Games with Strategic Complementarities: New Applications

by

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Games with Strategic Complementarities:
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Abstract
This paper surveys the theory of supermodular games and the lattice-theoretic approach to comparative statics up to recent developments. After the presentation of the basic theory, applications are developed for homogeneous and differentiated oligopolies, a general taxonomy of strategic behavior, Bayesian games, and global games and equilibrium selection.

Keywords: supermodular games, strategic complementarities, stability, monotone comparative statics, Bayesian games, global games, equilibrium selection

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1. Introduction

Games of strategic complementarities are those in which the best response of any player is increasing in the actions of the rivals. This is formalized in the class of supermodular games (Topkis (1979), Vives (1985a and 1990) and Milgrom and Roberts (1990)). Many games display strategic complementarities including those involving search, network externalities, oligopoly interaction or patent and arms races. Coordination failures in macroeconomics and financial markets provide more examples.1

The method is based on a lattice-theoretic approach and exploits order and monotonicity properties. For example, Tarski's fixed point theorem instead of Kakutani's fixed-point theorem, monotone comparative statics methods instead of the implicit function theorem. The approach has several advantages:

- Ensures the existence of equilibrium in pure strategies (without requiring quasiconcavity of payoffs).
- Allows a global analysis of the equilibrium set, which has an order structure with extremal elements.
- Equilibria have nice stability properties and there is an algorithm to compute extremal Equilibria.
- Monotone comparative statics results are obtained with minimal assumptions.

The plan of the paper is the following. Section 2 presents the basic theory. Section 3 develops some applications to oligopoly in homogenous and differentiated products environments. Section 4 extends the taxonomy of strategic behavior due to Fudenberg and Tirole (1985). Section 5 deals with Bayesian games and concluding remarks follow.2

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1 See, for example, Cooper and John (1988) and Diamond and Dybvig (1983).
2 For a more detailed overview of some of the topics covered in this paper and further references see Vives (1999).
2. Games with strategic complementarities

I will provide a definition of a game with strategic complementarities in a smooth context. This is by no means the most general way to define it. We will say that the game \((A_i, \pi_i; i \in N)\), where \(A_i\) is the strategy set and \(\pi_i\) the payoff of player \(i \in N\) (defined on the cross product of the strategy spaces of the players \(A\)), is smooth supermodular if each \(A_i\) is a compact cube in Euclidean space, and \(\pi_i(a_i, a_{-i})\) is twice continuously differentiable with

\[
\frac{\partial^2 \pi_i}{\partial a_i h \partial a_i k} \geq 0 \quad \text{for all } k \neq h, \quad \text{and}
\]

\[
\frac{\partial^2 \pi_i}{\partial a_i h \partial a_j k} \geq 0 \quad \text{for all } j \neq i \text{ and for all } h \text{ and } k
\]

where \(a_i h\) is action \(h\) of player \(i\).

Condition (i) is the strategic complementarity property (supermodularity) on own strategies \(a_i\). Condition (ii) is the strategic complementarity property on rivals' strategies \(a_{-i}\) (increasing differences in \((a_i, a_{-i})\)).

The game is strictly supermodular if \(\pi_i\) has strictly increasing differences in \((a_i, a_{-i})\):

\[
\frac{\partial^2 \pi_i}{\partial a_i h \partial a_j k} > 0 \quad \text{for all } j \neq i \text{ and for all } h \text{ and } k
\]

A useful transformation is the following. The game is log-supermodular if \(\log \pi_i\) fulfils conditions (i) and (ii).

Remark: The conditions can be weakened to strategy spaces that have an order structure (complete lattice) and differentiability is not needed (an “ordinal supermodular” game can be defined relaxing supermodularity to quasisupermodularity and increasing differences to a single crossing property (see Milgrom and Shanon (1994)). This means, for example, that strategy spaces with indivisibilities can be easily handled.

The following results hold in a supermodular game.

Result 1. Existence and order structure (Topkis (1979)): In a supermodular game there always exist extremal (largest \(\bar{a}\) and smallest \(\underline{a}\)) equilibria.
The result is shown using Tarski's fixed point theorem on the best-reply map, which is monotone because of the strategic complementarity assumptions. There is no reliance on quasi-concave payoffs and convex strategy sets to deliver convex-valued best replies as required when showing existence using Kakutani’s fixed point theorem.

The equilibrium set has further order properties. It is in fact a complete lattice (Vives (1990), Zhou (1994)). Furthermore, the equilibrium set of a strictly supermodular game with 3 or less players and totally ordered strategy spaces is totally ordered (Vives (1985) and problem 2.5 in Vives (1999)).

**Result 2. Symmetric games** (Vives (1990)): In a symmetric supermodular game (exchangeable against permutations of the players) symmetric equilibria exist (since $\bar{a}$ and $\underline{a}$ are symmetric). Therefore, if there is a unique symmetric equilibrium then the equilibrium is unique (since $\bar{a} = \underline{a}$).

**Result 3. Welfare** (Milgrom and Roberts (1990), Vives (1990)): In a supermodular game if the payoff to a player is increasing in the strategies of the other players then the largest (smallest) equilibrium point is the Pareto best (worst) equilibrium.

**Result 4. Stability** (Milgrom and Roberts (1990), Vives (1990)): In a supermodular game with continuous payoffs, best-reply dynamics:

(i) Approach the "box" defined by the smallest and the largest equilibrium points of the game (which correspond to the largest and smallest serially undominated strategies). Therefore, if the equilibrium is unique, the game is dominance solvable and the equilibrium globally stable.

(ii) Starting at any point in $A^+$ ($A^*$) converge monotonically downward (upward) to an equilibrium (see Figure 1).
Result 5. Duopoly with Strategic Substitutability (Vives (1990)): If $n = 2$ and there is strategic complementarity in own strategies, \( \partial^2 \pi_i / \partial a_i h \partial a_i k \geq 0 \) for all $k \neq h$, and strategic substitutability in rivals' strategies, \( \partial^2 \pi_i / \partial a_i h \partial a_j k \leq 0 \) for all $j \neq i$ and for all $h$ and $k$, then the transformed game with new strategies $s_1 = a_1$ and $s_2 = -a_2$ is smooth supermodular and the results above apply. (See Figure 2.)
**Result 6.** Properly mixed equilibria (i.e. Nash equilibria for which at least two players’ strategies are not pure strategies) in strictly supermodular games are unstable with respect to adaptive dynamics (Echenique and Edlin (2001)). Adaptive dynamics mean here that players that optimize myopically given their beliefs about the play of opponents and beliefs satisfy a weak monotonicity requirement (of the sort that weakly larger play is predicted when the player has observed play which is weakly larger than any action profile of rivals to which he attached positive probability). Examples of weakly monotone beliefs are beliefs generated by Cournot best reply dynamics, fictitious play, and Bayesian updating. In strict supermodular games weakly monotone beliefs will be self-confirming. A corollary is that, generically, in 2x2 games properly mixed equilibria are either unique (like matching pennies) or unstable. For example, the properly mixed equilibrium of the battle of the sexes (which is a game of strict strategic complementarities) is unstable. Furthermore, purified properly mixed equilibria are also unstable (i.e., pure strategy Bayesian equilibria of the nearby incomplete information game are unstable).

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3 If a 2x2 game has a unique equilibrium which is properly mixed, then it is globally stable (Fudenberg and Kreps (1993)).
Result 7. Comparative Statics (Milgrom and Shanon (1994)): Consider a supermodular game with parametrized payoffs $\pi_i (a_i, a_{-i}; t)$ with $t$ in a partially ordered set $T$. If $\partial^2 \pi_i /\partial a_i h \partial t \geq 0$ for all $h$ and $i$, then with an increase in $t$:

- the largest and smallest equilibrium points increase, and
- starting from any equilibrium, adaptive dynamics lead to a weakly larger equilibrium following the perturbation

Furthermore, (continuous) equilibrium selections that do not increase monotonically with $t$ predict unstable equilibria (Echenique (2000)).

In games with strategic complementarities, unambiguous monotone comparative statics obtain if we concentrate on stable equilibria. This is a multidimensional version of Samuelson’s (1947) Correspondence Principle, linking unambiguous comparative statics with stable equilibria, obtained with standard calculus methods applied to interior and stable one-dimensional models.

As an example suppose that each of $n$ firms can adopt a new technology at any period $t = 1, \ldots, T$ (as in Farrell and Saloner (1985)). The larger the number of adopters the more profitable is for a firm to switch to the new standard. Firms can be of different types, with larger types more likely to switch. This is a game of strategic complementarities with multiple equilibria some displaying “excess inertia”. That is, firms switch to the new technology only late in the game. If the cost of adopting the new technology is lowered, starting from an initial equilibrium, then adaptive dynamics will lead sequentially to increased levels of adoption of the new technology (see Echenique (2000)).

Scope of the Theory. Complementarities alone do not have much predictive power unless coupled with additional structure (Echenique (2001)). Define a game with strategic complementarities (GSC) as one in which there is a partial order on strategies (that can be chosen by the modeler) so that best responses are monotone increasing (and
with strategy sets having a lattice structure). Then (i) a game with a unique pure strategy equilibrium is a GSC if and only if Cournot best response dynamics (with unique or finite-valued best replies) have no cycles except for the equilibrium; (ii) a game with multiple pure strategy equilibria is always a GSC; and (iii) 2x2 games, generically, are either GSC or have no pure strategy equilibria (like matching pennies). Result (i) in particular means that a game with a unique and globally stable equilibrium is a GSC. Result (ii) is shown by taking one equilibrium to be the largest and another the smallest in a way that best responses are increasing. Indeed, a game with multiple equilibria always involves a coordination problem (i.e. coordinating on one equilibrium). We can find then an order on strategies that makes the game one of strategic complementarities.

3. Oligopoly

I present here some applications to oligopoly pricing: competition with differentiated products and comparative statics in Cournot markets.

3.1 Competition with differentiated products

Consider \( n \) firms competing in a differentiated product market with each firm producing a different variety. The demand for variety \( i \) is given by \( D_i(p_i, p_{-i}) \). In the Bertrand game firms compete in prices and in the Cournot game in quantities (to define the payoff then inverse demands are used).

**Bertrand oligopoly**

With constant marginal costs the profit function of firm \( i \), \( \pi_i = (p_i - c_i) D_i(p_i, p_{-i}) \) is log-supermodular in \( (p_i, p_{-i}) \) whenever \( \frac{\partial^2 \log D_i}{\partial p_i \partial p_j} \geq 0 \). This holds when \( \eta_i \) is increasing in \( p_{-i} \) as with constant elasticity, logit, or constant expenditure demand systems. We have then that extremal price equilibria do exist and that the largest and smallest price equilibrium vectors are increasing in an excise tax \( t \) (since then \( \pi_i = (p_i - t - c_i) D_i(p) \) and \( \frac{\partial^2 \pi_i}{\partial p_i \partial t} = -\frac{\partial D_i}{\partial p_i} > 0 \)). Furthermore, with constant elasticity or logit demands the

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4 Note that here the order is endogenous and it may be that in a GSC there is no equilibrium in pure
equilibrium is unique and therefore the game is dominance solvable and globally stable. The results can be extended to convex costs and multiproduct firms.

**Comparison of Cournot and Bertrand equilibria**

With gross substitute, or complementary, products if the price game is supermodular and quasi-concave then at any interior Cournot equilibrium prices are higher than the smallest Bertrand equilibrium price vector. The dual result is the following. With gross substitute, or complementary, products, if the quantity game is supermodular and quasi-concave, then at any interior Bertrand equilibrium outputs are higher than the smallest Cournot equilibrium quantity vector (Vives (1985b, 1990)).

To show the result apply Result 4(ii) to the price game starting at $A^+$ (Cournot prices must lie in this region because of the price cutting incentive at the Cournot level).

A corollary is that starting at any interior Cournot equilibrium if firms were to compete in prices they will cut prices until the market stabilizes at a Bertrand equilibrium.

3.2. Comparative statics in Cournot markets

Consider a Cournot market in which the profit function of firm $i$ is given by $\pi_i = P(Q) q_i - C_i(q_i)$, where $P$ is the inverse demand, $C_i$ the cost function of the firm and $q_i$ its output level.

The standard approach (Dixit (1986)) assumes quasiconcavity, downward sloping best replies, and uniqueness and stability of equilibrium. Let $P$ and $C_i$ be smooth with $P' < 0$, $P' + q_i P'' \leq 0$ (strategic substitutes), and $\frac{\partial C_i}{\partial q_i} - P' > 0$ for all $i$. Let $C_i(q_i; \theta_i)$ be such that $\frac{\partial C_i}{\partial \theta_i} > 0$ and $\frac{\partial^2 C_i}{\partial \theta_i \partial q_i} > 0$. Then with further conditions that ensure uniqueness and local stability of the equilibrium an increase in $\theta_i$ decreases $q_i$ and $\pi_i$ and increases $q_j$ and $\pi_j$, $j \neq i$. However, what if payoffs are not quasi-concave and/or there are multiple equilibria?

strategies because strategy sets may not be complete lattices.

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The lattice-theoretical approach (Amir (1996), Vives (1999)) makes the minimal assumptions to obtain the results (using variations of Result 8). At a Cournot duopoly with strategic substitututability we know that extremal equilibria equilibria exist and that an increase in $\theta_i$ decreases $q_i$ and $\pi_i$ and increases $q_j$ and $\pi_j, j \neq i$. Furthermore best reply dynamics lead to the result following the increase in $\theta_i$ starting at any equilibrium. At a Cournot oligopoly with strategic complementarity we have that if $0 \leq q_i \leq q^C \frac{C''}{P'}$ for all $i$ (or $\frac{\partial^2 \pi_i}{\partial q_i \partial \theta} \geq 0$) then extremal Cournot equilibria are increasing in $\theta$ and best reply dynamics lead to a larger equilibrium following the increase in $\theta$.

Restrict attention now to a symmetric Cournot oligopoly: $C_i = C, i = 1, \ldots, n$. In the standard approach (Seade (1980)) it is assumed that payoffs are quasiconcave and conditions are assumed (including $C'' - P' > 0$) so that there is a unique and locally stable symmetric equilibrium $q^*$. Let $0 \leq q_i \leq q^C \frac{C''}{P'}$. Then an increase in $\theta$ increases $q^*$. Total output increases and profits per firm decrease as $n$ increases. Output per firm may increase or decrease with $n$. The problem is that the approach is restrictive. For example, with multiple equilibria, changing $n$ either may make disappear the equilibrium considered or introduce more.

In the lattice-theoretic approach (Amir and Lambson (2000), Vives (1999)) it is assumed that $P' < 0$ and $C'' - P' > 0$ and it follows that a symmetric equilibrium (and no asymmetric equilibrium) exists. Let $0 \leq q_i \leq q^C \frac{C''}{P'}$. Then at extremal (symmetric) Cournot equilibria: Individual outputs are increasing in $\theta$, total output is increasing in $n$, profits per firm decrease with $n$, and individual outputs increase (decrease) with $n$ if demand is log-concave (log-convex and costs are zero)

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5 For example, a duopoly with $P(Q) = (Q+1)^\alpha$, for $\alpha > 2$ and zero costs.
4. Taxonomy of strategic behavior

Fudenberg and Tirole (1984) provided a taxonomy of strategic behavior in the context of a simple two-stage game between an incumbent and an entrant. At the first stage the incumbent (firm 1) can make an observable investment $k$ yielding at the market stage $\pi_i(x_1, x_2, k)$, where $x_i$ is the market action of firm $i$. The payoff of the entrant is $\pi_2(x_1, x_2)$. At a subgame-perfect equilibrium, we have that $\frac{\partial \pi_1}{\partial k} + \frac{\partial \pi_1}{\partial x_2} \frac{\partial x_2^*}{\partial k} = 0$ and $S = \frac{\partial \pi_1}{\partial x_2} \frac{\partial x_2^*}{\partial k}$ is the strategic effect. The standard approach assumes that at the second stage there are well-defined best-response functions for both firms, and that there is a unique and (locally) stable Nash equilibrium that depends smoothly on $k$, $x^*(k)$. Then $\text{Sign} \frac{\partial x_2^*}{\partial k} = \text{Sign} \left( \frac{\partial^2 \pi_2}{\partial x_1 \partial x_2} \right)$ and $\text{Sign} S = \frac{\partial \pi_1}{\partial x_2} \frac{\partial \pi_1}{\partial k} \frac{\partial x_1^*}{\partial x_2}$. If $\frac{\partial \pi_1}{\partial x_2} \frac{\partial \pi_1}{\partial k} < 0 (> 0)$, the investment makes firm 1 tough (soft) and a taxonomy can be provided depending on whether competition is of the strategic substitutes ($\frac{\partial^2 \pi_2}{\partial x_1 \partial x_2} \leq 0$) or complements ($\frac{\partial^2 \pi_2}{\partial x_1 \partial x_2} \geq 0$) variety.

In the lattice theoretic result (section 7.4.3, Vives (1999)) the taxonomy follows from minimal assumptions (the character of competition and investment) as applied to extremal equilibria. There is no need for the strong restrictions to obtain a unique and stable equilibrium. Indeed, if the market game is supermodular ($\frac{\partial^2 \pi_2}{\partial x_1 \partial x_2} \geq 0$) and
\[
\frac{\partial^2 \pi_1}{\partial k \partial x_1} \geq (\leq) 0 \text{ then extremal equilibria are increasing (decreasing) in } k. \text{ If the game is submodular } (\frac{\partial^2 \pi_2}{\partial x_1 \partial x_2} \leq 0) \text{ then extremal equilibrium strategies for player 1(2) are increasing (decreasing) in } k \text{ if } \frac{\partial^2 \pi_1}{\partial k \partial x_1} \geq 0 \text{ and the result is reversed if } \frac{\partial^2 \pi_1}{\partial k \partial x_1} \leq 0. \]
Therefore, $\text{Sign} \frac{\partial x_2^*}{\partial k} = \text{Sign} \left( \frac{\partial^2 \pi_2}{\partial x_1 \partial x_2} \frac{\partial \pi_2}{\partial k} \frac{\partial x_1^*}{\partial x_2} \right)$ when $x_2^*$ is an extremal equilibrium.
5. Bayesian games

Bayesian games provide another fertile ground for applications of the lattice-theoretical approach. I will detail some applications to the difficult existence issue in pure strategies, comparative statics under uncertainty and global games and equilibrium selection.

5.1 Existence in pure strategies

Very restrictive assumptions are needed to ensure existence of (pure strategy) Bayesian equilibria in games with a continuum of types and/or actions. Two approaches have been provided in the literature based on the lattice-theoretic approach.

First of all, supermodularity of the underlying certainty game is inherited by the Bayesian game (supermodularity is preserved by integration and strategy spaces can be shown to be complete lattices). Existence of pure strategy Bayesian equilibria follows then according to the results in Section 2 (Vives (1990, and Section 2.7.3, 1999).

Second, the lattice-theoretic methodology can be used also when each firm uses a strategy increasing in its type in response to increasing strategies of rivals (Athey (2000) and McAdams (2001)).

As an example, consider a differentiated Bertrand oligopoly with firm $i$ having random marginal costs $\theta_i$, which are potentially correlated. Now, the expected demand for firm $i$ $E(D_i(p_i, p_{-i}(\theta_,\theta_i))\theta_i)$ is log-supermodular in $(p_i, \theta_i)$ if both $D_i(p_i, p_{-i})$ and the joint density of $(\theta_j, ..., \theta_n)$ are log-supermodular and if the strategies of the rivals, $p_j(\cdot), j \neq i$, are increasing. It follows then that $E(\pi_i|\theta_i) = (p_i - \theta_i)E(D_i(p_i, p_{-i}(\theta_,\theta_i))|\theta_i)$ is log-supermodular in $(p_i, \theta_i)$ and that best replies of player $i$ are increasing in $\theta_i$. Existence of a Bayesian equilibrium (in pure strategies) follows provided $D_i$ is continuous using standard fixed points theorems.6

McAdams (2001) extends the results to multidimensional actions spaces adding the assumption that the payoff of each player is (quasi)-supermodular in his action given

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that other players use strategies increasing in their types. A Bayesian equilibrium in increasing pure strategies exists then in a class of multiunit auctions that includes Vickrey, uniform and discriminatory pricing auctions. Players are risk neutral, have one-dimensional independent types with marginal valuations increasing in their type, and there are decreasing differences of marginal values in own types and quantities of others. This extends the theory beyond games with strategic complementarities. For example, consider a Cournot duopoly in which each firm can expend some effort to boost demand and that higher types have lower marginal costs. The required conditions would be fulfilled provided effort increases the slope of the inverse demand function.

5.2 Comparative statics under uncertainty

The approach can be used fruitfully also in comparative static problems under uncertainty and incomplete information. For example, Athey (2001) extends the comparative static results of classical problems of investment under uncertainty using lattice-theoretic methods.

Another interesting application is provided by strategic information revelation when information is verifiable (Okuno, Postlewaite and Suzumura (1990)). Consider a two-stage duopoly where first firms can report their types (lying in a finite set) and in the second stage they compete in the market place (and assume that there is an interior equilibrium for any updated beliefs). Types are verifiable and therefore firms can conceal information but not lie. Suppose that we have a supermodular duopoly game in which best replies move monotonically with the types of the agents in such a way that each agent would like the rival to believe that he is of an extreme type (for example, lowest possible cost in Cournot or highest possible cost in Bertrand). This will be so with strategic substitutes if the payoff to a firm has strictly increasing differences in its action and type and decreasing differences in its action and the types of the rivals. With strategic complements the condition is that the payoff to a firm has strictly decreasing differences in its action and type and decreasing differences in its action and the types of the rivals. Under the assumptions any perfect Bayesian equilibrium of the two-stage game involves complete revelation of information. The intuition for the result is that in

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equilibrium inferences are skeptical: if a firm reports a set of types others believe the worst. Indeed, what the lattice-theoretic approach contributes is in isolating the minimal necessary assumptions to obtain the results. (See Vives (1999, p.257-260).)

5.3 Global games and equilibrium selection

Global games are games of incomplete information with type space determined by each player observing a noisy signal of the underlying state. The aim is equilibrium selection via perturbation of a complete information game. The basic idea is that when analyzing a complete information game with potentially multiple equilibria players have to entertain the “global picture” of slightly different possible games being played. Each player has a noisy estimate of the game being played and knows that the other players are also receiving noisy estimates.

Carlsson and van Damme (1993) show the following result. In 2x2 games if each player observes a noisy signal of the true payoffs and if ex ante feasible payoffs include payoffs that make each action strictly dominant then as noise becomes small, iterative strict dominance selects one equilibrium. The equilibrium selected is the Harsanyi-Selten (1988) risk dominant one if there are two equilibria in the complete information game. Carlsson and van Damme do not consider explicitly supermodular games but in the interesting case in which there are two equilibria in the complete information game then the game is one of strategic complementarities.

Frankel, Morris and Pauzner (2000) obtain a generalization of the limit uniqueness result to games of strategic complementarities. Consider the game with state of the world $\theta$ and each player $i$ observing signal $\theta + k \epsilon_i$ and having an ordered set of actions. Payoffs depend on the action profile and $\theta$. The authors assume that the game is of strategic complementarities (increasing best response of a player to the actions of rivals for any $\theta$), there is single crossing (the best response of a player to any given action profile of rivals is increasing in $\theta$) and there are limit dominant actions (for sufficiently high/low states each player has as dominant action the highest/lowest one). Under these
assumptions and under some technical conditions a limit uniqueness result (as $k$ tends to 0) is shown. However, the limit equilibrium may depend on the distribution of noise. Frankel, Morris and Pauzner give conditions for noise independent selection.

For symmetric binary action games of strategic complementarities with a continuum of players the following is an outline of proof of the existence of a unique equilibrium. First, symmetric GSC will have largest and smallest symmetric equilibria. Second, given that there are increasing differences in the action of a player and the state, extremal equilibrium strategies will be increasing with the type and can be taken to be of the cut off form (choose the first, “largest”, action if and only if the signal is below a threshold $t$). We know that the extremal equilibrium thresholds, $\tilde{t}$, bound the set of rationalizable strategies. The extremal equilibria can be found with the usual algorithm, starting at the extremal points of the strategy sets of players and iterating using the best responses. For example, to get $\tilde{t}$ start in the situation in which all the players choose the “largest” action for any state of the world (that is, start from $\tilde{t} = +\infty$) and applying iteratively the best response of a player obtain a decreasing sequence that converges to $\tilde{t}$. The rest is to show that when noise is small $\tilde{t} = \underline{t}$, and the equilibrium is unique. This can be shown to hold for any noise when the prior is uniform and then with very precise signals all equilibria will be close to the equilibrium with the uniform prior (indeed, with a very precise signal a Bayesian agent will put little weight to the prior and therefore this will look like a uniform prior).

A uniqueness result in switching strategies can be obtained also relaxing the strategic complementarity condition of actions to a single crossing condition (as in Athey (2001)) and assuming that signals fulfil the monotone likelihood ratio property. However, then it cannot be guaranteed that there are no other equilibria in non-monotone strategies.

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7 Payoffs have to be continuous with respect to actions and $\Theta$ and the sensitivity of payoffs to action profiles has a Lipschitz-type bound. The prior on the state as well as likelihoods of signals are given by a continuous and positive density (for signals with bounded support).

8 See, for example, Morris and Shin (2001).

There is also quite active research in dynamic global games or dynamic games with incomplete information aiming at obtaining a unique equilibrium (Frankel and Pauzner (1999), Burdy, Frankel and Pauzner (2001), Morris-Shin (1999a), Chamley (1999), Levin (2000), Toxvaerd (2001)).

6. Concluding remarks

In this paper I have surveyed briefly the theory and some applications of supermodular games. The survey has not been exhaustive. For example, applications to dynamic games have not been considered.9 I hope to have convinced the reader of the wide range of applications of the theory.

9 See, for example, Datta, Mirman, Morand and Reffet (2001) who characterize symmetric Markovian equilibria in a general version of the "Great Fish War" without requiring smoothness assumptions on the primitives, finding an algorithm for computing maximal equilibria and developing comparative static results with minimal assumptions. See also Hoppe and Lehmann-Grube (2002) for applications to innovation timing games.
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