INFORMATIVE ADVERTISING:
AN ALTERNATE VIEWPOINT AND IMPLICATIONS

by

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Abstract

Our objective is to broaden the current understanding of how horizontal differentiation interacts with both advertising and pricing by extending the analysis of Grossman and Shapiro (1984) to look at a full range of differentiation conditions. We seek to offer a useful perspective on the relationship between advertising and pricing by focusing on competitors whose essential difference prior to advertising and price decisions is product differentiation.

We construct a model where demand for a firm’s products is driven by three factors: consumers’ awareness of products and their attributes, pricing, and the degree of fit between a product’s attributes and the needs of the consumer. Following Salop (1979), differentiation is captured by representing the firms as equally spaced points in a unitary circular spatial market. We assume that product attributes are fixed and the firms make decisions about how much to advertise and what prices to set for their products.

A distinct element of the model is the mechanism by which advertising makes consumers aware of products. Similar to Grossman and Shapiro (1985), advertising is represented as a series of messages received randomly by consumers in the market and consumers only have interest in a product if they have seen advertising about it. It is important to underline that advertising only affects consumers’ awareness of a product and not their valuation of it. In addition, the probability of a consumer seeing a firm’s advertising is independent of the consumer’s location.

The primary finding of our analysis is that the impact of informative advertising on market prices and profits is a function of the pre-existing level of differentiation in the market. Advertising is observed to create distinct groups of consumers based on the advertising to which they have been exposed. The optimal pricing is a function of competing firms balancing the needs of each of the groups that have interest in their products.

When the level of differentiation between products is high, increases in advertising have no effect on observed prices. However, when the level of differentiation between products is moderate, increases in advertising tend to drive up prices. Finally, when the level of differentiation is low, we show that higher advertising leads to lower prices and profits.

We also find that total welfare can increase when higher advertising leads to higher prices. This highlights the risk of reaching conclusions about the anti-competitive effects of high advertising based solely on an observed relationship between advertising and pricing.
In a modified version of the model, we assume that the probability of a consumer seeing a firm’s advertising depends on that consumer’s location. More specifically, we consider situations in which firms can target heavier advertising to a) customers that are locationally close to them or b) customers that are locationally distant from them or. This captures the notion of two different types of markets, one in which firms aggressively pursue the competitor’s customers and the other in which firms focus their effort on loyal customers. We find that the targeting of advertising does affect the relationship between advertising and pricing. While the general pattern of results regarding the impact of differentiation on the advertising/price relationship is consistent across the three conditions examined, targeting has a particularly interesting effect in conditions of moderate differentiation. In fact, when distant consumers are targeted, the positive relationship observed with no targeting is reversed and prices fall with higher levels of advertising. However, the most interesting effect of targeted advertising is its effect on overall pricing. In conditions of low differentiation, targeting consumers who are nearby exacerbates price competition and reduces price below the no-targeting price. On the other hand, targeting consumers who are distant results in equilibrium prices that are higher than the no-targeting price. Exactly the opposite is observed when differentiation is moderate. These findings underline the importance of existing differentiation between firms for determining the effect that targeted advertising has on pricing. They also provide a potential explanation for offensive or defensive postures that firms employ in media buying that has not been considered previously.

Key Words: advertising/price competition, informative advertising, persuasive advertising, spatial competition, targeted advertising
1.0 Introduction

Tirole (1990) notes that advertising is one of the most important dimensions of non-price competition and economists and marketers alike have dedicated significant effort to understanding its role and impact across markets. While most would agree that the primary role of advertising is to provide information to potential consumers, there has been significant controversy regarding the nature of this information. Nevertheless, it is well accepted that a primary role of advertising is to generate awareness of products and also to make consumers aware of how alternative products are different. In the past, a marketer’s biggest challenge in selecting a media strategy was maximising the likelihood that “category users” were exposed to the commercials. However, the complexity of media strategy has increased and there are now many new advertising channels (see “The Monkey Puzzle”, The Economist, August 25, 2001, 54-55). Today, a firm can do more than target its ads to “category users”. Frequently, a firm can target heavier advertising to consumers who are more (or less) inclined to purchase its unique product.

A second important characteristic of modern markets is the existence of significant differences in product attributes across markets. Tirole (1990) notes that modern firms are well versed in the principle of differentiation, according to which firms do not want to produce identical products because of the intense price competition that results. Firms in markets from detergents to automobiles, make efforts to distinguish their products from competitors and establish unique clienteles (Kotler 1997). The ability to differentiate is not absolute and depends to an extent on the nature of the category (for example, it is easier to differentiate a complex product like an automobile than a simple product like laundry detergent). The most natural form of differentiation is perhaps based on quality (or vertical differences) where certain products are either more consistent, perform better, or last longer.

However, there are also significant differences between products that make them better for some customers than others. As noted by Tirole, this type of differentiation is known as horizontal differentiation. There is a tradition of modelling this form of differentiation using spatial models (Hotelling 1929, Salop 1979). While the relationship between product design and pricing has been examined in great deal, less attention has been devoted to understanding how differentiation interacts with both advertising and pricing. One exception is the model of Grossman and Shapiro (1984) where competing firms make decisions about advertising and pricing in a
market of horizontally differentiated firms. Importantly, however, Grossman and Shapiro restrict their analysis to situations of low differentiation where consumers in the market find any of the available products in the market to be “acceptable” alternatives.

The objective of this paper is to broaden our understanding of how horizontal differentiation interacts with both advertising and pricing by extending the analysis of Grossman and Shapiro (1984) to look at a full range of differentiation conditions. A secondary objective will be to provide an analysis of how the targeting of advertising by marketing managers affects the link between horizontal differentiation and the “advertising-price” relationship. For example, are findings regarding how differentiation affects the relationship between advertising and pricing altered when managers place heavier weights on consumers that have preferences more closely aligned with their respective products (i.e. heavier advertising is directed towards loyal consumers).

Our attention is focussed on advertising that generates awareness of products and provides factual information about product attributes. We recognize that advertising can increase a consumer’s willingness to pay for a product. However, our focus is to better understand the link between horizontal differentiation and the advertising-price relationship. In such conditions, there is not an automatic link between a consumer’s willingness to pay for a product and exposure to advertising.

The major insight provided by our analysis is that horizontal differentiation has a significant impact on the relationship between advertising and prices. This impact is based on how differentiation affects a) competition between firms and b) advertising’s role of allowing consumers to find products that better match their preferences. Advertising is observed to create distinct groups of consumers based on the advertising to which they are exposed. After advertising has created these groups of consumers, market pricing is a function of competing firms balancing the needs of each of the groups that have interest in their products.

When the level of differentiation between products is high, increases in advertising have no effect on observed prices. However, when the level of differentiation between products is moderate, increases in advertising tend to drive up prices. Finally, when the level of differentiation is low, we show that higher advertising leads to lower prices and profits.
The model allows us to demonstrate that a positive relationship between advertising and prices does not necessarily imply that increases in advertising generate losses in total welfare.

In addition, we find that the targeting of advertising can affect the relationship between advertising and pricing. In a modified version of the model, we look at two situations, one where firms target heavier advertising to customers that are nearby and the other where the firms target heavier advertising to customers that are distant. The objective is to look at two canonical cases: a market where firms vigorously defend their turf and a market where firms aggressively pursue the competitor’s customers. In general, the targeting of advertising does not alter the pattern of findings regarding the impact of differentiation on the advertising/price relationship. Similar to the no-targeting situation, in conditions of full differentiation, prices are unaffected by the level of advertising and in conditions of low differentiation, pricing is positively related to advertising levels. But when differentiation is moderate, targeting has a strong effect. In fact, when distant consumers are targeted, the positive relationship observed with no targeting is reversed and prices fall with higher levels of advertising.

The most notable effect of targeted advertising however, is its effect on overall pricing. When differentiation is low, targeting consumers who are nearby exacerbates price competition and reduces price below the no-targeting price. On the other hand, targeting consumers who are distant results in equilibrium prices that are higher than the no-targeting price. Exactly the opposite is observed in conditions of moderate differentiation. These findings underline the importance of existing differentiation between firms for determining the effect that targeted advertising has on pricing.

In the following section, we provide a review of the literature to summarize our current understanding of advertising and its relationship to pricing. In the third section, we present the modelling framework that is used to address our objective. In section 4, we present our analysis of how increased advertising affects pricing in the product market. In section 5, we discuss the implications of reductions in the cost of advertising on total welfare. In section 6, we present an analysis of how the targeting of advertising affects the relationship between advertising levels and pricing under different conditions of differentiation. In section 7, we provide a brief conclusion and discuss the managerial implications of our findings.
2.0 Literature Review

In many models, advertising is represented as an instrument which either increases the intensity of demand (at all price levels) or the amount consumers are willing to pay for a specific product (similar to a vertical quality improvement). Generally, the path through which advertising creates these effects is not addressed. One of the first attempts to build a true micro-model of advertising is found in Butters (1977). Here, advertising is represented as a series of messages sent to consumers to inform them about the existence and prices of products. Grossman and Shapiro (1984) extend this idea to a market with horizontal differentiation and analyse the impact that advertising has on the provision of variety in a market.

In many respects these models are realistic representations of advertising. After all, mass-media advertising can certainly be described as a series of messages directed towards a target audience defined by a series of demographic guidelines. Moreover, the media guidelines for firms in the same category are similar and the challenge for advertising agencies (with an allocated budget) is to achieve a desired number of exposures (frequency) with as large a fraction of the target audience as possible (reach) (Lilien, Kotler, and Moorthy, 1992). Interestingly, the notion of advertising as a series of messages is also used by Hertzendorf (1993) to analyse the predictions of Milgrom and Roberts (1986). Milgrom and Roberts suggest that the quantity of advertising for a given product can be a signal of quality when consumers cannot distinguish quality before buying (Nelson 1974). Their model predicts that high quality products will have higher prices and higher levels of advertising. Hertzendorf's findings are different from those of Milgrom and Roberts in that advertising is a signal only when the prices of high and low quality products are identical. While the work of Milgrom and Roberts and Hertzendorf is not directly related to our problem, it highlights the fact that analysing advertising on a micro-basis (and recognizing its message-sending character) can lead to findings that are unavailable from a model which is more general.

The actual impact that advertising has on consumers is also a subject of some controversy. Nelson (1974), Schmalensee (1978), Klein and Leffler (1981), Milgrom and Roberts (1986) and Bagwell and Ramey (1994) assume that the quantity of advertising is important because it signals hidden information to consumers (the advertisements themselves are assumed uninformative). Generally the hidden information that is signalled to consumers relates to the quality of the products, i.e.
higher quality products are advertised (Kirmani and Wright 1990). However, Bagwell and Ramey show that the information being signalled can even extend to the “value” that shoppers can expect by choosing an advertised retailer. Their findings are based on the existence of coordination economies (due to both the volume and variety of products sold at a retailer) and a concept of value that is based on the variety and prices offered to consumers. In contrast, the message-sending models (Butters 1977; Grossman and Shapiro 1984) posit that advertising makes consumers aware of the existence of products and the levels of certain product attributes. In these models, not only is the quantity of advertising important but the “content” of the advertisements is important too.

The idea that the content of advertising is important is also highlighted by Ehrlich and Fisher (1982) who suggest that a key role of advertising is to reduce the cost of consumption by providing consumers with information that allows them to realize the benefits of a product at reduced cost to themselves. Becker and Murphy (1993) even propose that advertising itself is a consumption good that has a positive effect on the valuation of the product advertised. These relations are then used to understand how much advertising will be provided to consumers and how much advertising will be consumed.

The work of Becker and Murphy highlights an important assumption that underlies many models of advertising. Advertising is observed to have a positive effect on the value that consumers place on the advertised product or service. Significant experimental research has demonstrated that advertising can in fact, lead to higher brand evaluations through mechanisms such as the effect of “mere exposure” (Anand, Holbrook and Stephens, 1988 and Heath, 1990).

Because the evidence is strong that advertising has multiple effects on consumers, a number of researchers have proposed dichotomous models of advertising in which two effects of advertising are represented. Boyer (1974) proposes that there are two forms of advertising: informative advertising which provides consumers with better information about products (especially pricing) and goodwill advertising which leads to increases in the “valuation” of products. This idea also underlies the work of Kotowitz and Mathewson (1979), Farris and Albion (1980) and Krisnamurhti and Raj (1985). In a laboratory setting, Mitra and Lynch (1995) show that both awareness (which affects how consumers form consideration sets) and the willingness of consumers to pay more for a brand (that is advertised) play a role in
determining the overall impact of advertising. Zhao (2000) considers advertising that can both signal quality and generate awareness for products. Not surprisingly, the lack of agreement on how advertising actually affects people is the source of a long discussion about whether advertising is generally good or bad for society.

Since the 1960's, researchers have considered this question from both a social welfare perspective and a moral perspective. Unfortunately, the question boiled down to an analysis of whether prices rise or fall with changes in the level of advertising. Some espoused the “partial view” which argues that advertising provides factual information to consumers allowing them to make rational choices. In other words, advertising is seen as providing information to consumers about the attributes (including price), quality and location of products. This view, first articulated by Telser (1964), suggests there is little evidence of anti-competitive effects of advertising in terms of pricing and profitability. It implies that advertising will tend to reduce product differentiation that is related to a lack of information. Studies in a number of industries (eyeglasses, pharmaceuticals, and toys) show that prices were significantly higher in states where advertising was prohibited (Benham 1972, Cady 1976 and Steiner 1973). This sanguine view of advertising emphasises the benefits of advertising in terms of better-informed consumers and lower prices.

The counterpoint known as the “adverse view” suggests that advertising is designed to persuade (and frequently fool) consumers into perceiving significant differences between products that are physically similar. This view emphasizes the anti-competitive nature of advertising (Bain 1956, Galbraith 1967, and Solow 1967). Comanor and Wilson (1974) suggest that advertising creates spurious product differentiation because the perceptions created by advertising lead consumers to pay premiums for products that are physically identical. Not surprisingly, numerous studies have been used to support this view of advertising by demonstrating a high correlation between advertising levels and prices (or profits) across a number of categories (see Comanor and Wilson 1979, Pokowski Leszczyc and Rao 1990 and Carlton and Perloff 1994 for a listing of relevant studies).

While the above controversy has not been resolved, it seems that the relationship of advertising levels to prices and the net effects of advertising on welfare

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1 The “partial view” is not inconsistent with circumstances (distributions of consumers that are discrete or non-uniform) that lead to a positive relationship between advertising and higher prices. For example, Meurer and Stahl (1994) propose a model of informative advertising that leads to price increases.
are highly dependent on the categories and circumstances under consideration. Nevertheless, it seems that a negative correlation between advertising and pricing is prevalent in situations where price information is an element of the advertising; more advertising reduces the ability of firms to take advantage of uninformed consumers. Conversely, the models of advertising that predict a positive correlation between advertising levels and pricing, generally involve messages that in some way enhance the value of the product for consumers (for example, through credibly signalling higher quality, through “mere exposure” or by reducing the cost of consumption).

These observations provide motivation for our analysis. Do commercials need to contain pricing information in order to generate negative correlations between advertising and pricing? Do commercials need to create “willingness to pay improvements” in order to generate positive correlations? Interestingly, the vast majority of broadcast advertising i.e. television, radio, magazine and outdoor advertising, does not contain pricing information (see Table 1).

Table 1
Television Advertising Content Mini-Survey*

<table>
<thead>
<tr>
<th></th>
<th>CBS (U.S.A.)</th>
<th>CTV (Canada)</th>
<th>TF1 (France)</th>
<th>Sky1 (Satellite UK)</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Commercials Observed</td>
<td>24</td>
<td>26</td>
<td>14</td>
<td>20</td>
</tr>
<tr>
<td># of Commercials Containing References to Pricing</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>% of Commercials with Pricing Information</td>
<td>4.1</td>
<td>7.7</td>
<td>7.1</td>
<td>15.0</td>
</tr>
</tbody>
</table>

*Information collected by the authors in August 1999 (based on approximately one hour of continuous viewing on each network).

In addition, many ads should not naturally lead to a positive correlation between advertising and pricing; they simply provide information about the characteristics of products. Our conjecture is that the inherent level of differentiation in a market may well provide an additional explanation for the types of correlations that are observed between advertising and pricing.

A secondary objective of our analysis is to see whether the observed relation in a market between advertising levels and pricing is affected by firms’ ability to target advertising messages. Because of the improved quality of consumer research and media buying due to information technology, finer targeting is now feasible. Accordingly, it is important to understand how markets are changed when technology...
allows firms to do better than simply target “high potential category users”. The marketing literature recognizes the importance of targeting, yet the work on targeting does not consider the targeting of advertising. Targeted pricing based on consumer-behaviour (past purchases) is considered in a two-period model by Fudenberg and Tirole (2000) and in a dynamic setting by Villas-Boas (1999). Chen and Iyer (2000) also consider the impact of location-based targeted pricing. In general, the ability to target prices to specific consumers allows higher prices to be charged. However, when the ability of firms to target individual consumers reaches an upper limit, prices can actually be driven downwards because the insulating impact of “infra-marginal” consumers disappears. Researchers have also considered the impact of targeted couponing (Shaffer and Zhang 1995) and targeted product modifications (Iyer and Soberman 2000). Here the findings echo the conclusions associated with targeted pricing. The general impact of targeted activities is to facilitate an overall rise in market pricing. Our objective will be to identify the effect that targeting advertising has on the relationship between advertising and pricing.

We now present the modelling framework for our analysis.

3.0 The Model
The model consists of firms (independently) directing advertising towards consumers and then setting prices for their products. Informed consumers then buy the firm's product that provides them with maximum surplus.

The Competitive Environment for Manufacturers and Consumers
The competitive environment consists of $N$ identical firms that produce competing products for sale to consumers with a constant marginal cost of production, $c$ (each firm produces at most a single brand). The products differ with respect to an attribute and each consumer is identified by an ideal point along this attribute that corresponds to her preferred brand. Following Salop (1979), consumers are uniformly distributed around a circle with density $\delta$ and the circle is assumed to have a circumference of unit length. Similar to Grossman and Shapiro (1984), Figure 1 illustrates the framework.
Each consumer is assumed to buy no more than one unit of product and places a value \( v \) on her most preferred product. Consumers however, cannot obtain their preferred product. A consumer located a distance \( x \) from firm \( n \) obtains a surplus \( v - tx - p_n \) by consuming firm \( n \)'s product, where \( t \) is the “preference” cost per unit distance and \( p_n \) is the price charged by firm \( n \).\footnote{The parameter \( t \) measures the sensitivity of consumers to product attributes given the locational interpretation of distance. In contrast to analytical models of persuasive advertising, advertising is purely informative in our framework. In particular, advertising is assumed to have no effect on \( v \) (the consumer’s willingness to pay) or \( t \) (the preference cost in the market). In addition, following our earlier discussion, advertising messages will contain information about attributes other than pricing (the objective here is to mirror the vast majority of media advertising for consumer goods). Accordingly, firms will make pricing decisions after advertising levels have been chosen. In this way (by definition), the advertising will not contain information about the pricing of products. This is different from models where advertising and pricing decisions are made simultaneously (Butters, 1977 and Grossman and Shapiro, 1984) and is similar to models such as that proposed by Meurer and Stahl (1994).} A consumer will only buy if she knows of a product offering positive surplus \( i.e. v - tx - p_n > 0 \). Without advertising, consumers are assumed to be uninformed about the existence or benefits offered by products and the only way a consumer can find out about a firm’s product is through a specific firm’s advertising. In particular, we

\footnote{Here we define \( x \) to be the shortest arc length between the consumer and the \( n \)th firm. This implies that consumers can travel in either a clockwise or counter-clockwise direction.}
assume that any consumer who has seen one or more messages from a given firm is informed about that firm’s products. If a consumer knows about more than one product offering positive surplus, she will buy the product offering the greatest surplus. As in Butters (1977), advertising is assumed to provide complete and truthful information about the attributes of a particular brand. This follows from FTC regulation that prohibits advertisers from making false or deceptive statements about their products (Peltzman 1981). Similar to Grossman and Shapiro (1984), consumers do not actively search for or experiment with brands they do not know about and we assume that advertising does not convey information about competing brands.

An important assumption of the model is that the advertising effort of a firm reaches participants in the category (consumers around the circle) in a random fashion. In other words, a firm does not have the ability to restrict its effort to those consumers who find its product most attractive (i.e. those consumers who are closer to the firm’s location). We relax this assumption in section 7. The advertising decision variable for firm \( n \) is \( \phi_n \), which can be interpreted as the “reach” of the advertising campaign in the total market. In essence, \( \phi_n \) is the fraction of all consumers in the market that have been exposed to one or more messages by firm \( n \), i.e. the total number of consumers reached by firm \( n \)’s marketing is \( \phi_n \delta \) and the probability of a random consumer being reached by firm \( n \)’s marketing is \( \phi_n \). An important implication of this representation of advertising is that it creates a second dimension of consumer heterogeneity based on the information consumers have about products (the first is location around the circle). In fact, after firms have conducted advertising, there are \( 2^N \) distinct groups of consumers uniformly distributed about the circular market based on whether they have been exposed to the advertising of each of the \( N \) firms. We now discuss the advertising technology that is employed by firms in this market.

**Advertising Technology**

As discussed above, advertising is modelled as a series of messages which are sent randomly to consumers around the circular market. In order for a consumer to be informed about a product, he must see at least one message from the firm in question. We assume without loss of generality that one message will reach a fraction \( f \) of the
The manufacturer must choose the number of messages $q$ and pays a price $\eta \delta$ for each message. The price per message $\eta \delta$ reflects the fact that media costs are generally based on the size of the population ($\delta$) and the percent of that population that receives the message ($\eta$ is a constant related to $f$, the reach of the media vehicle).\footnote{If TV advertising were the only marketing tool, this might be analogous to the expected viewership within a target market for given TV show.}

Now, the challenge is to relate the total cost of advertising $q \delta \eta$ to the total reach of the advertising campaign $\phi$. With one message of advertising, a fraction $1-f$ of the population does not receive the message. Thus, when a campaign consists of $q$ messages, a fraction $(1-f)^q$ of the population does not receive the message. This allows us to write the following expression for the reach of the campaign:

$$\phi = 1 - (1 - f)^q \quad (1)$$

Rearranging, we obtain the following expression for $q$, the number of messages:

$$q = \frac{\log(1 - \phi)}{\log(1 - f)} \quad (2)$$

Substituting for $q$, we write the following expression for the total cost of a campaign $C_a$ with reach $\phi$:

$$C_a = \eta \delta \frac{\log(1 - \phi)}{\log(1 - f)} \quad (3)$$

If we define a parameter $\alpha$ as follows,

$$\alpha = \frac{-\eta}{\log(1 - f)} \quad (4)$$

it is a positive parameter which captures the cost of marketing [$\log(1-f)<0$ always]. Then the cost of marketing can be written as:

$$C_a = -\delta \alpha \log(1 - \phi) \quad (5)$$

where $\alpha$ is a function of the cost per message ($\eta$) and the fraction of the population reached by each message ($f$). In section 5, we examine the impact of a reduction in the cost of advertising on total welfare. Since the relationship between the cost parameter $\alpha$ and $\eta$ is linear, a percentage reduction in the cost per spot (for a given media vehicle) can be interpreted directly as a percentage reduction in $\alpha$. In addition, a more efficient vehicle that delivers a greater fraction $f$ of the target population for a

\footnote{This is similar to the constant-reach, independent-readership technology outlined in Grossman and Shapiro (1984).}
given price per message can also be interpreted as a reduction in \( \alpha \). Here, the relation between a percentage increase in \( f \) and the corresponding decrease in \( \alpha \) is non-linear.

*Extensive Form of the Game*

The extensive form for the game is as follows:

*Step 1.* Firms choose advertising intensities \( \phi \).

*Step 2.* Firms choose prices \( p \), knowing the advertising intensities that all firms in the market have chosen. The prices chosen by firms are posted as retail prices.

*Step 3.* If a consumer has seen a message from one or more firms, the consumer will purchase the product that provides her with maximum surplus assuming that her participation constraint is satisfied.

As mentioned earlier, a consumer only buys if she is informed. This situation is intended to capture the idea that consumers need to be aware and have knowledge of the products that they buy. Because firms make simultaneous decisions to choose advertising intensities and then prices, the game described here is one of complete but imperfect information. To facilitate presentation of our analysis, we summarize the notation in the following table.

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\[ ^6 \text{Consumers obviously gain information about products from many sources other than advertising activity (e.g. word of mouth). However, as long as the information provided by external sources is proportional to the advertising effort, the findings of the model are unaffected.} \]
Table 2
Summary of Notation used in Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>The marginal cost of the product (constant across firms)</td>
</tr>
<tr>
<td>$C_a$</td>
<td>The total cost of advertising for each firm</td>
</tr>
<tr>
<td>$f$</td>
<td>The fraction of population reached by purchasing one advertising message</td>
</tr>
<tr>
<td>$N$</td>
<td>The number of firms in the circular market</td>
</tr>
<tr>
<td>$p^*$</td>
<td>The equilibrium price in the market</td>
</tr>
<tr>
<td>$p_n$</td>
<td>The price chosen at the focal (nth) firm</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>The equilibrium price chosen by other firms assuming symmetry</td>
</tr>
<tr>
<td>$q$</td>
<td>The number of messages purchased in a campaign</td>
</tr>
<tr>
<td>$t$</td>
<td>The transportation cost</td>
</tr>
<tr>
<td>$v$</td>
<td>The reservation value for a product that is ideally located for a consumer</td>
</tr>
<tr>
<td>$W_T$</td>
<td>Total welfare generated in the circular market</td>
</tr>
<tr>
<td>$x$</td>
<td>The distance from the focal firm of the indifferent consumer</td>
</tr>
<tr>
<td>$y$</td>
<td>The distance of the consumer from the focal firm who obtains zero surplus</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The advertising cost parameter that is constant across firms</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The density of consumers in the market</td>
</tr>
<tr>
<td>$\phi^*$</td>
<td>The equilibrium advertising level in the market</td>
</tr>
<tr>
<td>$\phi_n$</td>
<td>The advertising level chosen at the focal (nth) firm</td>
</tr>
<tr>
<td>$\bar{\phi}$</td>
<td>The equilibrium advertising level chosen by other firms assuming symmetry</td>
</tr>
<tr>
<td>$\eta$</td>
<td>The cost per message purchased (charged to each firm)</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>The equilibrium profit level</td>
</tr>
<tr>
<td>$\pi_n$</td>
<td>The profit of focal (nth) firm</td>
</tr>
</tbody>
</table>

Our objective is to identify non-cooperative Nash equilibria in prices and advertising intensities in three distinct situations. First, we consider the case where the number of firms $N$ in the market is large and the reservation price $v$ is such that any consumer obtains positive surplus from a firm whose message she observes. Second, we consider the case where the number of firms is more limited and the consumer can only afford products from “adjacent firms”. This is the case when $v-c \in \left[ \frac{t}{N}, \frac{2t}{N} \right]$. It is important to note that this range for $v-c$ is mutually exclusive from the first set of conditions. Finally, we look at the parametric conditions where $v-c < \frac{t}{N}$. In this situation, there are always a fraction of consumers between any pair of adjacent firms that cannot afford the products of both adjacent firms. To simplify our analysis, we normalize the density $\delta$ of the market to one.

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7 Following the logic of Grossman and Shapiro (1984), when $v+c > t$, this constraint is strictly satisfied.
4.0 The Relationship between Advertising and Pricing

We first present the analysis of conditions of low differentiation where every firm can attract customers located anywhere in the market.

4.1 The Low Differentiation Case (v-c>t)

When differentiation is low, the derivation of the demand curve for each firm is complex because a firm competes with every firm in the market (and not simply adjacent firms). In other words, a firm can potentially serve a consumer who is half an arc length away. These are the conditions studied by Grossman and Shapiro (1984) and following their reasoning, we divide a focal firm’s consumers into N groups, the Nth group being those consumers who have seen only firm n’s advertising and the remaining N-1 groups being those for whom firm n is the kth best alternative under full information.

We assume that the equilibrium involves symmetric prices and that the price of the N-1 rivals of firm n is $p\bar{p}$. We now write N-1 equations that describe the location of the indifferent consumer under full information:

Between firm n and firm $n+1$

$v - tx - p_n = v - t\left(\frac{1}{N} - x\right) - \bar{p}$  \hspace{1cm} (6)

Between firm n and firm $n+2$

$v - tx - p_n = v - t\left(\frac{2}{N} - x\right) - \bar{p}$  \hspace{1cm} (7)

This pattern is used to solve for $x_k$ where $x_k$ is the indifferent consumer between firm $n$ and the firm which is $k$ “firms” away from firm $n$.

$$x_k = \frac{\bar{p} - p_n}{2t} + \frac{k}{2N}$$  \hspace{1cm} (8)

When we account for consumers on both sides of firm n, the number of consumers ($K_1$) for whom firm n is the best alternative is given by $2x_j$. Similarly, the number of consumers ($K_2$) for whom firm n is the second best alternative is $2(x_2 - x_1)$, for $K_3$ by $2(x_3 - x_2)$ and so on. Substituting for $x_k$, we obtain $K_1 = \frac{\bar{p} - p_n}{t} + \frac{1}{N}$, $K_2 = \frac{1}{N}$ for $k=2…$
$N-1$. Given that consumers do not have full information, the number of consumers in the Nth group (who do not make a comparison) is $K_N = \frac{1}{N} - \frac{\overline{p} - p_n}{t}^N$.

These identities now provide sufficient information to write the demand curve for the firm $n$. The demand can be represented as the sum of demand from each of $N$ segments:

$$d_n = K_1\phi_1 + K_2\phi_2 + K_3\phi_3 + \ldots K_N\phi_N$$

Here $\phi_k$ is the probability that a consumer in the segment actually purchases from firm $n$. The probability $\phi_1$ that a consumer from the first group buys from firm $n$ is clearly $\phi_1$, since any consumer in this group who sees firm $n$’s advertising from firm will buy from firm $n$. The probability $\phi_2$ that a consumer from the second group buys from firm $n$ depends on that consumer having seen advertising from firm $n$ and not having a seen advertising from firm $n+1$. Thus, $\phi_2$ is given by the product $\phi_n (1 - \overline{\phi})$. Using similar reasoning, we obtain $\phi_k = \phi_n (1 - \overline{\phi})^{k-1}$. Substituting, we obtain the following expression for $d_n$ (a full derivation is available in a separate appendix).

$$d_n = \frac{\phi_1 (\overline{p} - p_n)}{t} (1 - (1 - \overline{\phi})^{N-1}) + \frac{\phi_n}{N\overline{\phi}} (1 - (1 - \overline{\phi})^N)$$

For $N$ sufficiently large, $(1 - \overline{\phi})^N$ and $(1 - \overline{\phi})^{N-1}$ are close to zero and we can write $d_n$ as:

$$d_n = \frac{\phi_1 (\overline{p} - p_n)}{t} + \frac{\phi_n}{N\overline{\phi}}$$

We now construct firm $n$’s profit function based on the modelling assumptions and equation 11 for $d_n$.

$$\pi_n = (p_n - c) \left[ \frac{\phi_1 (\overline{p} - p_n)}{t} + \frac{\phi_n}{N\overline{\phi}} \right] + \alpha \log (1 - \phi_n)$$

Taking the first order conditions of this expression in terms of price, we determine the optimal price in terms of advertising intensity. We then substitute for price and optimise with respect to advertising intensity. The equilibrium advertising intensity is found by assuming symmetry. This leads to our first proposition (the proof and reasoning for all propositions and results are provided in the technical appendix).

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8 This is obtained by noting that $K_N = 1 - \sum_{k=1}^{N-1} K_k$.

9 Detailed implications of this approximation are provided in Grossman and Shapiro (1984).
Proposition 1
When differentiation is low, the optimal price and advertising intensity for all $N$ firms in the market are

$$ p = c + \frac{2\alpha N t}{\sqrt{t^2 + 4\alpha N^2 t} - t} \quad \text{and} \quad \phi = \frac{\sqrt{t^2 + 4\alpha N^2 t} - t}{2\alpha N^2}. $$

Before proceeding to the main result of this section, it is useful to highlight two observations that obtain from Proposition 1. First, differentiating the expression for $p^*$, it is straightforward to show that pricing is positively related to $t$. Thus, the model has the reasonable property that the more differentiated firms are, the higher the equilibrium price. Second, it is easy to show that the higher is $N$ (the number of firms in the market), the lower is the advertising intensity. Thus, the model exhibits a second desirable property i.e. more firms reduce the incentive to invest in advertising because of less potential demand.

The objective of this section is to examine the relationship between advertising levels and pricing when differentiation is low. Because our focus is on equilibrium outcomes, we analyse the impact of structural changes that might lead to higher advertising. As discussed above, changes in the level of differentiation in a market or changes in the number of competitors certainly lead to different levels of advertising. However, in many industries, the level of differentiation and the number of legitimate competitors changes quite slowly. Thus, we assume that the most likely source of increases in advertising is changes in the cost of advertising.

The first order condition for price leads to the following proposition.

Proposition 2
When differentiation is low, the equilibrium price is a decreasing function of advertising intensity.

The intuition for Proposition 2 comes from the way advertising affects competition in the case of low differentiation. At low levels of advertising, all $N$ segments in equation 9 represent significant volume because $\phi_k$ for $k=2,3,...,N$ are significantly greater than zero. However, at higher levels of advertising, the focus of competition

---

$^{10}$ Fixing the number of firms is analogous to assuming that the fixed costs of entry to the market are high. Gilbert (1989) discusses the implications of this assumption that is common in many analytical models.
shifts from segments 2,3..N towards the segment where the focal firm is most preferred. This occurs because the probability of customers in segments 2,3…N not being informed of a superior alternative (to the focal firm) is lower at higher levels of advertising. Because advertising shifts competition from remote firms to adjacent firms (competitors that are less differentiated from the focal firm), price competition is intensified. This finding is consistent with Gatignon (1984) who, in an empirical study, finds that advertising tends to increase price sensitivity when competitors “confront each other directly” (as we might expect adjacent competitors to do).

We now analyse the effect that a reduction in the cost of advertising has on advertising intensity, prices and profits in the downstream market. This analysis is summarized in Proposition 3 and Result 1.

**Proposition 3**

*When differentiation is low:*

a) *A reduction in the cost of advertising leads to an increase in advertising.*

b) *A reduction in the cost of advertising leads to a decrease in prices*

**Result 1**

*A reduction in the cost of advertising leads to a decrease in firm profits.*

As expected, a reduction in the cost of advertising leads to higher equilibrium advertising intensities (in fact, this relationship is observed across all conditions we examine). Consistent with Proposition 2, this leads to lower prices in the market because of the way that advertising affects the “focus of competition”.

A further finding is that a reduction in the cost of advertising leads to lower profits for all firms. One might expect the profits of competing firms to increase when a key factor of production is available at a lower cost. What we find, however, is that firms find themselves in a Prisoners’ Dilemma: they all advertise more and generate less profit. When differentiation is low, firms have the “potential” to generate demand from consumers throughout the entire market (i.e. all around the circular market). However, as equilibrium advertising intensities rise, the likelihood of actually attracting consumers who are far away is low. When firms in the market advertise heavily, almost all consumers are informed of many “affordable” alternatives. But for a candidate consumer, the preferred alternatives will tend to be firms that are nearby (i.e. close to the consumer’s ideal point). Basically, advertising allows the candidate
consumer to find firms that are better suited to her taste. As a result, high advertising causes those firms to compete vigorously for the candidate consumer’s business. Hence, competition becomes localized between firms that are less differentiated so lower prices and lower firm profits are the outcome. This demonstrates that when differentiation is low, the relationships between “advertising and pricing” and “advertising and profits” are quite consistent with the “partial view” of advertising (despite the advertising not containing prices).

4.2 The Case of Moderate Differentiation $\left( \frac{I}{N} < v - c < \frac{2I}{N} \right)$

When differentiation is low, all firms compete with each other. However, when differentiation is moderate, there are N consecutive linear cities and firms only compete with their adjacent competitors (every firm has two adjacent competitors). The range identified for $v-c$ leads to this situation because, for all feasible levels of pricing, consumers who are located a distance greater than $\frac{1}{N}$ from a focal firm obtain less than zero surplus by purchasing the focal firm’s product (i.e. consumers are only interested in the products of adjacent firms). More specifically, we show that the feasible prices are bounded in the interval $\left\{ \frac{v+c}{2}, c + \frac{I}{N} \right\}$. Accordingly, a firm restricts its attention (in terms of profit maximization) to consumers located between it and its two competitors. In this range, there are four sets of consumers that are relevant for firm $n$. First, we have the consumers that are informed of firm $n$ and uninformed of firm $n-1$ but are located between firms $n$ and $n-1$. Second, we have the consumers who are informed of firm $n$ and uninformed of firm $n+1$ but are located between firms $n$ and $n+1$. Third, we have the consumers who are informed of firm $n$ and firm $n-1$ and finally, we have those who are informed of firm $n$ and firm $n+1$. The first and second groups of consumers are effectively “captive” consumers of firm $n$ since they are uninformed about the existence of firms $n-1$ and $n+1$ respectively. Because the reservation price $v$ is relatively lower when differentiation is moderate, the extent of demand is governed by the consumer for whom the individual rationality constraint binds. On either side of firm $n$, we define the consumer at $y_j$ (where $j=n-1$ or
\( n+1 \) as the demand from the first and second groups.

\[
v - ty_j - p_n = 0 \Rightarrow y_j = \frac{v - p_n}{t} \quad (13)
\]

For the third and fourth groups, the demand for firm \( n \) is found by identifying the indifferent consumer at \( x_{n-1} \) and \( x_{n+1} \) (\( x \) is the distance to the focal firm for the indifferent consumer).

\[
v - tx_{n-1} - p_n = v - \left( \frac{1}{N} - x_{n-1} \right) - \bar{p} \Rightarrow x_{n-1} = \frac{\bar{p} - p_n}{2t} + \frac{1}{2N} \quad (14)
\]

\[
v - tx_{n+1} - p = v - \left( \frac{1}{N} - x_{n+1} \right) - \bar{p} \Rightarrow x_{n+1} = \frac{\bar{p} - p_n}{2t} + \frac{1}{2N} \quad (15)
\]

Combining demands from these four groups, we can now write the objective function for firm \( n \). For purposes of exposition, we write \( \phi_{n-1}, \phi_n, \) and \( \phi_{n+1} \) as the advertising intensities chosen by firms \( n-1, n, \) and \( n+1 \) respectively.

\[
\pi_n = (p_n - c)[\phi_n (1-\phi_n) y_{n-1} + \phi_n (1-\phi_{n+1}) y_{n+1} + \phi_n \phi_{n-1} x_{n-1} + \phi_n \phi_{n+1} x_{n+1}] + \alpha \log (1-\phi_n) \quad (16)
\]

We now substitute for \( y_{n-1}, y_{n+1}, x_{n-1} \) and \( x_{n+1} \) and we replace \( \phi_{n-1} \) and \( \phi_{n+1} \) with \( \tilde{\phi} \).

The simplified objective function for firm \( n \) can be written as:

\[
\pi_n = (p_n - c) \left[ 2\phi_n (1 - \tilde{\phi}) \frac{v - p_n}{t} + 2\phi_n \tilde{\phi} \left( \frac{\bar{p} - p_n}{2t} + \frac{1}{2N} \right) \right] + \alpha \log (1-\phi_n) \quad (17)
\]

Similar to the case of low differentiation, we take the first order conditions of this expression in terms of price to determine the optimal price in terms of advertising intensity. We then substitute for price and optimise with respect to advertising intensity. This leads to Proposition 4.

**Proposition 4**

The equilibrium price when differentiation is moderate is equal to

\[
p^* = \frac{-2v + 2\bar{\phi}v - 2c + \bar{\phi}c - \bar{\phi} \sqrt{t/N}}{3\bar{\phi} - 4} \quad \text{This implies that } \frac{\partial p^*}{\partial \bar{\phi}} > 0 \text{ in the allowable range and as } \bar{\phi} \to 1, p^* \to c + \frac{t}{N} \text{ (the full information price)}. \]

Proposition 4 underlines the importance of differentiation in determining the relationship between advertising and pricing. When differentiation is moderate, higher advertising intensities lead to higher pricing and this stands in contrast to the
relationship observed when differentiation is low. The key difference between the two cases is that the level of differentiation ‘\( t \)’ relative to \( v \) is higher than in the case of moderate differentiation. The intuition for the finding obtains by considering how advertising affects competition.

As discussed earlier, when differentiation is low, the primary effect of higher advertising intensities is to shift the geographic focus of competition. At low levels of advertising, firms pick up demand throughout the market because consumers who have seen advertising from a focal firm are unlikely to have seen advertising from a competitor. As advertising levels increase, the demand that firms realize is primarily “local”, i.e. advertising “localizes” competition.

In contrast, when differentiation is moderate, competition is by definition “local” because consumers can only afford products from adjacent firms (if a consumer sees advertising from a non-adjacent firm, it does not affect her decisions because the product will not provide positive surplus). Here, advertising shifts the focus of demand for a focal firm from consumers who have only seen the focal firm’s advertising to consumers who have seen advertising from the focal firm and from the adjacent competitor. (In the above exposition, advertising shifts competition from the first and second group of consumers to the third and fourth groups.) This leads to a rise in price because when differentiation is moderate, the equilibrium price for consumers who have seen advertising from both firms is higher than the equilibrium price for consumers who have seen advertising from only one firm i.e. the competitive price, \( c + \frac{t}{N} \) is higher than the local monopoly price, \( \frac{v + c}{2} \). The optimal price for each firm involves choosing a price that is a compromise of the optimal price for each group of consumers being served. As advertising intensities increase, a greater percentage of all consumers in the market have seen advertising from both adjacent firms and this causes the equilibrium price to rise.

Similar to the case of low differentiation, we now analyse the effect that a reduction in the cost of advertising has on advertising intensity, prices and profits in the downstream market. This analysis is summarized in Proposition 5 and Result 2.
**Proposition 5**

*When differentiation is moderate:*

*a) A reduction in the cost of advertising leads to an increase in advertising.*

*b) A reduction in the cost of advertising leads to an increase in prices*

Similar to the case of low differentiation, a reduction in the cost of advertising leads to higher equilibrium advertising intensities. However when differentiation is moderate, a reduction in the cost of advertising leads to increases in pricing.

**Result 2**

*A reduction in the cost of advertising leads to an increase in firm profits.*

Result 2 indicates that a reduction in the cost of advertising leads to higher profits for all firms. In contrast to the case of low differentiation, firms are not in a Prisoners’ Dilemma: their profits increase when they find less expensive alternatives to send messages to consumers (i.e. a lower $\alpha$). This happens because a positive relationship that exists between advertising intensity and pricing. Not only do firms realize greater demand with lower advertising costs, they also charge customers higher prices. Here, the relationships between “advertising and pricing” and “advertising and profits” seem to follow the perspective of the “adverse view”. However, a fundamental premise of the adverse view is that total welfare is adversely affected (primarily as a result of higher prices). In section 5, we analyse this premise assuming that the number of firms is fixed.

The primary message of this section is that advertising has *different effects* on pricing depending on the level of differentiation. Moreover, these *different effects* happen without advertising having a dichotomous character. The different effects obtain because advertising affects competition differently depending on the level of differentiation. When differentiation is low, the focus of competition becomes more and more local as advertising increases and this creates an inverse relationship between advertising and pricing. When differentiation is moderate, competition is

---

11 Similar to Grossman and Shapiro (1984), the proposition holds as long as values of $\nu$ are not too high.

12 When entry and exit are costly, regulators seek to understand how total welfare is affected by the behaviour of existing firms and (where necessary) to regulate their conduct.
already local. Here, the main effect of advertising increases is to shift competition from consumers who have seen advertising from only one “feasible” alternative to consumers who have seen advertising from two feasible alternatives. This shifting of competitive focus across groups leads to a rise in price because the equilibrium price for consumers who have seen advertising for two feasible alternatives is higher.

4.3 The Fully Differentiated Case \( \left( v - c < \frac{t}{N} \right) \)

In the fully differentiated case, we assume that the equilibrium prices are such that an informed consumer can only afford the product of a firm that is less than a distance \( \frac{1}{2N} \) from her. We then confirm that this is the case using the price that is found to be an equilibrium in the final stage of the game. This assumption implies that there is no competition at the margin and firms are *de facto* local monopolies in their “areas” of the market. Using reasoning analogous to that utilized for the first and second groups of consumers in section 4.2, we write the objective function for firm \( n \).

\[
\pi_n = 2\phi_n (p_n - c) \frac{v - p_n}{t} + \alpha \log(1 - \phi_n) 
\]

Similarly, we solve for optimal prices and advertising intensities by differentiating and finding the maxima. The results of this analysis are summarized in Proposition 6:

**Proposition 6**

In the fully differentiated case, the equilibrium price, advertising intensities and profits are: \( p^* = \frac{v + c}{2}, \phi^* = 1 - \frac{2t\alpha}{(v - c)^2}, \pi^* = \frac{(v - c)^2}{2t} - \alpha + \alpha \log\left(\frac{2t\alpha}{(v - c)^2}\right) \).

Proposition 6 relates to a market where firms may have the opportunity to compete but they do not actually do so. The equilibrium pricing strategies create local monopolies where the firms in the market do not compete with each other. As expected, in these conditions, reductions in the cost of advertising lead to higher levels of advertising and profits. The primary insight provided by Proposition 6 is the independence of advertising intensities and price. This underlines the difference between this model and other models of advertising. Advertising provides information to consumers about products but clearly does not affect the amount consumers are
willing to pay for them (if it did, we would expect higher prices precisely when a firm is a monopoly). Therefore, in this model, any relationship that is observed between advertising and pricing (positive or negative) is entirely a function of the way firms compete.

The analysis of section 4 highlights the “non-monotonicity” of the relationship between advertising and pricing as a function of market differentiation. Low differentiation leads to a negative relationship between advertising and pricing but as differentiation becomes greater, the relationship changes. Increases in differentiation tend to reverse the negative relationship between advertising and pricing to a point at which the relationship actually becomes positive. Ultimately, when the degree of differentiation in the market is already significant, increasing differentiation can reduce and eventually eliminate the positive relationship between advertising and pricing. In the next section, we examine the impact of reductions in the cost of advertising on total welfare.

5.0 The Effect of Lower Advertising Costs on Total Welfare
Advocates of the “partial” view of advertising argue that higher levels of advertising are positive because they lead to lower prices (as noted earlier, the idea is that when consumers are better informed about pricing, high priced firms will have few customers). As advertising moves prices closer to marginal cost, it should reduce the welfare loss that is created by high prices (Telser 1964). In contrast, advocates of the “adverse” view of advertising argue that high levels of advertising are adverse because they lead to higher prices. These higher prices are assumed to create welfare losses because a greater number of consumers would have consumed the product were prices lower, i.e. consumption would have been closer to the level that is socially optimal (Carlton and Perloff 1994).

The objective is to examine these arguments in the context of our model where advertising is “truthful” non-price information about products. We propose to examine the impact of advertising increases that result from reductions in the per-unit cost of advertising. Such changes might be the result of firms finding more efficient ways to communicate with consumers or improved media buying.

13 As noted earlier, many structural changes can lead to increases or decreases in the “market level” of advertising. However, our interest is to examine possible negative welfare effects of informative
The standard approach to calculate total welfare is to add firm profits to consumer surplus (Tirole 1988). However, in this model, the profits of firms are simply a transfer of surplus from consumers to firms. Since the consumer surplus function and the firms’ profit functions are linear functions of price, total welfare is unaffected by transfers of funds between consumers and firms. Similar to Grossman and Shapiro (1984), we calculate total welfare by summing the gross benefit created through consumption (for each consumer this is \( v-c \) less transportation costs) and subtract the investments that firms make in advertising.

5.1 Total Welfare when Differentiation is Low

When differentiation is low, any consumer who observes a message from at least one firm will participate in the market and buy. The gross benefits to consumers are a function of the surplus created by each consumer consuming her ideal product less the average transportation cost incurred by a consumer in the market i.e. \( \bar{x}t \) where \( \bar{x} \) is the average distance travelled by a consumer.

\[
W^{Total} = \left[ \# of consumers \right] \left[ (v - \bar{x}t - c) - N \log \bar{\phi} \right]
\]

(19)

In this case, the number of consumers is 1 (because of our normalizations) less the percentage of consumers who have not seen any messages: \( (1 - \bar{\phi})^N \). The expression for \( \bar{\phi} \) is found in Proposition 1. Similar to the derivation in section 3.1, we divide consumers into \( N \) groups for a representative firm. If we assume that the likelihood of a consumer buying in the \( k \)th group is \( \phi_k \) and the average distance travelled by a consumer in the \( k \)th group is \( x_k \), we can write the average distance travelled as:

\[
\bar{x} = \frac{\sum_{k=1}^{N} \phi_k x_k}{\sum_{k=1}^{N} \phi_k}
\]

(20)

Assuming that \( (1 - \bar{\phi})^N \) is small, this can be approximated as: \( \bar{x} = \sum_{k=1}^{N} \phi_k x_k \). As before \( \phi_k \) is given by \( \bar{\phi} (1 - \bar{\phi})^{k-1} \) and it is easy to show that \( x_k = \frac{2k - 1}{4N} \). Following Grossman and Shapiro (1984), this can be simplified to yield a simple expression for advertising. If such effects do exist, making advertising less expensive ought to increase the negative effects.
\[ x = \frac{2 - \phi}{4N \phi} \]. We now substitute the expressions for \( x \), the number of informed consumers, and \( \phi \) into equation 19 to obtain a total welfare function expressed in terms of exogenous variables. This leads to Result 3:

**Result 3**

*When differentiation is low, reductions in the cost of advertising lead to increases in total welfare i.e. \( \frac{\partial W^T}{\partial \alpha} < 0 \).*

Result 3 confirms an expected result. When increases in advertising lead to reductions in price, reductions in the cost of advertising lead to increases in total welfare. Interestingly, the increase in total welfare is not due to an increase in the number of consumers who buy: a fundamental assumption we make is that \( (1 - \phi)^N \) is small i.e. every consumer in the market buys even before the reduction in the cost of advertising. The primary benefit that drives the welfare result is lower average “travel costs” for consumers. Higher advertising intensities help consumers find products that are better suited to their preferences and this benefit exceeds firms’ increased expenditures on advertising. We now consider the impact of reductions in the cost of advertising on total welfare when differentiation is moderate.

### 5.2 Total Welfare when Differentiation is Moderate

When differentiation is moderate, only consumers who observe a message from an adjacent firm will participate in the market and buy; many consumers receive messages from firms with products that are simply too different from their ideal products to be viable buying propositions. A useful simplification to this analysis is to consider the welfare generated by the activity of an individual firm (by symmetry, the total welfare for a single firm is \( \frac{1}{N} \)th of the total welfare in the market). We construct a welfare function based on the consumption of a focal firm’s product less the focal firm’s expenditures on advertising:
We now substitute the appropriate expressions for each component of the above function, recognizing that there are groups of consumers on both sides of firm $n$:

$$W^T = 2\theta_f(1 - \theta_f) \int_0^{v/p^*} v - tx - c \, dx + 2\theta_f^2 \int_0^{1/N} v - tx - c \, dx + \alpha \log(1 - \theta_f)$$

(21)

To simplify our analysis, we normalize $c$ to 0. Equation 21 can be written as:

$$W^T = 2\theta_f(1 - \theta_f) \frac{v - p^*}{t} \left( v - \frac{v - p^*}{2} \right) + 2\theta_f^2 \left( \frac{v}{2N} - \frac{t}{8N^2} \right) + \alpha \log(1 - \theta_f)$$

(22)

Proposition 3 allows us to substitute for $p^*$ providing an expression of the following form: $W^T = f(\theta_t(\alpha, N, t, v), \alpha, t, N, v)$. Given that $t, N,$ and $v$ are fixed, when we wish to examine the impact of a change in $\alpha$ on $W^T$, we proceed by writing the total derivative of $W^T$.

$$dW^T = \frac{\partial W^T}{\partial \theta} d\theta + \frac{\partial W^T}{\partial \alpha} d\alpha$$

(23)

This can be rewritten as:

$$\frac{dW^T}{d\alpha} = \frac{\partial W^T}{\partial \theta} \frac{d\theta}{d\alpha} + \frac{\partial W^T}{\partial \alpha}$$

(24)

The sign of this expression can be evaluated for a range of values and this leads to Result 4.

**Result 4**

*When differentiation is moderate and the number of firms is small, reductions in the cost of advertising lead to increases in total welfare i.e. $\frac{\partial W^T}{\partial \alpha} < 0$.\'*

When the number of firms is 10 or less, Result 4 demonstrates that total welfare increases even though prices rise. For the case of 2 firms, Figure 2a shows the equilibrium prices, advertising levels and welfare for decreasing $\alpha$. However, prices that increase with decreasing $\alpha$ suggest that some consumers, who could afford the products at a higher $\alpha$, would find the products unaffordable at a lower $\alpha$. This
Intuition is confirmed by Figure 2b which shows that the percentage of consumers, informed about one only firm who buy as a function of $\alpha$.

Figure 2a underlines the deadweight welfare loss that occurs when firms have price setting ability. Positive surplus would be generated by any informed consumer in Figure 2b actually consuming; however, equilibrium pricing prevents it. Nevertheless, total welfare increases (as shown in Figure 2a) and this is the result of two factors. First, the number of consumers who actually consume is higher when $\alpha$ is lower. Second, the group of consumers highlighted in Figure 2b is less important (a larger proportion of consumers have seen messages from two adjacent firms).

The value of this section is to show that increases in price are not *de facto* evidence of a reduction in total welfare. Here, prices do rise with increased advertising, and this leads to a lower percentage of consumers, who have seen messages from one adjacent firm, consuming. Yet, the analysis shows that the savings in advertising costs and the increased number of consumers who do consume more than offset this apparent loss.
5.3 Total Welfare in the Fully Differentiated Case

In the fully differentiated case, the total welfare function is constructed by adding the surplus created by consumption of consumers at each firm and then subtracting the cost of advertising:

\[
W^T = N \left[ 2\phi^* \int_0^{v-p^*} (v-tx-c) dx + \alpha \log(1-\phi^*) \right]
\]

Substituting the equilibrium values for \(p^*\) and \(\phi^*\) as per Proposition 6, we obtain:

\[
W^T = N \left[ \frac{v^2 - 2vc + c^2 - 2t\alpha + 2\alpha \log \left( \frac{2t\alpha}{(v-c)^2} \right)}{2t} \right]
\]

Differentiating this with respect to \(\alpha\) leads to Proposition 7.

**Proposition 7**

*When firms are fully differentiated, \(\frac{\partial W^T}{\partial \alpha} = N \log \left( \frac{2t\alpha}{(v-c)^2} \right)\), which is negative in the feasible range.*

Not surprisingly, in the absence of competition, the model generates the expected result: reducing the marginal cost of advertising leads to increases in total welfare.

To summarize, this section demonstrates that reductions in the cost of advertising lead to increases in total welfare independent of the relationship that is observed between advertising intensity and pricing. In particular, the results of section 5.2 show that higher prices (caused by higher advertising) can be associated with increases in total welfare.

6.0 Targeting Advertising and the Relationship of Advertising to Pricing

In this section, we consider a market where firms target heavier weights of advertising to consumers based on their location in the spatial market. Our objective is to determine how the targeting of advertising affects the relationship between advertising levels and pricing for the three differentiation conditions analysed in section 4. Because of the improved quality of consumer research and media buying...
due to information technology, much finer targeting is now feasible (see “Star Turn”, The Economist, March 9, 2000). Accordingly, it is important to understand how markets are changed when technology allows firms to do better than simply target “high potential category users”.

We will look at two cases: one where firms focus heavier advertising on consumers who are nearby and the other where firms focus heavier advertising on consumers who are distant. These polar situations will allow us to understand whether targeting affects the interaction of differentiation with the advertising/price relationship. Second, it will allow us to make observations about a) markets where firms seem to vigorously defend “their turf” and b) markets where firms are focussed on attracting the competitor’s customers. We start by considering targeted advertising under conditions of low differentiation.

6.1 Targeted Advertising when Differentiation is Low

In order to capture the effect of targeting, we consider two different forms of equation 9. First, we consider the case where firms target heavier efforts of advertising to consumers who are nearby.

\[ d_n = \phi_n \left( \frac{\bar{p} - p}{t} + \frac{1}{N} \right) + \rho \phi_n (1 - \bar{\phi}) \frac{1}{N} + \rho^2 \phi_n (1 - \bar{\phi})^2 \frac{1}{N} + \rho^3 \phi_n (1 - \bar{\phi})^3 \frac{1}{N} + ... + \rho^{N-1} \phi_n (1 - \bar{\phi})^{N-1} \left( \frac{1}{N} \frac{\bar{p} - p}{t} \right) \]  

The parameter \( \rho \in (0, 1) \) reflects degree to which firms focus their advertising nearby (when \( \rho=1 \) the demand function reduces to the case of no targeting). For sufficiently large \( N \), the demand function can be simplified to the expression in equation 28 (a full derivation is included in the technical appendix).

\[ d_n = \frac{\phi_n (\bar{p} - p_n)}{t} + \frac{\phi_n}{N(1 - \rho + \rho \bar{\phi})} \]  

Second, we consider the case where firms target heavier advertising to consumers who are far away. This situation is reflected in equation 29:

\[ d_n = (1 - \rho) \phi_n \left( \frac{\bar{p} - p}{t} + \frac{1}{N} \right) + (1 - \rho^2) \phi_n (1 - \bar{\phi}) \frac{1}{N} + (1 - \rho^3) \phi_n (1 - \bar{\phi})^2 \frac{1}{N} + ... + (1 - \rho^{N-1}) \phi_n (1 - \bar{\phi})^{N-1} \left( \frac{1}{N} \frac{\bar{p} - p}{t} \right) \]  

Here, the parameter \( \rho \in (0, 1) \) that reflects degree to which firms focus their advertising on customers far away (in contrast to equation 27, when \( \rho=0 \) the demand...
function reduces to the case of no targeting). For sufficiently large $N$, the demand function can be simplified to the expression in equation 30.

$$d_n = (1 - \rho) \frac{\phi_n (p - p_n)}{t} + \frac{\phi_n}{N(1 - \rho + \rho \phi)} - \frac{\rho \phi_n}{N(1 - \rho + \rho \phi)}$$  (30)

We construct firm $n$’s profit function for the two cases using the expressions derived for $d_n$ in equations 28 and 30 and optimise with respect to price. Proposition 8 summarizes the relationship of optimal price as a function of advertising level for the two situations in question. This allows us to make observations about how targeting affects both the overall level of pricing and the relationship of advertising to price.

**Proposition 8**

When differentiation is low and firms target heavier weight to customers nearby the optimal price is $p_{\text{nearby}}^* = c + \frac{t}{N(1 - \rho + \rho \phi)}$ and $\frac{dp^*}{d\phi} = -\rho \frac{t}{N(1 - \rho + \rho \phi)^2} < 0$. When firms target heavier weights to customers who are distant from their location the optimal price is $p_{\text{distant}}^* = c + \frac{t}{N\phi(1 - \rho + \rho \phi)}$ and $\frac{dp^*}{d\phi} = -\frac{t(1 - \rho + 2\rho \phi)}{N\phi^2(1 - \rho + \rho \phi)^2} < 0$.

Proposition 8 shows that the relationship between advertising levels and pricing is negative independent of whether firms are targeting heavier advertising to consumers nearby or far away. Therefore, when differentiation is low, the impact of differentiation on the relationship of advertising levels to pricing is unaffected by targeting. On the other hand, Proposition 8 also highlights the impact that targeting has on pricing in this region. When heavier advertising is targeted nearby, the equilibrium price is less than the no-targeting price of Proposition 1 $p^* = c + \frac{t}{N\phi}$. In fact, the finer is the targeting (i.e. $\rho \rightarrow 0$), the lower is the equilibrium price. In contrast, when advertising is targeted to customers who are distant, the equilibrium price is higher than the no target price of Proposition 1. In addition, the finer is the targeting (i.e. $\rho \rightarrow 1$), the higher is the equilibrium price. Thus, targeting has the counterintuitive effect of reducing equilibrium prices when consumers nearby receive heavier advertising and raising equilibrium prices when distant consumers receive heavier advertising. This obtains because of the how the targeting affects the locus of competition. When advertising is focussed on distant consumers, competition is more
remote and as a result higher prices are observed. The inverse is true when advertising is heavier on nearby consumers. This provides a possible explanation for why firms in highly competitive industries might focus their advertising on consumers who are natural consumers of competitive products. Not only is there the possibility of attracting these consumers but focusing advertising on the competitors’ natural consumers can indirectly lead to higher prices.

6.2 Targeted Advertising when Differentiation is Moderate

In order to capture the effect of targeting when differentiation is moderate, we assume that firms restrict their advertising to consumers between their location and the two adjacent firms. In the case of targeting customers that are nearby and distant, we assume that advertising intensity at each distance x from the firm is given by equations 31 and 32 respectively:

\[
\text{Target consumers nearby} \quad \phi(x) = \phi(1 - N_x) \tag{31}
\]

\[
\text{Target consumers near the competitor} \quad \phi(x) = \phi N_x \tag{32}
\]

The advertising intensities for adjacent competitors are the mirrors of these equations. In the case of targeting nearby, equation 31 implies that advertising is at a maximum at the focal firm’s location and drops linearly to zero at the location of the two adjacent competitors (the reverse applies for the case of targeting consumers near the competitor). This structure implies that advertising is only sent to consumers who are within one firm’s distance of the advertising firm. The objective function for the focal firm is written by integrating total demand over the region where the focal firm’s prices are attractive. This leads to equations 33 and 34 for the cases of targeting nearby and distant respectively.

\[
\pi = (p - c) \left[ 2 \phi_n \phi \left( \frac{N_x^2}{2} - \frac{N^2 x^3}{3} \right) + 2 \phi_n \left( y - \frac{Ny^2}{2} - \frac{Ny^2 \phi}{2} - \frac{N^2 y^3 \phi}{3} \right) \right] + \alpha \log(1 - \phi_n) \tag{33}
\]

\[
\pi = (p - c) \left[ 2 \phi_n \phi \left( \frac{N_x^2}{2} - \frac{N^2 x^3}{3} \right) + 2 \phi_n \left( \frac{Ny^2}{2} - \frac{Ny^2 \phi}{2} + \frac{N^2 y^3 \phi}{3} \right) \right] + \alpha \log(1 - \phi_n) \tag{34}
\]

where \( x = \bar{p} - p_n + \frac{1}{2N} \) and \( y = \frac{v - p_n}{t} \). A full derivation of the objective functions is provided in the technical appendix. The optimal price is obtained by differentiating with respect to \( p_n \) and then assuming a symmetric equilibrium (\( p_n = \bar{p} \)). The solutions are too long to be presented but the relationship of advertising level to
pricing can be simulated for any parameter conditions that satisfy conditions of moderate differentiation. We present results for N=50, v=1 and c=0 and transportation costs that span the allowable zone i.e. \( t \in (25,50) \).

**Figure 3**

*The Price-Advertising Relationship when Differentiation is Moderate*

The results show that prices are positively related to advertising levels when advertising is targeted to consumers nearby and negatively related when advertising is targeted to distant consumers. The results are robust since conditions of moderate differentiation are *fully determined* by the relationship of available surplus for an ideally located consumer \((v-c)\) to the travel cost between firms \((t/N)\). The explanation for the change in impact is as follows. When advertising weight is heaviest to nearby consumers, the optimal price for consumers who have only seen advertising from one firm is higher than the optimal price for consumers who have seen advertising from two adjacent firms (this is different from the case of no targeting). Ostensibly, as importance shifts from consumers who have only seen advertising from one firm to those who have seen advertising from both firms, this should drive prices downwards. However, because each firm’s maximum advertising occurs where the competitor’s advertising is at a minimum, targeting creates an upper bound on the fraction of the population that sees advertising from two adjacent firms. As a result, the over-riding factor that affects market pricing is the optimal price for consumers who have only seen advertising from one firm. When advertising is targeted on consumers nearby, higher advertising levels creates higher optimal prices.

---

14 In Figure 3, we show the results for two transportation costs but the lines have the same slope throughout the allowable space \( t \in (25,50) \).
for this group of consumers and this leads to the positive correlation\(^{15}\). Conversely, when firms target distant consumers, the optimal price for consumers who have only seen advertising from one firm is lower than the price for consumers who have seen advertising from both adjacent firms. Higher advertising levels create lower optimal prices for the group of consumers who have only seen advertising from one firm and this leads to the negative correlation. In sum, when differentiation is moderate and firms target their advertising, the likelihood that a consumer sees advertising from only one firm is high. As a result, the key determinant of equilibrium pricing is the optimal price for consumers who have only seen advertising from one firm.

The effect of targeting on the relationship between advertising and pricing is certainly strong in this region. Nevertheless, the most interesting effect of targeting appears to be how it impacts price levels. When differentiation is moderate, targeting consumers nearby causes an increase in prices, the exact opposite of what happens when differentiation is low! This happens because targeting reduces competition and a greater percent of a firm’s loyal consumers are willing to pay higher prices (more of them are located nearby). Conversely, when differentiation is moderate, targeting consumers who are distant causes a drop in prices. Here, individual rationality means that unless distant consumers are provided a low price they will not buy at all (their travel costs will exceed \(v\)). In sum, when differentiation is moderate, two factors (fewer feasible choices for each consumer and less relative surplus) completely reverse the impact that targeting has when differentiation is low. This suggests that the benefit of targeted advertising and the impact of targeting on pricing are highly dependent on the degree of differentiation in the market and who specifically is being targeted (loyal consumers or the “competitor’s consumers”).

6.3 Targeted Advertising in Fully Differentiated Conditions

In order to capture the effect of targeted advertising, in fully differentiated conditions, we assume that firms restrict their advertising to consumers between their location and the two adjacent firms (similar to section 6.2). The objective functions for the focal firms in the two conditions are:

\[
\text{Target nearby consumers} \quad \pi = 2\phi_y(p - \phi_y)\left(y - \frac{Ny^2}{2}\right) + \alpha \log(1 - \phi_y) \tag{35}
\]

\(^{15}\) A series of calculations can be performed easily to confirm this explanation.
Target distant consumers

\[ \pi = \phi_n (p - c)Ny^2 + \alpha \log(1 - \phi_n) \]  

(36)

where \( y = \frac{v - p}{t} \). Solving for optimal prices leads to Proposition 9:

**Proposition 9**

In fully differentiated conditions, the optimal price is

\[ p_{\text{nearby}}^* = \frac{1}{6N} \left( -4r + 4vN + 2cN + 2\sqrt{4t^2 - 2tvN + 2teN + v^2N^2} - 2vN^2c + c^2N^2 \right) \]

when consumers nearby receive heavier advertising and \( p_{\text{distant}}^* = \frac{1}{3}v + \frac{2}{3}c \) when distant consumers receive advertising. In addition, \( p_{\text{nearby}}^* > p_{\text{distant}}^* \) strictly.

Similar to the case of no targeting, advertising levels have no effect on the optimal price charged by firms when there are local monopoly conditions. In addition, targeting advertising to nearby consumers results in higher prices than targeting distant consumers. Of course, this is to be expected given that individual rationality is the only determinant of optimal pricing in fully differentiated conditions.

In summary, we find that the targeting of advertising does affect the relationship between advertising and pricing. While the general pattern of results regarding the impact of differentiation on the advertising/price relationship is consistent across the three conditions examined, targeting has a particularly interesting effect when differentiation is moderate. In fact, when distant consumers are targeted, the positive relationship of advertising levels to price is reversed and prices fall with higher levels of advertising. However, the most interesting effect of targeted advertising is how it affects overall pricing. When differentiation is low, targeting consumers who are nearby exacerbates price competition and reduces the price below the no-targeting price. On the other hand, targeting consumers who are distant results in equilibrium prices that are higher than the no-targeting price. Exactly the opposite is observed when differentiation is moderate. These findings underline the importance of existing differentiation between firms for determining the effect that targeted advertising has on pricing. The findings also provide a potential explanation for offensive or defensive postures that firms employ in media buying that has not been considered previously.
7.0 Conclusion, Managerial Implications, Limitations and Extensions

7.1 Conclusion

The objective of this analysis has been to show that advertising can lead to both increases or decreases in pricing when it is modelled as a series of purely informative messages. As shown in section 4.0, the critical factor that determines the relationship between advertising intensity and pricing is the pre-existing level of differentiation in the market. In fact, the model predicts a non-monotonic relationship between advertising and pricing as a function of differentiation. Specifically, the model shows that increases in advertising lead to decreases in pricing when levels of differentiation are low, increases in pricing when differentiation is moderate and no effect on pricing when differentiation is high. The lack of a relationship between advertising and pricing when differentiation is high underlines the role of advertising in our framework: it informs consumers about a product’s attributes (i.e. it creates awareness of a product and its characteristics) but does not affect consumers’ overall evaluation of the product.

The model also provides a vehicle for understanding why the relationship between advertising and pricing is non-monotonic. When the level of differentiation is low, increases in advertising tend to localise competition between firms in the market. As firms source a greater percent of their demand from consumers who are nearby, the level of competition between adjacent firms increases and this drives prices downwards.

When the level of differentiation is moderate, competition for consumers is already local. This is due to the fact that higher levels of differentiation make it infeasible for consumers who are “spatially distant” from a firm to patronise it. Nevertheless, the advertising still has the effect of shifting the focus of competition from one group of consumers to another. Specifically, increases in advertising tend to shift the focus of competition from consumers who are aware of only one “feasible firm” to consumers who are aware of two “feasible firms”. A unique characteristic of conditions of moderate differentiation is that the equilibrium price for consumers who are aware of two “feasible firms” is higher than the equilibrium price for consumers who are aware of only one “feasible firm”. Thus, the shift in focus (caused by increases in advertising) leads to higher pricing.
When the differentiation is “full”, firms effectively choose not to compete with each other. In these conditions, because of the informative nature of advertising, the level of advertising has no effect on pricing.

The advertising messages in our model contain information about product attributes and no information about pricing. This distinguishes our model from other models that represent advertising in a similar manner (Butters, 1977 and Grossman and Shapiro, 1984). This is important as the vast majority of media advertising for consumer goods does not contain pricing. In section 5, the analysis of the impact of “reductions in the cost of advertising” on total welfare is also important. Under very general conditions, the model demonstrates that total welfare can increase even when higher advertising leads to price increases.

Finally, the analysis of targeted advertising shows that even when targeting is possible, differentiation is important for understanding the effect that advertising has on pricing. Most importantly, the analysis highlights how targeting consumers who are nearby (i.e. loyal consumers) or distant (the competitor’s consumers) can have completely different effects on pricing depending on the existing level of differentiation between firms. The analysis suggests that focussing advertising on loyal consumers may be effective when firms are well differentiated. On the other hand, the findings provide a rationale for the churning of customers that appears endemic in many commoditized markets. Perhaps, focussing advertising on consumers whose preferences are more closely aligned with competitors’ products allows firms in commoditized industries to sustain higher price levels.

7.2 Managerial Implications
First, we discuss the implications for managers in competitive categories where advertising is important and primarily focussed on providing consumers with information about product attributes. Second, we discuss the relevance of our findings with regard to advertising regulation.

Managers who operate competitive firms in many industries are faced with regulations that prohibit them from advertising in certain media and at certain times of the day (consider for example, the significant regulations that apply to industries such as tobacco, alcohol, pharmaceuticals, lotteries and children’s toys). In general, these regulations increase the cost of advertising for firms and make it difficult to send messages to target consumers (Peltzman 1981). A further observation is that industry
associations in tobacco, alcohol, pharmaceuticals, lotteries and children’s toys frequently lobby and are involved in the drafting and enforcement of advertising regulations (Noll 1992). In markets like tobacco, where the level of category demand is relatively inelastic, the model suggests that industry associations have a strong incentive to support regulations that limit advertising activity\(^\text{16}\). Here, increases in the cost of advertising (the *de facto* impact of increased restrictions on advertising) lead to higher firm profits. This is reminiscent of the observations of Stigler (1971) that interest groups frequently benefit from regulation that is ostensibly enacted to protect consumers\(^\text{17}\). The model underlines the salience of differentiation as a basis for determining whether lobbying activity should be directed towards increasing or decreasing advertising regulation.

A second implication for managers relates to the benefit of finding cheaper media vehicles. The model suggests that new less-expensive media vehicles are more appealing when the level of differentiation between firms is significant. For example, in markets such as automobiles, where there are significant differences between brands (and differentiation is significant), new media vehicles have the potential to improve firm profitability because of the positive relationship observed between advertising and pricing. On the other hand, when differentiation is low, new and more efficient media vehicles are less attractive because a) they are unlikely to become a source of competitive advantage (it is difficult for a firm to restrict access to a new media vehicle) and b) the relationship between advertising levels and pricing is negative.

The model also has implications for regulators. In general, when advertising increases have no impact on prices or cause them to fall, higher advertising raises total welfare. However, the model also underlines the potential for welfare increases when prices rise in the face of higher advertising. In other words, the model highlights the need for careful analysis when examining the potential anti-competitive effects of advertising. In particular, the relationship between advertising levels and pricing is but one aspect of understanding the total effect of increased advertising on total welfare (Joskow and Rose 1992).

\(^{16}\) In this framework, inelastic category demand has a simple interpretation as a high \(v\) in relation to \(t\) (low differentiation) i.e. consumers will buy even if they cannot find a product that is perfectly suited to their tastes.

\(^{17}\) In spite of creating supra-normal profits for firms in these industries, regulations may be completely justified. Our model does not account for the negative externalities of products such as tobacco and alcohol (Gruenspecht and Lave 1992).
7.3 Limitations

We believe that our model sheds new light on advertising and the manner by which differentiation affects the advertising/price relationship. Nonetheless, the model has limitations and the insights of the paper have to be considered keeping these limitations in mind.

First, we assume that the number of firms in the industry is fixed and that firms neither exit nor enter the industry. Certainly in the short term, the number of firms in many industries does appear to be fixed; this observation may be due to high fixed costs of entering or exiting an industry, industry expertise, or limited resources (of some type). However, in the long term (and in some industries the long term is quite short), this assumption may be tenuous. If firms are making significant profits and the fixed costs of entry are small, we should expect new entrants. In addition, firms often exit an industry or merge with competitors (reducing the number of firms). Such actions would have effects on the observed levels of advertising, pricing and ultimately total welfare.

A second limitation is that our welfare analysis is restricted to variable factors such as consumption, production and advertising (holding the number of firms constant). In many cases, this is only half the picture: total welfare is also affected by society’s total expenditure on fixed costs. Accordingly, an interesting extension to this analysis would be to examine the equilibrium number of firms in an industry given a positive fixed cost (for establishing a firm) and a zero profit constraint. Similar to the analysis in Grossman and Shapiro (1984), such an extension would allow us to examine the welfare implications of environmental changes (such as a reduction in the cost of advertising) in monopolistically competitive industries.

A third limitation is that our welfare findings only relate to the effect of reductions in the cost of advertising. In reality, changes in the level of advertising may result from reductions in the marginal cost of production, changes in the perceived level of differentiation in the market or (as noted above) a change in the number of competing firms. We do not consider the impact of such changes on advertising intensity, pricing and ultimately, total welfare but believe them to be important. This framework certainly has the potential to facilitate such analysis.

Finally, we impose several constraints on the advertising represented in our model. For example, we assume that consumers only need exposure to one message to
be informed and we restrict our analysis to advertising that is truthful. In general, these restrictions seem broadly justified. The meaning of one message can easily be interpreted as the cost of “effective reach” within a media. Also, if advertising were truly false or misleading, one would expect consumers to start ignoring it (Peltzman 1981, Joskow 1981 and Nelson 1981).

7.4 Extensions

A useful extension to this study would be to empirically test the model across a number of categories where horizontal attributes are the main subject of advertising messages. In categories like sporting equipment, financial services, and communication services, firms frequently emphasize different product features in advertising; image or persuasive information seems to play less of a role. A first step would be to characterize the level of differentiation in a number of categories using a measure of average price cross-elasticity. Second, longitudinal data on overall advertising and category pricing, could be used to estimate the relationship between advertising and pricing in each category. The findings could then be used to determine whether differentiation is a driving force behind the relationship of advertising to pricing.

In section 3, we assume that advertising falls equally on all consumers in the market. As mentioned earlier, this may be limiting given recent advances in information technology and communication. The analysis of Section 6 provides a preliminary analysis of how targeting affects the relationship between advertising and pricing for three different conditions of differentiation. Of course, the analysis considers a situation where the targeting strategy of all firms is symmetric and not a decision variable. A natural question is to ask how firms will respond to each other when they can choose to place higher advertising weight on consumers that are either more closely aligned with their products (loyal consumers) or on consumers that are more closely aligned with the competitors’ products (the competitor’s loyal consumers). To tackle this problem, a simple model with two firms could be developed. By analysing the best responses of the firms to each other, equilibrium outcomes under a variety of differentiation conditions could be identified. Such an

18 For example, at a large North American brewery, effective reach is defined as the percentage of the target audience who sees 3 effective impressions (or 5 actual impressions) within a two-week period. Obviously, in our simplified model, the cost for such effective reach can be broadly interpreted as the cost per message with an effective reach of f.
analysis would allow us to shed light on the types of markets where firms are likely to defend their “own turf” vigorously versus those where firms aggressively pursue consumers throughout the market.
References


Technical Appendix: Informative Advertising: An Alternate Viewpoint and Implications

Derivation of Demand Function for Firm \( n \) in the Low Differentiation Case

Using equation 9 in the main text, \( \bar{p} \), we substitute for the values of \( K_k \) and \( \phi_k \) as described (note that the values of \( \phi_1, \phi_2, \phi_3 \ldots \phi_N \) are as described in the text):

\[
d_n = \phi_n \left( \frac{\bar{p} - p}{t} + \frac{1}{N} \right) + \phi_n(1-\bar{\phi})^2 + \phi_n(1-\bar{\phi})^3 + \phi_n(1-\bar{\phi})^4 + \ldots + \phi_n(1-\bar{\phi})^{N-1} \left( \frac{1}{N} - \frac{\bar{p} - p}{t} \right)
\]

Rearranging we obtain:

\[
d_n = \phi_n \left( 1 - (1-\bar{\phi})^{N-1} \right) + \frac{\phi_n}{N} + \frac{\phi_n(1-\bar{\phi})}{N} + \frac{\phi_n(1-\bar{\phi})^2}{N} + \frac{\phi_n(1-\bar{\phi})^3}{N} + \ldots + \frac{\phi_n(1-\bar{\phi})^{N-1}}{N} \quad (1)
\]

Let \( X = 1+(1-\bar{\phi})^2 + (1-\bar{\phi})^3 + \ldots + (1-\bar{\phi})^{N-1} \).

Then \( X(1-\bar{\phi}) = (1-\bar{\phi})^2 + (1-\bar{\phi})^3 + \ldots + (1-\bar{\phi})^{N} \). This implies that \( X - X(1-\bar{\phi}) = 1 - (1-\bar{\phi})^{N} \). Solving for \( X \), we obtain \( X = \frac{1 - (1-\bar{\phi})^N}{\bar{\phi}} \). Substituting into equation 1 above, we obtain the reduced expression for \( d_n \).

Proof of Proposition 1

When differentiation is low, the optimal price and advertising intensity for all \( N \) firms in the market are

\[
p = c + \frac{2\alpha N t}{\sqrt{t^2 + 4\alpha N^2 t} - t} \quad \text{and} \quad \phi = \frac{\sqrt{t^2 + 4\alpha N^2 t} - t}{2\alpha N^2}.
\]

The first order condition for price is obtained by differentiating equation 12 and setting it equal to zero. This implies that \( p_n = \frac{\bar{p} + c}{2} + \frac{t}{2N\phi} \) but \( p_n = \bar{p} \) in equilibrium which implies that \( \bar{p} = c + \frac{t}{N\phi} \). In equilibrium \( p_n = \bar{p} \) i.e. \( \bar{p} - p_n = 0 \) and the marginal change in \( \bar{p} - p_n \) as a function of \( \phi \) is of second order so we use the approximation that \( d_n = \frac{\phi_n}{N\phi} \). We now substitute back into equation 12 and obtain:

\[
\pi_n = \left( c + \frac{t}{N\phi} - c \right) \frac{\phi_n}{N\phi} + \alpha \log (1 - \phi_n)
\]

(2)

Taking the first order conditions with respect to marketing intensity, we obtain:

\[
\frac{\partial \pi_n}{\partial \phi_n} = \frac{t}{N^2\phi^2} + \frac{\alpha}{1 - \phi} = 0. \quad \text{Assuming symmetry and setting} \quad \phi_n = \bar{\phi}, \quad \text{we obtain:}
\]

\[
\frac{\partial \pi_n}{\partial \phi_n} = \frac{t}{N^2\phi^2} + \frac{\alpha}{1 - \phi} = 0.
\]

\[\text{Note that } p_n \text{ and the price of all competitors are strategic complements so any reduction in } \bar{p} \text{ due to an increase in } \phi \text{ will be offset by an equilibrium reduction in price enacted by firm } n.\]
\( 0 = \alpha N^2 \phi + t \phi - t \). This yields two roots and the positive root is given by
\[
\phi = \frac{\sqrt{t^2 + 4\alpha N^2} - t}{2\alpha N^2}.
\]
The expression for \( p \) is found by substituting the optimal \( \phi \) into
\[
\bar{p} = c + \frac{t}{N\phi}.
\]

**Q.E.D.**

**Proof of Proposition 2**

When differentiation is low, the equilibrium price is a decreasing function of advertising intensity.

As shown above, the first order condition for firm \( n \) in price and symmetry yields the following relationship between \( \bar{p} \) and \( \phi \):
\[
\bar{p} = c + \frac{t}{N\phi}.
\]
We differentiate this expression with respect to \( \phi \):
\[
\frac{\partial \bar{p}}{\partial \phi} = -\frac{t}{N\phi^2} < 0 \text{ for all } \phi.
\]

**Q.E.D.**

**Proof of Proposition 3**

When differentiation is low:

a) A reduction in the cost of advertising leads to an increase in advertising.

b) A reduction in the cost of advertising leads to a decrease in prices

**Part a**

\[
\frac{d\phi}{d\alpha} = -\frac{1}{2}\frac{t + 2\alpha N^2 - \sqrt{t + 4\alpha N^2}}{\alpha^2 N^2} \frac{\sqrt{t}}{\sqrt{t + 4\alpha N^2}} = -\frac{\sqrt{t}}{2\alpha^2 N^2 \sqrt{t + 4\alpha N^2}} \left( t + 2\alpha N^2 - \sqrt{t + 4\alpha N^2} \right)
\]
\[
\therefore \text{ sign } \left( \frac{d\phi}{d\alpha} \right) = -\text{ sign } \left( t + 2\alpha N^2 - \sqrt{t + 4\alpha N^2} \right). \text{ But } \left( t + 2\alpha N^2 - \sqrt{t + 4\alpha N^2} \right) > 0. \text{ Assume not. Then } \sqrt{t + 4\alpha N^2} > t + 2\alpha N^2 \Rightarrow \left( \sqrt{t + 4\alpha N^2} \right) > (t + 2\alpha N^2) \]
\[
\Rightarrow t^2 + 4\alpha N^2 > t^2 + 4\alpha N^2 t + 4\alpha^2 N^4 \Rightarrow 0 > 4\alpha^2 N^4 \text{ which is a contradiction. Therefore }
\]
\[
\frac{d\phi}{d\alpha} < 0.
\]

**Part b**

\[
\frac{dp}{d\alpha} = 2Nt \sqrt{t + 2\alpha N^2 - \sqrt{t + 4\alpha N^2}} \left( t + 2\alpha N^2 - \sqrt{t + 4\alpha N^2} \right)
\]
\[
\therefore \text{ sign } \left( \frac{dp}{d\alpha} \right) = \text{ sign } \left( t + 2\alpha N^2 - \sqrt{t + 4\alpha N^2} \right). \text{ But } t + 2\alpha N^2 - \sqrt{t + 4\alpha N^2} > 0 \text{ (as per Part a).}
\]

Therefore \( \frac{dp}{d\alpha} > 0. \)

**Q.E.D.**
**Basis for Result 1**

Using the expressions for $\bar{p}$ and $\bar{\phi}$, we obtain the following expression for $\pi_n$:

$$\pi_n = \frac{2\alpha}{(t^2 + 4N^2\alpha)^{\frac{1}{2}} - t} + \alpha \log \left(1 - \frac{(t^2 + 4N^2\alpha)^{\frac{1}{2}} - t}{2\alpha N^2}\right).$$

The derivative of this expression is long and available from the authors on request. The sign of the expression cannot be determined analytically. We conduct a simulation over a dense grid of values of $t$ and $\alpha$ for different values of $N$. Below we show the results of the simulation for $N=20$.

![Figure TA-1](image)

The surface has identical appearance for all values of $N$ between 10 and 200. No negative values were detected and this leads to the conclusion that a reduction in $\alpha$ leads to decreases in $\pi_n$ in the allowable zone.

**Proof of Proposition 4**

The equilibrium price when differentiation is moderate is given by

$$p^* = \frac{-2v + 2\bar{\phi}v - 2c + \bar{\phi}c - \bar{\phi} \frac{t}{N}}{3\bar{\phi} - 4}. \text{ This implies that } \frac{\partial p^*}{\partial \phi} > 0 \text{ in the allowable range and as } \bar{\phi} \to 1, p^* \to c + \frac{t}{N} \text{ (the full information price).}$$

Taking the objective function for firm $n$:

$$\pi_n = (p_n - c) \left[ 2\phi_n (1 - \bar{\phi}) \frac{v - p_n}{t} + 2\phi_n \bar{\phi} \left(\frac{p_n - \bar{p}_n}{2t} + \frac{1}{2N}\right) \right] + \alpha \log (1 - \phi_n)$$
and differentiating with respect to $p_n$, we obtain: 
\[
p_n = \frac{-2v + 2\phi v - 2c + \phi c - \phi p - \phi}{2\phi - 4} \frac{t}{N}.
\]

By symmetry $p^* = p_n = \bar{p}$ and solving we obtain: 
\[
p^* = \frac{-2v + 2\phi v - 2c + \phi c - \phi}{3\phi - 4} \frac{t}{N}.
\]

Differentiating $p^*$ with respect to $\phi$, we obtain: 
\[
\frac{\partial p^*}{\partial \phi} = \frac{-2(v - c - 2 \frac{t}{N})}{(-4 + 3\phi)^2}.
\]

Claim \(\frac{\partial p^*}{\partial \phi} > 0\) for all $v$ in the allowable zone. Assume not. Then \(v - c - 2 \frac{t}{N} > 0\)

\[\Rightarrow v - c > \frac{2t}{N}. \]

But \(v - c < \frac{2t}{N}\) (by assumption), which is a contradiction. Therefore \(\frac{\partial p^*}{\partial \phi} > 0\) as claimed. Substituting $\phi = 1$, we obtain $p^* = c + \frac{t}{N}$. This implies that $p^* < c + \frac{t}{N}$ when $\phi < 1$ confirming the assumption that every firm restricts its attention to consumers located between it and its two adjacent competitors.

Q.E.D.

Note: The expression for $p^*$ applies given that no gaps are created in segments 3 and 4 (i.e. the consumers who are aware of the products of both adjacent firms). Given the equilibrium prices, this is guaranteed for all values of $v - c \in \left\{\frac{3t}{2N}, \frac{2t}{N}\right\}$. However, when $v - c \in \left\{\frac{t}{N}, \frac{3t}{2N}\right\}$, and $\phi$ is sufficiently high, a gap will be created as prices rise towards $c + \frac{t}{N}$ (this is confirmed by evaluating consumer surplus for the consumer located at $x = \frac{1}{2N}$ for a price of $c + \frac{t}{N}$). In these conditions, prices will rise with advertising intensities to a level $\hat{\phi}$ when $\bar{p} = v - \frac{t}{2N}$ (this is strictly less than $c + \frac{t}{N}$ in the range as described). For advertising levels higher than $\hat{\phi}$, a corner solution of $\bar{p} = v - \frac{t}{2N}$ is observed in the pricing subgame.

**Proof of Proposition 5**

When differentiation is moderate:

a) A reduction in the cost of advertising leads to an increase in advertising.

b) A reduction in the cost of advertising leads to an increase in prices.
After having solved for optimal prices, we can write $\pi_n = f(\phi_n, \bar{\phi}, \alpha)$ and we know that
\[
\frac{\partial \pi_n}{\partial \phi_n} = 2(1 - \bar{\phi})(p_n - c)\frac{v - \bar{\phi}}{t} - \frac{\phi}{N} - (p_n - c) - \frac{\alpha}{1 - \phi_n} = 0.
\] Using the implicit function rule, we can write:
\[
0 = \frac{\partial}{\partial \phi_n} \left( \frac{\partial \pi_n}{\partial \phi_n} \right) d\phi_n + \frac{\partial}{\partial \bar{\phi}} \left( \frac{\partial \pi_n}{\partial \phi_n} \right) d\bar{\phi} + \frac{\partial}{\partial \alpha} \left( \frac{\partial \pi_n}{\partial \phi_n} \right) d\alpha.
\]
Note that because of symmetry $\phi_n = \bar{\phi} \Rightarrow d\phi_n = d\bar{\phi}$. Therefore by rearranging, we can write
\[
\frac{d\phi}{d\alpha} = -\frac{\frac{\partial^2 \pi_n}{\partial \phi_n^2} + \frac{\partial^2 \pi_n}{\partial \bar{\phi} \partial \phi_n}}{\frac{\partial^2 \pi_n}{\partial \alpha \partial \phi_n}}.
\]
Using the expression for $\frac{\partial \pi_n}{\partial \phi_n}$, we can calculate the terms of this expression: $\frac{\partial^2 \pi_n}{\partial \phi_n^2} = -\frac{1}{1 - \phi_n}$, $\frac{\partial^2 \pi_n}{\partial \bar{\phi} \partial \phi_n} = -\frac{1}{(1 - \phi_n)^2}$, and using the Envelope theorem regarding price i.e. $\left( \frac{\partial \pi_n}{\partial p_n} = 0 \right)$, $\frac{\partial^2 \pi_n}{\partial \phi_n \partial \bar{\phi}} = (p_n - c)\left( -2\frac{v - p_n}{t} + 1 \right)$.
Substituting for $p_n = \frac{-2v + 2\bar{\phi}v - 2c + \bar{\phi}}{N}$, we have
\[
\frac{\partial^2 \pi_n}{\partial \phi_n \partial \bar{\phi}} = \left( (2\bar{\phi}N - 2N)(v - c) - \frac{\phi}{N} \right) \frac{(2\bar{\phi}N - 4N)(v - c) - \phi_t + 4t}{N^2(3\bar{\phi} - 4)^2t}.
\]
Therefore,
\[
\frac{d\phi}{d\alpha} = -\frac{1}{1 - \phi_n} - \frac{1}{(1 - \phi_n)^2} - \left( (2\bar{\phi}N - 2N)(v - c) - \frac{\phi}{N} \right) \frac{(2\bar{\phi}N - 4N)(v - c) - \phi_t + 4t}{N^2(3\bar{\phi} - 4)^2t}.
\]
the numerator of this expression is positive so the sign of the expression is determined by the denominator. This denominator reaches its maximum when $v - c$ is at a minimum (this obtains because only the second term in the denominator can be positive) and this happens when $(2\bar{\phi}N - 4N)(v - c) - \phi_t + 4t$ is maximised. The minimum value for $(v - c) = \frac{t}{N}$ and because the solution is symmetric $\bar{\phi} = \phi_n$.
Therefore, the denominator is negative when
\[
-\frac{1}{N^2} \frac{9N^2\phi_n^2 - 24N^2\phi_n^3 + 16N^2 - 2\phi_t + 5\phi^2t - 4\phi^3t + \phi^4t}{(1 - \phi_n)^2(3\phi_n - 4)^2} < 0
\]
and this is satisfied for
\[ \frac{t}{N^2} < \frac{-9\phi_n^2 + 24\phi_n - 16}{-2\phi_n + 5\phi_n^2 - 4\phi_n^3 + \phi_n^4}. \] Between 0 and 1 the function \[ \frac{-9\phi_n^2 + 24\phi_n - 16}{-2\phi_n + 5\phi_n^2 - 4\phi_n^3 + \phi_n^4}, \] has the following shape and reaches a minimum at 33.02.

This implies that for all reasonable parameter combinations the proposition is true. It may be violated for very high transportation costs. For example, if \( N=10 \), then the proposition is true for all \( t < 3302 \). Note this is effectively a limit on the size of the reservation price \( v \) as well.

**Q.E.D.**

**Proof of Proposition 4**

In the fully differentiated case, the equilibrium price, advertising intensities and profits are:

\[ p^* = \frac{v + c}{2}, \quad \phi^* = 1 - \frac{2t\alpha}{(v - c)^2}, \quad \pi^* = \frac{(v - c)^2}{2t} - \alpha + \alpha \log \left( \frac{2t\alpha}{(v - c)^2} \right). \]

Assuming that there is no competition at the margin, we differentiate equation 18 in the main text and find the maximum.

\[ \frac{\partial \pi_n}{\partial p_n} = \frac{2\phi_n}{t} (-p_n + cv - p_n) = 0 \Rightarrow p^* = \frac{v + c}{2} \quad (3) \]

We now substitute into equation 18:

\[ \pi_n = 2\phi_n \left( \frac{v + c}{2} - c \right) \frac{v - \frac{v + c}{2}}{t} + \alpha \log (1 - \phi_n) \quad (4) \]

Differentiating with respect to \( \phi_n \) and finding the maximum, we obtain:

\[ \frac{\partial \pi_n}{\partial \phi_n} = \frac{(v - c)^2}{2t} - \alpha \frac{1 - \phi_n}{1 - \phi_n} = 0 \Rightarrow \phi^* = 1 - \frac{2t\alpha}{(v - c)^2} \quad (5) \]
Substituting back into the profit function, we obtain the desired result. The stability of this equilibrium depends on there being a gap of informed consumer that is not served between each pair of firms. Consider the surplus (from buying) for a consumer located $\frac{1}{2N}$ from the nearest firm (this is the consumer located exactly half way between two firms).

$$CS_{\frac{1}{2N}} = v - \frac{t}{2N} - p^* = v - \frac{t}{2N} - \frac{v + c}{2} = \frac{v - c}{2} - \frac{t}{2N}$$  \hspace{1cm} (6)

Assume $CS_{\frac{1}{2N}} > 0$. This implies that $v - c > \frac{t}{N}$, which is inconsistent with the boundary of the zone as described. Therefore, there is a group of informed consumers between every pair of firms that do not participate in the market. This ensures the stability of the equilibrium.

**Q.E.D.**

**Basis for Result 2**

*A reduction in the cost of advertising leads to an increase in firm profits*

From Proposition 3, we know that $p^* = \frac{-2v + 2\bar{\phi}v - 2c + \bar{\phi}c - \bar{\phi}t}{\sqrt{N}}$. We substitute this expression into the objective function for firm $n$ and obtain the following.

$$\pi_n = 2\phi_n(1 - \bar{\phi}) \left[ \frac{1}{N} \left( \frac{-2Nv + 2N\bar{\phi}v - 2Nc + N\bar{\phi}c - \bar{\phi}t}{3\bar{\phi} - 4} \right) - c \right] + \bar{\phi} \left( \frac{1}{N} \left( \frac{-2Nv + 2N\bar{\phi}v - 2Nc + N\bar{\phi}c - \bar{\phi}t}{3\bar{\phi} - 4} \right) - c \right) + \alpha \log(1 - \phi_n)$$

Taking the first order conditions of this expression with respect to $\phi_n$ we obtain:

$$\frac{\partial \pi_n}{\partial \phi_n} = 2(1 - \bar{\phi}) \left[ \frac{1}{N} \left( \frac{-2Nv + 2N\bar{\phi}v - 2Nc + N\bar{\phi}c - \bar{\phi}t}{3\bar{\phi} - 4} \right) - c \right] + \bar{\phi} \left( \frac{1}{N} \left( \frac{-2Nv + 2N\bar{\phi}v - 2Nc + N\bar{\phi}c - \bar{\phi}t}{3\bar{\phi} - 4} \right) - c \right) - \frac{\alpha}{1 - \phi_n} = 0$$

Assuming symmetry i.e. $\phi_n = \bar{\phi}$, we obtain:
The complexity of this expression makes it impossible to find an explicit symbolic solution for \( \phi \). However, it is possible to find explicit numerical solutions for \( \phi \) given a set of values for \( N, v, c, t \) and \( \alpha \). Thus, we conduct a numerical analysis for a set of parameter values that meet the conditions of moderate differentiation. We examine equilibrium values for \( \phi \) for a set of values of \( \alpha \) holding \( N, v, c \), and \( t \) constant. We normalize \( v \) to 1 and \( c \) to 0, and choose \( t \) such that \( v - c = \frac{3t}{2N} \). In Figure TA-2, we show the results of a numerical analysis for \( N=2 \) to 10 firms. Note that we choose a range of \( \alpha \) such that advertising levels are positive.

Using the set of optimal values for \( \phi \), we evaluate firm profits for a set of \( \alpha \) values given \( N, v, c, \) and \( t \). In Figure TA-3, we show the results of a numerical analysis for \( N=2 \) to 10 firms.

\[
0 = 2(1 - \bar{\phi}) \left[ \frac{1}{N} \left( -\frac{2Nv + 2N\bar{\phi}v - 2Nc + N\bar{\phi}c - \bar{\phi}t}{3\bar{\phi} - 4} \right) - c \right] + \frac{\bar{\phi}}{N} \left[ \frac{1}{N} \left( -\frac{2Nv + 2N\bar{\phi}v - 2Nc + N\bar{\phi}c - \bar{\phi}t}{3\bar{\phi} - 4} \right) - c \right] - \frac{\alpha}{1 - \phi} 
\]

1 The expression has four roots: two are complex, one is greater than one and the other lies in the allowable zone for \( \phi \).

2 Fixing \( t \) in this way for each \( N \) ensures that there is not a corner solution in prices for sufficiently low \( \alpha \) (see note that follows the proof of Proposition 3 in this appendix).

3 At \( \phi=0 \), the marginal cost of advertising is \( \alpha \). Therefore \( \alpha \) needs to be low enough such that marginal gain from positive demand exceeds the marginal cost of advertising.
The figure shows that as the cost of advertising ($\alpha$) decreases, the profits of each firm increase. The profit increase is greater at low levels of N but the increase in profits as $\alpha$ gets smaller for higher N is still positive.

**Basis for Result 3**

When differentiation is low, reductions in the cost of advertising lead to increases in total welfare i.e. $\frac{\partial W^T}{\partial \alpha} < 0$.

Assuming that $(1 - \bar{\phi})^v$ is small (i.e. close to zero), we substitute the values for $\bar{\phi}$ and $\bar{\alpha}$ into equation 19 in the main text and obtain the following expression for $W^T$.

$$W^T = (v-c) - N\alpha \log \left(1 - \frac{\sqrt{t} \sqrt{t + 4\alpha N^2} - t}{2\alpha N^2}\right) - \frac{1}{2} \left(\frac{tN}{\sqrt{t} \sqrt{t + 4\alpha N^2} - t}\right) \left(2 - \frac{\sqrt{t} \sqrt{t + 4\alpha N^2} - t}{2\alpha N^2}\right)$$

Taking the derivative of this expression with respect to $\alpha$, we obtain an expression that is too long for presentation purposes. In addition, because of the complexity of the expression, it is impossible to explicitly confirm its negative value. However, $v$ and $c$ do not figure in the expression for $\frac{\partial W^T}{\partial \alpha}$, so we can fully investigate possible values of $\frac{\partial W^T}{\partial \alpha}$ by looking at a set of values over $\alpha$ and $t$ for wide range of N. Below is a 3D plot for a range of $\alpha$ (.05 to .95) and $t$ (0.1 to 2.7) for N=20 firms.
The surface has identical appearance for all values of \( N \) between 10 and 200. No positive values were detected and this leads to the conclusion that \( \frac{\partial W_T}{\partial \alpha} < 0 \) for all parameter values when differentiation is low.

**Basis for Result 4**

*When differentiation is moderate and the number of firms is small, reductions in the cost of advertising lead to increases in total welfare i.e. \( \frac{\partial W_T}{\partial \alpha} < 0 \).*

To evaluate the sign of \( \frac{\partial W_T}{\partial \alpha} \), we examine the sign of equation 24 i.e.

\[
\frac{dW_T}{d\alpha} = \frac{\partial W_T}{\partial \phi} \frac{d\phi}{d\alpha} + \frac{\partial W_T}{\partial \alpha},
\]

for a range of values of \( N \) and \( \alpha \). Similar to the numerical analysis conducted for section 4.2 we normalize \( v \) to 1 and set \( t = \frac{N}{1.5} \) (this eliminates the possibility of corner solutions in price for sufficiently low \( \alpha \)). The second term of equation 24, \( \frac{\partial W_T}{\partial \alpha} = \log(1 - \bar{\phi}) \) is negative since \( \bar{\phi} < 1 \). The first term of equation 24 is the product of two derivatives \( \frac{\partial W_T}{\partial \phi} \) and \( \frac{d\phi}{d\alpha} \). Proposition 5 implies that the second term is negative. The first term needs to be calculated. Substituting for \( p^* \) in equation 22 (in the main text), differentiating with respect to \( \bar{\phi} < 1 \) and evaluating at \( v=1, c=0 \) and \( t = \frac{N}{1.5} \), we obtain:
\[
\frac{\partial W_T}{\partial \phi} = \frac{1}{2N} (880N^2\phi^2 - 472N^2\phi - 64\phi t^2 + 346\phi^3N^2 - 316\phi^4Nt - 790\phi^5N^2 + 640N\phi^6t
- 576N\phi^7t + 192N\phi t + 184\phi^2t^2 - 190\phi^3t^2 + 85\phi^4t^2 - 128\alpha N^2t^2 - 216\alpha N^2t\phi^2 + 288\alpha N^2t\phi
+ 96N^2 - 60\phi^2N^2 - 15\phi^3N^2 + 60\phi^4Nt + 54\alpha N^2t\phi^3)/(-4 + 3\phi)
\]

This expression is too complex to determine its sign analytically. Accordingly, we conduct a numerical analysis of its values for \(N=2\) to 10 firms. As before, we choose a range of \(\alpha\) such that advertising levels are positive. In Figure TA-5, we show the results of the analysis.

The analysis shows that \(\frac{\partial W_T}{\partial \phi}\) is positive throughout the range examined. Therefore, the product of \(\frac{\partial W_T}{\partial \phi}\) and \(\frac{d\phi}{d\alpha}\) is negative which confirms Result 4. Figure TA-6 is graph of \(\frac{dW_T}{d\alpha}\) for the zone analysed in Figure TA-5.
Derivation of Targeted Advertising Demand Function

Low Differentiation Targeting Consumers Nearby

From equation 27 in the paper:

\[
d_n = \phi_n\left(\frac{\overline{p} - p}{t} + \frac{1}{N}\right) + \rho \phi_n (1 - \overline{\phi}) \frac{1}{N} + \rho^2 \phi_n (1 - \overline{\phi})^2 \frac{1}{N} + \rho^3 \phi_n (1 - \overline{\phi})^3 \frac{1}{N} + ... \\
+ \rho^{n-1} \phi_n (1 - \overline{\phi})^{n-1} \left(\frac{1}{N} - \frac{\overline{p} - p}{t}\right)
\]

\[
d_n = \phi_n\left(\frac{\overline{p} - p}{t}\right) \left(n - (1 - \overline{\phi})^{n-1}\right) + \frac{\phi_n}{N} + \rho \phi_n (1 - \overline{\phi}) \frac{1}{N} + \rho^2 \phi_n (1 - \overline{\phi})^2 \frac{1}{N} + \rho^3 \phi_n (1 - \overline{\phi})^3 \frac{1}{N} + ... \\
+ \rho^{n-1} (1 - \overline{\phi})^{n-1} \left(\frac{\overline{p} - p}{t}\right)
\]

Let \( X = 1 + \rho (1 - \overline{\phi}) + \rho^2 (1 - \overline{\phi})^2 + ... + \rho^{n-1} (1 - \overline{\phi})^{n-1} \)

Then \( X = \frac{1 - \rho^n (1 - \overline{\phi})^n}{1 - \rho (1 - \overline{\phi})} = \frac{1 - \rho^n (1 - \overline{\phi})^n}{1 - \rho + \rho \overline{\phi}} \) substituting we obtain:

\[
d_n = \frac{\phi_n (\overline{p} - p_n)}{t} \left(n - (1 - \overline{\phi})^{n-1}\right) + \frac{\phi_n}{N} \left(1 - \rho^n (1 - \overline{\phi})^n\right) \frac{1}{N(1 - \rho + \rho \overline{\phi})} \text{ and for } N \text{ sufficiently large}
\]

this simplifies to: \( d_n = \frac{\phi_n (\overline{p} - p_n)}{t} + \frac{\phi_n}{N(1 - \rho + \rho \overline{\phi})} \).

Low Differentiation Targeting Distant Consumers

From equation 29 in the paper:

\[
d_n = (1 - \rho) \phi_n\left(\frac{\overline{p} - p}{t} + \frac{1}{N}\right) + (1 - \rho^2) \phi_n (1 - \overline{\phi}) \frac{1}{N} + (1 - \rho^3) \phi_n (1 - \overline{\phi})^2 \frac{1}{N} \\
+ (1 - \rho^4) \phi_n (1 - \overline{\phi})^3 \frac{1}{N} + ... + (1 - \rho^n) \phi_n (1 - \overline{\phi})^{n-1} \left(\frac{1}{N} - \frac{\overline{p} - p}{t}\right)
\]

\[
d_n = \phi_n\left(\frac{\overline{p} - p}{t}\right) \left(1 - (1 - \overline{\phi})^{n-1}\right) + \frac{\phi_n}{N} \left(1 - \rho^n (1 - \overline{\phi})^n\right) - \rho \phi_n \overline{\phi} (1 - \overline{\phi})^{n-1} \left(\frac{\overline{p} - p}{t}\right)
\]

\[
d_n = \phi_n\left(\frac{\overline{p} - p}{t}\right) \left(1 - (1 - \overline{\phi})^{n-1}\right) + \frac{\phi_n}{N} \left(1 - \rho^n (1 - \overline{\phi})^n\right) - \rho \phi_n \overline{\phi} (1 - \overline{\phi})^{n-1} \left(\frac{\overline{p} - p}{t}\right)
\]

For \( N \) sufficiently large this simplifies to:

\( d_n = (1 - \rho) \frac{\phi_n (\overline{p} - p_n)}{t} + \frac{\phi_n}{\overline{\phi}} - \frac{\rho \phi_n}{N(1 - \rho + \rho \overline{\phi})} \).
Moderate Differentiation Targeting Consumers Nearby

\[ d_a = \int \phi_n \phi (1 - Nx) \, dx + \int \phi_n (1 - Ny)(1 - \phi Ny) \, dy = \]
\[ \frac{\phi_n \phi N_x^2}{2} - \frac{\phi_n \phi N_x^3}{3} + \frac{\phi_n \phi N_y^2}{2} - \frac{\phi_n \phi N_y^3}{3} + \phi_n y \]

Substituting \( x = \frac{p - p_n}{2t} + \frac{1}{2N} \) and \( y = \frac{v - p_n}{t} \), we obtain the demand function in terms of price and advertising levels. To find the optimal price, we take first order conditions of the objective function (equation 33 in the text) and set \( \bar{p} = p_n \). This provides an expression for \( p_n \) in terms of the exogenous variables, \( \phi_n \) and \( \phi \). We now let \( \phi = \phi_n \) to obtain an implicit function that defines equilibrium price and advertising levels.

Recall that conditions of moderate differentiation are entirely defined by the relation of available surplus \( v - c \) to the travel cost between firms \( \frac{t}{N} \). Thus without any loss of generality, we can normalize \( v = 1 \), \( c = 0 \), and \( N = 50 \) and examine the relation of advertising levels to prices for the range of relevant travel costs i.e. \( t \in \{25, 50\} \). Note that we can change the parameters and the implicit function is not altered when the remaining parameters are scaled accordingly i.e. If the number of firms is doubled, then the transportation costs are doubled to obtain conditions of moderate differentiation and the implicit function is identical.

For example, with the normalizations identified above and \( t=33 \), we obtain the following implicit function.

\[ -\frac{1}{10781100} \phi (1101675p - 673200p - 409370p - 3015000\phi p^2 + 1485000p^2 + 2000000\phi p^3 - 158400) = 0 \]

Similarly for \( t=48 \), we obtain:

\[ -\frac{1}{1036800} \phi (9150p - 3600p + 314190p - 731250p^2 + 67500p^2 + 62500p^3 - 20700) = 0 \]

Graphing these two functions for the relevant range i.e. \( \phi \in \{0,1\} \), we obtain the picture presented in the paper. The functions are sloping upwards for all values of \( t \in \{25, 50\} \). Second order conditions have been checked in the relevant zone and they are satisfied.

Moderate Differentiation Targeting Distant Consumers

\[ d_a = \int \phi_n Nx \phi (1 - Nx) \, dx + \int \phi_n Ny (1 - \phi (1 - Ny)) \, dy = \]
\[ \frac{\phi_n \phi N_x^2}{2} - \frac{\phi_n \phi N_x^3}{3} + \frac{\phi_n \phi N_y^2}{2} - \frac{\phi_n \phi N_y^3}{3} + \phi_n y \]

Substituting \( x = \frac{\bar{p} - p_n}{2t} + \frac{1}{2N} \) and \( y = \frac{v - p_n}{t} \), we obtain the demand function in terms of price and advertising levels. To find the optimal price, we take first order conditions of the objective function (equation 34 in the text) and set \( \bar{p} = p_n \). This
provides an expression for \( p_n \) in terms of the exogenous variables, \( \phi_n \) and \( \phi \). We now let \( \phi = \phi_n \) to obtain an implicit function that defines equilibrium price and advertising levels.

Similar to the case of targeting nearby consumers, we implement the same normalizations and consider the relationship of advertising levels to pricing for \( t \in \{25, 50\} \).

With the normalizations identified above and \( t=33 \), we obtain the following implicit function.

\[
-\frac{1}{10781100} \phi (-40937 \phi - 495000 + 2000000 \phi^3 - 1485000 p^2 + 1980000 p - 3015000 \phi p^2 + 1101675 \phi p) = 0
\]

Similarly for \( t=48 \), we obtain:

\[
-\frac{1}{1036800} \phi (3419 \phi - 22500 + 62500 \phi^3 - 67500 p^2 + 90000 p - 73125 \phi p^2 + 91500 \phi p) = 0
\]

Graphing these two functions for the relevant range i.e. \( \phi \in \{0, 1\} \), we obtain the picture presented in the paper. The functions are sloping downwards for all values of \( t \in \{25, 50\} \). Second order conditions have been checked in the relevant zone and they are satisfied.

**Proof of Proposition 9**

In fully differentiated conditions, the optimal price is

\[
p_{\text{nearby}}^* = \frac{1}{6N} \left( -4t + 4vN + 2cN + 2\sqrt{4t^2 - 2tvN + 2tcN + v^2N^2 - 2vN^2c + c^2N^2} \right)
\]

when consumers nearby receive heavier advertising and

\[
p_{\text{distant}}^* = \frac{1}{3} v + \frac{2}{3} c
\]

when distant consumers receive advertising. In addition, \( p_{\text{nearby}}^* > p_{\text{distant}}^* \) strictly.

The optimal prices obtain by solving the first order conditions of the objective functions (equations 35 and 36 in the paper).

Proof that \( p_{\text{nearby}}^* > p_{\text{distant}}^* \). Assume not then:

\[
\frac{1}{3} v + \frac{2}{3} c - \frac{1}{6N} \left( -4t + 4vN + 2cN + 2\sqrt{4t^2 - 2tvN + 2tcN + v^2N^2 - 2vN^2c + c^2N^2} \right) > 0
\]

\[
\Rightarrow \frac{1}{3} \left( 2t - vN + cN - \sqrt{4t^2 - 2tvN + 2tcN + v^2N^2 - 2vN^2c + c^2N^2} \right) > 0
\]

\[
\Rightarrow 2t - vN + cN > \sqrt{4t^2 - 2tvN + 2tcN + v^2N^2 - 2vN^2c + c^2N^2}
\]

Squaring both sides implies that:

\[
4t^2 - 4tvN + 4tcN + v^2N^2 - 2vN^2c + c^2N^2 > 4t^2 - 2tvN + 2tcN + v^2N^2 - 2vN^2c + c^2N^2
\]

\[
- 4tN(v - c) > -2tN(v - c) \Rightarrow -4 > -2
\]

which is a contradiction.

Q.E.D.