the professional Chief Executive Officer of the firm. He is hired by shareholders to select and implement projects. He has an exogenous initial wealth, W, and his only other source of income comes from his option compensation package. He has a utility function, separable in total terminal wealth, x and his cost of participation, F.

$$U(x, F) = u(x) - F \tag{1}$$

He is risk averse, $x \geq 0$, $u' > 0$, $u'' < 0$, and his Participation for the second period demands a personal non-monetary cost of $F = Q > 0$. If he quits the firm at t_1 then he 'saves' this cost and so $F = 0$. We can think of Q as a Private Benefit to the manager which he enjoys if he quits the firm at t_1 and in Section 5 we shall consider the consequences of introducing costly effort.

Any endowment or cash payment he receives prior to t_2 (including that from any option exercises at t_1) is merely held at the risk free interest rate of zero until t_2.

3.1.2 The Firm

The firm is owned by shareholders who are either risk neutral or can diversify their portfolios sufficiently as to be indifferent to the riskiness of the firm's terminal liquidation value. There is no debt and we normalize the total number of shares in the firm to be 1. The shareholder objective is to maximize terminal cashflows, net of any compensation costs required to motivate the manager. We assume that shares are publicly traded on a stock market so that the t_1 stock price is observable and we give all of the bargaining power to the firm who can make a 'take-it-or-leave-it' offer to the manager, the latter accepting any offer which gives expected utility at least as high as that from quitting the firm.

3.2 Projects

3.2.1 Assets-In-Place

At t_1, the expected value of the firm's Assets-in-Place is denoted by V_1. We do not model any managerial actions taken at t_0 and so V_1 is just an exogenous parameter in our model whose importance is that it directly impacts the degree to which the previously granted options are In-The-Money. We shall consider values $V_1 > X_0$ to ensure that the 'old' options are already In-The-Money. For the purposes of the present paper, we merely note that no manager would want to exercise an Out-of-The-Money option and so the question of how many options should vest is irrelevant in this case.[8]

[8] Out-of-The-Money ('underwater') options do have incentive effects: they may make the manager take on excessive risk as he seeks to pull them back into the money or, conversely, they may lose all incentive value whatsoever as their value to a risk averse manager becomes insignificant. A burgeoning recent literature on stock option 'repricing' (Brenner, Sundaram and Yermack (2000), Acharya, John and Sundaram (2000), Carter and Lynch (2001)).has considered the problem of underwater options from many angles and we do not intend to contribute to that here. Indeed following the FASB proposals to tighten significantly the accounting treatment of repricings, it remains to be seen whether repricing will continue to be the controversial issue that it has been.

3.2.2 New projects

"Great deeds are usually wrought at great risks" Herodotus.

We stylize the t_1-project selection decision as being a choice between a 'Risky' or a 'Safe'[9] strategy, the Risky strategy having a higher expected NPV than the Safe strategy, and so being preferable from the shareholders' point of view, but having higher variance in cashflows, and so being potentially unattractive to a risk averse manager compensated via Performance Based Pay. In the present paper, we are implicitly assuming that the identification and implementation of risky projects is a 'good thing'. Of course, if the available risky projects have negative NPV then the firm would not find it appropriate to use large amounts of options. Numerous empirical studies[10] show that the use of stock options is positively associated with firms facing large Growth Opportunities and for this reason we model our manager facing a risky-but-profitable growth opportunity. We are interested in the design characteristics of options when used to promote risk-taking and all we have to say about firms that face a problem of over-investment in negative NPV risky projects is that such firms probably should not be using stock options in the first place.

In order to obtain transparent and tractable solutions, we assume that the safe strategy has a certain zero Net Present Value and so the final payoff will maintain the interim asset value, V_1. Choosing the risky project yields a mean improving spread of NPV, ε, at the cost of some uncertainty in payoffs which have dispersion $2H$. If the manager stays with the firm, his project selection decision is unverifiable. If the manager decides to quit the firm, then asset value falls to zero - the manager's continued presence is essential even for the realization of the assets in place (and their liquidation at t_1 is not possible). Quitting the firm is an observable verifiable event causing unexercised options to be forfeited.

We denote the manager's risky/safe project selection decision as choosing $q \in \{r, s\}$, his quit/participate decision as choosing $F \in \{0, Q\}$ and, depending on these t_1 decisions, the ter-

[9] Equivalently we could lable this as 'To Grow' or 'Not to Grow'.

[10] Smith and Watts (1992), Gaver and Gaver (1993), Mehran (1995).

minal t_2 gross payoffs are denoted as follows

Manager action	**Project Payoffs**
'risky'	$\tilde{V}_{risky} = \begin{cases} V_1 + H + \varepsilon & \text{w.p.} \quad 1/3 \\ V_1 + \varepsilon & \text{w.p.} \quad 1/3 \\ V_1 - H + \varepsilon & \text{w.p.} \quad 1/3 \end{cases}$
'safe'	$\tilde{V}_{safe} = V_1 \quad \text{w.p.} \quad 1$
'quit'	$\tilde{V}_{quit} = 0 \quad \text{w.p.} \quad 1$

Thus, playing 'safe' locks into the existing t_1 asset value whereas going 'risky' has NPV of ε. We normalize this NPV to $\delta = \frac{\varepsilon}{H}$, a measure of the NPV relative to the size of the project. The variance of the risky project payoff is $\sigma_r^2 = \frac{2}{3}H^2$.

3.2.3 Efficient decisions

We shall assume that the NPV, ε, is large enough that the firm will always find it efficient to induce the manager to choose 'risky'. This will be true providing the compensation cost of inducing 'risky' is not more than ε higher than the cost of inducing 'safe' and we will check that this is valid for any solution proposed. Moreover, we implicitly assume that the firm always finds it optimal to avoid the manager quitting the firm. We achieve this by assuming that the loss in firm value is unacceptably great should the manager depart e.g. $\tilde{V}_{quit} \equiv V_1 - L$, where L is 'large' (here, $L = V_1$) due to the costs of finding a replacement manager or the loss of vital knowledge held by the incumbent manager. Consideration of 'small' L would lead us into examination of management turnover and replacement, issues which are addressed in Brisley and Leshchinskii (2000, 2001).

With these assumptions, the firm's problem becomes one of motivating 'risky' at minimum compensation cost. Equivalently, it becomes a problem of minimizing compensation cost subject to the manager's Incentive Compatibility Constraint for Risky and the Participation Constraint. Letting $\overline{U}$ represent the manager's expected utility from his course of action, these are

'IC Risky' The manager prefers 'risky' to 'safe'

$$\overline{U}\,(risky) \geq \overline{U}\,(safe) \tag{2}$$

'Participation Constraint' The manager prefers 'risky' to 'quit'

$$\overline{U}\,(risky) \geq \overline{U}\,(quit) \tag{3}$$

3.2.4 Asset values and stock prices

We draw a careful distinction between the value of the firm's assets-in-place and the firm's stock price. This is because a rational market will impound into stock prices the value of assets-in-place *and* expected payoffs from future growth opportunities (providing the market believes these opportunities will be taken up). In our set-up we study the equilibrium where the market believes that the Firm will offer the optimal contract to the manager and believes that he will take the appropriate 'risky' decision and so we denote the equilibrium stock price by $P_1 = V_1 + \varepsilon$. For completeness we consider (and reject) an alternative equilibrium in Appendix B.

Furthermore, we invoke a 'small manager' assumption, namely that the variation of compensation costs in different final states of the world is small compared to the total value of the firm and so does not materially affect the value of the firm.[11]

3.3 Option remuneration

At t_0 the firm grants to the manager a number, N_0, of long-term (expiration date, t_2) stock options with exercise price X_0. This initial grant of options we will often refer to as 'old' options. We do not model how N_0, and X_0 are chosen by the firm, they were presumably chosen to ensure effective risk-taking decisions by the manager at t_0. The focus of our study is the optimal design of the *vesting characteristics* of these options - What proportion of these options should the firm allow the manager to exercise 'early' at t_1 and what proportion should remain unvested until t_2?

Firms using 'Calendar Vesting' would allow a fixed percentage, say 50%, of the options to vest at t_1. Firms using plain vanilla Performance Vesting might allow all options to vest at t_1, if and only if a stock price rise of 50% had been achieved. However, we permit a more sophisticated Performance

[11] In support of this assumption we quote from Hall and Liebman (1998): "Even if the typical CEO had in wealth...almost fourteen times annual total compensation at the median - the CEO could purchase only about 0.9% of the firm." Relaxing this assumption would generate an analytical 'warrant problem' because, as option exercise leads to dilution, stock prices adjust slightly downwards to take account of this. In turn, this changes slightly the prospective option payoffs for the manager and complicates enormously the incentives problem. We outline the problem in Appendix C, but claim that the slight quantitative adjustments necessary would lead to the same qualitative conclusions. Any gains in rigour are offset by problems of intractability and divert us from our main discussion of the incentive effects of option contracts.

Vesting Schedule, allowing the firm to condition fully upon the stock price gain achieved over the first period. The firm anticipates the incentives vesting will create for exercise, risk-taking, effort and participation managerial decisions to be taken at t_1 and takes into account the effect on costs of new option awards. Our vesting schedules will define, as a function of the t_1 stock price, the number, $N_0 - N_1$, of options which vest at t_1, leaving N_1 unvested until t_2. Equivalently, they will define the proportion, $k_1 = \frac{N_1}{N_0}$, of old options remaining unvested at t_1. We have already explained that we consider values $V_1 > X_0$ (equivalently $P_1 > X_0 + \varepsilon$) to ensure that the 'old' options are already In-The-Money because no manager would want to exercise an Out-of-The-Money option and so the question of how many options should vest is irrelevant in this case. We shall assume that no options vest when they are Out-of-The-Money.

In order to re-establish optimal risk-taking incentives, the firm may[12] also choose to grant additional options at t_1. We denote their number, n, their exercise price, X_1, and this final grant of options we will often refer to as 'new' options. They have expiry date t_2. For clarity of exposition, we assume that the Firm follows the almost universal practice of granting new options, if at all, At-The-Money and then we solve for n. Thus we fix the exercise price, X_1, equal to P_1, the t_1-stock price. As we shall see later, this assumption is not crucial to our analysis, indeed in our model the firm could generate the same risk-taking incentives at the same cost to the firm by issuing a greater number of Out-of-The-Money options ($X_1 > P_1$). Issuing In-The-Money options would certainly be suboptimal since their downside exposure is counter-productive for risk-taking incentives.

Based on this and the incentives offered by the original grant, the manager may decide to exercise his vested options at t_1. He also decides whether or not to remain with the firm for the second period. In equilibrium he chooses to stay and is faced with the project selection decision which will determine the distribution of terminal firm values.

3.3.1 Contract Form

We assume that all option exercises are settled in cash, so the number of outstanding shares remains unchanged and equal to 1. Option grant sizes N_0 and n will therefore be small fractions of 1, for example $N_0 = 0.01$ would represent the manager having a claim on 1% of the increases in firm

[12] Some compensation contracts specify *ex ante* at t_0 the number or value of options which will be granted at t_1. Hall (2000) points out that this may give the manager perverse incentives, leading him to 'game' the contract. In our model the t_1 grant of new options is a function of the t_1 share price and so could also theoretically be written into the t_0 contract.

value over the two periods. We assume that the firm uses only stock options to pay its manager. Clearly a richer model would include salary, bonus, restricted stock, pension rights etc. in the firm's armoury of compensation tools, however we are primarily interested in the convex incentives offered by options which now represent the most significant component of US CEO remuneration.[13] Our approach is to combat managerial risk aversion by rewarding high performance. A contractual alternative would be to insure the manager against low outcomes and indeed one way to ensure that he does not quit would be to offer a fixed salary for attendance during the second period. However, liquidity constrained firms often claim they have little choice but to grant options in order to preserve cash, shareholders 'paying' the manager via dilution of their claims. Even if liquidity is not a problem, Section 162(m) of the U.S. tax legislation caps the deductability of non-performance related pay at $1 million. Salaries represent a fixed cost to the firm and effectively amount to the issue of debt. ESO's allow the firm to reduce its financial risk by issuing equity instead of promising cash. Furthermore, a fixed salary may provide weak incentives to supply costly effort (which we introduce in Section 5 and options are argued by human resource professionals to be essential to attract, retain and motivate key employees.

In our model, the discrete payoff structure means that the terminal values at t_2 reveal perfectly the manager's actions at t_1. This could make optimal a 'forcing' contract which rewards the manager for any of the outcomes $V_1 + \varepsilon - H$, $V_1 + \varepsilon$, and $V_1 + \varepsilon - H$, but penalises the manager for the outcome V_1. Such a contract would be non-monotonic[14] since the manager would be better rewarded for achieving $V_1 + \varepsilon - H$ than for achieving V_1. In the present paper, however, we assume that the manager is able to unilaterally destroy firm value[15] at t_2. In the presence of costly effort, any such forcing contract is sub-optimal because the manager would have an incentive to choose 'safe' (or

[13] Watson Wyatt's 2000/2001 Annual Survey of Top Management Compensation (conducted on a broad sample of 1545 US firms, all industries) gives the average CEO base salary as $0.62 million compared to the average stock option compensation of $3.98 million.

[14] Core and Qian (2001) derive an optimal contract which is U-shaped, rewarding very poor performance higher than just mediocre performance.

[15] This could be by diverting funds to negative NPV projects, performing inappropriate mergers, burning down an uninsured factory or committing 'retail suicide'. The textbook example of the latter is perhaps the U.K. discount jewellery store Ratner's. In 1991 the CEO Gerald Ratner remarked in an interview that "the jewellery we sell is total crap". Retail sales fell immediately and the company went into liquidation a couple of years later.

Another attempt was made in July 2001 when David Shepherd, director of the U.K. clothes shop 'Topman', explained to a trade magazine why business suits did not make up a significant proportion of their retail sales: "Most of our customers are hooligans, the only time they wear a suit is for their first job interview or their first court appearance."

in Section 5, to be 'lazy') and then destroy sufficient value to 'achieve' $V_1 + \varepsilon - H$. Our assumption makes sub-optimal any contract which promises to reward the manager for very poor performance.

3.3.2 Option Payoffs

We denote the 'moneyness' of the old options at t_1 as $M = P_1 - X_0$, the 'intrinsic' (immediate exercise) value of the options. This we normalize to define a measure of 'relative moneyness', $\lambda = \frac{M}{H}$, and this parameter will play a central role in our analysis.

We have already assumed that $V_1 > X_0$ which implies that $M > \varepsilon$ (equivalently $\lambda > \delta$) and so the options are In-The-Money. Furthermore we rule out cases where $M > H$ (equivalently $\lambda > 1$) representing options that are so deep In-The-Money that they have no residual convexity since there is zero probability that the option can fall Out-of-The-Money. Such options have no remaining 'time value'. Thus, our $\lambda \in (\delta, 1]$ *measure of relative moneyness can also be considered as a measure of the amount of convexity that the old options have 'lost' due to stock price rises.*

Any 'old' option which vests *and is exercised early* at t_1 gives a cash payoff of M. The exercise price of the new options we choose $X_1 = P_1$ for notational economy and to accord with the widespread practice of granting options At-The-Money and so from the manager's t_1 perspective, prospective cash payoffs to each option *held to maturity* are:

	old options	**new options**
exercise price	X_0	X_1
'risky'	$\begin{cases} M+H & \text{w.p. } 1/3 \\ M & \text{w.p. } 1/3 \\ 0 & \text{w.p. } 1/3 \end{cases}$	$\begin{cases} H & \text{w.p. } 1/3 \\ 0 & \text{w.p. } 2/3 \end{cases}$
Present Value	$M + \frac{1}{3}(H - M)$	$H/3$
Intrinsic Value	M	0
Time Value	$\frac{1}{3}(H - M)$	$H/3$
'safe'	$M - \varepsilon$ w.p. 1	0 w.p. 1
'quit'	forfeited	forfeited

We notice that each new option is worthless in the 'safe' project and provides a payoff of H only in the upstate of the 'risky' project. The risk-taking incentives of n such At-The-Money options could be exactly replicated by instead granting $\frac{H}{H+P_1-X_1}n$ Out-of-The-Money options $(X_1 > P_1)$, each giving a payoff $H + P_1 - X_1$ in the upstate. To this extent, the choice of exercise price is arbitrary in our model and we choose $X_1 = P_1$ for notational simplicity and to accord with industry practice. The effect on effort provision incentives is more complex and so is the analysis when project outcomes follow a continuous distribution, however Hall and Murphy (2000) present an analysis of why granting options At-The-Money may be optimal.

3.4 Agent Objectives

3.4.1 Shareholders

We have seen that the firm wishes to motivate the choice of 'risky' at the lowest possible compensation cost to shareholders. From the above payoffs, we see that if the manager exercises $N_0 - N_1$ vested options at t_1, holds N_1 unvested options until t_2 and is granted n new options at t_1 then the expected total compensation cost to the firm is

$$(N_0 - N_1) M + N_1 \left(M + \frac{1}{3}(H - M) \right) + n\frac{H}{3} \tag{4}$$

whereupon the firm's programme can be written

$$\min_{N_1 \in [0, N_0], n \geq 0} N_0 M + N_1 \left(\frac{1}{3}(H - M) \right) + n\frac{H}{3} \tag{5}$$

The minimand is increasing in both N_1 and n. Subject to providing correct incentives the firm prefers to encourage strictly *higher* early exercise (early exercise reduces the expected cost of these options) and to grant strictly *fewer* new options. We shall see later (in Corollary 5) that these two goals are congruent and amount to reducing as far as possible the proportion $k_1 = \frac{N_1}{N_0}$ of old options remaining unexercised at t_1, subject to fulfilling the incentive compatibility constraints (2) and (3).

3.4.2 Manager

Given the terms of his incentive contract, the manager will choose his actions at t_1 to maximize his expected utility of remuneration and private benefits less effort. We assume that when the manager is indifferent between two courses of action, he is kind enough to choose that which shareholders prefer.

$$\max_{q \in \{r, s\}, F \in \{0, Q\}} \overline{U}(\tilde{x}(q, F), F) = E[u(\tilde{x})] - F \tag{6}$$

4 Solving the vesting problem

Now that we have reduced the firm's problem to one of using options to satisfy the Incentive Compatibility constraints in the most efficient manner, we consider each constraint in turn. We wish to focus on options as an antidote to managerial risk aversion and so we shall be most interested in cases where IC Risky is a binding constraint. We shall derive the conditions for there to exist a Risk Avoidance Agency Problem and show how different degrees of early vesting can be used in conjunction with the grant of new options to solve this problem. Armed with these potential solutions, we then optimise subject to satisfying the participation constraint. We shall show that the firm needs only to monotonically reduce the expected cost of the vesting/granting solution to the risk avoidance problem until the participation constraint binds.

We concentrate on the participation constraint (and introduce costly effort only as an extension in Section 5) because we take the view that CEO's cost of effort in the job is not high compared to his opportunity cost of accepting the job. We do not necessarily go as far as Woody Allen when he says "Showing up is 80% of life", but we do agree with Murphy (1999) when he says "... it is widely acknowledged that the fundamental shareholder-manager agency problem is *not* getting the CEO to work harder, but rather getting him to choose actions that increase rather than decrease shareholder value."

For reasons of tractability we give the manager a logarithmic utility function over his final wealth, $u(x) = \ln x$ which exhibits Decreasing Absolute Risk Aversion (DARA)[16] and Constant Relative Risk Aversion. His exogenous initial wealth, $W > 0$, ensures that this is defined even if all of his options expire worthless. When using the normalizations $\lambda = \frac{M}{H}$ and $\delta = \frac{\varepsilon}{H}$ we shall often find it convenient to normalize the manager's wealth to $\omega = \frac{W}{N_0 H}$, his wealth relative to the maximum potential further option gain from holding his initial options through the second period.

4.1 Incentive Compatibility: Risky versus Safe

Here we consider the manager's incentives to take the risky project rather than the safe one. As a benchmark case, we consider first the situation if none of the 'old' options are permitted to be exercised at t_1. If the manager participates in the second period, then IC Risky constraint (2)

[16] Whilst we do not vary the utility function, we can change the manager's degree of absolute risk aversion by varying W.

can be written

$$\begin{aligned}&\frac{1}{3}\ln\left(W+N_0\left(M+H\right)+nH\right)+\frac{1}{3}\ln\left(W+N_0M\right)+\frac{1}{3}\ln W\\ \geq\ &\ln\left(W+N_0\left(M-\varepsilon\right)\right)\end{aligned} \tag{7}$$

or

$$\rho\left(n,N_0,W,M\right)=\frac{\left(W+N_0M\right)\left(W+N_0\left(M+H\right)+nH\right)W}{\left(W+N_0\left(M-\varepsilon\right)\right)^3}\geq 1 \tag{8}$$

where we interpret $\rho\left(n,N_0,W,M\right)$ as a measure of the 'relative attractiveness of the risky project', (compared to the safe project).

4.1.1 Risk Avoidance Agency Problem

At t_1, the firm is not obliged to offer any new options, nor is the firm obliged to allow old options to vest early. However, this may cause the manager to reject the risky-but-profitable project in favour of the safe project and indeed IC Risky fails to hold whenever whenever $\rho\left(0,N_0,W,M\right)<1$. This enables us to derive conditions under which the manager will not choose the risky project unless old options are exercised and/or new options granted.

Lemma 1 *In the absence of early exercise and when no new options are granted*

i) There exists a threshold number, $\underline{N}_0\left(\lambda\right)$ *of old options such that IC Risky holds at* t_1 *if and only if*

$$N_0\leq\underline{N}_0\left(\lambda\right) \tag{9}$$

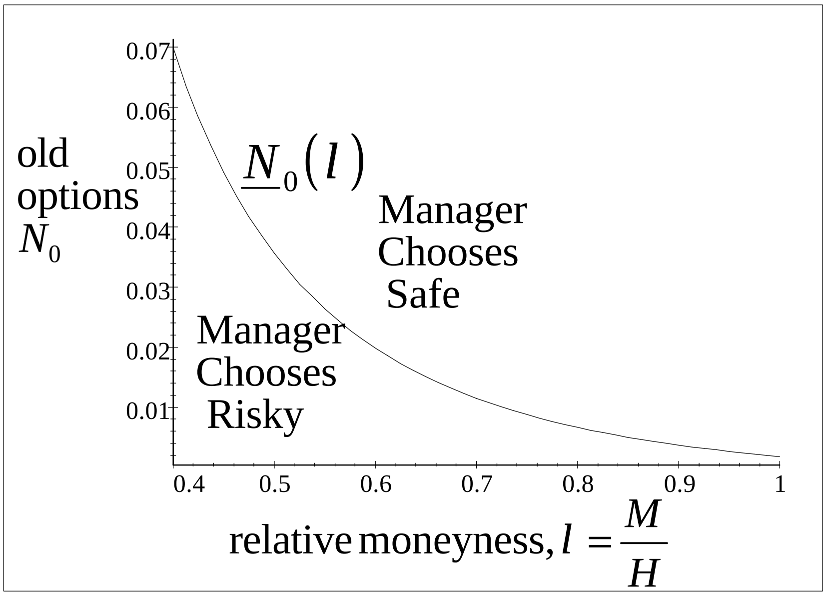
where $\underline{N}_0\left(\lambda\right)$ *is explicitly defined in the proof.*

ii) $\underline{N}_0\left(\lambda\right)$ *is decreasing in* λ*.*

This lemma and the following corollary illustrate the crux of our contention about the problems of options; options which gave optimal risk-taking incentives when they were granted at t_0 may give decidedly counter-productive incentives at t_1. They are more likely to incite risk avoidance the deeper In-The-Money they go because they lose their convexity and become more linear and stock-like in their payoffs. Options certainly impose risk on the manager, but once they have moved deep In-The-Money they may impose the 'wrong kind of risk' on him, failing to convexify enough his concave preferences and exposing him to painful downside risk.

Corollary 2 *If* $N_0>\underline{N}_0\left(1\right)$ *then*

Figure 2: IC Risky: Too many options, too far In-The-Money cause a risk avoidance problem.



i) there exists a threshold moneyness, $\underline{\lambda}(N_0) \in (\delta, 1)$ *such that IC Risky holds if and only if*

$$\lambda \leq \underline{\lambda}(N_0) \tag{10}$$

where $\underline{\lambda}(N_0)$ *is explicitly defined in the proof.*

ii) $\underline{\lambda}(N_0)$ *is decreasing in* N_0.

Figure 2 illustrates our result that 'too many options, too far In-The-Money, cause a risk avoidance agency problem' i.e., the manager will choose 'safe' instead of 'risky'. The following lemma shows that granting new options as a 'cure' for this problem becomes progressively more expensive for the firm the deeper In-The-Money are the old options.

Lemma 3 ***In the absence of early vesting.***

When $N_0 \geq \underline{N}_0$,

i) there exists a threshold number, n'_r, *of new options such that the problem can be solved by granting any* $n \geq n'_r$ *new options where*

$$n'_r = n'_r(N_0, \lambda) \tag{11}$$

is explicitly defined in the proof.

ii) n'_r is increasing and convex in N_0

iii) n'_r is increasing and convex in λ.

Note that, by construction, for each moneyness, λ, n'_r, becomes positive once $N_0 > \underline{N}_0$.

4.1.2 Allowing early vesting

We have shown how the existence of too many old options too deep In-The-Money causes an agency problem which can be solved by the granting of costly new options. And that the more old options in force, the more new options are required to 'cure' this risk avoidance problem. It is natural then to consider how the firm's position may be improved by somehow reducing the number of old options in force for the following period.[17] We claim that the early exercise provision in executive stock options can work to the firm's advantage by allowing the manager to remove option incentives which have become counter-productive. The firm can reduce the number of outstanding old options to N_1 by allowing $N_0 - N_1$ of them to be exercised and this also increases the manager's cash wealth by $(N_0 - N_1) M$.

Proposition 4 *If $N_0 \geq \underline{N}_0$ then*

i) the risk avoidance agency problem can be solved by letting $N_0 - N_1$ old options be exercised and granting $n \geq n_r$ new options where

$$n_r = \begin{cases} n_r\left(N_0, N_1, W, M\right) & if \quad N_1 > \underline{N}_1 \\ 0 & if \quad N_1 \leq \underline{N}_1 \end{cases} \tag{12}$$

where $n_r\left(N_0, N_1, W, M\right)$ and $\underline{N}_1$ are given explicitly in the proof and $\underline{N}_1$ represents a threshold below which no new options are required to solve the Risk Avoidance Agency Problem.

Putting $k_1 = \frac{N_1}{N_0}$ and expressing the number of new options as a proportion of the number of old options, this expression can be written explicitly

$$\frac{n_r\left(k_1\right)}{N_0} = \begin{cases} \frac{k_1\left(-k_1^2\delta^3 + \left(3\delta^2+\lambda\right)(\omega+\lambda)k_1 - (\omega+\lambda)^2(1+3\delta-\lambda)\right)}{(\omega+(1-k_1)\lambda)(\omega+\lambda)} & if \quad k_1 > \underline{k}_1 \\ 0 & if \quad k_1 \leq \underline{k}_1 \end{cases} \tag{13}$$

[17] Cancelling In-The-Money options without indemnifying the manager might just be theoretically possible if the firm could costlessly fire and re-employ the manager with a contract just sufficient to satisfy his reservation utility for the final period. However the reputational, human resource and ex ante incentive consequences of such a possibility would be very destructive so we assume that options are not granted with this Machiavelian intention.

where

$$\begin{aligned} \underline{k}_1 &= \frac{\underline{N}_1}{\underline{N}_0} \\ &= \frac{\omega+\lambda}{2\delta^3}\left(3\delta^2+\lambda-\sqrt{\lambda^2+\delta^2\left(6+4\delta\right)\lambda-\delta^3\left(4+3\delta\right)}\right) \end{aligned} \tag{14}$$

ii) n_r *is increasing and convex in* k_1

iii) n_r *is increasing in* λ

iv) $\underline{k}_1$ *is decreasing in* λ *and increasing in* δ

v) $\underline{N}_1 > \underline{N}_0$.

Proposition 4 gives us a recipe for solving the Risk Avoidance Agency Problem. If a proportion $1-k_1$ of the old options are exercised, then the firm needs to issue exactly $n_r(k_1)$ new options to restore risk-taking incentives. Figure 3 illustrates Part (ii) that n_r increases convexly with the proportion of unexercised old options. Part (iii) says that deeper moneyness increases the amount of old options that would need to be exercised in order to negate the need for *any* new options (for Risk Avoidance Agency Problem purposes, at least). Part (v) says that if $N_0 > \underline{N}_0$, then in order to negate the risk induced need for any new options, the firm does *not* need to reduce the old options outstanding down to the minimum level $\underline{N}_0$ that would have originally caused a problem. This is because of the wealth effect of cashed-in options on our DARA Manager.

As shown in Equation (5), the firm's compensation costs are decreasing in the exercise of 'old' options and increasing in the granting of 'new' options. The firms two objectives are therefore to minimise k_1 and to minimise n. Fortunately, these two objectives do not conflict - from Proposition 4 (ii), n_r is increasing in k_1 and so minimizing k_1 also minimizes the expected cash cost of new options. Thus we have

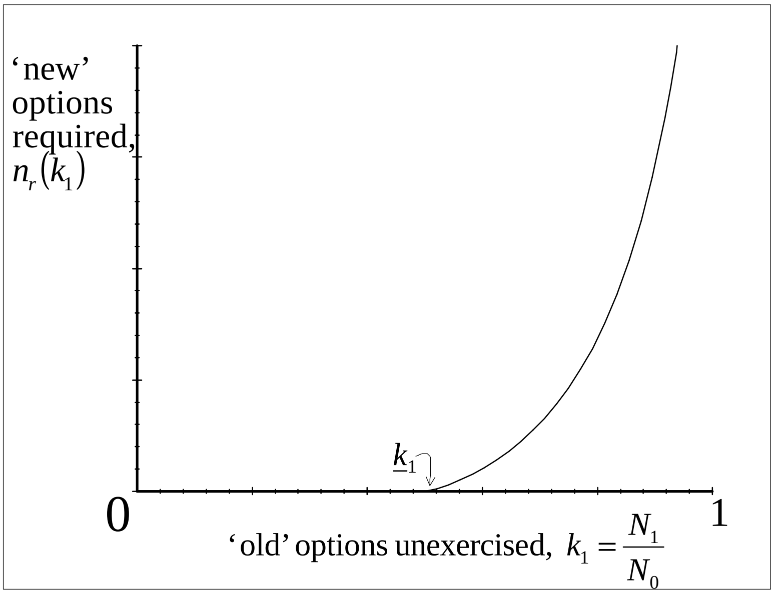
Corollary 5 *When the IC Risky constraint is binding and the firm allows* $1-k_1$ *options to vest and grants* $n_r(k_1)$ *new options, the firm's objective is*

$$\min k_1 \tag{15}$$

subject to the Participation Constraint.

We see now that the firm can motivate correct risk-taking incentives most cost effectively by sliding down and leftwards as far as possible along the curve of Figure 3. Indeed, were risk-taking incentives the only concern, the firm could simply choose $k_1 = 0$, $n_r = 0$. However we must not forget the Participation Constraint to which we now turn.

Figure 3: Number of new options required to solve risk avoidance problem, as function of number of old options unexercised.



4.2 Participation Constraint: solving subject to IC Risky

Since the firm will choose to satisfy the IC Risky constraint at the lowest possible cost, our approach will be to consider the satisfaction of the Participation Constraint *along the locus* $(k_1, n_r(k_1))$ which satisfies the IC Risky constraint. We shall show that participation incentives provided are monotonic along this locus and so for any particular level of moneyness, the firm will reduce k_1 (and hence $n_r(k_1)$) until the participation constraint starts to bind.

If the manager decides not to continue with the firm for the second period (to 'quit') then he 'saves' the personal cost of participation. His cash payoffs are then only those from exercising at t_1 any options which have already vested and he will forfeit any unvested options. To ensure that he prefers to stay and choose 'risky' we note that at t_1, if the manager exercises $N_0 - N_1$ old options, retains the remaining N_1 old options and is granted n new ones then his Participation Constraint (3) can be written

$$\begin{aligned}&\tfrac{1}{3}\ln\left(W + M\left(N_0 - N_1\right) + N_1\left(M+H\right) + nH\right)\\ &\quad +\tfrac{1}{3}\ln\left(W + M\left(N_0 - N_1\right) + N_1 M\right) \qquad \geq \ln\left(W + M\left(N_0 - N_1\right)\right) + Q \\ &\qquad +\tfrac{1}{3}\ln\left(W + M\left(N_0 - N_1\right)\right)\end{aligned} \tag{16}$$

Simplifying,

$$\frac{\left(\omega+\lambda+k_1+\frac{n}{N_0}\right)(\omega+\lambda)}{\left(\omega+\lambda\left(1-k_1\right)\right)^2} \geq \exp 3Q \tag{17}$$

From which we get

Lemma 6 *Along the locus $\left(k_1, n_r\left(k_1\right)\right)$ required to satisfy the IC Risky constraint*

i) the Participation Constraint becomes

$$\beta\left(k_1, n_r\left(k_1\right)\right) \geq \underline{\beta}(Q) \tag{18}$$

where $\underline{\beta}(Q)=\exp 3Q$ and

$$\beta\left(k_1, n_r\left(k_1\right)\right)=\begin{cases}\frac{\left(\omega+\lambda-k_1 \delta\right)^3}{\left(\omega+\lambda\left(1-k_1\right)\right)^3} & if \quad k_1 \geq \underline{k}_1 \\ \frac{\left(\omega+\lambda+k_1\right)(\omega+\lambda)}{\left(\omega+\lambda\left(1-k_1\right)\right)^2} & if \quad k_1<\underline{k}_1\end{cases} \tag{19}$$

ii) $\beta\left(k_1\right)$ is increasing in k_1.

iii) $\beta\left(k_1\right)$ is increasing in λ, $\forall \omega \geq 0.5$

iv) If $\omega<0.5$, then $\beta\left(k_1\right)$ is increasing in λ $\forall k_1 \geq \min \left\{\underline{k}_1, \frac{(1-2\omega)(\omega+\lambda)}{(2\omega+\lambda)}\right\}$ but decreasing in λ for $k_1<\min \left\{\underline{k}_1, \frac{(1-2\omega)(\omega+\lambda)}{(2\omega+\lambda)}\right\}$.

We interpret $\beta\left(k_1\right)$ as a measure of the participation incentives generated for the manager when a proportion k_1 of the old options remain unexercised and are augmented by the number $n_r\left(k_1\right)$ of new options required to install sufficient risk-taking incentives. Part (i) says that these incentives must be at least as high as a constant threshold level. Part (ii) expresses the intuition that the manager is more likely to stay with the firm the more of his option wealth remains unvested until t_2. The analytical importance of this is that the set $\left\{\left(k_1, n_r\left(k_1\right)\right) \in[0,1] \times[0, \infty): \beta\left(k_1, n_r\left(k_1\right)\right) \geq \underline{\beta}(Q)\right\}$ is continuous and connected. Thus, to satisfy the IC Risky and Participation Constraints at minimum cost, the firm need just reduce the value of k_1 until the participation constraint binds. Part (iii) says that participation incentives increase with the moneyness of options and again corresponds to the intuition that the manager is more likely to stay with the firm the more valuable his unvested option wealth. The caveat to this is in Part (iv) when the manager is at particularly low relative exogenous wealth levels ($\omega<0.5$), then the wealth effect of moneyness on his *vested* option wealth can reduce his marginal utility for wealth sufficiently that the participation incentives are decreasing in moneyness. However even this effect can only dominate for 'small' k_1, not high enough to trigger a grant of new options.

Proposition 7 ***The firm's optimal vesting schedule and granting of new options.***

The firm's optimal proportion, k_{part}, of unexercised old options (i.e., allowing $1 - k_{part}$ to be exercised at t_1) and corresponding number, n_{part}, of new options together ensure that the manager does not quit and does choose the risky project. These optima depend on the manager's total cost of participation, Q, as follows:

i) If $\underline{\beta}(Q) > \beta(1)$

Then

$$k_{part} = 1 \tag{20}$$

And $n_{part} > n_r(1)$ and can be written explicitly

$$n_{part} = \left(\frac{\omega^2}{(\omega + \lambda)} \underline{\beta} - (\omega + \lambda + 1) \right) \tag{21}$$

ii) If $\underline{\beta}(Q) \in (\beta(\underline{k}_1), \beta(1))$

Then $k_{part} \in (\underline{k}_1, 1)$ and can be written explicitly

$$k_{part} = \frac{(\omega + \lambda)\left(e^Q - 1\right)}{e^Q \lambda - \delta} \tag{22}$$

and $n_{part} = n_r(k_{part}) > 0$ which can be written explicitly

$$\frac{\left(e^{2Q}\lambda^2 - e^Q\lambda - 2\lambda\delta e^{2Q} - 2e^Q\lambda\delta + \delta^2 e^{2Q} + \delta + \delta^2 e^Q + \delta^2\right)(\omega + \lambda)\left(e^Q - 1\right)}{\left(e^Q\lambda - \delta\right)^2} N_0 \tag{23}$$

iii) If $\underline{\beta}(Q) \leq \beta(\underline{k}_1)$

Then $k_{part} \in (0, \underline{k}_1]$ and can be written explicitly

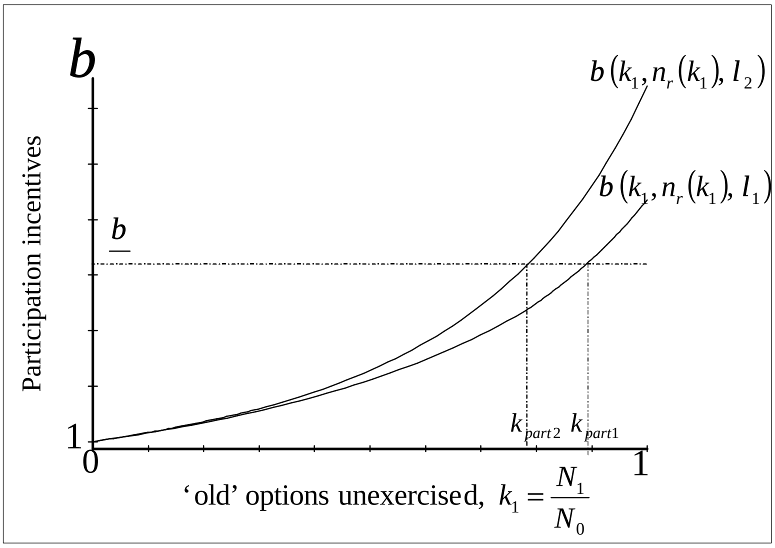
$$k_{part} = \frac{(\omega + \lambda)}{2\underline{\beta}\lambda^2} \left(1 + 2\underline{\beta}\lambda - \sqrt{\left(1 + 4\underline{\beta}\lambda(1 + \lambda)\right)} \right) \tag{24}$$

and

$$n_{part} = n_r(k_{part}) = 0 \tag{25}$$

This proposition uses the recipe of Proposition 4 to ensure risk-taking whilst satisfying the participation constraint at minimum cost. Part (i) corresponds to the manager's participation requirements being so high that even keeping all old options unvested and giving him $n_r(1)$ new ones would be insufficient to make him stay. Thus, nothing vests and an even higher number of new options are required (though fewer are required the higher is λ because moneyness 'helps' the old options provide participation incentives). Risk-taking is not even a binding constraint in this case and so we shall pursue it no further. Risk-taking is binding in parts (ii) and (iii) where we have found explicit solutions for the vesting schedule and the number of new options that should be granted. These solutions have the following properties.

Figure 4: Participation incentives provided when fraction k_1 of old options remain unexercised and number $n_r(k_1)$ of new options are granted. Shown for two different levels of moneyness, $\lambda_1 < \lambda_2$.



Proposition 8 *i) k_{part} is decreasing and convex in λ.*

ii) n_{part} is increasing in λ whenever $\beta(1) < \underline{\beta}$.

This is the central result of our paper: *the deeper In-The-Money are the manager's old options, the more the firm will want him to exercise and so the more it should allow to vest.* This suggests that the firm should consider a vesting schedule which is increasing in stock price attained at t_1. Such a vesting policy we shall term 'Progressive Performance Vesting' and we expand upon this in Section 6. Re-optimizing risk incentives is made easier by the fact that the more valuable are the unvested options, the fewer are required to bind the manager to the firm. Given that we have painted the old options as the risk avoiding menace which needs to be removed, this result is quite intuitive. Less obvious, however, is how the size of the new option grant will be related: Increased moneyness increases the problem of old options (tending to increase n_{part}) but also increases the vesting of old options (tending to decrease n_{part}). Our result shows that the former effect dominates - when the Risk-taking constraint also binds, (parts (ii) and (iii) of Proposition 7) then the *number of new options granted is increasing as a function of the moneyness of old options.* Thus the exercise of old options is positively associated with the granting of new options and may give us a different interpretation of the results of Ofek and Yermack (2000) to which we return in Section 6.

Figure 5: Optimal vesting fraction, $1 - k_{part}$, and number of new options, $n_r\left(k_{part}\right)$ shown for two different levels of moneyness, $\lambda_1 < \lambda_2$.

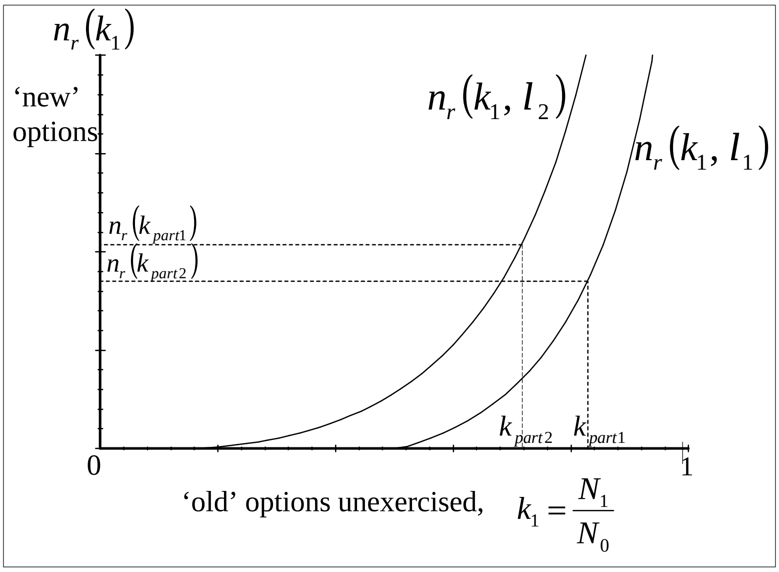


Figure 4 illustrates the result that the higher the moneyness, $(\lambda_2 > \lambda_1)$, the fewer old options should be left unvested $(k_{part2} < k_{part1})$ because the required Participation Incentives are achieved with more vesting. Figure 5 shows that despite this, higher moneyness still implies the granting of more new options, i.e., $n_r\left(k_{part2}\right) > n_r\left(k_{part1}\right)$.

4.2.1 Early Exercise of Vested Options

To complete this section, we note that our derivations of optimal vesting schedules have been made on the assumption that the manager exercises options when they vest. The following lemma confirms that this is indeed the case.

Lemma 9 *When the firm follows the optimal strategy of vesting old options and granting new options in accordance with Proposition 7, the manager does indeed maximize his own utility by exercising the whole* $1 - k_{part}$ *proportion of options that the firm has allowed to vest.*

If the manager were to leave some vested options unexercised (i.e. holding some fraction $k_1 > k_{part}$) then he would will not have correct incentives to choose 'risky' so would choose an alternative course of action. Yet choosing 'safe' he would want to exercise as many options as possible at the

t_1 stock price because the effect of this out-of-equilibrium project decision would be a fall in stock price, reducing the moneyness of any unexercised options. Similarly, choosing 'quit' he would want to exercise as many options as possible at t_1 stock price because he will forfeit any unexercised options on departure.

5 Introducing Costly Effort

Up to now we have assumed that the manager's only relevant personal cost is the 'cost of participation', Q. In this section we extend the basic model by requiring that the manager supply costly effort at t_1 if the profitable growth opportunity is to be available. We decompose the manager's total cost of participation into two components, $Q = B + E$. If the manager does not quit at t_1 then it 'costs' him B which we can think of as the Private Benefit he could obtain elsewhere by quitting the firm. If, in addition, he supplies 'effort' in the job then this costs him a further E. The supply of effort is non-observable and non-verifiable.

The project payoffs $\tilde{V}_{risky}$ and $\tilde{V}_{safe}$ remain as defined in section 3.2.2, but are available only if the manager supplies effort. If he does not supply effort then the project payoffs at t_2 are as follows

$$\tilde{V}_{lazy} = \begin{cases} V_1 + H & \text{w.p.} \quad 1/3 \\ V_1 & \text{w.p.} \quad 1/3 \\ V_1 - H & \text{w.p.} \quad 1/3 \end{cases} \tag{26}$$

This has mean V_1 and variance $\frac{2}{3}H^2$. If supplied, effort can be directed towards increasing cashflows by ε, ('risky') or towards removing the uncertainty in $\tilde{V}$ ('safe'). We have interpreted this as representing the choice between accepting or rejecting a risky-but-profitable project. An alternative interpretation could be that the manager has the opportunity to use a costly, variance reducing hedge.

In our revised set-up we again assume that the efficient decision is for the firm to induce 'risky with effort', requiring that E be 'not so large' that the costs of extracting it become prohibitive. The Incentive Compatibility constraints for risky and for participation remain as before but an additional Incentive Compatibility constraint now exists. Since we are most interested in the role of options in risk-taking we concentrate on situations where IC Risky binds. In the previous section we saw how the firm reduces k_1 and $n_r(k_1)$ as much as possible until the participation constraint binds at k_{part}. In the present section we follow the same approach to discover the k_{eff} at which the IC effort constraint binds. Typically $k_{eff} \neq k_{part}$ and so in order to solve the problem completely, the firm chooses $k_* = \max \{k_{part}, k_{eff}\}$ whereupon the most stringent of these two constraints binds and the other is typically slack.

Ensuring that the manager prefers 'risky with effort' to 'lazy' we have

IC Effort

$$\overline{U}(risky) \geq \overline{U}(lazy) \tag{27}$$

Now at t_1, if the manager exercises $N_0 - N_1$ old options, retains the remaining N_1 old options and is granted n new ones then we can write the payoffs for IC Effort (see proof of Lemma 10). Note here that the 'lazy' option payoffs are quite different to the 'quit' option payoffs seen in the Participation Constraint. In both cases, early exercise of vested options gives the same t_1 cash benefit, however 'quit' leads to the forfeiture of all unvested options whereas 'lazy' still leaves the manager with old and new options which will vest at t_2 and *do* stand some chance of falling In-The-Money.

Lemma 10 *Along the locus* $(k_1, n_r(k_1))$ *required to satisfy the IC Risky constraint*

i) the IC Effort Constraint becomes

$$g(k_1, n_r(k_1)) \geq \underline{g}(E) \tag{28}$$

where $\underline{g}(E) = \exp 3E$ *and* $g(k_1, n_r(k_1)) =$

$$\begin{cases} \frac{(\omega+\lambda-\delta k_1)^2(\omega+\lambda)}{-\delta^3(1-\delta)k_1^3+3\delta^2(1-\delta)(\omega+\lambda)k_1^2-\delta(\omega+\lambda)^2(3(1-\delta)+\lambda)k_1+(\omega+\lambda)^3} & if \quad k_1 \geq \underline{k}_1 \\ \frac{(\omega+\lambda+k_1)(\omega+\lambda)}{(\omega+\lambda+(1-\delta)k_1)(\omega+\lambda-\delta k_1)} & if \quad k_1 < \underline{k}_1 \end{cases} \tag{29}$$

ii) $g(k_1)$ *is increasing in* k_1*.*

We interpret $g(k_1)$ as a measure of the effort incentives generated when a proportion k_1 of the old options remain unexercised and are augmented by the number $n_r(k_1)$ of new options required to efficiently install correct risk-taking incentives.

Part ii) says that, at any level of moneyness, λ, the more of the manager's option wealth that remains unvested until t_2, the greater the manager's returns to supplying effort at t_1. The analytical importance of this lemma is that the set $\{(k_1, n_r(k_1)) \in [0,1] \times [0, \infty) : g(k_1, n_r(k_1)) \geq \underline{g}(E)\}$ is continuous and connected. Thus, to satisfy the IC Risky and IC Effort constraints at minimum cost, the firm need simply reduce the value of k_1 until the Effort constraint binds.The next proposition derives explicit vesting schedules when Effort is the binding constraint. Trivially this is the case when $B = 0$. More generally, it the case when the cost of supplying effort in the job is 'high' compared to the opportunity cost of accepting the job.

Proposition 11 ***The firm's optimal vesting schedule and granting of new options***

The firm's optimal proportion, k_{eff}, of unexercised old options (i.e., allowing $1 - k_{eff}$ to be exercised at t_1) and corresponding number, n_{eff}, of new options together ensure that the manager does supply effort and does choose the risky project. These optima depend on the manager's cost of effort, E, as follows

i) If $\underline{g}(E) > g(1)$

Then

$$k_{eff} = 1 \tag{30}$$

And $n_{eff} > n_r(1)$ and can be written explicitly

$$n_{eff} = \frac{(\omega + \lambda + 1)(\omega + \lambda) - (\omega + \lambda + 1 - \delta)(\omega + \lambda - \delta)\underline{g}}{\left(\underline{g}(1 - \delta)(\omega + \lambda - \delta) - (\omega + \lambda)\right)} N_0 \tag{31}$$

ii) If $\underline{g}(E) \in (g(\underline{k}_1), g(1))$

Then $k_{eff} \in (\underline{k}_1, 1)$ and is the positive real root of the cubic

$$\begin{aligned} &\underline{g}\delta^3(1-\delta)k_1^3 + \delta^2\left(1 - 3\underline{g}(1-\delta)\right)(\omega+\lambda)k_1^2 \\ &+\delta(\omega+\lambda)^2\left(\underline{g}(3(1-\delta)+\lambda) - 2\right)k_1 - (\omega+\lambda)^3\left(\underline{g} - 1\right) \quad = 0 \end{aligned} \tag{32}$$

to which there exists a unique real solution.

And

$$n_{eff} = n_r(k_{eff}) > 0 \tag{33}$$

iii) If $\underline{g}(E) \leq g(\underline{k}_1)$

Then $k_{eff} \in (0, \underline{k}_1]$ and can be written explicitly

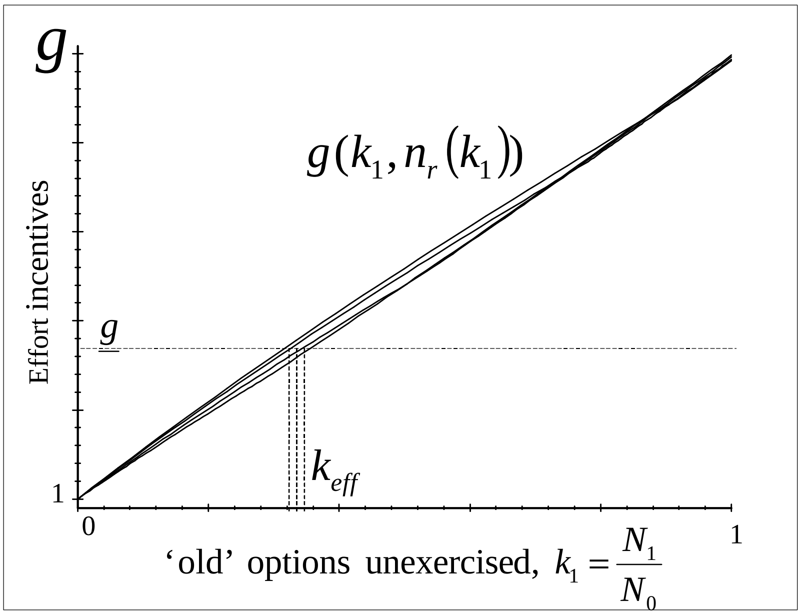
$$k_{eff} = \frac{(\omega + \lambda)}{2\underline{g}\delta^2}\left(1 + 2\underline{g}\delta - \sqrt{\left(1 + 4\underline{g}\delta(1 + \delta)\right)}\right) \tag{34}$$

And

$$n_{eff} = n_r(k_{eff}) = 0 \tag{35}$$

This proposition uses the recipe of Proposition 4 to ensure risk-taking whilst satisfying the Effort constraint at minimum cost. Part i) corresponds to the manager's effort requirements being so high that even keeping all old options unvested and giving him $n_r(1)$ new ones would be insufficient to make him supply effort. Thus, nothing vests and an even higher number of new options are

Figure 6: Effort incentives provided when fraction k_1 of old options remain unexercised and number $n_r(k_1)$ of new options are granted. Shown for several different levels of moneyness, λ.



required. Risk-taking is not even a binding constraint in this case and so we shall pursue it no further. Risk-taking is binding in parts ii) and iii) where we have found explicit solutions for the vesting schedule and the number of new options that should be granted. In seeking to derive the Effort equivalent of the Participation Proposition 8, we find that the function $g(k_1)$ is non-monotonic in λ and hence k_{eff} is potentially non-monotonic in λ. We relegate to the Appendix a couple of regularities in the behaviour of k_{eff} but more illuminating is an inspection of Figure 6.

A striking feature (particularly compared to the corresponding Figure 4 for $\beta(k_1)$) is that $g(k_1)$ is remarkably insensitive to λ. This can be explained by the fact that when deciding between 'risky with effort' or 'lazy', our manager takes into account the 'extra ε' he will enjoy in the upstate (unvested old and new options) and middle state (unvested old options only) of the trinomial payoffs he is contemplating. In our model, this extra payoff has the same cash value whatever the moneyness of the old options[18]. It is true that the moneyness will affect the utility valuation that the manager puts on these extra payoffs, but as the following Lemma shows, the relative effect of

[18] The discrete nature of our payoffs means that the delta of our options is effectively constant on the interval $\lambda \in (\delta, 1)$. In a model with continuous payoff distributions (e.g., Black-Scholes) the delta would increase, the deeper In-The-Money were the old options.

this on the vesting schedule is tiny.

Lemma 12 *If* $\underline{g}(E) \in (g(\underline{k}_1), g(1))$, *then* k_{eff} *is virtually constant on* $\lambda \in [\underline{\lambda}, 1]$. *When the Effort constraint binds, increasing moneyness,* λ, *has economically insignificant impact on the optimal vesting schedule.*

Lemma 12 suggests that we should cap the vesting schedule if or when the moneyness has increased to a level that makes effort bind. In the Appendix we derive bounds on the relative costs of B and E which determine whether the Participation or Effort constrant binds. At low moneyness, the participation constraint binds providing E is 'not too large'. As moneyness increases, k_{part} decreases and the slack effort constraint may eventually bind (if E is 'not too small') or may never bind (if E is 'small enough'). The proportion of options that the firm should allow to vest is $1 - k_*$, where $k_* = \min\{k_{part}, k_{eff}\}$. Thus the performance vesting schedule will be an increasing function, perhaps capped at some constant level if there exists a moneyness at which the effort constraint binds.

Early Exercise of Vested Options As in the previous section, we should now check that the new set-up still leads to the manager exercising all the options that the firm vests. However in the appendix we show that there do potentially exist some circumstances when the manager would prefer to leave some vested options unexercised and to be 'lazy' in the second period. His overall utility gain from doing so is very small and we claim that if the manager potentially needs to be 'forced' to exercise, then the firm could protect itself by putting an early expiration clause into its option grants so that whenever vesting is accelerated by the achievement of the stock price target, expiration is also accelerated. In this way the manager must exercise immediately in order to cash in his gains.

6 Executive Stock Option Plans

In this section, we interpret our results and discuss their empirical and normative implications for the design of option schemes. We point out what can 'go wrong' with calendar vesting and outline the empirical and economic implications. We discuss the use of performance vesting options and show how their design may be refined, then we discuss briefly the use of reload options.

Calendar vesting Having derived optimal vesting schedules, it is now clear that traditional calendar vesting schedules are unlikely to offer 'correct' incentives for risk taking. If the calendar vesting schedule permits a fixed proportion $1 - k_c$ to vest at t_1, leaving k_c unvested until t_1 then the following scenarios are possible:

i) $k_c > k_*$ and the firm has allowed 'not enough' options to vest. From Lemma 4 and Figure 5 we can see that to re-establish risk-taking incentives, the firm needs to grant an unnecessarily high number of new options, $n_r(k_c) > n_r(k_*)$.

ii) $k_c < k_*$ and the firm has allowed 'too many' options to vest. If the manager exercises all of the $1 - k_c$ vested options then the firm needs to grant an unnecessarily high number of new options, not in order to ensure risk taking but in order to satisfy the binding participation or effort constraint. It may be that the manager chooses rationally to exercise less than $1 - k_c$ options and this may mitigate the firms failure to implement the optimal vesting schedule. As we show in Appendix D, because the manager rationally exercises more options the deeper they are In-The-Money, *it is possible that rational managerial exercise decisions may approximate the outcomes of our Progressive Performance Vesting Schedule*. However, if he knows that the firm will inevitably re-establish risk-taking incentives, the manager has strategic incentives to exercise the full $1 - k_c$.

Clearly, vesting a sub-optimal number of options can be expensive for the firm in terms of costly new option grants, the alternative being missed growth opportunities.

Empirically, it would be interesting to document how risk-taking behaviour and subsequent option grants are affected dynamically, as options move In-The-Money. We know that managers rationally exercise more, the deeper In-The-Money and Heath, Huddart and Lang (1999) find evidence of particularly intense exercise activity immediately following vesting dates. This suggests the existence of a pent-up desire to exercise and is evidence that vesting restrictions are often binding. We have seen no study that examines the effect on risk-taking when managers are constrained by vesting requirements. Our results predict that when this is the case, risk-taking will decrease unless the firm issues large numbers of new options to reconvexify the incentives. Numerous studies have shown how options (but not restricted stock) raise volatility. We conjecture that after a period of increasing stock prices, the risk-taking incentives of options may weaken (and even become disincentives) unless renewed and reinvigorated by the mechanism of early exercise and granting of further options that we analyse in this paper. Future research should look at the dynamic portfolio incentives held by their managers and its interaction with temporal volatility evolution.

Our Proposition 8 implies that stock price run-ups may be followed by particularly large grants of new options, even once allowance is made for the high level of exercise of old options. This result has echoes of "You can pay me now and you can pay me later..." (Boschen and Smith (1995)). Their results on pre-1990 data show how managers reap the rewards for good performance not only contemporaneously, but also up to five years following an innovation in firm performance. Our results provide further support for the notion that options are granted both as a current reward for past success *and* to provide future incentives. Empirically, Ofek and Yermack (2000) find that in years when boards grant equity-based compensation to managers, the latter tend to reduce their previously owned equity holdings, thereby counteracting the board's attempts to increase their personal exposure to firm value. Our results suggest that another interpretation may be that there is an implicit recognition of the mutual benefit of option exercise, sale of stock and granting of new options to reoptimise incentives.

Performance Vesting Options The normative conclusion of the present paper is that when risk-taking incentives are important, the firm might be advised to use Progressive Performance Vesting rather than Calender Vesting Options. However, the characteristics of firms actually adopting performance vesting is an unexplored empirical question well worthy of investigation.

A justification sometimes given for performance vesting options is that they strengthen the link between pay and performance, addressing the concerns of shareholder groups who resent managers receiving large option payouts for often mediocre performance. With a performance target, vesting becomes a reward for increasing stock price significantly rather than just an inevitable consequence of the passage of time. Performance targets are generally simple step functions of stock price e.g., no options vest unless a 50% price gain is achieved, whereupon 100% of the options vest. In future research, we intend to document variation in the terms offered and relate this to the characteristics of the firms and CEO's involved. Theoretically, there is no limit to the complexity that could be built into a vesting schedule. Whilst we would not advise practioners to adopt our equation (22) in the design of their vesting schedules, the principle of 'the higher the stock price, the more options vest' might be incorporated more imaginatively into option contracts. Figure 7 shows qualitatively how our proposed Progressive Performance Vesting differs from typical Calendar Vesting and plain vanilla Performance Vesting schemes.

A limitation of our model is that we do not model the decisions and agency problems existing at t_0. However, we can say that the prospect of performance vesting gives the manager additional incentives ex ante at t_0. Managers presiding over successful firms know that they will be rewarded in

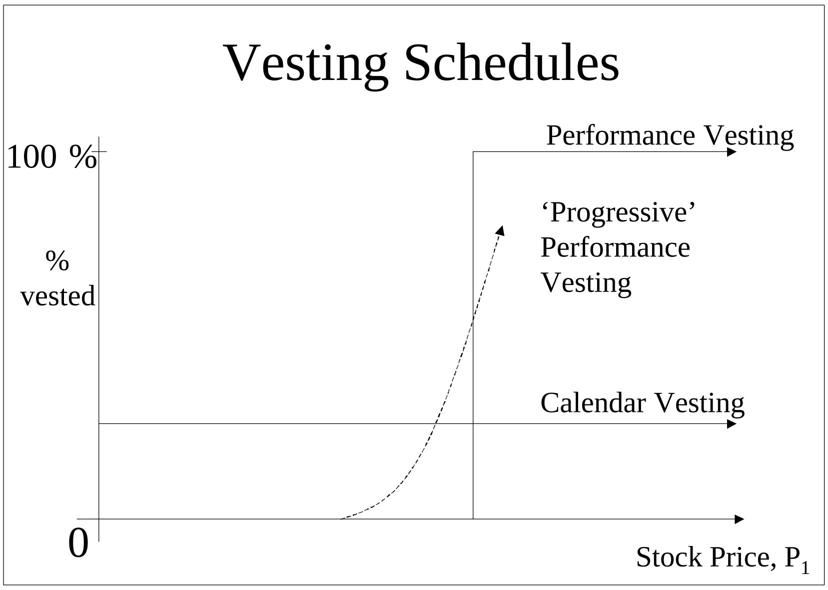


Figure 7:

three ways: their currently held options will each be more valuable (explicit Option Value incentive), a greater number will vest enabling them to cash in significant gains (explicit Performance Vesting incentive) and they will be rewarded yet again by the granting of a larger number of new options (implicit 'Repeat Award' incentive). In ongoing research we provide a fuller characterisation of the ex ante incentives provided by these instruments.

Reload Options The focus of our study has been on the way that firms may seek to manage the convexity of the incentives offered to management. In the context of risk-taking incentives we have seen the problem of ensuring sufficient convexity in the manager's incentives at the current share price. During the 1990's, compensation consultants have popularised 'Reload Options' which have the characteristic that they involve the sequential exercise and 'reloading' of executive options. Each time that the manager exercises reload options, the firm issues him new options At-The-Money. At first glance, this mechanism appears to perfectly address our identified problem of 'lost convexity' - absent large stock price falls, they enable the manager to keep his options always close-to-the-money. However, the reload feature only works if the manager 'pays' the exercise price of the option with previously held shares. Firms tend to use reloads to encourage managers to hold stock and again, it is of empirical interest what sort of firms choose this approach. Effectively the manager is obliged to maintain a store of stocks if he is to make best use of his reloads and we have already shown what risk-reducing incentives this may create. In an accompanying note (Brisley (2001)) we expand upon the risk incentives provided by reloads and our conclusion is contained in the title: "Reload options don't have option payoffs...".

7 Conclusion

This paper has a strong practitioner-oriented message to compensation consultants, boards of directors and anyone else involved in designing executive compensation contracts. It says: "Make sure you think about the risk-taking incentives you are setting for management. Not just at the beginning, but be aware how they may evolve over time". A firm that cares about risk taking must care about the convexity of the compensation contracts offered to managers. In particular, when options are deep In-The-Money, they lose their convexity and this may dramatically reduce risk-taking as managers concentrate on preserving the paper gains they have made. To firms with many valuable-but-risky growth opportunities, the dangers of management conservatism are obvious. The deeper In-The-Money are the options, the more counter-productive may become this effect. We suggest this is one reason for allowing partial early vesting of stock options. Traditional calendar vesting may go some way to achieving this - because managers are risk averse, the deeper In-The-Money are the options, the more the manager will wish to exercise. This can act in the interests of the firm, who may therefore rely on the manager to voluntarily remove incentives when they have become counter-productive. Whilst this coincidence points in the general direction that the firm is aiming, it cannot be relied upon to always give the right incentives, particularly when the fixed vesting schedule leaves the manager holding large amounts of unvested deep In-The-Money options. This is where performance vesting may prove superior because linking vesting to stock price gains allows the manager to remove incentives when and only when they have lost their risk-inducing properties and makes more efficient the firm's attempts to re-establish 'correct' incentives. Performance vesting has the additional advantage that it costs the firm less in terms of dilution - forcing the manager to bear risk is costly because he demands a risk premium, forcing the manager to bear the 'wrong kind' of risk is doubly expensive. Permitting exercise and re-optimising incentives can reduce this cost because the firm needs to give the manager less dollar value but succeeds in rewarding him with reduced uncertainty. When the firm is keen to reduce media and shareholder perceptions of inflated option windfalls, this may be one step towards achieving that objective.

Whilst firms are free to design their compensation schemes within limits, Accounting, Taxation and other regulations certainly impact upon the compensation design choices that firms make. Performance *contingent* options vest if and only if the performance target is met, whereas performance *accelerated* options vest when the target is met and otherwise vest on some fixed calendar date. The latter qualify for favourable accounting treatment and are more commonly used. To the extent that differences in accounting treatment favour one contract solution over another, a level playing field would seem preferable. However compensation consultants and accountants are usually creative enough to approximate the desired outcome with a contract which achieves favourable accounting treatment.[19] In the United States, obtention of the most favourable tax treatment for Incentive Stock Options[20] requires the employee to hold the options for at least one year before exercise *and* to hold the ensuing stock for a further year after exercise. In France the requirement is even stronger and applies to all forms of ESO, namely four years from grant *plus* two years holding the stock after exercise[21]. In the light of our results, we believe that policy-makers eager to encourage risk-taking should be aware of the potentially counter-productive incentive effects of such holding period requirements.

[19] To approximate the incentives and outcomes of a performance contingent option, one might use a performance accelerated option with similar terms except that if the performance target is not met then the option eventually vests at year 9.

[20] Incentive Stock Options account for a very small part of CEO pay. Most of CEO pay comes from Non-Qualifying Stock Options for which the one year stock-holding period after exercise is not relevant.

[21] "La reforme tant annoncé du regime des stock-options", Option Finance, #646, 11 June 2001

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Appendix

A Summary of Notation

Dates, Asset Values and Projects

t_0	initial date
t_1	interim decision date
t_2	terminal liquidation date
P_1	equilibrium stock price at t_1
V_1	asset value at t_1 (parameter)
$\tilde{V}_{qF}$	liquidation asset value at t_2, (random variable depending on manager's decisions at t_1)
q	$\in \{r, s\}$, Manager Project choice, 'risky' or 'safe'
F	$\in \{0, B, Q\}$, Manager's personal cost of choosing, 'quit', 'lazy' or 'effort'

$$\tilde{V}_{risky} = \left\{ \begin{array}{cc} V_1 + H + \varepsilon & \text{w.p. } \frac{1}{3} \\ V_1 + \varepsilon & \text{w.p. } \frac{1}{3} \\ V_1 - H + \varepsilon & \text{w.p. } \frac{1}{3} \end{array} \right| \quad \text{payoffs to 'risky with effort' (equilibrium manager decisions)}$$

$$\tilde{V}_{safe} = V_1 \quad \text{w.p.} \quad 1 \;, \quad \text{certain payoff to 'safe with effort'}$$

$$\tilde{V}_{lazy} = \left\{ \begin{array}{cc} V_1 + H & \text{w.p. } \frac{1}{3} \\ V_1 & \text{w.p. } \frac{1}{3} \\ V_1 - H & \text{w.p. } \frac{1}{3} \end{array} \right| \text{, payoffs to 'lazy'}$$

$$\tilde{V}_{quit} = 0 \quad \text{w.p.} \quad 1 \;, \quad \text{certain payoff to 'quit'}$$

ε	Net Present Value of 'risky with effort' $\tilde{V}_{risky}$ project
H	half of the range of the 'risky with effort' $\tilde{V}_{risky}$ project outcomes
δ	$= \frac{\varepsilon}{H} \in (0, 1)$, normalisation of NPV
σ_r^2	$= \frac{2}{3} H^2$, variance of the 'risky with effort' $\tilde{V}_{risky}$ project payoffs

Options

	# of outstanding shares $= 1$
N_0	# of 'old' options granted at t_0 (exog.)
X_0	exercise price of options granted at t_0
N_1	# of these 'old' options kept throughout the second period to t_2
$N_0 - N_1$	# of these 'old' options exercised early at t_1
k_1	$\frac{N_1}{N_0}$, proportion of 'old' options retained to maturity, (normalisation of N_1)
M	$= P_1 - X_0$ the 'moneyness' at t_1 of options granted at t_0
λ	$= \frac{M}{H} \in [\delta, 1]$, relative moneyness, (normalisation of M)
n	# of 'new' options granted at t_1 (endog.)
X_1	exercise price of any options granted at t_1

Manager

$U(x, F)$	$= u(x) - F$ utility function of total t_2-wealth, x, and effort cost, F
$u(x)$	$= \ln x$, logarithmic utility for wealth
W	fixed outside wealth of manager (exog.)
ω	$= \frac{W}{HN_0}$ normalisation of W

Derived Formulae

$\underline{N}_0$	# of old options *above which* there is a Risk Avoidance Problem at t_1
n'_r	# of new options required to solve that problem, in *absence* of early exercise
n_r	# of new options required to solve Risk Avoidance problem, in *presence* of early exercise
$\underline{N}_1$	# of unexercised old options (after exercise at t_1), below which *no* new options are required for risk-taking
$\underline{k}_1$	$= \frac{\underline{N}_1}{N_0}$, normalisation of $\underline{N}_1$
N_{part}	optimal # of unexercised old options to solve participation and risk constraints
k_{part}	$= \frac{N_{part}}{N_0}$, optimal proportion of unexercised old options, normalization of N_{part}
n_{part}	corresponding # of new options to solve participation and risk constraints
N_{eff}	optimal # of unexercised old options to solve effort and risk constraints
k_{eff}	$= \frac{N_{eff}}{N_0}$, optimal proportion of unexercised old options, normalization of N_{eff}
n_{eff}	corresponding # of new options to solve effort and risk constraints

B An Alternative Equilibrium?

In the analysis of our model we have chosen to study the equilibrium in which the market believes the manager *will* take the positive NPV project (and so share price $P_1 = V_1 + \varepsilon$) and in which the manager does find it optimal to take the positive NPV project. Market and manager take the optimal contract as given since they know it will be in the firm's interests to grant it.

In the interests of completeness and transparency, we now consider the potential existence of an alternative equilibria in which the market believes the manager will *not* take the positive NPV project (and so share price $P_1 = V_1$) and in which the manager would *not* find it optimal to take the positive NPV project

Assuming that the firm still issues new options with exercise price, $X_1 = V_1 + \varepsilon$, the IC Risky, Participation and Effort constraints would each be slightly changed because the manager would have less instant cash reward for early exercise at t_1. It is not clear whether or not our optimal solution to our studied equilibrium would also support this alternative equilibrium. However we claim that even if it did, the equilibrium could be broken by the competitive market pressure of an arbitrageur/raider buying all outstanding shares at $P_1 = V_1$, arranging a private option-like contract with the manager and implementing the optimal solution of our preferred equilibrium.

C The Warrant Problem

We have employed a 'small manager' assumption in order to simplify the analysis. We outline here the complications it would cause if we did not make this approximation.

Strictly speaking, the stock prices at t_2 will not exactly equal the final gross asset value, but will adjust downwards to take account of the share taken by the manager through his exercise(s) at t_2 and/or t_1. Furthermore, the stock price at t_1 must also take account of current and future dilution caused by manager exercise.

Thus, if the interim gross asset value is V_1 and interim stock price is P_1 and if the manager exercises $N_0 - N_1$ options, leaves N_1 unexercised, receives n new options and takes the 'risky with effort' project, then at t_2, the rational final stock price *after* the market has seen the realization of V_2 and before (and after) the manager has exercised his outstanding options is given by

$$P_2 + n\left(P_2 - X_1\right)^+ + N_1\left(P_2 - X_0\right)^+ + \left(N_0 - N_1\right)\left(P_1 - X_0\right) = V_2 \tag{36}$$

where $V_2 = \begin{cases} V_1 + \varepsilon + H & \text{in the 'Up' state} \\ V_1 + \varepsilon & \text{in the 'Middle' state} \\ V_1 + \varepsilon - H & \text{in the 'Down' state} \end{cases}$

Denoting P_{2U}, P_{2M}, P_{2D}, as the values of P_2 in the Up, Middle and Down terminal states respectively

$$P_1 = \frac{1}{3}\left(P_{2U} + P_{2M} + P_{2D}\right) \tag{37}$$

and assuming $\left(P_2 - X_1\right)^+ = 0$ in the Middle state (which we can do by choosing X_1 'high enough'), then

$$\begin{aligned} P_{2U}\left(1 + n + N_1\right) &= V_1 + \varepsilon + H + nX_1 + X_0N_0 - \left(N_0 - N_1\right)P_1 \\ P_{2M}\left(1 + N_1\right) &= V_1 + \varepsilon + X_0N_0 - \left(N_0 - N_1\right)P_1 \\ P_{2D} &= V_1 + \varepsilon - H + X_0\left(N_0 - N_1\right) - \left(N_0 - N_1\right)P_1 \end{aligned} \tag{38}$$

giving

$$P_1 = \frac{\left(\frac{1}{(1+n+N_1)} + \frac{1}{(1+N_1)} + 1\right)\left(V_1 + \varepsilon + X_0N_0\right) + \frac{nX_1 - (n+N_1)H}{(1+n+N_1)} - X_0N_1}{3 + \left(\frac{1}{(1+n+N_1)} + \frac{1}{(1+N_1)} + 1\right)\left(N_0 - N_1\right)} \tag{39}$$

from which we can recover P_{2U}, P_{2M} and P_{2D} each in terms of V_1, ε, H, X_0, N_0, n and N_1. These solutions themselves determine the payoffs to the manager's options which is an input to the derivation of the optimal numbers of options to grant and to vest...Effectively the above system of equations would represent yet another condition on the derivations of N_* and n_*.

D Rational Early Exercise

In equilibrium, the manager's expected utility can be written

$$\overline{U}\left(risky\right) = \ln HN_0 + \frac{1}{3}\ln\left(\left(\omega + \lambda + k_* + \frac{n_*}{N_0}\right)\left(\omega + \lambda\right)\left(\omega + \lambda\left(1 - k_*\right)\right)\right) - Q \qquad (40)$$

We have shown in Lemma 9 the circumstances in which the manager will exercise the options that the firm allows to vest. This exercise decision is partly driven by the fact that the manager knows that to *not* exercise would change his incentives, leading him to supply the 'wrong' decisions and making it optimal to exercise. We have, quite properly, ensured that our manager make his exercise/project decisions simultaneously. However, to show the effect of the manager's risk aversion in isolation, we now take the 'risky with effort' decision as given and ask how many options the manager would exercise *abstracting from its effects on his incentives within the firm.* This is the approach taken in Huddart (1994) and Detemple and Sundaresan (1999).

Taking n_* as given, the manager's exercise decision would be to choose k to maximise the following objective

$$\left(\omega + \lambda + k + \frac{n_*}{N_0}\right)\left(\omega + \lambda\left(1 - k\right)\right) \qquad (41)$$

Expanding, the maximand is quadratic in k,

$$-\lambda k^2 + \left(-\omega\lambda + \lambda - \lambda^2 + \omega - \frac{n_*}{N_0}\lambda\right)k + \omega^2 + 2\omega\lambda + \lambda^2 + \frac{n_*}{N_0}\omega + \frac{n_*}{N_0}\lambda \qquad (42)$$

This function has a maximum at

$$k_{manager} = \frac{1}{2}\left(\left(\frac{1-\lambda}{\lambda}\right)\left(\omega + \lambda\right) - \frac{n_*}{N_0}\right) \qquad (43)$$

and we proceed to examine whether this solution is interior to the range of $k \in (0, 1)$.

First take $n_* = 0$, then $k_{manager} = \frac{1}{2}\left(\frac{1-\lambda}{\lambda}\right)\left(\omega + \lambda\right) \geq 0$ and $k_{manager} < 1$ if and only if $\lambda + \omega\lambda + \lambda^2 - \omega > 0$ i.e.,

$$\lambda > \frac{1}{2}\left(\sqrt{\left(1 + 6\omega + \omega^2\right)} - \left(1 + \omega\right)\right) \qquad (44)$$

Otherwise there is a corner solution at $k = 1$.

Also $\frac{\partial k_{manager}}{\partial \lambda} = -\frac{1}{2}\frac{\lambda^2 + \omega}{\lambda^2}$ which is strictly negative so the manager exercises more options, the deeper they are In-The-Money

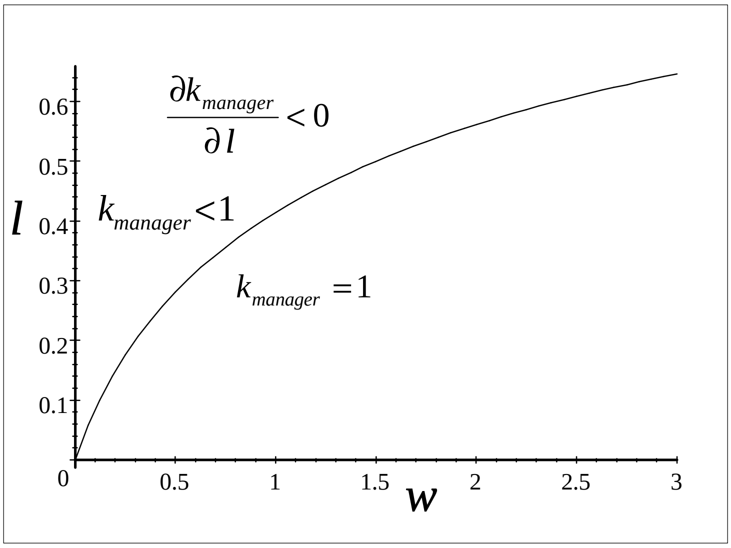


Figure 8:

$$k_{manager} = \begin{cases} \frac{1}{2}\left(\frac{1-\lambda}{\lambda}\right)(\omega + \lambda) & if \quad \lambda > \frac{1}{2}\left(\sqrt{(1+6\omega+\omega^2)} - (1+\omega)\right) \\ 1 & otherwise \end{cases} \tag{45}$$

So, for low λ he exercises nothing, exercising more as λ increases. Intuitively, higher wealth reduces the risk aversion of our DARA manager, increasing the threshold moneyness at which the manager will start to exercise early.

For $n_* > 0$, we know that n_* is increasing in λ, so $\frac{\partial k_{manager}}{\partial \lambda}$ is still negative and manager chooses to exercising more, the deeper In-The-Money. Substituting in the solution for n_* from equation 23, and the expression for k_* from equation 22

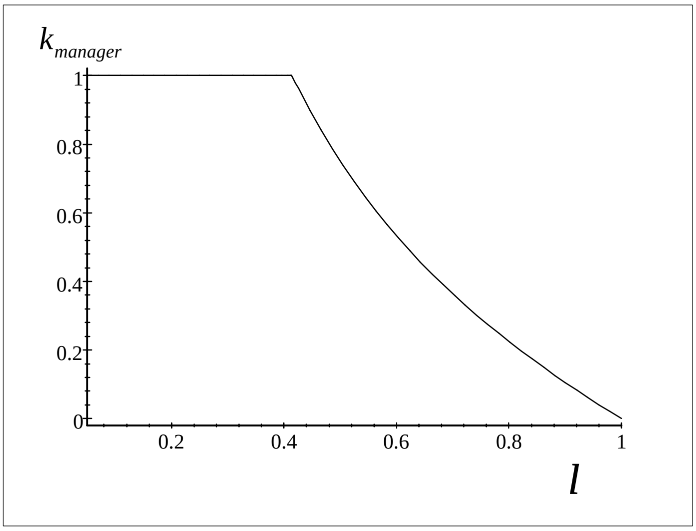
$$k_{manager} = k_{part}\left[1 - \frac{1}{2}\frac{(\lambda-\delta)\left(e^{3Q}\lambda^2 - \left(\delta e^{2Q}+1\right)e^{Q}\lambda + \delta\right)}{\lambda\left(e^{Q}-1\right)\left(e^{Q}\lambda - \delta\right)}\right] \tag{46}$$

To the extent that the term $\frac{1}{2}\frac{(\lambda-\delta)\left(e^{3Q}\lambda^2 - \left(\delta e^{2Q}+1\right)e^{Q}\lambda + \delta\right)}{\lambda\left(e^{Q}-1\right)\left(e^{Q}\lambda - \delta\right)}$ is 'small', the manager may exercise approximately the 'right number' of options from the firm's point of view.

E Proofs of Lemmas and Propositions

Proof of Lemma 1. i) IC Risky binds when

Figure 9: with $\omega = 1$, $k_{manager} < 1$ once $\lambda > \sqrt{2} - 1$. Note also that $k_{manager} = 0$ once $\lambda = 1$ since then the options have lost all time value and so lose nothing in early exercise.



$$\frac{(W + NM)(W + N(M + H))W}{(W + N(M - \varepsilon))^3} = 1 \tag{47}$$

Simplifying

$$(W + N(M - \varepsilon))^3 - (W + NM)(W + N(M + H))W = 0 \tag{48}$$

Which gives the following equation in N

$$\left[(M - \varepsilon)^3 N^2 + \left(2M^2 - 6M\varepsilon + 3\varepsilon^2 - MH\right) WN - (H - M + 3\varepsilon) W^2\right] N = 0 \tag{49}$$

which has solutions

$N = 0$, and $\frac{W\left(MH - 2M^2 + 6M\varepsilon - 3\varepsilon^2 \pm \sqrt{(M^2H^2 + 6MH\varepsilon^2 + 4M\varepsilon^3 - 3\varepsilon^4 - 4\varepsilon^3 H)}\right)}{2(M-\varepsilon)^3}$

The two non-zero roots are respectively positive and negative. We choose the positive one, namely

$$\underline{N}_0 = \frac{W\left(MH - 2M^2 + 6M\varepsilon - 3\varepsilon^2 + \sqrt{(M^2H^2 + 6MH\varepsilon^2 + 4M\varepsilon^3 - 3\varepsilon^4 - 4\varepsilon^3 H)}\right)}{2(M - \varepsilon)^3} \tag{50}$$

equivalently

$$\underline{N}_0(\lambda) = \frac{W\left(-2\lambda^2 + \lambda(1 + 6\delta) - 3\delta^2 + \sqrt{\lambda^2 + 2\lambda\delta^2(3 + 2\delta) - \delta^3(4 + 3\delta)}\right)}{2H(\lambda - \delta)^3} \tag{51}$$

Now equation 49 has positive leading coefficient, so for $N_0 > \underline{N}_0(\lambda)$ we have

$$(W + N(M - \varepsilon))^3 - (W + NM)(W + N(M + H))W > 0 \tag{52}$$

i.e.,

$$1 > \frac{(W + NM)(W + N(M + H))W}{(W + N(M - \varepsilon))^3} \tag{53}$$

breaking the IC Risky constraint.

ii)To find the sign of $\frac{\partial}{\partial\lambda}\underline{N}_0(\lambda)$, we transform equation 51 using $\lambda = \delta + y$ and calculate $\frac{\partial}{\partial y}\left(\frac{\delta^2+2\delta y-2y^2+\delta+y+\sqrt{\left(\delta^2+2\delta y+y^2+2\delta^3+6\delta^2 y+\delta^4+4\delta^3 y\right)}}{y^3}\right) =$
$= -\left(\frac{\left(4y\delta+2y(1-y)+3\delta^2+3\delta\right)}{y^4} + \frac{5\delta y+2y^2+15\delta^2 y+10\delta^3 y+3\delta^2+6\delta^3+3\delta^4}{\sqrt{\left(\delta^2+2\delta y+y^2+2\delta^3+6\delta^2 y+\delta^4+4\delta^3 y\right)}y^4}\right)$ which is certainly negative.

Proof of Corollary2. i) The condition says that there is a Risk Avoidance Agency Problem at least when the options are 100% In-The-Money, $\underline{N}_0(1) < N_0$.

However, we have shown that $\underline{N}_0(\lambda)$ is decreasing in λ and we can see that $\underline{N}_0(\lambda) \nearrow \infty > N_0$ as $\lambda \searrow \delta$.

Thus, $\exists\underline{\lambda} \in (\delta, 1)$ s.t. $\underline{N}_0(\underline{\lambda}) = N_0$ whereupon $\underline{N}_0(\lambda) < N_0, \forall\lambda > \underline{\lambda}$.

$\underline{\lambda}$ is the solution to

$$\frac{\left(-2\lambda^2 + \lambda(1 + 6\delta) - 3\delta^2 + \sqrt{\lambda^2 + 2\lambda\delta^2(3 + 2\delta) - \delta^3(3\delta + 4)}\right)}{2(\lambda - \delta)^3} = \frac{1}{\omega} \tag{54}$$

i.e., to

$$\lambda^3 + (2\omega - 3\delta)\lambda^2 + \left(-6\omega\delta + 3\delta^2 - \omega + \omega^2\right)\lambda - \delta^3 - 3\omega^2\delta + 3\omega\delta^2 - \omega^2 = 0 \tag{55}$$

ii) We have shown that the LHS of equation 54 is monotonic decreasing in λ. Increasing N_0 causes the RHS to increase and so reduces the $\underline{\lambda}$ at which equality is achieved.

Proof of Lemma 3. From equation 8, without early exercise, if there is a Risk Avoidance Agency Problem when $n = 0$, then we need to increase n until equation 8 is satisfied, whereupon $\rho(n_r', N_0, W, M) = 1$ which gives

i)

$$n_r' = \left(\frac{(\omega + (\lambda - \delta))^3}{\omega(\omega + \lambda)} - (\omega + \lambda + 1)\right)N_0 \tag{56}$$

ii) $\frac{\partial}{\partial N_0}\left(\frac{(W+N_0(M-\varepsilon))^3}{WH(W+N_0 M)} - \frac{(W+N_0(M+H))}{H}\right) =$

$$-(\lambda+1)+\frac{(W+N_0(M-\varepsilon))^2}{(W+N_0M)^2}\frac{W(2M-3\varepsilon)+2N_0M(M-\varepsilon)}{WH} \tag{57}$$

which is positive for $N_0 = \underline{N}_0$

And $\frac{\partial^2}{\partial N_0^2}\left(\frac{(W+N_0(M-\varepsilon))^3}{WH(W+N_0M)}-\frac{(W+N_0(M+H))}{H}\right)$

$= 2(W+N_0(M-\varepsilon))\frac{M^2(M-\varepsilon)^2N_0^2+MW(2M-3\varepsilon)(M-\varepsilon)N_0+W^2\left(\left(M-\frac{3\varepsilon}{2}\right)^2+\frac{3\varepsilon^2}{4}\right)}{WH(W+N_0M)^3}$ which is positive

iii) transforming equation 56, with $\lambda = y+\delta$, $y>0$

$$\begin{aligned}&\frac{\partial}{\partial y}\left(\frac{(\omega+y)^3}{\omega(\omega+\delta+y)}-\omega-\delta-y-1\right)\\ =\;&\frac{\omega^3+\omega^2\delta+4\omega^2y+4\omega y\delta+5\omega y^2+3y^2\delta+2y^3-\omega\delta^2}{\omega(\omega+\delta+y)^2}>0\end{aligned} \tag{58}$$

and

$$\begin{aligned}&\frac{\partial^2}{\partial y^2}\left(\frac{(\omega+y)^3}{\omega(\omega+\delta+y)}-\omega-\delta-y-1\right)\\ =\;&2(\omega+y)\frac{\omega^2+3\omega\delta+2\omega y+3\delta^2+3\delta y+y^2}{\omega(\omega+\delta+y)^3}>0\end{aligned} \tag{59}$$

Proof of Proposition 4. With the cashin of (N_0-N_1) old options, each of moneyness M, and the granting of n new options, condition 2 becomes

$$\begin{aligned}&(W+MN_0+N_1H+nH)(W+MN_0)(W+M(N_0-N_1))\\ \geq\;&(W+MN_0-\varepsilon N_1)^3\end{aligned} \tag{60}$$

i.e.,

$$\left(\omega+\lambda+k_1+\frac{n}{N_0}\right)(\omega+\lambda)(\omega+\lambda(1-k_1))\geq(\omega+\lambda-\delta k_1)^3 \tag{61}$$

i) When $n=0$, if this condition is satisfied then no new options need be granted.

The inequality is breached when

$$(\omega+\lambda+k_1)(\omega+\lambda)(\omega+\lambda(1-k_1))<(\omega+\lambda-\delta k_1)^3 \tag{62}$$

i.e.,

$$k_1\left[k_1^2\delta^3-\left(\lambda+3\delta^2\right)(\omega+\lambda)k_1+(\omega+\lambda)^2(1-\lambda+3\delta)\right]<0 \tag{63}$$

The quadratic in k_1 has solutions

$k_1=\frac{(\omega+\lambda)}{2\delta^3}\left(3\delta^2+\lambda\pm\sqrt{\left(-3\delta^4+6\lambda\delta^2+\lambda^2+4\delta^3\lambda-4\delta^3\right)}\right)$

which are both positive so we take the smaller and our inequality is breached when

$$k_1 > \underline{k}_1 = \frac{(\omega+\lambda)}{2\delta^3}\left(3\delta^2+\lambda-\sqrt{\lambda^2+\delta^2(6+4\delta)\lambda-(4+3\delta)\delta^3}\right) \tag{64}$$

whereupon then increasing n achieves equality when

$$\begin{aligned}\frac{n}{N_0} &= \frac{(\omega+\lambda-\delta k_1)^3}{(\omega+\lambda)(\omega+\lambda(1-k_1))}-(\omega+\lambda+k_1)\\ &= \frac{k_1\left(-\frac{\delta^3}{(\omega+\lambda)}k_1^2+\left(3\delta^2+\lambda\right)k_1-(\omega+\lambda)(1-\lambda+3\delta)\right)}{(\omega+\lambda(1-k_1))}\end{aligned} \tag{65}$$

ii)

The first derivative

$$\frac{\partial}{\partial k_1}\left(\frac{n}{N_0}\right)=\frac{2\delta^3k_1^3\lambda-\left(3\delta^3+3\delta^2\lambda+\lambda^2\right)(\omega+\lambda)k_1^2+2(\omega+\lambda)^2\left(3\delta^2+\lambda\right)k_1-(\omega+\lambda)^3(1-\lambda+3\delta)}{(\omega+\lambda)(-\omega-\lambda+\lambda k_1)^2} \tag{66}$$

Now when $k_1=\underline{k}_1$, this has the sign of

$$\begin{aligned}&\left(\lambda^2+3\delta^2\lambda-\delta^3\right)\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)\\ &+\left(-\lambda^3-3\lambda\delta^4-2\lambda^2\delta^3+3\delta^3\lambda-6\lambda^2\delta^2+3\delta^5\right)\sqrt{\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)}\end{aligned} \tag{67}$$

To show this is positive, we need

$$\begin{aligned}&\left(\left(\lambda^2+3\delta^2\lambda-\delta^3\right)\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)\right)^2\\ >\ &\left(-\lambda^3-3\lambda\delta^4-2\lambda^2\delta^3+3\delta^3\lambda-6\lambda^2\delta^2+3\delta^5\right)^2\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)\end{aligned} \tag{68}$$

i.e.

$$\begin{aligned}0\ <\ &\left(\left(\lambda^2+3\delta^2\lambda-\delta^3\right)\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)\right)^2\\ &-\left(-\lambda^3-3\lambda\delta^4-2\lambda^2\delta^3+3\delta^3\lambda-6\lambda^2\delta^2+3\delta^5\right)^2\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)\end{aligned} \tag{69}$$

but the RHS $=4\delta^6(1-\lambda+3\delta)\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)(\lambda-\delta)^3$ which is indeed positive

Furthermore, the second derivative, $\frac{\partial^2}{\partial k_1^2}\left(\frac{n}{N_0}\right)$ has the sign of $\left(\delta\lambda k_1+\frac{(\lambda-3\delta)(\omega+\lambda)}{2}\right)^2+\frac{3}{4}(\lambda-\delta)^2(\omega+\lambda)^2$ which is positive.

iii) $\frac{\partial}{\partial \lambda}\left(\frac{n}{N_0}\right)$ can be found as an expression which has the sign of

$$\begin{aligned}
&\delta^2 k_1^2 (\omega+\lambda)(\lambda+2\omega+2\delta)+\delta^2 k_1^2 (2\lambda+\omega)(\lambda-\delta k_1+\omega) \\
&+(\omega+\lambda)^2\left(\omega(\omega+2\lambda-3\delta)-3\delta^2\right)+(\omega+\lambda)^2\left(\lambda^2+3\delta^2+3\delta\omega\right)(1-k_1)
\end{aligned} \tag{70}$$

which is positive.

Proof of Lemma6 i) substituting equation 13 into inequality 17, we see that

If $k_1 \leq \underline{k}_1$, then $\beta(k_1)=\frac{(\omega+\lambda+k_1)(\omega+\lambda)}{(\omega+\lambda(1-k_1))^2}$

and if $k_1 > \underline{k}_1$, then

$$\begin{aligned}
\beta(k_1) &= \frac{\left(\omega+\lambda+k_1+\frac{k_1\left(-k_1^2\delta^3+\left(3\delta^2+\lambda\right)(\omega+\lambda)k_1-(\omega+\lambda)^2(1+3\delta-\lambda)\right)}{(\omega+(1-k_1)\lambda)(\omega+\lambda)}\right)(\omega+\lambda)}{(\omega+\lambda(1-k_1))^2} \\
&= \frac{(\omega+\lambda-\delta k_1)^3}{(\omega+\lambda(1-k_1))^3}=\left(1+\frac{(\lambda-\delta)k_1}{\omega+\lambda(1-k_1)}\right)^3
\end{aligned} \tag{71}$$

ii) If $k_1 \leq \underline{k}_1$, then

$$\frac{\partial}{\partial k_1}\beta(k_1)=(\omega+\lambda)\frac{\omega+\lambda+\lambda k_1+2\lambda\omega+2\lambda^2}{(\omega+\lambda-\lambda k_1)^3} \tag{72}$$

which is positive.

If $k_1 > \underline{k}_1$, then

$$\frac{\partial}{\partial k_1}\beta(k_1)=3(\omega+\lambda-\delta k_1)^2\frac{(\lambda-\delta)(\omega+\lambda)}{(\omega+\lambda-\lambda k_1)^4} \tag{73}$$

which is also positive

iii) and iv) If $k_1 > \underline{k}_1$, then

$$\frac{\partial}{\partial \lambda}\beta(k_1)=3(\omega+\lambda-\delta k_1)^2 k_1\frac{\omega+\delta-\delta k_1}{(\omega+\lambda-\lambda k_1)^4} \tag{74}$$

which is positive

If $k_1 \leq \underline{k}_1$, then

$$\frac{\partial}{\partial \lambda}\beta(k_1)=k_1\frac{2\lambda\omega-\omega-\lambda+\lambda k_1+2\omega^2+2\omega k_1}{(\omega+\lambda-\lambda k_1)^3} \tag{75}$$

which is positive if and only if

$$k_1 > \frac{(1-2\omega)(\omega+\lambda)}{(\lambda+2\omega)} \tag{76}$$

Aside: $\beta(1)$ and $\beta(\underline{k}_1)$

$$\beta(1,n_r(1))=\frac{(\omega+\lambda-\delta)^3}{\omega^3} \tag{77}$$

and

$$\beta\left(\underline{k}_1\right)=\frac{\left(1+\frac{1}{2\delta^3}\left(3\delta^2+\lambda-\sqrt{\lambda^2+\delta^2\left(6+4\delta\right)\lambda-\delta^3\left(4+3\delta\right)}\right)\right)}{\left(1-\frac{\lambda}{2\delta^3}\left(3\delta^2+\lambda-\sqrt{\lambda^2+\delta^2\left(6+4\delta\right)\lambda-\delta^3\left(4+3\delta\right)}\right)\right)^2} \tag{78}$$

Note that whilst $\beta\left(k_1\right)$ is increasing in λ, $\beta\left(\underline{k}_1\right)$ is decreasing in λ (because $\underline{k}_1$ is decreasing in λ).

Proof of Proposition 7 *i)* If $\exp 3Z > \beta\left(1\right)$, then $k_{part}=1$ and we need to grant strictly more than $n_r\left(1\right)$ new options to satisfy

$$\frac{\left(\omega+\lambda+1+\frac{n}{N_0}\right)\left(\omega+\lambda\right)}{\omega^2}=\exp\left(3Q\right) \tag{79}$$

which gives directly equation 21.

ii) If $\exp 3Q \in \left(\beta\left(\underline{k}_1\right),\beta\left(1\right)\right)$

Then $k_{part}\in\left(\underline{k}_1,1\right)$ and $\beta\left(k_1\right)=e^{3Q}$ yields the cubic

$$\begin{aligned}&\left(e^{3Q}\lambda^3-\delta^3\right)k_1^3-3\left(e^{3Q}\lambda^2-\delta^2\right)\left(\omega+\lambda\right)k_1^2\\&+3\left(\omega+\lambda\right)^2\left(e^{3Q}\lambda-\delta\right)k_1-\left(\omega+\lambda\right)^3\left(e^{3Q}-1\right)\quad=0\end{aligned} \tag{80}$$

to which $k_{part}=\left(\frac{(\omega+\lambda)\left(e^{Q}-1\right)}{e^{Q}\lambda-\delta}\right)$ is the unique real solution.

Furthermore, $\frac{n_{part}}{N_0}=\frac{n_r(k_{part})}{N_0}$

$$=\frac{\left(\frac{(\omega+\lambda)\left(e^{Z}-1\right)}{e^{Z}\lambda-\delta}\right)\left(-\left(\frac{(\omega+\lambda)\left(e^{Z}-1\right)}{e^{Z}\lambda-\delta}\right)^2\delta^3+\left(3\delta^2+\lambda\right)(\omega+\lambda)\left(\frac{(\omega+\lambda)\left(e^{Z}-1\right)}{e^{Z}\lambda-\delta}\right)-(\omega+\lambda)^2(1+3\delta-\lambda)\right)}{\left(\omega+\left(1-\left(\frac{(\omega+\lambda)\left(e^{Z}-1\right)}{e^{Z}\lambda-\delta}\right)\right)\lambda\right)(\omega+\lambda)}$$

giving $n_{part}=\frac{\left(e^{2Z}\lambda^2-e^{Z}\lambda-2\lambda\delta e^{2Z}-2e^{Z}\lambda\delta+\delta^2e^{2Z}+\delta+\delta^2e^{Z}+\delta^2\right)(\omega+\lambda)\left(e^{Z}-1\right)}{\left(e^{Z}\lambda-\delta\right)^2}N_0$

iii) If $\exp 3Z \leq \beta\left(\underline{k}_1\right)$, then $n_{part}=0$ and we need to retain only enough old options to satisfy

$$\frac{\left(\omega+\lambda+k_1\right)\left(\omega+\lambda\right)}{\left(\omega+\lambda\left(1-k_1\right)\right)^2}=\exp 3Q \tag{81}$$

the quadratic

$$e^{3Q}\lambda^2k_1^2+\left(-2e^{3Q}\omega\lambda-2e^{3Q}\lambda^2-\omega-\lambda\right)k_1+e^{3Q}\omega^2+e^{3Q}\lambda^2+2e^{3Q}\omega\lambda-2\omega\lambda-\lambda^2-\omega^2=0 \tag{82}$$

Then $k_{part}\in\left(0,\underline{k}_1\right]$ and can be written explicitly

$$k_{part}=\frac{\left(\omega+\lambda\right)}{2e^{3Q}\lambda^2}\left(1+2e^{3Q}\lambda-\sqrt{\left(1+4e^{3Q}\lambda\left(1+\lambda\right)\right)}\right) \tag{83}$$

Proof of Proposition 8 i)$\frac{\partial}{\partial\lambda}\left(k_{part}\right) = -\left(e^{Q}-1\right)\frac{\delta+e^{Z}\omega}{\left(e^{Q}\lambda-\delta\right)^{2}}$ which is certainly negative

so $\left(1-k_{part}\right)$ is increasing

$\frac{\partial^2}{\partial\lambda^2}\left(k_{part}\right) = \frac{\partial^2}{\partial\lambda^2}\left(\frac{(\omega+\lambda)\left(e^{Q}-1\right)}{e^{Q}\lambda-\delta}\right) = 2\left(e^{Q}-1\right)e^{Q}\frac{\delta+e^{Q}\omega}{\left(e^{Q}\lambda-\delta\right)^{3}}$ which is certainly positive

so $\left(1-k_{part}\right)$ is concave

ii)

$\frac{\partial}{\partial\lambda}\left(n_{part}\right)$ has the sign of the expression

$$\begin{aligned}
&e^{3Q}(\lambda-\delta)^3 + 3e^{2Q}\delta\left(e^{Q}-1\right)(\lambda-\delta)^2 + \\
&\left(\left(e^{Q}-1\right)2\delta^2 e^{2Q} + \left(\omega+2e^{Q}\delta\omega-\delta^2\right)e^{2Q} + e^{Q}\delta(1+3\delta)\right)(\lambda-\delta) \\
&+\left(e^{Q}-1\right)\left(\delta^2+\delta+\left(\omega-\delta^2\right)e^{Q}\right)\delta
\end{aligned} \tag{84}$$

which is positive.

Proof of Lemma 9 Imagine first that the manager does as is hoped of him and exercises exactly $N_0 - N_{part}$ old options. He then has the correct incentives for 'risky' and looks forward to expected utility of

$$\begin{aligned}
\overline{U}(risky) &= \frac{1}{3}\ln\left(W + M\left(N_0 - N_{part}\right) + (M+H)N_{part} + Hn_{part}\right) \\
&+\frac{1}{3}\ln\left(W + M\left(N_0 - N_{part}\right) + MN_{part}\right) \\
&+\frac{1}{3}\ln\left(W + M\left(N_0 - N_{part}\right)\right) \\
&-Q
\end{aligned} \tag{85}$$

$$\begin{aligned}
\overline{U}(risky) &= \ln HN_0 \\
&+\frac{1}{3}\ln\left(\left(\omega+\lambda+k_{part}+\frac{n_{part}}{N_0}\right)(\omega+\lambda)\left(\omega+\lambda\left(1-k_{part}\right)\right)\right) - Q
\end{aligned} \tag{86}$$

Now imagine the contraire, that he decides to keep some $N_1 > N_{part}$ old options 'in play' for the second period. Since the IC Risky constraint was binding at $N_1 = N_{part}$, he will no longer have correct incentives to choose 'risky' so will choose an alternative course of action.

The 'safe' expected utility is

$$\overline{U}(safe) = \ln HN_0 + \ln\left(\omega+\lambda-\delta k_1\right) - Q \tag{87}$$

which is strictly decreasing in k_1 and so the manager will not choose to leave vested options unexercise (effectively he would be failing to exercise at the t_1 stock price when he knows that the stock price will fall if he takes the out-of-equilibrium decision 'safe'.

The 'quit' expected utility is

$$\overline{U}\,(quit) = \ln HN_0 + \ln\left(\omega + \lambda\left(1 - k_1\right)\right) \tag{88}$$

which is strictly decreasing in k_1 and so the manager will not do this (intuitively, to resign leaving vested options unexercised would be irrational).

Proof of Lemma 10 IC Effort can be written

$$\begin{bmatrix} \frac{1}{3}\ln\left(W + M\left(N_0 - N_1\right) + N_1\left(M + H\right) + nH\right) \\ +\frac{1}{3}\ln\left(W + M\left(N_0 - N_1\right) + N_1 M\right) \\ +\frac{1}{3}\ln\left(W + M\left(N_0 - N_1\right)\right) - E \end{bmatrix}$$

$$\geq \begin{bmatrix} \frac{1}{3}\ln\left(W + M\left(N_0 - N_1\right) + N_1\left(M + H - \varepsilon\right) + n\left(H - \varepsilon\right)\right) \\ +\frac{1}{3}\ln\left(W + M\left(N_0 - N_1\right) + N_1\left(M - \varepsilon\right)\right) \\ +\frac{1}{3}\ln\left(W + M\left(N_0 - N_1\right)\right) \end{bmatrix} \tag{89}$$

Simplifying

$$\frac{\left(\omega + \lambda + k_1 + \frac{n}{N_0}\right)\left(\omega + \lambda\right)}{\left(\omega + \lambda + \left(1 - \delta\right)k_1 + \frac{n(1-\delta)}{N_0}\right)\left(\omega + \lambda - \delta k_1\right)} \geq \exp 3E \tag{90}$$

i) substituting equation 13 into inequality 90, we see that

If $k_1 \leq \underline{k}_1$, then

$$g\left(k_1, n_r\left(k_1\right)\right) = \frac{\left(\omega + \lambda + k_1\right)\left(\omega + \lambda\right)}{\left(\omega + \lambda + \left(1 - \delta\right)k_1\right)\left(\omega + \lambda - \delta k_1\right)} \tag{91}$$

and if $k_1 > \underline{k}_1$, then

$$g\left(k_1, n_r\left(k_1\right)\right) = \frac{\left(\omega + \lambda - \delta k_1\right)^2\left(\omega + \lambda\right)}{-\delta^3\left(1 - \delta\right)k_1^3 + 3\delta^2\left(1 - \delta\right)\left(\omega + \lambda\right)k_1^2 - \delta\left(\omega + \lambda\right)^2\left(3\left(1 - \delta\right) + \lambda\right)k_1 + \left(\omega + \lambda\right)^3} \tag{92}$$

ii) If $k_1 \leq \underline{k}_1$, then $\frac{\partial g}{\partial k} = \frac{\partial}{\partial k}\left(\frac{(\omega+\lambda+k)(\omega+\lambda)}{(\omega+\lambda+(1-\delta)k)(\omega+\lambda-\delta k)}\right)$
$= \left(\omega + \lambda\right)\delta\frac{2\lambda^2+2\omega^2+4\omega\lambda+2k\lambda-2\lambda\delta k+2k\omega-\delta k^2+k^2-2\omega\delta k}{(\omega+\lambda+k-\delta k)^2(\omega+\lambda-\delta k)^2}$

which has the sign of $\left(\lambda - \delta k\right)2\lambda + 2\omega^2 + \left(2\lambda - \delta k\right)2\omega + 2k\lambda + 2k\omega + k^2\left(1 - \delta\right)$ which is certainly positive.

If $k_1 > \underline{k}_1$, then $\frac{\partial g}{\partial k} = \frac{\partial}{\partial k}\left(\frac{(\omega+\lambda-\delta k)^2(\omega+\lambda)}{-\delta^3(1-\delta)k^3+3\delta^2(1-\delta)(\omega+\lambda)k^2-\delta(\omega+\lambda)^2(3(1-\delta)+\lambda)k+(\omega+\lambda)^3}\right) =$

which has the sign of

$3\omega^2\lambda + 3\omega\lambda^2 + \lambda^3 - 9\omega\lambda^2\delta - 3\delta^3k^2\omega - 3\delta k\lambda^2 + 3\delta^2k\omega^2 - 3\delta k\omega^2 + \delta k\lambda^3 + 3\delta^2k\lambda^2 + 3\delta^2k^2\omega - 3\delta^3k^2\lambda + 3\delta^2k^2\lambda + 2\delta k\omega\lambda^2 + \delta k\omega^2\lambda - 6\delta k\omega\lambda + 6\delta^2k\omega\lambda + \delta^4k^3 - \delta^3k^3 - 3\omega^3\delta + \omega^3\lambda + 3\omega^2\lambda^2 + 3\omega\lambda^3 + \lambda^4 + \omega^3 - 3\lambda^3\delta - 9\omega^2\lambda\delta =$

which we can show is positive except for very high delta.

Proof of Proposition 11. *i*) If $\exp(3E) > g(1)$ then $k_{eff} = 1$ and we need to grant strictly more than $n_r(1)$ new options to satisfy

$$\frac{\left(\omega+\lambda+1+\frac{n}{N_0}\right)(\omega+\lambda)}{\left(\omega+\lambda+(1-\delta)+\frac{n(1-\delta)}{N_0}\right)(\omega+\lambda-\delta)} = \exp(3E) \tag{93}$$

which gives directly a linear equation in n, yielding solution 31.

ii) If $\exp(3E) \in (g(\underline{k}_1), g(1))$, then $k_{eff} \in (\underline{k}_1, 1)$ and solves

$$\frac{(\omega+\lambda-\delta k_1)^2(\omega+\lambda)}{-\delta^3(1-\delta)k_1^3+3\delta^2(1-\delta)(\omega+\lambda)k_1^2-\delta(\omega+\lambda)^2(3(1-\delta)+\lambda)k_1+(\omega+\lambda)^3} = e^{3E} \tag{94}$$

which yields the cubic

$$e^{3E}\delta^3(1-\delta)k_1^3+\delta^2\left(1-3e^{3E}(1-\delta)\right)(\omega+\lambda)k_1^2+\delta(\omega+\lambda)^2\left(e^{3E}(3(1-\delta)+\lambda)-2\right)k_1-(\omega+\lambda)^3\left(e^{3E}-1\right) \tag{95}$$

iii) If $\underline{g} \leq g(\underline{k}_1)$ then $n_{eff} = 0$ and k_{eff} solves

$$\frac{(\omega+\lambda+k_1)(\omega+\lambda)}{(\omega+\lambda+(1-\delta)k_1)(\omega+\lambda-\delta k_1)} = \underline{g} \tag{96}$$

giving the quadratic

$$\underline{g}\delta(1-\delta)k_1^2+\left(1+2\underline{g}\delta-\underline{g}\right)(\omega+\lambda)k_1-(\omega+\lambda)^2\left(\underline{g}-1\right)=0 \tag{97}$$

which has just one positive solution

$$k_{eff} = \frac{(\omega+\lambda)}{2\underline{g}\delta(1-\delta)}\left(-2\underline{g}\delta+\underline{g}-1+\sqrt{\left(\underline{g}^2-2\underline{g}+1+4\underline{g}\delta^2\right)}\right) \tag{98}$$